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# سایت کنکور



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پاسخ به همه سوالات شما در تمامی مقاطع تحصیلی، در انجمن کنکور

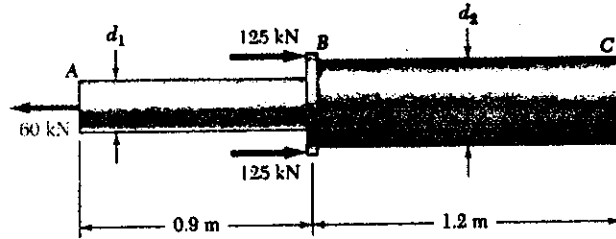
مدیریت سایت کنکور : آراز و فراز رهبر

# CHAPTER 1

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PROBLEM 1.1

1.1 Two solid cylindrical rods  $AB$  and  $BC$  are welded together at  $B$  and loaded as shown. Knowing that  $d_1 = 30$  mm and  $d_2 = 50$  mm, find the average normal stress in the mid section of (a) rod  $AB$ , (b) rod  $BC$ .



SOLUTION

(a) rod  $AB$

Force:  $P = 60 \times 10^3$  N tension

Area:  $A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30 \times 10^{-3})^2 = 706.86 \times 10^{-6} \text{ m}^2$

Normal stress:  $\sigma_{AB} = \frac{P}{A} = \frac{60 \times 10^3}{706.86 \times 10^{-6}} = 84.88 \times 10^6 \text{ Pa}$

$\sigma_{AB} = 84.9 \text{ MPa}$  ◀

(b) rod  $BC$

Force:  $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$  N

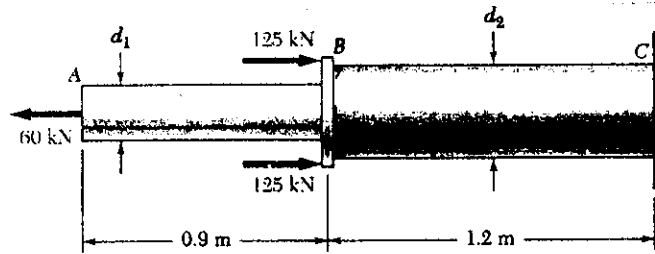
Area:  $A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} (50 \times 10^{-3})^2 = 1.9635 \times 10^{-3} \text{ m}^2$

Normal stress:  $\sigma_{BC} = \frac{P}{A} = \frac{-190 \times 10^3}{1.9635 \times 10^{-3}} = -96.77 \times 10^6 \text{ Pa}$

$\sigma_{BC} = -96.8 \text{ MPa}$  ◀

PROBLEM 1.2

1.2 Two solid cylindrical rods  $AB$  and  $BC$  are welded together at  $B$  and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters  $d_1$  and  $d_2$ .



SOLUTION

rod AB

Force:  $P = 60 \times 10^3 \text{ N}$

Stress:  $\sigma_{AB} = 150 \times 10^6 \text{ Pa}$

Area:  $A = \frac{\pi}{4} d_1^2$

$$\sigma_{AB} = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma_{AB}}$$

$$\frac{\pi}{4} d_1^2 = \frac{P}{\sigma_{AB}}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(60 \times 10^3)}{\pi (150 \times 10^6)} = 509.3 \times 10^{-6} \text{ m}^2$$

$$d_1 = 22.56 \times 10^{-3} \text{ m}$$

$$d_1 = 22.6 \text{ mm} \quad \blacktriangleleft$$

rod BC

Force  $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3 \text{ N}$

Stress:  $\sigma_{BC} = -150 \times 10^6 \text{ Pa}$

Area:  $A = \frac{\pi}{4} d_2^2$

$$\sigma_{BC} = \frac{P}{A} = \frac{4P}{\pi d_2^2}$$

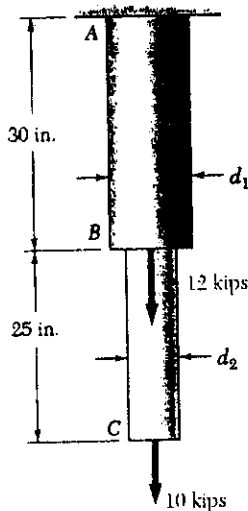
$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(-190 \times 10^3)}{\pi (-150 \times 10^6)} = 1.6128 \times 10^{-3} \text{ m}^2$$

$$d_2 = 40.16 \times 10^{-3} \text{ m}$$

$$d_2 = 40.2 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 1.3**

1.3 Two solid cylindrical rods  $AB$  and  $BC$  are welded together at  $B$  and loaded as shown. Knowing that  $d_1 = 1.25$  in. and  $d_2 = 0.75$  in., find the normal stress at the midpoint of (a) rod  $AB$ , (b) rod  $BC$ .



**SOLUTION**

(a) rod  $AB$

$$P = 12 + 10 = 22 \text{ kips}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (1.25)^2 = 1.2272 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{22}{1.2272} = 17.93 \text{ ksi}$$

(b) rod  $BC$

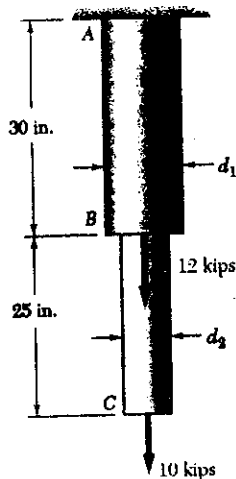
$$P = 10 \text{ kips}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} (0.75)^2 = 0.4418 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{10}{0.4418} = 22.6 \text{ ksi}$$

**PROBLEM 1.4**

1.4 Two solid cylindrical rods  $AB$  and  $BC$  are welded together at  $B$  and loaded as shown. Knowing that the normal stress must not exceed 25 ksi in either rod, determine the smallest allowable values of the diameters  $d_1$  and  $d_2$ .



**SOLUTION**

rod  $AB$ :

$$P = 12 + 10 = 22 \text{ kips}$$

$$\sigma_{AB} = 25 \text{ ksi} \quad A_{AB} = \frac{\pi}{4} d_1^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{4P}{\pi d_1^2}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(22)}{\pi(25)} = 1.1205 \text{ in}^2$$

$$d_1 = 1.059 \text{ in}$$

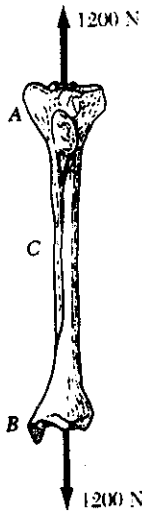
rod  $BC$ :  $P = 10 \text{ kips} \quad \sigma_{BC} = 25 \text{ ksi} \quad A_{BC} = \frac{\pi}{4} d_2^2$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(10)}{\pi(25)} = 0.5093 \text{ in}^2$$

$$d_2 = 0.714 \text{ in}$$

PROBLEM 1.5

1.5 A strain gage located at  $C$  on the surface of bone  $AB$  indicates that the average normal stress in the bone is  $3.80 \text{ MPa}$  when the bone is subjected to two  $1200\text{-N}$  forces as shown. Assuming the cross section of the bone at  $C$  to be annular and knowing that its outer diameter is  $25 \text{ mm}$ , determine the inner diameter of the bone's cross section at  $C$ .



SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

$$\text{Geometry: } A = \frac{\pi}{4} (d_1^2 - d_2^2)$$

$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

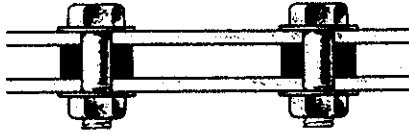
$$= 222.9 \times 10^{-6} \text{ m}^2$$

$$d_1 = 14.93 \times 10^{-3} \text{ m}$$

$$d_1 = 14.93 \text{ mm} \blacktriangleleft$$

## PROBLEM 1.6

1.6 Two steel plates are to be held together by means of  $\frac{1}{4}$ -in.-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 30 ksi in the bolts and 18 ksi in the spacers, determine the outer diameter of the spacers which yields the most economical and safe design.



## SOLUTION

At each bolt location the upper plate is pulled down by the tensile force  $P_b$  of the bolt. At the same time the spacer pushes that plate upward with a compressive force  $P_s$ . In order to maintain equilibrium

$$P_b = P_s$$

$$\text{For the bolt} \quad \sigma_b = \frac{P_b}{A_b} = \frac{4P_b}{\pi d_b^2} \quad \text{or} \quad P_b = \frac{\pi}{4} \sigma_b d_b^2$$

$$\text{For the spacer} \quad \sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)} \quad \text{or} \quad P_s = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

Equating  $P_b$  and  $P_s$

$$\frac{\pi}{4} \sigma_b d_b^2 = \frac{\pi}{4} \sigma_s (d_s^2 - d_b^2)$$

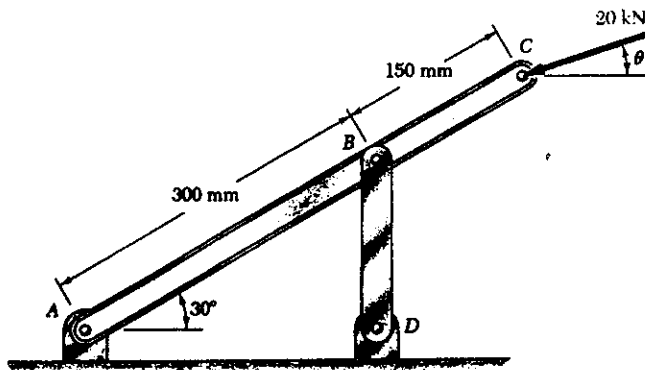
$$d_s^2 = d_b^2 + \frac{\sigma_b}{\sigma_s} d_b^2 = \left(1 + \frac{\sigma_b}{\sigma_s}\right) d_b^2$$

$$d_s = \left(1 + \frac{30}{18}\right) \left(\frac{1}{4}\right)^2 = 0.16667 \text{ in}^2$$

$$d_s = 0.408 \text{ in.}$$

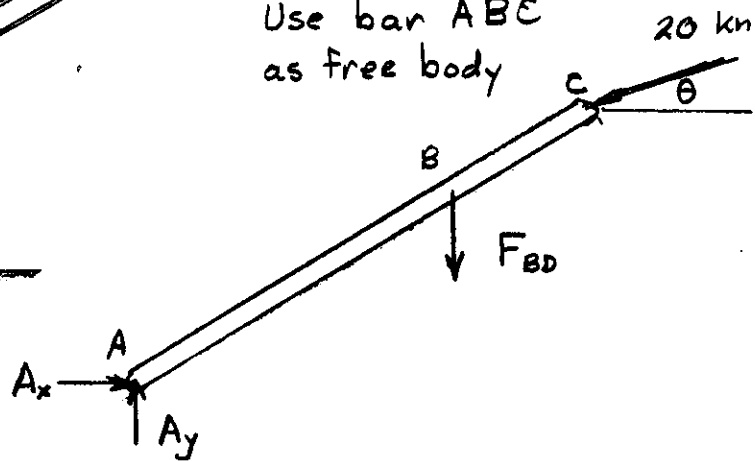
PROBLEM 1.7

1.7 Link  $BD$  consists of a single bar 30 mm wide and 12 mm thick. Knowing that each pin has a 10-mm diameter, determine the maximum value of the average normal stress in link  $BD$  if (a)  $\theta = 0$ , (b)  $\theta = 90$



SOLUTION

Use bar  $ABC$  as free body



$$\Sigma M_A = 0$$

$$(a) \quad \theta = 0^\circ \quad (0.450 \sin 30^\circ)(20 \times 10^3) - (0.300 \cos 30^\circ) F_{BD} = 0$$

$$F_{BD} = 17.32 \times 10^3 \text{ N}$$

$$(b) \quad \theta = 90^\circ \quad (0.450 \cos 30^\circ)(20 \times 10^3) - (0.300 \cos 30^\circ) F_B = 0$$

$$F_{BD} = -30 \times 10^3 \text{ N}$$

Areas

$$(a) \quad \text{tension loading} \quad A = (0.030 - 0.010)(0.012) = 240 \times 10^{-6} \text{ m}^2$$

$$(b) \quad \text{compression} \quad A = (0.030)(0.012) = 360 \times 10^{-6} \text{ m}^2$$

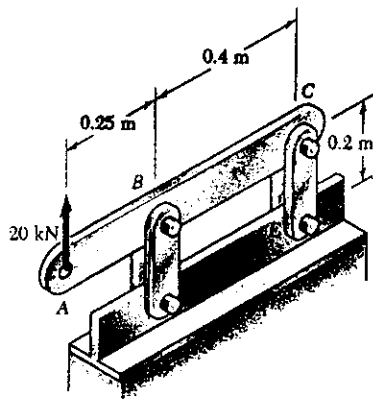
Stresses

$$(a) \quad \sigma = \frac{F_{BC}}{A} = \frac{17.32 \times 10^3}{240 \times 10^{-6}} = 72.2 \times 10^6 \quad 72.2 \text{ MPa} \blacktriangleleft$$

$$(b) \quad \sigma = \frac{F_{BC}}{A} = \frac{-30 \times 10^3}{360 \times 10^{-6}} = -83.3 \times 10^6 \quad -83.3 \text{ MPa} \blacktriangleleft$$



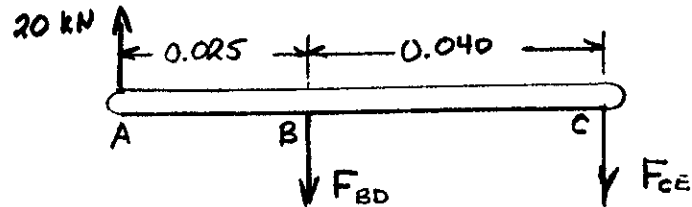
PROBLEM 1.8



1.8 Each of the four vertical links has an  $8 \times 36$ -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (a) points B and D, (b) points C and E.

SOLUTION

Use bar ABC as a free body.



$$\Sigma M_C = 0 \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N} \quad \text{Link BD is in tension}$$

$$\Sigma M_B = 0 \quad -(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{ N} \quad \text{Link CE is in compression.}$$

Net area of one link for tension =  $(0.008)(0.036 - 0.016)$

=  $160 \times 10^{-6} \text{ m}^2$ . For two parallel links  $A_{net} = 320 \times 10^{-6} \text{ m}^2$

Tensile stress in link BD

$$(a) \quad \sigma_{BD} = \frac{F_{BD}}{A_{net}} = \frac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.56 \times 10^6 \text{ or } 101.6 \text{ MPa} \quad \blacktriangleleft$$

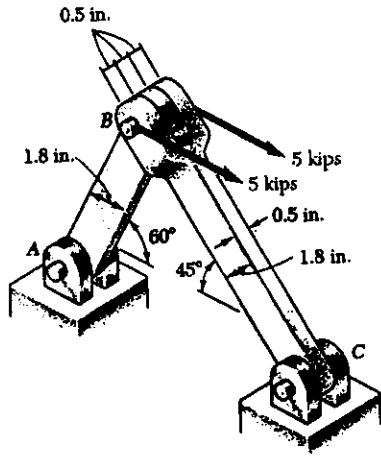
Area for one link in compression =  $(0.008)(0.036)$

=  $288 \times 10^{-6} \text{ m}^2$ . For two parallel links  $A = 576 \times 10^{-6} \text{ m}^2$

$$(b) \quad \sigma_{CE} = \frac{F_{CE}}{A} = \frac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.70 \times 10^6 \text{ or } -21.7 \text{ MPa} \quad \blacktriangleleft$$

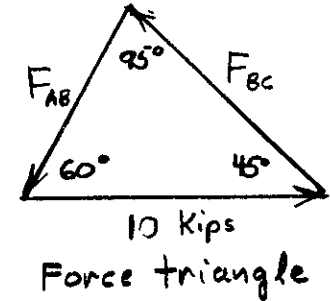
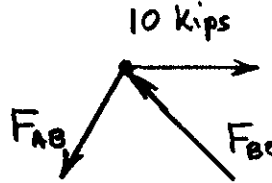
**PROBLEM 1.9**

1.9 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link *AB*, (b) in link *BC*.



**SOLUTION**

Use joint *B* as free body.



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$$F_{AB} = 7.3205 \text{ kips}$$

$$F_{BC} = 8.9658 \text{ kips.}$$

Link *AB* is a tension member

Minimum section at pin  $A_{int} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

(a) Stress in *AB*  $\sigma_{AB} = \frac{F_{AB}}{A_{int}} = \frac{7.3205}{0.5} = 14.64 \text{ ksi}$

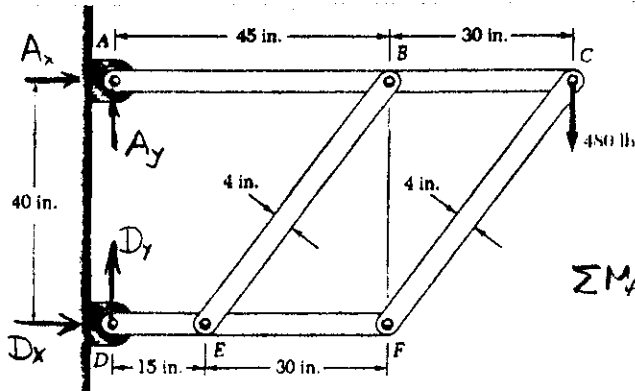
Link *BC* is a compression member

Cross sectional area is  $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(b) Stress in *BC*  $\sigma_{BC} = \frac{-F_{BC}}{A} = \frac{-8.9658}{0.9} = -9.96 \text{ ksi}$

PROBLEM 1.10

1.10 The frame shown consists of four wooden members,  $ABC$ ,  $DEF$ ,  $BE$ , and  $CF$ . Knowing that each member has a  $2 \times 4$ -in. rectangular cross section and that each pin has a  $\frac{1}{2}$ -in. diameter, determine the maximum value of the average normal stress (a) in member  $BE$ , (b) in member  $CF$ .



SOLUTION

Add support reactions to figure as shown.

Using entire frame as free body

$$\sum M_A = 0 \quad 40 D_x - (45 + 30)(480) = 0$$

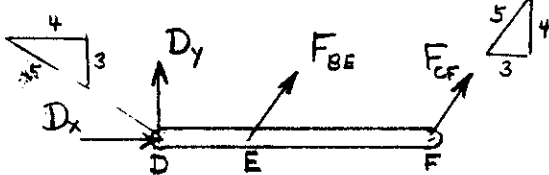
$$D_x = 900 \text{ lb.}$$

Use member  $DEF$  as free body

$$\sum F_x = 0$$

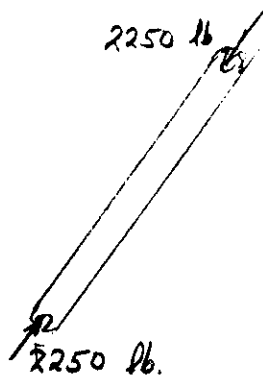
$$\frac{3}{5} D_y - \frac{4}{5} D_x = 0$$

$$D_y = \frac{4}{3} D_x = 1200 \text{ lb.}$$



$$\sum M_F = 0 \quad -(30)\left(\frac{4}{3} F_{BE}\right) - (30 + 15) D_y = 0 \quad F_{BE} = -2250 \text{ lb.}$$

$$\sum M_E = 0 \quad (30)\left(\frac{4}{3} F_{CF}\right) - (15) D_y = 0 \quad F_{CF} = 750 \text{ lb.}$$



Stress in compression member  $BE$

$$\text{Area } A = 2 \text{ in} \times 4 \text{ in} = 8 \text{ in}^2$$

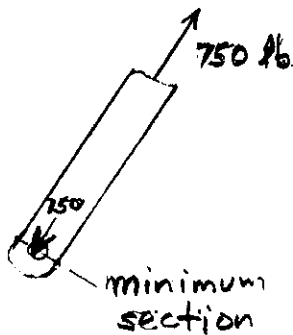
$$(a) \sigma_{BE} = \frac{F_{BE}}{A} = \frac{-2250}{8} = -281 \text{ psi}$$

Stress in tension member  $CF$

Minimum section area occurs at pin.

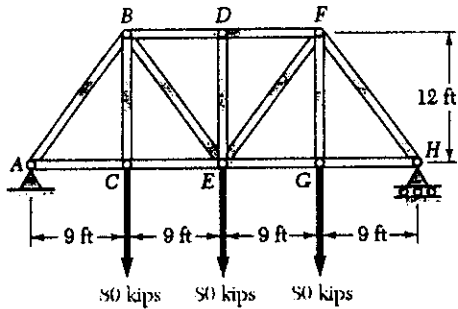
$$A_{\min} = (2)(4.0 - 0.5) = 7.0 \text{ in}^2$$

$$(b) \sigma_{CF} = \frac{F_{CF}}{A_{\min}} = \frac{750}{7.0} = 107.1 \text{ psi}$$



**PROBLEM 1.11**

1.11 For the Pratt bridge truss and loading shown, determine the average normal stress in member  $BE$ , knowing that the cross-sectional area of that member is  $5.87 \text{ in}^2$ .



**SOLUTION**

Use entire truss as free body

$$\sum M_H = 0$$

$$(9)(80) + (18)(80) + (27)(80) - 36 A_y = 0$$

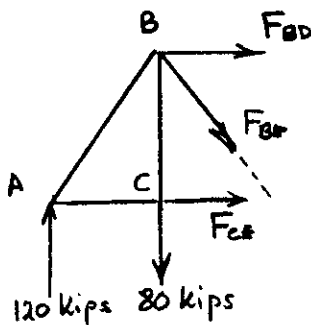
$$A_y = 120 \text{ kips}$$

Use portion of truss to the left of a section cutting members  $BD$ ,  $BE$ , and  $CE$ .

$$+\uparrow \sum F_y = 0$$

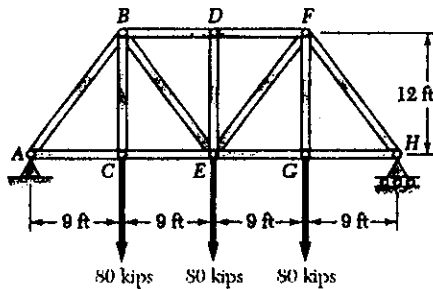
$$120 - 80 - \frac{12}{15} F_{BE} = 0 \quad \therefore F_{BE} = 50 \text{ kips}$$

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50}{5.87} = 8.52 \text{ ksi}$$



**PROBLEM 1.12**

1.12 Knowing that the average normal stress in member  $CE$  of the Pratt bridge truss shown must not exceed  $21 \text{ ksi}$  for the given loading, determine the cross-sectional area of that member which will yield the most economical and safe design. Assume that both ends of the member will be adequately reinforced.



**SOLUTION**

Use entire truss as free body

$$\sum M_H = 0$$

$$(9)(80) + (18)(80) + (27)(80) - 36 A_y = 0$$

$$A_y = 120 \text{ kips}$$

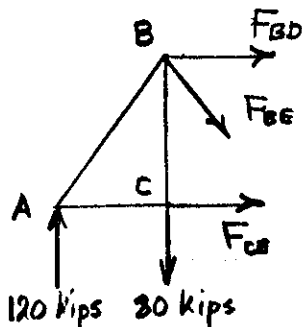
Use portion of truss to the left of a section cutting members  $BD$ ,  $BE$ , and  $CE$

$$\sum M_B = 0$$

$$12 F_{CE} - (9)(120) = 0 \quad \therefore F_{CE} = 90 \text{ kips}$$

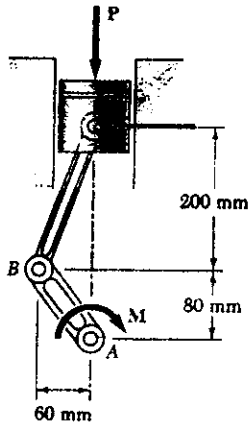
$$\sigma_{CE} = \frac{F_{CE}}{A_{CE}}$$

$$A_{CE} = \frac{F_{CE}}{\sigma_{CE}} = \frac{90}{21} = 4.29 \text{ in}^2$$



PROBLEM 1.13

1.13 A couple  $M$  of magnitude  $1500 \text{ N}\cdot\text{m}$  is applied to the crank of an engine. For the position shown, determine (a) the force  $P$  required to hold the engine system in equilibrium, (b) the average normal stress in the connecting rod  $BC$ , which has a  $450\text{-mm}^2$  uniform cross section.



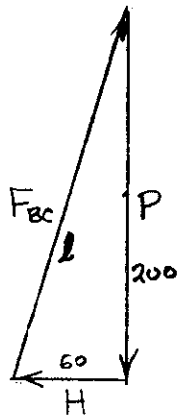
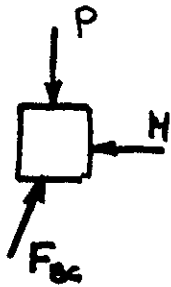
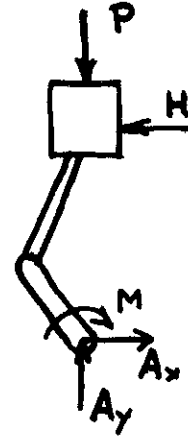
SOLUTION

Use piston, rod, and crank together as free body. Add wall reaction  $H$  and bearing reactions  $A_x$  and  $A_y$ .

$$\sum M_A = 0$$

$$(0.280 \text{ m})H - 1500 \text{ N}\cdot\text{m} = 0$$

$$H = 5.3571 \times 10^3 \text{ N}$$



Use piston alone as free body. Note that rod is a two-force member; hence the direction of force  $F_{bc}$  is known. Draw the force triangle and solve for  $P$  and  $F_{bc}$  by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$$

$$P = 17.86 \text{ kN} \quad \blacktriangleleft$$

$$\frac{F_{bc}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{bc} = 18.643 \times 10^3 \text{ N}$$

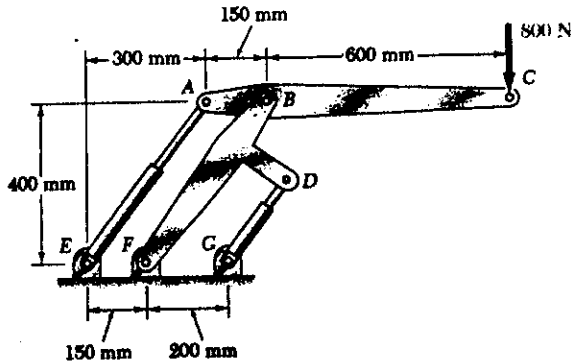
Rod  $BC$  is a compression member. Its area is  $450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$

$$\text{Stress} \quad \sigma_{bc} = \frac{-F_{bc}}{A} = \frac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6 \text{ Pa}$$

$$(b) \quad \sigma_{bc} = -41.4 \text{ MPa} \quad \blacktriangleleft$$

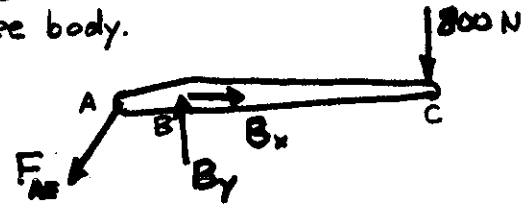
PROBLEM 1.14

1.14 Two hydraulic cylinders are used to control the position of the robotic arm ABC. Knowing that the control rods attached at A and D each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (a) member AE, (b) member DG.



SOLUTION

Use member ABC as free body.

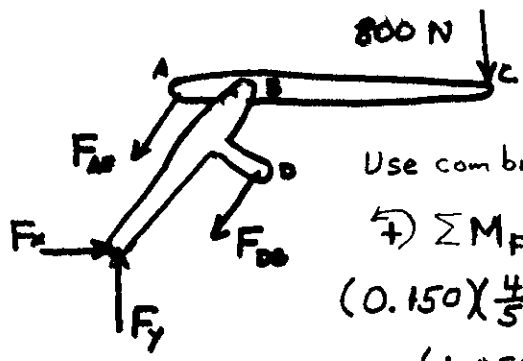


$$\sum M_B = 0 \quad (0.150) \frac{4}{5} F_{AE} - (0.600)(800) = 0 \quad F_{AE} = 4 \times 10^3 \text{ N}$$

Area of rod in member AE is  $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{ m}^2$

Stress in rod AE:  $\sigma_{AE} = \frac{F_{AE}}{A} = \frac{4 \times 10^3}{314.16 \times 10^{-6}} = 12.73 \times 10^6 \text{ Pa}$

(a)  $\sigma_{AE} = 12.73 \text{ MPa}$



Use combined members ABC and BFD as free body.

$$\sum M_F = 0$$

$$(0.150) \left( \frac{4}{5} F_{AE} \right) - (0.200) \left( \frac{4}{5} F_{DG} \right) - (1.050 - 0.350)(800) = 0 \quad F_{DG} = -1500 \text{ N}$$

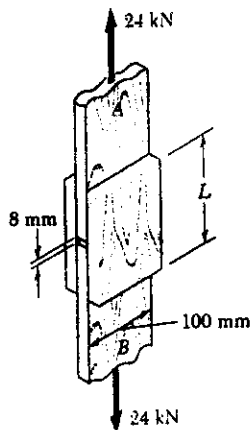
Area in rod DG is  $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{ m}^2$

Stress in rod DG:  $\sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416 \times 10^{-6}} = -4.77 \times 10^6 \text{ Pa}$

(b)  $\sigma_{DG} = -4.77 \text{ MPa}$

PROBLEM 1.15

1.15 The wooden members *A* and *B* are to be joined by plywood splice plates which will be fully glued on the surfaces in contact. As part of the design of the joint and knowing that the clearance between the ends of the members is to be 8 mm, determine the smallest allowable length *L* if the average shearing stress in the glue is not to exceed 800 kPa.



SOLUTION

There are four separate areas of glue. Each area must transmit half of the 24 kN load.

Therefore  $F = 12 \text{ kN} = 12 \times 10^3 \text{ N}$

Shearing stress in glue  $\tau = 800 \times 10^3 \text{ Pa}$

$\tau = \frac{F}{A} \therefore A = \frac{F}{\tau} = \frac{12 \times 10^3}{800 \times 10^3} = 15 \times 10^{-3} \text{ m}^2$

Let  $l =$  length of glue area and  $w =$  width  $= 100 \text{ mm} = 0.1 \text{ m}$

$A = lw \therefore l = \frac{A}{w} = \frac{15 \times 10^{-3}}{0.1} = 150 \times 10^{-3} \text{ m} = 150 \text{ mm}$

$L = 2l + \text{gap} = (2)(150) + 8 = 308 \text{ mm}$

PROBLEM 1.16

1.16 Determine the diameter of the largest circular hole which can be punched into a sheet of polystyrene 6-mm thick, knowing that the force exerted by the punch is 45 kN and that a 55-MPa average shearing stress is required to cause the material to fail.

SOLUTION

$A = \pi dt$  for cylindrical failure surface

Shearing stress  $\tau = \frac{P}{A} \therefore A = \frac{P}{\tau}$

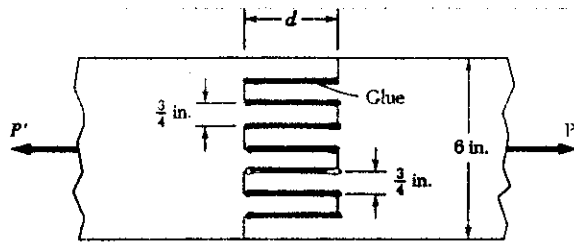
Equating A's  $\pi dt = \frac{P}{\tau}$

Solving for  $d$ :  $d = \frac{P}{\pi t \tau} = \frac{45 \times 10^3}{\pi (0.006)(55 \times 10^6)} = 43.4 \times 10^{-3} \text{ m}$

$d = 43.4 \text{ mm}$

**PROBLEM 1.17**

1.17 Two wooden planks, each  $\frac{7}{8}$ -in. thick and 6 in. wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 120 psi, determine the smallest allowable length  $d$  of the cuts if the joint is to withstand an axial load of magnitude  $P = 1200$  lb.



**SOLUTION**

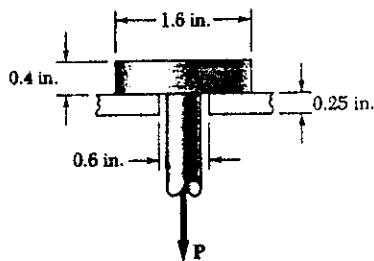
Seven surfaces carry the total load  $P = 1200$  lb.

$$\text{Area } A = (7)\left(\frac{7}{8}\right) d = \frac{49}{8} d$$

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} \quad \frac{49}{8} d = \frac{1200}{120} \quad d = 1.633 \text{ in} \quad \blacktriangleleft$$

**PROBLEM 1.18**

1.18 A load  $P$  is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load  $P$  which may be applied to the rod.



**SOLUTION**

For steel  $A_1 = \pi dt = \pi(0.6)(0.4) = 0.7540 \text{ in}^2$

$$\tau_1 = \frac{P}{A_1} \quad \therefore \quad P = A_1 \tau_1 = (0.7540)(18) = 13.57 \text{ kips}$$

For aluminum  $A_2 = \pi dt = \pi(1.6)(0.25) = 1.2566 \text{ in}^2$

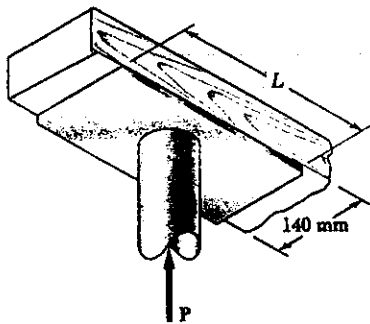
$$\tau_2 = \frac{P}{A_2} \quad \therefore \quad P = A_2 \tau_2 = (1.2566)(10) = 12.57 \text{ kips}$$

Limiting value of  $P$  is the smaller value  $\therefore P = 12.57 \text{ kips} \quad \blacktriangleleft$



**PROBLEM 1.19**

1.19 The axial force in the column supporting the timber beam shown is  $P = 75 \text{ kN}$ . Determine the smallest allowable length  $L$  of the bearing plate if the bearing stress in the timber is not to exceed  $3.0 \text{ MPa}$ .



**SOLUTION**

$$\sigma_b = \frac{P}{A} = \frac{P}{LW}$$

Solving for  $L$ :

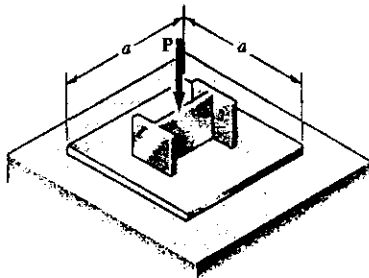
$$L = \frac{P}{\sigma_b W} = \frac{75 \times 10^3}{(3.0 \times 10^6)(0.140)}$$

$$= 178.6 \times 10^{-3} \text{ m}$$

$$L = 178.6 \text{ mm}$$

**PROBLEM 1.20**

1.20 An axial load  $P$  is supported by a short  $W250 \times 67$  column of cross-sectional area  $A = 8580 \text{ mm}^2$  and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed  $150 \text{ MPa}$  and that the bearing stress on the concrete foundation must not exceed  $12.5 \text{ MPa}$ , determine the side  $a$  of the plate which will provide the most economical and safe design.



**SOLUTION**

Area of column:  $A = 8580 \text{ mm}^2 = 8580 \times 10^{-6} \text{ m}^2$

Normal stress in column:  $\sigma = 150 \times 10^6 \text{ Pa}$

$$\sigma = \frac{P}{A} \therefore P = A\sigma = (8580 \times 10^{-6})(150 \times 10^6)$$

$$= 1.287 \times 10^6 \text{ N}$$

Bearing plate:  $\sigma_b = \frac{P}{A_b}$  and  $A_b = a^2$  for square plate.

$$A_b = a^2 = \frac{P}{\sigma_b} = \frac{1.287 \times 10^6}{12.5 \times 10^6} = 321 \times 10^{-3} \text{ m} \text{ or } 321 \text{ mm}$$

**PROBLEM 1.21**



1.21 Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is  $\frac{1}{2}$  in. and the inner diameter of each washer is  $\frac{5}{8}$  in., which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter  $d$  of the washers, knowing that the average normal stress in the bolts is 5 ksi and that the bearing stress between the washers and the planks must not exceed 12 ksi.

**SOLUTION**

$$\text{Bolt: } A_{\text{bolt}} = \frac{\pi}{4} d_b^2 = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$\sigma_b = \frac{P}{A} \therefore \text{Tensile force in bolt } P = \sigma_b A = (5)(0.19635) = 0.98175 \text{ kips}$$

Washer: inside diameter =  $d_i = \frac{5}{8}$  in., outside diameter =  $d_o$

$$\text{Bearing area } A_w = \frac{\pi}{4} (d_o^2 - d_i^2) \text{ and } A_w = \frac{P}{\sigma_b}$$

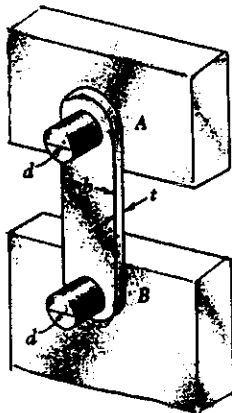
$$\text{Equating } \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{P}{\sigma_b}$$

$$d_o^2 = d_i^2 + \frac{4P}{\pi \sigma_b} = \left(\frac{5}{8}\right)^2 + \frac{(4)(0.98175)}{\pi (12.5)} = 1.4323 \text{ in}^2$$

$$d_o = 1.197 \text{ in}$$

**PROBLEM 1.22**

1.22 Link AB, of width  $b = 2$  in. and thickness  $t = \frac{1}{4}$  in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is -20 ksi and that the average shearing stress in each of the two pins is 12 ksi, determine (a) the diameter  $d$  of the pins, (b) the average bearing stress in the link.



**SOLUTION**

Rod AB is in compression.

$$A = bt \text{ where } b = 2 \text{ in and } t = \frac{1}{4} \text{ in}$$

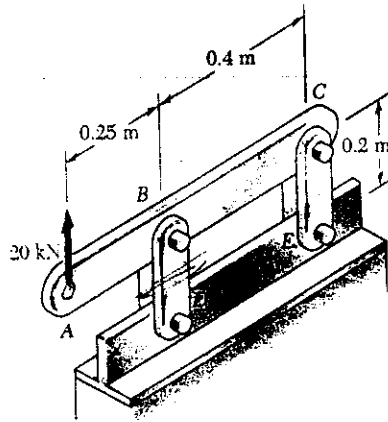
$$P = -\sigma A = -(-20)(2)\left(\frac{1}{4}\right) = 10 \text{ kips}$$

$$\text{Pin: } \tau_p = \frac{P}{A_p} \text{ and } A_p = \frac{\pi}{4} d^2$$

$$(a) d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{4P}{\pi \tau_p}} = \sqrt{\frac{(4)(10)}{\pi (12)}} = 1.030 \text{ in}$$

$$(b) \sigma_b = \frac{P}{dt} = \frac{10}{(1.030)(0.25)} = 38.8 \text{ ksi}$$

PROBLEM 1.23

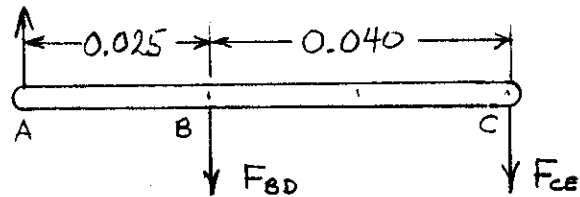


1.8 Each of the four vertical links has an  $8 \times 36$ -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter.

1.23 For the assembly and loading of Prob. 1.8, determine (a) the average shearing stress in the pin at B, (b) the average bearing stress at B in link BD, (c) the average bearing stress at B in member ABC, knowing that this member has a  $10 \times 50$ -mm uniform rectangular cross section.

SOLUTION

Use bar ABC as a free body



$$\sum M_C = 0 \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B  $\tau = \frac{F_{BD}}{2A}$  for double shear

where  $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \quad 80.8 \text{ MPa} \quad \blacktriangleleft$$

(b) Bearing link BD  $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

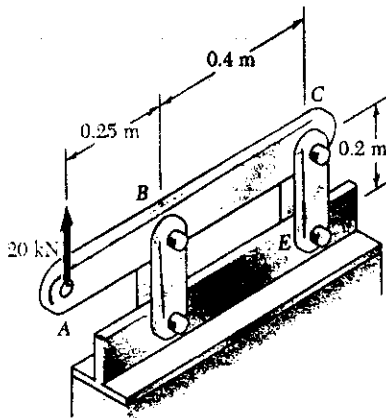
$$\sigma_b = \frac{\frac{1}{2} F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \quad 127.0 \text{ MPa} \quad \blacktriangleleft$$

(c) Bearing in ABC at B

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \quad 203 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 1.24

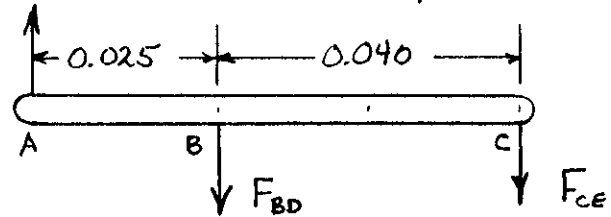


1.8 Each of the four vertical links has an  $8 \times 36$ -mm uniform rectangular cross section and each of the four pins has a 16-mm diameter.

1.24 For the assembly and loading of Prob. 1.8, determine (a) the average shearing stress in the pin at C, (b) the average bearing stress at C in link CE, (c) the average bearing stress at C in member ABC, knowing that this member has a  $10 \times 50$ -mm uniform rectangular cross section.

SOLUTION

Use bar ABC as a free body



$$\sum M_B = 0 \quad -(0.040)F_{CE} - (0.025)(20 \times 10^3) = 0 \quad F_{CE} = -12.5 \times 10^3$$

(a) Shear in pin at C

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$\text{Double shear} \quad \tau = \frac{F_{CE}}{2A} = \frac{12.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 31.1 \times 10^6 \quad 31.1 \text{ MPa} \quad \blacktriangleleft$$

(b) Bearing in link CE at C

$$A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{\frac{1}{2} F_{CE}}{A} = \frac{(0.5)(12.5 \times 10^3)}{128 \times 10^{-6}} = 48.8 \times 10^6 \quad 48.8 \text{ MPa} \quad \blacktriangleleft$$

(c) Bearing in ABC at C

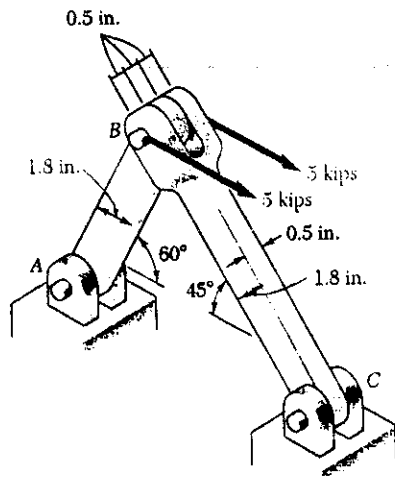
$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{CE}}{A} = \frac{12.5 \times 10^3}{160 \times 10^{-6}} = 78.1 \times 10^6 \quad 78.1 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 1.25**

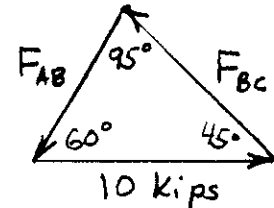
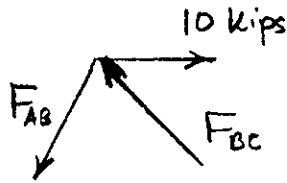
1.9 Two horizontal 5-kip forces are applied to pin *B* of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link *AB*, (*b*) in link *BC*.

1.25 For the assembly and loading of Prob. 1.9, determine (*a*) the average shearing stress in the pin at *A*, (*b*) the average bearing stress at *A* in member *AB*.



**SOLUTION**

Use joint *B* as free body.



Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \quad F_{AB} = 7.3205 \text{ kips}$$

(a) Shearing stress in pin at *A*       $\tau = \frac{F_{AB}}{2A_p}$

where  $A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.8)^2 = 0.5026 \text{ in}^2$

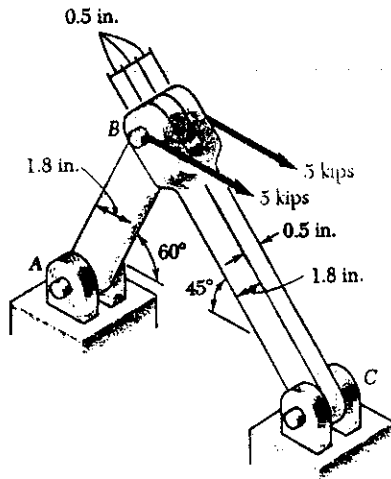
$$\tau = \frac{7.3205}{(2)(0.5026)} = 7.28 \quad 7.28 \text{ ksi} \quad \blacktriangleleft$$

(b) Bearing stress at *A* in member *AB*

$$A_b = t d = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.3205}{0.4} = 18.30 \quad 18.30 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 1.26**

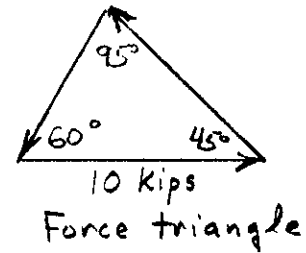
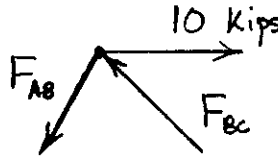


1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.

1.26 For the assembly and loading of Prob. 1.9, determine (a) the average shearing stress in the pin at C, (b) the average bearing stress at C in member BC, (c) the average bearing stress at B in member BC.

**SOLUTION**

Use joint B as free body



Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \quad F_{BC} = 8.9658 \text{ kips}$$

(a) Shearing stress in pin at C  $\tau = \frac{F_{BC}}{2A_p}$

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \quad 8.92 \text{ ksi} \quad \blacktriangleleft$$

(b) Bearing stress at C in member BC  $\sigma_b = \frac{F_{BC}}{A}$

$$A = t d = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \quad 22.4 \text{ ksi} \quad \blacktriangleleft$$

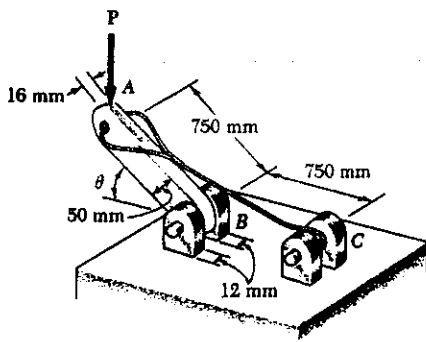
(c) Bearing stress at B in member BC  $\sigma_b = \frac{F_{BC}}{A}$

$$A = 2t d = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \quad 11.21 \text{ ksi} \quad \blacktriangleleft$$

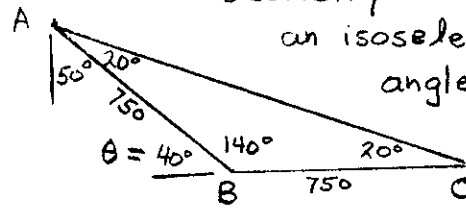
PROBLEM 1.27

1.27 Knowing that  $\theta = 40^\circ$  and  $P = 9$  kN, determine (a) the smallest allowable diameter of the pin at B if the average shearing stress in the pin is not to exceed 120 MPa, (b) the corresponding average bearing stress in member AB at B, (c) the corresponding average bearing stress in each of the support brackets at B.

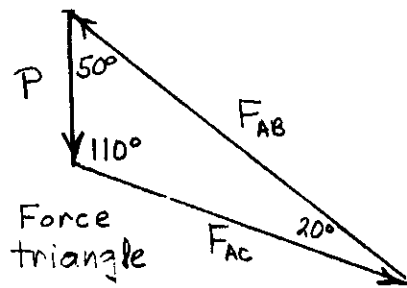
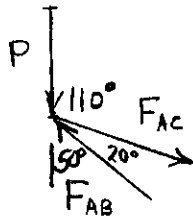


SOLUTION

Geometry: Triangle ABC is an isosceles triangle with angles shown here.



Use joint A as a free body.



Law of Sines applied to force triangle

$$\frac{P}{\sin 20^\circ} = \frac{F_{AB}}{\sin 110^\circ} = \frac{F_{AC}}{\sin 50^\circ}$$

$$F_{AB} = \frac{P \sin 110^\circ}{\sin 20^\circ} = \frac{(9) \sin 110^\circ}{\sin 20^\circ} = 24.73 \text{ kN}$$

(a) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_p} = \frac{F_{AB}}{2 \frac{\pi}{4} d^2} = \frac{2F_{AB}}{\pi d^2} \quad \text{where } F_{AB} = 24.73 \times 10^3 \text{ N}$$

$$d^2 = \frac{2F_{AB}}{\pi \tau} = \frac{(2)(24.73 \times 10^3)}{\pi (120 \times 10^6)} = 131.18 \times 10^{-6} \text{ m}^2$$

$$d = 11.45 \times 10^{-3} \text{ m} \quad 11.45 \text{ mm} \blacktriangleleft$$

(b) Bearing stress in AB at A.

$$A_b = t d = (0.016)(11.45 \times 10^{-3}) = 183.26 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.73 \times 10^3}{183.26 \times 10^{-6}} = 134.9 \times 10^6 \quad 134.9 \text{ MPa} \blacktriangleleft$$

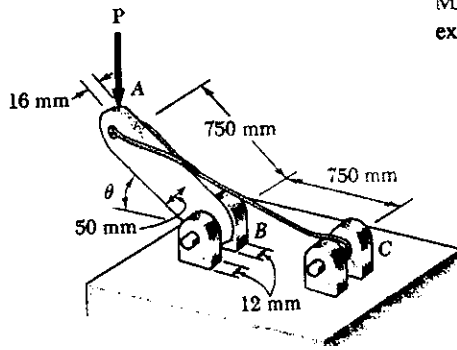
(c) Bearing stress in support brackets at B

$$A = t d = (0.012)(11.45 \times 10^{-3}) = 137.4 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{\frac{1}{2} F_{AB}}{A} = \frac{(0.5)(24.73 \times 10^3)}{137.4 \times 10^{-6}} = 90.0 \times 10^6 \quad 90.0 \text{ MPa} \blacktriangleleft$$

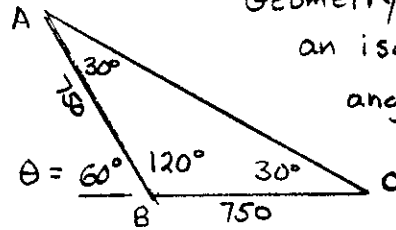
PROBLEM 1.28

1.28 Determine the largest load  $P$  which may be applied at  $A$  when  $\theta = 60^\circ$ , knowing that the average shearing stress in the 10-mm-diameter pin at  $B$  must not exceed 120 MPa and that the average bearing stress in member  $AB$  and in the bracket at  $B$  must not exceed 90 MPa.

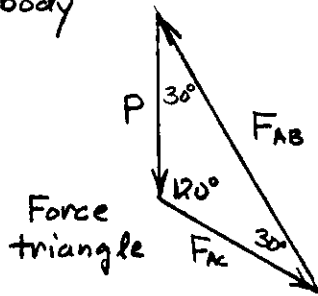
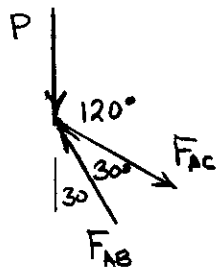


SOLUTION

Geometry: Triangle ABC is an isosceles triangle with angles shown here



Use joint A as free body



Law of sines applied to force triangle

$$\frac{P}{\sin 30^\circ} = \frac{F_{AB}}{\sin 120^\circ} = \frac{F_{AC}}{\sin 30^\circ}$$

$$P = \frac{F_{AB} \sin 30^\circ}{\sin 120^\circ} = 0.57735 F_{AB}$$

$$P = \frac{F_{AC} \sin 30^\circ}{\sin 30^\circ} = F_{AC}$$

If shearing stress in pin at B is critical

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.010)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = 2 A_p \tau = (2)(78.54 \times 10^{-6})(120 \times 10^6) = 18.850 \times 10^3 \text{ N}$$

If bearing stress in member AB at bracket at A is critical

$$A_b = t d = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = A_b \sigma_b = (160 \times 10^{-6})(90 \times 10^6) = 14.40 \times 10^3 \text{ N}$$

If bearing stress in the bracket at B is critical

$$A_b = 2 t d = (2)(0.012)(0.010) = 240 \times 10^{-6} \text{ m}^2$$

$$F_{AB} = A_b \sigma_b = (240 \times 10^{-6})(90 \times 10^6) = 21.6 \times 10^3 \text{ N}$$

Allowable  $F_{AB}$  is the smallest, i.e.  $14.40 \times 10^3 \text{ N}$

Then, from Statics  $P_{allow} = (0.57735)(14.40 \times 10^3)$

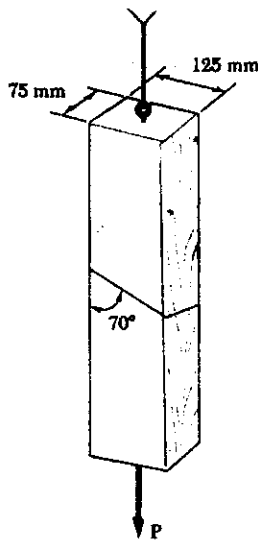
$$= 8.31 \times 10^3 \text{ N}$$

$$8.31 \text{ kN}$$



**PROBLEM 1.29**

1.29 The 6- kN load  $P$  is supported by two wooden members of  $75 \times 125$ - mm uniform rectangular cross section which are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.



**SOLUTION**

$$P = 6 \times 10^3 \text{ N}$$

$$\theta = 90^\circ - 70^\circ = 20^\circ$$

$$A_o = (0.075)(0.125) = 9.375 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{(6 \times 10^3) \cos^2 20^\circ}{9.375 \times 10^{-3}} = 565 \times 10^3$$

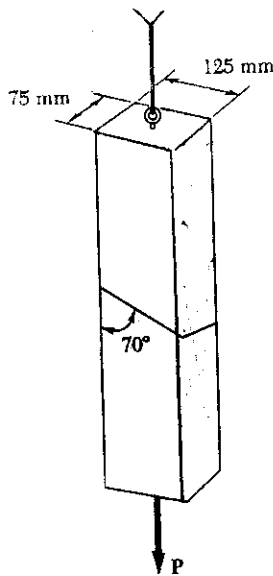
$$\sigma = 565 \text{ kPa}$$

$$\tau = \frac{P}{2A_o} \sin 2\theta = \frac{(6 \times 10^3) \sin 40^\circ}{(2)(9.375 \times 10^{-3})} = 206 \times 10^3$$

$$\tau = 206 \text{ kPa}$$

**PROBLEM 1.30**

1.30 Two wooden members of  $75 \times 125$ - mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 500 kPa, determine (a) the largest load  $P$  which can be safely supported, (b) the corresponding shearing stress in the splice.



**SOLUTION**

$$A_o = (0.075)(0.125) = 9.375 \times 10^{-3} \text{ m}^2$$

$$\theta = 90^\circ - 70^\circ = 20^\circ \quad \sigma = 500 \times 10^3 \text{ Pa}$$

$$\sigma = \frac{P}{A_o} \cos^2 \theta$$

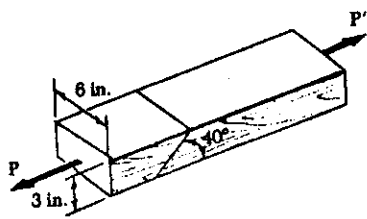
$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(9.375 \times 10^{-3})(500 \times 10^3)}{\cos^2 20^\circ} = 5.3085 \times 10^3$$

$$(a) \quad P = 5.31 \text{ kN}$$

$$\tau = \frac{P \sin 2\theta}{2A_o} = \frac{(5.3085 \times 10^3) \sin 40^\circ}{(2)(9.375 \times 10^{-3})} = 181.99 \times 10^3$$

$$(b) \quad \tau = 182.0 \text{ kPa}$$

PROBLEM 1.31



1.31 Two wooden members of 3 × 6-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 90 psi, determine (a) the largest load P which can be safely applied, (b) the corresponding tensile stress in the splice.

SOLUTION

$$\theta = 90^\circ - 40^\circ = 50^\circ$$

$$A_o = (3)(6) = 18 \text{ in}^2$$

$$\tau = \frac{P}{2A} \sin 2\theta$$

$$P = \frac{2A\tau}{\sin 2\theta} = \frac{(2)(18)(90)}{\sin 100^\circ} = 3290$$

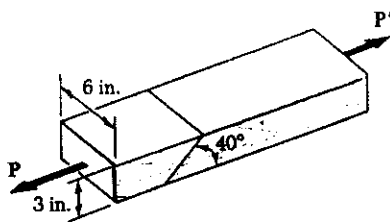
(a)

$$P = 3290 \text{ lb.}$$

(b)  $\sigma = \frac{P \cos^2 \theta}{A_o} = \frac{3290 \cos^2 50^\circ}{18} = 75.5$

$$\sigma = 75.5 \text{ psi}$$

PROBLEM 1.32



1.32 Two wooden members of 3 × 6-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that P = 2400 lb, determine the normal and shearing stresses in the glued splice.

SOLUTION

$$\theta = 90^\circ - 40^\circ = 50^\circ \quad P = 2400 \text{ lb.}$$

$$A_o = (3)(6) = 18 \text{ in}^2$$

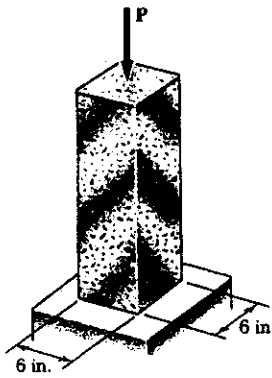
$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{(2400) \cos^2 50^\circ}{18} = 55.1$$

$$\sigma = 55.1 \text{ psi}$$

$$\tau = \frac{P}{2A} \sin 2\theta = \frac{(2400) \sin 100^\circ}{(2)(18)} = 65.7$$

$$\tau = 65.7 \text{ psi}$$

PROBLEM 1.33



1.33 A centric load  $P$  is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (a) the magnitude of  $P$ , (b) the orientation of the surface on which the maximum shearing stress occurs, (c) the normal stress exerted on that surface, (d) the maximum value of the normal stress in the block.

SOLUTION

$$A_0 = (6)(6) = 36 \text{ in}^2 \quad \tau_{\max} = 2.5 \text{ ksi}$$

$$\theta = 45^\circ \text{ for plane of } \tau_{\max}$$

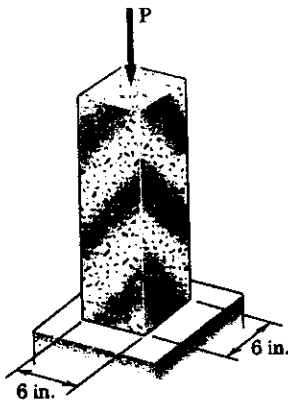
$$(a) \quad \tau_{\max} = \frac{|P|}{2A_0} \therefore |P| = 2A_0 \tau_{\max} = (2)(36)(2.5) = 180 \text{ Kips}$$

$$(b) \quad \sin 2\theta = 1 \quad 2\theta = 90^\circ \quad \theta = 45^\circ$$

$$(c) \quad \sigma_{45} = \frac{P}{A_0} \cos^2 45^\circ = \frac{P}{2A_0} = \frac{-180}{(2)(36)} = -2.5 \text{ ksi}$$

$$(d) \quad \sigma_{\max} = \frac{P}{A_0} = \frac{-180}{36} = -5 \text{ ksi}$$

PROBLEM 1.34



1.34 A 240-kip load  $P$  is applied to the granite block shown. Determine the resulting maximum value of (a) the normal stress, (b) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$$A_0 = (6)(6) = 36 \text{ in}^2$$

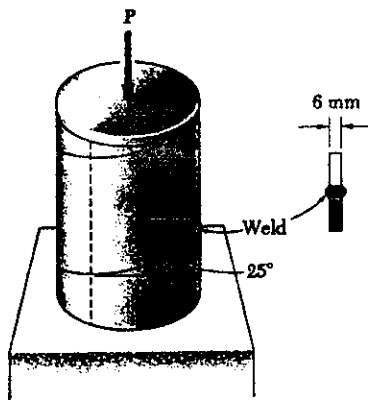
$$\sigma = \frac{P}{A_0} \cos^2 \theta = \frac{-240}{36} \cos^2 \theta = -6.67 \cos^2 \theta$$

$$(a) \quad \text{max tensile stress} = 0 \text{ at } \theta = 90^\circ$$

$$\text{max. compressive stress} = 6.67 \text{ ksi} \text{ at } \theta = 0^\circ$$

$$(b) \quad \tau_{\max} = \frac{P}{2A_0} = \frac{240}{(2)(36)} = 3.33 \text{ ksi} \text{ at } \theta = 45^\circ$$

PROBLEM 1.35



1.35 A steel pipe of 300- mm outer diameter is fabricated from 6- mm- thick plate by welding along a helix which forms an angle of  $25^\circ$  with a plane perpendicular to the axis of the pipe. Knowing that a 250- kN axial force  $P$  is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$$d_o = 0.300 \text{ m} \quad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$$

$$= 5.54 \times 10^{-3} \text{ m}^2$$

$$\theta = 25^\circ$$

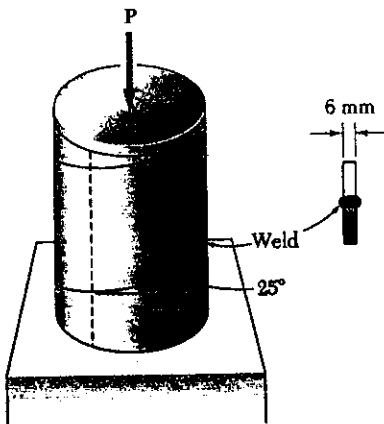
$$\sigma = \frac{P}{A_o} \cos^2 \theta = \frac{-250 \times 10^3 \cos^2 25^\circ}{5.54 \times 10^{-3}}$$

$$= -37.1 \times 10^6 \quad \sigma = -37.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau = \frac{P}{2A_o} \sin 2\theta = \frac{-250 \times 10^3 \sin 50^\circ}{(2)(5.54 \times 10^{-3})}$$

$$= -17.28 \times 10^6 \quad \tau = 17.28 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 1.36



1.36 A steel pipe of 300- mm outer diameter is fabricated from 6- mm- thick plate by welding along a helix which forms an angle of  $25^\circ$  with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in directions respectively normal and tangential to the weld are  $\sigma = 50 \text{ MPa}$  and  $\tau = 30 \text{ MPa}$ , determine the magnitude  $P$  of the largest axial force that can be applied to the pipe.

SOLUTION

$$d_o = 0.300 \text{ m} \quad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$$

$$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$$

$$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$$

$$= 5.54 \times 10^{-3} \text{ m}^2$$

$$\theta = 25^\circ$$

Based on  $|\sigma| = 50 \text{ MPa}$ :  $\sigma = \frac{P}{A_o} \cos^2 \theta$

$$P = \frac{A_o \sigma}{\cos^2 \theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25^\circ} = 337 \times 10^3$$

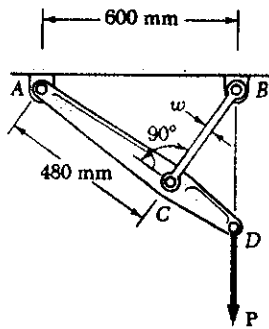
Based on  $|\tau| = 30 \text{ MPa}$ :  $\tau = \frac{P}{2A_o} \sin 2\theta$

$$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50^\circ} = 434 \times 10^3$$

Smaller value is the allowable value of  $P$   $\therefore P = 337 \text{ kN} \quad \blacktriangleleft$

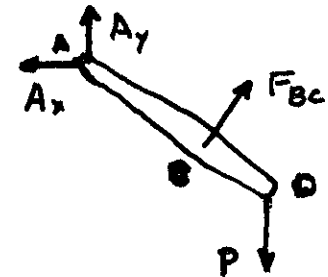
PROBLEM 1.37

1.37 Link BC is 6 mm thick, has a width  $w = 25$  mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load  $P$ ?



SOLUTION

Use bar ACD as a free body and note that member BD is a two-force member



$$\sum M_A = 0$$

$$(480) F_{bc} - (600) P = 0$$

$$F_{bc} = \frac{600}{480} P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{ N}$$

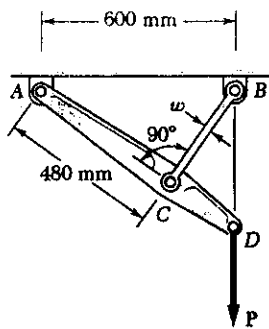
Ultimate load for member BC  $F_u = \sigma_u A$

$$F_u = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{ N}$$

Factor of safety  $F.S. = \frac{F_u}{F_{bc}} = \frac{72 \times 10^3}{20 \times 10^3} = 3.60$

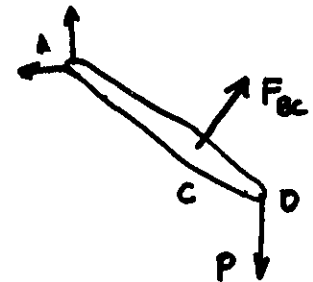
PROBLEM 1.38

1.38 Link BC is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width  $w$  if the structure shown is being designed to support a 20-kN load  $P$  with a factor of safety of 3?



SOLUTION

Use bar ACD as a free body and note that member BC is a two-force member.



$$\sum M_A = 0$$

$$480 F_{bc} - 600 P = 0$$

$$F_{bc} = \frac{600 P}{480} = \frac{(600)(20 \times 10^3)}{480} = 25 \times 10^3 \text{ N}$$

For a factor of safety  $F.S. = 3$ , the ultimate load of member BC

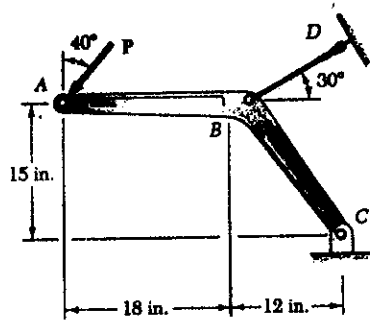
$$F_u = (F.S.)(F_{bc}) = (3)(25 \times 10^3) = 75 \times 10^3 \text{ N}$$

But  $F_u = \sigma_u A \therefore A = \frac{F_u}{\sigma_u} = \frac{75 \times 10^3}{450 \times 10^6} = 166.67 \times 10^{-6} \text{ m}^2$

For a rectangular section  $A = wt$  or  $w = \frac{A}{t} = \frac{166.67 \times 10^{-6}}{0.006}$

$$w = 27.8 \times 10^{-3} \text{ m or } 27.8 \text{ mm}$$

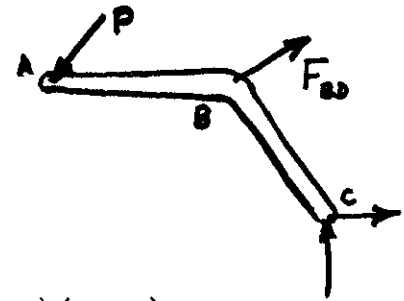
**PROBLEM 1.39**



1.39 Member *ABC*, which is supported by a pin and bracket at *C* and a cable *BD*, was designed to support the 4-kip load *P* as shown. Knowing that the ultimate load for cable *BD* is 25 kips, determine the factor of safety with respect to cable failure.

**SOLUTION**

Use member *ABC* as a free body and note that member *BD* is a two-force member.



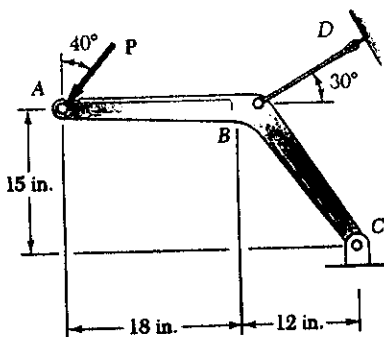
$$\sum M_C = 0$$

$$(P \cos 40^\circ)(30 \text{ in.}) + (P \sin 40^\circ)(15 \text{ in.}) - (F_{BD} \cos 30^\circ)(15 \text{ in.}) - (F_{BD} \sin 30^\circ)(12 \text{ in.}) = 0$$

$$F_{BD} = \frac{32.623}{18.990} P = \frac{(32.623)(4)}{18.990} = 6.8715 \text{ kips}$$

Factor of safety for cable *BD*       $F.S. = \frac{F_{ult}}{F_{BD}} = \frac{25}{6.8715} = 3.64$  ◀

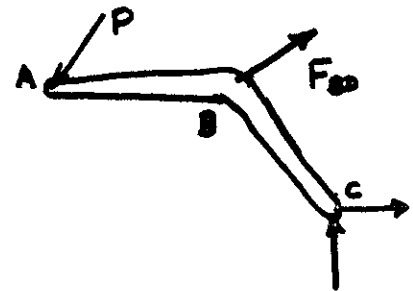
**PROBLEM 1.40**



1.40 Knowing that the ultimate load for cable *BD* is 25 kips and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force *P* which can be safely applied as shown to member *ABC*.

**SOLUTION**

Use member *ABC* as a free body and note that member *BD* is a two-force member.



$$\sum M_C = 0$$

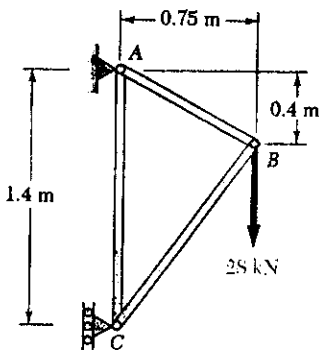
$$(P \cos 40^\circ)(30 \text{ in.}) + (P \sin 40^\circ)(15 \text{ in.}) - (F_{BD} \cos 30^\circ)(15 \text{ in.}) - (F_{BD} \sin 30^\circ)(12 \text{ in.}) = 0$$

$$P = \frac{18.990}{32.623} F_{BD} = 0.58216 F_{BD}$$

Allowable load for member *BD* is  $F_{BD} = \frac{F_{ult}}{F.S.} = \frac{25}{3.2} = 7.8125 \text{ kips}$

Allowable load  $P = (0.58216)(7.8125) = 4.55 \text{ kips}$  ◀

PROBLEM 1.41



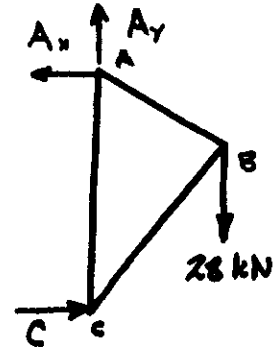
1.41 Members  $AB$  and  $AC$  of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar  $AB$  has a 15-mm-square cross section, determine (a) the factor of safety for bar  $AB$ , (b) the dimensions of the cross section of bar  $AC$  if it is to have the same factor of safety as bar  $AB$ .

SOLUTION

Length of member  $AB$

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

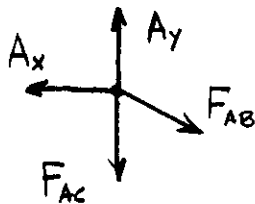
Use entire truss as a free body



$$\sum M_C = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$$

$$\sum F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$$

Use joint A as free body



$$\sum F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$\sum F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar  $A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_u = 120 \times 10^3 \text{ N}$

For the material  $\sigma_u = \frac{P_u}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For bar  $AB \quad F.S. = \frac{F_u}{F_{AB}} = \frac{\sigma_u A}{F_{AB}} = \frac{(300 \times 10^6)(0.015)^2}{17 \times 10^3} = 3.97$

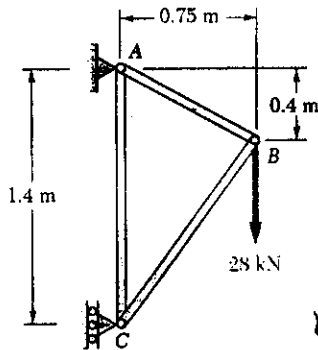
(b) For bar  $AC \quad F.S. = \frac{F_u}{F_{AC}} = \frac{\sigma_u A}{F_{AC}} = \frac{\sigma_u a^2}{F_{AC}}$

$$a^2 = \frac{(F.S.) F_{AC}}{\sigma_u} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$$a = 16.27 \times 10^{-3} \text{ m}$$

$$16.27 \text{ mm}$$

PROBLEM 1.42



1.42 Members  $AB$  and  $AC$  of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required dimensions of the cross section of (a) bar  $AB$ , (b) bar  $AC$ .

SOLUTION

Length of member  $AB$

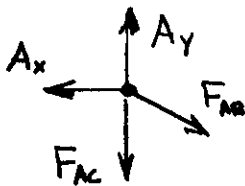
$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

Use entire truss as a free body

$$\circlearrowleft \sum M_C = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$$

$$+\uparrow \sum F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$$

Use joint  $A$  as free body

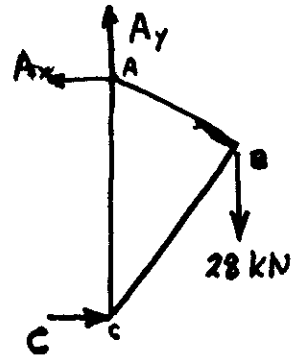


$$\rightarrow \sum F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$$+\uparrow \sum F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$



For the test bar  $A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_u = 120 \times 10^3 \text{ N}$

For the material  $\sigma_u = \frac{P_u}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For member  $AB \quad F.S. = \frac{P_u}{F_{AB}} = \frac{\sigma_u A}{F_{AB}} = \frac{\sigma_u a^2}{F_{AB}}$

$$a^2 = \frac{(F.S.) F_{AB}}{\sigma_u} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{ m}^2$$

$$a = 13.47 \times 10^{-3} \text{ m}$$

13.47 mm ◀

(b) For member  $AC \quad F.S. = \frac{P_u}{F_{AC}} = \frac{\sigma_u A}{F_{AC}} = \frac{\sigma_u b^2}{F_{AC}}$

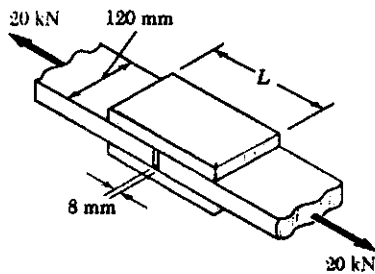
$$b^2 = \frac{(F.S.) F_{AC}}{\sigma_u} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6} \text{ m}^2$$

$$b = 14.61 \times 10^{-3} \text{ m}$$

14.61 mm ◀



**PROBLEM 1.43**



1.43 The two wooden members shown, which support a 20-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.8 MPa and the clearance between the members is 8 mm. Determine the factor of safety, knowing that the length of each splice is  $L = 200$  mm.

**SOLUTION**

There are 4 separate areas of glue. Each glue area must transmit 10 kN of shear load.

$$P = 10 \times 10^3 \text{ N}$$

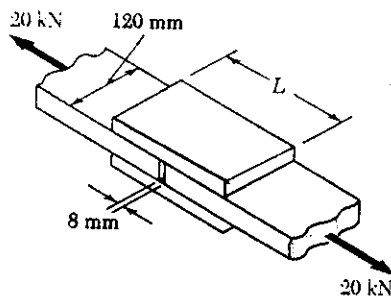
Length of splice  $L = 2l + c$  where  $l =$  length of glue and  $c =$  clearance.  
 $l = \frac{1}{2}(L - c) = \frac{1}{2}(0.200 - 0.008) = 0.096 \text{ m.}$

Area of glue  $A = lw = (0.096)(0.120) = 11.52 \times 10^{-3} \text{ m}^2$

Ultimate load  $P_u = \tau_u A = (2.8 \times 10^6)(11.52 \times 10^{-3}) = 32.256 \times 10^3 \text{ N}$

Factor of safety  $F.S. = \frac{P_u}{P} = \frac{32.256 \times 10^3}{10 \times 10^3} = 3.23$  ◀

**PROBLEM 1.44**



1.43 The two wooden members shown, which support a 20-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.8 MPa and the clearance between the members is 8 mm.

1.44 For the joint and loading of Prob. 1.43, determine the required length  $L$  of each splice if a factor of safety of 3.5 is to be achieved.

**SOLUTION**

There are 4 separate areas of glue. Each glue area must transmit 10 kN of shear load.

$$P = 10 \times 10^3 \text{ N}$$

Required ultimate load  $P_u = (F.S.)(P) = (3.5)(10 \times 10^3) = 35 \times 10^3 \text{ N}$

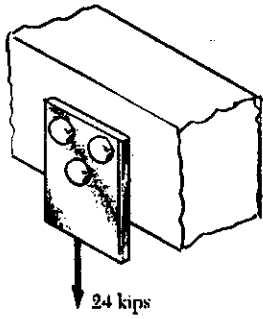
Required length  $l$  of each glue area

$$P_u = \tau_u A = \tau_u lw \quad l = \frac{P_u}{\tau_u w} = \frac{35 \times 10^3}{(2.8 \times 10^6)(0.120)} = 104.17 \times 10^{-3} \text{ m}$$

Length of splice  $L = 2l + c = (2)(104.17 \times 10^{-3}) + 0.008$   
 $= 216.3 \times 10^{-3} \text{ m} \quad 216 \text{ mm}$  ◀

**PROBLEM 1.45**

1.45 Three  $\frac{3}{4}$ -in.-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 24-kip load and that the ultimate shearing stress for the steel used is 52 ksi, determine the factor of safety for this design.



**SOLUTION**

$$\text{For each bolt } A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

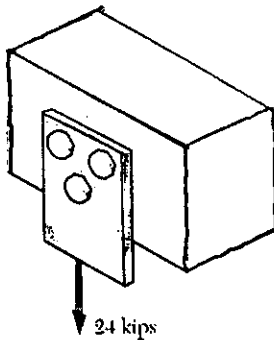
$$P_u = A\tau_u = (0.4418)(52) = 22.97 \text{ kips}$$

$$\text{Per bolt } P = \frac{24}{3} = 8 \text{ kips}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{22.97}{8} = 2.87$$

**PROBLEM 1.46**

1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 24-kip load, that the ultimate shearing stress for the steel used is 52 ksi, and that a factor of safety of 3.37 is desired, determine the required diameter of the bolts.



**SOLUTION**

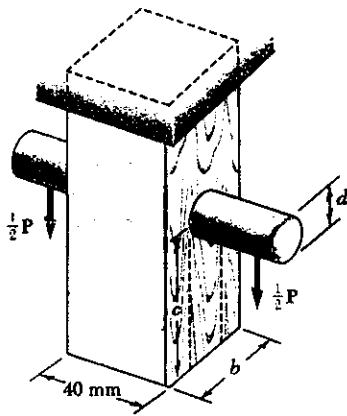
$$\text{For each bolt } P = \frac{24}{3} = 8 \text{ kips}$$

$$\text{Required } P_u = (\text{F.S.})P = (3.37)(8) = 26.96 \text{ kips}$$

$$\tau_u = \frac{P_u}{A} \therefore A = \frac{P_u}{\tau_u} = \frac{26.96}{52} = 0.51846 \text{ in}^2$$

$$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.51846)}{\pi}} = 0.8125 \text{ in.}$$

PROBLEM 1.47



1.47 A load  $P$  is supported as shown by a steel pin which has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 150 MPa in shear. Knowing that the diameter of the pin is  $d = 16$  mm and that the magnitude of the load is  $P = 20$  kN, determine (a) the factor of safety for the pin, (b) the required values of  $b$  and  $c$  if the factor of safety for the wooden member is to be the same as that found in part a for the pin.

SOLUTION

$$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

(a) Pin:  $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

Double shear  $\tau = \frac{P}{2A}$   $\tau_u = \frac{P_u}{2A}$

$$P_u = 2A\tau_u = (2)(201.06 \times 10^{-6})(150 \times 10^6)$$

$$= 60.319 \times 10^3 \text{ N}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{60.319 \times 10^3}{20 \times 10^3} = 3.02$$

(b) Tension in wood  $P_u = 60.319 \times 10^3 \text{ N}$  for same F.S.

$$\sigma_u = \frac{P_u}{A} = \frac{P_u}{w(b-d)} \quad \text{where } w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_u}{w\sigma_u} = 0.016 + \frac{60.319 \times 10^3}{(0.040)(60 \times 10^6)} = 41.1 \times 10^{-3} \text{ m}$$

$$b = 41.1 \text{ mm}$$

Shear in wood  $P_u = 60.319 \times 10^3 \text{ N}$  for same F.S.

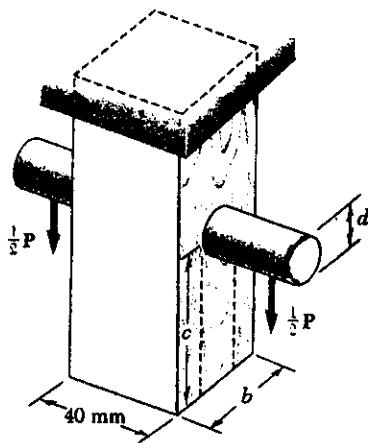
Double shear; each area is  $A = wc$

$$\tau_u = \frac{P_u}{2A} = \frac{P_u}{2wc}$$

$$c = \frac{P_u}{2w\tau_u} = \frac{60.319 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 100.5 \times 10^{-3} \text{ m}$$

$$c = 100.5 \text{ mm}$$

PROBLEM 1.48



1.47 A load  $P$  is supported as shown by a steel pin which has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 150 MPa in shear.

1.48 For the support of Prob. 1.47, knowing that  $b = 40$  mm,  $c = 55$  mm and  $d = 12$  mm, determine the allowable load  $P$  if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin

$$P_u = 2A\tau_u = 2\left(\frac{\pi}{4}d^2\right)\tau_u$$

$$= \frac{\pi}{4}(2)(0.012)^2(150 \times 10^6) = 33.93 \times 10^3 \text{ N}$$

Based on tension in wood

$$P_u = A\sigma_u = w(b-d)\sigma_u$$

$$= (0.040)(0.040 - 0.012)(60 \times 10^6)$$

$$= 67.2 \times 10^3 \text{ N}$$

Based on double shear in the wood

$$P_u = 2A\tau_u = 2wc\tau_u = (2)(0.040)(0.055)(7.5 \times 10^6)$$

$$= 33.0 \times 10^3 \text{ N}$$

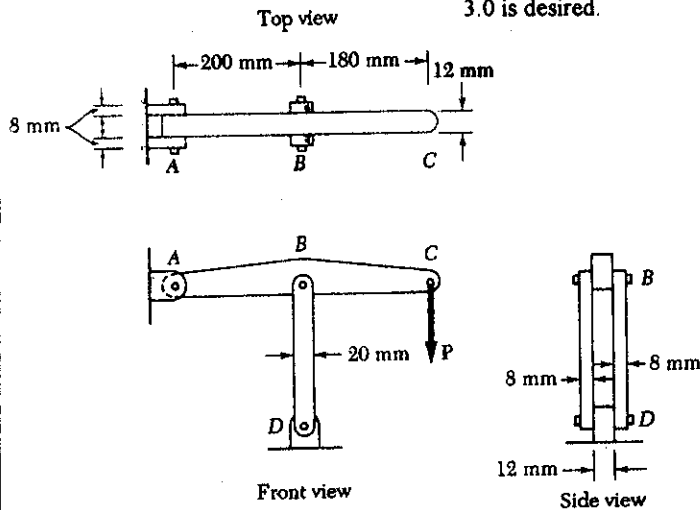
Use smallest  $P_u = 33.0 \times 10^3 \text{ N}$

Allowable  $P = \frac{P_u}{\text{F.S.}} = \frac{33.0 \times 10^3}{3.2} = 10.31 \times 10^3 \text{ N}$

10.31 kN

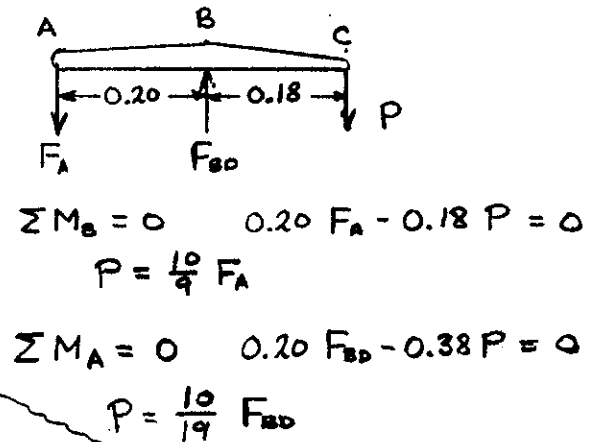
PROBLEM 1.49

1.49 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired.



SOLUTION

Statics: Use ABC as free body.



Based on double shear in pin A

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.008)^2 = 50.266 \times 10^{-6} \text{ m}^2$$

$$F_A = \frac{2\sigma_u A}{\text{F.S.}} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 3.72 \times 10^3 \text{ N}$$

Based on double shear in pins at B and D

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$$

$$F_{B0} = \frac{2\sigma_u A}{\text{F.S.}} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{B0} = 3.97 \times 10^3 \text{ N}$$

Based on compression in links BD

$$\text{For one link } A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2\sigma_u A}{\text{F.S.}} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3 \text{ N}$$

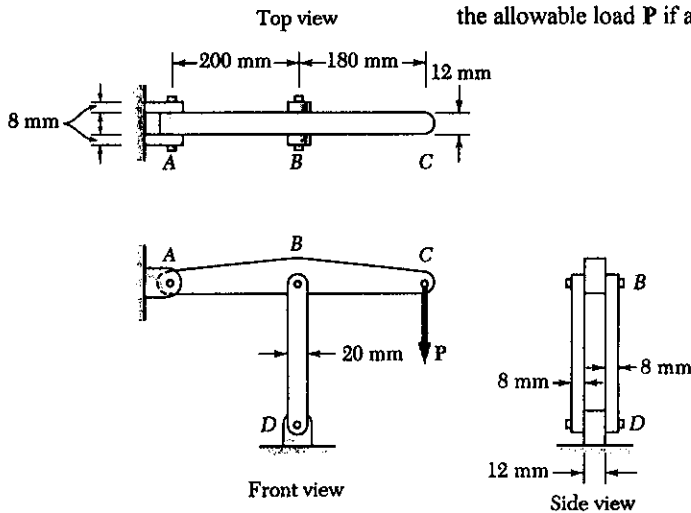
Allowable value of P is smallest  $\therefore P = 3.72 \times 10^3 \text{ N}$

3.72 kN

**PROBLEM 1.50**

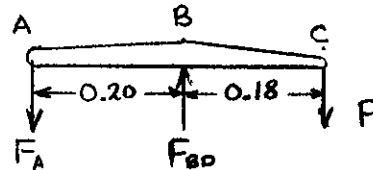
1.49 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load P if an overall factor of safety of 3.0 is desired

1.50 In an alternative design for the structure of Prob. 1.49, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load P if an overall factor of safety of 3.0 is desired.



**SOLUTION**

Statics: Use ABC as free body.



$$\sum M_B = 0 \quad 0.20 F_A - 0.18 P = 0$$

$$P = \frac{10}{9} F_A$$

$$\sum M_A = 0 \quad 0.20 F_{BD} - 0.38 P = 0$$

$$P = \frac{10}{19} F_{BD}$$

Based on double shear in pin A

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.010)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$F_A = \frac{2\tau_u A}{F.S.} = \frac{(2)(100 \times 10^6)(78.54 \times 10^{-6})}{3.0} = 5.236 \times 10^3 \text{ N}$$

$$P = \frac{10}{9} F_A = 5.82 \times 10^3 \text{ N}$$

Based on double shear in pins at B and D

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.012)^2 = 113.10 \times 10^{-6} \text{ m}^2$$

$$F_{BD} = \frac{2\tau_u A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3 \text{ N}$$

Based on compression in links BD

$$\text{For one link } A = (0.020)(0.008) = 160 \times 10^{-6} \text{ m}^2$$

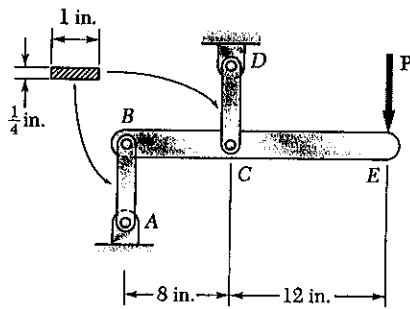
$$F_{BD} = \frac{2\sigma_u A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3 \text{ N}$$

$$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3 \text{ N}$$

Allowable value of P is smallest  $\therefore P = 3.97 \times 10^3 \text{ N}$

3.97 kN ◀

PROBLEM 1.51



1.51 Each of the steel links  $AB$  and  $CD$  is connected to a support and to member  $BCE$  by  $\frac{1}{2}$ -in.-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 24 ksi for the steel used in the pins and that the ultimate normal stress is 60 ksi for the steel used in the links, determine the allowable load  $P$  if an overall factor of safety of 3.2 is desired. (Note that the links are not reinforced around the pin holes.)

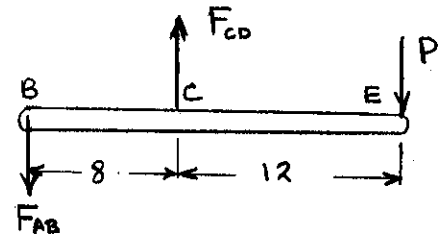
SOLUTION

Use  $BCE$  as free body

$$\sum M_B = 0$$

$$8F_{CD} - 20P = 0$$

$$P = \frac{2}{5} F_{CD}$$



$$\sum M_C = 0 \quad 8F_{AB} - 12P = 0 \quad P = \frac{2}{3} F_{AB}$$

Both links have the same area and same pin diameter; hence, being of the same material, they will have the same ultimate load.

Based on pin in single shear

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$F_u = \tau_u A = (24)(0.19635) = 4.7124 \text{ kips}$$

Based on tension in link

$$A = (b-d)t = \left(1 - \frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in}^2$$

$$F_u = \sigma_u A = (60)(0.125) = 7.50 \text{ kips}$$

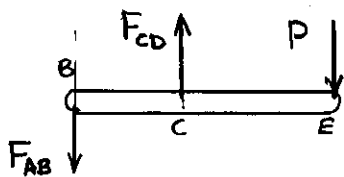
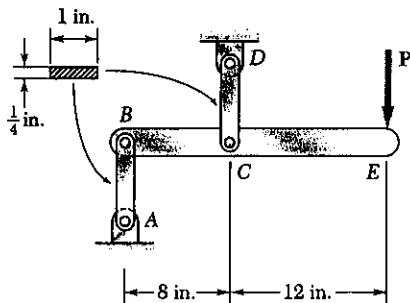
Ultimate load for link is smallest:  $F_u = 4.7124 \text{ kips}$ .

Allowable load for link  $F = \frac{F_u}{F.S.} = \frac{4.7124}{3.2} = 1.4726 \text{ kips}$

Allowable load for structure  $P = \frac{2}{3} F = 0.9817 \text{ kips}$

$$F = 589 \text{ lb}$$

**PROBLEM 1.52**



1.51 Each of the steel links  $AB$  and  $CD$  is connected to a support and to member  $BCE$  by  $\frac{1}{2}$ -in.-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 24 ksi for the steel used in the pins and that the ultimate normal stress is 60 ksi for the steel used in the links, determine the allowable load  $P$  if an overall factor of safety of 3.2 is desired. (Note that the links are not reinforced around the pin holes.)

1.52 An alternative design is being considered to support member  $BCE$  of Prob. 1.51 in which link  $CD$  will be replaced by two links, each of  $\frac{1}{8} \times 1$ -in. cross section, causing the pins at  $C$  and  $D$  to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load  $P$  if an overall factor of safety of 3.2 is desired.

**SOLUTION**

Use member  $BCE$  as free body

$$\begin{aligned} \sum M_B = 0 \quad 8F_{CD} - 20P = 0 \quad P &= \frac{2}{5} F_{CD} \\ \sum M_C = 0 \quad 8F_{AB} - 12P = 0 \quad P &= \frac{2}{3} F_{AB} \end{aligned}$$

Based on pin  $A$  in single shear

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$F_u = \tau_u A = (24)(0.19635) = 4.7124 \text{ kips}$$

Based on tension in link  $AB$

$$A = (b-d)t = \left(1 - \frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in}^2$$

$$F_u = \sigma_u A = (60)(0.125) = 7.50 \text{ kips}$$

Ultimate load for link  $AB$  is smallest, i.e.  $F_u = 4.7124$  kips

Corresponding ultimate load for structure:  $P_u = \frac{2}{3} F_u = 3.1416$  kips

Based on pins at  $C$  and  $D$  in double shear

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$F_u = 2\tau_u A = (2)(24)(0.19635) = 9.4248 \text{ kips}$$

Based on tension in links  $BC$

$$A = (b-d)t = \left(1 - \frac{1}{2}\right)\left(\frac{1}{8}\right) = 0.0625 \text{ in}^2 \text{ (one link)}$$

$$F_u = 2\sigma_u A = (2)(60)(0.0625) = 7.50 \text{ kips (total, both links)}$$

Ultimate load for links  $BC$  is smallest, i.e.  $F_u = 7.50$  kips

Corresponding ultimate load for structure  $P_u = \frac{2}{3} F_u = 3.00$  kips.

Actual ultimate load is smallest, i.e.  $P_u = 3.00$  kips

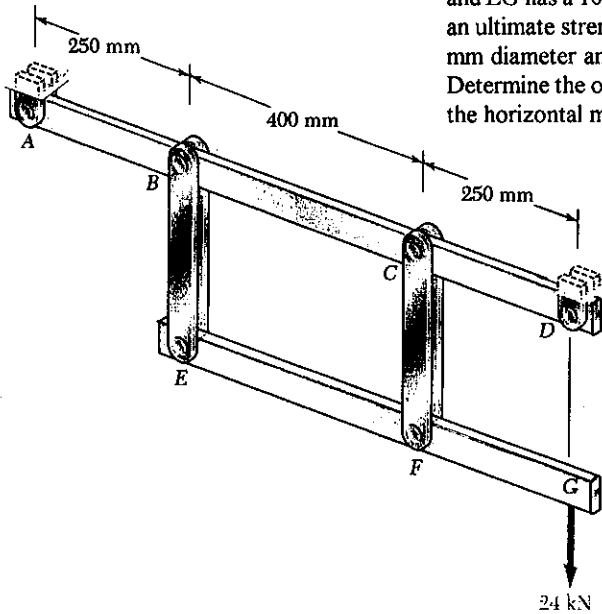
Allowable load for structure  $P = \frac{P_u}{F.S.} = \frac{3.00}{3.2} = 0.938$  kip

$$P = 938 \text{ lb.}$$



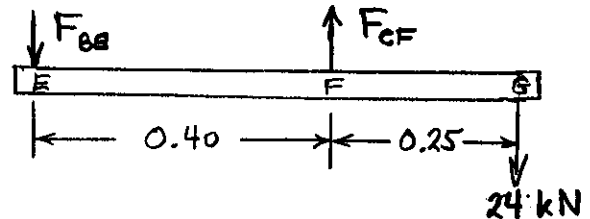
**PROBLEM 1.53**

1.53 Each of the two vertical links  $CF$  connecting the two horizontal members  $AD$  and  $EG$  has a  $10 \times 40$ -mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at  $C$  and  $F$  has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links  $CF$  and the pins connecting them to the horizontal members.



**SOLUTION**

Use member  $EFG$  as free body.



$$\sum M_E = 0$$

$$0.40 F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links  $CF$

$$A = (b - d)t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6} \text{ m}^2 \text{ (one link)}$$

$$F_u = 2\sigma_u A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3 \text{ N}$$

Based on double shear in pins

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$F_u = 2\tau_u A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3 \text{ N}$$

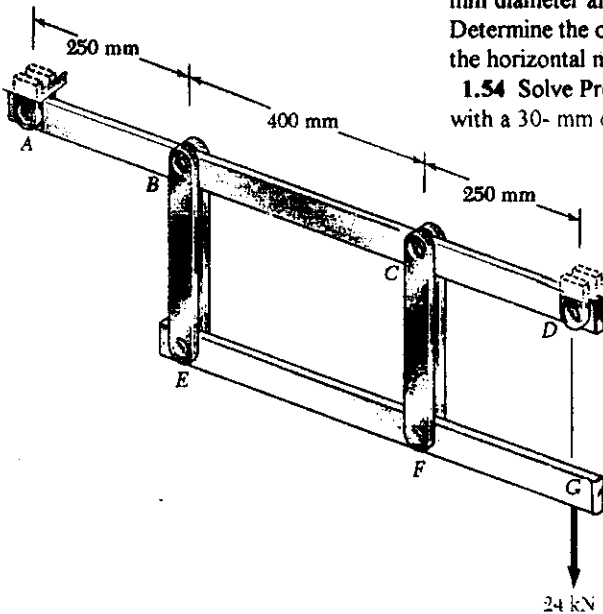
Actual  $F_u$  is smaller value, i.e.  $F_u = 94.248 \times 10^3 \text{ N}$

$$\text{Factor of safety } F.S. = \frac{F_u}{F_{CF}} = \frac{94.248 \times 10^3}{39 \times 10^3} = 2.42$$

**PROBLEM 1.54**

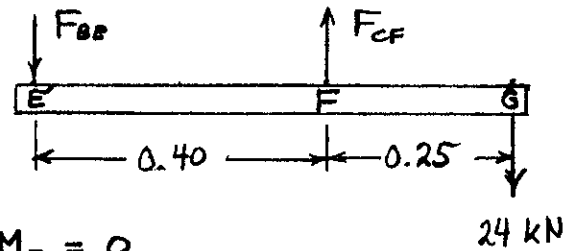
**1.53** Each of the two vertical links  $CF$  connecting the two horizontal members  $AD$  and  $EG$  has a  $10 \times 40$ -mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at  $C$  and  $F$  has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links  $CF$  and the pins connecting them to the horizontal members.

**1.54** Solve Prob. 1.53, assuming that the pins at  $C$  and  $F$  have been replaced by pins with a 30-mm diameter.



**SOLUTION**

Use member  $EFG$  as free body.



$$\sum M_E = 0$$

$$0.40 F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links  $CF$

$$A = (b - d)t = (0.040 - 0.030)(0.010) = 100 \times 10^{-6} \text{ m}^2 \text{ (one link)}$$

$$F_u = 25_u A = (2)(400 \times 10^6)(100 \times 10^{-6}) = 80.0 \times 10^3 \text{ N}$$

Based on double shear in pins

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.030)^2 = 706.86 \times 10^{-6} \text{ m}^2$$

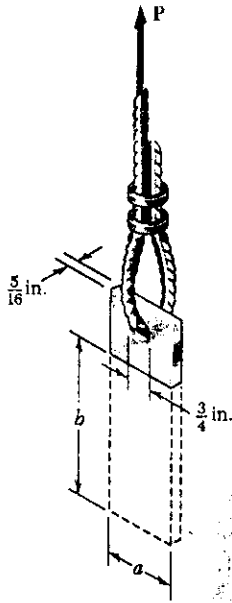
$$F_u = 2\tau_u A = (2)(150 \times 10^6)(706.86 \times 10^{-6}) = 212.06 \times 10^3 \text{ N}$$

Actual  $F_u$  is smaller value, i.e.  $F_u = 80.0 \times 10^3 \text{ N}$

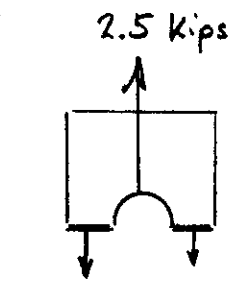
Factor of safety  $F.S. = \frac{F_u}{F_{CF}} = \frac{80.0 \times 10^3}{39 \times 10^3} = 2.05$

PROBLEM 1.55

1.55 A steel plate  $\frac{5}{16}$  in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is  $\frac{3}{4}$  in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when  $P = 2.5$  kips, determine (a) the required width  $a$  of the plate, (b) the minimum depth  $b$  to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



SOLUTION



Based on tension in plate

$$A = (a - d)t$$

$$P_u = \sigma_u A$$

$$F.S. = \frac{P_u}{P} = \frac{\sigma_u (a - d)t}{P}$$

Solving for  $b$

$$a = d + \frac{(F.S.)P}{\sigma_u t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)(\frac{5}{16})}$$

$$a = 1.550 \text{ in.}$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b$$

$$\tau_u = 0.300 \text{ ksi}$$

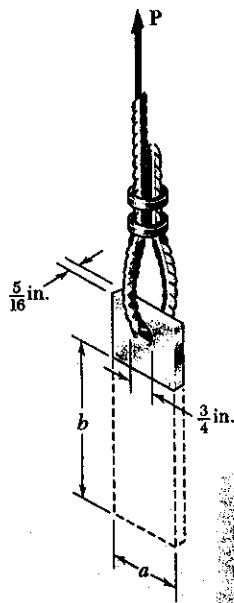
$$P_u = \tau_u A = 2\tau_u (a + t)b$$

$$F.S. = \frac{P_u}{P}$$

$$\text{Solving for } b \quad b = \frac{(F.S.)P}{2(a + t)\tau_u} = \frac{(3.6)(2.5)}{(2)(1.550 + \frac{5}{16})(0.300)}$$

$$b = 8.05 \text{ in.}$$

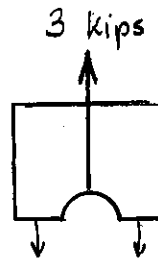
**PROBLEM 1.56**



1.55 A steel plate  $\frac{5}{16}$  in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is  $\frac{3}{4}$  in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi.

1.56 Determine the factor of safety for the cable anchor of Prob. 1.55 when  $P = 3$  kips, knowing that  $a = 2$  in. and  $b = 7.5$  in.

**SOLUTION**



Based on tension in plate

$$A = (a - d)t$$

$$= (2 - \frac{3}{4})(\frac{5}{16}) = 0.3906 \text{ in}^2$$

$$P_u = \sigma_u A$$

$$= (36)(0.3906) = 14.06 \text{ kips}$$

$$F.S. = \frac{P_u}{P} = \frac{14.06}{3} = 4.69$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a + t)b = 2(2 + \frac{5}{16})(7.5)$$

$$A = 34.69 \text{ in}^2 \quad \tau_u = 0.300 \text{ ksi}$$

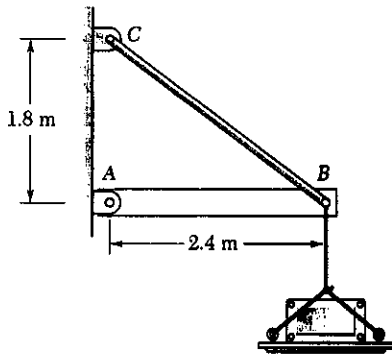
$$P_u = \tau_u A = (0.300)(34.69) = 10.41 \text{ kips}$$

$$F.S. = \frac{P_u}{P} = \frac{10.41}{3} = 3.47$$

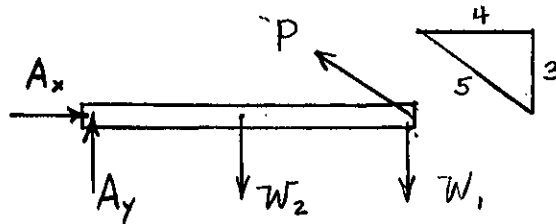
Actual factor of safety is the smaller value  $F.S. = 3.47$   $\blacktriangleleft$

PROBLEM 1.57

\*1.57 A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (a) Using the Load and Resistance Factor Design method with a resistance factor  $\phi = 0.90$  and load factors  $\gamma_D = 1.25$  and  $\gamma_L = 1.6$ , determine the largest load which can be safely placed on the platform. (b) What is the corresponding conventional factor of safety for rod BC?



SOLUTION



$$\sum M_A = 0 \quad (2.4) \frac{3}{5} P - 2.4 W_1 - 1.2 W_2 \therefore P = \frac{5}{3} W_1 + \frac{5}{6} W_2$$

For dead loading  $W_1 = (40)(9.81) = 392.4 \text{ N}$

$$W_2 = (50)(9.81) = 490.5 \text{ N}$$

$$P_D = \left(\frac{5}{3}\right)(392.4) + \left(\frac{5}{6}\right)(490.5) = 1.0628 \times 10^3 \text{ N}$$

For live loading  $W_1 = mg$   $W_2 = 0$

$$P_L = \frac{5}{3} mg \quad \text{from which} \quad m = \frac{3}{5} \frac{P_L}{g}$$

Design criterion

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12 \times 10^3) - (1.25)(1.0628 \times 10^3)}{1.6}$$

$$= 5.920 \times 10^3 \text{ N}$$

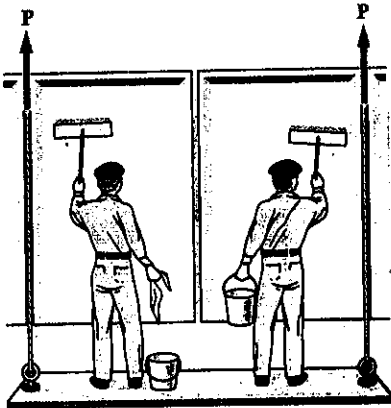
Allowable load  $m = \frac{3}{5} \frac{5.92 \times 10^3}{9.81} = 362 \text{ kg}$

Conventional factor safety

$$P = P_D + P_L = 1.0628 \times 10^3 + 5.920 \times 10^3 = 6.983 \times 10^3 \text{ N}$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{12 \times 10^3}{6.983 \times 10^3} = 1.718$$

**PROBLEM 1.58**



\*1.58 The Load and Resistance Factor Design method is to be used to select the two cables which will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with his equipment. Since these workers are free to move on the platform, 75% of their total weight and of the weight of their equipment will be used as the design live load of each cable. (a) Assuming a resistance factor  $\phi = 0.85$  and load factors  $\gamma_D = 1.2$  and  $\gamma_L = 1.5$ , determine the required minimum ultimate load of one cable. (b) What is the conventional factor of safety for the selected cables?

**SOLUTION**

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)(\frac{1}{2} \times 160) + (1.5)(\frac{3}{4} \times 2 \times 195)}{0.85}$$

$$= 629 \text{ lb.}$$

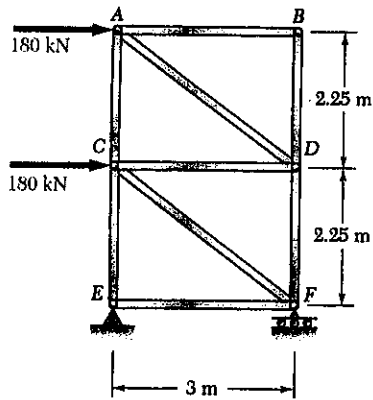
Conventional factor of safety

$$P = P_D + P_L = \frac{1}{2} \times 80 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

$$F.S. = \frac{P_U}{P} = \frac{629}{372.5} = 1.689$$

PROBLEM 1.59

1.59 For the truss and loading shown, determine the average normal stress in member  $DF$ , knowing that the cross-sectional area of that member is  $2500 \text{ mm}^2$ .

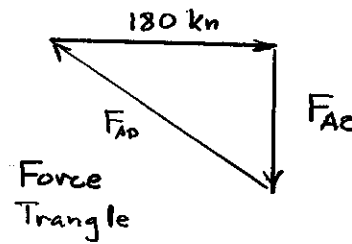
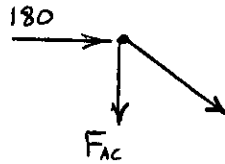


SOLUTION

Using method of joints to find member forces

Joint B:  $AB$  and  $BD$  are zero force members.

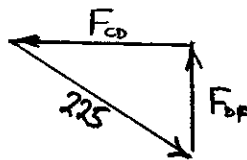
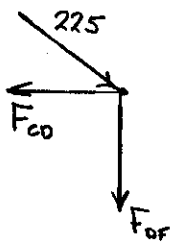
Joint A:  $l_{AD} = \sqrt{3^2 + 2.25^2} = 3.75 \text{ m}$



By similar triangles

$$\frac{F_{AD}}{3.75} = \frac{180}{3} \therefore F_{AD} = 225 \text{ lb. (compression)}$$

Joint D



By similar triangles

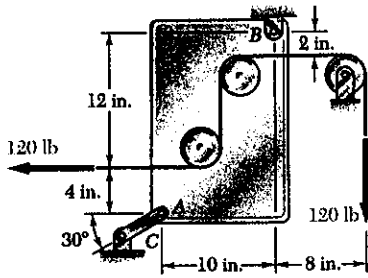
$$\frac{F_{DF}}{2.25} = \frac{225}{3.75}$$

$$F_{DF} = 135 \text{ kN (comp)} = 135 \times 10^3 \text{ N}$$

$$\text{Area: } A_{DF} = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$$

$$\text{Stress: } \sigma_{DF} = -\frac{135 \times 10^3}{2500 \times 10^{-6}} = -54 \times 10^6 \text{ Pa} = -54.0 \text{ MPa} \leftarrow$$

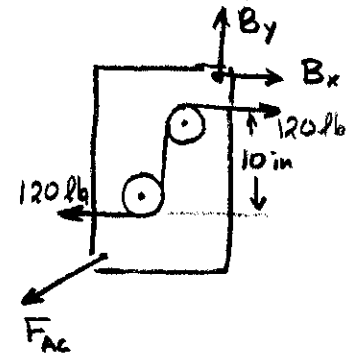
**PROBLEM 1.60**



1.60 Link AC has a uniform rectangular cross section  $\frac{1}{8}$  in. thick and 1 in. wide. Determine the normal stress in the central portion of that link.

**SOLUTION**

Use the plate together with two pulleys as a free body. Note that the cable tension causes a 1200 lb-in clockwise couple to act on the body.



$$\sum M_B = 0$$

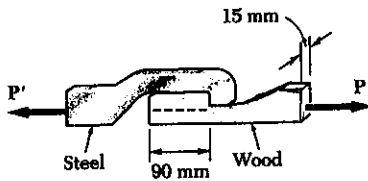
$$-(12 + 4)(F_{AC} \cos 30^\circ) + (10)(F_{AC} \sin 30^\circ) - 1200 = 0$$

$$F_{AC} = -\frac{1200}{16 \cos 30^\circ - 10 \sin 30^\circ} = -135.50 \text{ lb.}$$

Area of link AC:  $A = 1 \text{ in} \times \frac{1}{8} \text{ in} = 0.125 \text{ in}^2$

Stress in link AC:  $\sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = 1084 \text{ psi} = 1.084 \text{ ksi} \blacktriangleleft$

**PROBLEM 1.61**



1.61 When the force P reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

Area being sheared

$$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$$

Force  $P = 8 \times 10^3 \text{ N}$

Shearing stress  $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^3 \text{ Pa} = 5.93 \text{ MPa} \blacktriangleleft$

**PROBLEM 1.62**

**SOLUTION**

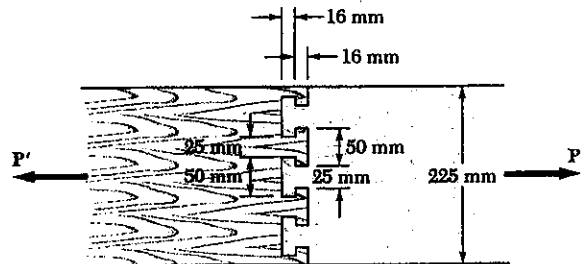
Six areas, each 16 mm x 12 mm are being sheared.

$$\text{Area: } A = (6)(16)(12) = 1152 \text{ mm}^2 = 1152 \times 10^{-6} \text{ m}^2$$

$$\tau = \frac{P}{A}$$

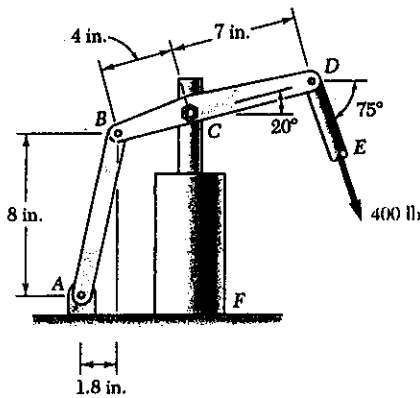
$$P = \tau A = (8 \times 10^6)(1152 \times 10^{-6}) = 9.22 \times 10^3 \text{ N} = 9.22 \text{ kN} \blacktriangleleft$$

1.62 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.





**PROBLEM 1.63**

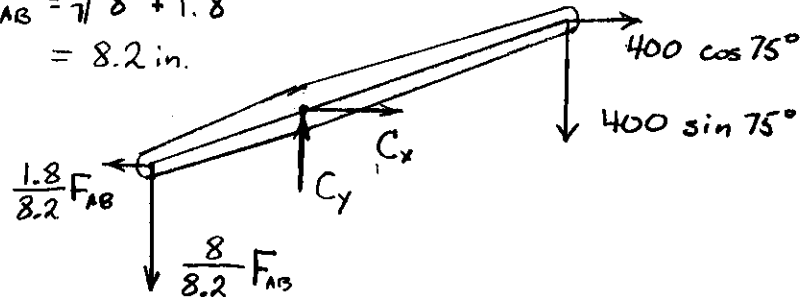


1.63 The hydraulic cylinder  $CF$ , which partially controls the position of rod  $DE$ , has been locked in the position shown. Member  $BD$  is  $\frac{3}{8}$  in. thick and is connected to the vertical rod by a  $\frac{3}{8}$ -in.-diameter bolt. Determine (a) the average shearing stress in the bolt, (b) the bearing stress at  $C$  in member  $BD$ .

**SOLUTION**

Use member  $BCD$  as a free body, and note that  $AB$  is a two force member.

$$l_{AB} = \sqrt{8^2 + 1.8^2} = 8.2 \text{ in.}$$



$$\begin{aligned} \sum M_C = 0 & \quad (4 \cos 20^\circ) \left( \frac{8}{8.2} F_{AB} \right) - (4 \sin 20^\circ) \left( \frac{1.8}{8.2} F_{AB} \right) \\ & \quad - (7 \cos 20^\circ) (400 \sin 75^\circ) - (7 \sin 20^\circ) (400 \cos 75^\circ) = 0 \\ & \quad 3.36678 F_{AB} - 2789.35 = 0 \quad \therefore F_{AB} = 828.49 \text{ lb.} \end{aligned}$$

$$\begin{aligned} \sum F_x = 0 & \quad -\frac{1.8}{8.2} F_{AB} + C_x + 400 \cos 75^\circ = 0 \\ C_x & = \frac{(1.8)(828.49)}{8.2} - 400 \cos 75^\circ = 78.34 \text{ lb.} \end{aligned}$$

$$\begin{aligned} \sum F_y = 0 & \quad -\frac{8}{8.2} F_{AB} + C_y - 400 \sin 75^\circ = 0 \\ C_y & = \frac{(8)(828.49)}{8.2} + 400 \sin 75^\circ = 1197.65 \text{ lb.} \\ C & = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb.} \end{aligned}$$

(a) Shearing stress in the bolt:  $P = 1197.2 \text{ lb.}$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left( \frac{3}{8} \right)^2 = 0.11045 \text{ in}^2$$

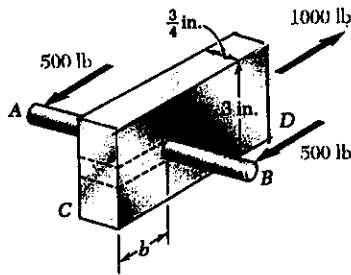
$$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{ psi} = 10.84 \text{ ksi} \quad \blacktriangleleft$$

(b) Bearing stress at  $C$  in member  $BCD$ :  $P = 1197.2 \text{ lb.}$

$$A_b = dt = \left( \frac{3}{8} \right) \left( \frac{5}{8} \right) = 0.234375 \text{ in}^2$$

$$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{ psi} = 5.11 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 1.64**



1.64 A  $\frac{1}{2}$ -in.- diameter steel rod  $AB$  is fitted to a round hole near end  $C$  of the wooden member  $CD$ . For the loading shown, determine (a) the maximum average normal stress in the wood, (b) the distance  $b$  for which the average shearing stress is 90 psi on the surfaces indicated by the dashed lines, (c) the average bearing stress on the wood.

**SOLUTION**

(a) Maximum normal stress in the wood

$$A_{net} = \frac{3}{4} (3 - \frac{1}{2}) = 1.875 \text{ in}^2$$

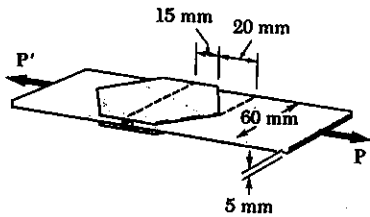
$$\sigma = \frac{P}{A_{net}} = \frac{1000}{1.875} = 533 \text{ psi}$$

(b)  $\tau = \frac{P}{A} = \frac{P}{2bt}$

$$b = \frac{P}{2t\tau} = \frac{1000}{(2)(\frac{3}{4})(90)} = 7.41 \text{ in.}$$

(c)  $\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1000}{(\frac{1}{2})(\frac{3}{4})} = 2667 \text{ psi}$

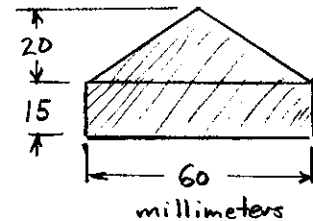
**PROBLEM 1.65**



**SOLUTION**

Bond area: (See figure)

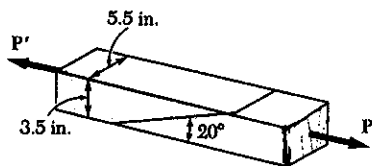
$$A = \frac{1}{2}(60)(20) + (15)(60) = 1500 \text{ mm}^2 = 1500 \times 10^{-6} \text{ m}^2$$



$$P_u = 2A\tau_u = (2)(1500 \times 10^{-6})(900 \times 10^3) = 2700 \text{ N}$$

$$\text{F.S.} = \frac{P_u}{P} = \frac{2700}{1500} = 1.800$$

**PROBLEM 1.66**



1.66 Two wooden members of  $3.5 \times 5.5$ -in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 75 psi, determine the largest axial load  $P$  which can be safely applied.

**SOLUTION**

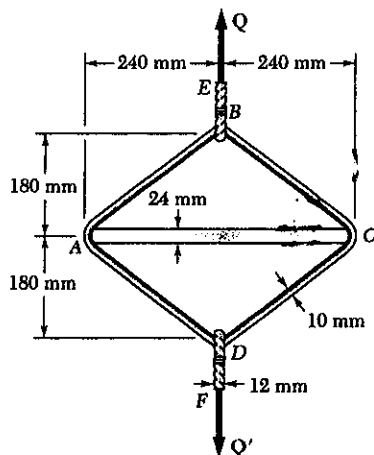
$$A_o = (3.5)(5.5) = 19.25 \text{ in}^2$$

$$\theta = 90^\circ - 20^\circ = 70^\circ$$

$$\tau = \frac{P}{A_o} \sin \theta \cos \theta = \frac{P}{2A_o} \sin 2\theta$$

$$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(19.25)(75)}{\sin 140^\circ} = 4492 \text{ lb} = 4.49 \text{ kips}$$

PROBLEM 1.67



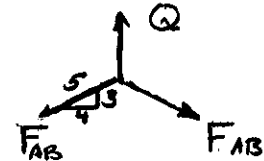
1.67 A steel loop  $ABCD$  of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod  $AC$ . Cables  $BE$  and  $DF$ , each of 12-mm diameter, are used to apply the load  $Q$ . Knowing that the ultimate strength of the aluminum used for the rod is 260 MPa and that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load  $Q$  which can be applied if an overall factor of safety of 3 is desired.

SOLUTION

Using joint B as a free body and considering symmetry

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

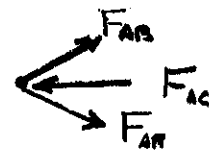
$$Q = \frac{6}{5} F_{AB}$$



Using joint A as a free body and considering symmetry

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$



Based on strength of cable BE

$$Q_u = \sigma_u A = \sigma_u \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop

$$\begin{aligned} Q_u &= \frac{6}{5} F_{AB,u} = \frac{6}{5} \sigma_u A = \frac{6}{5} \sigma_u \frac{\pi}{4} d^2 \\ &= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N} \end{aligned}$$

Based on strength of rod AC

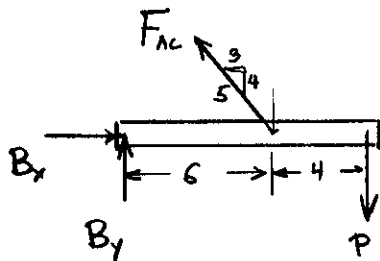
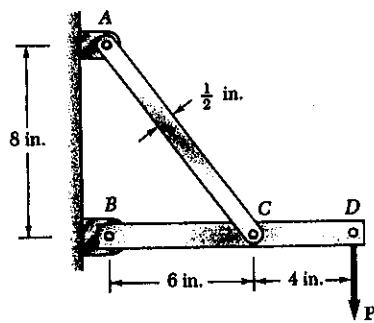
$$\begin{aligned} Q_u &= \frac{3}{4} F_{AC,u} = \frac{3}{4} \sigma_u A = \frac{3}{4} \sigma_u \frac{\pi}{4} d^2 \\ &= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N} \end{aligned}$$

Actual ultimate load  $Q_u$  is the smallest  $\therefore Q_u = 45.24 \times 10^3 \text{ N}$

$$\text{Allowable load } Q = \frac{Q_u}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3 \text{ N}$$

$$= 15.08 \text{ kN}$$

## PROBLEM 1.68



1.68 Link  $AC$  has a uniform  $\frac{1}{4} \times \frac{1}{2}$ -in. uniform rectangular cross section and is made of a steel with a 60-ksi ultimate normal stress. It is connected to a support at  $A$  and to member  $BCD$  at  $C$  by  $\frac{3}{8}$ -in.-diameter pins, while member  $BCD$  is connected to a support at  $B$  by a  $\frac{5}{16}$ -in.-diameter pin. All of the pins are in single shear and are made of a steel with a 25-ksi ultimate shearing stress. Knowing that an overall factor of safety of 3.25 is desired, determine the largest load  $P$  which can be safely applied at  $D$ . Note that link  $AC$  is not reinforced around the pin holes.

## SOLUTION

$$\curvearrowright \sum M_B = 0 \quad (6) \left( \frac{3}{5} F_{AC} \right) - 10 P = 0$$

$$F_{AC} = 2.0833 P \quad P = 0.480 F_{AC}$$

$$\rightarrow \sum F_x = 0 \quad B_x - \frac{3}{5} F_{AC} = 0$$

$$B_x = \frac{3}{5} F_{AC} = \left( \frac{3}{5} \right) (2.0833 P) = 1.25 P$$

$$\uparrow \sum F_y = 0 \quad B_y + \frac{4}{5} F_{AC} - P = 0$$

$$B_y = P - \frac{4}{5} (2.0833 P) = -0.66667 P$$

$$B = \sqrt{B_x^2 + B_y^2} = 1.41667 P, \quad P = 0.70588 B$$

Based on strength of link  $AC$ :  $\sigma_u = 60 \text{ ksi}$

$$A_{net} = \left( \frac{1}{4} \right) \left( \frac{1}{2} - \frac{3}{8} \right) = 0.03125 \text{ in}^2, \quad F_{AC,u} = \sigma_u A_{net} = (60)(0.03125) = 1.875 \text{ kip.}$$

$$P_u = (0.480)(1.875) = 0.900 \text{ kip.}$$

Based on strength of pin at  $C$ :  $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left( \frac{3}{8} \right)^2 = 0.11045 \text{ in}^2$

$$\tau_u = 25 \text{ ksi} \quad F_{AC,u} = \tau_u A_{pin} = (25)(0.11045) = 2.761 \text{ kip.}$$

$$P_u = (0.480)(2.761) = 1.325 \text{ kip.}$$

Based on strength of pin at  $B$ :  $A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left( \frac{5}{16} \right)^2 = 0.07670 \text{ in}^2$

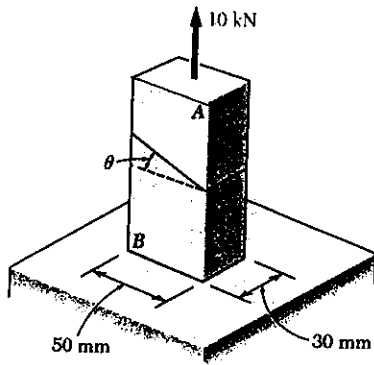
$$B_u = \tau_u A_{pin} = (25)(0.07670) = 1.9175 \text{ kip.}$$

$$P_u = (0.70588)(1.9175) = 1.3535 \text{ kip}$$

Actual  $P_u$  is the smallest:  $P_u = 0.900 \text{ kip.}$

$$\text{Allowable value for } P: \quad P = \frac{P_u}{FS} = \frac{0.900}{3.25} = 0.277 \text{ kip} = 277 \text{ lb.} \quad \blacktriangleleft$$

**PROBLEM 1.69**



1.69 The two portions of member  $AB$  are glued together along a plane forming an angle  $\theta$  with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of  $\theta$  for which the factor of safety of the member is at least 3.0.

**SOLUTION**

$$A_0 = (0.030)(0.050) = 1.50 \times 10^{-3} \text{ m}^2$$

$$P = 10 \times 10^3 \text{ N} \quad P_0 = (F.S.)P = 30 \times 10^3 \text{ N}$$

Based on tensile stress

$$\sigma_v = \frac{P_0}{A_0} \cos^2 \theta$$

$$\cos^2 \theta = \frac{\sigma_u A_0}{P_0} = \frac{(17 \times 10^6)(1.50 \times 10^{-3})}{30 \times 10^3} = 0.85$$

$$\cos \theta = 0.92195$$

$$\theta = 22.79^\circ$$

$$\theta \geq 22.79^\circ$$

Based on shearing stress  $\tau_v = \frac{P_0}{A_0} \sin \theta \cos \theta = \frac{P_0}{2A_0} \sin 2\theta$

$$\sin 2\theta = \frac{2A_0 \tau_u}{P_0} = \frac{(2)(1.50 \times 10^{-3})(9 \times 10^6)}{30 \times 10^3} = 0.900$$

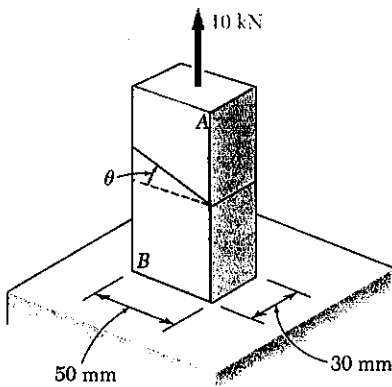
$$2\theta = 64.16^\circ$$

$$\theta = 32.08^\circ$$

$$\theta \leq 32.08^\circ$$

Hence  $22.79^\circ \leq \theta \leq 32.08^\circ$

**PROBLEM 1.70**



1.70 The two portions of member  $AB$  are glued together along a plane forming an angle  $\theta$  with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (a) the value of  $\theta$  for which the factor of safety of the member is maximum, (b) the corresponding value of the factor of safety. (Hint: Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

**SOLUTION**

$$A_0 = (0.030)(0.050) = 1.50 \times 10^{-3} \text{ m}^2$$

At the optimum angle  $(F.S.)_\sigma = (F.S.)_\tau$

$$\text{Normal stress: } \sigma = \frac{P}{A_0} \cos^2 \theta \quad \therefore P_0 \sigma = \frac{\sigma_u A_0}{\cos^2 \theta}$$

$$(F.S.)_\sigma = \frac{P_0 \sigma}{P} = \frac{\sigma_u A_0}{P \cos^2 \theta}$$

$$\text{Shearing stress: } \tau = \frac{P}{A_0} \sin \theta \cos \theta \quad \therefore P_0 \tau = \frac{\tau_u A_0}{\sin \theta \cos \theta}$$

$$(F.S.)_\tau = \frac{P_0 \tau}{P} = \frac{\tau_u A_0}{P \sin \theta \cos \theta}$$

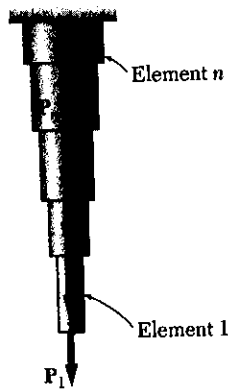
$$\text{Equating: } \frac{\sigma_u A_0}{P \cos^2 \theta} = \frac{\tau_u A_0}{P \sin \theta \cos \theta}$$

$$\text{Solving: } \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{\tau_u}{\sigma_u} = \frac{9}{17} = 0.52941; \quad \theta_{opt} = 27.9^\circ$$

$$\text{Then } P_0 = \frac{\sigma_u A_0}{\cos^2 \theta} = \frac{(17 \times 10^6)(1.50 \times 10^{-3})}{\cos^2 27.9^\circ} = 32.65 \times 10^3$$

$$F.S. = \frac{P_0}{P} = \frac{32.65 \times 10^3}{10 \times 10^3} = 3.26$$

**PROBLEM 1.C1**



**1.C1** A solid steel rod consisting of  $n$  cylindrical elements welded together is subjected to the loading shown. The diameter of element  $i$  is denoted by  $d_i$  and the load applied to its lower end by  $P_i$ , with the magnitude  $P_i$  of this load being assumed positive if  $P_i$  is directed downward as shown and negative otherwise. (a) Write a computer program which can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (b) Use this program to solve Probs. 1.1 and 1.3.

**SOLUTION**

FORCE IN ELEMENT  $i$ :

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^i P_k$$

AVERAGE STRESS IN ELEMENT  $i$ :

Area =  $A_i = \frac{1}{4} \pi d_i^2$       Ave stress =  $\frac{F_i}{A_i}$

PROGRAM OUTPUTS

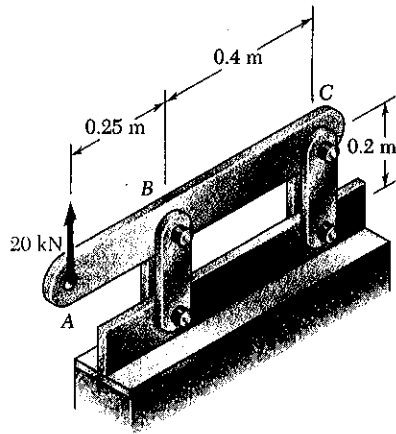
Problem 1.1  
Element Stress (MPa)

1	84.883
2	-96.766

Problem 1.3  
Element Stress (ksi)

1	22.635
2	17.927

**PROBLEM 1.C2**

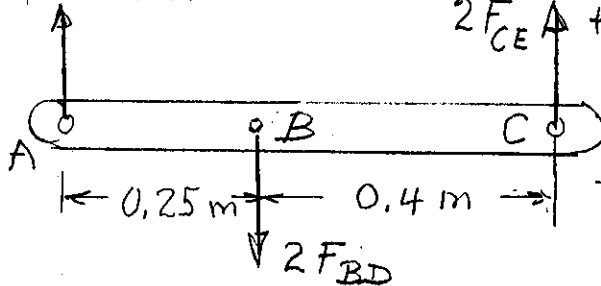


**1.C2** A 20-kN force is applied as shown to the horizontal member ABC. Member ABC has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at A, B, C, and D has the same diameter  $d$  and is in double shear. (a) Write a computer program to calculate for values of  $d$  from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins B and D, (2) the average normal stress in the links connecting pins C and E, (3) the average shearing stress in pin B, (4) the average shearing stress in pin C, (5) the average bearing stress at B in member ABC, (6) the average bearing stress at C in member ABC. (b) Check your program by comparing the values obtained for  $d = 16$  mm with the answers given for Probs. 1.8, 1.23, and 1.24. (c) Use this program to find the permissible values of the diameter  $d$  of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (d) Solve part c, assuming that the thickness of member ABC has been reduced from 10 to 8 mm.

**SOLUTION**

FORCES IN LINKS

$P = 20 \text{ kN}$



F. B. DIAGRAM OF ABC :

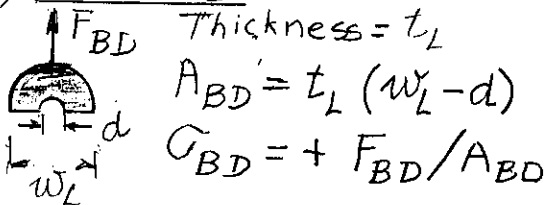
$\sum M_C = 0 : 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$  (TENSION)

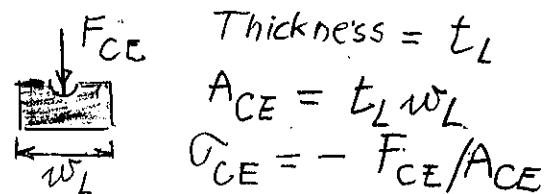
$\sum M_B = 0 : 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$  (COMP.)

(1) LINK BD



(2) LINK CE



(3) PIN B

$\tau_B = F_{BD} / (\pi d^2 / 4)$

(4) PIN C

$\tau_C = F_{CE} / (\pi d^2 / 4)$

(5) BEARING STRESS AT B

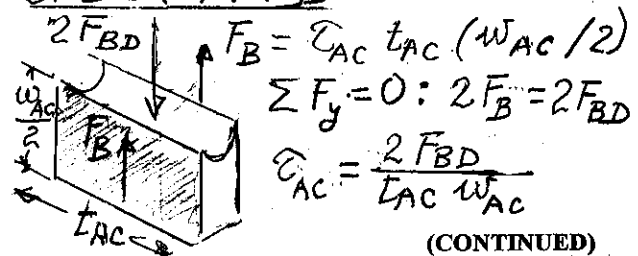
Thickness of member AC =  $t_{AC}$

$\sigma_{\text{Bear B}} = F_{BD} / (d t_{AC})$

(6) BEARING STRESS AT C

$\sigma_{\text{Bear C}} = F_{CE} / (d t_{AC})$

SHEARING STRESS IN ABC UNDER PIN B



(CONTINUED)

PROBLEM 1.C2 CONTINUED

PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m,  
AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	<del>206.90</del>	79.58	<del>325.00</del>	125.00
11.00	81.25	-21.70	<del>170.99</del>	65.77	<del>295.45</del>	113.64
12.00	84.64	-21.70	<del>143.68</del>	55.26	<del>270.83</del>	104.17
13.00	88.32	-21.70	<del>122.43</del>	47.09	<del>250.00</del>	96.15
14.00	92.33	-21.70	<del>105.56</del>	40.60	<del>232.14</del>	89.29
15.00	96.73	-21.70	<del>91.96</del>	35.37	<del>216.67</del>	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 ← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	<del>156.25</del>	-21.70	39.11	15.04	141.30	54.35
24.00	<del>169.27</del>	-21.70	35.92	13.82	135.42	52.08
25.00	<del>184.66</del>	-21.70	33.10	12.73	130.00	50.00
26.00	<del>203.13</del>	-21.70	30.61	11.77	125.00	48.08
27.00	<del>225.69</del>	-21.70	28.38	10.92	120.37	46.30
28.00	<del>253.91</del>	-21.70	26.39	10.15	116.07	44.64
29.00	<del>290.18</del>	-21.70	24.60	9.46	112.07	43.10
30.00	<del>338.54</del>	-21.70	22.99	8.84	108.33	41.67

(c) ANSWER :  $16 \text{ mm} \leq d \leq 22 \text{ mm}$  (c)

CHECK: For  $d = 22 \text{ mm}$ ,  $\text{Tau AC} = 65 \text{ MPa} < 90 \text{ MPa}$  O.K.

INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m,  
AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

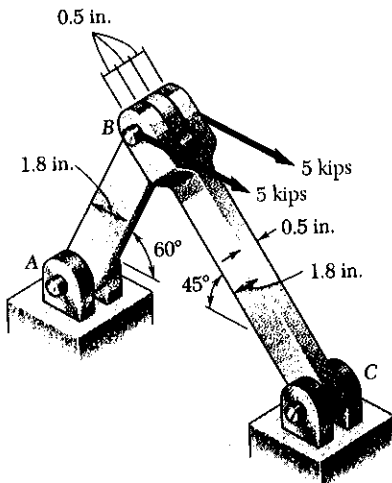
d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	<del>206.90</del>	79.58	<del>406.25</del>	156.25
11.00	81.25	-21.70	<del>170.99</del>	65.77	<del>369.32</del>	142.05
12.00	84.64	-21.70	<del>143.68</del>	55.26	<del>338.54</del>	130.21
13.00	88.32	-21.70	<del>122.43</del>	47.09	<del>312.50</del>	120.19
14.00	92.33	-21.70	<del>105.56</del>	40.60	<del>290.18</del>	111.61
15.00	96.73	-21.70	<del>91.96</del>	35.37	<del>270.83</del>	104.17
16.00	101.56	-21.70	80.82	31.08	<del>253.91</del>	97.66
17.00	106.91	-21.70	71.59	27.54	<del>238.97</del>	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	<del>156.25</del>	-21.70	39.11	15.04	176.63	67.93
24.00	<del>169.27</del>	-21.70	35.92	13.82	169.27	65.10
25.00	<del>184.66</del>	-21.70	33.10	12.73	162.50	62.50
26.00	<del>203.13</del>	-21.70	30.61	11.77	156.25	60.10
27.00	<del>225.69</del>	-21.70	28.38	10.92	150.46	57.87
28.00	<del>253.91</del>	-21.70	26.39	10.15	145.09	55.80
29.00	<del>290.18</del>	-21.70	24.60	9.46	140.09	53.88
30.00	<del>338.54</del>	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER :  $18 \text{ mm} \leq d \leq 22 \text{ mm}$  (d)

CHECK: For  $d = 22 \text{ mm}$ ,  $\text{Tau AC} = 81.25 \text{ MPa} < 90 \text{ MPa}$  O.K.



**PROBLEM 1.C3**

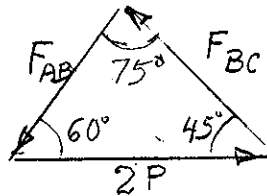
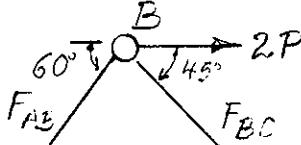


**1.C3** Two horizontal 5-kip forces are applied to pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter  $d$  and is in double shear. (a) Write a computer program to calculate for values of  $d$  from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in pin A, (4) the average shearing stress in pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (b) Check your program by comparing the values obtained for  $d = 0.8$  in. with the answers given for Probs. 1.9, 1.25, and 1.26. (c) Use this program to find the permissible values of the diameter  $d$  of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (d) Solve part c, assuming that a new design is being investigated, in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 in. to 2.4 in.

**SOLUTION**

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B



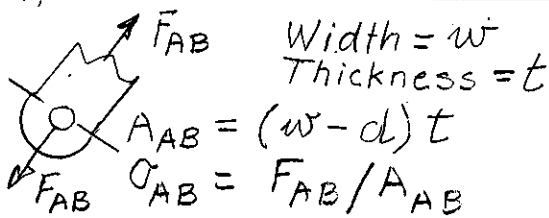
FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{2P}{\sin 75^\circ}$$

$$F_{AB} = 2P(\sin 45^\circ / \sin 75^\circ)$$

$$F_{BC} = 2P(\sin 60^\circ / \sin 75^\circ)$$

(1) MAX. AVE. STRESS IN AB



(3) PIN A

$$\tau_A = (F_{AB}/2) / (\pi d^2/4)$$

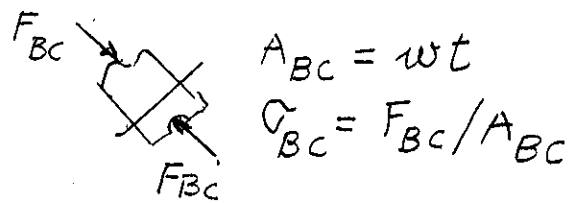
(5) BEARING STRESS AT A

$$\text{Sig Bear A} = F_{AB}/dt$$

(7) BEARING STRESS AT B IN MEMBER BC

$$\text{Sig Bear B} = F_{BC}/2dt$$

(2) AVE. STRESS IN BC



(4) PIN C

$$\tau_C = (F_{BC}/2) / (\pi d^2/4)$$

(6) BEARING STRESS AT C

$$\text{Sig Bear C} = F_{BC}/dt$$


(CONTINUED)

PROBLEM 1.C3 CONTINUED

PROGRAM OUTPUTS


INPUT DATA FOR PARTS (a), (b), (c):  $P = 5$  kips,  $w = 1.8$  in.,  $t = 0.5$  in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	<del>18.642</del>	<del>22.831</del>	29.282	35.863	17.932
0.550	11.713	-9.962	<del>15.406</del>	<del>18.869</del>	26.620	32.603	16.301
0.600	12.201	-9.962	<del>12.945</del>	<del>15.855</del>	24.402	29.886	14.943
0.650	12.731	-9.962	11.030	<del>12.510</del>	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	<del>11.649</del>	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151
1.150	<del>22.525</del>	-9.962	3.524	4.316	12.731	15.593	7.796
1.200	<del>24.402</del>	-9.962	3.236	3.964	12.201	14.943	7.471
1.250	<del>26.620</del>	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	<del>29.282</del>	-9.962	2.758	3.377	11.262	13.793	6.897
1.350	<del>32.536</del>	-9.962	2.557	3.132	10.845	13.283	6.641
1.400	<del>36.603</del>	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	<del>41.831</del>	-9.962	2.217	2.715	10.097	12.367	6.183
1.500	<del>48.803</del>	-9.962	2.071	2.537	9.761	11.954	5.977

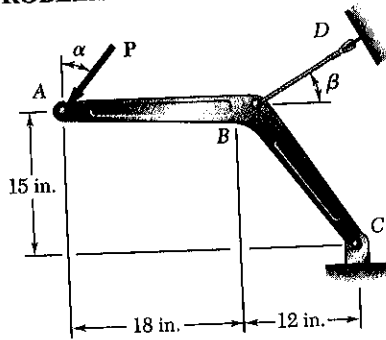
(c) ANSWER:  $0.70$  in.  $\leq d \leq 1.10$  in.  (c)

INPUT DATA FOR PART (d):  $P = 5$  kips,  $w = 2.4$  in.,  $t = 0.3$  in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	<del>18.642</del>	<del>22.831</del>	<del>48.803</del>	<del>59.772</del>	29.886
0.550	13.190	-12.452	<del>15.406</del>	<del>18.869</del>	<del>44.367</del>	<del>54.338</del>	27.169
0.600	13.556	-12.452	<del>12.945</del>	<del>15.855</del>	<del>40.669</del>	<del>49.810</del>	24.905
0.650	13.944	-12.452	11.030	<del>13.510</del>	<del>37.541</del>	<del>45.978</del>	22.989
0.700	14.354	-12.452	9.511	<del>11.649</del>	<del>34.860</del>	<del>42.684</del>	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	<del>39.848</del>	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	<del>37.357</del>	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	<del>22.183</del>	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	<del>23.240</del>	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	<del>24.402</del>	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	<del>25.686</del>	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	<del>27.113</del>	-12.452	2.071	2.537	16.268	19.924	9.962

(d) ANSWER:  $0.85$  in.  $\leq d \leq 1.25$  in.  (d)

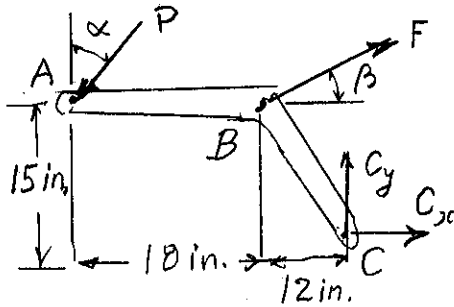
**PROBLEM 1.C4**



**1.C4** A 4-kip force  $P$  forming an angle  $\alpha$  with the vertical is applied as shown to member  $ABC$ , which is supported by a pin and bracket at  $C$  and by a cable  $BD$  forming an angle  $\beta$  with the horizontal. (a) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of  $\alpha$  and  $\beta$  from 0 to  $45^\circ$ , using increments in  $\alpha$  and  $\beta$  corresponding to 0.1 increments in  $\tan \alpha$  and  $\tan \beta$ . (b) Check that for any given value of  $\alpha$  the maximum value of the factor of safety is obtained for  $\beta = 38.66^\circ$  and explain why. (c) Determine the smallest possible value of the factor of safety for  $\beta = 38.66^\circ$ , as well as the corresponding value of  $\alpha$ , and explain the result obtained.

**SOLUTION**

(a) DRAW F. B. DIAGRAM OF ABC:



$$+\circlearrowleft \sum M_C = 0: (P \sin \alpha)(15 \text{ in.}) + (P \cos \alpha)(30 \text{ in.}) - (F \cos \beta)(15 \text{ in.}) - (F \sin \beta)(12 \text{ in.}) = 0$$

$$F = P \frac{15 \sin \alpha + 30 \cos \alpha}{15 \cos \beta + 12 \sin \beta}$$

$$F. S. = F_{ult} / F$$

OUTPUT FOR  $P = 4$  kips AND  $F_{ult} = 20$  kips

VALUES OF FS  
BETA

ALPHA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

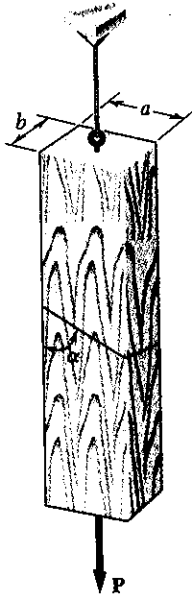
(b) When  $\beta = 38.66^\circ$ ,  $\tan \beta = 0.8$  and cable  $BD$  is perpendicular to the lever arm  $BC$ .

(c)  $F. S. = 3.579$  for  $\alpha = 26.6^\circ$ ;  $P$  is perpendicular to the lever arm  $AC$ .

NOTE:

The value  $F. S. = 3.579$  is the smallest of the values of  $F. S.$  corresponding to  $\beta = 38.66^\circ$  and the largest of those corresponding to  $\alpha = 26.6^\circ$ . The point  $\alpha = 26.6^\circ$ ,  $\beta = 38.66^\circ$  is a "saddle point", or "minimax" of the function  $F. S.(\alpha, \beta)$ .

**PROBLEM 1.C5**

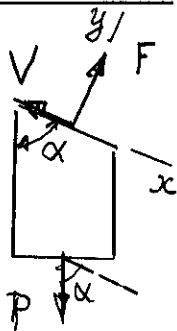


**1.C5** A load  $P$  is supported as shown by two wooden members of uniform rectangular cross section which are joined by a simple glued scarf splice. (a) Denoting by  $\sigma_U$  and  $\tau_U$ , respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of  $a$ ,  $b$ ,  $P$ ,  $\sigma_U$ , and  $\tau_U$ , expressed in either SI or U.S. customary units, and for values of  $\alpha$  from  $5^\circ$  to  $85^\circ$  at  $5^\circ$  intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (b) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.32, knowing that  $\sigma_U = 1.26$  MPa and  $\tau_U = 1.50$  MPa for the glue used in Prob. 1.29, and that  $\sigma_U = 150$  psi and  $\tau_U = 214$  psi for the glue used in Prob. 1.32. (c) Verify in each of these two cases that the shearing stress is maximum for  $\alpha = 45^\circ$ .

**SOLUTION**

(1) and (2)

Draw the F.B. diagram of lower member:



$$+\downarrow \sum F_x = 0: -V + P \cos \alpha = 0 \quad V = P \cos \alpha$$

$$+\uparrow \sum F_y = 0: F - P \sin \alpha = 0 \quad F = P \sin \alpha$$

$$\text{Area} = ab / \sin \alpha$$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab) \sin^2 \alpha$$

$$\text{Shearing stress: } \tau = \frac{V}{\text{Area}} = (P/ab) \sin \alpha \cos \alpha$$

(3) F.S. for tension (normal stresses)

$$F_{SN} = \sigma_U / \sigma$$

(4) F.S. for shear:

$$F_{SS} = \tau_U / \tau$$

(c) OVERALL F.S.:

$$FS = \text{The smaller of } F_{SN} \text{ and } F_{SS}.$$

(CONTINUED)

PROBLEM 1.C5 CONTINUED

PROGRAM OUTPUTS

For Prob. 1.29:  $P = 6 \text{ kN}$   
 $a = 125 \text{ mm}$ ,  $b = 75 \text{ mm}$ ,  $\alpha = 70^\circ$ ,  $\sigma_u = 1.26 \text{ MPa}$ ,  $\tau_u = 1.50 \text{ MPa}$ ,

ALPHA	SIG(MPa)	TAU(MPa)	FSN	FSS	FS
5.0000	0.0049	0.0556	259.1782	26.9942	26.9942
10.0000	0.0193	0.1094	65.2905	13.7053	13.7053
15.0000	0.0429	0.1600	29.3899	9.3750	9.3750
20.0000	0.0749	0.2057	16.8301	7.2925	7.2925
25.0000	0.1143	0.2451	11.0229	6.1191	6.1191
30.0000	0.1600	0.2771	7.8750	5.4127	5.4127
35.0000	0.2106	0.3007	5.9842	4.9883	4.9883
40.0000	0.2644	0.3151	4.7649	4.7598	4.7598
45.0000	0.3200	0.3200	3.9375	4.6875	3.9375
50.0000	0.3756	0.3151	3.3549	4.7598	3.3549
55.0000	0.4294	0.3007	2.9340	4.9883	2.9340
60.0000	0.4800	0.2771	2.6250	5.4127	2.6250
65.0000	0.5257	0.2451	2.3968	6.1191	2.3968
70.0000	0.5651	0.2057	2.2296	7.2925	2.2296
75.0000	0.5971	0.1600	2.1101	9.3750	2.1101
80.0000	0.6207	0.1094	2.0300	13.7053	2.0300
85.0000	0.6351	0.0556	1.9838	26.9942	1.9838

▲ (c)

▲ (b)

For Prob. 1.32:  $P = 2400 \text{ lb}$   
 $a = 6 \text{ in.}$ ,  $b = 3 \text{ in.}$ ,  $\alpha = 40^\circ$ ,  $\sigma_u = 150 \text{ psi}$ ,  $\tau_u = 214 \text{ psi}$ .

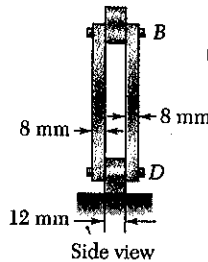
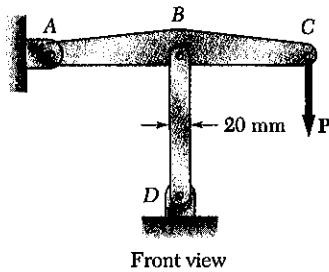
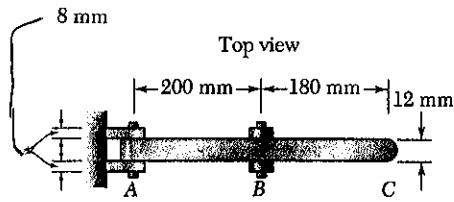
ALPHA	SIG(psi)	TAU(psi)	FSN	FSS	FS
5.0000	1.0128	11.5765	148.1018	18.4857	18.4857
10.0000	4.0205	22.8013	37.3089	9.3854	9.3854
15.0000	8.9316	33.3333	16.7942	6.4200	6.4200
20.0000	15.5970	42.8525	9.6172	4.9939	4.9939
25.0000	23.8142	51.0696	6.2988	4.1904	4.1904
30.0000	33.3333	57.7350	4.5000	3.7066	3.7066
35.0000	43.8653	62.6462	3.4196	3.4160	3.4160
40.0000	55.0901	65.6538	2.7228	3.2595	2.7228
45.0000	66.6667	66.6667	2.2500	3.2100	2.2500
50.0000	78.2432	65.6538	1.9171	3.2595	1.9171
55.0000	89.4680	62.6462	1.6766	3.4160	1.6766
60.0000	100.0000	57.7350	1.5000	3.7066	1.5000
65.0000	109.5192	51.0696	1.3696	4.1904	1.3696
70.0000	117.7363	42.8525	1.2740	4.9939	1.2740
75.0000	124.4017	33.3333	1.2058	6.4200	1.2058
80.0000	129.3128	22.8013	1.1600	9.3854	1.1600
85.0000	132.3205	11.5765	1.1336	18.4857	1.1336

▲ (b)

▲ (c)

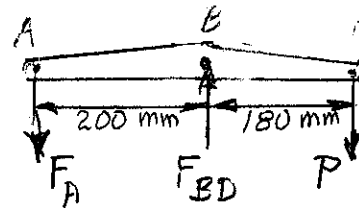
**PROBLEM 1.C6**

**1.C6** Member *ABC* is supported by a pin and bracket at *A* and by two links which are pin-connected to the member at *B* and to a fixed support at *D*. (a) Write a computer program to calculate the allowable load  $P_{all}$  for any given values of (1) the diameter  $d_1$  of the pin at *A*, (2) the common diameter  $d_2$  of the pins at *B* and *D*, (3) the ultimate normal stress  $\sigma_U$  in each of the two links, (4) the ultimate shearing stress  $\tau_U$  in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (b and c) Check your program by using the data of Probs. 1.49 and 1.50, respectively, and comparing the answers obtained for  $P_{all}$  with those given in the text. (d) Use your program to determine the allowable load  $P_{all}$ , as well as which of the stresses is critical, when  $d_1 = d_2 = 15$  mm,  $\sigma_U = 110$  MPa for aluminum links,  $\tau_U = 100$  MPa for steel pins, and *F.S.* = 3.2.



**SOLUTION**

(a) F.B. DIAGRAM OF ABC:



$$\begin{aligned} \sum M_A = 0: & P = \frac{200F_{BD}}{380} \\ \sum M_B = 0: & P = \frac{200}{180} F_A \end{aligned}$$

- (1) For given  $d_1$  of pin A:  $F_A = 2(\sigma_U / F.S.) (\pi d_1^2 / 4)$ ,  $P_1 = \frac{200}{180} F_A$
- (2) For given  $d_2$  of pins B and D:  $F_{BD} = 2(\sigma_U / F.S.) (\pi d_2^2 / 4)$ ,  $P_2 = \frac{200}{380} F_{BD}$
- (3) For ultimate stress in links BD:  $F_{BD} = 2(\sigma_U / F.S.) (0.02)(0.008)$ ,  $P_3 = \frac{200}{380} F_{BD}$
- (4) For ult. shearing stress in pins:  $P_4$  is the smaller of  $P_1$  and  $P_2$
- (5) For desired overall *F.S.*:  $P_5$  is the smaller of  $P_3$  and  $P_4$

If  $P_3 < P_4$ , stress is critical in links

If  $P_4 < P_3$  and  $P_1 < P_2$ , stress is critical in pin A

If  $P_4 < P_3$  and  $P_2 < P_1$ , stress is critical in pins B and D

PROGRAM OUTPUTS

(b) Prob. 1.49. DATA:  $d_1 = 8$  mm,  $d_2 = 12$  mm,  $\sigma_U = 250$  MPa,  $\tau_U = 100$  MPa, *F.S.* = 3.0  
 $P_{all} = 3.72$  kN. Stress in pin A is critical  $\blacktriangleleft$

(c) Prob. 1.50. DATA:  $d_1 = 10$  mm,  $d_2 = 12$  mm,  $\sigma_U = 250$  MPa,  $\tau_U = 100$  MPa, *F.S.* = 3.0  
 $P_{all} = 5.97$  kN. Stress in pins B and D is critical  $\blacktriangleleft$

(d) DATA:  $d_1 = d_2 = 15$  mm,  $\sigma_U = 110$  MPa,  $\tau_U = 100$  MPa, *F.S.* = 3.2  
 $P_{all} = 5.79$  kN. Stress in links is critical  $\blacktriangleleft$

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# CHAPTER 2

### PROBLEM 2.1

2.1 A steel rod is 2.2 m long and must not stretch more than 1.2 mm when a 8.5 kN load is applied to it. Knowing that  $E = 200$  GPa, determine (a) the smallest diameter rod which should be used, (b) the corresponding normal stress caused by the load.

#### SOLUTION

$$(a) \quad \delta = \frac{PL}{AE} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(8.5 \times 10^3)(2.2)}{(200 \times 10^9)(1.2 \times 10^{-3})} = 77.92 \times 10^{-6} \text{ m}^2$$

$$A = \frac{\pi}{4} d^2 \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(77.92 \times 10^{-6})}{\pi}} = 9.96 \times 10^{-3} \text{ m} \\ = 9.96 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{8.5 \times 10^3}{77.92 \times 10^{-6}} = 109.1 \times 10^6 \text{ Pa} = 109.1 \text{ MPa} \quad \blacktriangleleft$$

### PROBLEM 2.2

2.2 A 4.8-ft-long steel wire of  $\frac{1}{4}$ -in. diameter steel wire is subjected to a 750-lb tensile load. Knowing that  $E = 29 \times 10^6$  psi, determine (a) the elongation of the wire, (b) the corresponding normal stress.

#### SOLUTION

$$(a) \quad L = 4.8 \text{ ft} = 57.6 \text{ in.} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{1}{4}\right)^2 = 49.087 \times 10^{-3} \text{ in}^2$$

$$\delta = \frac{PL}{AE} = \frac{(750)(57.6)}{(49.087 \times 10^{-3})(29 \times 10^6)} = 30.3 \times 10^{-3} \text{ in} = 0.0303 \text{ in} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{750}{49.087 \times 10^{-3}} = 15.28 \times 10^3 \text{ psi} = 15.28 \text{ ksi} \quad \blacktriangleleft$$

### PROBLEM 2.3

2.3 Two gage marks are placed exactly 10 inches apart on a  $\frac{1}{2}$ -in.-diameter aluminum rod with  $E = 10.1 \times 10^6$  psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (a) the stress in the rod, (b) the factor of safety.

#### SOLUTION

$$(a) \quad \delta = 10.009 - 10.000 = 0.009 \text{ in.}$$

$$\frac{\delta}{L} = \frac{\sigma}{E} \quad \therefore \quad \sigma = \frac{E\delta}{L} = \frac{(10.1 \times 10^6)(0.009)}{10} = 9.09 \times 10^3 \text{ psi} \\ = 9.09 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad \text{F.S.} = \frac{\sigma_u}{\sigma} = \frac{16}{9.09} = 1.760 \quad \blacktriangleleft$$

### PROBLEM 2.4

2.4 A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that  $E = 105$  GPa and that the maximum allowable normal stress is 180 MPa, determine (a) the smallest diameter that can be selected for the rod, (b) the corresponding maximum length of the rod.

#### SOLUTION

$$(a) \quad \sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{4 \times 10^3}{180 \times 10^6} = 22.222 \times 10^{-6} \text{ m}^2$$

$$A = \frac{\pi}{4} d^2 \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(22.222 \times 10^{-6})}{\pi}} = 5.32 \times 10^{-3} \text{ m} \\ = 5.32 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \delta = \frac{PL}{AE} \quad \therefore \quad L = \frac{AES}{P} = \frac{(22.222 \times 10^{-6})(105 \times 10^9)(3 \times 10^{-3})}{4 \times 10^3} \\ = 1.750 \text{ m} \quad \blacktriangleleft$$



## PROBLEM 2.5

2.5 A 9-m length of 6-mm-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 18 mm when a tensile force  $P$  is applied. Knowing that  $E = 200$  GPa, determine (a) the magnitude of the force  $P$ , (b) the corresponding normal stress in the wire.

## SOLUTION

$$(a) \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.006)^2 = 28.274 \times 10^{-6} \text{ m}^2$$

$$\delta = \frac{PL}{AE} \quad \therefore \quad P = \frac{AES}{L} = \frac{(28.274 \times 10^{-6})(200 \times 10^9)(18 \times 10^{-3})}{9}$$

$$= 11.31 \times 10^3 \text{ N} = 11.31 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{11.31 \times 10^3}{28.274 \times 10^{-6}} = 400 \times 10^6 \text{ Pa} = 400 \text{ MPa} \quad \blacktriangleleft$$

## PROBLEM 2.6

2.6 A 4.5-ft. aluminum pipe should not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that  $E = 10.1 \times 10^6$  psi and that the allowable tensile strength is 14 ksi., determine (a) the maximum allowable length of the pipe, (b) the required area of the pipe if the tensile load is 127.5 kips.

## SOLUTION

$$(a) \quad \delta = \frac{PL}{AE} \quad \therefore \quad L = \frac{EAS}{P} = \frac{ES}{\sigma} = \frac{(10.1 \times 10^6)(0.05)}{14 \times 10^3} = 36.1 \text{ in} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{127.5 \times 10^3}{14 \times 10^3} = 9.11 \text{ in}^2 \quad \blacktriangleleft$$

## PROBLEM 2.7

2.7 A nylon thread is subjected to a 8.5-N tension force. Knowing that  $E = 3.3$  GPa and that the length of the thread increases by 1.1 %, determine (a) the diameter of the thread, (b) the stress in the thread.

## SOLUTION

$$(a) \quad \frac{\delta}{L} = \frac{1.1}{100} \quad \therefore \quad \frac{L}{\delta} = 90.909$$

$$\delta = \frac{PL}{AE} \quad \therefore \quad A = \frac{PL}{ES} = \frac{(8.5)(90.909)}{3.3 \times 10^9} = 234.16 \times 10^{-9} \text{ m}^2$$

$$A = \frac{\pi}{4} d^2 \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = 0.546 \times 10^{-3} \text{ m} = 0.546 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} = \frac{8.5}{234.16 \times 10^{-9}} = 36.3 \times 10^6 \text{ Pa} = 36.3 \text{ MPa} \quad \blacktriangleleft$$

## PROBLEM 2.8

2.8 A cast-iron tube is used to support a compressive load. Knowing that  $E = 10 \times 10^6$  psi and that the maximum allowable change in length is 0.025 percent, determine (a) the maximum normal stress in the tube, (b) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

## SOLUTION

$$(a) \quad \frac{\delta}{L} = \frac{0.025}{100} = 0.00025$$

$$\sigma = \frac{ES}{L} = (10 \times 10^6)(0.00025) = 2.5 \times 10^3 \text{ psi} = 2.5 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{1600}{2.5 \times 10^3} = 0.640 \text{ in}^2$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$d_i^2 = d_o^2 - \frac{4A}{\pi} = 2.0^2 - \frac{(4)(0.64)}{\pi} = 3.1851 \text{ in}^2 \quad \therefore \quad d_i = 1.7847 \text{ in}$$

$$t = \frac{1}{2} (d_o - d_i) = \frac{1}{2} (2.0 - 1.7847) = 0.1077 \text{ in.} \quad \blacktriangleleft$$

★ PROBLEM 2.9

SOLUTION

2.9 A block of 10-in. length and  $1.8 \times 1.6$  in. cross section is to support a centric compressive load  $P$ . The material to be used is a bronze for which  $E = 14 \times 10^6$  psi. Determine the largest load which can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12 percent of its original length.

Considering allowable stress  $\sigma = 18 \text{ ksi} = 18 \times 10^3 \text{ psi}$

$$A = (1.8)(1.6) = 2.88 \text{ in}^2 \quad \sigma = \frac{P}{A}$$

$$P = \sigma A = (18 \times 10^3)(2.88) = 51.8 \times 10^3 \text{ lb}$$

Considering allowable deformation  $\frac{\delta}{L} = \frac{0.12}{100} = 0.0012$

$$\delta = \frac{PL}{AE} \therefore P = AE \frac{\delta}{L} = (2.88)(14 \times 10^6)(0.0012) = 48.4 \times 10^3 \text{ lb}$$

Smaller value governs  $P = 48.4 \times 10^3 \text{ lb} = 48.4 \text{ kips}$  ◀

PROBLEM 2.10

SOLUTION

2.10 A 9-kN tensile load will be applied to a 50-m length of steel wire with  $E = 200$  GPa. Determine the smallest diameter wire which can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in the length of the wire should be at most 25 mm.

Considering allowable stress  $\sigma = 150 \times 10^6 \text{ Pa}$

$$\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{9 \times 10^3}{150 \times 10^6} = 60 \times 10^{-6} \text{ m}^2$$

Considering allowable elongation  $\delta = 25 \times 10^{-3} \text{ m}$

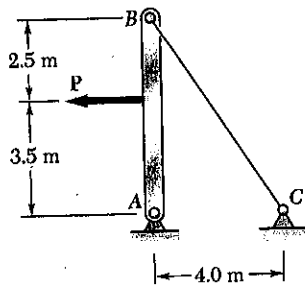
$$\delta = \frac{PL}{AE} \therefore A = \frac{PL}{E\delta} = \frac{(9 \times 10^3)(50)}{(200 \times 10^9)(25 \times 10^{-3})} = 90 \times 10^{-6} \text{ m}^2$$

Larger area governs  $A = 90 \times 10^{-6} \text{ m}^2$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(90 \times 10^{-6})}{\pi}} = 10.70 \times 10^{-3} \text{ m} \\ = 10.70 \text{ mm} \quad \blacktriangleleft$$

PROBLEM 2.11

2.11 The 4-mm-diameter cable BC is made of a steel with  $E = 200$  GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load  $P$  that can be applied as shown.



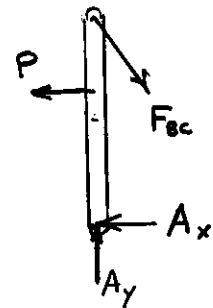
SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body

$$\sum M_A = 0 \quad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$$

$$P = 0.9509 F_{BC}$$



Considering allowable stress  $\sigma = 190 \times 10^6 \text{ Pa}$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

Considering allowable elongation  $\delta = 6 \times 10^{-3} \text{ m}$

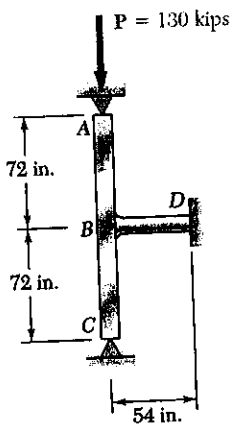
$$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AES}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs  $F_{BC} = 2.091 \times 10^3 \text{ N}$

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N} = 1.988 \text{ kN}$$

PROBLEM 2.12

2.12 Rod BD is made of steel ( $E = 29 \times 10^6$  psi) and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is  $0.02P$ . If the stress must not exceed 18 ksi and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest diameter rod that can be used for member BD.



SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6 \times 10^3 \text{ lb}$$

Considering stress  $\sigma = 18 \text{ ksi} = 18 \times 10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \therefore A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

Considering deformation  $\delta = (0.001)(144) = 0.144 \text{ in}$ .

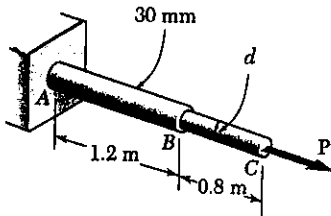
$$\delta = \frac{F_{BD} L_{BD}}{AE} \therefore A = \frac{F_{BD} L_{BD}}{E\delta} = \frac{(2.6 \times 10^3)(54)}{(29 \times 10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs  $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4} d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}} = 0.429 \text{ in}$$

**PROBLEM 2.13**

2.13 A single axial load of magnitude  $P = 58 \text{ kN}$  is applied at end  $C$  of the brass rod  $ABC$ . Knowing that  $E = 105 \text{ GPa}$ , determine the diameter  $d$  of portion  $BC$  for which the deflection of point  $C$  will be  $3 \text{ mm}$ .



**SOLUTION**

$$\delta_c = \sum \frac{P_i L_i}{A_i E} = \frac{P}{E} \left\{ \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right\}$$

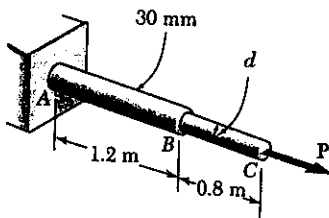
$$\frac{L_{BC}}{A_{BC}} = \frac{E \delta_c}{P} - \frac{L_{AB}}{A_{AB}} = \frac{(105 \times 10^9)(3 \times 10^{-3})}{58 \times 10^3} - \frac{1.2}{\frac{\pi}{4}(0.030)^2} = 3.7334 \times 10^3 \text{ m}^{-1}$$

$$A_{BC} = \frac{L_{BC}}{3.7334 \times 10^3} = \frac{0.8}{3.7334 \times 10^3} = 214.28 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 \therefore d_{BC} = \sqrt{\frac{4A_{BC}}{\pi}} = \sqrt{\frac{(4)(214.28 \times 10^{-6})}{\pi}} = 16.52 \times 10^{-3} \text{ m} = 16.52 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 2.14**

2.14 Both portions of the rod  $ABC$  are made of an aluminum for which  $E = 73 \text{ GPa}$ . Knowing that the diameter of portion  $BC$  is  $d = 20 \text{ mm}$ , determine the largest force  $P$  that can be applied if  $\sigma_{all} = 160 \text{ MPa}$  and the corresponding deflection at point  $C$  is not to exceed  $4 \text{ mm}$ .



**SOLUTION**

$$A_{AB} = \frac{\pi}{4} (0.030)^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

Considering allowable stress  $\sigma = 160 \times 10^6 \text{ Pa}$

$$\sigma = \frac{P}{A} \therefore P = A\sigma$$

Portion AB  $P = (706.86 \times 10^{-6})(160 \times 10^6) = 113.1 \times 10^3 \text{ N}$

Portion BC  $P = (314.16 \times 10^{-6})(160 \times 10^6) = 50.3 \times 10^3 \text{ N}$

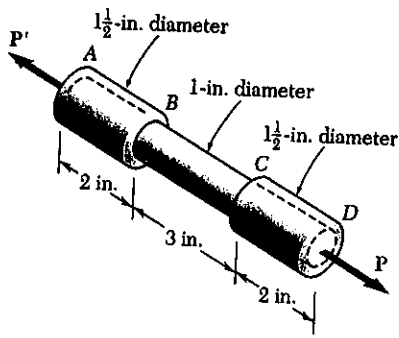
Considering allowable deflection  $\delta_c = 4 \times 10^{-3} \text{ m}$

$$\delta_c = \sum \frac{P L_i}{A E} = \frac{P}{E} \left( \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right)$$

$$P = E \delta_c \left( \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right)^{-1} = (73 \times 10^9)(4 \times 10^{-3}) \left( \frac{1.2}{706.86 \times 10^{-6}} + \frac{0.8}{314.16 \times 10^{-6}} \right)^{-1} = 68.8 \times 10^3 \text{ N}$$

Smallest value for  $P$  governs  $P = 50.3 \times 10^3 \text{ N} = 50.3 \text{ kN} \quad \blacktriangleleft$

**PROBLEM 2.15**



2.15 The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that  $E = 29 \times 10^6$  psi, determine (a) the load  $P$  so that the total deformation is 0.002 in., (b) the corresponding deformation of the central portion  $BC$ .

**SOLUTION**

$$(a) \quad \delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$$

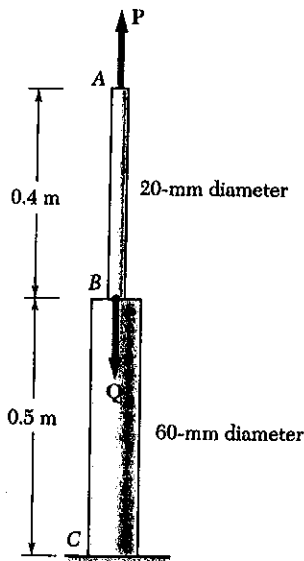
$$P = ES \left( \sum \frac{L_i}{A_i} \right)^{-1} \quad A_i = \frac{\pi}{4} d_i^2$$

	$L, \text{in.}$	$d, \text{in.}$	$A, \text{in}^2$	$L/A, \text{in}^{-1}$
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$$P = (29 \times 10^6)(0.002)(6.083)^{-1} = 9.535 \times 10^3 \text{ lb.} = 9.53 \text{ kips}$$

$$(b) \quad \delta_{BC} = \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E} \frac{L_{BC}}{A_{BC}} = \frac{9.535 \times 10^3}{29 \times 10^6} (3.8197) = 1.254 \times 10^{-3} \text{ in.}$$

**PROBLEM 2.16**



2.16 Both portions of the rod  $ABC$  are made of an aluminum for which  $E = 70$  GPa. Knowing that the magnitude of  $P$  is 4 kN, determine (a) the value of  $Q$  so that the deflection at  $A$  is zero, (b) the corresponding deflection of  $B$ .

**SOLUTION**

$$(a) \quad A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

Force in member  $AB$  is  $P$  tension

$$\text{Elongation } \delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})}$$

$$= 72.756 \times 10^{-6} \text{ m}$$

Force in member  $BC$  is  $Q - P$  compression

$$\text{Shortening } \delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})}$$

$$= 2.5263 \times 10^{-9} (Q-P)$$

For zero deflection at  $A$   $\delta_{BC} = \delta_{AB}$

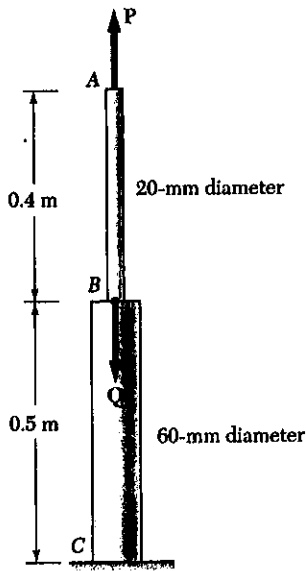
$$2.5263 \times 10^{-9} (Q-P) = 72.756 \times 10^{-6} \quad \therefore Q-P = 28.8 \times 10^3 \text{ N}$$

$$Q = 28.8 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3 \text{ N} = 32.8 \text{ kN}$$

$$(b) \quad \delta_{AB} = \delta_{BC} = \delta_B = 72.756 \times 10^{-6} \text{ m} = 0.0728 \text{ mm}$$

★ PROBLEM 2.17

2.17 The rod  $ABC$  is made of an aluminum for which  $E = 70$  GPa. Knowing that  $P = 6$  kN and  $Q = 42$  kN, determine the deflection of (a) point  $A$ , (b) point  $B$ .



SOLUTION

$$(a) \quad A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (0.020)^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (0.060)^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$P_{AB} = P = 6 \times 10^3 \text{ N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3 \text{ N}$$

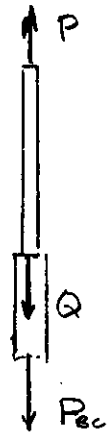
$$L_{AB} = 0.4 \text{ m} \quad L_{BC} = 0.5 \text{ m}$$

$$\begin{aligned} \delta_{AB} &= \frac{P_{AB} L_{AB}}{A_{AB} E} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} \\ &= 109.135 \times 10^{-6} \text{ m} \end{aligned}$$

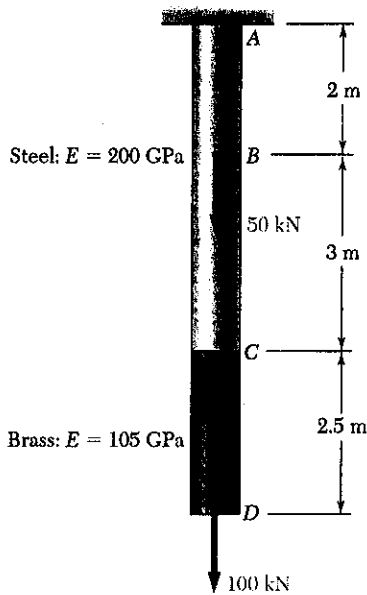
$$\begin{aligned} \delta_{BC} &= \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} \\ &= -90.947 \times 10^{-6} \text{ m} \end{aligned}$$

$$\begin{aligned} \delta_A &= \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6} \text{ m} = 18.19 \times 10^{-6} \text{ m} \\ &= 0.01819 \text{ mm} \end{aligned}$$

$$(b) \quad \delta_B = \delta_{BC} = -90.9 \times 10^{-6} \text{ m} = -0.0909 \text{ mm}$$



**PROBLEM 2.18**



2.18 The 36-mm-diameter steel rod  $ABC$  and a brass rod  $CD$  of the same diameter are joined at point  $C$  to form the 7.5-m rod  $ABCD$ . For the loading shown, and neglecting the weight of the rod, determine the deflection of (a) point  $C$ , (b) point  $D$ .

**SOLUTION**

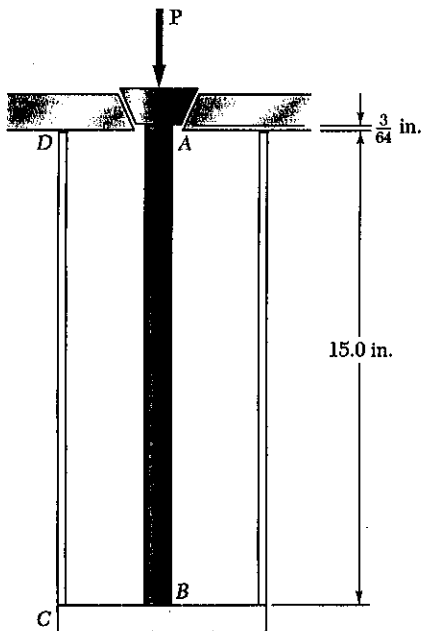
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.036)^2 = 1.01787 \times 10^{-3} \text{ m}^2$$

Portion	$P_i$	$L_i$	$E_i$	$P_i L_i / A E_i$
AB	150 kN	2 m	200 GPa	$1.474 \times 10^{-3} \text{ m}$
BC	100 kN	3 m	200 GPa	$1.474 \times 10^{-3} \text{ m}$
CD	100 kN	2.5 m	105 GPa	$2.339 \times 10^{-3} \text{ m}$

$$(a) \quad \delta_c = \delta_{AB} + \delta_{BC} = 1.474 \times 10^{-3} + 1.474 \times 10^{-3} = 2.948 \times 10^{-3} \text{ m} = 2.95 \text{ mm} \blacktriangleleft$$

$$(b) \quad \delta_D = \delta_c + \delta_{CD} = 2.948 \times 10^{-3} + 2.339 \times 10^{-3} = 5.287 \times 10^{-3} \text{ m} = 5.29 \text{ mm} \blacktriangleleft$$

**PROBLEM 2.19**



2.19 The brass tube  $AB$  ( $E = 15 \times 10^6$  psi) has a cross-sectional area of  $0.22 \text{ in}^2$  and is fitted with a plug at  $A$ . The tube is attached at  $B$  to a rigid plate which is itself attached at  $C$  to the bottom of an aluminum cylinder ( $E = 10.4 \times 10^6$  psi) with a cross-sectional area of  $0.40 \text{ in}^2$ . The cylinder is then hung from a support at  $D$ . In order to close the cylinder, the plug must move down through  $\frac{3}{64} \text{ in}$ . Determine the force  $P$  that must be applied to the cylinder.

Shortening of brass tube  $AB$

$$L_{AB} = 15 + \frac{3}{64} = 15.047 \text{ in} \quad A_{AB} = 0.22 \text{ in}^2$$

$$E_{AB} = 15 \times 10^6 \text{ psi}$$

$$\delta_{AB} = \frac{P L_{AB}}{E_{AB} A_{AB}} = \frac{P (15.047)}{(15 \times 10^6)(0.22)} = 4.5597 \times 10^{-6} P$$

Lengthening of aluminum cylinder  $CD$

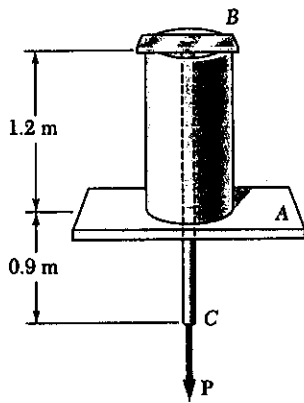
$$L_{CD} = 15 \text{ in}, \quad A_{CD} = 0.40 \text{ in}^2, \quad E_{CD} = 10.4 \times 10^6 \text{ psi}$$

$$\delta_{CD} = \frac{P L_{CD}}{E_{CD} A_{CD}} = \frac{P (15)}{(10.4 \times 10^6)(0.40)} = 3.6058 \times 10^{-6} P$$

$$\text{Total deflection} \quad \delta_A = \delta_{AB} + \delta_{CD}$$

$$\frac{3}{64} = (4.5597 \times 10^{-6} + 3.6058 \times 10^{-6}) P \quad \therefore P = 5.74 \times 10^3 \text{ lb.} = 5.74 \text{ kips} \blacktriangleleft$$

**PROBLEM 2.20**



2.20 A 1.2-m section of aluminum pipe of cross-sectional area  $1100 \text{ mm}^2$  rests on a fixed support at A. The 15-mm-diameter steel rod BC hangs from a rigid bar that rests on the top of the pipe at B. Knowing that the modulus of elasticity is 200 GPa for steel and 72 GPa for aluminum, determine the deflection of point C when a 60 kN force is applied at C.

**SOLUTION**

Rod BC  $L_{BC} = 2.1 \text{ m}, E_{BC} = 200 \times 10^9 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.015)^2 = 176.715 \times 10^{-6} \text{ m}^2$$

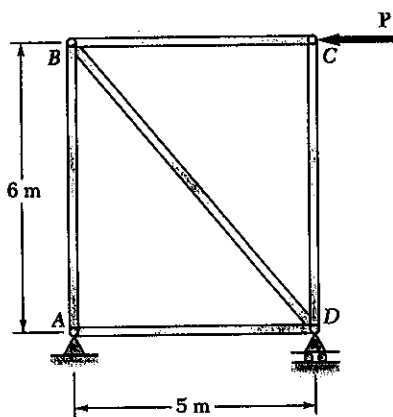
$$\begin{aligned} \delta_{C/B} &= \frac{PL_{BC}}{E_{BC}A_{BC}} = \frac{(60 \times 10^3)(2.1)}{(200 \times 10^9)(176.715 \times 10^{-6})} \\ &= 3.565 \times 10^{-3} \text{ m} \end{aligned}$$

Pipe AB:  $L_{AB} = 1.2 \text{ m}, E_{AB} = 72 \times 10^9 \text{ Pa}, A_{AB} = 1100 \text{ mm}^2 = 1100 \times 10^{-6} \text{ m}^2$

$$\delta_{B/A} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{(60 \times 10^3)(1.2)}{(72 \times 10^9)(1100 \times 10^{-6})} = 909.1 \times 10^{-6} \text{ m}^2$$

$$\delta_C = \delta_{B/A} + \delta_{C/B} = 909.1 \times 10^{-6} + 3.565 \times 10^{-3} = 4.47 \times 10^{-3} \text{ m} = 4.47 \text{ mm} \blacktriangleleft$$

**PROBLEM 2.21**



2.21 The steel frame ( $E = 200 \text{ GPa}$ ) shown has a diagonal brace BD with an area of  $1920 \text{ mm}^2$ . Determine the largest allowable load P if the change in length of member BD is not to exceed 1.6 mm.

**SOLUTION**

$$\delta_{BC} = 1.6 \times 10^{-3} \text{ m}, A_{BD} = 1920 \text{ mm}^2 = 1920 \times 10^{-6} \text{ m}^2$$

$$L_{BC} = \sqrt{5^2 + 6^2} = 7.810 \text{ m}, E_{BC} = 200 \times 10^9 \text{ Pa}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}}$$

$$\begin{aligned} F_{BC} &= \frac{E_{BC} A_{BC} \delta_{BC}}{L_{BC}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81} \\ &= 78.67 \times 10^3 \text{ N} \end{aligned}$$

Use joint B as a free body:  $\sum F_x = 0$

$$\frac{5}{7.810} F_{BC} - P = 0$$

$$P = \frac{5}{7.810} F_{BC} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

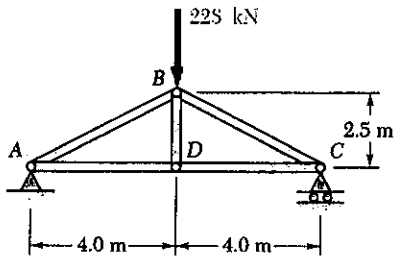
$$= 50.4 \times 10^3 \text{ N} = 50.4 \text{ kN} \blacktriangleleft$$





**PROBLEM 2.22**

2.22 For the steel truss ( $E = 200 \text{ GPa}$ ) and loading shown, determine the deformations of members  $AB$  and  $AD$ , knowing that their cross-sectional areas are  $2400 \text{ mm}^2$  and  $1800 \text{ mm}^2$ , respectively.



**SOLUTION**

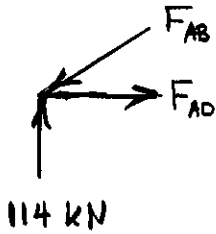
Statics: Reactions are 114 kN upward at A & C.

Member  $BD$  is a zero force member

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint A as a free body:  $\uparrow \sum F_y = 0 \quad 114 - \frac{2.5}{4.717} F_{AB} = 0$

$$F_{AB} = 215.10 \text{ kN}$$



$$\pm \sum F_x = 0 \quad F_{AD} - \frac{4}{4.717} F_{AB} = 0$$

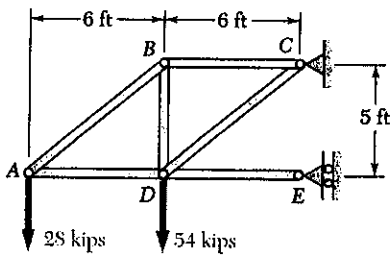
$$F_{AD} = \frac{(4)(215.10)}{4.717} = 182.4 \text{ kN}$$

Member  $AB$ : 
$$S_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})} = 2.11 \times 10^{-3} \text{ m} = 2.11 \text{ mm}$$

$$S_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})} = 2.03 \times 10^{-3} \text{ m} = 2.03 \text{ mm}$$

**PROBLEM 2.23**

2.23 Members  $AB$  and  $BC$  are made of steel ( $E = 29 \times 10^6 \text{ psi}$ ) with cross-sectional areas of  $0.80 \text{ in}^2$  and  $0.64 \text{ in}^2$ , respectively. For the loading shown, determine the elongation of (a) member  $AB$ , (b) member  $BC$ .



**SOLUTION**

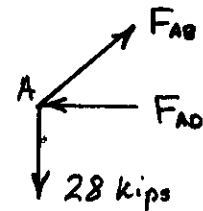
(a) 
$$L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in}$$

Use joint A as a free body

$$\uparrow \sum F_y = 0 \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{ lb}$$

$$S_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)} = 0.1767 \text{ in}$$



(b) Use joint B as a free body

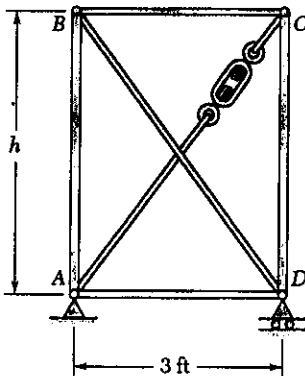
$$\pm \sum F_x = 0 \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{ lb}$$

$$S_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)} = 0.1304 \text{ in}$$

**PROBLEM 2.24**

2.24 Members  $AB$  and  $CD$  are  $1\frac{1}{8}$ -in.-diameter steel rods, and members  $BC$  and  $AD$  are  $\frac{7}{8}$ -in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member  $AC$  is put in tension. Knowing that  $E = 29 \times 10^6$  psi and  $h = 4$  ft, determine the largest allowable tension in  $AC$  so that the deformations in members  $AB$  and  $CD$  do not exceed 0.04 in.



**SOLUTION**

$$\delta_{AB} = \delta_{CD} = 0.04 \text{ in} \quad h = 4 \text{ ft} = 48 \text{ in.} = L_{CD}$$

$$A_{CD} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

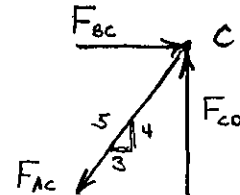
$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(0.99402)(0.04)}{48} = 24.022 \times 10^3 \text{ lb.}$$

Use joint  $C$  as a free body

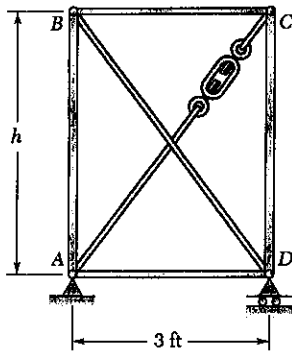
$$+\uparrow \sum F_y = 0 : F_{CD} - \frac{4}{5} F_{AC} = 0 \quad \therefore F_{AC} = \frac{5}{4} F_{CD}$$

$$F_{AC} = \frac{5}{4} (24.022 \times 10^3) = 30.0 \times 10^3 \text{ lb.}$$

$$30.0 \text{ kips}$$



PROBLEM 2.25



2.24 Members  $AB$  and  $CD$  are  $1\frac{1}{8}$ -in.-diameter steel rods, and members  $BC$  and  $AD$  are  $\frac{7}{8}$ -in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member  $AC$  is put in tension. Knowing that  $E = 29 \times 10^6$  psi and  $h = 4$  ft, determine the largest allowable tension in  $AC$  so that the deformations in members  $AB$  and  $CD$  do not exceed 0.04 in.

2.25 For the structure in Prob. of 2.24, determine (a) the distance  $h$  so that the deformations in members  $AB$ ,  $BC$ ,  $CD$  and  $AD$  are all equal to 0.04 in., (b) the corresponding tension in member  $AC$ .

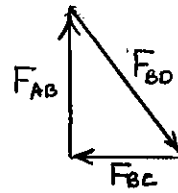
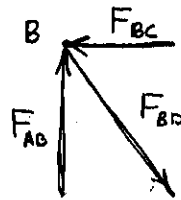
SOLUTION

(a) Statics: Use joint B as a free body

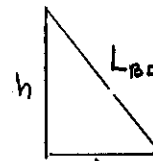
From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BD}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$



Force Triangle



Geometry

For equal deformations

$$\delta_{AB} = \delta_{BC} \therefore \frac{F_{AB} h}{EA_{AB}} = \frac{F_{BC} b}{EA_{BC}}$$

$$F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for  $F_{AB}$

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC}$$

$$\frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{9/8}{7/8} = \frac{9}{7}$$

$$b = 3 \text{ ft} = 36 \text{ in.}$$

$$h = \frac{9}{7} b = \frac{9}{7} (3) = 3.86 \text{ ft} = 46.3 \text{ in.}$$

(b) Setting  $\delta_{AB} = \delta_{BC} = 0.04$  in.

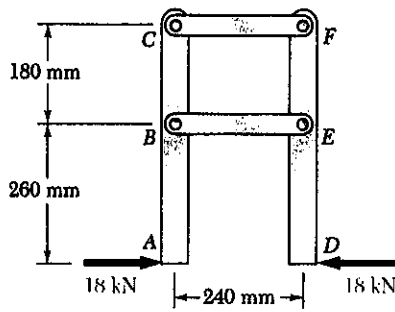
$$\delta_{BC} = \frac{F_{BC} b}{EA_{BC}} \therefore F_{BC} = \frac{EA_{BC} \delta_{BC}}{b} = \frac{(29 \times 10^6) \frac{\pi}{4} (\frac{7}{8})^2 (0.04)}{36} = 19.376 \times 10^3 \text{ lb.}$$

$$F_{AB} = \frac{h}{b} F_{BC} = \frac{9}{7} (19.376 \times 10^3) = 24.912 \times 10^3 \text{ lb}$$

From the force triangle

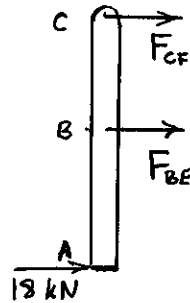
$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 31.6 \times 10^3 \text{ lb.}$$

**PROBLEM 2.26**



2.26 Members  $ABC$  and  $DEF$  are joined with steel links ( $E = 200$  GPa). Each of the links is made of a pair of  $25 \times 35$ -mm plates. Determine the change in length of (a) member  $BE$ , (b) member  $CF$ .

**SOLUTION**



Use member  $ABC$  as a free body

$$\sum M_B = 0$$

$$(0.260)(18 \times 10^3) - (0.180)F_{CF} = 0$$

$$F_{CF} = \frac{(0.260)(18 \times 10^3)}{0.180} = 26 \times 10^3 \text{ N}$$

$$\sum M_C = 0 \quad (0.440)(18 \times 10^3) + (0.180)F_{BE} = 0$$

$$F_{BE} = -\frac{(0.440)(18 \times 10^3)}{0.180} = -44 \times 10^3 \text{ N}$$

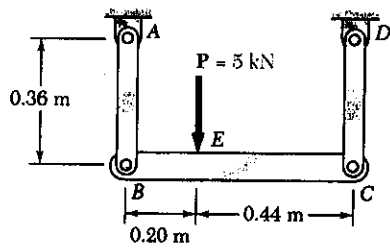
Area for link made of two plates

$$A = (2)(0.025)(0.035) = 1.75 \times 10^{-3} \text{ m}^2$$

$$(a) \quad \delta_{BE} = \frac{F_{BE} L_{BE}}{EA} = \frac{(-44 \times 10^3)(0.240)}{(200 \times 10^9)(1.75 \times 10^{-3})} = -30.2 \times 10^{-6} \text{ m} = -0.0302 \text{ mm} \quad \blacktriangleleft$$

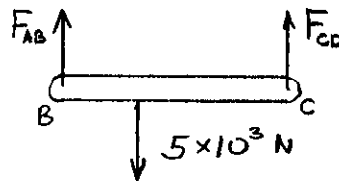
$$(b) \quad \delta_{CF} = \frac{F_{CF} L_{CF}}{EA} = \frac{(26 \times 10^3)(0.240)}{(200 \times 10^9)(1.75 \times 10^{-3})} = 17.83 \times 10^{-6} \text{ m} = 0.01783 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 2.27**



2.27 Each of the links  $AB$  and  $CD$  is made of aluminum ( $E = 75 \text{ GPa}$ ) and has a cross-sectional area of  $125 \text{ mm}^2$ . Knowing that they support the rigid member  $BC$ , determine the deflection of point  $E$ .

**SOLUTION**



Use member  $BC$  as a free body

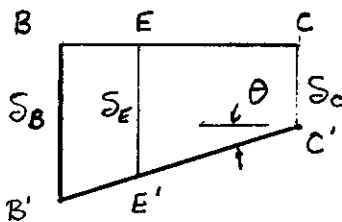
$$\sum M_C = 0 \quad -(0.64) F_{AB} + (0.44)(5 \times 10^3) = 0 \quad F_{AB} = 3.4375 \times 10^3 \text{ N}$$

$$\sum M_B = 0 \quad (0.64) F_{CD} - (0.20)(5 \times 10^3) = 0 \quad F_{CD} = 1.5625 \times 10^3 \text{ N}$$

For links  $AB$  and  $CD$   $A = 125 \text{ mm}^2 = 125 \times 10^{-6} \text{ m}^2$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA} = \frac{(3.4375 \times 10^3)(0.36)}{(75 \times 10^9)(125 \times 10^{-6})} = 132.00 \times 10^{-6} \text{ m} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{EA} = \frac{(1.5625 \times 10^3)(0.36)}{(75 \times 10^9)(125 \times 10^{-6})} = 60.00 \times 10^{-6} \text{ m} = \delta_C$$

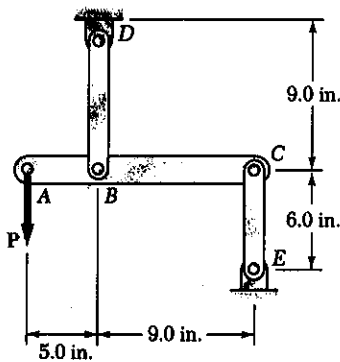


Deformation diagram

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{EC}} = \frac{72.00 \times 10^{-6}}{0.64} = 112.5 \times 10^{-6} \text{ rad}$$

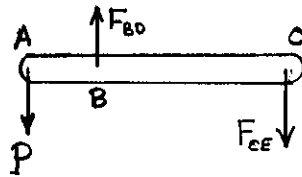
$$\begin{aligned} \delta_E &= \delta_C + l_{EC} \theta \\ &= 60.00 \times 10^{-6} + (0.44)(112.5 \times 10^{-6}) \\ &= 109.5 \times 10^{-6} \text{ m} = 0.1095 \text{ mm} \end{aligned}$$

★ PROBLEM 2.28



2.28 Link  $BD$  is made of brass ( $E = 15 \times 10^6$  psi) and has a cross-sectional area of  $0.40 \text{ in}^2$ . Link  $CE$  is made of aluminum ( $E = 10.4 \times 10^6$  psi) and has a cross-sectional area of  $0.50 \text{ in}^2$ . Determine the maximum force  $P$  that can be applied vertically at point  $A$  if the deflection of  $A$  is not to exceed  $0.014 \text{ in}$ .

SOLUTION



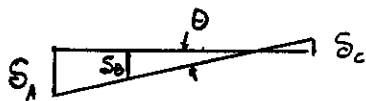
Use member  $ABC$  as a free body.

$$\sum M_C = 0, \quad 14P - 9F_{BD} = 0, \quad F_{BD} = 1.5556P$$

$$\sum M_B = 0, \quad 5P - 9F_{CE} = 0, \quad F_{CE} = 0.5556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}} = \frac{(1.5556P)(9.0)}{(15 \times 10^6)(0.40)} = 2.3333 \times 10^{-6} P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE} L_{CE}}{E_{CE} A_{CE}} = \frac{(0.5556P)(6.0)}{(10.4 \times 10^6)(0.50)} = 0.6410 \times 10^{-6} P \uparrow$$



Deformation Diagram

From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{l_{BC}} = \frac{2.9743 \times 10^{-6} P}{9} = 0.3305 \times 10^{-6} P$$

$$\begin{aligned} \delta_A &= \delta_B + l_{AB} \theta \\ &= 2.3333 \times 10^{-6} P + (5)(0.3305 \times 10^{-6}) P \\ &= 3.9858 \times 10^{-6} P \end{aligned}$$

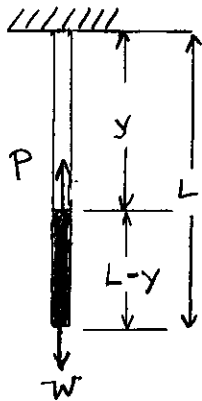
Apply displacement limit  $\delta_A = 0.014 \text{ in} = 3.9858 \times 10^{-6} P$

$$P = \frac{0.014}{3.9858 \times 10^{-6}} = 3.51 \times 10^3 \text{ lb} = 3.51 \text{ kips}$$

**PROBLEM 2.29**

2.29 A homogeneous cable of length  $L$  and uniform cross section is suspended from one end. (a) Denoting by  $\rho$  the density (mass per unit volume) of the cable and by  $E$  its modulus of elasticity, determine the elongation of the cable due to its own weight. (b) Assuming now the cable to be horizontal, determine the force that should be applied to each end of the cable to obtain the same elongation as in part a.

**SOLUTION**



(a) For element at point identified by coordinate  $y$

$$P = \text{weight of portion below the point} \\ = \rho g A (L - y)$$

$$dS = \frac{P dy}{EA} = \frac{\rho g A (L - y) dy}{EA} = \frac{\rho g (L - y)}{E} dy$$

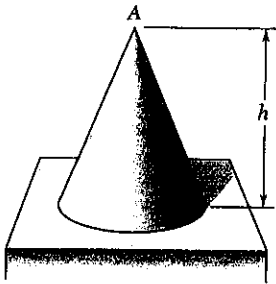
$$S = \int_0^L \frac{\rho g (L - y)}{E} dy = \frac{\rho g}{E} \left( Ly - \frac{1}{2} y^2 \right) \Big|_0^L \\ = \frac{\rho g}{E} \left( L^2 - \frac{L^2}{2} \right) = \frac{1}{2} \frac{\rho g L^2}{E}$$

(b) For  $S = \frac{PL}{EA}$

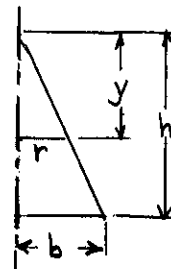
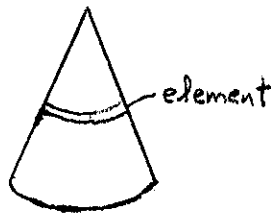
$$P = \frac{EAS}{L} = \frac{EA}{L} \frac{\rho g L^2}{2E} = \frac{1}{2} \rho g L = \frac{1}{2} W$$

**PROBLEM 2.30**

2.30 Determine the deflection of the apex  $A$  of a homogeneous circular cone of height  $h$ , density  $\rho$ , and modulus of elasticity  $E$ , due to its own weight.



**SOLUTION**



Let  $b$  = radius of the base and  $r$  = radius at section with coordinate  $y$ .

$$r = \frac{b}{h} y$$

Volume of portion above element  $V = \frac{1}{3} \pi r^2 y = \frac{1}{3} \pi \frac{b^2}{h^2} y^3$

$$P = \rho g V = \frac{\pi \rho g b^2 y^3}{3h^2}$$

$$A = \pi r^2 = \frac{\pi b^2}{h^2} y^2$$

$$S = \sum \frac{P \Delta y}{EA} = \int_0^h \frac{P dy}{EA} = \int_0^h \frac{\pi \rho g b^2 y^3}{3h^2} \cdot \frac{h^2}{E \pi b^2 y^2} dy = \int_0^h \frac{\rho g y}{3E} dy$$

$$= \frac{\rho g}{3E} \frac{y^2}{2} \Big|_0^h = \frac{\rho g h^2}{6E}$$

**PROBLEM 2.31**

2.31 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is  $d_1$ , show that when the diameter is  $d$ , the true strain is  $\epsilon_t = 2 \ln(d_1/d)$ .

**SOLUTION**

If the volume is constant  $\frac{\pi}{4} d^2 L = \frac{\pi}{4} d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\epsilon_t = \ln \frac{L}{L_0} = \ln \left(\frac{d_1}{d}\right)^2 = 2 \ln \frac{d_1}{d}$$

**PROBLEM 2.32**

2.32 Denoting by  $\epsilon$  the "engineering strain" in a tensile specimen, show that the true strain is  $\epsilon_t = \ln(1 + \epsilon)$ .

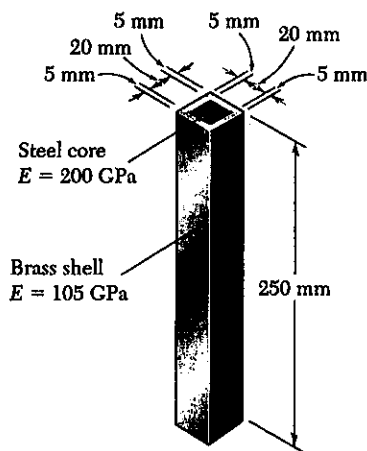
**SOLUTION**

$$\epsilon_t = \ln \frac{L}{L_0} = \ln \frac{L_0 + \Delta L}{L_0} = \ln \left(1 + \frac{\Delta L}{L_0}\right) = \ln(1 + \epsilon)$$

Thus  $\epsilon_t = \ln(1 + \epsilon)$



**PROBLEM 2.33**



2.33 An axial force of 60 kN is applied to the assembly shown by means of rigid end plates. Determine (a) the normal stress in the brass shell, (b) the corresponding deformation of the assembly.

**SOLUTION**

Let  $P_b$  = portion of axial force carried by brass shell

$P_s$  = portion of axial force carried by steel core

$$\delta = \frac{P_b L}{A_b E_b} \quad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \quad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s) \frac{\delta}{L}$$

$$\frac{\delta}{L} = \epsilon = \frac{P}{E_b A_b + E_s A_s}$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

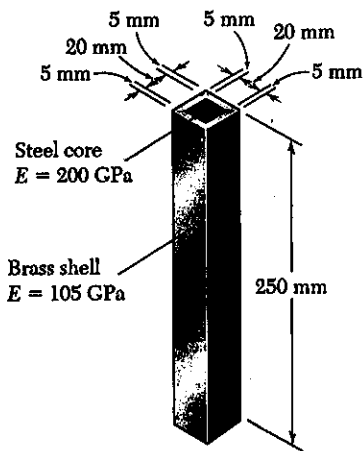
$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\frac{\delta}{L} = \epsilon = \frac{60 \times 10^3}{(105 \times 10^9)(500 \times 10^{-6}) + (200 \times 10^9)(400 \times 10^{-6})} = 452.83 \times 10^{-6}$$

$$(a) \quad \sigma_b = E_b \epsilon = (105 \times 10^9)(452.83 \times 10^{-6}) = 47.5 \times 10^6 \text{ Pa} \\ = 47.5 \text{ MPa}$$

$$(b) \quad \delta = L \epsilon = (250 \times 10^{-3})(452.83 \times 10^{-6}) = 113.2 \times 10^{-6} \text{ m} \\ = 0.1132 \times 10^{-3} \text{ m} \\ = 0.1132 \text{ mm}$$

**PROBLEM 2.34**



2.34 The length of the assembly decreases by 0.15 mm when an axial force is applied by means of rigid end plates. Determine (a) the magnitude of the applied force, (b) the corresponding stress in the steel core.

**SOLUTION**

Let  $P_b$  = portion of axial force carried by brass shell.

$P_s$  = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b}$$

$$P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s}$$

$$P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s) \frac{\delta}{L}$$

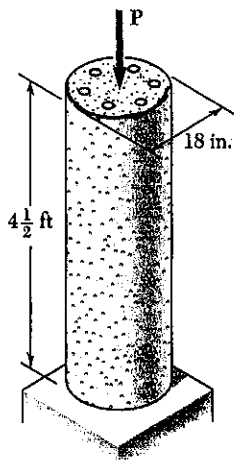
$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\begin{aligned} (a) \quad P &= [(105 \times 10^9)(500 \times 10^{-6}) + (200 \times 10^9)(400 \times 10^{-6})] \frac{0.15 \times 10^{-3}}{250 \times 10^{-3}} \\ &= 79.5 \times 10^3 \text{ N} \qquad = 79.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} (b) \quad \sigma_s &= E_s \epsilon = \frac{E_s \delta}{L} = \frac{(200 \times 10^9)(0.15 \times 10^{-3})}{250 \times 10^{-3}} = 120 \times 10^6 \text{ Pa} \\ &= 120 \text{ MPa} \end{aligned}$$

PROBLEM 2.35



2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a  $1\frac{1}{8}$ -in. diameter. Knowing that  $E_s = 29 \times 10^6$  psi and  $E_c = 4.2 \times 10^6$  psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force  $P$  is applied to the post.

SOLUTION

Let  $P_c$  = portion of axial force carried by concrete  
 $P_s$  = portion carried by the six steel rods

$$\delta = \frac{P_c L}{E_c A_c} \quad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \quad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s) \frac{\delta}{L}$$

$$\epsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6 \frac{\pi}{4} d_s^2 = \frac{6\pi}{4} (1.125)^2 = 5.964 \text{ in}^2$$

$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4} (18)^2 - 5.964 = 248.5 \text{ in}^2$$

$$L = 4.5 \text{ ft} = 54 \text{ in}$$

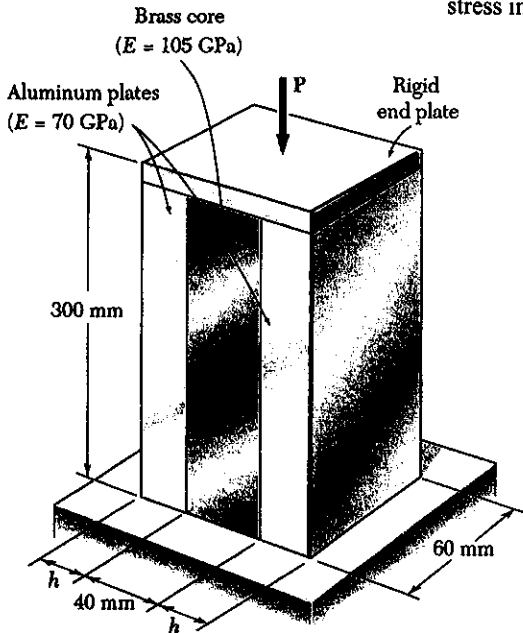
$$\epsilon = \frac{-350 \times 10^3}{(4.2 \times 10^6)(248.5) + (29 \times 10^6)(5.964)} = -287.67 \times 10^{-6}$$

$$\sigma_s = E_s \epsilon = (29 \times 10^6)(-287.67 \times 10^{-6}) = -8.34 \times 10^3 \text{ psi} = -8.34 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_c = E_c \epsilon = (4.2 \times 10^6)(-287.67 \times 10^{-6}) = -1.208 \times 10^3 \text{ psi} = -1.208 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 2.36**

2.36 An axial centric force of magnitude  $P = 450 \text{ kN}$  is applied to the composite block shown by means of a rigid end plate. Knowing that  $h = 10 \text{ mm}$ , determine the normal stress in (a) the brass core, (b) the aluminum plates.



**SOLUTION**

Let  $P_b =$  portion of axial force carried by brass core

$P_a =$  portion carried by two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \quad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \quad P_a = \frac{E_a A_a \delta}{L}$$

$$P = P_b + P_a = (E_b A_b + E_a A_a) \frac{\delta}{L}$$

$$\epsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

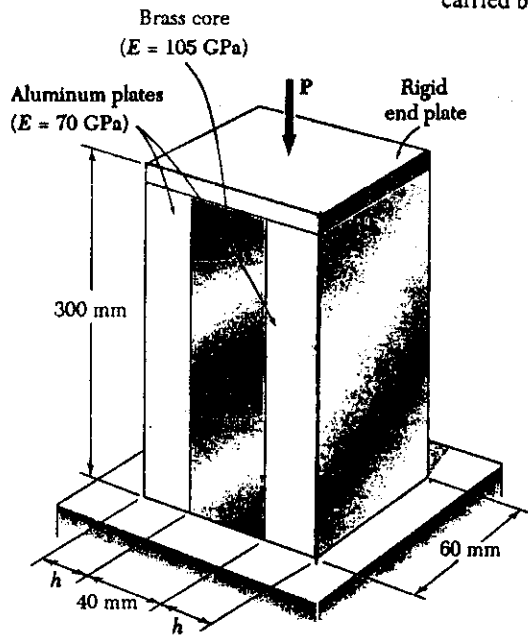
$$A_a = (2)(60)(10) = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

$$\epsilon = \frac{450 \times 10^3}{(105 \times 10^9)(2400 \times 10^{-6}) + (70 \times 10^9)(1200 \times 10^{-6})} = 1.3393 \times 10^{-3}$$

$$(a) \quad \sigma_b = E_b \epsilon = (105 \times 10^9)(1.3393 \times 10^{-3}) = 140.6 \times 10^6 \text{ Pa} = 140.6 \text{ MPa} \blacktriangleleft$$

$$(b) \quad \sigma_a = E_a \epsilon = (70 \times 10^9)(1.3393 \times 10^{-3}) = 93.75 \times 10^6 \text{ Pa} = 93.75 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.37



2.37 For the composite block shown in Prob. 2.36, determine (a) the value of  $h$  if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (b) the total load if the stress in the brass is 80 MPa.

SOLUTION

Let  $P_b$  = portion of axial force carried by brass core

$P_a$  = portion carried by the two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \quad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \quad P_a = \frac{E_a A_a \delta}{L}$$

(a) Given  $P_a = \frac{1}{2} P_b$

$$\frac{E_a A_a \delta}{L} = \frac{1}{2} \frac{E_b A_b \delta}{L}$$

$$A_a = \frac{1}{2} \frac{E_b}{E_a} A_b$$

$$A_b = (40)(60) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{1}{2} \frac{105 \times 10^9}{70 \times 10^9} 2400 = 1800 \text{ mm}^2 = (2)(60)h$$

$$h = \frac{1800}{(2)(60)} = 15 \text{ mm}$$

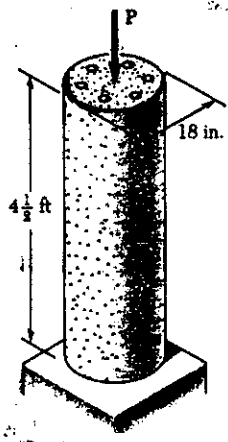
(b)  $\sigma_b = \frac{P_b}{A_b}$

$$P_b = A_b \sigma_b = (2400 \times 10^{-6})(80 \times 10^6) = 192 \times 10^3 \text{ N}$$

$$P_a = \frac{1}{2} P_b = 96 \times 10^3 \text{ N}$$

$$P = P_b + P_a = 288 \times 10^3 \text{ N} = 288 \text{ kN}$$

PROBLEM 2.38



2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a  $1\frac{1}{8}$ -in. diameter. Knowing that  $E_s = 29 \times 10^6$  psi and  $E_c = 4.2 \times 10^6$  psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force  $P$  is applied to the post.

2.38 For the post of Prob. 2.35, determine the maximum centric force which may be applied if the allowable normal stress is 20 ksi in the steel and 2.4 ksi in the concrete.

SOLUTION

Determine allowable strain in each material

$$\text{Steel: } \epsilon_s = \frac{\sigma_s}{E_s} = \frac{20 \times 10^3}{29 \times 10^6} = 689.97 \times 10^{-6}$$

$$\text{Concrete: } \epsilon_c = \frac{\sigma_c}{E_c} = \frac{2.4 \times 10^3}{4.2 \times 10^6} = 571.43 \times 10^{-6}$$

$$\text{Smaller value governs } \epsilon = \frac{\delta}{L} = 571.43 \times 10^{-6}$$

Let  $P_c$  = portion of load carried by concrete

$P_s$  = portion carried by six steel rods

$$\delta = \frac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \frac{\delta}{L} = E_c A_c \epsilon$$

$$\delta = \frac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \frac{\delta}{L} = E_s A_s \epsilon$$

$$P = P_c + P_s = (E_c A_c + E_s A_s) \epsilon$$

$$A_s = 6 \frac{\pi}{4} d_s^2 = \frac{6\pi}{4} (1.125)^2 = 5.964 \text{ in}^2$$

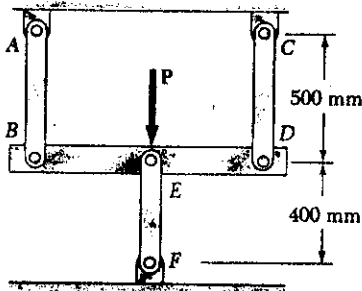
$$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4} (18)^2 - 5.964 = 248.5 \text{ in}^2$$

$$P = [(4.2 \times 10^6)(248.5) + (29 \times 10^6)(5.964)](571.43 \times 10^{-6})$$

$$= 695 \times 10^3 \text{ lb} = 695 \text{ kips}$$

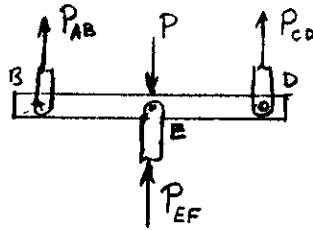
PROBLEM 2.39

2.39 Three steel rods ( $E = 200 \text{ GPa}$ ) support a 36-kN load  $P$ . Each of the rods  $AB$  and  $CD$  has a  $200\text{-mm}^2$  cross-sectional area and rod  $EF$  has a  $625\text{-mm}^2$  cross-sectional area. Determine the (a) the change in length of rod  $EF$ , (b) the stress in each rod.



SOLUTION

Use member  $BED$  as a free body



By symmetry, or by  $\sum M_E = 0$

$$P_{CD} = P_{AB}$$

$$\sum F_y = 0$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}}, \quad \delta_{CD} = \frac{P_{CD} L_{CD}}{E A_{CD}}, \quad \delta_{EF} = \frac{P_{EF} L_{EF}}{E A_{EF}}$$

Since  $L_{AB} = L_{CD}$  and  $A_{AB} = A_{CD}$ ,  $\delta_{AB} = \delta_{CD}$

Since points  $A, C,$  and  $E$  are fixed  $\delta_B = \delta_{AB}, \delta_D = \delta_{CD}, \delta_E = \delta_{EF}$

Since member  $BED$  is rigid  $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB} L_{AB}}{E A_{AB}} = \frac{P_{EF} L_{EF}}{E A_{EF}} \quad \therefore P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF} = 0.256 P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2 \times 0.256) P_{EF} + P_{EF} = 1.512 P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

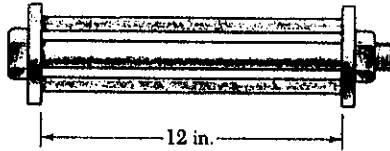
(a)  $\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m} = 0.0762 \text{ mm} \blacktriangleleft$

or  $\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$

(b)  $\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa} \blacktriangleleft$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = 38.1 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.40



2.40 A brass bolt ( $E_b = 15 \times 10^6$  psi) with a  $\frac{3}{8}$ -in. diameter is fitted inside a steel tube ( $E_s = 29 \times 10^6$  psi) with a  $\frac{7}{8}$ -in. outer diameter and  $\frac{1}{8}$ -in. wall thickness. After the nut has been fit snugly, it is tightened one quarter of a full turn. Knowing that the bolt is single-threaded with a 0.1-in. pitch, determine the normal stress ( $a$ ) in the bolt, ( $b$ ) in the tube.

SOLUTION

The movement of the nut along the bolt after a quarter turn is equal to  $\frac{1}{4} \times$  pitch.

$$\delta = \left(\frac{1}{4}\right)(0.1) = 0.025 \text{ in}$$

Also  $\delta = \delta_{\text{bolt}} + \delta_{\text{tube}}$  where  $\delta_{\text{bolt}}$  = elongation of the bolt  
and  $\delta_{\text{tube}}$  = shortening of the tube

Let  $P_{\text{bolt}}$  = axial tensile force in the bolt  
 $P_{\text{tube}}$  = axial compressive force in the tube

For equilibrium of each end plate  $P_{\text{bolt}} = P_{\text{tube}} = P$

$$A_{\text{bolt}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

$$A_{\text{tube}} = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} \left(\left(\frac{7}{8}\right)^2 - \left(\frac{5}{8}\right)^2\right) = 0.29452 \text{ in}^2$$

$$\delta_{\text{bolt}} = \frac{P_{\text{bolt}} L}{E A_{\text{bolt}}} = \frac{(P)(12)}{(15 \times 10^6)(0.11045)} = 7.2431 \times 10^{-6} P$$

$$\delta_{\text{tube}} = \frac{P_{\text{tube}} L}{E A_{\text{tube}}} = \frac{(P)(12)}{(29 \times 10^6)(0.29452)} = 1.4050 \times 10^{-6} P$$

$$0.025 = 7.2431 \times 10^{-6} P + 1.4050 \times 10^{-6} P = 8.6481 \times 10^{-6} P$$

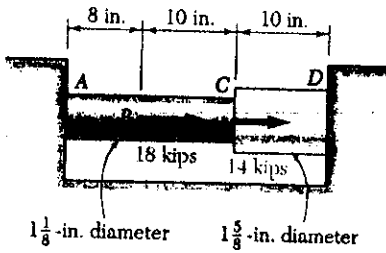
$$P = 2.8908 \times 10^3 \text{ lb}$$

$$(a) \sigma_{\text{bolt}} = \frac{P}{A_{\text{bolt}}} = \frac{2.8908 \times 10^3}{0.11045} = 26.2 \times 10^3 \text{ psi} = 26.2 \text{ ksi}$$

$$(b) \sigma_{\text{tube}} = -\frac{P}{A_{\text{tube}}} = -\frac{2.8908 \times 10^3}{0.29452} = -9.82 \times 10^3 \text{ psi} = -9.82 \text{ ksi}$$



PROBLEM 2.41



2.41 Two cylindrical rods, CD made of steel ( $E = 29 \times 10^6$  psi) and AC made of aluminum ( $E = 10.4 \times 10^6$  psi), are joined at B and restrained by rigid supports at A and D. Determine (a) the reactions at A and D, (b) the deflection of point C.

SOLUTION

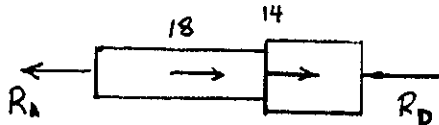
AB:  $P = R_A$ ,  $L_{AB} = 8$  in

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{AB} = \frac{PL}{EA}$$

$$= \frac{R_A (8)}{(10.4 \times 10^6)(0.99402)}$$

$$= 0.77386 \times 10^{-6} R_A$$



BC:  $P = R_A - 18 \times 10^3$ ,  $L = 10$  in,  $A = 0.99402 \text{ in}^2$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 18 \times 10^3)(10)}{(10.4 \times 10^6)(0.99402)} = 0.96732 \times 10^{-6} R_A - 17.412 \times 10^{-3}$$

CD:  $P = R_A - 18 \times 10^3 - 14 \times 10^3 = R_A - 32 \times 10^3$

$L = 10$  in  $A = \frac{\pi}{4} d_{CD}^2 = \frac{\pi}{4} (1.625)^2 = 2.0739 \text{ in}^2$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 32 \times 10^3)(10)}{(29 \times 10^6)(2.0739)} = 0.16627 \times 10^{-6} R_A - 5.321 \times 10^{-3}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 1.9075 \times 10^{-6} R_A - 22.733 \times 10^{-3}$$

Since point D cannot move relative to A  $\delta_{AD} = 0$

(a)  $1.9075 \times 10^{-6} R_A - 22.733 \times 10^{-3} = 0$   $R_A = 11.92 \times 10^3 \text{ lb.}$  ←

$R_D = 32 \times 10^3 - R_A = 20.08 \times 10^3 \text{ lb.}$  ←

(b)  $\delta_C = \delta_{AB} + \delta_{CD}$

$$= 1.7412 \times 10^{-6} R_A - 17.412 \times 10^{-3}$$

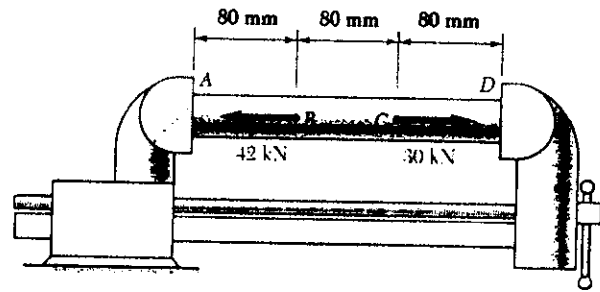
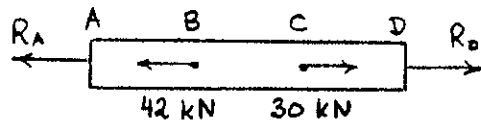
$$= (1.7412 \times 10^{-6})(11.92 \times 10^3) - 17.412 \times 10^{-3} = 3.34 \times 10^{-3} \text{ in.}$$
 ←

or  $\delta_C = \frac{R_D L_{CD}}{E_{CD} A_{CD}} = \frac{(20.08 \times 10^3)(10)}{(29 \times 10^6)(2.0739)} = 3.34 \times 10^{-3} \text{ in.}$  ←

PROBLEM 2.42

2.42 A steel tube ( $E = 200 \text{ GPa}$ ) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at  $A$  and  $D$ , (b) the change in length of the portion  $BC$  of the tube.

SOLUTION



For the tube  $d_i = d_o - 2t$   
 $= 32 - (2)(4) = 24 \text{ mm}$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (32^2 - 24^2)$$

$$= 351.86 \text{ mm}^2 = 351.86 \times 10^{-6} \text{ m}^2$$

AB:  $P = R_A, L = 0.080 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.080)}{(200 \times 10^9)(351.86 \times 10^{-6})} = 1.1368 \times 10^{-9} R_A$$

BC:  $P = R_A + 42 \times 10^3, L = 0.080 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 42 \times 10^3)(0.080)}{(200 \times 10^9)(351.86 \times 10^{-6})} = 1.1368 \times 10^{-9} R_A + 47.746 \times 10^{-6}$$

CD:  $P = R_A + 12 \times 10^3, L = 0.080$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 12 \times 10^3)(0.080)}{(200 \times 10^9)(351.86 \times 10^{-6})} = 1.1368 \times 10^{-9} R_A + 13.642 \times 10^{-6}$$

Total:  $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 3.4104 \times 10^{-9} R_A + 61.388 \times 10^{-6}$

Given jaw movement  $\delta_{AD} = -0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ m}$

(a)  $-0.2 \times 10^{-3} = 3.4104 \times 10^{-9} R_A + 61.388 \times 10^{-6} \therefore R_A = -76.6 \times 10^3 \text{ N}$   
 $= -76.6 \text{ kN}$

$R_D = R_A + 12 \times 10^3$   
 $R_D = -64.6 \times 10^3 \text{ N}$   
 $= -64.6 \text{ kN}$

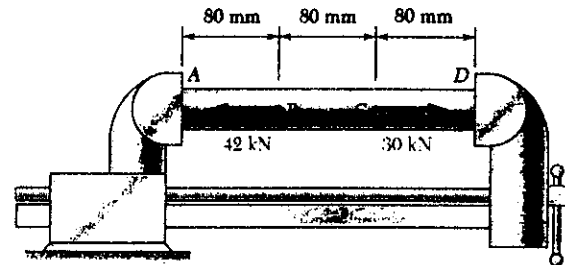
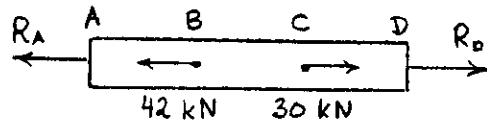
(b)  $\delta_{BC} = (1.1368 \times 10^{-9})(-76.6 \times 10^3) + 47.746 \times 10^{-6} = -39.4 \times 10^{-6} \text{ m}$   
 $= -0.0394 \text{ mm}$

PROBLEM 2.43

SOLUTION

2.42 A steel tube ( $E = 200 \text{ GPa}$ ) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at A and D, (b) the change in length of the portion BC of the tube.

2.43 Solve Prob. 2.42, assuming that after the forces have been applied, the vise is adjusted to decrease the distance between its jaws by 0.1 mm.



For the tube  $d_i = d_o - 2t$   
 $= 32 - (2)(4) = 24 \text{ mm}$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (32^2 - 24^2)$$

$$= 351.86 \text{ mm}^2 = 351.86 \times 10^{-6} \text{ m}^2$$

AB:  $P = R_A$ ,  $L = 0.080 \text{ m}$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.080)}{(200 \times 10^9)(351.86 \times 10^{-6})} = 1.1368 \times 10^{-9} R_A$$

BC:  $P = R_A + 42 \times 10^3$ ,  $L = 0.080 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 42 \times 10^3)(0.080)}{(200 \times 10^9)(351.86 \times 10^{-6})} = 1.1368 \times 10^{-9} R_A + 47.746 \times 10^{-6}$$

CD:  $P = R_A + 12 \times 10^3$ ,  $L = 0.080$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 12 \times 10^3)(0.080)}{(200 \times 10^9)(351.86 \times 10^{-6})} = 1.1368 \times 10^{-9} R_A + 13.642 \times 10^{-6}$$

Total:  $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 3.4104 \times 10^{-9} R_A + 61.388 \times 10^{-6}$

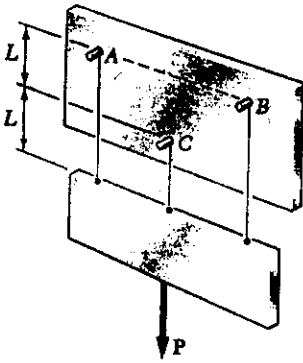
Due to the movement of the jaws  $\delta_{AD} = -0.1 \text{ mm} = -0.1 \times 10^{-3} \text{ m}$

(a)  $-0.1 \times 10^{-3} = 3.4104 \times 10^{-9} R_A + 61.388 \times 10^{-6}$   $R_A = -47.322 \times 10^3 \text{ N}$   
 $= -47.3 \text{ kN}$   $\blacktriangleleft$

$R_D = R_A + 12 \times 10^3$   $= -35.3 \times 10^3 \text{ N}$   
 $= -35.3 \text{ kN}$   $\blacktriangleleft$

(b)  $\delta_{BC} = (1.1368 \times 10^{-9})(-47.322 \times 10^3) + 47.746 \times 10^{-6} = -6.05 \times 10^{-6} \text{ m}$   
 $= -0.00605 \text{ mm}$   $\blacktriangleleft$

PROBLEM 2.44



2.44 Three wires are used to suspend the plate shown. Aluminum wires are used at A and B with a diameter of  $\frac{1}{8}$  in. and a steel wire is used at C with a diameter of  $\frac{1}{12}$  in. Knowing that the allowable stress for aluminum ( $E = 10.4 \times 10^6$  psi) is 14 ksi and that the allowable stress for steel ( $E = 29 \times 10^6$  psi) is 18 ksi, determine the maximum load P that may be applied.

SOLUTION

By symmetry  $P_A = P_B$ , and  $\delta_A = \delta_B$

Also,  $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire

$$\epsilon_A = \epsilon_B = \frac{\delta}{2L}, \quad \epsilon_C = \frac{\delta}{L} = 2\epsilon_A$$

Determine allowable strain

$$A \ \& \ B \quad \epsilon_A = \frac{\sigma_A}{E_A} = \frac{14 \times 10^3}{10.4 \times 10^6} = 1.3462 \times 10^{-3}$$

$$\epsilon_C = 2\epsilon_A = 2.6924 \times 10^{-3}$$

$$C \quad \epsilon_C = \frac{\sigma_C}{E_C} = \frac{18 \times 10^3}{29 \times 10^6} = 0.6207 \times 10^{-3}$$

$$\epsilon_A = \epsilon_B = \frac{1}{2}\epsilon_C = 0.3103 \times 10^{-3}$$

Allowable strain for wire C governs  $\therefore \sigma_C = 18 \times 10^3$  psi

$$\sigma_A = E_A \epsilon_A \quad P_A = A_A E_A \epsilon_A = \frac{\pi}{4} \left(\frac{1}{8}\right)^2 (10.4 \times 10^6) (0.3103 \times 10^{-3})$$

$$= 139.61 \text{ lb}$$

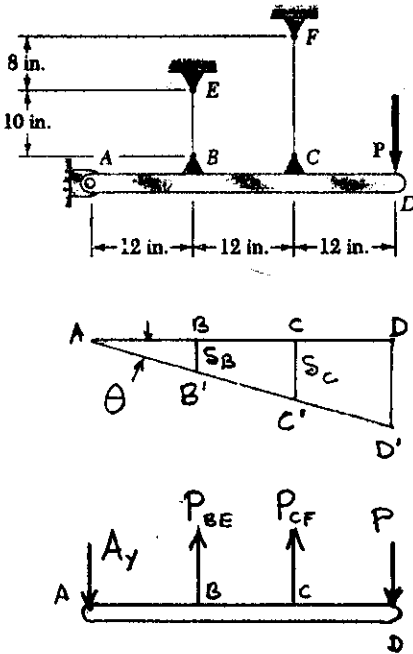
$$P_B = 139.61 \text{ lb}$$

$$\sigma_C = E_C \epsilon_C \quad P_C = A_C \sigma_C = \frac{\pi}{4} \left(\frac{1}{12}\right)^2 (18 \times 10^3) = 98.17 \text{ lb}$$

For equilibrium of the plate

$$P = P_A + P_B + P_C = 177.4 \text{ lb}$$

PROBLEM 2.45



2.45 The rigid bar  $AD$  is supported by two steel wires of  $\frac{1}{16}$ -in. diameter ( $E = 29 \times 10^6$  psi) and a pin and bracket at  $D$ . Knowing that the wires were initially taut, determine (a) the additional tension in each wire when a 220-lb load  $P$  is applied at  $D$ , (b) the corresponding deflection of point  $D$ .

SOLUTION

Let  $\theta$  be the rotation of bar  $ABCD$

Then  $\Delta_B = 12\theta$

$\Delta_C = 24\theta$

$P_{BE} = \frac{P_{BE} L_{BE}}{AE}$

$P_{BE} = \frac{EA \Delta_{BE}}{L_{BE}} = \frac{(29 \times 10^6) \frac{\pi}{4} (\frac{1}{16})^2 (12\theta)}{10}$

$= 106.77 \times 10^3 \theta$

$\Delta_C = \frac{P_{CF} L_{CF}}{EA}$

$P_{CF} = \frac{EA \Delta_{CF}}{L_{CF}} = \frac{(29 \times 10^6) \frac{\pi}{4} (\frac{1}{16})^2 (24\theta)}{18}$

$= 118.63 \times 10^3 \theta$

Using free body  $ABCD$

$\sum M_A = 0 \quad 12 P_{BE} + 24 P_{CF} - 36 P = 0$

$(12)(106.77 \times 10^3 \theta) + (24)(118.63 \times 10^3 \theta) - (36)(220) = 0$

$4.1283 \times 10^6 \theta = (36)(220)$

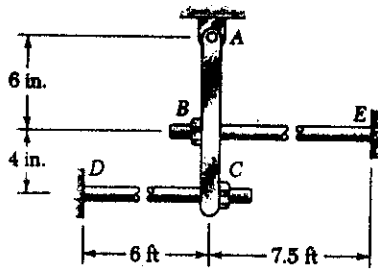
$\theta = 1.9185 \times 10^{-3} \text{ rad}$

(a)  $P_{BE} = (106.77 \times 10^3)(1.9185 \times 10^{-3}) = 204.8 \text{ lb}$

$P_C = (118.63 \times 10^3)(1.9185 \times 10^{-3}) = 227.6 \text{ lb}$

(b)  $\Delta_D = 36\theta = (36)(1.9185 \times 10^{-3}) = 69.1 \times 10^{-3} \text{ in}$   
 $= 0.0691 \text{ in}$

PROBLEM 2.46



2.46 The steel rods  $BE$  and  $CD$  each have a diameter of  $\frac{5}{8}$  in. ( $E = 29 \times 10^6$  psi). The ends are threaded with a pitch of 0.1 in. Knowing that after being snugly fit, the nut at  $B$  is tightened one full turn, determine (a) the tension in rod  $CD$ , (b) the deflection of point  $C$  of the rigid member  $ABC$ .

SOLUTION

Let  $\theta$  be the rotation of bar  $ABC$  as shown

Then,  $S_B = 6\theta$  and  $S_C = 10\theta$

But  $S_B = S_{turn} - \frac{P_{BE} L_{BE}}{E_{st} A_{BE}}$

$P_{BE} = (E_{st} A_{BE})(S_{turn} - S_B) / L_{BE}$

$L_{BE} = 7.5 \text{ ft} = 90 \text{ in.}, \therefore S_{turn} = 0.1 \text{ in}$

$A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (\frac{5}{8})^2 = 0.3068 \text{ in}^2$

$P_{BE} = \frac{(29 \times 10^6)(0.3068)(0.1 - 6\theta)}{90}$

$= 9.886 \times 10^3 - 593.15 \times 10^3 \theta$

$S_C = \frac{P_{CD} L_{CD}}{E A_{CD}} \therefore P_{CD} = \frac{E A S_C}{L_{CD}}$

$L_{CD} = 6 \text{ ft} = 72 \text{ in.}, A_{CD} = 0.3068 \text{ in}^2$

$P_{CD} = \frac{(29 \times 10^6)(0.3068)(10\theta)}{72}$

$= 1.23572 \times 10^6 \theta$

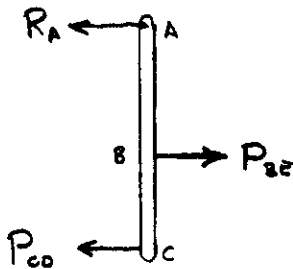
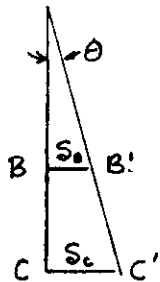
$\sum M_A = 0 \quad 6 P_{BE} - 10 P_{CD} = 0$

$(6)(9.886 \times 10^3 - 593.16 \times 10^3 \theta) - (10)(1.23572 \times 10^6) \theta = 0$

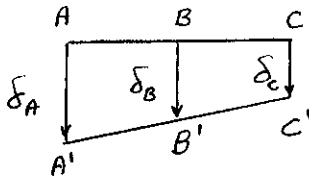
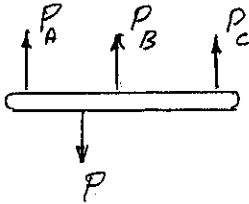
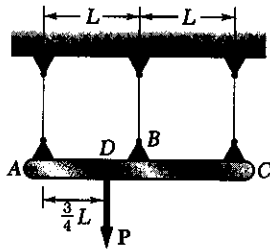
$59.316 \times 10^3 - 15.916 \times 10^6 \theta = 0 \quad \theta = 3.7268 \times 10^{-3} \text{ rad}$

(a)  $P_{CD} = (1.23572 \times 10^6)(3.7268 \times 10^{-3}) = 4.61 \times 10^3$   
 $= 4.61 \text{ kips}$

(b)  $S_C = 10 \theta = (10)(3.7268 \times 10^{-3}) = 37.3 \times 10^{-3} \text{ in}$   
 $= 0.0373 \text{ in}$



**PROBLEM 2.47**



2.47 The rigid rod  $ABCD$  is suspended from three wires of the same material. The cross-sectional area of the wire at  $B$  is equal to half of the cross-sectional area of the wires  $A$  and  $C$ . Determine the tension in each wire caused by the load  $P$ .

**SOLUTION**

$$\uparrow \sum M_A = 0 \quad 2LP_C + LP_B - \frac{3}{4}LP = 0$$

$$P_C = \frac{3}{8}P - \frac{1}{2}P_B$$

$$\uparrow \sum M_C = 0 \quad -2LP_A - LP_B + \frac{5}{4}LP = 0$$

$$P_A = \frac{5}{8}P - \frac{1}{2}P_B$$

LET  $l$  BE THE LENGTH OF THE WIRES

$$\delta_A = \frac{P_A l}{EA} = \frac{l}{EA} \left( \frac{5}{8}P - \frac{1}{2}P_B \right)$$

$$\delta_B = \frac{P_B l}{E(A/2)} = \frac{2l}{EA} P_B$$

$$\delta_C = \frac{P_C l}{EA} = \frac{l}{EA} \left( \frac{3}{8}P - \frac{1}{2}P_B \right)$$

FROM THE DEFORMATION DIAGRAM:

$$\delta_A - \delta_B = \delta_B - \delta_C$$

OR

$$\delta_B = \frac{1}{2}(\delta_A + \delta_C)$$

$$\frac{l}{E(A/2)} P_B = \frac{1}{2} \frac{l}{EA} \left( \frac{5}{8}P - \frac{1}{2}P_B + \frac{3}{8}P - \frac{1}{2}P_B \right)$$

$$\frac{5}{2} P_B = \frac{1}{2} P; \quad P_B = \frac{1}{5} P$$

$$P_B = 0.200P \quad \blacktriangleleft$$

$$P_A = \frac{5}{8}P - \frac{1}{2} \left( \frac{P}{5} \right) = \frac{21}{40} P$$

$$P_A = 0.525P \quad \blacktriangleleft$$

$$P_C = \frac{3}{8}P - \frac{1}{2} \left( \frac{P}{5} \right) = \frac{11}{40} P$$

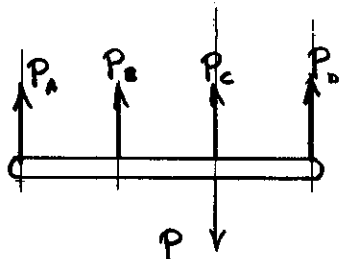
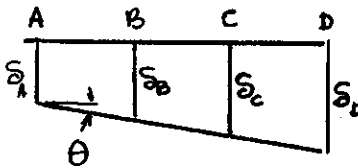
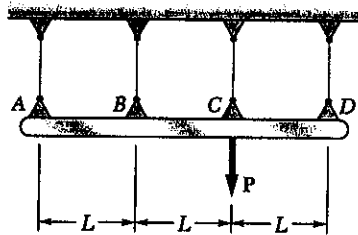
$$P_C = 0.275P \quad \blacktriangleleft$$

CHECK:

$$P_A + P_B + P_C = 1.000P \quad \text{OK}$$

PROBLEM 2.48

2.48 The rigid bar  $ABCD$  is suspended from four identical wires. Determine the tension in each wire caused by the load  $P$ .



SOLUTION

Let  $\theta$  be the slope of bar  $ABCD$  after deformation

$$S_B = S_A + L\theta$$

$$S_C = S_A + 2L\theta$$

$$S_D = S_A + 3L\theta$$

$$P_A = \frac{EA}{l} S_A$$

$$P_B = \frac{EA}{l} S_B = \frac{EA}{l} S_A + \frac{EAL}{l} \theta$$

$$P_C = \frac{EA}{l} S_C = \frac{EA}{l} S_A + \frac{2EAL}{l} \theta$$

$$P_D = \frac{EA}{l} S_D = \frac{EA}{l} S_A + \frac{3EAL}{l} \theta$$

$$\uparrow \sum F_y = 0$$

$$P_A + P_B + P_C + P_D - P = 0$$

$$\frac{4EA}{l} S_A + \frac{6EA}{l} L\theta = P$$

$$4S_A + 6L\theta = \frac{Pl}{EA} \quad (1)$$

$$\curvearrowright \sum M_A = 0$$

$$LP_B + 2LP_C + 3LP_D - 2LP = 0$$

$$\frac{6EAL}{l} S_A + \frac{14EAL}{l} L\theta = 2LP$$

$$6S_A + 14L\theta = \frac{2Pl}{EA} \quad (2)$$

Solving (1) and (2) simultaneously

$$L\theta = \frac{1}{10} \frac{Pl}{EA}$$

$$S_A = -\frac{1}{10} \frac{Pl}{EA}$$

$$P_A = \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} = \frac{1}{10} P$$

$$P_B = \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} + \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} = \frac{1}{5} P$$

$$P_C = \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} + 2 \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} = \frac{3}{10} P$$

$$P_D = \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} + 3 \frac{EA}{l} \cdot \frac{1}{10} \frac{Pl}{EA} = \frac{2}{5} P$$



**PROBLEM 2.49**

2.49 A steel railroad track ( $E = 200 \text{ GPa}$ ,  $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$ ) was laid out at a temperature of  $6^\circ\text{C}$ . Determine the normal stress in the rails when the temperature reaches  $48^\circ\text{C}$ , assuming that the rails (a) are welded to form a continuous track, (b) are 10 m long with 3-mm gaps between them.

**SOLUTION**

$$(a) \quad \delta_T = \alpha(\Delta T)L = (11.7 \times 10^{-6})(48 - 6)(10) = 4.914 \times 10^{-3} \text{ m}$$

$$\delta_p = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200 \times 10^9} = 50 \times 10^{-12} \sigma$$

$$\delta = \delta_T + \delta_p = 4.914 \times 10^{-3} + 50 \times 10^{-12} \sigma = 0 \quad \therefore \sigma = -98.3 \times 10^6 \text{ Pa} \\ = -98.3 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \delta = \delta_T + \delta_p = 4.914 \times 10^{-3} + 50 \times 10^{-12} \sigma = 3 \times 10^{-3}$$

$$\sigma = \frac{3 \times 10^{-3} - 4.914 \times 10^{-3}}{50 \times 10^{-12}} = -38.3 \times 10^6 \text{ Pa} = -38.3 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 2.50**



Brass core  
 $E = 15 \times 10^6$  psi  
 $\alpha = 11.6 \times 10^{-6}/^\circ\text{F}$

Aluminum shell  
 $E = 10.6 \times 10^6$  psi  
 $\alpha = 12.9 \times 10^{-6}/^\circ\text{F}$

**2.50** The aluminum shell is fully bonded to the brass core, and the assembly is unstressed at a temperature of  $78^\circ\text{F}$ . Considering only axial deformations, determine the stress when the temperature reaches  $180^\circ\text{F}$  (a) in the brass core, (b) in the aluminum shell.

**SOLUTION**

$$\Delta T = 180 - 78 = 102 \text{ F}$$

Let  $P_b$  be the tensile force developed in the brass core

For equilibrium with zero total force, the compressive force in the aluminum shell is  $P_b$

Strains 
$$\epsilon_b = \frac{P_b}{E_b A_b} + \alpha_b(\Delta T), \quad \epsilon_a = -\frac{P_b}{E_a A_a} + \alpha_a(\Delta T)$$

Matching 
$$\epsilon_b = \epsilon_a \quad \frac{P_b}{E_b A_b} + \alpha_b(\Delta T) = -\frac{P_b}{E_a A_a} + \alpha_a(\Delta T)$$

$$\left( \frac{1}{E_b A_b} + \frac{1}{E_a A_a} \right) P_b = (\alpha_a - \alpha_b)(\Delta T)$$

$$A_b = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1)^2 = 0.7854 \text{ in}^2$$

$$A_a = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (2.5^2 - 1.0^2) = 4.1233 \text{ in}^2$$

$$\alpha_a - \alpha_b = 1.3 \times 10^{-6} /^\circ\text{F}$$

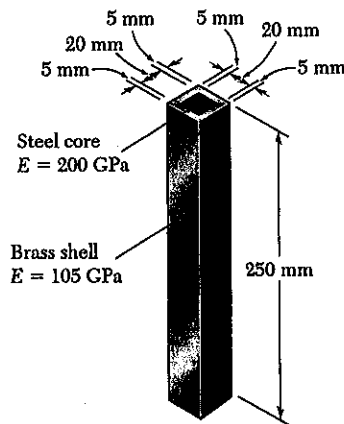
$$\left[ \frac{1}{(15 \times 10^6)(0.7854)} + \frac{1}{(10.6 \times 10^6)(4.1233)} \right] P_b = (1.3 \times 10^{-6})(102)$$

$$P_b = 1.2305 \times 10^3 \text{ lb}$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{1.2305 \times 10^3}{0.7854} = 1.567 \times 10^3 \text{ psi} = 1.567 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_a = -\frac{P_b}{A_a} = -\frac{1.2305 \times 10^3}{4.1233} = -0.298 \times 10^3 \text{ psi} = -0.298 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 2.51**



2.51 The brass shell ( $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$ ) is fully bonded to the steel core ( $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$ ). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 55 MPa.

**SOLUTION**

Let  $P_s$  = axial force developed in the steel core  
 For equilibrium with zero total force, the compressive force in the brass shell is  $P_s$ .

Strains  $\epsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T)$

$\epsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$

Matching  $\epsilon_s = \epsilon_b$

$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$

$(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}) P_s = (\alpha_b - \alpha_s)(\Delta T)$

$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$

$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$

$\alpha_b - \alpha_s = 9.2 \times 10^{-6} / ^\circ\text{C}$

$P_s = \sigma_s A_s = (55 \times 10^6)(400 \times 10^{-6}) = 22 \times 10^3 \text{ N}$

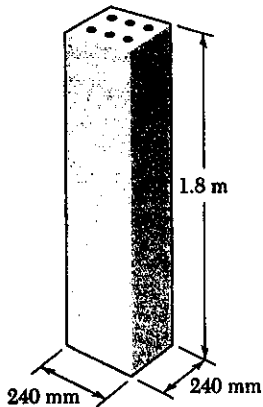
$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \times 10^9)(400 \times 10^{-6})} + \frac{1}{(105 \times 10^9)(500 \times 10^{-6})} = 31.55 \times 10^{-9} \text{ N}^{-1}$

$(31.55 \times 10^{-9})(22 \times 10^3) = (9.2 \times 10^{-6})(\Delta T)$

$\Delta T = 75.4 \text{ } ^\circ\text{C}$

PROBLEM 2.52

2.52 The concrete post ( $E_c = 25 \text{ GPa}$  and  $\alpha_c = 9.9 \times 10^{-6}/^\circ\text{C}$ ) is reinforced with six steel bars, each of 22-mm diameter ( $E_s = 200 \text{ GPa}$  and  $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$ ). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of  $35^\circ\text{C}$ .



SOLUTION

$$A_s = 6 \cdot \frac{\pi}{4} d^2 = 6 \cdot \frac{\pi}{4} (22)^2 = 2.2808 \times 10^3 \text{ mm}^2 = 2.2808 \times 10^{-3} \text{ m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808 \times 10^3 = 55.32 \times 10^3 \text{ mm}^2 = 55.32 \times 10^{-3} \text{ m}^2$$

Let  $P_c$  = tensile force developed in the concrete

For equilibrium with zero total force, the compressive force in the six steel rods is  $P_c$

Strains:  $\epsilon_s = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$  ,  $\epsilon_c = \frac{P_c}{E_c A_c} + \alpha_c (\Delta T)$

Matching:  $\epsilon_c = \epsilon_s$   $\frac{P_c}{E_c A_c} + \alpha_c (\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s (\Delta T)$

$$\left( \frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right) P_c = (\alpha_s - \alpha_c) (\Delta T)$$

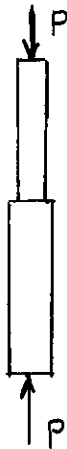
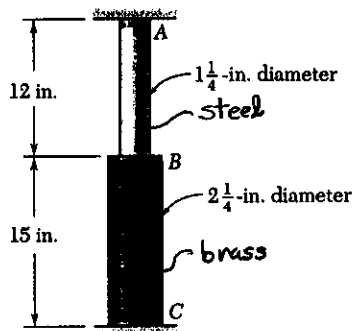
$$\left[ \frac{1}{(25 \times 10^9)(55.32 \times 10^{-3})} + \frac{1}{(200 \times 10^9)(2.2808 \times 10^{-3})} \right] P_c = (11.7 \times 10^{-6} - 9.9 \times 10^{-6})(35)$$

$$P_c = 21.61 \times 10^3 \text{ N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61 \times 10^3}{55.32 \times 10^{-3}} = 0.391 \times 10^6 \text{ Pa} = 0.391 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_s = -\frac{P_c}{A_s} = \frac{-21.61 \times 10^3}{2.2808 \times 10^{-3}} = -9.47 \times 10^6 \text{ Pa} = -9.47 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 2.53**



2.53 A rod consisting of two cylindrical portions  $AB$  and  $BC$  is restrained at both ends. Portion  $AB$  is made of steel ( $E_s = 29 \times 10^6$  psi,  $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$ ) and portion  $BC$  is made of brass ( $E_b = 15 \times 10^6$  psi,  $\alpha_b = 10.4 \times 10^{-6}/^\circ\text{F}$ ). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions  $AB$  and  $BC$  by a temperature rise of  $65^\circ\text{F}$ , (b) the corresponding deflection of point  $B$ .

**SOLUTION**

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (1.25)^2 = 1.2272 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (2.25)^2 = 3.9761 \text{ in}^2$$

Free thermal expansion

$$\begin{aligned} \delta_T &= L'_{AB} \alpha_s (\Delta T) + L_{BC} \alpha_b (\Delta T) \\ &= (12)(6.5 \times 10^{-6})(65) + (15)(10.4 \times 10^{-6})(65) \\ &= 15.21 \times 10^{-3} \text{ in} \end{aligned}$$

Shortening due to induced compressive force  $P$

$$\begin{aligned} \delta_P &= \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}} \\ &= \frac{12P}{(29 \times 10^6)(1.2272)} + \frac{15P}{(15 \times 10^6)(3.9761)} = 588.69 \times 10^{-9} P \end{aligned}$$

For zero net deflection  $\delta_P = \delta_T$

$$(588.69 \times 10^{-9}) P = 15.21 \times 10^{-3}$$

$$P = 25.84 \times 10^3 \text{ lb}$$

$$(a) \quad \sigma_{AB} = - \frac{P}{A_{AB}} = - \frac{25.84 \times 10^3}{1.2272} = -21.1 \times 10^3 \text{ psi} = -21.1 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{BC} = - \frac{P}{A_{BC}} = - \frac{25.84 \times 10^3}{3.9761} = -6.50 \times 10^3 \text{ psi} = -6.50 \text{ ksi} \quad \blacktriangleleft$$

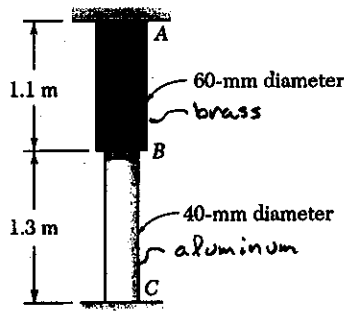
$$(b) \quad \delta_B = + \frac{PL_{AB}}{E_s A_{AB}} - L_{AB} \alpha_s (\Delta T)$$

$$= + \frac{(25.84 \times 10^3)(12)}{(29 \times 10^6)(1.2272)} + (12)(6.5 \times 10^{-6})(65) = +3.64 \times 10^{-3} \text{ in } \uparrow$$

$$\text{i.e. } 3.64 \times 10^{-3} \text{ in } \uparrow$$

$$= 0.00364 \text{ in } \uparrow \quad \blacktriangleleft$$

**PROBLEM 2.54**



2.54 A rod consisting of two cylindrical portions  $AB$  and  $BC$  is restrained at both ends. Portion  $AB$  is made of brass ( $E_b = 105 \text{ GPa}$ ,  $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$ ) and portion  $BC$  is made of aluminum ( $E_a = 72 \text{ GPa}$ ,  $\alpha_a = 23.9 \times 10^{-6}/^\circ\text{C}$ ). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions  $AB$  and  $BC$  by a temperature rise of  $42^\circ\text{C}$ , (b) the corresponding deflection of point  $B$ .

**SOLUTION**

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\begin{aligned} \delta_T &= L_{AB} \alpha_b (\Delta T) + L_{BC} \alpha_a (\Delta T) \\ &= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42) \\ &= 2.2705 \times 10^{-3} \text{ m} \end{aligned}$$

Shortening due to induced compressive force

$$\begin{aligned} \delta_P &= \frac{P L_{AB}}{E_b A_{AB}} + \frac{P L_{BC}}{E_a A_{BC}} \\ &= \frac{1.1 P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3 P}{(72 \times 10^9)(1.2566 \times 10^{-3})} \\ &= 18.074 \times 10^{-9} P \end{aligned}$$

For zero net deflection  $\delta_P = \delta_T$

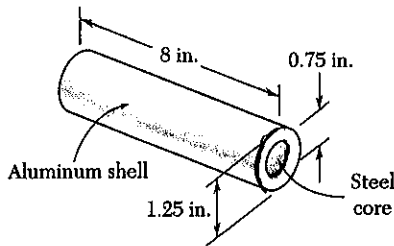
$$\begin{aligned} 18.074 \times 10^{-9} P &= 2.2705 \times 10^{-3} \\ P &= 125.62 \times 10^3 \text{ N} \end{aligned}$$

$$(a) \quad \sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$

$$\begin{aligned} (b) \quad \delta_B &= +\frac{P L_{AB}}{E_b A_{AB}} - L_{AB} \alpha_b (\Delta T) \\ &= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42) \\ &= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm} \\ &\quad \text{i.e. } 0.500 \text{ mm } \downarrow \end{aligned}$$

PROBLEM 2.55



2.55 The assembly shown consists of an aluminum shell ( $E_a = 10.6 \times 10^6$  psi,  $\alpha_a = 12.9 \times 10^{-6}/^\circ\text{F}$ ) fully bonded to a steel core ( $E_s = 29 \times 10^6$  psi,  $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$ ) and is unstressed. Determine (a) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (b) the corresponding change in length of the assembly.

SOLUTION

Since  $\alpha_a > \alpha_s$ , the shell is in compression for a positive temperature rise

Let  $\sigma_a = -6 \text{ ksi} = -6 \times 10^3 \text{ psi}$

$$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$P = -\sigma_a A_a = \sigma_s A_s$  where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\epsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

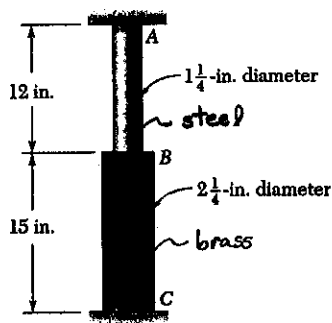
(a)  $\Delta T = 145.91 \text{ }^\circ\text{F}$

(b)  $\epsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$

or  $\epsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$

$\delta = L \epsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053 \text{ in.}$

PROBLEM 2.56



2.53 A rod consisting of two cylindrical portions  $AB$  and  $BC$  is restrained at both ends. Portion  $AB$  is made of steel ( $E_s = 29 \times 10^6$  psi,  $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$ ) and portion  $BC$  is made of brass ( $E_b = 15 \times 10^6$  psi,  $\alpha_b = 10.4 \times 10^{-6}/^\circ\text{F}$ ). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions  $AB$  and  $BC$  by a temperature rise of  $65^\circ\text{F}$ , (b) the corresponding deflection of point  $B$ .

2.56 For the rod of Prob. 2.53, determine the maximum allowable temperature change if the stress in the steel portion  $AB$  is not to exceed 18 ksi and if the stress in the brass portion  $CB$  is not to exceed 7 ksi.

SOLUTION

Allowable force in each portion

$$AB: \sigma_{AB} = -18 \text{ ksi} = -18 \times 10^3 \text{ psi}, \quad A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (1.25)^2 = 1.2272 \text{ in}^2$$

$$P = \sigma_{AB} A_{AB} = (-18 \times 10^3)(1.2272) = -22.090 \times 10^3 \text{ lb.}$$

$$BC: \sigma_{BC} = -7 \text{ ksi} = -7 \times 10^3 \text{ psi}, \quad A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (2.25)^2 = 3.9761 \text{ in}^2$$

$$P = \sigma_{BC} A_{BC} = (-7 \times 10^3)(3.9761) = -27.833 \times 10^3 \text{ lb.}$$

Smaller absolute value governs  $\therefore P = -22.090 \times 10^3 \text{ lb.}$

Deformation due to  $P$

$$\delta_P = \frac{P L_{AB}}{E_{AB} A_{AB}} + \frac{P L_{BC}}{E_{BC} A_{BC}} = -\frac{(22.090 \times 10^3)(12)}{(29 \times 10^6)(1.2272)} - \frac{(22.090 \times 10^3)(15)}{(15 \times 10^6)(3.9761)}$$

$$= -13.004 \times 10^{-3} \text{ in}$$

Free thermal expansion

$$\delta_T = L_{AB} \alpha_s (\Delta T) + L_{BC} \alpha_b (\Delta T) = (12)(6.5 \times 10^{-6})(\Delta T) + (15)(10.4 \times 10^{-6})(\Delta T)$$

$$= (234 \times 10^{-6})(\Delta T)$$

Total deformation is zero

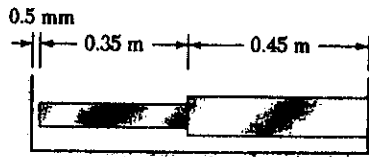
$$\delta_T + \delta_P = (234 \times 10^{-6})(\Delta T) - 13.004 \times 10^{-3} = 0$$

$$\Delta T = 55.6^\circ\text{F}$$



PROBLEM 2.57

2.57 Determine (a) the compressive force in the bars shown after a temperature rise of 96°C, (b) the corresponding change in length of the bronze bar.

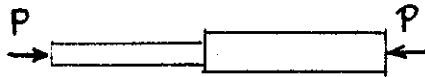


Bronze	Aluminum
$A = 1500 \text{ mm}^2$	$A = 1800 \text{ mm}^2$
$E = 105 \text{ GPa}$	$E = 73 \text{ GPa}$
$\alpha = 21.6 \times 10^{-6}/^\circ\text{C}$	$\alpha = 23.2 \times 10^{-6}/^\circ\text{C}$

SOLUTION

Calculate free thermal expansion

$$\begin{aligned} \delta_T &= L_b \alpha_b (\Delta T) + L_a \alpha_a \Delta T \\ &= (0.35)(21.6 \times 10^{-6})(96) + (0.45)(23.2 \times 10^{-6})(96) \\ &= 1.728 \times 10^{-3} \text{ m} \end{aligned}$$



Constrained expansion

$$\delta = 0.5 \text{ mm} = 0.500 \times 10^{-3} \text{ m}$$

Shortening due to induced compressive force P

$$\delta_P = 1.728 \times 10^{-3} - 0.500 \times 10^{-3} = 1.228 \times 10^{-3} \text{ m}$$

But, in terms of P

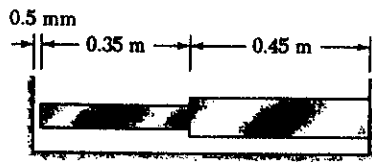
$$\begin{aligned} \delta_P &= \frac{PL_b}{A_b E_b} + \frac{PL_a}{A_a E_a} = \left( \frac{L_b}{A_b E_b} + \frac{L_a}{A_a E_a} \right) P \\ &= \left( \frac{0.35}{(1500 \times 10^{-6})(105 \times 10^9)} + \frac{0.45}{(1800 \times 10^{-6})(73 \times 10^9)} \right) P \\ &= 5.6496 \times 10^{-9} P \end{aligned}$$

(a) Equating  $5.6496 \times 10^{-9} P = 1.228 \times 10^{-3} \therefore P = 217.46 \times 10^3 \text{ N} = 217 \text{ kN}$  ◀

(b)  $\delta_b = L_b \alpha_b (\Delta T) - \frac{PL_b}{A_b E_b}$

$$\begin{aligned} &= (0.35)(21.6 \times 10^{-6})(96) - \frac{(217.46 \times 10^3)(0.35)}{(1500 \times 10^{-6})(105 \times 10^9)} \\ &= 725.76 \times 10^{-6} - 483.24 \times 10^{-6} = 242.5 \times 10^{-6} \text{ m} \\ &= 0.2425 \text{ mm} \end{aligned}$$
 ◀

## PROBLEM 2.58



Bronze	Aluminum
$A = 1500 \text{ mm}^2$	$A = 1800 \text{ mm}^2$
$E = 105 \text{ GPa}$	$E = 73 \text{ GPa}$
$\alpha = 21.6 \times 10^{-6}/^\circ\text{C}$	$\alpha = 23.2 \times 10^{-6}/^\circ\text{C}$

2.58 Knowing that a 0.5-mm gap exists when the temperature is  $20^\circ\text{C}$ , determine (a) the temperature at which the normal stress in the aluminum bar will be equal to  $-90 \text{ MPa}$ , (b) the corresponding exact length of the aluminum bar.

## SOLUTION

$$\sigma_a = -90 \times 10^6 \text{ Pa} \quad A_a = 1800 \times 10^{-6} \text{ m}^2$$

$$P = -\sigma_a A_a = (90 \times 10^6)(1800 \times 10^{-6}) = 162 \times 10^3 \text{ N}$$

Shortening due to  $P$

$$\begin{aligned} \delta_P &= \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a} \\ &= \frac{(162 \times 10^3)(0.35)}{(105 \times 10^9)(1500 \times 10^{-6})} + \frac{(162 \times 10^3)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})} \\ &= 914.79 \times 10^{-6} \text{ m} = 0.91479 \text{ mm} \end{aligned}$$

Available length for thermal expansion

$$\delta_T = 0.5 \text{ mm} + 0.91479 \text{ mm} = 1.41479 \text{ mm} = 1.41479 \times 10^{-3} \text{ m}$$

$$\text{But } \delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$$

$$= (0.35)(21.6 \times 10^{-6}) \Delta T + (0.45)(23.2 \times 10^{-6}) \Delta T$$

$$= 18.00 \times 10^{-6} (\Delta T)$$

$$\text{Equating } 18.00 \times 10^{-6} (\Delta T) = 1.41479 \times 10^{-3} \quad \therefore \Delta T = 78.6^\circ\text{C}$$

(a)

$$\begin{aligned} T_{\text{hot}} &= T_{\text{cold}} + \Delta T \\ &= 20 + 78.6 = 98.6^\circ\text{C} \end{aligned}$$

$$(b) \quad \delta_a = L_a \alpha_a (\Delta T) - \frac{PL_a}{E_a A_a}$$

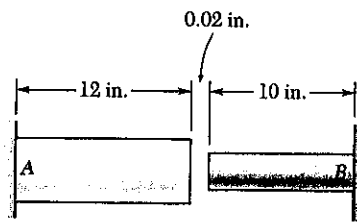
$$= (0.45)(23.2 \times 10^{-6})(78.6) - \frac{(162 \times 10^3)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})}$$

$$= 820.58 \times 10^{-6} - 554.79 \times 10^{-6} = 265.78 \times 10^{-6} \text{ m}$$

$$L_{\text{exact}} = L_a + \delta_a = 0.45 \text{ m} + 265.78 \times 10^{-6} \text{ m}$$

$$= 0.450266 \text{ m} = 450.0266 \text{ mm}$$

**PROBLEM 2.59**



Aluminum	Stainless steel
$A = 2.8 \text{ in}^2$	$A = 1.2 \text{ in}^2$
$E = 10.4 \times 10^6 \text{ psi}$	$E = 28.0 \times 10^6 \text{ psi}$
$\alpha = 13.3 \times 10^{-6}/^\circ\text{F}$	$\alpha = 9.6 \times 10^{-6}/^\circ\text{C}$

2.59 At room temperature ( $70^\circ\text{F}$ ) a 0.02-in. gap exists between the ends of the rods shown. At a later time when the temperature has reached  $320^\circ\text{F}$ , determine (a) the normal stress in the aluminum rod, (b) the change in length of the aluminum rod.

**SOLUTION**

$$\Delta T = 320 - 70 = 250^\circ\text{F}$$

Free thermal expansion

$$\begin{aligned} S_T &= L_a \alpha_a (\Delta T) + L_s \alpha_s (\Delta T) \\ &= (12)(13.3 \times 10^{-6})(250) + (10)(9.6 \times 10^{-6})(250) \\ &= 63.9 \times 10^{-3} \text{ in} = 0.0639 \text{ in.} \end{aligned}$$



Shortening due to P to meet constraint

$$S_P = 0.0639 - 0.02 = 0.0439 \text{ in.}$$

$$\begin{aligned} S_P &= \frac{PL_a}{A_a E_a} + \frac{PL_s}{A_s E_s} = \left( \frac{L_a}{A_a E_a} + \frac{L_s}{A_s E_s} \right) P \\ &= \left( \frac{12}{(2.8)(10.4 \times 10^6)} + \frac{10}{(1.2)(28.0 \times 10^6)} \right) P = 709.71 \times 10^{-9} P \end{aligned}$$

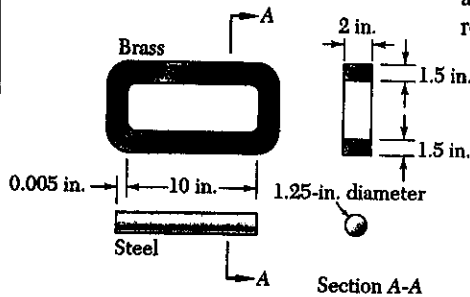
Equating  $709.71 \times 10^{-9} P = 0.0439$   $P = 61.857 \times 10^3 \text{ lb}$

(a)  $\sigma_a = -\frac{P}{A_a} = -\frac{61.857 \times 10^3}{2.8} = -22.09 \times 10^3 \text{ psi}$   
 $= -22.09 \text{ ksi}$

(b)  $S_a = L_a \alpha_a (\Delta T) - \frac{PL_a}{A_a E_a}$   
 $= (12)(13.3 \times 10^{-6})(250) - \frac{(61.857 \times 10^3)(12)}{(2.8)(10.4 \times 10^6)}$   
 $= 39.90 \times 10^{-3} - 25.49 \times 10^{-3} = 14.41 \times 10^{-3} \text{ in}$   
 $= 0.01441 \text{ in}$

PROBLEM 2.60

2.60 A brass link ( $E_b = 15 \times 10^6$  psi,  $\alpha_b = 10.4 \times 10^{-6}/^\circ\text{F}$ ) and a steel rod ( $E_s = 29 \times 10^6$  ksi,  $\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$ ) have the dimensions shown at a temperature of  $65^\circ\text{F}$ . The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to  $100^\circ\text{F}$ . Determine (a) the final normal stress in the steel rod, (b) the final length of the steel rod.



SOLUTION

$\Delta T$  associated with difference between final and initial dimensions

$$\Delta T = 100 - 65 = 35^\circ\text{F}$$

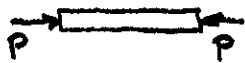
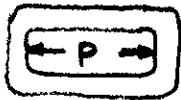
Free thermal expansion of each part

Brass link  $(\delta_T)_b = \alpha_b(\Delta T)(L) = (10.4 \times 10^{-6})(35)(10) = 3.64 \times 10^{-3}$  in

Steel rod  $(\delta_T)_s = \alpha_s(\Delta T)(L) = (6.5 \times 10^{-6})(35)(10) = 2.275 \times 10^{-3}$  in

At the final temperature the free length of the steel rod  
 $10.005 + 2.275 \times 10^{-3} - 3.64 \times 10^{-3} = 3.635 \times 10^{-3}$  in  
 longer than the brass link

Add equal but opposite forces  $P$  to elongate the brass link and contract the steel rod.



Brass link  $(\delta_P)_b = \frac{PL}{AE} = \frac{P(10)}{(2)(1.5)(2)(15 \times 10^6)}$   
 $= 111.11 \times 10^{-9} P$

Steel rod  $(\delta_P)_s = \frac{PL}{AE} = \frac{P(10)}{\frac{\pi}{4}(1.25)^2(29 \times 10^6)}$   
 $= 280.99 \times 10^{-9} P$

$$(\delta_P)_b + (\delta_P)_s = 3.635 \times 10^{-3}$$

$$(392.10 \times 10^{-9}) P = 3.635 \times 10^{-3} \quad P = 9.2705 \times 10^3 \text{ lb}$$

(a) Final stress in steel rod  $\sigma_s = -\frac{P}{A_s} = -\frac{9.2705 \times 10^3}{\frac{\pi}{4}(1.25)^2}$   
 $= -7.55 \times 10^3 \text{ psi} = -7.55 \text{ ksi} \blacktriangleleft$

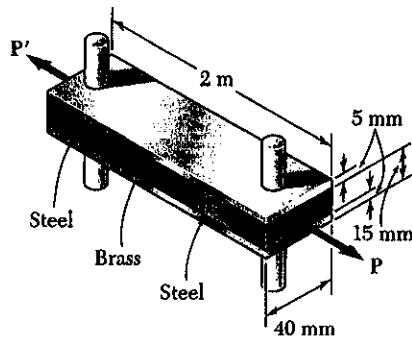
(b) Final length of steel rod

$$L_f = 10.000 + 0.005 + (\delta_T)_s - (\delta_P)_s$$

$$= 10.005 + 2.275 \times 10^{-3} - (280.99 \times 10^{-9})(9.2705 \times 10^3)$$

$$= 10.00467 \text{ in.} \blacktriangleleft$$

PROBLEM 2.61



2.61 Two steel bars ( $E_s = 200 \text{ GPa}$  and  $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$ ) are used to reinforce a brass bar ( $E_b = 105 \text{ GPa}$ ,  $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$ ) which is subjected to a load  $P = 25 \text{ kN}$ . When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made  $0.5 \text{ mm}$  smaller than the  $2 \text{ m}$  needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

SOLUTION

(a) Required temperature change for fabrication

$$\delta_T = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

(a) Temperature change required to expand steel bar by this amount

$$\delta_T = L \alpha_s \Delta T \quad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$$

$$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$

$$\Delta T = 21.368 \text{ }^\circ\text{C}$$

$$21.4 \text{ }^\circ\text{C}$$

(b) Once assembled, a tensile force  $P^*$  develops in the steel and a compressive force  $P^*$  develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel:  $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_s = \frac{FL}{A_s E_s} = \frac{P(2.00)}{(400 \times 10^{-6})(200 \times 10^9)} = 25 \times 10^{-9} P^*$$

Contraction of brass:  $A_b = (40)(15) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$$(\delta_p)_b = \frac{P^*L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^9)} = 31.746 \times 10^{-9} P^*$$

But  $(\delta_p)_s + (\delta_p)_b$  is equal to the initial amount of misfit

$$(\delta_p)_s + (\delta_p)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.811 \times 10^3 \text{ N}$$

Stresses due to fabrication

Steel:  $\sigma_s^* = \frac{P^*}{A_s} = \frac{8.811 \times 10^3}{400 \times 10^{-6}} = 22.03 \times 10^6 \text{ Pa} = 22.03 \text{ MPa}$

Brass:  $\sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811 \times 10^3}{600 \times 10^{-6}} = -14.68 \times 10^6 \text{ Pa} = -14.68 \text{ MPa}$

To these stresses must be added the stresses due to the  $25 \text{ kN}$  load.

continued

## Problem 2.61 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let  $\delta'$  be the additional displacement. Also, let  $P_s$  and  $P_b$  be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_b = \frac{A_s E_s}{L} \delta' = \frac{(400 \times 10^{-6})(200 \times 10^9)}{2.00} \delta' = 40 \times 10^6 \delta'$$

$$P_b = \frac{A_b E_b}{L} \delta' = \frac{(600 \times 10^{-6})(105 \times 10^9)}{2.00} \delta' = 31.5 \times 10^6 \delta'$$

$$\text{Total } P = P_s + P_b = 25 \times 10^3 \text{ N}$$

$$40 \times 10^6 \delta' + 31.5 \times 10^6 \delta' = 25 \times 10^3 \quad \delta' = 349.65 \times 10^{-6} \text{ m}$$

$$P_s = (40 \times 10^6)(349.65 \times 10^{-6}) = 13.986 \times 10^3 \text{ N}$$

$$P_b = (31.5 \times 10^6)(349.65 \times 10^{-6}) = 11.140 \times 10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986 \times 10^3}{400 \times 10^{-6}} = 34.97 \times 10^6 \text{ Pa}$$

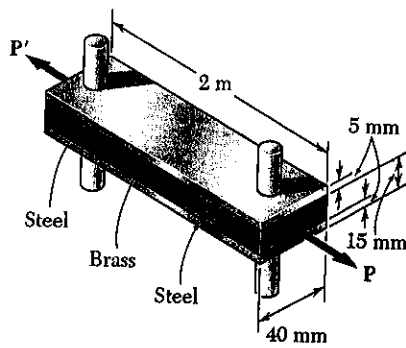
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140 \times 10^3}{600 \times 10^{-6}} = 18.36 \times 10^6 \text{ Pa}$$

Add stress due to fabrication

$$\sigma_s = 34.97 \times 10^6 + 22.03 \times 10^6 = 57.0 \times 10^6 \text{ Pa} = 57.0 \text{ MPa}$$

$$\sigma_b = 18.36 \times 10^6 - 14.68 \times 10^6 = 3.68 \times 10^6 \text{ Pa} = 3.68 \text{ MPa} \blacktriangleleft$$

**PROBLEM 2.62**



2.61 Two steel bars ( $E_s = 200 \text{ GPa}$  and  $\alpha_s = 11.7 \times 10^{-6}/^\circ\text{C}$ ) are used to reinforce a brass bar ( $E_b = 105 \text{ GPa}$ ,  $\alpha_b = 20.9 \times 10^{-6}/^\circ\text{C}$ ) which is subjected to a load  $P = 25 \text{ kN}$ . When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

2.62 Determine the maximum load  $P$  that may be applied to the brass bar of Prob. 2.61 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

**SOLUTION**

See solution to PROBLEM 3.61 to obtain the fabrication stresses

$$\sigma_s^* = 22.03 \text{ MPa} \qquad \sigma_b^* = -14.68 \text{ MPa}$$

Allowable stresses:  $\sigma_{s, \text{all}} = 30 \text{ MPa}$  ,  $\sigma_{b, \text{all}} = 25 \text{ MPa}$

Available stress increase from load

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$

$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains

$$\epsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$

$$\epsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs  $\therefore \epsilon = 39.85 \times 10^{-6}$

Areas:  $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$   
 $A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

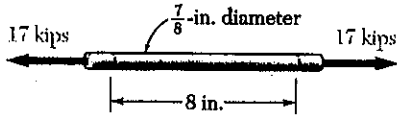
$$P_s = E_s A_s \epsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$$

$$P_b = E_b A_b \epsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^3 \text{ N}$$

Total allowable additional force

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N} \\ = 5.70 \text{ kN}$$

**PROBLEM 2.63**



2.63 In a standard tensile test a steel rod of  $\frac{7}{8}$ -in. diameter is subjected to a tension force of 17 kips. Knowing that  $\nu = 0.3$  and  $E = 29 \times 10^6$  psi, determine (a) the elongation of the rod in an 8-in. gage length, (b) the change in diameter of the rod.

**SOLUTION**

$$P = 17 \text{ kip} = 17 \times 10^3 \text{ lb.} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$

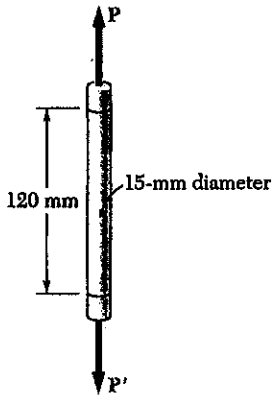
$$\sigma = \frac{P}{A} = \frac{17 \times 10^3}{0.60132} = 28.27 \times 10^3 \text{ psi} \quad \epsilon_x = \frac{\sigma}{E} = \frac{28.27 \times 10^3}{29 \times 10^6} = 974.9 \times 10^{-6}$$

$$\delta_x = L \epsilon_x = (8.0)(974.9 \times 10^{-6}) = 7.80 \times 10^{-3} \text{ in} = 0.00780 \text{ in.} \quad \blacktriangleleft$$

$$\epsilon_y = -\nu \epsilon_x = -(0.3)(974.9 \times 10^{-6}) = -292.5 \times 10^{-6}$$

$$\delta_y = d \epsilon_y = \left(\frac{7}{8}\right)(-292.5 \times 10^{-6}) = -256 \times 10^{-6} \text{ in} = -0.000256 \text{ in.} \quad \blacktriangleleft$$

**PROBLEM 2.64**



2.64 A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a 15-mm-diameter rod and it is subjected to a 3.5 kN tensile force. Knowing that an elongation of 11 mm and a decrease in diameter of 0.62 mm are observed in a 120-mm gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio of the material.

**SOLUTION**

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (15)^2 = 176.715 \text{ mm}^2 = 176.715 \times 10^{-6} \text{ m}^2$$

$$P = 3.5 \times 10^3 \text{ N}$$

$$\sigma = \frac{P}{A} = \frac{3.5 \times 10^3}{176.715 \times 10^{-6}} = 19.806 \times 10^6 \text{ Pa}$$

$$\epsilon_x = \frac{\delta_x}{L} = \frac{11}{120} = 91.667 \times 10^{-3}$$

$$E = \frac{\sigma}{\epsilon_x} = \frac{19.806 \times 10^6}{91.667 \times 10^{-3}} = 216 \times 10^6 \text{ Pa} = 216 \text{ MPa} \quad \blacktriangleleft$$

$$\delta_y = -0.62 \text{ mm}$$

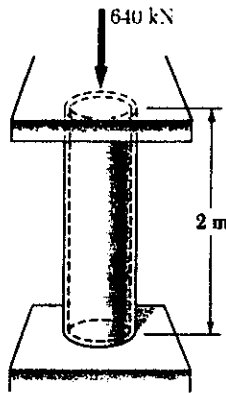
$$\epsilon_y = \frac{\delta_y}{d} = -\frac{0.62}{15} = -41.333 \times 10^{-3}$$

$$\nu = -\frac{\epsilon_y}{\epsilon_x} = \frac{41.333 \times 10^{-3}}{91.667 \times 10^{-3}} = 0.4509 \quad \blacktriangleleft$$

$$G = \frac{E}{2(1+\nu)} = \frac{216 \times 10^6}{2(1+0.4509)} = 74.5 \times 10^6 \text{ Pa} = 74.5 \text{ MPa} \quad \blacktriangleleft$$



**PROBLEM 2.65**



2.65 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that  $E = 73$  GPa and  $\nu = 0.33$ , determine (a) the change in length of the pipe, (b) the change in its outer diameter, (c) the change in its wall thickness.

**SOLUTION**

$$d_o = 240 \text{ mm} \quad t = 10 \text{ mm} \quad d_i = d_o - 2t = 220 \text{ mm}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3 \text{ mm}^2 \\ = 7.2257 \times 10^{-3} \text{ m}^2$$

$$P = 640 \times 10^3 \text{ N}$$

$$(a) \delta = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3} \text{ m} \\ = -2.43 \text{ mm}$$

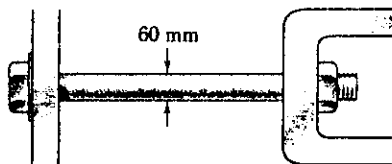
$$\epsilon = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$$

$$\epsilon_{\text{lat}} = -\nu \epsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$$

$$(b) \Delta d_o = d_o \epsilon_{\text{lat}} = (240)(400.4 \times 10^{-6}) = 0.0961 \text{ mm}$$

$$(c) \Delta t = t \epsilon_{\text{lat}} = (10)(400.4 \times 10^{-6}) = 0.00400 \text{ mm}$$

**PROBLEM 2.66**



2.66 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that  $E = 200$  GPa and  $\nu = 0.29$ , determine the internal force in the bolt, if the diameter is observed to decrease by 13  $\mu\text{m}$ .

**SOLUTION**

$$\delta_y = -13 \times 10^{-6} \text{ m} \quad d = 60 \times 10^{-3} \text{ m}$$

$$\epsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$$

$$\nu = -\frac{\epsilon_y}{\epsilon_x} \quad \therefore \epsilon_x = -\frac{\epsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$$

$$\sigma_x = E \epsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6 \text{ Pa}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (60)^2 = 2.827 \times 10^3 \text{ mm}^2 = 2.827 \times 10^{-3} \text{ m}^2$$

$$F = \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3 \text{ N} \\ = 422 \text{ kN}$$

**PROBLEM 2.67**



2.67 An aluminum plate ( $E = 74 \text{ GPa}$ ,  $\nu = 0.33$ ) plate is subjected to a centric axial load which causes a normal stress  $\sigma$ . Knowing that before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when  $\sigma = 125 \text{ MPa}$ .

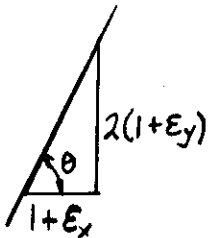
**SOLUTION**

The slope after deformation is  $\tan \theta = \frac{2(1+\epsilon_y)}{1+\epsilon_x}$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{125 \times 10^6}{74 \times 10^9} = 1.6892 \times 10^{-3}$$

$$\epsilon_y = -\nu \epsilon_x = -(0.33)(1.6892 \times 10^{-3}) = 0.5574 \times 10^{-3}$$

$$\tan \theta = \frac{2(1 - 0.0005574)}{1 + 0.0016892} = 1.99551$$



**PROBLEM 2.68**

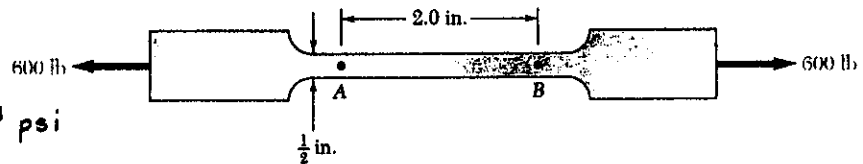
2.68 A 600 lb tensile load is applied to a test coupon made from  $\frac{1}{16}$  in. flat steel plate ( $E = 29 \times 10^6 \text{ psi}$ ,  $\nu = 0.30$ ). Determine the resulting change (a) in the 2.00-in. gage length, (b) in the width of portion AB of the test coupon, (c) in the thickness of portion AB, (d) in the cross-sectional area of portion AB.

**SOLUTION**

$$A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi}$$

$$\epsilon_x = \frac{\sigma}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$



$$(a) \Delta L_x = L_0 \epsilon_x = (2.0)(662.07 \times 10^{-6}) = 1.324 \times 10^{-3} \text{ in.}$$

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

$$(b) \Delta w_{width} = w_0 \epsilon_y = \left(\frac{1}{2}\right)(-198.62 \times 10^{-6}) = -99.3 \times 10^{-6} \text{ in.}$$

$$(c) \Delta t_{thickness} = t_0 \epsilon_z = \left(\frac{1}{16}\right)(-198.62 \times 10^{-6}) = -12.41 \times 10^{-6} \text{ in.}$$

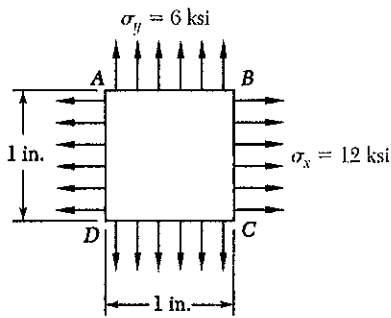
$$(d) A = wt = w_0(1 + \epsilon_y)t_0(1 + \epsilon_z) \\ = w_0 t_0 (1 + \epsilon_y + \epsilon_z + \epsilon_y \epsilon_z)$$

$$\Delta A = A - A_0 = w_0 t_0 (\epsilon_y + \epsilon_z + \epsilon_y \epsilon_z)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{16}\right)(-198.62 \times 10^{-6} - 198.62 \times 10^{-6} + \text{negligible term})$$

$$= -12.41 \times 10^{-6} \text{ in}^2$$

**PROBLEM 2.69**



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ , determine the change in length of (a) side  $AB$ , (b) side  $BC$ , (c) diagonal  $AC$ .

**SOLUTION**

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{29 \times 10^6} [12 \times 10^3 - (0.30)(6 \times 10^3)]$$

$$= 351.72 \times 10^{-6}$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{29 \times 10^6} [6 \times 10^3 - (0.30)(12 \times 10^3)]$$

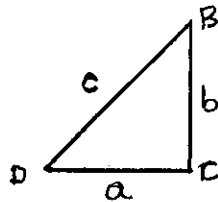
$$= 82.76 \times 10^{-6}$$

(a)  $\delta_{AB} = (\overline{AB})_0 \epsilon_x = (1.00)(351.72 \times 10^{-6}) = 351.7 \times 10^{-6}$  in. ▶

(b)  $\delta_{BC} = (\overline{BC})_0 \epsilon_y = (1.00)(82.76 \times 10^{-6}) = 82.8 \times 10^{-6}$  in. ▶

(c)  $(\overline{AC}) = \sqrt{(\overline{AB})^2 + (\overline{BC})^2} = \sqrt{(\overline{AB}_0 + \delta_{AB})^2 + (\overline{BC}_0 + \delta_{BC})^2}$   
 $= \sqrt{(1 + 352 \times 10^{-6})^2 + (1 + 82.8 \times 10^{-6})^2}$   
 $= 1.41452$   
 $(\overline{AC})_0 = \sqrt{2} \quad \overline{AC} - (\overline{AC})_0 = 307 \times 10^{-6}$  in. ▶

or use calculus as follows:



Label sides using  $a$ ,  $b$ , and  $c$  as shown

$$c^2 = a^2 + b^2$$

Obtain differentials  $2c \, dc = 2a \, da + 2b \, db$

from which  $dc = \frac{a}{c} da + \frac{b}{c} db$

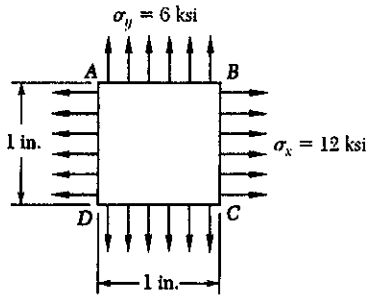
But  $a = 1.00$  in,  $b = 1.00$  in,  $c = \sqrt{2}$  in.

$$da = \delta_{AB} = 351.72 \times 10^{-6}$$
 in,  $db = \delta_{BC} = 82.8 \times 10^{-6}$  in

$$\delta_{AC} = dc = \frac{1.00}{\sqrt{2}} (351.7 \times 10^{-6}) + \frac{1.00}{\sqrt{2}} (82.8 \times 10^{-6})$$

$$= 307 \times 10^{-6}$$
 in. ▶

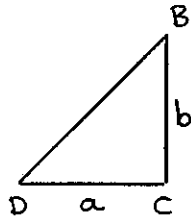
**PROBLEM 2.70**



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ , determine the change in length of (a) side  $AB$ , (b) side  $BC$ , (c) diagonal  $AC$ .

2.70 For the square of Prob. 2.69, determine the percent change in the slope of diagonal  $DB$  due to the pressurization of the vessel.

**SOLUTION**



Label sides  $a$  and  $b$  as shown.

The slope is  $s = \frac{b}{a}$

The change in slope is calculated from differential calculus

$$ds = \frac{a db - b da}{a^2} \quad \text{or} \quad \frac{ds}{s} = \frac{a}{b} \frac{adb - bda}{a^2} = \frac{db}{b} - \frac{da}{a}$$

$$\% \text{ change in slope} = \frac{ds}{s} \times 100\%$$

$$\frac{da}{a} = \epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{29 \times 10^6} [12 \times 10^3 - (0.29)(6 \times 10^3)]$$

$$= 351.72 \times 10^{-6}$$

$$\frac{db}{b} = \epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{29 \times 10^6} [6 \times 10^3 - (0.29)(12 \times 10^3)]$$

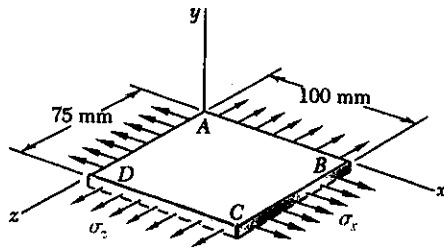
$$= 82.76 \times 10^{-6}$$

$$\frac{ds}{s} = 351.72 \times 10^{-6} - 82.76 \times 10^{-6} = 268.96 \times 10^{-6}$$

$$\% \text{ change in slope} = 268.96 \times 10^{-4} \%$$

$$= 0.0269 \%$$

PROBLEM 2.71



2.71 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses  $\sigma_x = 120 \text{ MPa}$  and  $\sigma_y = 160 \text{ MPa}$ . Knowing that the properties of the fabric can be approximated as  $E = 87 \text{ GPa}$  and  $\nu = 0.34$ , determine the change in length of (a) side  $AB$ , (b) side  $BC$ , (c) diagonal  $AC$ .

SOLUTION

$$\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}$$

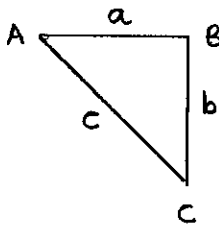
$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \\ &= \frac{1}{87 \times 10^9} [120 \times 10^6 - (0.34)(160 \times 10^6)] \\ &= 754.02 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) = \frac{1}{87 \times 10^9} [-(0.34)(120 \times 10^6) + 160 \times 10^6] \\ &= 1.3701 \times 10^{-3} \end{aligned}$$

(a)  $\delta_{AB} = (\overline{AB}) \epsilon_x = (100 \text{ mm})(754.02 \times 10^{-6}) = 0.0754 \text{ mm}$  ▶

(b)  $\delta_{BC} = (\overline{BC}) \epsilon_z = (75 \text{ mm})(1.3701 \times 10^{-3}) = 0.1028 \text{ mm}$  ▶

(c)



Label sides of right triangle  $ABC$  as  $a$ ,  $b$ , and  $c$

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus

$$2c \, dc = 2a \, da + 2b \, db$$

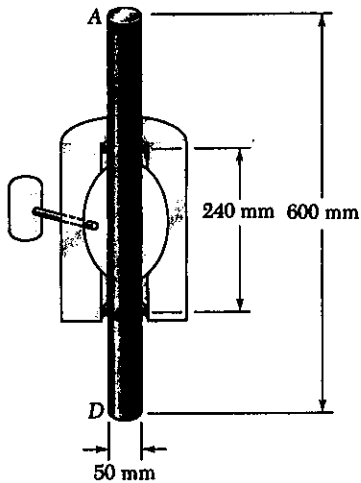
$$dc = \frac{a}{c} \, da + \frac{b}{c} \, db$$

But  $a = 100 \text{ mm}$ ,  $b = 75 \text{ mm}$ ,  $c = \sqrt{100^2 + 75^2} = 125 \text{ mm}$

$$da = \delta_{AB} = 0.0754 \text{ mm} \quad db = \delta_{BC} = 0.1028 \text{ mm}$$

$$\delta_{AC} = dc = \frac{100}{125} (0.0754) + \frac{75}{125} (0.1028) = 0.1220 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 2.72**



2.72 The brass rod  $AD$  is fitted with a jacket that is used to apply an hydrostatic pressure of 48 MPa to the 250-mm portion  $BC$  of the rod. Knowing that  $E = 105$  GPa, and  $\nu = 0.33$ , determine (a) the change in the total length  $AD$ , (b) the change in diameter of portion  $BC$  of the rod.

**SOLUTION**

$$\sigma_x = \sigma_z = -p = -48 \times 10^6 \text{ Pa}, \quad \sigma_y = 0$$

$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) \\ &= \frac{1}{105 \times 10^9} [-48 \times 10^6 - (0.33)(0) - (0.33)(-48 \times 10^6)] \\ &= -306.29 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) \\ &= \frac{1}{105 \times 10^9} [-(0.33)(-48 \times 10^6) + 0 - (0.33)(-48 \times 10^6)] \\ &= -301.71 \times 10^{-6} \end{aligned}$$

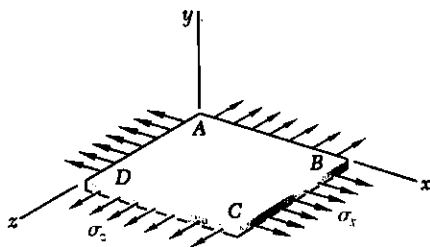
(a) Change in length: Only portion  $BC$  is strained.  $L = 240$  mm

$$\Delta y = L \epsilon_y = (240)(-301.71 \times 10^{-6}) = -0.0724 \text{ mm}$$

(b) Change in diameter:  $d = 50$  mm

$$\Delta x = \Delta z = d \epsilon_x = (50)(-306.29 \times 10^{-6}) = -0.01531 \text{ mm}$$

**PROBLEM 2.73**



2.73 The homogeneous plate  $ABCD$  is subjected to a biaxial loading as shown. It is known that  $\sigma_x = \sigma_0$  and that the change in length of the plate in the  $x$  direction must be zero, that is,  $\epsilon_x = 0$ . Denoting by  $E$  the modulus of elasticity and by  $\nu$  Poisson's ratio, determine (a) the required magnitude of  $\sigma_2$ , (b) the ratio  $\sigma_0 / \epsilon_2$ .

$$\sigma_x = \sigma_0, \quad \sigma_y = 0, \quad \epsilon_x = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{1}{E} (\sigma_0 - \nu \sigma_2)$$

$$(a) \quad \sigma_x = \nu \sigma_2$$

$$(b) \quad \epsilon_2 = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_2) = \frac{1}{E} (-\nu^2 \sigma_0 - 0 + \sigma_2) = \frac{1-\nu^2}{E} \sigma_0$$

$$\frac{\sigma_0}{\epsilon_2} = \frac{E}{1-\nu^2}$$

**PROBLEM 2.74**

2.74 For a member under axial loading, express the normal strain  $\epsilon'$  in a direction forming an angle of  $45^\circ$  with the axis of the load in terms of the axial strain  $\epsilon_x$  by (a) comparing the hypotenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (b) using the values of the corresponding stresses  $\sigma'$  and  $\sigma_x$  shown in Fig. 1.40, and the generalized Hooke's law.

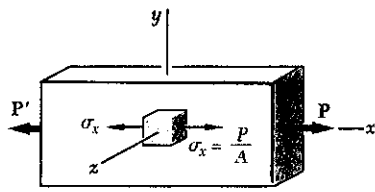


Fig 1.40 (a)

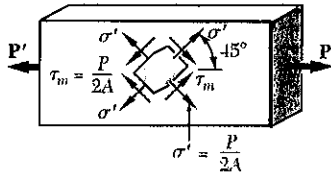
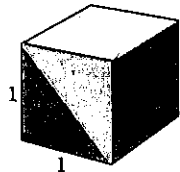
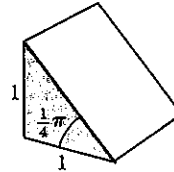


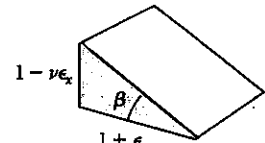
Fig 1.40 (b)



(a)



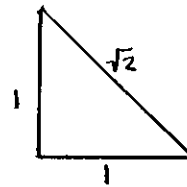
(b)



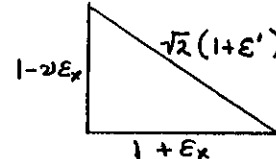
(c)

Fig 2.54

**SOLUTION**



Before deformation



After deformation

(a)

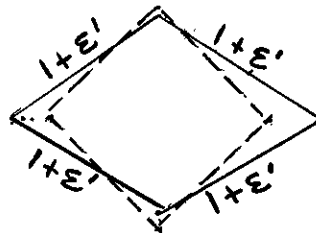
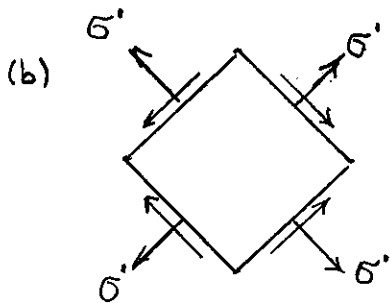
$$[\sqrt{2}(1 + \epsilon')]^2 = (1 + \epsilon_x)^2 + (1 - 2\nu\epsilon_x)^2$$

$$2(1 + 2\epsilon' + \epsilon'^2) = 1 + 2\epsilon_x + \epsilon_x^2 + 1 - 2\nu\epsilon_x + \nu^2\epsilon_x^2$$

$$4\epsilon' + 2\epsilon'^2 = 2\epsilon_x + \epsilon_x^2 - 2\nu\epsilon_x + \nu^2\epsilon_x^2$$

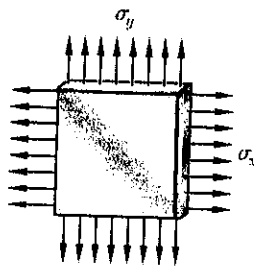
Neglect squares as small  $4\epsilon' = 2\epsilon_x - 2\nu\epsilon_x$

$$\epsilon' = \frac{1 - \nu}{2} \epsilon_x$$



$$\begin{aligned} \epsilon' &= \frac{\sigma'}{E} - \frac{\nu\sigma'}{E} \\ &= \frac{1 - \nu}{E} \cdot \frac{P}{2A} \\ &= \frac{1 - \nu}{2E} \sigma_x \\ &= \frac{1 - \nu}{2} \epsilon_x \end{aligned}$$

**PROBLEM 2.75**



2.75 In many situations it is known that the normal stress in a given direction is zero, for example  $\sigma_z = 0$  in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains  $\epsilon_x$  and  $\epsilon_y$  have been determined experimentally, we can express  $\sigma_x$ ,  $\sigma_y$  and  $\epsilon_z$  as follows:

$$\sigma_x = E \frac{\epsilon_x + \nu \epsilon_y}{1 - \nu^2} \quad \sigma_y = E \frac{\epsilon_y + \nu \epsilon_x}{1 - \nu^2} \quad \epsilon_z = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y)$$

**SOLUTION**

$$\sigma_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) \quad (1) \quad \epsilon_y = \frac{1}{E} (-\nu \sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by  $\nu$  and adding to (1)

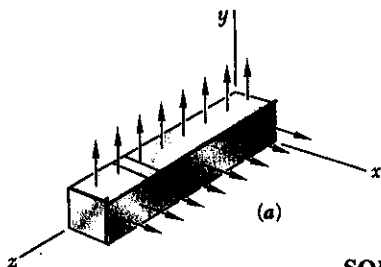
$$\epsilon_x + \nu \epsilon_y = \frac{1 - \nu^2}{E} \sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y) \quad \blacktriangleleft$$

Multiplying (1) by  $\nu$  and adding to (2)

$$\epsilon_y + \nu \epsilon_x = \frac{1 - \nu^2}{E} \sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1 - \nu^2} (\epsilon_y + \nu \epsilon_x) \quad \blacktriangleleft$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y) = -\frac{\nu}{E} \cdot \frac{E}{1 - \nu^2} (\epsilon_x + \nu \epsilon_y + \epsilon_y + \nu \epsilon_x) \\ &= -\frac{\nu(1 + \nu)}{1 - \nu^2} (\epsilon_x + \epsilon_y) = -\frac{\nu}{1 - \nu} (\epsilon_x + \epsilon_y) \quad \blacktriangleleft \end{aligned}$$

**PROBLEM 2.76**



2.76 In many situations physical constraints prevent strain from occurring in a given direction, for example  $\epsilon_z = 0$  in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express  $\sigma_z$ ,  $\epsilon_x$  and  $\epsilon_y$  as follows:

$$\begin{aligned} \sigma_z &= \nu(\sigma_x + \sigma_y) \\ \epsilon_x &= \frac{1}{E} [(1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y] \quad \epsilon_y = \frac{1}{E} [(1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x] \end{aligned}$$

**SOLUTION**

$$\epsilon_z = 0 = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y) \quad \blacktriangleleft$$

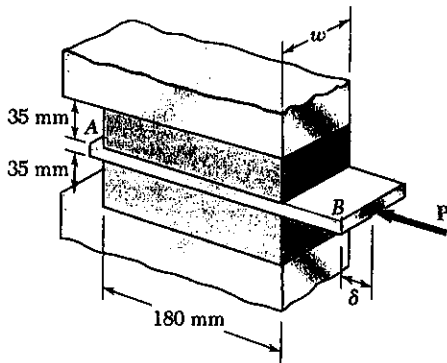
$$\begin{aligned} \epsilon_x &= \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{1}{E} [\sigma_x - \nu \sigma_y - \nu^2 (\sigma_x + \sigma_y)] \\ &= \frac{1}{E} [(1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y] \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = \frac{1}{E} [-\nu \sigma_x + \sigma_y - \nu^2 (\sigma_x + \sigma_y)] \\ &= \frac{1}{E} [(1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x] \quad \blacktriangleleft \end{aligned}$$



**PROBLEM 2.77**

2.77 Two blocks of rubber, each of width  $w = 60$  mm, are bonded to rigid supports and to the movable plate  $AB$ . Knowing that a force of magnitude  $P = 19$  kN causes a deflection  $\delta = 3$  mm, determine the modulus of rigidity of the rubber used.



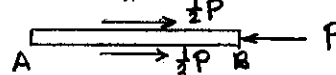
**SOLUTION**

Consider upper block of rubber. The force carried is  $\frac{1}{2}P$ .



The shearing stress is

$$\tau = \frac{\frac{1}{2}P}{A}$$



where  $A = (180 \text{ mm})(60 \text{ mm}) = 10.8 \times 10^3 \text{ mm}^2 = 10.8 \times 10^{-3} \text{ m}^2$

$P = 19 \times 10^3 \text{ N}$

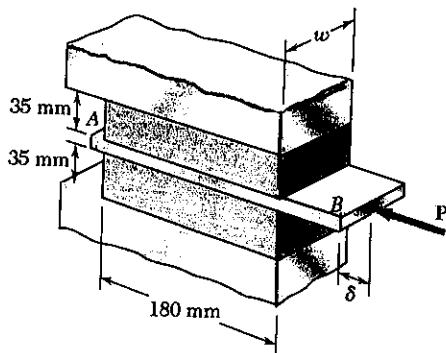
$\tau = \frac{(\frac{1}{2})(19 \times 10^3)}{10.8 \times 10^{-3}} = 0.87963 \times 10^6 \text{ Pa}$

$\gamma = \frac{S}{h} = \frac{3 \text{ mm}}{35 \text{ mm}} = 0.085714$

$G = \frac{\tau}{\gamma} = \frac{0.87963 \times 10^6}{0.085714} = 10.26 \times 10^6 \text{ Pa} = 10.26 \text{ MPa}$

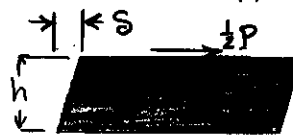
**PROBLEM 2.78**

2.78 Two blocks of rubber, for which  $G = 7.5$  MPa, are bonded to rigid supports and to the movable plate  $AB$ . Knowing that the width of each block is  $w = 80$  mm, determine the effective spring constant,  $k = P/\delta$ , of the system.



**SOLUTION**

Consider the upper block of rubber. The force carried is  $\frac{1}{2}P$ .



The shearing stress is

$$\tau = \frac{\frac{1}{2}P}{A}$$

from which

$$P = 2A\tau$$

The shearing strain is  $\gamma = \frac{S}{h}$  from which  $S = h\gamma$

Effective spring constant  $k = \frac{P}{\delta} = \frac{2A\tau}{h\gamma}$

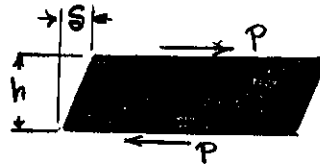
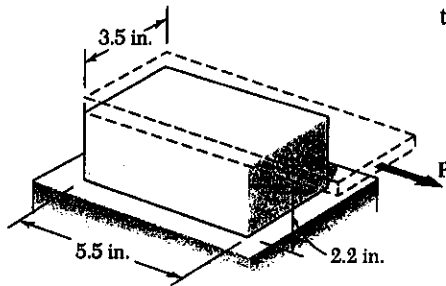
Noting that  $\tau = G\gamma$ ,

$k = \frac{2AG}{h} = \frac{(2)(0.180)(0.080)(7.5 \times 10^6)}{0.035} = 6.17 \times 10^6 \text{ N/m}$

$6.17 \times 10^3 \text{ kN/m}$

**PROBLEM 2.79**

2.79 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force  $P$  is applied. Knowing that for the plastic used  $G = 55$  ksi, determine the deflection of the plate when  $P = 9$  kips.



Consider the plastic block. The shearing force carried is  $P = 9 \times 10^3$  lb.

The area is  $A = (3.5)(5.5) = 19.25 \text{ in}^2$

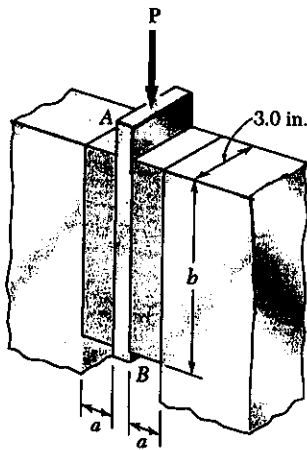
Shearing stress  $\tau = \frac{P}{A} = \frac{9 \times 10^3}{19.25} = 467.52 \text{ psi}$

Shearing strain  $\gamma = \frac{\tau}{G} = \frac{467.52}{55 \times 10^3} = 0.0085006$

But  $\gamma = \frac{S}{h} \therefore S = h \gamma = (2.2)(0.0085006) = 0.0187 \text{ in.}$

**PROBLEM 2.80**

2.80 A vibration isolation unit consists of two blocks of hard rubber bonded to plate  $AB$  and to rigid supports as shown. For the type and grade of rubber used  $\tau_{all} = 220$  psi and  $G = 1800$  psi. Knowing that a centric vertical force of magnitude  $P = 3.2$  kips must cause a 0.1 in. vertical deflection of the plate  $AB$ , determine the smallest allowable dimensions  $a$  and  $b$  of the block.



**SOLUTION**

Consider the rubber block on the right. It carries a shearing force equal to  $\frac{1}{2}P$ .

The shearing stress is  $\tau = \frac{\frac{1}{2}P}{A}$

or required  $A = \frac{P}{2\tau} = \frac{3.2 \times 10^3}{(2)(220)} = 7.2727 \text{ in}^2$

But  $A = (3.0)b$

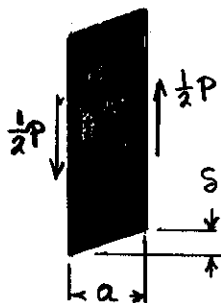
Hence  $b = \frac{A}{3.0} = 2.42 \text{ in.}$

Use  $b = 2.42 \text{ in}$  and  $\tau = 220 \text{ psi}$

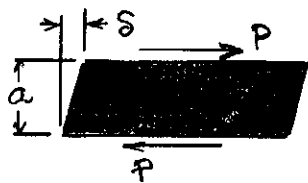
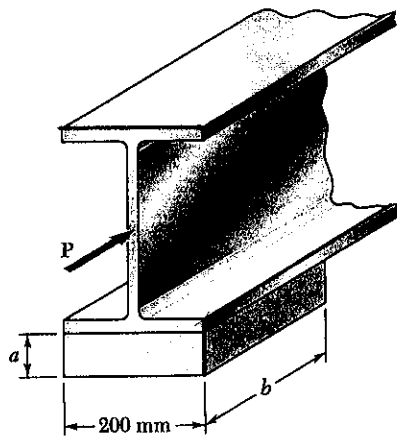
Shearing strain  $\gamma = \frac{\tau}{G} = \frac{220}{1800} = 0.12222$

But  $\gamma = \frac{S}{a}$

Hence  $a = \frac{S}{\gamma} = \frac{0.1}{0.12222} = 0.818 \text{ in.}$



PROBLEM 2.81



But  $\gamma = \frac{\delta}{a} \therefore$

2.81 An elastomeric bearing ( $G = 0.9 \text{ MPa}$ ) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN lateral load is applied as shown. Determine (a) the smallest allowable dimension  $b$ , (b) the smallest required thickness  $a$  if the maximum allowable shearing stress is 420 kPa.

SOLUTION

Shearing force  $P = 22 \times 10^3 \text{ N}$

Shearing stress  $\tau = 420 \times 10^3 \text{ Pa}$

$$\tau = \frac{P}{A} \therefore A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3} \text{ m}^2$$

$$= 52.381 \times 10^3 \text{ mm}^2$$

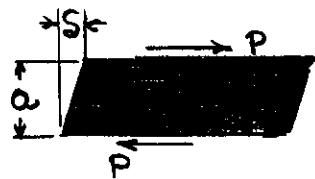
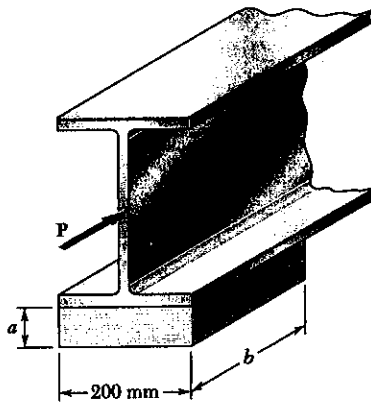
$A = (200 \text{ mm})(b)$

$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262 \text{ mm}$

$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$

$a = \frac{\delta}{\gamma} = \frac{10 \text{ mm}}{466.67 \times 10^{-3}} = 21.4 \text{ mm}$

PROBLEM 2.82



2.82 For the elastomeric bearing in Prob. 2.81 with  $b = 220 \text{ mm}$  and  $a = 30 \text{ mm}$ , determine the shearing modulus  $G$  and the shear stress  $\tau$  for a maximum lateral load  $P = 19 \text{ kN}$  and a maximum displacement  $\delta = 12 \text{ mm}$ .

SOLUTION

Shearing force  $P = 19 \times 10^3 \text{ N}$

Area  $A = (200 \text{ mm})(220 \text{ mm}) = 44 \times 10^3 \text{ mm}^2$   
 $= 44 \times 10^{-3} \text{ m}^2$

$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^3 \text{ Pa}$   
 $= 431 \text{ kPa}$

Shearing strain  $\gamma = \frac{\delta}{a} = \frac{12 \text{ mm}}{30 \text{ mm}} = 0.400$

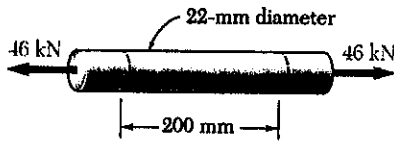
Shearing modulus

$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6 \text{ Pa}$

$= 1.080 \text{ MPa}$

**PROBLEM 2.83**

\*2.83 Determine the dilatation  $e$  and the change in volume of the 200-mm length of the rod shown if (a) the rod is made of steel with  $E = 200$  GPa and  $\nu = 0.30$ , (b) the rod is made of aluminum with  $E = 70$  GPa and  $\nu = 0.35$ .



**SOLUTION**

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$P = 46 \times 10^3 \text{ N} \quad \sigma_x = \frac{P}{A} = 121.01 \times 10^6 \text{ Pa} \quad \sigma_y = \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x}{E} \quad \epsilon_y = \epsilon_z = -\nu \epsilon_x = -\nu \frac{\sigma_x}{E}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E} (\sigma_x - \nu \sigma_x - \nu \sigma_x) = \frac{(1 - 2\nu) \sigma_x}{E}$$

$$\text{Volume } \mathcal{V} = AL = (380.13 \text{ mm}^2)(200 \text{ mm}) = 76.026 \times 10^3 \text{ mm}^3$$

$$\Delta \mathcal{V} = \mathcal{V} e$$

(a) steel:  $e = \frac{(0.4)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6}$

$$\Delta \mathcal{V} = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40 \text{ mm}^3$$

(b) aluminum:  $e = \frac{(0.3)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6}$

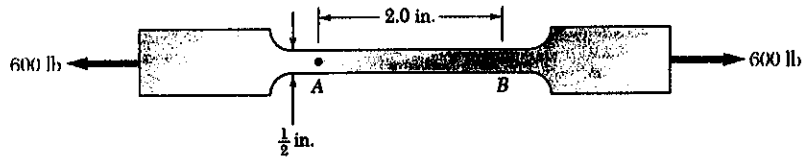
$$\Delta \mathcal{V} = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4 \text{ mm}^3$$

PROBLEM 2.84

\*2.84 Determine the change in volume of the 2-in. gage length segment  $AB$  in Prob. 2.68 (a) by computing the dilatation of the material, (b) by subtracting the original volume of portion  $AB$  from its final volume.

From PROBLEM 2.68

thickness =  $\frac{1}{16}$  in  
 $E = 29 \times 10^6$  psi  
 $\nu = 0.30$



SOLUTION

(a)  $A = (\frac{1}{2})(\frac{1}{16}) = 0.03125 \text{ in}^2$

Volume:  $\mathcal{V}_0 = A L_0 = (0.03125)(2.00) = 0.0625 \text{ in}^3$

$\sigma_x = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi}$        $\sigma_y = \sigma_z = 0$

$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = \frac{\sigma_x}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$

$\epsilon_y = \epsilon_z = -\nu \epsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$

$e = \epsilon_x + \epsilon_y + \epsilon_z = 264.83 \times 10^{-6}$

$\Delta \mathcal{V} = \mathcal{V}_0 e = (0.0625)(264.83 \times 10^{-6}) = 16.55 \times 10^{-6} \text{ in}^3$       ◀

(b) From the solution to PROBLEM 2.68

$\delta_x = 1.324 \times 10^{-3} \text{ in}$ ,     $\delta_y = -99.3 \times 10^{-6} \text{ in}$      $\delta_z = -12.41 \times 10^{-6} \text{ in}$ .

The dimensions when under a 600 lb tensile load are:

length  $L = L_0 + \delta_x = 2 + 1.324 \times 10^{-3} = 2.001324 \text{ in}$ .

width  $W = W_0 + \delta_y = \frac{1}{2} - 99.3 \times 10^{-6} = 0.4999007 \text{ in}$ .

thickness  $t = t_0 + \delta_z = \frac{1}{16} - 12.41 \times 10^{-6} = 0.06248759 \text{ in}$

volume  $\mathcal{V} = L W t = 0.062516539 \text{ in}^3$

$\Delta \mathcal{V} = \mathcal{V} - \mathcal{V}_0 = 0.062516539 - 0.0625 = 16.54 \times 10^{-6} \text{ in}^3$       ◀

## PROBLEM 2.85

\*2.85 A 6-in. diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ , determine (a) the decrease in diameter of the sphere, (b) the decrease in volume of the sphere, (c) the percent increase in the density of the sphere.

## SOLUTION

$$\text{For a solid sphere } V_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6} (6.00)^3 = 113.097 \text{ in}^3$$

$$\sigma_x = \sigma_y = \sigma_z = -p = -7.1 \times 10^3 \text{ psi}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y - \nu \sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(7.1 \times 10^3)}{29 \times 10^6} = -97.93 \times 10^{-6}$$

$$\text{Likewise } \epsilon_y = \epsilon_z = -97.93 \times 10^{-6}$$

$$e = \epsilon_x + \epsilon_y + \epsilon_z = -293.79 \times 10^{-6}$$

$$(a) \quad -\Delta d = -d_0 \epsilon_x = -(6.00)(97.93 \times 10^{-6}) = 588 \times 10^{-6} \text{ in.} \quad \blacktriangleleft$$

$$(b) \quad -\Delta V = -V_0 e = -(113.097)(-293.79 \times 10^{-6}) = 33.2 \times 10^{-3} \text{ in}^3 \quad \blacktriangleleft$$

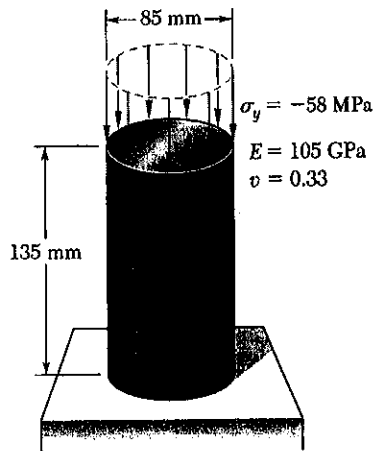
$$(c) \quad \text{Let } m = \text{mass of sphere} \quad m = \text{constant.}$$

$$m = \rho_0 V_0 = \rho V = \rho V_0 (1+e)$$

$$\begin{aligned} \frac{\rho - \rho_0}{\rho_0} &= \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)} \cdot \frac{V_0}{m} - 1 = \frac{1}{1+e} - 1 \\ &= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots \\ &\approx -e = 293.79 \times 10^{-6} \end{aligned}$$

$$\frac{\rho - \rho_0}{\rho_0} \times 100\% = (293.79 \times 10^{-6})(100\%) = 0.0294\% \quad \blacktriangleleft$$

**PROBLEM 2.86**



\*2.86 (a) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (b) Solve part a assuming that the loading is hydrostatic with  $\sigma_x = \sigma_y = \sigma_z = -70 \text{ MPa}$ .

**SOLUTION**

$$h_0 = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4} (85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_0 = A_0 h_0 = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

$$(a) \quad \sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = \frac{\sigma_y}{E} \\ &= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6} \end{aligned}$$

$$\Delta h = h_0 \epsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm} \quad \blacktriangleleft$$

$$\begin{aligned} e &= \frac{1-2\nu}{E} (\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} \\ &= -187.81 \times 10^{-6} \end{aligned}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3 \quad \blacktriangleleft$$

$$(b) \quad \sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \quad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E} (-\nu \sigma_x + \sigma_y - \nu \sigma_z) = \frac{1-2\nu}{E} \sigma_y \\ &= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6} \end{aligned}$$

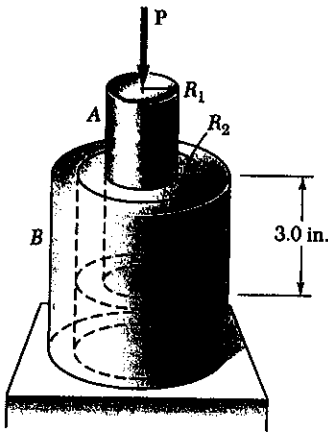
$$\Delta h = h_0 \epsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm} \quad \blacktriangleleft$$

$$e = \frac{1-2\nu}{E} (\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3 \quad \blacktriangleleft$$

**PROBLEM 2.87**

\*2.87 A vibration isolation support consists of a rod *A* of radius  $R_1 = \frac{3}{8}$ -in. and a tube *B* of inner radius  $R_2 = 1$  in. bonded to a 3-in.-long hollow rubber cylinder with a modulus of rigidity  $G = 1.8$  ksi. Determine the largest allowable force *P* which may be applied to rod *A* if its deflection is not to exceed 0.1 in.



**SOLUTION**

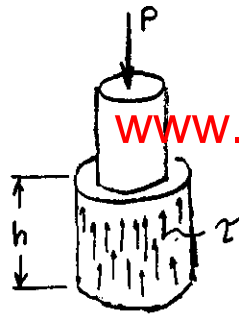
Let *r* be a radial coordinate. Over the hollow rubber cylinder  $R_1 \leq r \leq R_2$

Shearing stress  $\tau$  acting on a cylindrical surface of radius *r* is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length *dr*

$$\frac{dS}{dr} = \gamma$$

$$dS = \gamma dr = \frac{P}{2\pi G h} \frac{dr}{r}$$

Total deformation

$$S = \int_{R_1}^{R_2} dS = \frac{P}{2\pi G h} \int_{R_1}^{R_2} \frac{dr}{r}$$

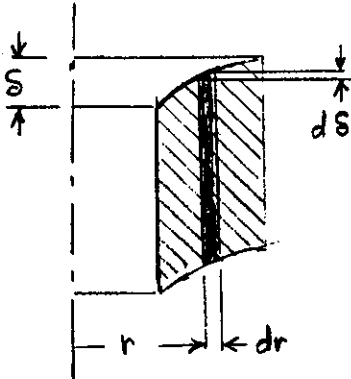
$$= \frac{P}{2\pi G h} \ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h} (\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h} \ln \frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h S}{\ln(R_2/R_1)}$$

Data:  $R_1 = \frac{3}{8}$  in = 0.375 in.,  $R_2 = 1.0$  in.,  $h = 3.0$  in

$G = 1.8 \times 10^3$  psi,  $S = 0.1$  in

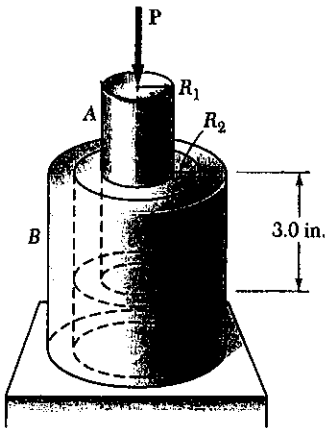
$$P = \frac{(2\pi)(1.8 \times 10^3)(3.0)(0.1)}{\ln(1.0/0.375)} = 3.46 \times 10^3 \text{ lb} = 3.46 \text{ kips}$$





**PROBLEM 2.88**

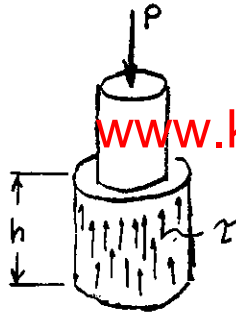
\*2.88 A vibration isolation support consists of a rod  $A$  of radius  $R_1$  and a tube  $B$  of inner radius  $R_2$  bonded to a 3-in.-long hollow rubber cylinder with a modulus of rigidity  $G = 1.6$  ksi. Determine the required value of the ratio  $R_2/R_1$  if a 2-kip force  $P$  is to cause a 0.12-in. deflection of rod  $A$ .



**SOLUTION**

Let  $r$  be a radial coordinate. Over the hollow rubber cylinder  $R_1 \leq r \leq R_2$

Shearing stress  $\tau$  acting on a cylindrical surface of radius  $r$  is



$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$

Shearing deformation over radial length  $dr$

$$\frac{dS}{dr} = \gamma$$

$$dS = \gamma dr = \frac{P}{2\pi G h} \frac{dr}{r}$$

Total deformation

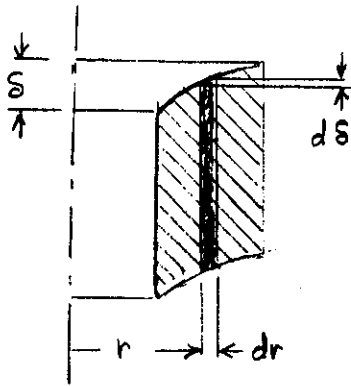
$$S = \int_{R_1}^{R_2} dS = \frac{P}{2\pi G h} \int_{R_1}^{R_2} \frac{dr}{r}$$

$$= \frac{P}{2\pi G h} \ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi G h} (\ln R_2 - \ln R_1)$$

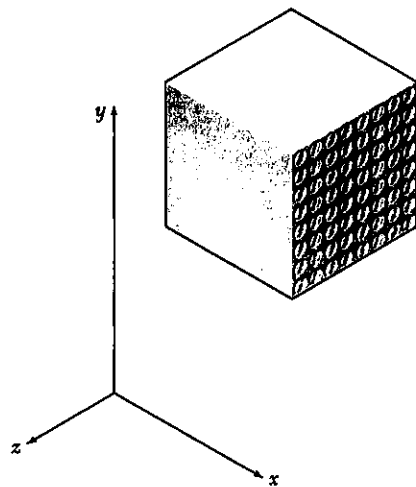
$$= \frac{P}{2\pi G h} \ln \frac{R_2}{R_1}$$

$$\ln \frac{R_2}{R_1} = \frac{2\pi G h S}{P} = \frac{(2\pi)(1.6 \times 10^3)(3.0)(0.12)}{2 \times 10^3} = 1.8096$$

$$\frac{R_2}{R_1} = \exp(1.8096) = 6.11$$



**PROBLEM 2.89**



\*2.89 A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the  $x$  direction. The cube is constrained against deformations in the  $y$  and  $z$  directions and is subjected to a tensile load of 65 kN in the  $x$  direction. Determine (a) the change in the length of the cube in the  $x$  direction, (b) the stresses  $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_z$ .

$$\begin{aligned} E_x &= 50 \text{ GPa} & \nu_{xz} &= 0.254 \\ E_y &= 15.2 \text{ GPa} & \nu_{xy} &= 0.254 \\ E_z &= 15.2 \text{ GPa} & \nu_{zy} &= 0.428 \end{aligned}$$

**SOLUTION**

Stress-to-strain equations are

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1) \qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2) \qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3) \qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

The constraint conditions are  $\epsilon_y = 0$  and  $\epsilon_z = 0$ .

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y} \sigma_y - \frac{\nu_{zy}}{E_z} \sigma_z = \frac{\nu_{yx}}{E_x} \sigma_x \quad (7)$$

$$-\frac{\nu_{yz}}{E_y} \sigma_y + \frac{1}{E_z} \sigma_z = \frac{\nu_{xz}}{E_x} \sigma_x \quad (8)$$

$$\begin{aligned} \frac{1}{15.2} \sigma_y - \frac{0.428}{15.2} \sigma_z &= \frac{0.254}{50} \sigma_x \quad \text{or} \quad \sigma_y - 0.428 \sigma_z = 0.077216 \sigma_x \\ -\frac{0.428}{15.2} \sigma_y + \frac{1}{15.2} \sigma_z &= \frac{0.254}{50} \sigma_x \quad \text{or} \quad -0.428 \sigma_y + \sigma_z = 0.077216 \sigma_x \end{aligned}$$

Solving simultaneously  $\sigma_y = \sigma_z = 0.134993 \sigma_x$

Using (4) and (5) in (1)  $\epsilon_x = \frac{1}{E_x} \sigma_x - \frac{\nu_{yx}}{E_x} \sigma_y - \frac{\nu_{zx}}{E_x} \sigma_z$

$$\begin{aligned} \epsilon_x &= \frac{1}{E_x} \left[ 1 - (0.254)(0.134993) - (0.254)(0.134993) \right] \sigma_x \\ &= \frac{0.93142}{E_x} \sigma_x \end{aligned}$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65 \times 10^3}{1600 \times 10^{-6}} = 40.625 \times 10^6 \text{ Pa}$$

continued

Problem 2.89 continued

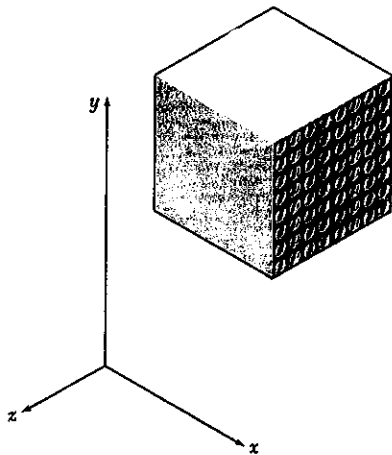
$$\epsilon_x = \frac{(0.93142)(40.625 \times 10^3)}{50 \times 10^9} = 756.78 \times 10^{-6}$$

$$(a) \delta_x = L_x \epsilon_x = (40 \text{ mm})(756.78 \times 10^{-6}) = 0.0303 \text{ mm}$$

$$(b) \sigma_x = 40.625 \times 10^6 \text{ Pa} = 40.6 \text{ MPa}$$

$$\sigma_y = \sigma_z = (0.134993)(40.625 \times 10^6) = 5.48 \times 10^6 \text{ Pa} = 5.48 \text{ MPa}$$

PROBLEM 2.90



\*2.90 The composite cube of Prob. 2.89 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (a) the stresses  $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_z$ , (b) the change in the dimension in the y direction.

$$\begin{aligned} E_x &= 50 \text{ GPa} & \nu_{xz} &= 0.254 \\ E_y &= 15.2 \text{ GPa} & \nu_{xy} &= 0.254 \\ E_z &= 15.2 \text{ GPa} & \nu_{zy} &= 0.428 \end{aligned}$$

SOLUTION

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \quad (1)$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad (4)$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \quad (2)$$

$$\frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \quad (5)$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \quad (3)$$

$$\frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \quad (6)$$

Constraint condition  $\epsilon_z = 0$

Load condition  $\sigma_y = 0$

From equation (3)  $0 = -\frac{\nu_{xz}}{E_x} \sigma_x + \frac{1}{E_z} \sigma_z$

$$\sigma_z = \frac{\nu_{xz} E_z}{E_x} \sigma_x = \frac{(0.254)(15.2)}{50} \sigma_x = 0.077216 \sigma_x$$

continued

## Problem 2.90 continued

From equation (1) with  $\sigma_y = 0$

$$\begin{aligned}\epsilon_x &= \frac{1}{E_x} \sigma_x - \frac{\nu_{zx}}{E_z} \sigma_z = \frac{1}{E_x} \sigma_x - \frac{\nu_{zx}}{E_x} \sigma_z \\ &= \frac{1}{E_x} [\sigma_x - 0.254 \sigma_z] = \frac{1}{E_x} [1 - (0.254)(0.077216)] \sigma_x \\ &= \frac{0.98039}{E_x} \sigma_x\end{aligned}$$

$$\sigma_x = \frac{E_x \epsilon_x}{0.98039}$$

But  $\epsilon_x = \frac{\delta_x}{L_x} = \frac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(a)  $\sigma_x = \frac{(50 \times 10^9)(875 \times 10^{-6})}{0.98039} = 44.625 \times 10^3 \text{ Pa} = 44.6 \text{ MPa}$   $\blacktriangleleft$

$\sigma_y = 0$   $\blacktriangleleft$

$\sigma_z = (0.077216)(44.625 \times 10^6) = 3.446 \times 10^6 \text{ Pa} = 3.45 \text{ MPa}$   $\blacktriangleleft$

From (2) 
$$\begin{aligned}\epsilon_y &= -\frac{\nu_{xy}}{E_x} \sigma_x + \frac{1}{E_y} \sigma_y - \frac{\nu_{zy}}{E_z} \sigma_z \\ &= -\frac{(0.254)(44.625 \times 10^6)}{50 \times 10^9} + 0 - \frac{(0.428)(3.446 \times 10^6)}{15.2 \times 10^9} \\ &= -323.73 \times 10^{-6}\end{aligned}$$

$\delta_y = L_y \epsilon_y = (40 \text{ mm})(-323.73 \times 10^{-6}) = -0.0129 \text{ mm}$   $\blacktriangleleft$

**PROBLEM 2.91**

**\*2.91** Show that for any given material, the ratio  $G/E$  of the modulus of rigidity over the modulus of elasticity is always less than  $\frac{1}{2}$  but more than  $\frac{1}{3}$  [Hint: Refer to Eq. (2.43) and to Sec. 2.13.]

**SOLUTION**

$$G = \frac{E}{2(1+\nu)} \quad \text{or} \quad \frac{E}{G} = 2(1+\nu)$$

Assume  $\nu \geq 0$  for almost all materials and  $\nu < \frac{1}{2}$  for a positive bulk modulus

Applying the bounds  $2 \leq \frac{E}{G} < 2(1+\frac{1}{2}) = 3$

Taking the reciprocals  $\frac{1}{2} \geq \frac{G}{E} \geq \frac{1}{3}$

or  $\frac{1}{3} \leq \frac{G}{E} \leq \frac{1}{2}$

**PROBLEM 2.92**

**\*2.92** The material constants  $E$ ,  $G$ ,  $k$ , and  $\nu$  are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (a)  $k = GE/(9G - 3E)$  and (b)  $\nu = (3k - 2G)/(6k + 2G)$ .

**SOLUTION**

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

(a)  $1+\nu = \frac{E}{2G}$  or  $\nu = \frac{E}{2G} - 1$

$$k = \frac{E}{3[1 - 2(\frac{E}{2G} - 1)]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 3E}$$

(b)  $\frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$

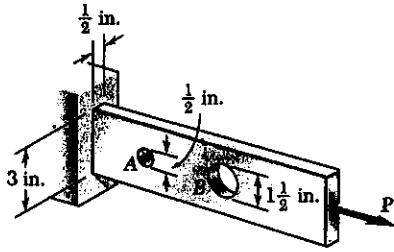
$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k\nu$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

**PROBLEM 2.93**

2.93 Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For  $P = 6.5$  kips, determine the maximum value of the stress ( $\sigma$ ) at A, (b) at B.



**SOLUTION**

(a) At hole A  $r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$  in

$d = 3 - \frac{1}{2} = 2.50$  in

$A_{net} = dt = (2.50)(\frac{1}{2}) = 1.25$  in<sup>2</sup>

$\sigma_{nom} = \frac{P}{A_{net}} = \frac{6.5}{1.25} = 5.2$  ksi

$\frac{r}{d} = \frac{1/4}{2.50} = 0.10$  From Fig 2.64 a  $K = 2.70$

$\sigma_{max} = K\sigma_{nom} = (2.70)(5.2) = 14.04$  ksi

(b) At hole B  $r = \frac{1}{2}(1.5) = 0.75$ ,  $d = 3 - 1.5 = 1.5$  in

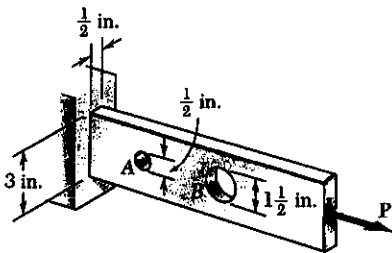
$A_{net} = dt = (1.5)(\frac{1}{2}) = 0.75$  in<sup>2</sup>,  $\sigma_{nom} = \frac{P}{A_{net}} = \frac{6.5}{0.75} = 8.667$  ksi

$\frac{r}{d} = \frac{0.75}{1.5} = 0.5$  From Fig 2.64 a  $K = 2.10$

$\sigma_{max} = K\sigma_{nom} = (2.10)(8.667) = 18.2$  ksi

**PROBLEM 2.94**

2.94 Knowing that  $\sigma_{all} = 16$  ksi, determine the maximum allowable value of the centric axial load P.



**SOLUTION**

At hole A  $r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$  in

$d = 3 - \frac{1}{2} = 2.50$  in.

$A_{net} = dt = (2.50)(\frac{1}{2}) = 1.25$  in<sup>2</sup>

$\frac{r}{d} = \frac{1/4}{2.50} = 0.10$  From Fig 2.64 a,  $K = 2.70$

$\sigma_{max} = \frac{KP}{A_{net}} \therefore P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.25)(16)}{2.70} = 7.41$  kips

At hole B  $r = \frac{1}{2}(1.5) = 0.75$  in,  $d = 3 - 1.5 = 1.5$  in.

$A_{net} = dt = (1.5)(\frac{1}{2}) = 0.75$  in<sup>2</sup>,

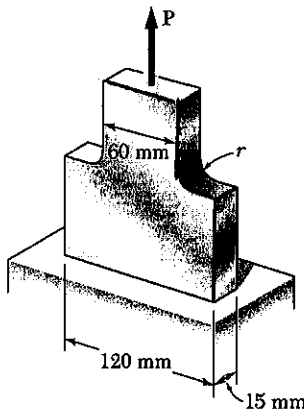
$\frac{r}{d} = \frac{0.75}{1.5} = 0.5$  From Fig 2.64 a  $K = 2.10$

$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(0.75)(16)}{2.10} = 5.71$  kips

Smaller value for P controls  $P = 5.71$  kips

**PROBLEM 2.95**

2.95 Knowing that, for the plate shown, the allowable stress is 125 MPa, determine the maximum allowable value of  $P$  when (a)  $r = 12$  mm, (b)  $r = 18$  mm.



**SOLUTION**

$$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$$

(a)  $r = 12$  mm  $\frac{r}{d} = \frac{12 \text{ mm}}{60 \text{ mm}} = 0.2$

From Fig. 2.64 b  $K = 1.92$   $\sigma_{\max} = K \frac{P}{A}$

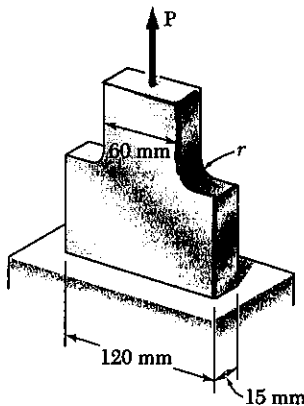
$$P = \frac{A \sigma_{\max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^6)}{1.92} = 58.6 \times 10^3 \text{ N} = 58.3 \text{ kN}$$

(b)  $r = 18$  mm,  $\frac{r}{d} = \frac{18 \text{ mm}}{60 \text{ mm}} = 0.30$ , From Fig 2.64 b  $K = 1.75$

$$P = \frac{A \sigma_{\max}}{K} = \frac{(900 \times 10^{-6})(125 \times 10^6)}{1.75} = 64.3 \times 10^3 \text{ N} = 64.3 \text{ kN}$$

**PROBLEM 2.96**

2.96 Knowing that  $P = 38$  kN, determine the maximum stress when (a)  $r = 10$  mm, (b)  $r = 16$  mm, (c)  $r = 18$  mm.



**SOLUTION**

$$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$$

(a)  $r = 10$  mm  $\frac{r}{d} = \frac{10 \text{ mm}}{60 \text{ mm}} = 0.1667$

From Fig 2.64 b  $K = 2.06$   $\sigma_{\max} = \frac{KP}{A}$

$$\sigma_{\max} = \frac{(2.06)(38 \times 10^3)}{900 \times 10^{-6}} = 87.0 \times 10^6 \text{ Pa} = 87.0 \text{ MPa}$$

(b)  $r = 16$  mm  $\frac{r}{d} = \frac{16 \text{ mm}}{60 \text{ mm}} = 0.2667$

From Fig 2.64 b  $K = 1.78$

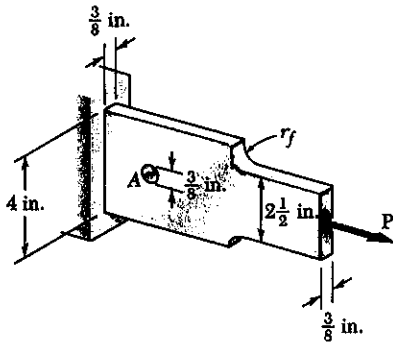
$$\sigma_{\max} = \frac{(1.78)(38 \times 10^3)}{900 \times 10^{-6}} = 75.2 \times 10^6 \text{ Pa} = 75.2 \text{ MPa}$$

(c)  $r = 18$  mm  $\frac{r}{d} = \frac{18 \text{ mm}}{60 \text{ mm}} = 0.30$

From Fig 2.64 b  $K = 1.75$

$$\sigma_{\max} = \frac{(1.75)(38 \times 10^3)}{900 \times 10^{-6}} = 73.9 \times 10^6 \text{ Pa} = 73.9 \text{ MPa}$$

**PROBLEM 2.97**



2.97 Knowing that the hole has a diameter of  $\frac{3}{8}$  in., determine (a) the radius  $r_f$  of the fillets for which the same maximum stress occurs at the hole  $A$  and at the fillets, (b) the corresponding maximum allowable load  $P$  if the allowable stress is 15 ksi.

**SOLUTION**

For the circular hole  $r = (\frac{1}{2})(\frac{3}{8}) = 0.1875$  in  
 $d = 4 - \frac{3}{8} = 3.625$  in  $\frac{r}{d} = \frac{0.1875}{3.625} = 0.0517$   
 $A_{net} = dt = (3.625)(\frac{3}{8}) = 1.3594$  in<sup>2</sup>

From Fig 2.64 a  $K_{hole} = 2.82$

$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$

(b)  $P = \frac{A_{net} \sigma_{max}}{K_{hole}} = \frac{(1.3594)(15)}{2.82} = 7.23$  kips

(a) For fillet  $D = 4$  in,  $d = 2.5$  in  $\frac{D}{d} = \frac{4.0}{2.5} = 1.60$

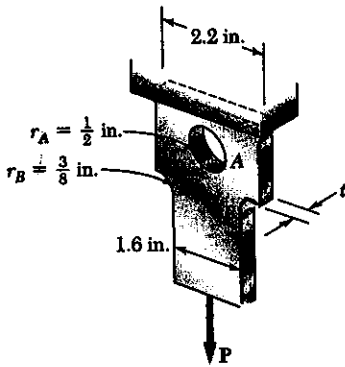
$A_{min} = dt = (2.5)(\frac{3}{8}) = 0.9375$  in<sup>2</sup>

$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \therefore K_{fillet} = \frac{A_{min} \sigma_{max}}{P} = \frac{(0.9375)(15)}{7.23} = 1.945$

From Fig 2.64 b  $\frac{r_f}{d} \approx 0.17 \therefore r_f \approx 0.17d = (0.17)(2.5) = 0.43$  in

**PROBLEM 2.98**

2.98 For  $P = 8.5$  kips, determine the minimum plate thickness  $t$  required if the allowable stress is 18 ksi.



**SOLUTION**

At the hole:  $r_A = \frac{1}{2}$  in  $d_A = 2.2 - 1.0 = 1.2$  in.

$\frac{r_A}{d_A} = \frac{1/2}{1.2} = 0.417$

From Fig 2.64 a  $K = 2.22$

$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \therefore t = \frac{KP}{d_A \sigma_{max}}$

$t = \frac{(2.22)(8.5)}{(1.2)(18)} = 0.87$  in.

At the fillet  $D = 2.2$  in,  $d_B = 1.6$  in  $\frac{D}{d_B} = \frac{2.2}{1.6} = 1.375$

$r_B = \frac{3}{8} = 0.375$  in  $\frac{r_B}{d_B} = \frac{0.375}{1.6} = 0.2344$

From Fig 2.64 b  $K = 1.70$   $\sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

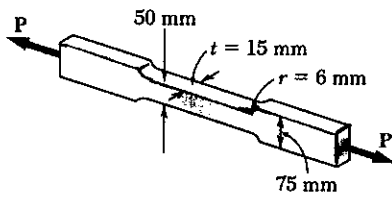
$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(8.5)}{(1.6)(18)} = 0.50$  in

The larger value is the required minimum plate thickness

$t = 0.87$  in



**PROBLEM 2.99**



2.99 (a) Knowing that the allowable stress is 140 MPa, determine the maximum allowable magnitude of the centric load P. (b) Determine the percent change in the maximum allowable magnitude of P if the raised portions are removed at the ends of the specimen.

**SOLUTION**

$$\frac{D}{d} = \frac{75 \text{ mm}}{50 \text{ mm}} = 1.50, \quad \frac{r}{d} = \frac{6 \text{ mm}}{50 \text{ mm}} = 0.12$$

From Fig 2.64 b  $K = 2.10$

$$A_{\min} = t d = (15)(50) = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2$$

$$(a) \quad \sigma_{\max} = \frac{KP}{A_{\min}} \therefore P = \frac{A_{\min} \sigma_{\max}}{K} = \frac{(750 \times 10^{-6})(140 \times 10^6)}{2.10} = 50 \times 10^3 \text{ N} = 50 \text{ kN}$$

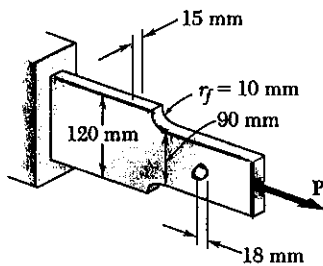
(b) Without raised section  $K = 1.00$

$$P = A_{\min} \sigma_{\max} = (750 \times 10^{-6})(140 \times 10^6) = 105 \times 10^3 = 105 \text{ kN}$$

$$\% \text{ change} = \left( \frac{105 - 50}{50} \right) \times 100\% = 110\%$$

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**PROBLEM 2.100**



2.100 A centric axial force is applied to the steel bar shown. Knowing that  $\sigma_{\text{all}}$  is 135 MPa, determine the maximum allowable load P.

**SOLUTION**

At the hole:  $r = 9 \text{ mm}$   $d = 90 - 18 = 72 \text{ mm}$

$$\frac{r}{d} = 0.125 \quad \text{From Fig 2.64 a} \quad K = 2.65$$

$$A_{\text{net}} = t d = (15)(72) = 1.08 \times 10^3 \text{ mm}^2 = 1.08 \times 10^{-3} \text{ m}^2$$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}}$$

$$P = \frac{A_{\text{net}} \sigma_{\max}}{K} = \frac{(1.08 \times 10^{-3})(135 \times 10^6)}{2.65} = 55 \times 10^3 \text{ N} = 55 \text{ kN}$$

At the fillet  $D = 120 \text{ mm}$ ,  $d = 90 \text{ mm}$ ,  $\frac{D}{d} = \frac{120}{90} = 1.333$

$$r = 10 \text{ mm} \quad \frac{r}{d} = \frac{10}{90} = 0.1111 \quad \text{From Fig 2.64 b} \quad K = 2.02$$

$$A_{\min} = t d = (15)(90) = 1.35 \times 10^3 \text{ mm}^2 = 1.35 \times 10^{-3} \text{ m}^2$$

$$\sigma_{\max} = \frac{KP}{A_{\min}}$$

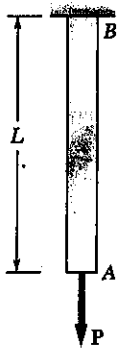
$$P = \frac{A_{\min} \sigma_{\max}}{K} = \frac{(1.35 \times 10^{-3})(135 \times 10^6)}{2.02} = 90 \times 10^3 \text{ N} = 90 \text{ kN}$$

Smaller value for P controls

$$P = 55 \text{ kN}$$

PROBLEM 2.101

2.101 The 30-mm square bar  $AB$  has a length  $L = 2.2$  m; it is made of a mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_Y = 345$  MPa. A force  $P$  is applied to the bar until end  $A$  has moved down by an amount  $\delta_m$ . Determine the maximum value of the force  $P$  and the permanent set of the bar after the force has been removed, knowing that (a)  $\delta_m = 4.5$  mm, (b)  $\delta_m = 8$  mm.



SOLUTION

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L \epsilon_Y = \frac{L \sigma_Y}{E} = \frac{(2.2)(345 \times 10^6)}{200 \times 10^9} = 3.795 \times 10^{-3} = 3.795 \text{ mm}$$

IF  $\delta_m \geq \delta_Y$   $P_m = A \sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N}$

Unloading  $\delta' = \frac{P_m L}{AE} = \frac{\sigma_Y L}{E} = \delta_Y = 3.795 \text{ mm}$

$$\delta_p = \delta_m - \delta'$$

(a)  $\delta_m = 4.5 \text{ mm} > \delta_Y$   $P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$  ▶

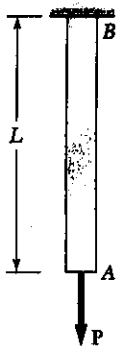
$\delta_{\text{perm}} = 4.5 \text{ mm} - 3.795 \text{ mm} = 0.705 \text{ mm}$  ▶

(b)  $\delta_m = 8 \text{ mm} > \delta_Y$   $P_m = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$  ▶

$\delta_p = 8.0 \text{ mm} - 3.795 \text{ mm} = 4.205 \text{ mm}$  ▶

PROBLEM 2.102

2.102 The 30-mm square bar  $AB$  has a length  $L = 2.5$  m; it is made of mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_Y = 345$  MPa. A force  $P$  is applied to the bar and then removed to give it a permanent set  $\delta_p$ . Determine the maximum value of the force  $P$  and the maximum amount  $\delta_m$  by which the bar should be stretched if the desired value of  $\delta_p$  is (a) 3.5 mm, (b) 6.5 mm.



SOLUTION

$$A = (30)(30) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\delta_Y = L \epsilon_Y = \frac{L \sigma_Y}{E} = \frac{(2.5)(345 \times 10^6)}{200 \times 10^9} = 4.3125 \times 10^{-3} \text{ m} = 4.3125 \text{ mm}$$

When  $\delta_m$  exceeds  $\delta_Y$ , thus producing a permanent stretch of  $\delta_p$ , the maximum force is

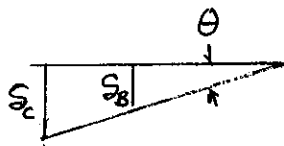
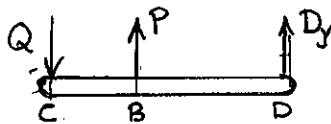
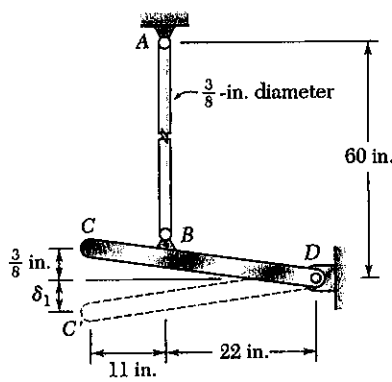
$$P_m = A \sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3 \text{ N} = 310.5 \text{ kN}$$
 ▶

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(a)  $\delta_p = 3.5 \text{ mm}$   $\delta_m = 3.5 \text{ mm} + 4.3125 \text{ mm} = 7.81 \text{ mm}$  ▶

(b)  $\delta_p = 6.5 \text{ mm}$   $\delta_m = 6.5 \text{ mm} + 4.3125 \text{ mm} = 10.81 \text{ mm}$  ▶

**PROBLEM 2.103**



2.103 Rod  $AB$  is made of a mild steel that is assumed to be elastoplastic with  $E = 29 \times 10^6$  ksi and  $\sigma_y = 36$  ksi. After the rod has been attached to a rigid lever  $CD$ , it is found that end  $C$  is  $\frac{3}{8}$ -in. too high. A vertical force  $Q$  is then applied at  $C$  until this point has moved to position  $C'$ . Determine the required magnitude of  $Q$  and the deflection  $\delta$ , if the lever is to *snap* back to a horizontal position after  $Q$  is removed.

**SOLUTION**

Since the rod  $AB$  is to be stretched permanently, the peak force in the rod is  $P = P_y$ , where

$$P_y = A \sigma_y = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 (36) = 3.976 \text{ kips}$$

Referring the free body diagram of lever  $CD$

$$\sum M_D = 0 \quad 33Q - 22P = 0$$

$$Q = \frac{22}{33} P = \frac{(22)(3.976)}{33} = 2.65 \text{ kips}$$

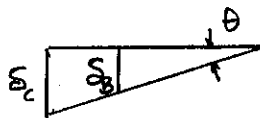
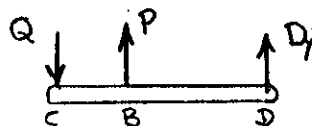
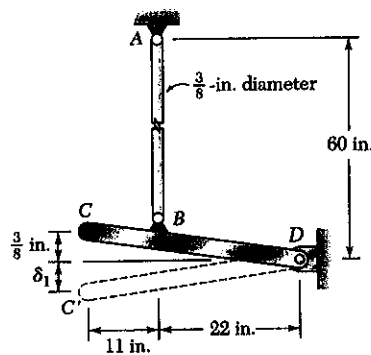
During unloading, the springback at  $B$  is

$$\delta_B = L_{AB} \epsilon_r = \frac{L_{AB} \sigma_r}{E} = \frac{(60)(36 \times 10^3)}{29 \times 10^6} = 0.0745 \text{ in}$$

From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B}{22} = \frac{\delta_c}{33} \therefore \delta_c = \frac{33}{22} \delta_B = 0.1117 \text{ in}$$

**PROBLEM 2.104**



2.103 Rod  $AB$  is made of a mild steel that is assumed to be elastoplastic with  $E = 29 \times 10^6$  ksi and  $\sigma_y = 36$  ksi. After the rod has been attached to a rigid lever  $CD$ , it is found that end  $C$  is  $\frac{3}{8}$ -in. too high. A vertical force  $Q$  is then applied at  $C$  until this point has moved to position  $C'$ . Determine the required magnitude of  $Q$  and the deflection  $\delta$ , if the lever is to *snap* back to a horizontal position after  $Q$  is removed.

2.104 Solve Prob. 2.103, assuming that the yield point of the mild steel used is 50 ksi.

**SOLUTION**

Since the rod  $AB$  is to be stretched permanently, the peak force in the rod is  $P = P_y$ , where

$$P_y = A \sigma_y = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 (50) = 5.522 \text{ kips}$$

Referring to the free body diagram of lever  $CD$

$$\sum M_D = 0 \quad 33Q - 22P = 0$$

$$Q = \frac{22}{33} P = \frac{(22)(5.522)}{33} = 3.68 \text{ kips}$$

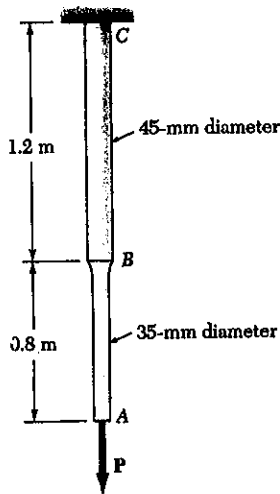
During unloading, the springback at  $B$  is

$$\delta_B = L_{AB} \epsilon_r = \frac{L_{AB} \sigma_r}{E} = \frac{(60)(50 \times 10^3)}{29 \times 10^6} = 0.1034 \text{ in}$$

From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B}{22} = \frac{\delta_c}{33} \therefore \delta_c = \frac{33}{22} \delta_B = 0.1552 \text{ in}$$

**PROBLEM 2.105**



2.105 Rods  $AB$  and  $BC$  are made of a mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_y = 345$  MPa. The rods are stretched until end has moved down 9 mm. Neglecting stress concentrations, determine (a) the maximum value of the force  $P$ , (b) the permanent set measured at points  $A$  and  $B$  after the force has been removed.

**SOLUTION**

$$A_{AB} = \frac{\pi}{4}(0.035)^2 = 962.1 \times 10^{-6} \text{ m}^2 \quad A_{BC} = \frac{\pi}{4}(0.045)^2 = 1.5904 \times 10^{-3} \text{ m}^2$$

$$(a) P_{max} = A_{min} \sigma_y = (962.1 \times 10^{-6})(345 \times 10^6) = 331.93 \times 10^3 \text{ N} = 332 \text{ kN}$$

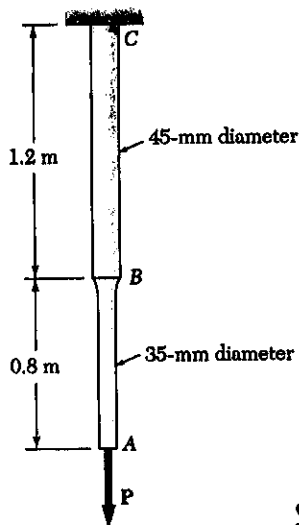
$$(b) \text{ Spring back } S' = \frac{PL_{AB}}{EA_{AB}} + \frac{PL_{BC}}{EA_{BC}} = \frac{P}{E} \left( \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right)$$

$$S' = \frac{331.98 \times 10^3}{200 \times 10^9} \left( \frac{0.8}{962.1 \times 10^{-6}} + \frac{1.2}{1.5904 \times 10^{-3}} \right) = 2.63 \times 10^{-3} \text{ m} = 2.63 \text{ mm}$$

At point  $A$   $S_p = S_m - S' = 9 \text{ mm} - 2.63 \text{ mm} = 6.37 \text{ mm}$

At point  $B$ : No yielding in  $BC$ ; hence  $S_p = 0$

**PROBLEM 2.106**



2.105 Rods  $AB$  and  $BC$  are made of a mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_y = 345$  MPa. The rods are stretched until end has moved down 9 mm. Neglecting stress concentrations, determine (a) the maximum value of the force  $P$ , (b) the permanent set measured at points  $A$  and  $B$  after the force has been removed.

2.106 Solve Prob. 2.105, assuming that the yield point of the mild steel used is 250 MPa.

**SOLUTION**

$$A_{AB} = \frac{\pi}{4}(0.035)^2 = 962.1 \times 10^{-6} \text{ m}^2, \quad A_{BC} = \frac{\pi}{4}(0.045)^2 = 1.5904 \times 10^{-3} \text{ m}^2$$

$$(a) P_{max} = A_{min} \sigma_y = (962.1 \times 10^{-6})(250 \times 10^6) = 240.53 \times 10^3 \text{ N} = 241 \text{ kN}$$

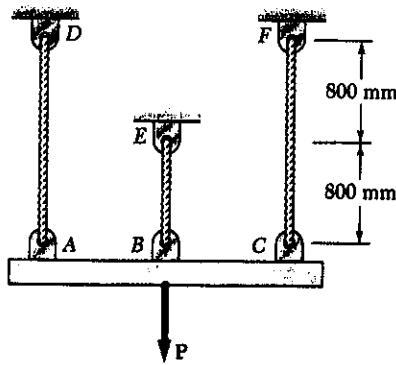
$$(b) \text{ Spring back } S' = \frac{PL_{AB}}{EA_{AB}} + \frac{PL_{BC}}{EA_{BC}} = \frac{P}{E} \left( \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right)$$

$$S' = \frac{240.53 \times 10^3}{200 \times 10^9} \left( \frac{0.8}{962.1 \times 10^{-6}} + \frac{1.2}{1.5904 \times 10^{-3}} \right) = 1.908 \times 10^{-3} \text{ m} = 1.908 \text{ mm}$$

At point  $A$   $S_p = S_m - S' = 9 \text{ mm} - 1.908 \text{ mm} = 7.09 \text{ mm}$

At point  $B$ , no yielding in  $BC$ ; hence  $S_p = 0$

PROBLEM 2.107



2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which  $\sigma_y = 345 \text{ MPa}$  and  $E = 200 \text{ GPa}$ . A force  $P$  is applied to the rigid bar  $ABC$  until the bar has moved downward a distance  $\delta = 2 \text{ mm}$ . Knowing that the cables were initially taut, determine (a) the maximum value of  $P$ , (b) the maximum stress that occurs in cable  $AD$ , (c) the final displacement of the bar after the load is removed. (Hint: In part c, cable  $BE$  is not taut.)

SOLUTION

For each cable  $A = \frac{\pi}{4}(0.006)^2 = 28.274 \times 10^{-6} \text{ m}^2$

Strain at initial yielding

$$\epsilon_y = \frac{\sigma_y}{E} = \frac{345 \times 10^6}{200 \times 10^9} = 1.725 \times 10^{-3}$$

Strain in cables  $AD$  and  $CF$ :  $\epsilon_{AD} = \epsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2 \text{ mm}}{1600 \text{ mm}} = 1.25 \times 10^{-3}$

Strain in cable  $BE$ :  $\epsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2 \text{ mm}}{800 \text{ mm}} = 2.50 \times 10^{-3}$

Since  $\epsilon_{AD} < \epsilon_y$ ,  $\sigma_{AD} = E \epsilon_{AD} = (200 \times 10^9)(1.25 \times 10^{-3}) = 250 \times 10^6 \text{ Pa}$

Since  $\epsilon_{BE} > \epsilon_y$ ,  $\sigma_{BE} = \sigma_y = 345 \times 10^6 \text{ Pa}$

Forces:  $P_{AD} = P_{CF} = A \sigma_{AD} = (28.274 \times 10^{-6})(250 \times 10^6) = 7.0685 \times 10^3 \text{ N}$

$P_{BE} = A \sigma_{BE} = (28.274 \times 10^{-6})(345 \times 10^6) = 9.7545 \times 10^3 \text{ N}$

For equilibrium of bar  $ABC$   $P_{AD} + P_{BE} + P_{CF} - P = 0$

(a)  $P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685) \times 10^3 \text{ N}$   
 $= 23.9 \times 10^3 \text{ N} = 23.9 \text{ kN}$

(b)  $\sigma_{AD} = 250 \times 10^6 \text{ Pa} = 250 \text{ MPa}$

After unloading  $P = 0$

Cable  $BE$  is not taut  $P_{BE} = 0$

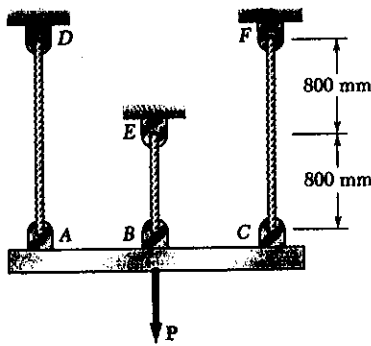
By symmetry  $P_{AD} = P_{CF}$

For equilibrium  $P_{AD} = P_{CF} = 0$

(c) Final displacement  $\delta$  is controlled by the final lengths of cables  $AD$  and  $CF$ . Since these cables were never permanently deformed, the final displacement is

$$\delta = \delta_{AD} = \delta_{CF} = 0$$

PROBLEM 2.108



2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which  $\sigma_y = 345 \text{ MPa}$  and  $E = 200 \text{ GPa}$ . A force  $P$  is applied to the rigid bar  $ABC$  until the bar has moved downward a distance  $\delta = 2 \text{ mm}$ . Knowing that the cables were initially taut, determine (a) the maximum value of  $P$ , (b) the maximum stress that occurs in cable  $AD$ , (c) the final displacement of the bar after the load is removed. (Hint: In part c, cable  $BE$  is not taut.)

2.108 Solve Prob. 2.107, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

SOLUTION

For each rod  $A = \frac{\pi}{4}(0.006)^2 = 28.274 \times 10^{-6} \text{ m}^2$

Strain at initial yielding  $\epsilon_y = \frac{\sigma_y}{E} = \frac{345 \times 10^6}{200 \times 10^9} = 1.725 \times 10^{-3}$

Strain in rods  $AD$  and  $CF$ :  $\epsilon_{AD} = \epsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2 \text{ mm}}{1600 \text{ mm}} = 1.25 \times 10^{-3}$

Strain in rod  $BE$ :  $\epsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2 \text{ mm}}{800 \text{ mm}} = 2.50 \times 10^{-3}$

Since  $\epsilon_{AD} < \epsilon_y$ ,  $\sigma_{AD} = E\epsilon_{AD} = (200 \times 10^9)(1.25 \times 10^{-3}) = 250 \times 10^6 \text{ Pa}$

Since  $\epsilon_{BE} > \epsilon_y$ ,  $\sigma_{BE} = \sigma_y = 345 \times 10^6 \text{ Pa}$

Forces:  $P_{AD} = P_{CF} = A\sigma_{AD} = (28.274 \times 10^{-6})(250 \times 10^6) = 7.0685 \times 10^3 \text{ N}$

$P_{BE} = A\sigma_{BE} = (28.274 \times 10^{-6})(345 \times 10^6) = 9.7545 \times 10^3 \text{ N}$

For equilibrium of bar  $ABC$   $P_{AD} + P_{BE} + P_{CF} - P = 0$

(a)  $P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685) \times 10^3 \text{ N} = 23.9 \text{ kN}$

(b)  $\sigma_{AD} = 250 \times 10^6 \text{ Pa} = 250 \text{ MPa}$

Let  $S'$  = change in displacement during unloading

$P'_{AD} = \frac{EA}{L_{AD}} S' = \frac{(200 \times 10^9)(28.274 \times 10^{-6})}{1600 \times 10^{-3}} S' = 3.534 \times 10^6 S' = P'_{CF}$

$P'_{BE} = \frac{EA}{L_{BE}} S' = \frac{(200 \times 10^9)(28.274 \times 10^{-6})}{800 \times 10^{-3}} S' = 7.0685 \times 10^6 S'$

For equilibrium  $P' = P'_{AD} + P'_{BE} + P'_{CF} = 14.137 \times 10^6 S'$

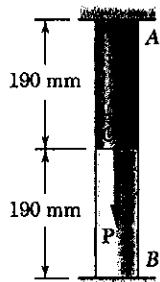
But  $P - P' = 0$   $P' = P = 23.89 \times 10^3 \text{ N}$

$S' = \frac{23.89 \times 10^3}{14.137 \times 10^6} = 1.690 \times 10^{-3} \text{ m}$

Permanent displacement of bar

$S_{\text{final}} = S_{\text{max}} - S' = 2 \times 10^{-3} - 1.690 \times 10^{-3} = 0.310 \times 10^{-3} \text{ m}$   
 $= 0.310 \text{ mm}$

PROBLEM 2.109



2.109 Rod  $AB$  consists of two cylindrical portions  $AC$  and  $BC$ , each with a cross-sectional area of  $1750 \text{ mm}^2$ . Portion  $AC$  is made of a mild steel with  $E = 200 \text{ GPa}$  and  $\sigma_y = 250 \text{ MPa}$ , and portion  $BC$  is made of a high-strength steel with  $E = 200 \text{ GPa}$  and  $\sigma_y = 345 \text{ MPa}$ . A load  $P$  is applied at  $C$  as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of  $C$  if  $P$  is gradually increased from zero to  $975 \text{ kN}$  and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of  $C$ .

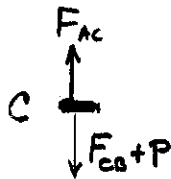
SOLUTION

Displacement at  $C$  to cause yielding of  $AC$

$$\delta_{C,y} = L_{AC} \epsilon_{y,AC} = \frac{L_{AC} \sigma_{y,AC}}{E} = \frac{(0.190)(250 \times 10^6)}{200 \times 10^9} = 0.2375 \times 10^{-3} \text{ m}$$

Corresponding force  $F_{AC} = A \sigma_{y,AC} = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^3 \text{ N}$

$$F_{CB} = -\frac{E A \delta_c}{L_{CB}} = -\frac{(200 \times 10^9)(1750 \times 10^{-6})(0.2375 \times 10^{-3})}{0.190} = -437.5 \times 10^3 \text{ N}$$



For equilibrium of element at  $C$

$$F_{AC} - (F_{CB} + P) = 0 \quad P_y = F_{AC} - F_{CB} = 875 \times 10^3 \text{ N}$$

Since applied load  $P = 975 \times 10^3 \text{ N} > 875 \times 10^3 \text{ N}$ , portion  $AC$  yields.

$$F_{CB} = F_{AC} - P = 437.5 \times 10^3 - 975 \times 10^3 \text{ N} = -537.5 \times 10^3 \text{ N}$$

(a)  $\delta_c = -\frac{F_{CB} L_{CB}}{E A} = \frac{(537.5 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.29179 \times 10^{-3} \text{ m} = 0.292 \text{ mm}$

(b) Maximum stresses  $\sigma_{AB} = \sigma_{y,AB} = 250 \text{ MPa}$

$$\sigma_{BC} = \frac{F_{CB}}{A} = -\frac{537.5 \times 10^3}{1750 \times 10^{-6}} = -307.14 \times 10^6 \text{ Pa} = -307 \text{ MPa}$$

(c) Deflection and forces for unloading

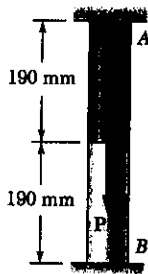
$$S' = \frac{F_{AC}' L_{AC}}{E A} = -\frac{F_{CB}' L_{CB}}{E A} \quad \therefore F_{CB}' = -F_{AC}' \frac{L_{AC}}{L_{CB}} = -F_{AC}'$$

$$P' = 975 \times 10^3 = P_{AC}' - P_{CB}' = 2 P_{AC}' \quad P_{AC}' = 487.5 \times 10^3 \text{ N}$$

$$\delta' = \frac{(487.5 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26464 \times 10^{-3} \text{ m}$$

$$\delta_p = \delta_m - \delta' = 0.29179 \times 10^{-3} - 0.26464 \times 10^{-3} = 0.02715 \times 10^{-3} \text{ m} = 0.027 \text{ mm}$$

PROBLEM 2.110



2.109 Rod  $AB$  consists of two cylindrical portions  $AC$  and  $BC$ , each with a cross-sectional area of  $1750 \text{ mm}^2$ . Portion  $AC$  is made of a mild steel with  $E = 200 \text{ GPa}$  and  $\sigma_y = 250 \text{ MPa}$ , and portion  $BC$  is made of a high-strength steel with  $E = 200 \text{ GPa}$  and  $\sigma_y = 345 \text{ MPa}$ . A load  $P$  is applied at  $C$  as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of  $C$  if  $P$  is gradually increased from zero to  $975 \text{ kN}$  and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of  $C$ .

2.110 For the composite rod of Prob. 2.109, if  $P$  is gradually increased from zero until the deflection of point  $C$  reaches a maximum value of  $\delta_m = 0.3 \text{ mm}$  and then decreased back to zero, determine, (a) the maximum value of  $P$ , (b) the maximum stress in each portion of the rod, (c) the permanent deflection of  $C$  after the load is removed.

SOLUTION

Displacement at  $C$  is  $\delta_m = 0.30 \text{ mm}$ . The corresponding strains are

$$\epsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\epsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding

$$\epsilon_{y,AC} = \frac{\sigma_{y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \quad (\text{yielding})$$

$$\epsilon_{y,CB} = -\frac{\sigma_{y,CB}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \quad (\text{elastic})$$

(a) Forces:  $F_{AC} = A\sigma_y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^3 \text{ N}$

$$F_{CB} = EA\epsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^3 \text{ N}$$

For equilibrium of element at  $C$   $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CB} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} = 990 \text{ kN}$$

(b) Stresses: AC  $\sigma_{AC} = \sigma_{y,AC} = 250 \text{ MPa}$

CB  $\sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6 \text{ Pa} = -316 \text{ MPa}$

(c) Deflection and forces for unloading

$$\delta' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} \quad \therefore P_{CB}' = -P_{AC}' \frac{L_{AC}}{L_{CB}} = -P_{AC}'$$

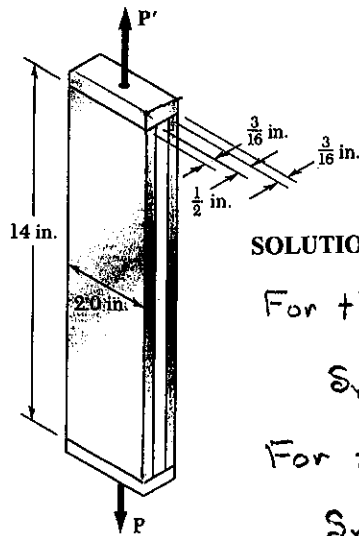
$$P' = P_{AC}' - P_{CB}' = 2P_{AC}' = 990.1 \times 10^3 \text{ N} \quad \therefore P_{AC}' = 495.05 \times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26874 \times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm} = 0.031 \text{ mm}$$



PROBLEM 2.111



2.111 Two tempered-steel bars, each  $\frac{3}{16}$ -in. thick, are bonded to a  $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude  $P$ . Both steels are elastoplastic with  $E = 29 \times 10^6$  psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load  $P$  is gradually increased from zero until the deformation of the bar reaches a maximum value  $\delta_m = 0.04$  in. and then decreased back to zero. Determine (a) the maximum value of  $P$ , (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

SOLUTION

For the mild steel  $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{r1} = \frac{L \sigma_{r1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in.}$$

For the tempered steel  $A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{r2} = \frac{L \sigma_{r2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in.}$$

Total area:  $A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{r1} < \delta_m < \delta_{r2}$  The mild steel yields. Tempered steel is elastic.

(a) Forces  $P_1 = A_1 \sigma_{r1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$

$$P_2 = \frac{EA_2 \delta_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb.}$$

$$P = P_1 + P_2 = 112.14 \times 10^3 \text{ lb} = 112.1 \text{ kips} \quad \blacktriangleleft$$

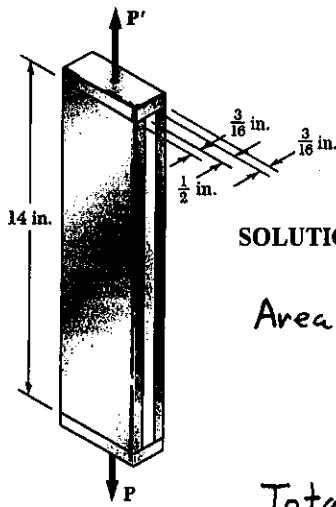
(b) Stresses  $\sigma_1 = \frac{P_1}{A_1} = \sigma_{r1} = 50 \times 10^3 \text{ psi} = 50 \text{ ksi}$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi} = 82.86 \text{ ksi} \quad \blacktriangleleft$$

Unloading  $\delta' = \frac{PL}{EA} = \frac{(112.14 \times 10^3)(14)}{(29 \times 10^6)(1.75)} = 0.03094 \text{ in.}$

(c) Permanent set  $\delta_p = \delta_m - \delta' = 0.04 - 0.03094 = 0.00906 \text{ in.} \quad \blacktriangleleft$

**PROBLEM 2.112**



2.111 Two tempered-steel bars, each  $\frac{3}{16}$ -in. thick, are bonded to a  $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude  $P$ . Both steels are elastoplastic with  $E = 29 \times 10^6$  psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

2.112 For the composite bar of Prob. 2.111, if  $P$  is gradually increased from zero to 98 kips and then decreased back to zero, determine (a) the maximum deformation of the bar, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

**SOLUTION**

Areas: Mild steel  $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$   
 Tempered steel  $A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$   
 Total:  $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \therefore P_Y = A \sigma_{Y1} = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_Y$ , therefore mild steel yields.

Let  $P_1 =$  force carried by mild steel  
 $P_2 =$  force carried by tempered steel

$$P_1 = A_1 \sigma_1 = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

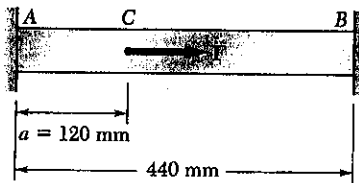
(a)  $S_m = \frac{P_2 L}{EA_2} = \frac{(48 \times 10^3)(14)}{(29 \times 10^6)(0.75)} = 0.03090 \text{ in.}$  ▶

(b)  $\sigma_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi} = 64 \text{ ksi}$  ▶

Unloading  $S' = \frac{PL}{EA} = \frac{(98 \times 10^3)(14)}{(29 \times 10^6)(1.75)} = 0.02703 \text{ in}$

(c)  $S_p = S_m - S' = 0.03090 - 0.02703 = 0.00387 \text{ in.}$  ▶

PROBLEM 2.113

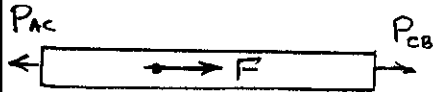


2.113 Bar  $AB$  has a cross-sectional area of  $1200 \text{ mm}^2$  and is made of a steel that is assumed to be elastoplastic with  $E = 200 \text{ GPa}$  and  $\sigma_Y = 250 \text{ MPa}$ . Knowing that the force  $F$  increases from 0 to  $520 \text{ kN}$  and then decreases to zero, determine (a) the permanent deflection of point  $C$ , (b) the residual stress in the bar.

SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion  $AC$  :  $P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$   
 $= 300 \times 10^3 \text{ N}$



For equilibrium  $F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$

$$= -220 \times 10^3 \text{ N}$$

$$s_c = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.293333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$s'_c = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} = \frac{(F - P'_{AC}) L_{CB}}{EA}$$

$$P'_{AC} \left( \frac{L_{AC}}{EA} + \frac{L_{CB}}{EA} \right) = \frac{F L_{CB}}{EA}$$

$$P'_{AC} = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.120)}{0.440} = 378.182 \times 10^3 \text{ N}$$

$$P'_{CB} = P'_{AC} - F = 378.182 \times 10^3 - 520 \times 10^3 = -141.818 \times 10^3 \text{ N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.182 \times 10^3}{1200 \times 10^{-6}} = 315.152 \times 10^6 \text{ Pa}$$

$$\sigma'_{BC} = \frac{P'_{BC}}{A} = -\frac{141.818 \times 10^3}{1200 \times 10^{-6}} = -118.182 \times 10^6 \text{ Pa}$$

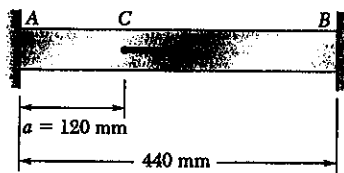
$$s'_c = \frac{(378.182)(0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.189091 \times 10^{-3} \text{ m}$$

(a)  $s_{cp} = s_c - s'_c = 0.293333 \times 10^{-3} - 0.189091 \times 10^{-3} = 0.1042 \times 10^{-3} \text{ m}$   
 $= 0.1042 \text{ mm}$   $\blacktriangleleft$

(b)  $\sigma_{AC, res} = \sigma_Y - \sigma'_{AC} = 250 \times 10^6 - 315.152 \times 10^6 = -65.2 \times 10^6 \text{ Pa}$   
 $= -65.2 \text{ MPa}$   $\blacktriangleleft$

$\sigma_{CB, res} = \sigma_{CB} - \sigma'_{CB} = -183.333 \times 10^6 + 118.182 \times 10^6 = -65.2 \times 10^6 \text{ Pa}$   
 $= -65.2 \text{ MPa}$   $\blacktriangleleft$

**PROBLEM 2.114**



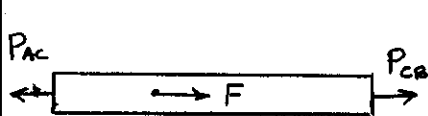
2.113 Bar  $AB$  has a cross-sectional area of  $1200 \text{ mm}^2$  and is made of a steel that is assumed to be elastoplastic with  $E = 200 \text{ GPa}$  and  $\sigma_y = 250 \text{ MPa}$ . Knowing that the force  $F$  increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point  $C$ , (b) the residual stress in the bar.

2.114 Solve Prob. 2.113, assuming that  $a = 180 \text{ mm}$ .

**SOLUTION**

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

$$\text{Force to yield portion } AC: P_{Ac} = A\sigma_y = (1200 \times 10^{-6})(250 \times 10^6) = 300 \times 10^3 \text{ N}$$



$$\text{For equilibrium } F + P_{CB} - P_{Ac} = 0$$

$$P_{CB} = P_{Ac} - F = 300 \times 10^3 - 520 \times 10^3 = -220 \times 10^3 \text{ N}$$

$$\delta_c = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.238333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta_c' = \frac{P_{Ac}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{Ac}') L_{CB}}{EA} \therefore P_{Ac}' \left( \frac{L_{AC}}{EA} + \frac{L_{CB}}{EA} \right) = \frac{F L_{CB}}{EA}$$

$$P_{Ac}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.180)}{0.440} = 307.273 \times 10^3 \text{ N}$$

$$P_{CB}' = P_{Ac}' - F = 307.273 \times 10^3 - 520 \times 10^3 = -212.727 \times 10^3 \text{ N}$$

$$\delta_c' = \frac{(307.273 \times 10^3)(0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.230455 \times 10^{-3} \text{ m}$$

$$\sigma_{Ac}' = \frac{P_{Ac}'}{A} = \frac{307.273 \times 10^3}{1200 \times 10^{-6}} = 256.061 \times 10^6 \text{ Pa}$$

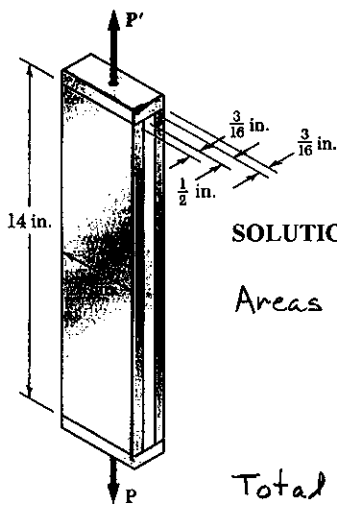
$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727 \times 10^3}{1200 \times 10^{-6}} = -177.273 \times 10^6 \text{ Pa}$$

$$(a) \delta_{cp} = \delta_c - \delta_c' = 0.238333 \times 10^{-3} - 0.230455 \times 10^{-3} = 0.00788 \times 10^{-3} \text{ m} = 0.00788 \text{ mm} \blacktriangleleft$$

$$(b) \sigma_{Ac, res} = \sigma_{Ac} - \sigma_{Ac}' = 250 \times 10^6 - 256.061 \times 10^6 = -6.06 \times 10^6 \text{ Pa} = -6.06 \text{ MPa} \blacktriangleleft$$

$$\sigma_{CB, res} = \sigma_{CB} - \sigma_{CB}' = -183.333 \times 10^6 + 177.273 \times 10^6 = -6.06 \times 10^6 \text{ Pa} = -6.06 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.115



2.111 Two tempered-steel bars, each  $\frac{3}{16}$ -in. thick, are bonded to a  $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude  $P$ . Both steels are elastoplastic with  $E = 29 \times 10^6$  psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

\*2.115 For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if  $P$  is gradually increased from zero to 98 kips and then decreased back to zero.

SOLUTION

Areas : Mild steel  $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$   
 Tempered steel  $A_2 = (2)(\frac{3}{16})(2) = 0.75 \text{ in}^2$   
 Total :  $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \therefore P_Y = A\sigma_{Y1} = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_Y$ ; therefore mild steel yields

let  $P_1$  = force carried by mild steel  
 $P_2$  = force carried by tempered steel

$$P_1 = A_1 \sigma_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi}$$

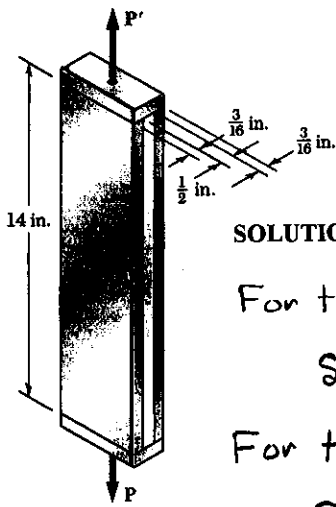
$$\text{Unloading } \sigma' = \frac{P}{A} = \frac{98 \times 10^3}{1.75} = 56 \times 10^3 \text{ psi}$$

Residual stresses

$$\begin{aligned} \text{mild steel } \sigma_{1, \text{res}} &= \sigma_1 - \sigma' = 50 \times 10^3 - 56 \times 10^3 = -6 \times 10^3 \text{ psi} \\ &= -6 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{tempered steel } \sigma_{2, \text{res}} &= \sigma_2 - \sigma' = 64 \times 10^3 - 56 \times 10^3 \\ &= 8 \times 10^3 \text{ psi} = 8 \text{ ksi} \end{aligned}$$

PROBLEM 2.116



2.111 Two tempered-steel bars, each  $\frac{3}{16}$ -in. thick, are bonded to a  $\frac{1}{2}$ -in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude  $P$ . Both steels are elastoplastic with  $E = 29 \times 10^6$  psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

\*2.116 For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if  $P$  is gradually increased from zero until the deformation of the bar reaches a maximum value  $\delta_m = 0.04$  in. and is then decreased back to zero.

SOLUTION

For the mild steel  $A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$S_{Y1} = \frac{L \sigma_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in}$$

For the tempered steel  $A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$S_{Y2} = \frac{L \sigma_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in}$$

Total area:  $A = A_1 + A_2 = 1.75 \text{ in}^2$

$S_{Y1} < S_m < S_{Y2}$  The mild steel yields. Tempered steel is elastic.

Forces  $P_1 = A_1 \sigma_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb}$

$$P_2 = \frac{EA_2 S_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb}$$

Stresses  $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50 \times 10^3 \text{ psi}$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi}$$

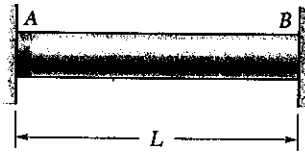
Unloading  $\sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08 \times 10^3 \text{ psi}$

Residual stresses

$$\begin{aligned} \sigma_{1, \text{res}} &= \sigma_1 - \sigma' = 50 \times 10^3 - 64.08 \times 10^3 = -14.08 \times 10^3 \text{ psi} \\ &= -14.08 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \sigma_{2, \text{res}} &= \sigma_2 - \sigma' = 82.86 \times 10^3 - 64.08 \times 10^3 = 18.78 \times 10^3 \text{ psi} \\ &= 18.78 \text{ ksi} \end{aligned}$$

## PROBLEM 2.117



2.117 A uniform steel rod of cross-sectional area  $A$  is attached to rigid supports and is unstressed at a temperature of  $8^\circ\text{C}$ . The steel is assumed to be elastoplastic with  $\sigma_y = 250\text{ MPa}$  and  $G = 200\text{ GPa}$ . Knowing that  $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$ , determine the stress in the bar (a) when the temperature is raised to  $165^\circ\text{C}$ , (b) after the temperature has returned to  $8^\circ\text{C}$ .

## SOLUTION

Determine temperature change to cause yielding

$$S = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_y L}{E} + L\alpha(\Delta T)_y = 0$$

$$(\Delta T)_y = \frac{\sigma_y}{E\alpha} = \frac{250 \times 10^6}{(200 \times 10^9)(11.7 \times 10^{-6})} = 106.838^\circ\text{C}$$

$$\text{But } \Delta T = 165 - 8 = 157^\circ\text{C}$$

$$(a) \text{ Yielding occurs} = \sigma = -\sigma_y = -250\text{ MPa}$$

$$\text{Cooling } (\Delta T)' = 157^\circ\text{C}$$

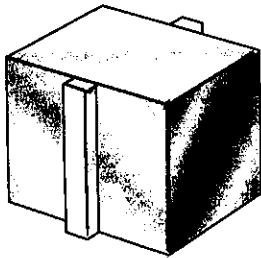
$$S' = S'_p + S'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(200 \times 10^9)(11.7 \times 10^{-6})(157) = -367.38 \times 10^6\text{ Pa}$$

$$(b) \sigma_{\text{res}} = -\sigma_y - \sigma' = -250 \times 10^6 + 367.38 \times 10^6 = 117.38 \times 10^6\text{ Pa} \\ = 117.4\text{ MPa}$$

PROBLEM 2.118



2.118 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at  $T_1 = 20^\circ\text{C}$ , all stresses are zero. Knowing that the temperature will be slowly raised to  $T_2$  and then reduced to  $T_1$ , determine (a) the highest temperature  $T_2$  that does *not* result in residual stresses, (b) the temperature  $T_2$  that will result in a residual stress in the aluminum equal to 100 MPa. Assume  $\alpha_s = 23.6 \times 10^{-6}/^\circ\text{C}$  for the aluminum and  $\alpha_a = 11.7 \times 10^{-6}/^\circ\text{C}$  for the steel. Further assume that the aluminum is elastoplastic, with  $E = 70 \text{ GPa}$  and  $\sigma_y = 100 \text{ MPa}$ . (Hint: Neglect the small stresses in the plate.)

SOLUTION

Determine temperature change to cause yielding

$$s = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{100 \times 10^6}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 120.04^\circ\text{C}$$

(a)  $T_{2Y} = T_1 + (\Delta T)_Y = 20 + 120.04 = 140.04^\circ\text{C}$

After yielding

$$s = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$s' = \frac{P'L}{AE} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set  $\sigma_{res} = -\sigma_Y$

$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

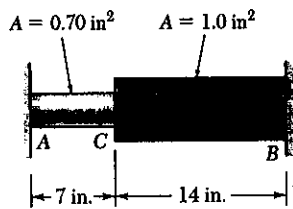
$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(100 \times 10^6)}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 240.1^\circ\text{C}$$

(b)  $T_2 = T_1 + \Delta T = 20 + 240.1 = 260.1^\circ\text{C}$

If  $T_2 > 260.1^\circ\text{C}$ , the aluminum bar will most likely yield in compression.



PROBLEM 2.119



2.119 The steel rod  $ABC$  is attached to rigid supports and is unstressed at a temperature of  $38^\circ\text{F}$ . The steel is assumed elastoplastic, with  $\sigma_y = 36 \text{ ksi}$  and  $E = 29 \times 10^6 \text{ psi}$ . The temperature of both portions of the rod is then raised to  $250^\circ\text{F}$ . Knowing that  $\alpha = 6.5 \times 10^{-6}/^\circ\text{F}$ , determine (a) the stress in portion  $AC$ , (b) the deflection of point  $C$ .

SOLUTION

$$S_{B/A} = S_{B/A,P} + S_{B/A,T} = 0 \quad (\text{constraint})$$

Determine  $\Delta T$  to cause yielding in  $AC$ .

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} + L_{AB} \alpha (\Delta T) = 0$$

$$(\Delta T) = \frac{P}{L_{AB} E \alpha} \left( \frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right)$$

At yielding  $P = A_{AC} \sigma_y$

$$\begin{aligned} (\Delta T)_y &= \frac{A_{AC} \sigma_y}{L_{AB} E \alpha} \left( \frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right) = \frac{(0.70)(36 \times 10^3)}{(21)(29 \times 10^6)(6.5 \times 10^{-6})} \left( \frac{7}{0.70} + \frac{14}{1.0} \right) \\ &= 152.785^\circ\text{F} \end{aligned}$$

Actual  $\Delta T = 250 - 38 = 212^\circ\text{F} > (\Delta T)_y \therefore$  yielding occurs.

$$\sigma_{AC} = -\sigma_y = -36 \text{ ksi}$$

$$P = \sigma_y A_{AC} = (36 \times 10^3)(0.70) = 25.2 \times 10^3 \text{ lb}$$

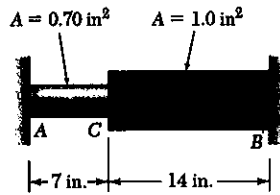
$$s_c = -s_{c/B} = \frac{PL_{CB}}{EA_{CB}} - L_{CB} \alpha (\Delta T)$$

$$= \frac{(25.2 \times 10^3)(14)}{(29 \times 10^6)(1.0)} - (14)(6.5 \times 10^{-6})(212)$$

$$= 0.012176 - 0.019292 = -0.007116 \text{ in}$$

$$s_c = 0.00712 \text{ in} \leftarrow$$

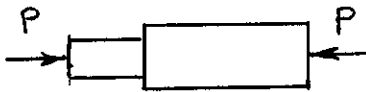
**PROBLEM 2.120**



2.119 The steel rod  $ABC$  is attached to rigid supports and is unstressed at a temperature of  $38^\circ\text{F}$ . The steel is assumed elastoplastic, with  $\sigma_y = 36 \text{ ksi}$  and  $E = 29 \times 10^6 \text{ psi}$ . The temperature of both portions of the rod is then raised to  $250^\circ\text{F}$ . Knowing that  $\alpha = 6.5 \times 10^{-6}/^\circ\text{F}$ , determine (a) the stress in portion  $AC$ , (b) the deflection of point  $C$ .

\*2.120 Solve Prob. 2.119, assuming that the temperature of the rod is raised to  $250^\circ\text{F}$  and then returned to  $38^\circ\text{F}$ .

**SOLUTION**



$$S_{B/A} = S_{B/A,P} + S_{B/A,T} = 0 \quad (\text{constraint})$$

Determine  $\Delta T$  to cause yielding in  $AC$ .

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} + L_{AB}\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{L_{AB}E\alpha} \left( \frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}} \right) = \frac{P}{(21)(29 \times 10^6)(6.5 \times 10^{-6})} \left( \frac{7}{0.70} + \frac{14}{1.0} \right)$$

$$= 6.0629 \times 10^{-3} P \quad \text{At yielding } P_y = \sigma_y A_{AC} = (36 \times 10^3)(0.7) = 25.2 \times 10^3 \text{ lb}$$

$$(\Delta T)_y = (6.0629 \times 10^{-3})(25.2 \times 10^3) = 152.785^\circ\text{F}$$

Actual  $\Delta T = 250 - 38 = 212^\circ\text{F} > (\Delta T)_y \therefore$  yielding occurs.

$$\sigma_{AC} = -\sigma_y = -36 \times 10^3 \text{ psi}$$

$$s_c = -s_{B/C} = \frac{PL_{CB}}{EA_{CB}} - L_{CB}\alpha(\Delta T) = \frac{(25.2 \times 10^3)(14)}{(29 \times 10^6)(1.0)} - (14)(6.5 \times 10^{-6})(212)$$

$$= 0.012176 - 0.019292 = -0.007116 \text{ in}$$

Cooling  $\Delta T' = 212^\circ\text{F} \quad P' = \frac{\Delta T}{6.0629 \times 10^{-3}} = \frac{212}{6.0629 \times 10^{-3}}$

$$P' = \frac{\Delta T'}{6.0629 \times 10^{-3}} = \frac{212}{6.0629 \times 10^{-3}} = 34.967 \times 10^3 \text{ lb}$$

(a) Residual stress in  $AC$

$$\sigma_{AC, \text{res}} = -\sigma_y + \frac{P'}{A_{AC}} = -36 \times 10^3 + \frac{34.967 \times 10^3}{0.7} = 13.95 \times 10^3 \text{ psi} = 13.95 \text{ ksi}$$

$$s_c' = -s_{B/C}' = -\frac{P'L_{CB}}{EA_{CB}} + L_{CB}\alpha(\Delta T')$$

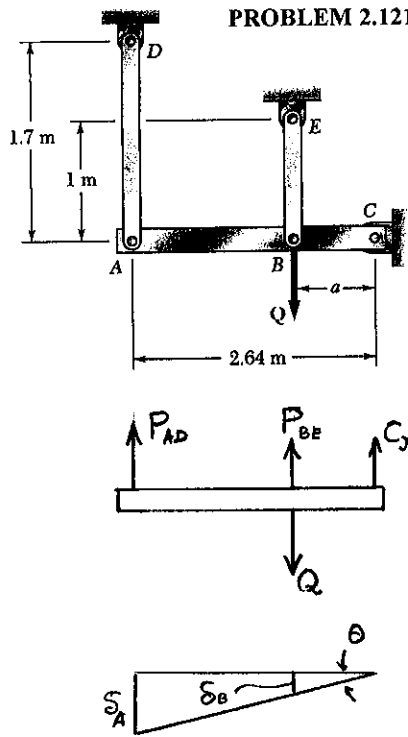
$$= -\frac{(34.967 \times 10^3)(14)}{(29 \times 10^6)(1.0)} + (14)(6.5 \times 10^{-6})(212)$$

$$= -0.016881 + 0.019292 = 0.002411 \text{ in}$$

$$s_{c,p} = s_c + s_c' = -0.007116 + 0.002411 = -0.00471 \text{ in}$$

$$0.00471 \text{ in} \leftarrow$$

PROBLEM 2.121



2.121 The rigid bar  $ABC$  is supported by two links,  $AD$  and  $BE$ , of uniform  $37.5 \times 6$ -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_y = 250$  MPa. The magnitude of the force  $Q$  applied at  $B$  is gradually increased from zero to  $260$  kN. Knowing that  $a = 0.640$  m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point  $B$ .

SOLUTION

Statics:  $\Sigma M_C = 0 \quad 0.640(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation:  $\delta_A = 2.64 \theta, \quad \delta_B = a\theta = 0.640 \theta$

Elastic Analysis:

$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$

$P_{AD} = \frac{EA}{L_{AD}} \delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} \delta_A = 26.47 \times 10^6 \delta_A$   
 $= (26.47 \times 10^6)(2.64 \theta) = 69.88 \times 10^6 \theta$

$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$

$P_{BE} = \frac{EA}{L_{BE}} \delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} \delta_B = 45 \times 10^6 \delta_B$   
 $= (45 \times 10^6)(0.640 \theta) = 28.80 \times 10^6 \theta$

$\sigma_{BE} = \frac{P_{BE}}{A} = 128 \times 10^9 \theta$

From Statics  $Q = P_{BE} + \frac{2.64}{0.640} P_{AD} = P_{BE} + 4.125 P_{AD}$   
 $= [28.80 \times 10^6 + (4.125)(69.88 \times 10^6)] \theta = 317.06 \times 10^6 \theta$

$Q_y$  at yielding of link  $AD \quad \sigma_{AD} = \sigma_y = 250 \times 10^6 = 310.6 \times 10^9 \theta$

$\theta_y = 804.89 \times 10^{-6}$

$Q_y = (317.06 \times 10^6)(804.89 \times 10^{-6}) = 255.2 \times 10^3 \text{ N}$

Since  $Q = 260 \times 10^3 > Q_y$ , link  $AD$  yields.  $\sigma_{AD} = 250 \text{ MPa}$   $\blacktriangleleft$

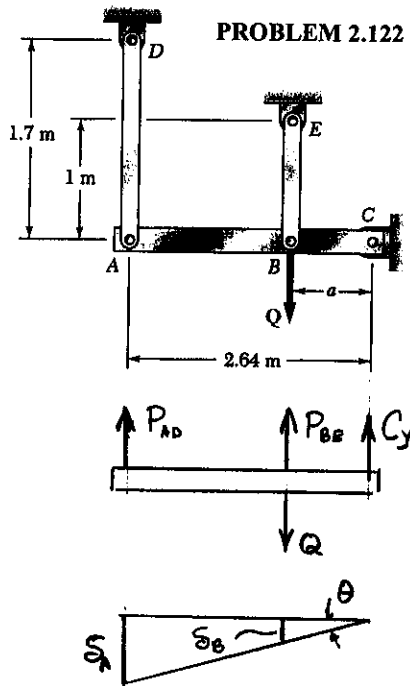
$P_{AD} = A \sigma_y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$

From Statics  $P_{BE} = Q - 4.125 P_{AD} = 260 \times 10^3 - (4.125)(56.25 \times 10^3)$

$P_{BE} = 27.97 \times 10^3 \text{ N} \quad \sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97 \times 10^3}{225 \times 10^{-6}} = 124.3 \times 10^6 \text{ Pa}$   
 $= 124.3 \text{ MPa}$   $\blacktriangleleft$

$\delta_B = \frac{P_{BE} L_{BE}}{EA} = \frac{(27.97 \times 10^3)(1.0)}{(200 \times 10^9)(225 \times 10^{-6})} = 621.53 \times 10^{-6} \text{ m}$   
 $= 0.622 \text{ mm}$   $\blacktriangleleft$

PROBLEM 2.122



2.121 The rigid bar  $ABC$  is supported by two links,  $AD$  and  $BE$ , of uniform  $37.5 \times 6$ -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_Y = 250$  MPa. The magnitude of the force  $Q$  applied at  $B$  is gradually increased from zero to  $260$  kN. Knowing that  $a = 0.640$  m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point  $B$ .

2.122 Solve Prob. 2.121, knowing that  $a = 1.76$  m and that the magnitude of the force  $Q$  applied at  $B$  is gradually increased from zero to  $135$  kN.

SOLUTION

Statics:  $\sum M_C = 0 \quad 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation:  $\delta_A = 2.64 \theta, \quad \delta_B = 1.76 \theta$

Elastic Analysis

$A = (37.5)(6) = 225 \text{ mm}^2 = 225 \times 10^{-6} \text{ m}^2$

$P_{AD} = \frac{EA}{L_{AD}} \delta_A = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.7} \delta_A = 26.47 \times 10^6 \delta_A$   
 $= (26.47 \times 10^6)(2.64 \theta) = 69.88 \times 10^6 \theta$

$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6 \times 10^9 \theta$

$P_{BE} = \frac{EA}{L_{BE}} \delta_B = \frac{(200 \times 10^9)(225 \times 10^{-6})}{1.0} \delta_B = 45 \times 10^6 \delta_B = (45 \times 10^6)(1.76 \theta)$   
 $= 79.2 \times 10^6 \theta$

$\sigma_{BE} = \frac{P_{BE}}{A} = 352 \times 10^9 \theta$

From Statics  $Q = P_{BE} + \frac{2.64}{1.76} P_{AD} = P_{BE} + 1.500 P_{AD}$   
 $= [79.2 \times 10^6 + (1.500)(69.88 \times 10^6)] \theta = 178.62 \times 10^6 \theta$

$\theta_Y$  at yielding of link BE  $\sigma_{BE} = \sigma_Y = 250 \times 10^6 = 352 \times 10^9 \theta_Y$

$\theta_Y = 710.23 \times 10^{-6}$

$Q_Y = (178.62 \times 10^6)(710.23 \times 10^{-6}) = 126.86 \times 10^3 \text{ N}$

Since  $Q = 135 \times 10^3 \text{ N} > Q_Y$ , link BE yields  $\sigma_{BE} = \sigma_Y = 250 \text{ MPa}$  ←

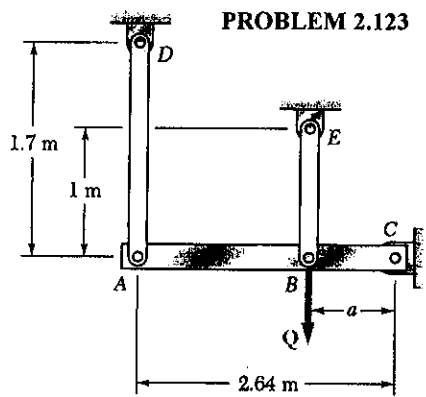
$P_{BE} = A \sigma_Y = (225 \times 10^{-6})(250 \times 10^6) = 56.25 \times 10^3 \text{ N}$

From Statics  $P_{AD} = \frac{1}{1.500} (Q - P_{BE}) = 52.5 \times 10^3 \text{ N}$

$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5 \times 10^3}{225 \times 10^{-6}} = 233.3 \times 10^6 = 233 \text{ MPa}$  ←

From elastic analysis of AD  $\theta = \frac{P_{AD}}{69.88 \times 10^6} = 751.29 \times 10^{-5} \text{ rad}$

$\delta_B = 1.76 \theta = 1.322 \times 10^{-3} \text{ m} = 1.322 \text{ mm}$  ←



2.121 The rigid bar  $ABC$  is supported by two links,  $AD$  and  $BE$ , of uniform  $37.5 \times 6$ -mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_y = 250$  MPa. The magnitude of the force  $Q$  applied at  $B$  is gradually increased from zero to 260 kN. Knowing that  $a = 0.640$  m, determine (a) the value of the normal stress in each link, (b) the ~~maximum~~ **final** deflection of point  $B$ .

\*2.123 Solve Prob. 2.121, assuming that the magnitude of the force  $Q$  applied at  $B$  is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that  $a = 0.640$  m, determine (a) the residual stress in each link, (b) the final deflection of point  $B$  the residual stress in each link. Assume that the links are braced so that they can carry compressive forces without buckling.

**SOLUTION**

See solution to PROBLEM 2.121 for the normal stresses in each link and the deflection of point  $B$  after loading

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa} \quad \sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$

$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to PROBLEM 2.121 applies to the unloading

$$Q = 317.06 \times 10^6 \theta'$$

$$\theta' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

$$\sigma'_{AD} = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$

$$\sigma'_{BE} = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$

$$\delta'_B = 0.640 \theta' = 524.82 \times 10^{-6} \text{ m}$$

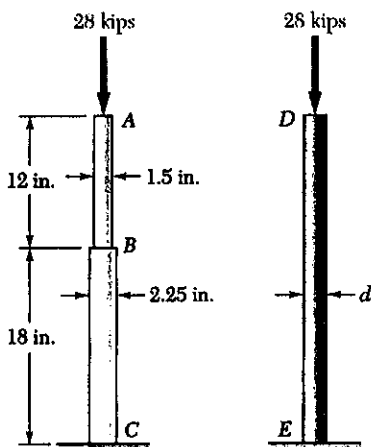
(a) Residual stresses

$$\begin{aligned} \sigma_{AD, \text{res}} &= \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^6 = -4.70 \times 10^6 \text{ Pa} \\ &= -4.70 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma_{BE, \text{res}} &= \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \\ &= 19.34 \text{ MPa} \end{aligned}$$

$$\begin{aligned} (b) \delta_{B, \text{P}} &= \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} \\ &= 96.71 \times 10^{-6} \text{ m} = 0.0967 \text{ mm} \end{aligned}$$

## PROBLEM 2.124



2.124 The aluminum rod  $ABC$  ( $E = 10.1 \times 10^6$  psi), which consists of two cylindrical portions  $AB$  and  $BC$ , is to be replaced with a cylindrical steel rod  $DE$  ( $E = 29 \times 10^6$  psi) of the same overall length. Determine the minimum required diameter  $d$  of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

## SOLUTION

Deformation of aluminum rod

$$\begin{aligned} \delta_A &= \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E} \left( \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right) \\ &= \frac{28 \times 10^3}{10.1 \times 10^6} \left( \frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2} \right) = 0.031376 \text{ in} \end{aligned}$$

Steel rod  $\delta = 0.031376 \text{ in}$

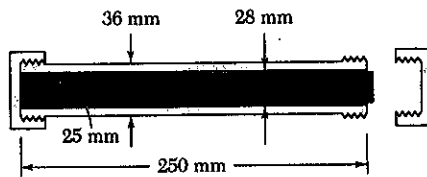
$$\delta = \frac{PL}{EA} \quad \therefore A = \frac{PL}{E\delta} = \frac{(28 \times 10^3)(30)}{(29 \times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore A = \frac{P}{\sigma} = \frac{28 \times 10^3}{24 \times 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value  $A = 1.1667 \text{ in}^2$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(1.1667)}{\pi}} = 1.219 \text{ in.}$$

PROBLEM 2.125



2.125 A 250-mm-long aluminum tube ( $E = 70 \text{ GPa}$ ) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ( $E = 105 \text{ GPa}$ ) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$S_{\text{tube}} = \frac{PL}{E_{\text{tube}} A_{\text{tube}}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$S_{\text{rod}} = -\frac{PL}{E_{\text{rod}} A_{\text{rod}}} = \frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} = -4.8505 \times 10^{-9} P$$

$$S^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$S_{\text{tube}} = S^* + S_{\text{rod}} \quad \text{or} \quad S_{\text{tube}} - S_{\text{rod}} = S^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

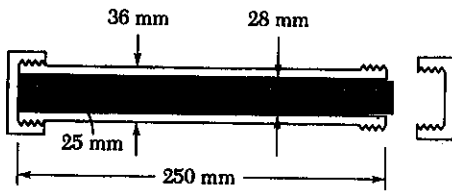
$$(a) \quad \sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308 \cdot 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa} = 67.9 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa} = -55.6 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \delta_{\text{tube}} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m} = 0.2425 \text{ mm} \quad \blacktriangleleft$$

$$\delta_{\text{rod}} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m} = -0.1325 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 2.126**



2.125 A 250-mm-long aluminum tube ( $E = 70$  GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ( $E = 105$  GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

2.126 In Prob. 2.125, determine the average normal stress in the tube and the rod, assuming that the temperature was  $15^\circ\text{C}$  when the nuts were snugly fitted and that the final temperature is  $55^\circ\text{C}$ . (For aluminum,  $\alpha = 23.6 \times 10^{-6}/^\circ\text{C}$ ; for brass,  $\alpha = 20.9 \times 10^{-6}/^\circ\text{C}$ .)

**SOLUTION**

$$A_{\text{tube}} = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\Delta T = 55 - 15 = 40^\circ\text{C}$$

$$\begin{aligned} \delta_{\text{tube}} &= \frac{PL}{E_{\text{tube}} A_{\text{tube}}} + L \alpha_{\text{tube}} (\Delta T) = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} + (0.250)(23.6 \times 10^{-6})(40) \\ &= 8.8815 \times 10^{-9} P + 236 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} \delta_{\text{rod}} &= -\frac{PL}{E_{\text{rod}} A_{\text{rod}}} + L \alpha_{\text{rod}} (\Delta T) = -\frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} + (0.250)(20.9 \times 10^{-6})(40) \\ &= -4.8505 \times 10^{-9} P + 209 \times 10^{-6} \end{aligned}$$

$$\delta^* = \frac{1}{4} \text{turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{\text{tube}} = \delta_{\text{rod}} + \delta^*$$

$$8.8815 \times 10^{-9} P + 236 \times 10^{-6} = -4.8505 \times 10^{-9} P + 209 \times 10^{-6} + 375 \times 10^{-6}$$

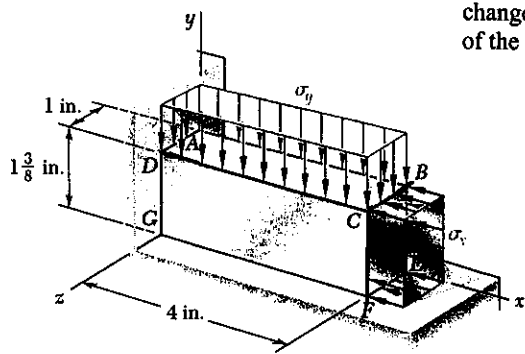
$$13.732 \times 10^{-9} P = 348 \times 10^{-6} \quad P = 25.342 \times 10^3 \text{ N}$$

$$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{25.342 \times 10^3}{402.12 \times 10^{-6}} = 63.0 \times 10^6 \text{ Pa} = 63.0 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{25.342 \times 10^3}{490.87 \times 10^{-6}} = -51.6 \times 10^6 \text{ Pa} = -51.6 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 2.127



2.127 The block shown is made of a magnesium alloy for which  $E = 6.5 \times 10^6$  psi and  $\nu = 0.35$ . Knowing that  $\sigma_x = -20$  ksi, determine (a) the magnitude of  $\sigma_y$  for which the change in the height of the block will be zero, (b) the corresponding change in the area of the face ABCD, (c) the corresponding change in the volume of the block.

SOLUTION

$$\delta_y = 0 \quad \epsilon_y = 0$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = 0$$

$$(a) \quad \sigma_y = \nu \sigma_x = (0.35)(-20 \times 10^3)$$

$$= -7 \times 10^3 \text{ psi} = -7 \text{ ksi}$$

$$(b) \quad \epsilon_z = \frac{1}{E} (-\nu \sigma_x - \nu \sigma_y) = -\frac{\nu(\sigma_x + \sigma_y)}{E}$$

$$= \frac{(0.35)(-20 \times 10^3 - 7 \times 10^3)}{6.5 \times 10^6} = 1.4538 \times 10^{-3}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{-20 \times 10^3 - (0.35)(-7 \times 10^3)}{6.5 \times 10^6}$$

$$= -2.7 \times 10^{-3}$$

$$A_0 + \Delta A = L_x(1 + \epsilon_x)L_z(1 + \epsilon_z) = L_x L_z (1 + \epsilon_x + \epsilon_z + \epsilon_x \epsilon_z)$$

$$\text{But } A_0 = L_x L_z$$

$$\Delta A = L_x L_z (\epsilon_x + \epsilon_z + \epsilon_x \epsilon_z)$$

$$= (4.0)(1.0)(1.4538 \times 10^{-3} - 2.7 \times 10^{-3} + \text{small term})$$

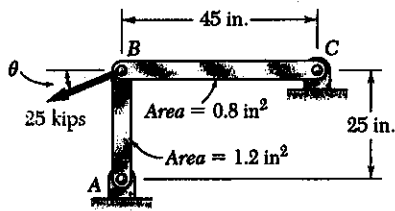
$$= -4.98 \times 10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2$$

(c) Since  $L_y$  is constant

$$\Delta V = L_y (\Delta A) = (1.375)(-4.98 \times 10^{-3}) = -6.85 \times 10^{-3} \text{ in}^3$$

$$= -0.00685 \text{ in}^3$$

**PROBLEM 2.128**



2.128 The uniform rods  $AB$  and  $BC$  are made of steel and are loaded as shown. Knowing that  $E = 29 \times 10^6$  psi, determine the magnitude and direction of the deflection of point  $B$  when  $\theta = 22^\circ$ .

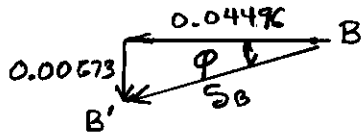
**SOLUTION**

$$P_{BC} = P \cos \theta = (25 \times 10^3) \cos 22^\circ = 23.18 \times 10^3 \text{ lb}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{E A_{BC}} = \frac{(23.18 \times 10^3)(45)}{(29 \times 10^6)(0.8)} = 0.04496 \text{ in}$$

$$P_{AB} = P \sin \theta = (25 \times 10^3) \sin 22^\circ = 9.365 \times 10^3 \text{ lb}$$

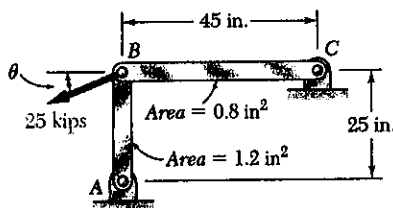
$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}} = \frac{(9.365 \times 10^3)(25)}{(29 \times 10^6)(1.2)} = 0.00673 \text{ in}$$



$$\tan \phi = \frac{0.00673}{0.04496} = 0.1496 \quad \phi = 8.51^\circ \quad \blacktriangleleft$$

$$\delta = \sqrt{0.04496^2 + 0.00673^2} = 0.0455 \text{ in.} \quad \blacktriangleleft$$

**PROBLEM 2.129**



2.129 Knowing that  $E = 29 \times 10^6$  psi, determine (a) the value of  $\theta$  for which the deflection of point  $B$  is down and to the left along a line forming an angle of  $36^\circ$  with the horizontal, (b) the corresponding magnitude of the deflection of  $B$ .

**SOLUTION**

$$\delta_{BC} = \delta \cos 36^\circ$$

$$P_{BC} = \frac{E A_{BC} \delta_{BC}}{L_{BC}} = \frac{(29 \times 10^6)(0.8) \delta \cos 36^\circ}{45} = 417.09 \times 10^3 \delta$$

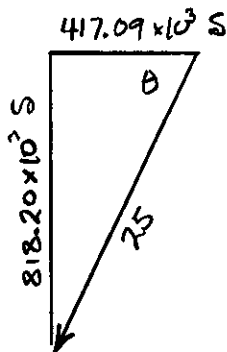
$$\delta_{AB} = \delta \sin 36^\circ$$

$$P_{AC} = \frac{E A_{AC} \delta_{AC}}{L_{AC}} = \frac{(29 \times 10^6)(1.2) \delta \sin 36^\circ}{25} = 818.20 \times 10^3 \delta$$

$$\tan \theta = \frac{818.20 \times 10^3 \delta}{417.09 \times 10^3 \delta} = 1.9617 \quad \theta = 63.0^\circ \quad \blacktriangleleft$$

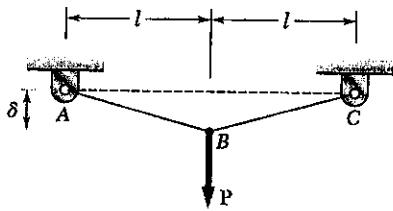
$$P = 25 \times 10^3 = \sqrt{(417.09 \times 10^3 \delta)^2 + (818.20 \times 10^3 \delta)^2} = 918.38 \times 10^3 \delta$$

$$\delta = \frac{25 \times 10^3}{918.38 \times 10^3} = 0.0272 \text{ in.} \quad \blacktriangleleft$$



PROBLEM 2.130

2.130 The uniform wire  $ABC$ , of unstretched length  $2l$ , is attached to the supports shown and a vertical load  $P$  is applied at the midpoint  $B$ . Denoting by  $A$  the cross-sectional area of the wire and by  $E$  the modulus of elasticity, show that, for  $\delta \ll l$ , the deflection at the midpoint  $B$  is

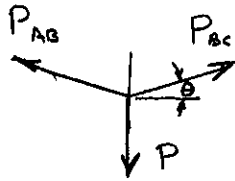


$$\delta = l \sqrt{\frac{P}{AE}}$$

SOLUTION

Use approximation

$$\sin \theta \approx \tan \theta \approx \frac{\delta}{l}$$

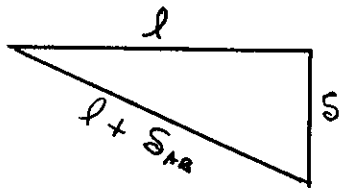


Statics  $\sum F_y = 0 \quad 2P_{AB} \sin \theta - P = 0$

$$P_{AB} = \frac{P}{2 \sin \theta} \approx \frac{Pl}{2\delta}$$

Elongation  $S_{AB} = \frac{P_{AB}l}{AE} = \frac{Pl^2}{2AES}$

Deflection



From the right triangle

$$(l + S_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = l^2 + 2l S_{AB} + S_{AB}^2 - l^2$$

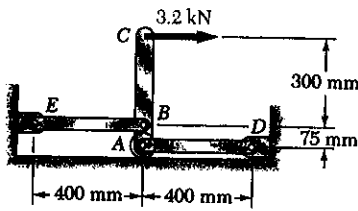
$$= 2l S_{AB} \left(1 + \frac{1}{2} \frac{S_{AB}}{l}\right) \approx 2l S_{AB}$$

$$\approx \frac{Pl^3}{AES}$$

$$S^3 \approx \frac{Pl^3}{AE} \quad \therefore S \approx l \sqrt[3]{\frac{P}{AE}}$$

**PROBLEM 2.131**

2.131 The steel bars  $BE$  and  $AD$  each have a  $6 \times 18$ -mm cross section. Knowing that  $E = 200$  GPa, determine the deflections of points  $A$ ,  $B$ , and  $C$  of the rigid bar  $ABC$ .



**SOLUTION**

Use rigid bar  $ABC$  as a free body

$$\sum M_B = 0 \quad (75)P_{AD} - (300)(3.2) = 0$$

$$P_{AD} = 12.8 \text{ kN}$$

$$\sum F_x = 0 \quad -P_{BE} + 3.2 + P_{AD} = 0$$

$$P_{BE} = 16 \text{ kN}$$

**Deformations**

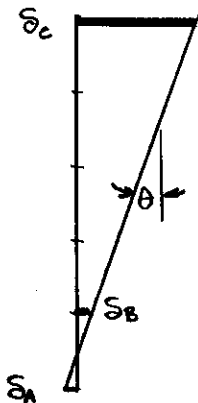
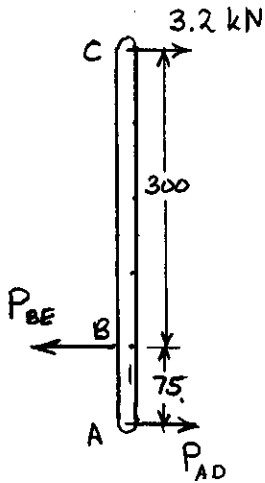
$$A = (6)(18) = 108 \text{ mm}^2 = 108 \times 10^{-6} \text{ m}^2$$

$$\begin{aligned} \leftarrow \delta_A = \delta_{AD} &= \frac{P_{AD} L_{AD}}{EA} = \frac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} \\ &= 237.04 \times 10^{-6} \text{ m} = 0.237 \text{ mm} \leftarrow \end{aligned}$$

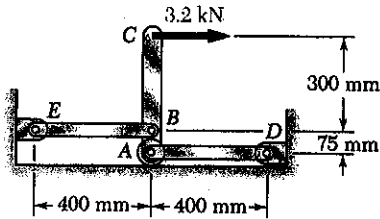
$$\begin{aligned} \leftarrow \delta_B = \delta_{BE} &= \frac{P_{BE} L_{BE}}{EA} = \frac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} \\ &= 296.30 \times 10^{-6} \text{ m} = 0.296 \text{ mm} \leftarrow \end{aligned}$$

$$\begin{aligned} \theta &= \frac{\delta_A + \delta_B}{L_{AB}} = \frac{(237.04 + 296.30)(10^{-6})}{75 \times 10^{-3}} \\ &= 7.1112 \times 10^{-3} \end{aligned}$$

$$\begin{aligned} \delta_C &= \delta_B + L_{BC} \theta \\ &= 296.30 \times 10^{-6} + (300 \times 10^{-3})(7.1112 \times 10^{-3}) \\ &= 2.4297 \times 10^{-3} \text{ m} = 2.43 \text{ mm} \rightarrow \end{aligned}$$



**PROBLEM 2.132**



2.131 The steel bars  $BE$  and  $AD$  each have a  $6 \times 18$ -mm cross section. Knowing that  $E = 200$  GPa, determine the deflections of points  $A$ ,  $B$ , and  $C$  of the rigid bar  $ABC$ .

2.132 In Prob. 2.131, the 3.2-kN force caused point  $C$  to deflect to the right. Using  $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$ , determine the (a) the overall change in temperature that causes point  $C$  to return to its original position, (b) the corresponding total deflection of points  $A$  and  $B$ .

**SOLUTION**

Use rigid  $ABC$  as a free body

$$\circlearrowleft \sum M_B = 0 \quad 75 P_{AD} - (300)(3.2) = 0$$

$$P_{AD} = 12.8 \text{ kN}$$

$$\rightarrow \sum F_x = 0 \quad -P_{BE} + 3.2 + P_{AD} = 0$$

$$P_{BE} = 16 \text{ kN}$$

Deformations:

$$\begin{aligned} \leftarrow \delta_A = \delta_{AD} &= \frac{P_{AD} L_{AD}}{EA} + L_{AD} \alpha (\Delta T) \\ &= \frac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} + (400 \times 10^{-3})(11.7 \times 10^{-6})(\Delta T) \\ &= 237.04 \times 10^{-6} + 4.68 \times 10^{-6} (\Delta T) \end{aligned}$$

$$\begin{aligned} \rightarrow \delta_B = \delta_{BE} &= \frac{P_{BE} L_{BE}}{EA} + L_{BE} \alpha (\Delta T) \\ &= \frac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})} + (400 \times 10^{-3})(11.7 \times 10^{-6})(\Delta T) \\ &= 296.30 \times 10^{-6} + 4.68 \times 10^{-6} (\Delta T) \end{aligned}$$

$$\delta_C = 0 \quad \delta_B = 0.300 \theta \quad -\delta_A = 0.375 \theta$$

$$-\delta_A = \frac{0.375}{0.300} \delta_B = 1.25 \delta_B$$

$$-(237.04 \times 10^{-6} + 4.68 \times 10^{-6} (\Delta T)) = (1.25) [296.30 \times 10^{-6} + 4.68 \times 10^{-6} (\Delta T)]$$

$$-10.53 \times 10^{-6} (\Delta T) = 607.415 \times 10^{-6} \dots \Delta T = -57.684 \text{ } ^\circ\text{C}$$

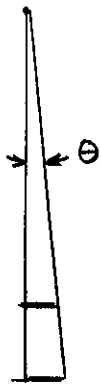
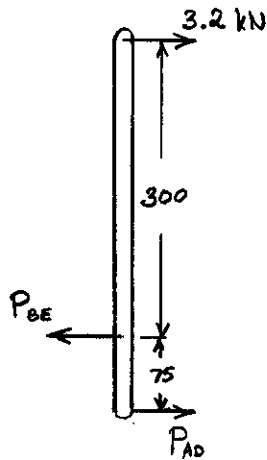
$$= -57.7 \text{ } ^\circ\text{C}$$

$$\delta_A = 237.04 \times 10^{-6} + (4.68 \times 10^{-6})(-57.684) = -32.92 \times 10^{-6} \text{ m}$$

$$\delta_A = 0.0329 \text{ mm} \rightarrow$$

$$\delta_B = 296.30 \times 10^{-6} - (4.68 \times 10^{-6})(57.684) = +26.34 \times 10^{-6} \text{ m}$$

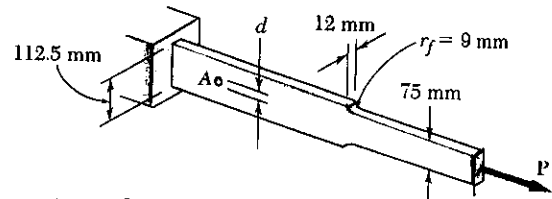
$$\delta_B = 0.0263 \text{ mm} \rightarrow$$



PROBLEM 2.133

2.133 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from 9 to 27 mm in 3-mm increments. (a) Determine the diameter  $d$  of the largest bit that can be used if the allowable load at the hole is not to exceed that at the fillets. (b) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load  $P$ ?

SOLUTION



At the fillets,  $r = 9 \text{ mm}$   $d = 75 \text{ mm}$

$$D = 112.5 \text{ mm} \quad \frac{D}{d} = \frac{112.5}{75} = 1.5$$

$$\frac{r}{d} = \frac{9}{75} = 0.12 \quad \text{From Fig 2.64 b} \quad K = 2.10$$

$$A_{\min} = (75)(12) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\max} = K \frac{P_{\text{all}}}{A_{\min}} = \sigma_{\text{all}} \quad \therefore P_{\text{all}} = \frac{A_{\min} \sigma_{\text{all}}}{K} = \frac{(900 \times 10^{-6})(145 \times 10^6)}{2.10}$$

$$= 62.1 \times 10^3 \text{ N} = 62.1 \text{ kN}$$

At the hole:  $A_{\text{net}} = (D - 2r)t$ ,  $\frac{r}{d} = \frac{r}{D - 2r}$

where  $D = 112.5 \text{ mm}$   $r = \text{radius of circle}$   $t = 12 \text{ mm}$

$K$  is taken from Fig 2.64 a

$$\sigma_{\max} = K \frac{P}{A_{\text{net}}} = \sigma_{\text{all}} \quad \therefore P_{\text{all}} = \frac{A_{\text{net}} \sigma_{\text{all}}}{K}$$

Hole diam	$r$	$d = D - 2r$	$r/d$	$K$	$A_{\text{net}}$	$P_{\text{all}}$
9 mm	4.5 mm	103.5 mm	0.0435	2.87	$1242 \times 10^{-6} \text{ m}^2$	$62.7 \times 10^3 \text{ N}$
15 mm	7.5 mm	97.5 mm	0.077	2.75	$1170 \times 10^{-6} \text{ m}^2$	$61.7 \times 10^3 \text{ N}$
21 mm	10.5 mm	91.5 mm	0.115	2.67	$1098 \times 10^{-6} \text{ m}^2$	$59.6 \times 10^3 \text{ N}$
27 mm	13.5 mm	85.5 mm	0.158	2.57	$1026 \times 10^{-6} \text{ m}^2$	$57.9 \times 10^3 \text{ N}$

largest hole with  $P_{\text{all}} > 62 \text{ kN}$  is the 9 mm diameter hole.  $\blacktriangleleft$

Allowable force  $P_{\text{all}} = 62 \text{ kN}$   $\blacktriangleleft$

## PROBLEM 2.134

2.134 (a) For  $P = 58 \text{ kN}$  and  $d = 12 \text{ mm}$ , determine the maximum stress in the plate shown. (b) Solve part a, assuming that the hole at A is not drilled.

## SOLUTION

Maximum stress at hole

Use Fig. 2.64 a for values of  $K$

$$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \quad K = 2.80$$

$$A_{\text{net}} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{\text{max}} = K \frac{P}{A_{\text{net}}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \quad \frac{D}{d} = \frac{112.5}{75} = 1.50, \quad K = 2.10$$

$$A_{\text{min}} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

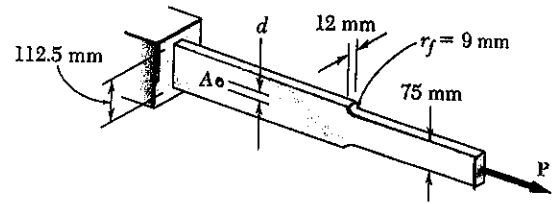
$$\sigma_{\text{max}} = K \frac{P}{A_{\text{min}}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets

$$\sigma_{\text{max}} = 134.7 \text{ MPa}$$

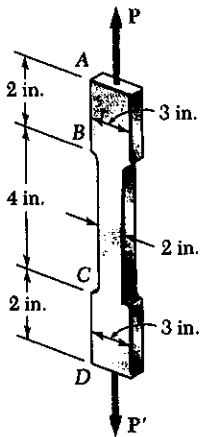
(b) Without hole

$$\sigma_{\text{max}} = 135.3 \text{ MPa}$$



PROBLEM 2.135

2.135 The steel tensile specimen  $ABCD$  ( $E = 29 \times 10^6$  psi and  $\sigma_y = 50$  ksi) is loaded in tension until the maximum tensile strain is  $\epsilon = 0.0025$ . (a) Neglecting the effect of the fillets on the change in length of the specimen, determine the resulting overall length  $AD$  of the specimen after the load is removed. (b) Following the removal of the load in part (b), a compressive load is applied until the maximum compressive strain is  $\epsilon = 0.0020$ . Determine the resulting overall length  $AD$  after the load is removed.



SOLUTION

$$(a) \epsilon_y = \frac{\sigma_y}{E} = \frac{50 \times 10^3}{29 \times 10^6} = 0.001724$$

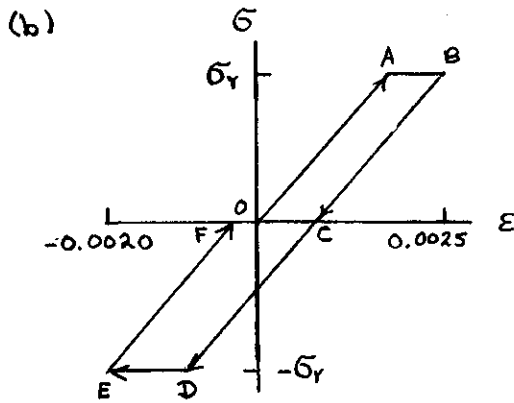
$$\epsilon_{max} = 0.0025 > \epsilon_y \quad \text{Yielding occurs in portion BC}$$

$$\sigma_{BC} = \sigma_y = 50 \times 10^3 \text{ psi}$$

Permanent strain in BC

$$\epsilon_{BC} = \epsilon_{max} - \epsilon_y = 0.0025 - 0.001724 = 0.000776$$

$$\delta_{BC} = L_{BC} \epsilon_{BC} = (4)(0.000776) = 0.00310 \text{ in.}$$



In reversed loading, at point E on stress-strain plot

$$\epsilon = -0.0020$$

as given. During removal of the reversed load, the change in strain is  $\sigma_y / E = 0.001724$ .

The permanent strain in BC is

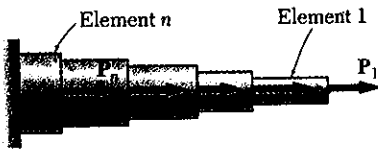
$$\epsilon_{BC} = -0.0020 + 0.001724 = -0.000276$$

$$\delta_{BC} = L_{BC} \epsilon_{BC} = (4)(-0.000276) = -0.001104 \text{ in.}$$

Note that portions AB and CD are always elastic, thus their deformations during loading and unloading do not contribute to any permanent deformation.



**PROBLEM 2.C1**



**2.C1** A rod consisting of  $n$  elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element  $i$  is denoted by  $L_i$ , its cross-sectional area by  $A_i$ , its modulus of elasticity by  $E_i$ , and the load applied to its right end by  $P_i$ , the magnitude  $P_i$  of this load being assumed to be positive if  $P_i$  is directed to the right and negative otherwise. (a) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (b) Use this program to solve Probs. 2.17 and 2.18.

**SOLUTION**

FOR EACH ELEMENT, ENTER

$$L_i, A_i, E_i$$

COMPUTE DEFORMATION

$$\text{UPDATE AXIAL LOAD } P = P + P_i$$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_i E_i$$

TOTAL DEFORMATION:

UPDATE THROUGH  $n$  ELEMENTS

$$\delta = \delta + \delta_i$$

PROGRAM OUTPUT

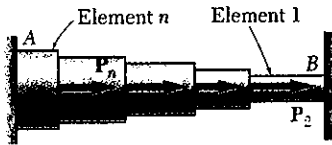
Problem 2.17

Element	Stress (MPa)	Deformation (mm)
1	19.0986	.1091
2	-12.7324	-.0909
Total Deformation =		.0182 mm

Problem 2.18

Element	Stress (MPa)	Deformation (mm)
1	98.2438	2.3391
2	98.2438	1.4737
3	147.3657	1.4737
Total Deformation =		5.2865 mm

**PROBLEM 2.C2**



**2.C2** Rod AB is horizontal with both ends fixed; it consists of  $n$  elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element  $i$  is denoted by  $L_i$ , its cross-sectional area by  $A_i$ , its modulus of elasticity by  $E_i$ , and the load applied to its right end by  $P_i$ , the magnitude  $P_i$  of this load being assumed to be positive if  $P_i$  is directed to the right and negative otherwise. (Note that  $P_1 = 0$ .) (a) Write a computer program that can be used to determine the reactions at A and B, the average normal stress in each element, and the deformation of each element. (b) Use this program to solve Prob. 2.41.

**SOLUTION**

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE  $\delta_B$  WITH  $R_B = 0$

FOR EACH ELEMENT, ENTER  
 $L_i, A_i, E_i$

UPDATE AXIAL LOAD

$$P = P + P_i$$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_i E_i$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE  $\delta_B$  DUE TO UNIT LOAD AT B

$$\text{UNIT } \sigma_i = 1/A_i$$

$$\text{UNIT } \delta_i = L_i/A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT B = ZERO

$$\delta_B + R_B \text{ UNIT } \delta_B = 0$$

SOLVING:

$$R_B = -\delta_B / \text{UNIT } \delta_B$$

THEN:

$$R_A = \sum P_i + R_B$$

CONTINUED

PROBLEM 2.C2 CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

Problem 2.41

RA = -11.909 kips

RB = -20.091 kips

Element	Stress (ksi)	Deformation (in.)
---------	--------------	-------------------

1	12.002	-.00923
2	-6.128	-.00589
3	-9.687	-.00334

**PROBLEM 2.C3**

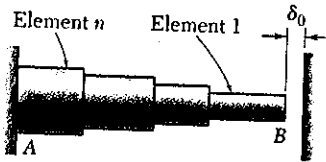


Fig. P2.C3

**2.C3** Rod  $AB$  consists of  $n$  elements, each of which is homogeneous and of uniform cross section. End  $A$  is fixed, while initially there is a gap  $\delta_0$  between end  $B$  and the fixed vertical surface on the right. The length of element  $i$  is denoted by  $L_i$ , its cross-sectional area by  $A_i$ , its modulus of elasticity by  $E_i$ , and its coefficient of thermal expansion by  $\alpha_i$ . After the temperature of the rod has been increased by  $\Delta T$ , the gap at  $B$  is closed and the vertical surfaces exert equal and opposite forces on the rod. (a) Write a computer program that can be used to determine the magnitude of the reactions at  $A$  and  $B$ , the normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.53, 2.54, 2.57, and 2.59.

**SOLUTION**

WE COMPUTE THE DISPLACEMENTS AT  $B$   
ASSUMING THERE IS NO SUPPORT AT  $B$ :

ENTER  $L_i, A_i, E_i, \alpha_i$

ENTER TEMPERATURE CHANGE  $T$

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE  $\delta_B$  DUE TO UNIT LOAD AT  $B$

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = -R_B$$

FOR EACH ELEMENT

$$\sigma_i = -R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

CONTINUED

PROBLEM 2.C3 CONTINUED

PROGRAM OUTPUT

Problem 2.53

R = 25.837 kips

Element	Stress (ksi)	Deform. (10 <sup>-3</sup> in.)
1	-21.054	-3.642
2	-6.498	3.642

Problem 2.54

R = 125.628 kN

Element	Stress (MPa)	Deform. (microm)
1	-44.432	500.104
2	-99.972	-500.104

Problem 2.57

R = 217.465 kN

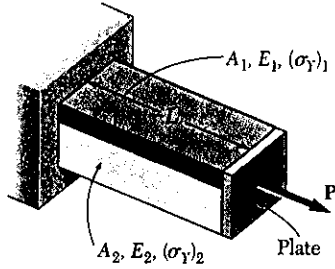
Element	Stress (MPa)	Deform. (microm)
1	-144.977	242.504
2	-120.814	257.496

Problem 2.59

R = 61.857 kips

Element	Stress (ksi)	Deform. (10 <sup>-3</sup> in.)
1	-22.092	14.410
2	-51.547	5.590

**PROBLEM 2.C4**



**2.C4** Bar  $AB$  has a length  $L$  and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load  $P$  which is gradually increased from zero until the deformation of the bar has reached a maximum value  $\delta_m$  and then decreased back to zero. (a) Write a computer program that, for each of 25 values of  $\delta_m$  equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value  $P_m$  of the load, the maximum normal stress in each material, the permanent deformation  $\delta_p$  of the bar, and the residual stress in each material. (b) Use this program to solve Probs. 2.109, 2.111, and 2.112.

**SOLUTION**

**NOTE: THE FOLLOWING ASSUMES  $(\sigma_Y)_1 < (\sigma_Y)_2$**   
DISPLACEMENT INCREMENT

$$\delta_m = 0.05 (\sigma_Y)_2 L / E_2$$

DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1 L / E_1 \quad \delta_B = (\sigma_Y)_2 L / E_2$$

FOR EACH DISPLACEMENT

IF  $\delta_m < \delta_A$ :

$$\sigma_1 = \delta_m E_1 / L$$

$$\sigma_2 = \delta_m E_2 / L$$

$$P_m = (\delta_m / L) (A_1 E_1 + A_2 E_2)$$

IF  $\delta_A < \delta_m < \delta_B$ :

$$\sigma_1 = (\sigma_Y)_1$$

$$\sigma_2 = \delta_m E_2 / L$$

$$P_m = A_1 \sigma_1 + (\delta_m / L) A_2 E_2$$

IF  $\delta_m > \delta_B$ :

$$\sigma_1 = (\sigma_Y)_1 \quad \sigma_2 = (\sigma_Y)_2$$

$$P_m = A_1 \sigma_1 + A_2 \sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES

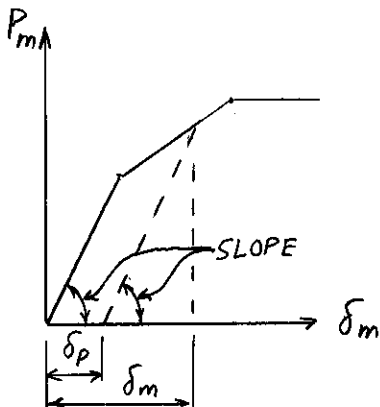
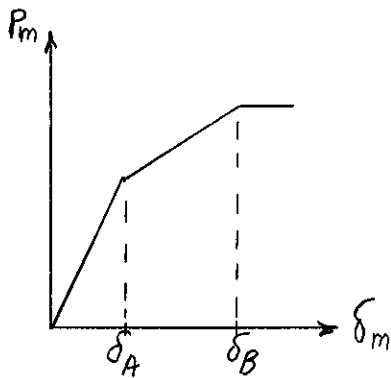
SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1 E_1 + A_2 E_2) / L$$

$$\delta_p = \delta_m - (P_m / \text{SLOPE})$$

$$(\sigma_1)_{res} = \sigma_1 - (E_1 P_m / (L \text{ SLOPE}))$$

$$(\sigma_2)_{res} = \sigma_2 - (E_2 P_m / (L \text{ SLOPE}))$$



CONTINUED

**PROBLEM 2.C4 CONTINUED**

PROGRAM OUTPUT

Problem 2.109

DM MUm	PM kN	SIGM(1) MPa	SIGM(2) MPa	DP MUm	SIGR(1) MPa	SIG(2) MPa
.000	.000	.000	.000	.000	.000	.000
16.387	60.375	17.250	17.250	.000	.000	.000
32.775	120.750	34.500	34.500	.000	.000	.000
49.162	181.125	51.750	51.750	.000	.000	.000
65.550	241.500	69.000	69.000	.000	.000	.000
81.938	301.875	86.250	86.250	.000	.000	.000
98.325	362.250	103.500	103.500	.000	.000	.000
114.713	422.625	120.750	120.750	.000	.000	.000
131.100	483.000	138.000	138.000	.000	.000	.000
147.487	543.375	155.250	155.250	.000	.000	.000
163.875	603.750	172.500	172.500	.000	.000	.000
180.262	664.125	189.750	189.750	.000	.000	.000
196.650	724.500	207.000	207.000	.000	.000	.000
213.037	784.875	224.250	224.250	.000	.000	.000
229.425	845.250	241.500	241.500	.000	.000	.000
245.812	890.312	250.000	258.750	4.156	-4.375	4.375
262.200	920.500	250.000	276.000	12.350	-13.000	13.000
278.587	950.687	250.000	293.250	20.544	-21.625	21.625
294.975	980.875	250.000	310.500	28.737	-30.250	30.250
311.362	1011.062	250.000	327.750	36.931	-38.875	38.875
327.750	1041.250	250.000	345.000	45.125	-47.500	47.500
344.137	1041.250	250.000	345.000	61.512	-47.500	47.500
360.525	1041.250	250.000	345.000	77.900	-47.500	47.500
376.912	1041.250	250.000	345.000	94.287	-47.500	47.500
393.300	1041.250	250.000	345.000	110.675	-47.500	47.500

Problems 2.111 and 2.112

DM 10**-3 in.	PM kips	SIGM(1) ksi	SIGM(2) ksi	DP 10**-3 in.	SIGR(1) ksi	SIG(2) ksi
.000	.000	.000	.000	.000	.000	.000
2.414	8.750	5.000	5.000	.000	.000	.000
4.828	17.500	10.000	10.000	.000	.000	.000
7.241	26.250	15.000	15.000	.000	.000	.000
9.655	35.000	20.000	20.000	.000	.000	.000
12.069	43.750	25.000	25.000	.000	.000	.000
14.483	52.500	30.000	30.000	.000	.000	.000
16.897	61.250	35.000	35.000	.000	.000	.000
19.310	70.000	40.000	40.000	.000	.000	.000
21.724	78.750	45.000	45.000	.000	.000	.000
24.138	87.500	50.000	50.000	.000	.000	.000
26.552	91.250	50.000	55.000	1.379	-2.143	2.857
28.966	95.000	50.000	60.000	2.759	-4.286	5.714
31.379	98.750	50.000	65.000	4.138	-6.429	8.571
33.793	102.500	50.000	70.000	5.517	-8.571	11.429
36.207	106.250	50.000	75.000	6.897	-10.714	14.286
38.621	110.000	50.000	80.000	8.276	-12.857	17.143
41.034	113.750	50.000	85.000	9.655	-15.000	20.000
43.448	117.500	50.000	90.000	11.034	-17.143	22.857
45.862	121.250	50.000	95.000	12.414	-19.286	25.714
48.276	125.000	50.000	100.000	13.793	-21.429	28.571
50.690	125.000	50.000	100.000	16.207	-21.429	28.571
53.103	125.000	50.000	100.000	18.621	-21.429	28.571
55.517	125.000	50.000	100.000	21.034	-21.429	28.571
57.931	125.000	50.000	100.000	23.448	-21.429	28.571

**PROBLEM 2.C5**

**2.C5** The stress concentration factor for a flat bar with a centric hole under axial loading can be expressed as:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where  $r$  is the radius of the hole and  $D$  is the width of the bar. (a) Write a computer program that can be used to determine the allowable load  $P$  for given values of  $r$ ,  $D$ , the thickness  $t$  of the bar, and the allowable stress  $\sigma_{all}$  of the material. (b) Use this program to solve Prob. 2.94.

**SOLUTION**

ENTER

$$r, D, t, \sigma_{all}$$

COMPUTE  $K$ 

$$RD = 2.0 \ r/D$$

$$K = 3.00 - 3.13 RD + 3.66 RD^2 - 1.53 RD^3$$

COMPUTE AVERAGE STRESS

$$\sigma_{ave} = \sigma_{all}/K$$

ALLOWABLE LOAD

$$P_{all} = \sigma_{ave} (D - 2.0r)t$$

PROGRAM OUTPUT

Problem 2.94

Hole at A:

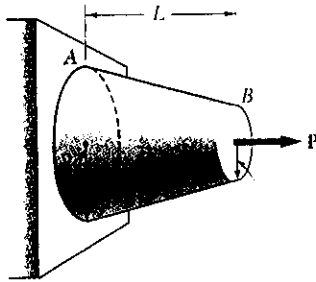
$$K = 2.573 \quad P = 7.773 \text{ Kips}$$

Hole at B:

$$K = 2.159 \quad P = 5.559 \text{ Kips}$$

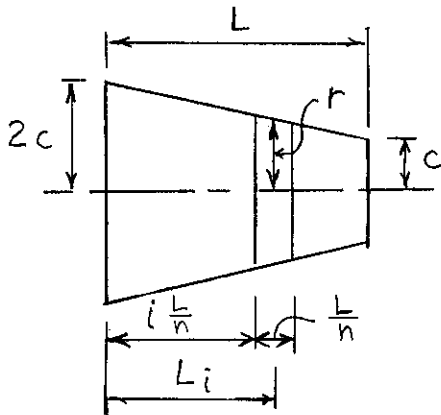


**PROBLEM 2.C6**



**2.C6** A solid truncated cone is subjected to an axial force  $P$  as shown. Write a computer program that can be used to obtain an approximation of the elongation of the cone by replacing it by  $n$  circular cylinders of equal thickness and of radius equal to the mean radius of the portion of cone they replace. Knowing that the exact value of the elongation of the cone is  $(PL)/(2\pi c^2 E)$  and using for  $P$ ,  $L$ ,  $c$ , and  $E$  values of your choice, determine the percentage error involved when the program is used with (a)  $n = 6$ , (b)  $n = 12$ , (c)  $n = 60$ .

**SOLUTION**



FOR  $i = 1$  TO  $n$ :

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$

AREA:

$$A = \pi r_i^2$$

DISPLACEMENT:

$$\delta = \delta + P(L/n)/(AE)$$

EXACT DISPLACEMENT:

$$\delta_{EXACT} = PL/(2.0\pi c^2 E)$$

PERCENTAGE ERROR:

$$PERCENT = 100(\delta - \delta_{EXACT})/\delta_{EXACT}$$

PROGRAM OUTPUT

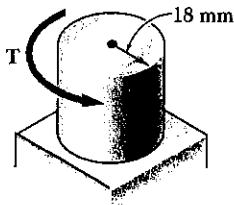
n	Approximate	Exact	Percent
6	0.15852	0.15915	-.40083
12	0.15899	0.15915	-.10100
60	0.15915	0.15915	-.00405

# CHAPTER 3

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**PROBLEM 3.1**

3.1 Determine the torque  $T$  which causes a maximum shearing stress of 70 MPa in the steel cylindrical shaft shown.



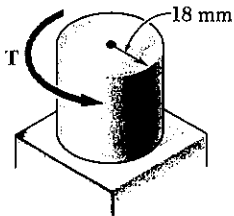
SOLUTION

$$\tau_{max} = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4$$

$$T = \frac{\pi}{2}c^3\tau_{max} = \frac{\pi}{2}(0.018)^3(70 \times 10^6) = 641 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 3.2**

3.2 Determine the maximum shearing stress caused by a torque of magnitude  $T = 800 \text{ N}\cdot\text{m}$ .



SOLUTION

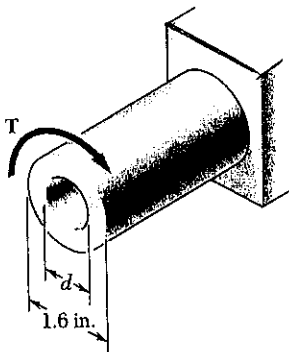
$$\tau_{max} = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.018)^3} = 87.3 \times 10^6 \text{ Pa}$$

$$87.3 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 3.3**

3.3 Knowing that the internal diameter of the hollow shaft shown is  $d = 0.9 \text{ in.}$ , determine the maximum shearing stress caused by a torque of magnitude  $T = 9 \text{ kip}\cdot\text{in.}$



SOLUTION

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \quad c = 0.8 \text{ in.}$$

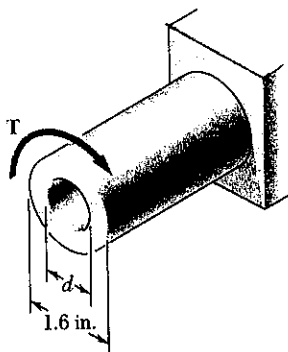
$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(0.9) = 0.45 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(9)(0.8)}{0.5790} = 12.44 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 3.4**

3.4 Knowing that  $d = 1.2 \text{ in.}$ , determine the torque  $T$  which causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.



SOLUTION

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in.} \quad c = 0.8 \text{ in.}$$

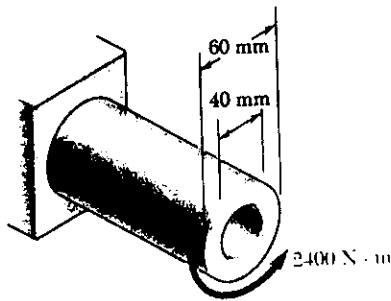
$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(1.2) = 0.6 \text{ in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398 \text{ in}^4$$

$$\tau_{max} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{max}}{c} = \frac{(0.4398)(7.5)}{0.8} = 4.12 \text{ kip}\cdot\text{in.} \quad \blacktriangleleft$$

**PROBLEM 3.5**



3.5 (a) For the hollow shaft and loading shown, determine the maximum shearing stress. (b) Determine the diameter of a solid shaft for which the maximum shearing stress is the same as in part a.

**SOLUTION**

$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(0.040) = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(0.060) = 0.030 \text{ m} \quad c = 0.030 \text{ m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020^4) \\ = 1.0210 \times 10^{-6} \text{ m}^4$$

$$(a) \quad \tau_{\max} = \frac{Tc}{J} = \frac{(2400)(0.03)}{1.0210 \times 10^{-6}} = 70.52 \times 10^6 \text{ Pa}$$

$$70.5 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \tau = \frac{Tc_3}{J}, \quad J = \frac{\pi}{2}c_3^4, \quad \tau = \frac{2T}{\pi c_3^3}$$

$$c_3^3 = \frac{2T}{\pi\tau} = \frac{(2)(2400)}{\pi(70.52 \times 10^6)} = 21.67 \times 10^{-6} \text{ m}^3$$

$$c_3 = 27.88 \times 10^{-3} \text{ m} \quad d_3 = 2c_3 = 55.8 \times 10^{-3} \text{ m}$$

$$55.8 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 3.6**

3.6 (a) Determine the torque which may be applied to a solid shaft of 90-mm outer diameter without exceeding an allowable shearing stress of 75 MPa. (b) Solve part a, assuming that the solid shaft is replaced by a hollow shaft of the same mass and of 90-mm inner diameter.

**SOLUTION**

$$(a) \quad \text{For the solid shaft} \quad c = \frac{1}{2}d = \left(\frac{1}{2}\right)(0.090) = 0.045 \text{ m}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\pi}{2}(0.045)^3 = 143.14 \times 10^{-6} \text{ m}^3$$

$$\tau_{\max} = \frac{Tc}{J} \quad \therefore T = \frac{\tau_{\max}J}{c} = \frac{(75 \times 10^6)(143.14 \times 10^{-6})}{0.045} = 10.74 \times 10^3 \text{ N}\cdot\text{m}$$

$$10.74 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \quad \text{Hollow shaft} \quad c_1 = \frac{1}{2}d_i = \left(\frac{1}{2}\right)(0.090) = 0.045 \text{ m}$$

For equal masses the cross sectional areas must be equal

$$A = \pi c^2 = \pi(c_2^2 - c_1^2) \quad \text{or} \quad c_2 = \sqrt{c_1^2 + c^2}$$

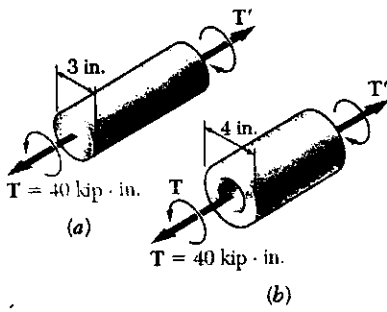
$$c_2 = \sqrt{0.045^2 + 0.045^2} = 0.0636396 \text{ m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 19.3237 \times 10^{-6} \text{ m}^4$$

$$T = \frac{\tau_{\max}J}{c_2} = \frac{(75 \times 10^6)(19.3237 \times 10^{-6})}{0.0636396} = 22.77 \times 10^3 \text{ N}\cdot\text{m}$$

$$22.8 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 3.7**



3.7 (a) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (b) Determine the inner diameter of the hollow cylinder, of 4-in. outer diameter, for which the maximum stress is the same as in part a.

**SOLUTION**

(a) Solid shaft  $c = \frac{1}{2}d = \frac{1}{2}(3.0) = 1.5 \text{ in.}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(40)}{\pi(1.5)^3} = 7.545 \text{ ksi}$$

(b) Hollow shaft  $c_2 = \frac{1}{2}d = \frac{1}{2}(4.0) = 2.0 \text{ in.}$

$$\frac{J}{c_2} = \frac{\frac{\pi}{2}(c_2^4 - c_1^4)}{c_2} = \frac{T}{\tau_{max}}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi \tau_{max}} = 2.0^4 - \frac{(2)(40)(2.0)}{\pi(7.545)} = 9.25 \text{ in}^4$$

$$c_1 = 1.74395 \text{ in} \quad d_1 = 2c_1 = 3.49 \text{ in}$$

**PROBLEM 3.8**

3.8 (a) Determine the torque which may be applied to a solid shaft of 0.75-in. diameter without exceeding an allowable shearing stress of 10 ksi. (b) Solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half its outer diameter..

**SOLUTION**

(a) Solid shaft:  $c = \frac{1}{2}d = (\frac{1}{2})(0.75) = 0.375 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.375)^4 = 0.031063 \text{ in}^4 \quad \tau_{max} = 10 \text{ ksi}$$

$$T = \frac{J \tau_{max}}{c} = \frac{(0.031063)(10)}{0.375} = 0.828 \text{ kip·in or } 828 \text{ lb·in}$$

(b) Hollow shaft

For the same area as the solid shaft

$$A = \pi(c_2^2 - c_1^2) = \pi[c_2^2 - (\frac{1}{2}c_2)^2] = \frac{3}{4}\pi c_2^2 = \pi c^2$$

$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.375) = 0.433013 \text{ in}$$

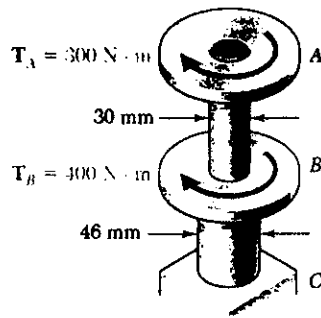
$$c_1 = \frac{1}{2}c_2 = 0.216506$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.433013^4 - 0.216506^4) = 0.051772 \text{ in}^4$$

$$T = \frac{\tau_{max} J}{c_2} = \frac{(10)(0.051772)}{0.433013} = 1.196 \text{ kip·in or } 1196 \text{ in·lb.}$$

**PROBLEM 3.9**

3.9 The torques shown are exerted on pulleys *A* and *B*. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *BC*.



**SOLUTION**

Shaft *AB*:  $T_{AB} = 300 \text{ N}\cdot\text{m}$ ,  $d = 0.030 \text{ m}$ ,  $c = 0.015 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi (0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa} \blacktriangleleft$$

Shaft *BC*:  $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$

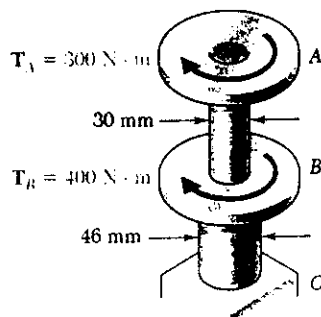
$d = 0.046 \text{ m}$ ,  $c = 0.023 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa} \blacktriangleleft$$

**PROBLEM 3.10**

3.10 The torques shown are exerted on pulleys *A* and *B* which are attached to solid circular shafts *AB* and *BC*. In order to reduce the total mass of the assembly, determine the smallest diameter of shaft *BC* for which the largest shearing stress in the assembly is not increased.



**SOLUTION**

Shaft *AB*:  $T_{AB} = 300 \text{ N}\cdot\text{m}$ ,  $d = 0.030 \text{ m}$ ,  $c = 0.015 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi (0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft *BC*:  $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$

$d = 0.046 \text{ m}$ ,  $c = 0.023 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi (0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress ( $56.588 \times 10^6 \text{ Pa}$ ) occurs in portion AB

Reduce the diameter of *BC* to provide the same stress.

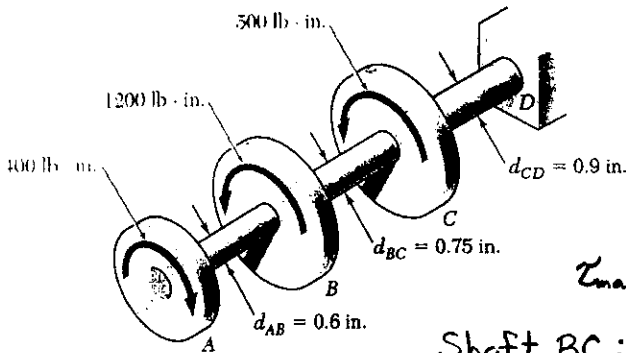
$$T_{BC} = 700 \text{ N}\cdot\text{m} \quad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau_{max}} = \frac{(2)(700)}{\pi (56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{ m}^3$$

$$c = 19.895 \times 10^{-3} \text{ m} \quad d = 2c = 39.79 \times 10^{-3} \text{ m} \quad 39.8 \text{ mm} \blacktriangleleft$$

**PROBLEM 3.11**

3.11 Knowing that each portion of the shaft *AD* consists of a solid circular rod, determine (a) the portion of the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



**SOLUTION**

Shaft AB:  $T = 400 \text{ lb}\cdot\text{in}$

$c = \frac{1}{2}d = 0.30 \text{ in}$

$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{\max} = \frac{(2)(400)}{\pi (0.30)^3} = 9431 \text{ psi}$

Shaft BC:  $T = -400 + 1200 = 800 \text{ lb}\cdot\text{in}$

$c = \frac{1}{2}d = 0.375 \text{ in}$

$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi (0.375)^3} = 9658 \text{ psi}$

Shaft CD:  $T = -400 + 1200 + 500 = 1300 \text{ lb}\cdot\text{in}$

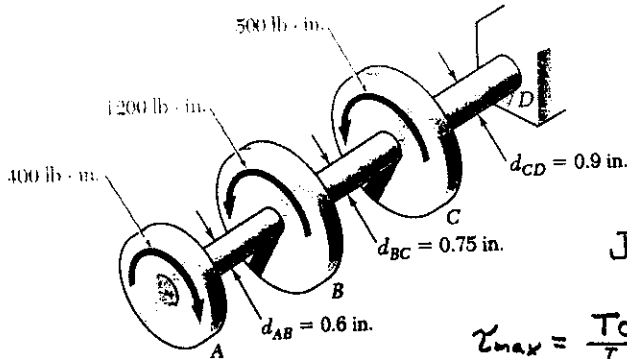
$c = \frac{1}{2}d = 0.45 \text{ in}$

$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1300)}{\pi (0.45)^3} = 9082 \text{ psi}$

Answers: (a) shaft BC (b) 9.66 ksi

**PROBLEM 3.12**

3.12 Knowing that a 0.30-in.-diameter hole has been drilled through each portion of shaft *AD*, determine (a) the portion of the shaft in which the maximum shearing stress occurs, (b) the magnitude of that stress.



**SOLUTION**

Hole:  $c_1 = \frac{1}{2}d_1 = 0.15 \text{ in}$

Shaft AB:  $T = 400 \text{ lb}\cdot\text{in}$

$c_2 = \frac{1}{2}d_2 = 0.30 \text{ in}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.30^4 - 0.15^4) = 0.011928 \text{ in}^4$

$\tau_{\max} = \frac{Tc_2}{J} = \frac{(400)(0.30)}{0.011928} = 10600 \text{ psi}$

Shaft BC:  $T = -400 + 1200 = 800 \text{ lb}\cdot\text{in}$

$c_2 = \frac{1}{2}d_2 = 0.375 \text{ in}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.375^4 - 0.15^4) = 0.030268 \text{ in}^4$

$\tau_{\max} = \frac{Tc_2}{J} = \frac{(800)(0.375)}{0.030268} = 9911 \text{ psi}$

Shaft CD:  $T = -400 + 1200 + 500 = 1300 \text{ lb}\cdot\text{in}$

$c_2 = \frac{1}{2}d_2 = 0.45 \text{ in}$

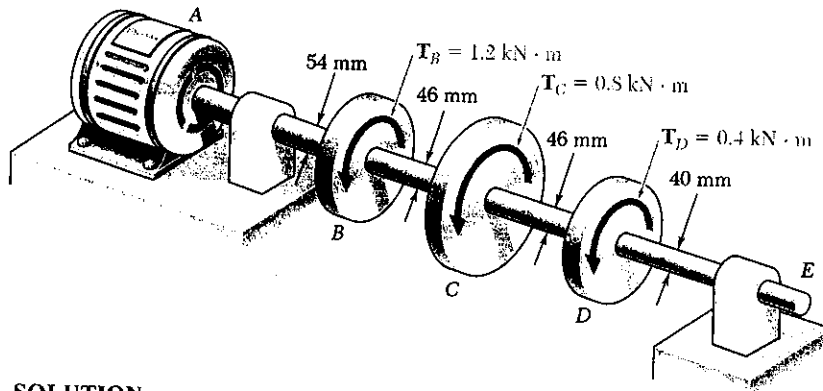
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.45^4 - 0.15^4) = 0.063617 \text{ in}^4$

$\tau_{\max} = \frac{Tc_2}{J} = \frac{(1300)(0.45)}{0.063617} = 9196 \text{ psi}$

Answers: (a) shaft AB (b) 10.06 ksi

**PROBLEM 3.13**

3.13 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. Knowing that each shaft is solid, determine the maximum shearing stress (a) in shaft AB, (b) in shaft BC, (c) in shaft CD.



**SOLUTION**

Shaft AB:  $T_{AB} = 2.4 \times 10^3 \text{ N}\cdot\text{m}$ ,  $c = \frac{1}{2}d = 0.027 \text{ m}$

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.4 \times 10^3)}{\pi (0.027)^3} = 77.625 \times 10^6 \text{ Pa} \quad 77.6 \text{ MPa} \blacktriangleleft$$

Shaft BC:  $T_{BC} = 2.4 \text{ kN}\cdot\text{m} - 1.2 \text{ kN}\cdot\text{m} = 1.2 \text{ kN}\cdot\text{m}$ ,  $c = \frac{1}{2}d = 0.023 \text{ m}$

$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2 \times 10^3)}{\pi (0.023)^3} = 62.788 \times 10^6 \text{ Pa} \quad 62.8 \text{ MPa} \blacktriangleleft$$

Shaft CD:  $T_{CD} = 0.4 \times 10^3 \text{ N}\cdot\text{m}$ ,  $c = \frac{1}{2}d = 0.023 \text{ m}$

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4 \times 10^3)}{\pi (0.023)^3} = 20.929 \times 10^6 \text{ Pa} \quad 20.9 \text{ MPa} \blacktriangleleft$$

**PROBLEM 3.14**

3.14 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. In order to reduce the mass of the assembly, determine the smallest diameter of shaft BC for which the largest shearing stress in the assembly is not increased.

**SOLUTION**

See solution to problem 3.13 for figure and for maximum shearing stresses in portions AB, BC, and CD of the shaft. The largest value is  $\tau_{max} = 77.625 \times 10^6 \text{ Pa}$  occurring in AB.

Adjust diameter of BC to obtain the same value of stress

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

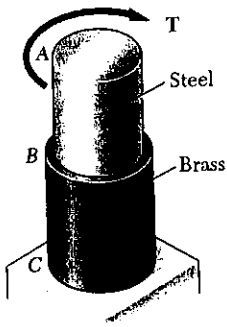
$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(1.2 \times 10^3)}{\pi (77.625 \times 10^6)} = 9.8415 \times 10^{-6} \text{ m}^3$$

$$c = 21.43 \times 10^{-3} \text{ m} \quad d = 2c = 42.8 \times 10^{-3} \text{ m} \quad 42.8 \text{ mm} \blacktriangleleft$$



PROBLEM 3.15

3.15 The allowable stress is 15 ksi in the 1.5-in.-diameter rod  $AB$  and 8 ksi in the 1.8-in.-diameter rod  $BC$ . Neglecting the effect of stress concentrations, determine the largest torque that may be applied at  $A$ .



SOLUTION

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad T = \frac{\pi}{2} c^3 \tau_{max}$$

Shaft  $AB$ :  $\tau_{max} = 15 \text{ ksi}$       $c = \frac{1}{2}d = 0.75 \text{ in}$

$$T = \frac{\pi}{2} (0.75)^3 (15) = 9.94 \text{ kip}\cdot\text{in}$$

Shaft  $BC$ :  $\tau_{max} = 8 \text{ ksi}$       $c = \frac{1}{2}d = 0.90 \text{ in}$

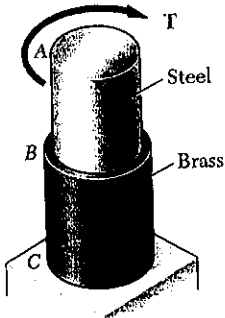
$$T = \frac{\pi}{2} (0.90)^3 (8) = 9.16 \text{ kip}\cdot\text{in}$$

The allowable torque is the smaller value

$$T = 9.16 \text{ kip}\cdot\text{in} \quad \blacktriangleleft$$

PROBLEM 3.16

3.16 The allowable stress is 15 ksi in the steel rod  $AB$  and 8 ksi in the brass rod  $BC$ . Knowing that a torque  $T = 10 \text{ kip}\cdot\text{in}$ . is applied at  $A$ , determine the required diameter of (a) rod  $AB$ , (b) for  $BC$ .



SOLUTION

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2} c^4, \quad c^3 = \frac{2T}{\pi \tau_{max}}$$

Shaft  $AB$ :  $T = 10 \text{ kip}\cdot\text{in}$       $\tau_{max} = 15 \text{ ksi}$

$$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244 \text{ in}^3$$

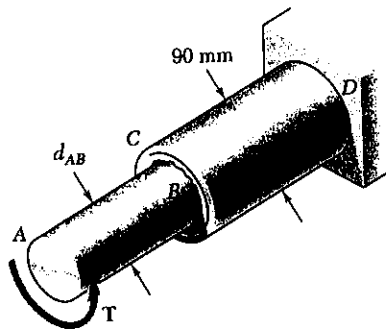
$$c = 0.7515 \text{ in} \quad d = 2c = 1.503 \text{ in} \quad \blacktriangleleft$$

Shaft  $BC$ :  $T = 10 \text{ kip}\cdot\text{in}$       $\tau_{max} = 8 \text{ ksi}$

$$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577 \text{ in}^3$$

$$c = 0.9267 \text{ in} \quad d = 2c = 1.853 \text{ in} \quad \blacktriangleleft$$

**PROBLEM 3.17**



3.17 The solid rod  $AB$  has a diameter  $d_{AB} = 60$  mm. The pipe  $CD$  has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of a steel for which the allowable shearing stress is 75 MPa, determine the largest torque  $T$  which may be applied at  $A$ .

**SOLUTION**

$$\tau_{all} = 75 \times 10^6 \text{ Pa} \quad T_{all} = \frac{J \tau_{all}}{C}$$

Rod  $AB$ :  $C = \frac{1}{2}d = 0.030 \text{ m}$ ,  $J = \frac{\pi}{2}C^4$

$$T_{all} = \frac{\pi}{2}C^3 \tau_{all} = \frac{\pi}{2}(0.030)^3(75 \times 10^6) = 3.181 \times 10^3 \text{ N}\cdot\text{m}$$

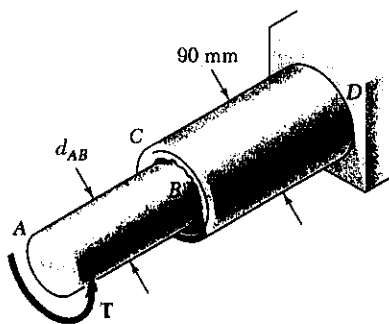
Pipe  $CD$ :  $C_2 = \frac{1}{2}d_2 = 0.045 \text{ m}$   $C_1 = C_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{all} = \frac{(2.8073 \times 10^{-6})(75 \times 10^6)}{0.045} = 4.679 \times 10^3 \text{ N}\cdot\text{m}$$

Allowable torque is the smaller value ( $3.18 \times 10^3 \text{ N}\cdot\text{m}$ )  $3.18 \text{ kN}\cdot\text{m}$  ◀

**PROBLEM 3.18**



3.18 The solid rod  $AB$  has a diameter  $d_1 = 60$  mm and is made of a steel for which the allowable shearing stress is 85 MPa. The pipe  $CD$  has an outer diameter of 90 mm and a wall thickness of 6 mm; it is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque  $T$  which may be applied at  $A$ .

**SOLUTION**

Rod  $AB$ :  $\tau_{all} = 85 \times 10^6 \text{ Pa}$ ,  $C = \frac{1}{2}d = 0.030 \text{ m}$

$$T_{all} = \frac{J \tau_{all}}{C} = \frac{\pi}{2}C^3 \tau_{all}$$

$$= \frac{\pi}{2}(0.030)^3(85 \times 10^6) = 3.605 \times 10^3 \text{ N}\cdot\text{m}$$

Pipe  $CD$ :  $\tau_{all} = 54 \times 10^6 \text{ Pa}$   $C_2 = \frac{1}{2}d_2 = 0.045 \text{ m}$

$$C_1 = C_2 - t = 0.045 - 0.006 = 0.039 \text{ m}$$

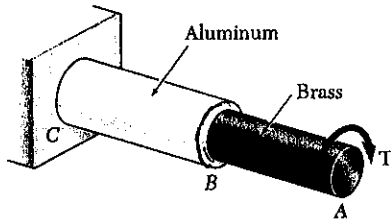
$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6} \text{ m}^4$$

$$T_{all} = \frac{J \tau_{all}}{C_2} = \frac{(2.8073 \times 10^{-6})(54 \times 10^6)}{0.045} = 3.369 \times 10^3 \text{ N}\cdot\text{m}$$

Allowable torque is smaller value  $T_{all} = 3.369 \times 10^3 \text{ N}\cdot\text{m}$

$3.37 \text{ kN}\cdot\text{m}$  ◀

**PROBLEM 3.19**



3.19 The allowable stress is 50 MPa in the brass rod *AB* and 25 MPa in the aluminum rod *BC*. Knowing that a torque  $T = 1250 \text{ N}\cdot\text{m}$  is applied at *A*, determine the required diameter of (a) rod *AB*, (b) for *BC*.

SOLUTION

$$\tau_{\max} = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4 \quad c^3 = \frac{2T}{\pi \tau_{\max}}$$

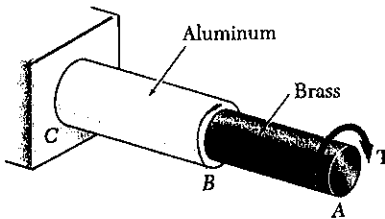
Rod *AB*:  $c^3 = \frac{(2)(1250)}{\pi(50 \times 10^6)} = 15.915 \times 10^{-6} \text{ m}^3$

$c = 25.15 \times 10^{-3} \text{ m} = 25.15 \text{ mm} \quad d_{AB} = 2c = 50.3 \text{ mm}$  ▶

Rod *BC*:  $c^3 = \frac{(2)(1250)}{\pi(25 \times 10^6)} = 31.831 \times 10^{-6} \text{ m}^3$

$c = 31.69 \times 10^{-3} \text{ m} = 31.69 \text{ mm} \quad d_{BC} = 2c = 63.4 \text{ mm}$  ▶

**PROBLEM 3.20**



3.20 The solid rod *BC* has a diameter of 30 mm and is made an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is hollow and has an outer diameter of 25 mm; it is made a brass for which the allowable shearing stress is 50 MPa. Determine (a) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (b) the largest torque that may be applied at *A*.

SOLUTION

Solid rod *BC*:  $\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2} c^4$

$\tau_{\text{all}} = 25 \times 10^6 \text{ Pa} \quad c = \frac{1}{2} d = 0.015 \text{ m}$

$T_{\text{all}} = \frac{\pi}{2} c^3 \tau_{\text{all}} = \frac{\pi}{2} (0.015)^3 (25 \times 10^6) = 132.536 \text{ N}\cdot\text{m}$

Hollow rod *AB*:  $\tau_{\text{all}} = 50 \times 10^6 \text{ Pa} \quad T_{\text{all}} = 132.536 \text{ N}\cdot\text{m}$

$c_2 = \frac{1}{2} d_2 = \frac{1}{2} (0.025) = 0.0125 \text{ m} \quad c_1 = ?$

$T_{\text{all}} = \frac{J \tau_{\text{all}}}{c_2} = \frac{\pi}{2} (c_2^4 - c_1^4) \frac{\tau_{\text{all}}}{c_2}$

$c_1^4 = c_2^4 - \frac{2T_{\text{all}} c_2}{\pi \tau_{\text{all}}} = 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$

$c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm} \quad d_1 = 2c_1 = 15.18 \text{ mm}$  ▶

Allowable torque  $T_{\text{all}} = 132.5 \text{ N}\cdot\text{m}$  ▶

**PROBLEM 3.21**

3.21 A torque of magnitude  $T = 1000 \text{ N}\cdot\text{m}$  is applied at  $D$  as shown. Knowing that the diameter of shaft  $AB$  is  $56 \text{ mm}$  and the diameter of shaft  $CD$  is  $42 \text{ mm}$ , determine the maximum shearing stress in (a) shaft  $AB$ , (b) shaft  $CD$ .

**SOLUTION**

$$T_{CD} = 1000 \text{ N}\cdot\text{m}$$

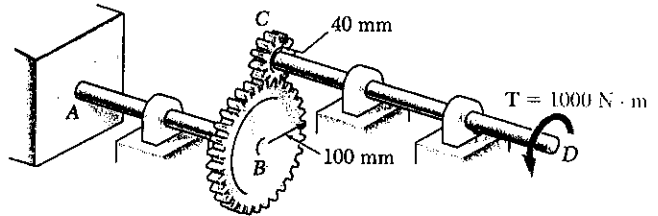
$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40} (1000) = 2500 \text{ N}\cdot\text{m}$$

Shaft  $AB$ :  $c = \frac{1}{2}d = 0.028 \text{ m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi (0.028)^3} = 72.50 \times 10^6 \quad \mathbf{72.5 \text{ MPa} \blacktriangleleft}$$

Shaft  $BC$ :  $c = \frac{1}{2}d = 0.020 \text{ m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi (0.020)^3} = 68.7 \times 10^6 \quad \mathbf{68.7 \text{ MPa} \blacktriangleleft}$$



**PROBLEM 3.22**

3.22 A torque of magnitude  $T = 1000 \text{ N}\cdot\text{m}$  is applied at  $D$  as shown. Knowing that the allowable shearing stress is  $60 \text{ MPa}$  in each shaft, determine the required diameter of (a) shaft  $AB$ , (b) shaft  $CD$ .

**SOLUTION**

$$T_{CD} = 1000 \text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40} (1000) = 2500 \text{ N}\cdot\text{m}$$

Shaft  $AB$ :  $\tau_{all} = 60 \times 10^6 \text{ Pa}$

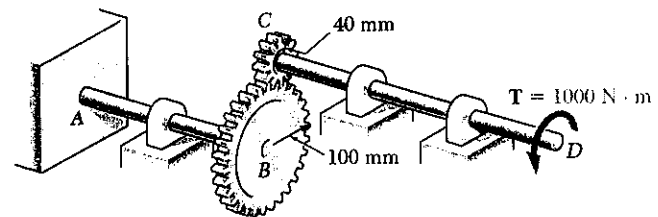
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi (60 \times 10^6)} = 26.526 \times 10^{-6} \text{ m}^3$$

$$c = 29.82 \times 10^{-3} = 29.82 \text{ mm} \quad d = 2c = 59.6 \text{ mm} \quad \mathbf{\blacktriangleleft}$$

Shaft  $CD$ :  $\tau_{all} = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(1000)}{\pi (60 \times 10^6)} = 10.610 \times 10^{-6} \text{ m}^3$$

$$c = 21.97 \times 10^{-3} \text{ m} = 21.97 \text{ mm} \quad d = 2c = 43.9 \text{ mm} \quad \mathbf{\blacktriangleleft}$$



**PROBLEM 3.23**

3.23 and 3.24 Under normal operating conditions a motor exerts a torque of magnitude  $T_F = 1200 \text{ lb}\cdot\text{in}$ . at  $F$ . Knowing that the allowable shearing stress is 10.5 ksi in each shaft, for the given data, determine the required diameter of (a) shaft CDE, (b) shaft FGH.

3.23  $r_D = 8 \text{ in.}$ ,  $r_G = 3 \text{ in.}$

**SOLUTION**

$$T_F = 1200 \text{ lb}\cdot\text{in}$$

$$T_E = \frac{r_D}{r_G} T_F = \frac{8}{3} (1200) = 3200 \text{ lb}\cdot\text{in}$$

$$\tau_{\text{all}} = 10.5 \text{ ksi} = 10500 \text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \quad c^3 = \frac{2T}{\pi \tau}$$

(a) Shaft CDE

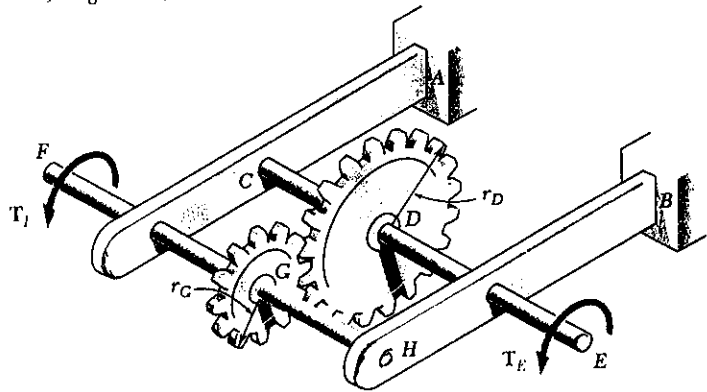
$$c^3 = \frac{(2)(3200)}{\pi (10500)} = 0.194012 \text{ in}^3$$

$$c = 0.5789 \quad d_{DE} = 2c = 1.158 \text{ in.}$$

(b) Shaft FGH

$$c^3 = \frac{(2)(1200)}{\pi (10500)} = 0.072757 \text{ in}^3$$

$$c = 0.4174 \text{ in} \quad d_{FG} = 2c = 0.835 \text{ in}$$



**PROBLEM 3.24**

3.23 and 3.24 Under normal operating conditions a motor exerts a torque of magnitude  $T_F = 1200 \text{ lb}\cdot\text{in}$ . at  $F$ . Knowing that the allowable shearing stress is 10.5 ksi in each shaft, for the given data, determine the required diameter of (a) shaft CDE, (b) shaft FGH.

3.24  $r_D = 3 \text{ in.}$ ,  $r_G = 8 \text{ in.}$

**SOLUTION**

$$T_F = 1200 \text{ lb}\cdot\text{in}$$

$$T_E = \frac{r_D}{r_G} T_F = \frac{3}{8} (1200) = 450 \text{ lb}\cdot\text{in.}$$

$$\tau_{\text{all}} = 10.5 \text{ ksi} = 10500 \text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \quad c^3 = \frac{2T}{\pi \tau}$$

Shaft CDE

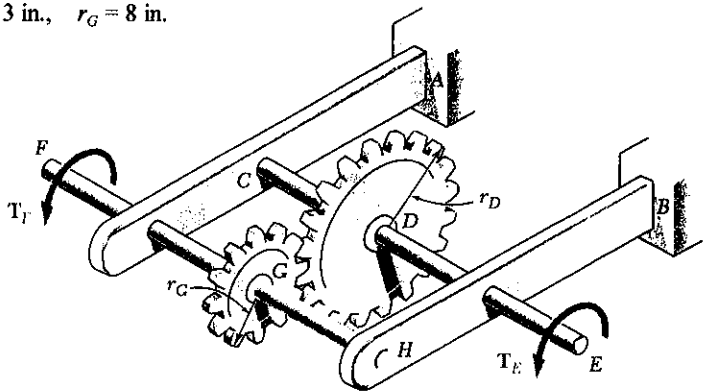
$$c^3 = \frac{(2)(450)}{\pi (10500)} = 0.027284 \text{ in}^3$$

$$c = 0.30105 \text{ in} \quad d_{DE} = 2c = 0.602 \text{ in}$$

Shaft FGH

$$c^3 = \frac{(2)(1200)}{\pi (10500)} = 0.072757 \text{ in}^3$$

$$c = 0.4174 \text{ in.} \quad d_{FG} = 2c = 0.835 \text{ in}$$



**PROBLEM 3.25**

3.25 Under normal operating conditions a motor exerts a torque of magnitude  $T_F$  at  $F$ . The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters of  $d_{CDE} = 0.900$  in. and  $d_{FGH} = 0.800$  in. Knowing that  $r_D = 6.5$  in. and  $r_G = 4.5$  in., determine the largest torque  $T_F$  which may be exerted at  $F$ .

**SOLUTION**

$$\tau_{all} = 12 \text{ ksi}$$

Shaft FG:  $c = \frac{1}{2}d = 0.400$  in

$$T_{F,all} = \frac{J \tau_{all}}{c} = \frac{\pi}{2} c^3 \tau_{all}$$

$$= \frac{\pi}{2} (0.400)^3 (12) = 1.206 \text{ kip}\cdot\text{in.}$$

Shaft DE:  $c = \frac{1}{2}d = 0.450$  in

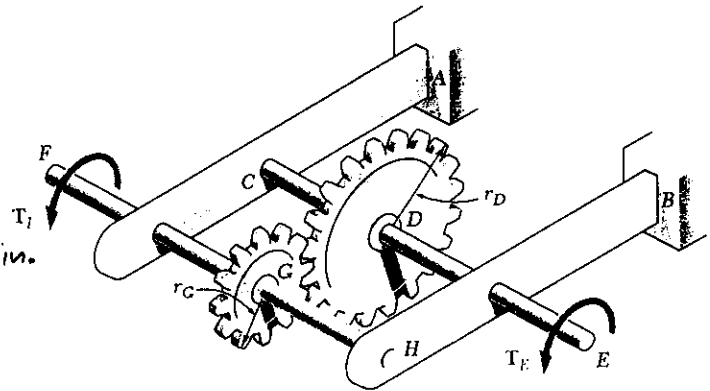
$$T_{E,all} = \frac{\pi}{2} c^3 \tau_{all}$$

$$= \frac{\pi}{2} (0.450)^3 (12) = 1.7177 \text{ kip}\cdot\text{in}$$

$$T_F = \frac{r_G}{r_D} T_E \quad T_{F,all} = \frac{4.5}{6.5} (1.7177) = 1.189 \text{ kip}\cdot\text{in}$$

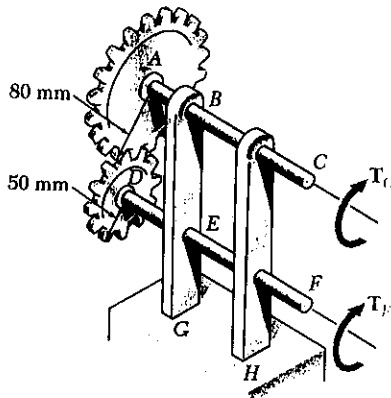
Allowable value of  $T_F$  is the smaller

$$T_{F,all} = 1.189 \text{ kip}\cdot\text{in} \blacktriangleleft$$



**PROBLEM 3.26**

3.26 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 60 MPa. Knowing that a 600 N·m-torque  $T_C$  is applied at  $C$ , determine the required diameter of (a) shaft BC, (b) shaft EF.



**SOLUTION**

Shaft BC:  $T_C = 600 \text{ N}\cdot\text{m}$ ,  $\tau_{max} = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(600)}{\pi(60 \times 10^6)} = 6.3662 \times 10^{-6} \text{ m}^3$$

$$c = 18.53 \times 10^{-3} \text{ m} = 18.53 \text{ mm}, \quad d_{BC} = 2c = 37.1 \text{ mm} \blacktriangleleft$$

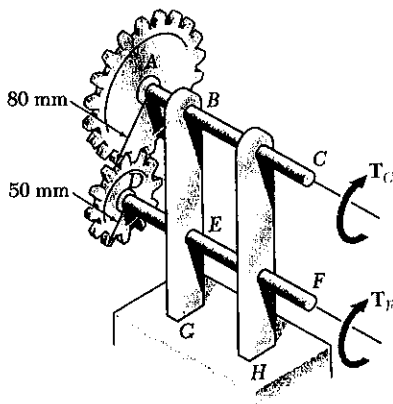
Shaft EF:  $T_F = \frac{r_B}{r_A} T_C = \frac{50}{80} (600) = 375 \text{ N}\cdot\text{m}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(375)}{\pi(60 \times 10^6)} = 3.9787 \times 10^{-6} \text{ m}^3$$

$$c = 15.85 \times 10^{-3} \text{ m} = 15.85 \text{ mm}, \quad d_{EF} = 2c = 31.7 \text{ mm} \blacktriangleleft$$

**PROBLEM 3.27**



3.27 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 50 MPa. Knowing that the diameters of the two shafts are, respectively,  $d_{BC} = 40$  mm and  $d_{EF} = 32$  mm, determine the largest torque  $T_C$  which may be applied at C.

**SOLUTION**

Shaft AC:  $\tau_{max} = 50 \times 10^6 \text{ Pa}$ ,  $c = \frac{1}{2}d = 0.020 \text{ m}$

$$T_C = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.020)^3(50 \times 10^6) = 628.3 \text{ N}\cdot\text{m}$$

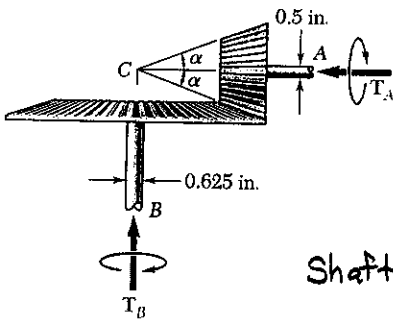
Shaft DF:  $\tau_{max} = 50 \times 10^6 \text{ Pa}$ ,  $c = \frac{1}{2}d = 0.016 \text{ m}$

$$T_F = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.016)^3(50 \times 10^6) = 321.7 \text{ N}\cdot\text{m}$$

From Statics:  $T_C = \frac{r_A}{r_D} T_F = \frac{80}{50} (321.7) = 514.7 \text{ N}\cdot\text{m}$

Allowable value of  $T_C$  is the smaller, i.e.  $T_F = 515 \text{ N}\cdot\text{m}$

**PROBLEM 3.28**



3.28 In the bevel-gear system shown  $\alpha = 18.43^\circ$ . Knowing that the allowable shearing stress is 8 ksi in each shaft, determine the largest torque  $T_A$  which may be applied at A.

**SOLUTION**

Shaft A:  $\tau = 8 \text{ ksi}$ ,  $c = \frac{1}{2}d = 0.25 \text{ in}$

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.25)^3(8) = 0.19635 \text{ kip}\cdot\text{in}$$

Shaft B:  $\tau = 8 \text{ ksi}$ ,  $c = \frac{1}{2}d = 0.3125 \text{ in}$

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.3125)^3(8) = 0.3885 \text{ kip}\cdot\text{in}$$

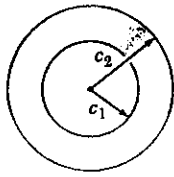
From Statics:  $T_A = \frac{r_A}{r_B} T_B = (\tan \alpha) T_B = (\tan 18.43^\circ)(0.3885) = 0.12779 \text{ kip}\cdot\text{in}$

Allowable value of  $T_A$  is the smaller

$$T_A = 0.1278 \text{ kip}\cdot\text{in} = 127.8 \text{ lb}\cdot\text{in}$$

PROBLEM 3.29

3.29 (a) For a given allowable stress, determine the ratio  $T/w$  of the maximum allowable torque  $T$  and the weight per unit length  $w$  for the hollow shaft shown. (b) Denoting by  $(T/w)_0$  the value of this ratio computed for a solid shaft of the same radius  $c_2$ , express the ratio  $T/w$  for the hollow shaft in terms of  $(T/w)_0$  and  $c_1/c_2$ .



SOLUTION

$w$  = weight per unit length,  $\gamma$  = specific weight

$W$  = total weight,  $L$  = length

$$w = \frac{W}{L} = \frac{\gamma LA}{L} = \gamma A = \gamma \pi (c_2^2 - c_1^2)$$

$$T = \frac{J \tau_{al}}{c_2} = \frac{\pi}{2} \frac{c_2^4 - c_1^4}{c_2} \tau_{al} = \frac{\pi}{2} \frac{(c_2^2 + c_1^2)(c_2^2 - c_1^2)}{c_2} \tau_{al}$$

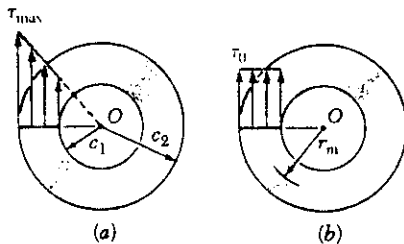
$$\left(\frac{T}{w}\right)_h = \frac{(c_2^2 + c_1^2) \tau_{al}}{2\gamma c_2} = \frac{c_2 \tau_{al}}{2\gamma} \left(1 + \frac{c_1^2}{c_2^2}\right) \quad (\text{hollow shaft}) \quad \blacktriangleleft$$

$$c_1 = 0 \text{ for solid shaft} \quad \left(\frac{T}{w}\right)_0 = \frac{c_2 \tau_{al}}{2\gamma} \quad (\text{solid shaft})$$

$$\frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2} \quad \left(\frac{T}{w}\right)_h = \left(\frac{T}{w}\right)_0 \left(1 + \frac{c_1^2}{c_2^2}\right) \quad \blacktriangleleft$$

PROBLEM 3.30

3.30 While the exact distribution of the shearing stresses in a hollow cylinder shaft is shown in Fig. (1), an approximate value may be obtained for  $\tau_{max}$  by assuming the stresses to be uniformly distributed over the area  $A$  of the cross section, as shown in Fig. (2), and then further assuming that all the elementary shearing forces act a distance from  $O$  equal to the mean radius  $r_m = \frac{1}{2}(c_1 + c_2)$  of the cross section. This approximate value is  $\tau_0 = T/Ar_m$ , where  $T$  is the applied torque. Determine the ratio  $\tau_{max}/\tau_0$  of the true value of the maximum shearing stress and its approximate value  $\tau_0$  for values of  $c_1/c_2$  respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.



SOLUTION

$$\text{For a hollow shaft: } \tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)} = \frac{2Tc_2}{\pi(c_2^2 - c_1^2)(c_2^2 + c_1^2)}$$

$$= \frac{2Tc_2}{A(c_2^2 + c_1^2)}$$

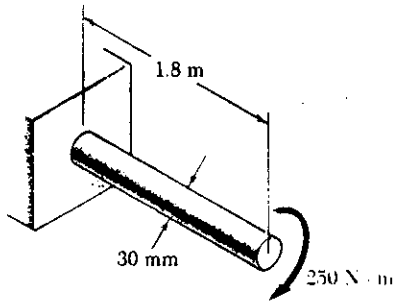
$$\text{By definition } \tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

$$\text{Dividing } \frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2} \quad \blacktriangleleft$$

$c_1/c_2$	1.0	0.95	0.75	0.5	0.0
$\tau_{max}/\tau_0$	1.0	1.025	1.120	1.200	1.0



PROBLEM 3.31



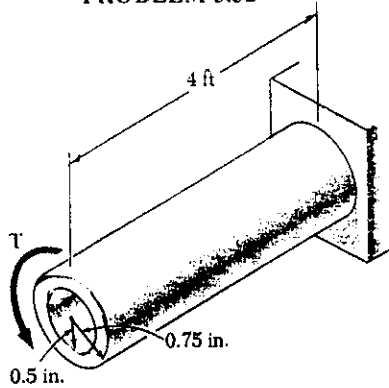
3.31 (a) For the solid steel shaft shown ( $G = 77 \text{ GPa}$ ), determine the angle of twist at  $A$ . (b) Solve part  $a$ , assuming that the steel shaft is hollow with a 30-mm outer diameter and a 20-mm inner diameter.

SOLUTION

(a)  $c = \frac{1}{2}d = 0.015 \text{ m}$ ,  $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4$   
 $J = 79.522 \times 10^{-9} \text{ m}^4$ ,  $L = 1.8 \text{ m}$ ,  $G = 77 \times 10^9 \text{ Pa}$   
 $T = 250 \text{ N}\cdot\text{m}$        $\phi = \frac{TL}{GJ}$   
 $\phi = \frac{(250)(1.8)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$   
 $\phi = \frac{(73.49 \times 10^{-3})180}{\pi} = 4.21^\circ$

(b)  $c_2 = 0.015 \text{ m}$ ,  $c_1 = \frac{1}{2}d_1 = 0.010 \text{ m}$ ,  $J = \frac{\pi}{2}(c_2^4 - c_1^4)$   
 $J = \frac{\pi}{2}(0.015^4 - 0.010^4) = 63.814 \times 10^{-9} \text{ m}^4$        $\phi = \frac{TL}{GJ}$   
 $\phi = \frac{(250)(1.8)}{(77 \times 10^9)(63.814 \times 10^{-9})} = 91.58 \times 10^{-3} \text{ rad} = \frac{180}{\pi}(91.58 \times 10^{-3}) = 5.25^\circ$

PROBLEM 3.32



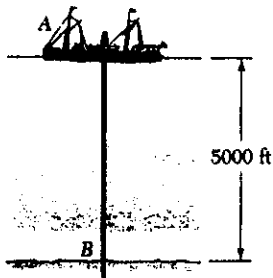
3.32 For the aluminum shaft shown ( $G = 3.9 \times 10^6 \text{ psi}$ ), determine (a) the torque  $T$  which causes an angle of twist of  $5^\circ$ , (b) the angle of twist caused by the same torque  $T$  in a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(a)  $\phi = \frac{TL}{GJ}$ ,  $T = \frac{GJ\phi}{L}$   
 $\phi = 5^\circ = 87.266 \times 10^{-3} \text{ rad}$ ,  $L = 4 \text{ ft} = 48 \text{ in}$   
 $J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.75^4 - 0.5^4) = 0.39884 \text{ in}^4$   
 $T = \frac{(3.9 \times 10^6)(0.39884)(87.266 \times 10^{-3})}{48}$   
 $= 2.8279 \times 10^3 \text{ lb}\cdot\text{in} = 2.83 \text{ kip}\cdot\text{in}$

(b) Hollow shaft  $A = \pi(c_2^2 - c_1^2)$       Solid shaft  $A = \pi c^2$   
 Matching areas  $c^2 = c_2^2 - c_1^2 = 0.75^2 - 0.5^2 = 0.3125 \text{ in}^2$   
 $c = 0.5590 \text{ in}$ ,  $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.5590)^4 = 153.398 \times 10^{-3} \text{ in}^4$   
 $\phi = \frac{TL}{GJ} = \frac{(2.8279 \times 10^3)(48)}{(3.9 \times 10^6)(153.398 \times 10^{-3})} = 226.89 \times 10^{-3} \text{ rad}$   
 $= 13.00^\circ$

**PROBLEM 3.33**



3.33 The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. Knowing that the top of the 8-in.-diameter steel drill pipe ( $G = 11.2 \times 10^6$  psi) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.

**SOLUTION**

$$\phi = \frac{T L}{G J} \quad T = \frac{G J \phi}{L}$$

$$\tau = \frac{T c}{J} = \frac{G J \phi c}{J L} = \frac{G \phi c}{L}$$

$$\phi = 2 \text{ rev} = (2)(2\pi) = 12.566 \text{ rad}, \quad c = \frac{1}{2}d = 4.0 \text{ in}$$

$$L = 5000 \text{ ft} = 60000 \text{ in} \quad \tau = \frac{(11.2 \times 10^6)(12.566)(4.0)}{60000}$$

$$= 9.3826 \times 10^3 \text{ psi} = 9.38 \text{ ksi}$$

**PROBLEM 3.34**

3.34 Determine the largest allowable diameter of a 3-m-long steel rod ( $G = 77$  GPa) if the rod is to be twisted through  $30^\circ$  without exceeding a shearing stress of 80 MPa.

**SOLUTION**

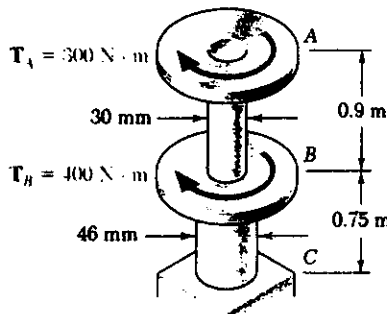
$$L = 3 \text{ m}, \quad \phi = \frac{30 \pi}{180} = 523.6 \times 10^{-3} \text{ rad}, \quad \tau = 80 \times 10^6 \text{ Pa}$$

$$\phi = \frac{T L}{G J}, \quad T = \frac{G J \phi}{L}, \quad \tau = \frac{T c}{J} = \frac{G J \phi c}{J L} = \frac{G \phi c}{L}, \quad c = \frac{\tau L}{G \phi}$$

$$c = \frac{(80 \times 10^6)(3.0)}{(77 \times 10^9)(523.6 \times 10^{-3})} = 5.953 \times 10^{-3} \text{ m} = 5.953 \text{ mm}$$

$$d = 2c = 11.91 \text{ mm}$$

**PROBLEM 3.35**



3.35 The torques shown are exerted on pulleys A and B. Knowing that the shafts are solid and made of aluminum ( $G = 77$  GPa), determine the angle of twist between (a) A and B, (b) A and C.

**SOLUTION**

(a)  $T_{AB} = 300 \text{ N}\cdot\text{m}, \quad L_{AB} = 0.9 \text{ m}, \quad c_{AB} = \frac{1}{2}d = 0.015 \text{ m}$

$$J_{AB} = \frac{\pi}{2} (0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$\phi_{AB} = \frac{T_{AB} L_{AB}}{G J} = \frac{(300)(0.9)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 44.095 \times 10^{-3} \text{ rad}$$

$$\phi_{AB} = 2.53^\circ$$

(b)  $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}, \quad L_{BC} = 0.75 \text{ m}, \quad c_{BC} = \frac{1}{2}d = 0.023 \text{ m}$

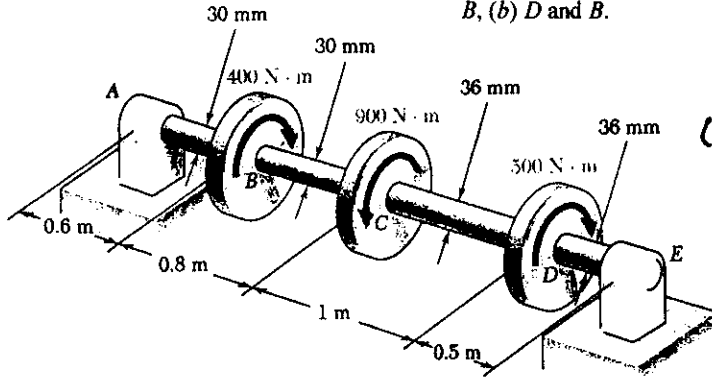
$$J_{BC} = \frac{\pi}{2} (0.023)^4 = 439.573 \times 10^{-9} \text{ m}^4$$

$$\phi_{BC} = \frac{T_{BC} L_{BC}}{G J_{BC}} = \frac{(700)(0.75)}{(77 \times 10^9)(439.573 \times 10^{-9})} = 15.511 \times 10^{-3} \text{ rad}$$

$$\phi_{AC} = \phi_{AB} + \phi_{BC} = 59.606 \times 10^{-3} \text{ rad} = 3.42^\circ$$

**PROBLEM 3.36**

3.36 The torques shown are exerted on pulleys B, C and D. Knowing that the entire shaft is made of steel ( $G = 27 \text{ GPa}$ ), determine the angle of twist between (a) C and B, (b) D and B.



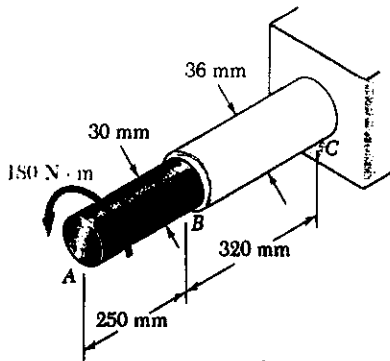
**SOLUTION**

(a) Shaft BC:  $c = \frac{1}{2}d = 0.015 \text{ m}$   
 $J_{bc} = \frac{\pi}{2}c^4 = 79.522 \times 10^{-9} \text{ m}^4$   
 $L_{bc} = 0.8 \text{ m}, G = 27 \times 10^9 \text{ Pa}$   
 $\phi_{bc} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27 \times 10^9)(79.522 \times 10^{-9})}$   
 $= 0.149904 \text{ rad} = 8.54^\circ$

(b) Shaft CD:  $c = \frac{1}{2}d = 0.018 \text{ m}$      $J_{cd} = \frac{\pi}{2}c^4 = 164.896 \times 10^{-9} \text{ m}^4$   
 $L_{cd} = 1.0 \text{ m}$      $T_{cd} = 400 - 900 = -500 \text{ N}\cdot\text{m}$   
 $\phi_{cd} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27 \times 10^9)(164.896 \times 10^{-9})} = -0.11230 \text{ rad}$   
 $\phi_{BD} = \phi_{bc} + \phi_{cd} = 0.14904 - 0.11230 = 0.03674 \text{ rad} = 2.11^\circ$

**PROBLEM 3.37**

3.37 The solid brass rod AB ( $G = 39 \text{ GPa}$ ) is bonded to the solid aluminum rod BC ( $G = 27 \text{ GPa}$ ). Determine the angle of twist (a) at B, (b) at A.



**SOLUTION**

Shaft AB:  $c = \frac{1}{2}d = 0.015 \text{ m}$      $L = 0.250 \text{ m}$   
 $G = 39 \times 10^9 \text{ Pa}$      $T = 180 \text{ N}\cdot\text{m}$   
 $J = \frac{\pi}{2}c^4 = 79.522 \times 10^{-9} \text{ m}^4$

$\phi_{AB} = \frac{TL}{GJ} = \frac{(180)(0.250)}{(39 \times 10^9)(79.522 \times 10^{-9})} = 14.510 \times 10^{-3} \text{ rad}$

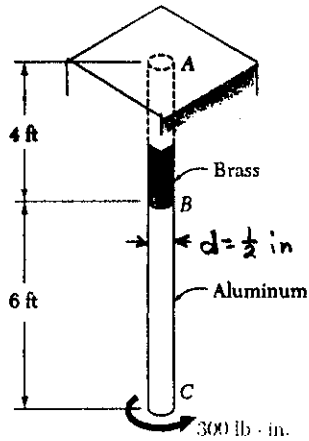
Shaft BC:  $c = \frac{1}{2}d = 0.018 \text{ m}, L = 0.320 \text{ m}$   
 $G = 27 \times 10^9 \text{ Pa}, T = 180 \text{ N}\cdot\text{m}$   
 $J = \frac{\pi}{2}c^4 = 164.896 \times 10^{-9} \text{ m}^4$

$\phi_{BC} = \frac{(180)(0.320)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 12.937 \times 10^{-3} \text{ rad}$

Answers: (a)  $\phi_B = \phi_{BC} = 12.937 \times 10^{-3} \text{ rad} = 0.741^\circ$

(b)  $\phi_A = \phi_{BC} + \phi_{AB} = 27.447 \times 10^{-3} \text{ rad} = 1.573^\circ$

PROBLEM 3.38



3.38 The brass rod  $AB$  ( $G = 5.6 \times 10^6$  psi) is bonded to the aluminum rod  $BC$  ( $G = 3.9 \times 10^6$  psi). Knowing that each rod is solid, determine the angle of twist (a) at  $B$ , (b) at  $C$ .

SOLUTION

Both portions  $c = \frac{1}{2}d = 0.25$  in

$$J = \frac{\pi}{2}c^4 = 6.1359 \times 10^{-3} \text{ in}^4 \quad T = 300 \text{ lb}\cdot\text{in}$$

Shaft  $AB$ :  $G_{AB} = 5.6 \times 10^6$  psi  $L_{AB} = 4 \text{ ft} = 48$  in

$$\begin{aligned} \phi_B = \phi_{AB} &= \frac{T L_{AB}}{G_{AB} J} = \frac{(300)(48)}{(5.6 \times 10^6)(6.1359 \times 10^{-3})} \\ &= 0.419 \text{ rad} = 24.0^\circ \end{aligned}$$

Shaft  $BC$ :  $G = 3.9 \times 10^6$  psi  $L_{BC} = 6 \text{ ft} = 72$  in

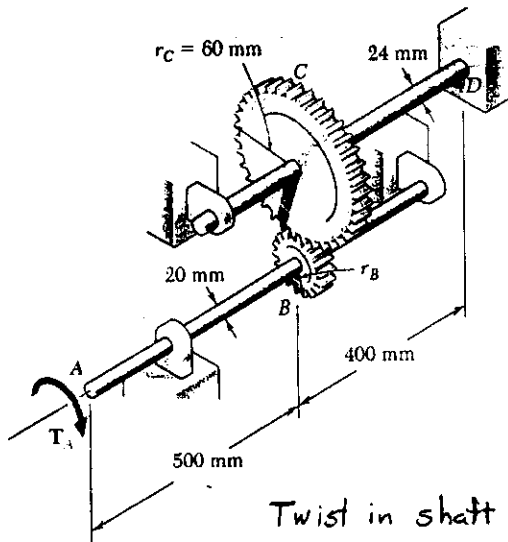
$$\begin{aligned} \phi_{BC} &= \frac{T L_{BC}}{G_{BC} J} = \frac{(300)(72)}{(3.9 \times 10^6)(6.1359 \times 10^{-3})} \\ &= 0.903 \text{ rad} = 51.7^\circ \end{aligned}$$

$$\phi_C = \phi_B + \phi_{BC}$$

$$= 0.417 + 0.903 = 1.320 \text{ rad} = 75.6^\circ$$

PROBLEM 3.39

3.39 Two solid steel shafts ( $G = 77 \text{ GPa}$ ) are connected by the gears shown. Knowing that the radius of gear B is  $r_B = 20 \text{ mm}$ , determine the angle through which end A rotates when  $T_A = 75 \text{ N}\cdot\text{m}$ .



SOLUTION

Calculation of torques.

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad \therefore T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{0.060}{0.020} (75) = 225 \text{ N}\cdot\text{m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} C_{CD}^4 = \frac{\pi}{2} (0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \quad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \phi_{CD} = \frac{TL}{GJ} = \frac{(225)(0.400)}{(77 \times 10^9)(32.572 \times 10^{-9})} = 35.885 \times 10^{-3} \text{ rad.}$$

Rotation angle at C  $\phi_C = \phi_{CD} = 35.885 \times 10^{-3} \text{ rad}$

Circumferential displacement at contact points of gears B and C

$$S = r_C \phi_C = r_B \phi_B$$

Rotation angle at B:  $\phi_B = \frac{r_C}{r_B} \phi_C = \frac{0.060}{0.020} (35.885 \times 10^{-3}) = 107.654 \times 10^{-3} \text{ rad}$

Twist in shaft AB:

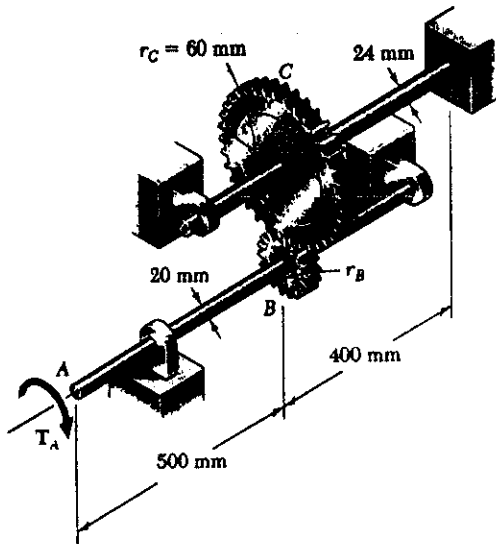
$$J_{AB} = \frac{\pi}{2} C_{AB}^4 = \frac{\pi}{2} (0.010)^4 = 15.708 \times 10^{-9} \text{ m}^4, \quad L_{AB} = 0.500 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \phi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77 \times 10^9)(15.708 \times 10^{-9})} = 31.004 \times 10^{-3} \text{ rad}$$

Rotation at A  $\phi_A = \phi_B + \phi_{AB} = 138.7 \times 10^{-3} \text{ rad} = 7.94^\circ$

PROBLEM 3.40

3.40 Solve Prob. 3.39, assuming that a change in design of the assembly resulted in the radius of gear B being increased to 30 mm.



$G = 77 \text{ GPa}$ ,  $r_B = 30 \text{ mm}$ ,  $T_A = 75 \text{ N}\cdot\text{m}$   
Determine the angle through which end A rotates.

SOLUTION

Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CO}}{r_C} \therefore T_{CO} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ N}\cdot\text{m}$$

$$T_{CO} = \frac{0.060}{0.030} (75) = 150 \text{ N}\cdot\text{m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} C_{CD}^4 = \frac{\pi}{2} (0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \quad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \phi_{CD} = \frac{TL}{GJ} = \frac{(150)(0.400)}{(77 \times 10^9)(32.572 \times 10^{-9})} = 23.923 \times 10^{-3} \text{ rad}$$

Rotation angle at C  $\phi_C = \phi_{CD} = 23.923 \times 10^{-3} \text{ rad}$ .

Circumferential displacement at contact points of gears B and C.

$$s = r_C \phi_C = r_B \phi_B$$

Rotation angle at B  $\phi_B = \frac{r_C}{r_B} \phi_C = \frac{0.060}{0.030} (23.923 \times 10^{-3}) = 47.846 \times 10^{-3} \text{ rad}$

Twist in shaft AB

$$J_{AB} = \frac{\pi}{2} C_{AB}^4 = \frac{\pi}{2} (0.010)^4 = 15.708 \times 10^{-9} \text{ m}^4, \quad L_{AB} = 0.500 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \phi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77 \times 10^9)(15.708 \times 10^{-9})} = 31.004 \times 10^{-3} \text{ rad}$$

Rotation at A  $\phi_A = \phi_B + \phi_{AB} = 78.85 \times 10^{-3} \text{ rad} = 4.52^\circ$  ◀

PROBLEM 3.41

3.41 Two shafts, each of  $\frac{3}{4}$ -in. diameter, are connected by the gears shown. Knowing that  $G = 11.2 \times 10^6$  psi and that the shaft at  $F$  is fixed, determine the angle through which end  $A$  rotates when a 750 lb-in. torque is applied at  $A$ .

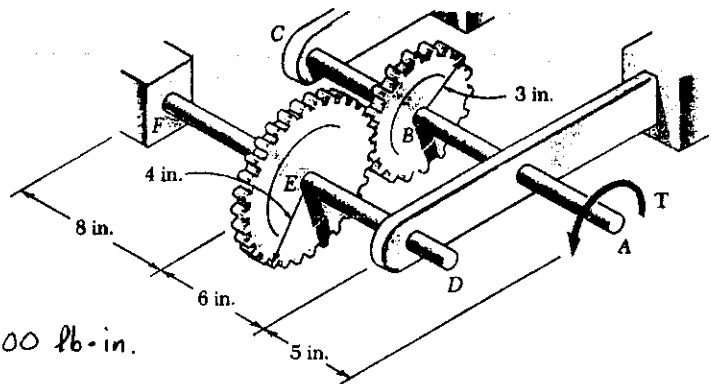
SOLUTION

Calculation of torques

Circumferential contact force between gears  $B$  and  $E$

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E}$$

$$\therefore T_{EF} = \frac{r_E}{r_B} T_{AB} = \frac{4}{3} (750) = 1000 \text{ lb-in.}$$



Twist in shaft  $FE$

$$L_{FE} = 8 \text{ in, } J_{FE} = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.375)^4 = 31.063 \times 10^{-3} \text{ in}^4, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\phi_{FE} = \frac{T_{FE} L_{FE}}{G J_{FE}} = \frac{(1000)(8)}{(11.2 \times 10^6)(31.063 \times 10^{-3})} = 22.995 \times 10^{-3} \text{ rad}$$

Rotation at  $E$   $\phi_E = 22.995 \times 10^{-3} \text{ rad}$

Tangential displacement at gear circle  $s = r_E \phi_E = r_B \phi_B$

Rotation at  $B$   $\phi_B = \frac{r_E}{r_B} \phi_E = \frac{4}{3} (22.995 \times 10^{-3}) = 30.660 \times 10^{-3} \text{ rad}$

Twist in shaft  $BA$

$$L_{BA} = 6 + 5 = 11 \text{ in, } J_{BA} = 31.063 \times 10^{-3} \text{ in}^4$$

$$\phi_{AB} = \frac{T_{AB} L_{BA}}{G J_{AB}} = \frac{(750)(11)}{(11.2 \times 10^6)(31.063 \times 10^{-3})} = 23.713 \times 10^{-3} \text{ rad}$$

Rotation at  $A$

$$\phi_A = \phi_B + \phi_{AB} = 30.660 \times 10^{-3} + 23.713 \times 10^{-3} = 54.373 \times 10^{-3} \text{ rad}$$

$$= 3.12^\circ$$

PROBLEM 3.42

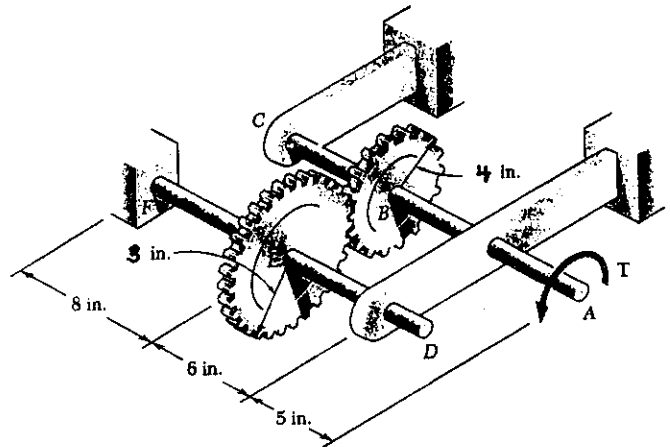
SOLUTION

Calculation of torques

Tangential contact force between gears B and E

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E}$$

$$T_{EF} = \frac{r_E}{r_B} T_{AB} = \frac{3}{4} (750) = 562.5 \text{ lb}\cdot\text{in}$$



Twist in shaft FE

$$L_{FE} = 8 \text{ in}, J_{FE} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.375)^4 = 31.063 \times 10^{-3} \text{ in}^4 \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\phi_{FE} = \frac{T_{FE} L_{FE}}{G J_{FE}} = \frac{(562.5)(8)}{(11.2 \times 10^6)(31.063 \times 10^{-3})} = 12.935 \times 10^{-3} \text{ rad}$$

Rotation at E  $\phi_E = 12.935 \times 10^{-3} \text{ rad}$

Tangential displacement at gear circle  $S = r_E \phi_E = r_B \phi_B$

Rotation at B  $\phi_B = \frac{r_E}{r_B} \phi_E = \frac{3}{4} (12.935 \times 10^{-3}) = 9.701 \times 10^{-3} \text{ rad}$

Twist shaft AB

$$L_{BA} = 6 + 5 = 11 \text{ in}, J_{AB} = 31.063 \times 10^{-3} \text{ in}^4$$

$$\phi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{(750)(11)}{(11.2 \times 10^6)(31.063 \times 10^{-3})} = 23.713 \times 10^{-3} \text{ rad}$$

Rotation at A  $\phi_A = \phi_B + \phi_{AB}$

$$= 9.701 \times 10^{-3} + 23.713 \times 10^{-3} = 33.414 \times 10^{-3} \text{ rad} = 1.914^\circ$$



PROBLEM 3.43

3.43 A coder  $F$ , used to record in digital form the rotation of shaft  $A$ , is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter  $d$ . Two of the gears have a radius  $r$  and the other two a radius  $nr$ . If the rotation of the coder  $F$  is prevented, determine in terms of  $T$ ,  $l$ ,  $G$ ,  $J$ , and  $n$  the angle through which end  $A$  rotates.

SOLUTION

$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\phi_E = \phi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\phi_D = \frac{r_E}{r_D} \phi_E = \frac{\phi_E}{n} = \frac{T_A l}{n^3 GJ}$$

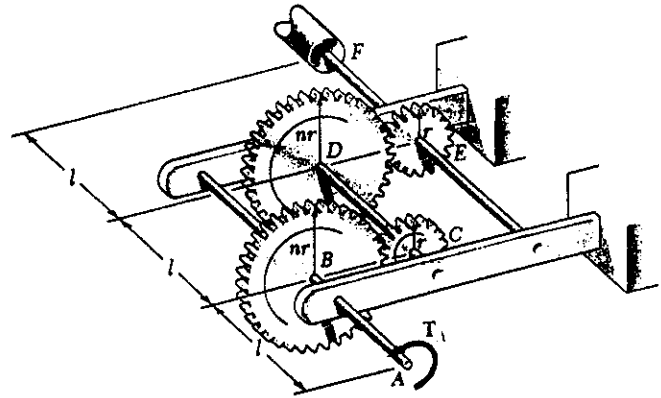
$$\phi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{n GJ}$$

$$\phi_C = \phi_D + \phi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{n GJ} = \frac{T_A l}{GJ} \left( \frac{1}{n^3} + \frac{1}{n} \right)$$

$$\phi_B = \frac{r_C}{r_B} \phi_C = \frac{\phi_C}{n} = \frac{T_A l}{GJ} \left( \frac{1}{n^4} + \frac{1}{n^2} \right)$$

$$\phi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\phi_A = \phi_B + \phi_{AB} = \frac{T_A l}{GJ} \left( \frac{1}{n^4} + \frac{1}{n^2} + 1 \right)$$



PROBLEM 3.44

3.43 A coder  $F$ , used to record in digital form the rotation of shaft  $A$ , is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter  $d$ . Two of the gears have a radius  $r$  and the other two a radius  $nr$ . If the rotation of the coder  $F$  is prevented, determine in terms of  $T$ ,  $l$ ,  $G$ ,  $J$ , and  $n$  the angle through which end  $A$  rotates.

SOLUTION

See solution to PROBLEM 3.43 for development of equation for  $\phi_A$

3.44 For the gear train described in Prob. 3.43, determine the angle through which end  $A$  rotates when  $T = 0.75 \text{ N}\cdot\text{m}$ ,  $l = 60 \text{ mm}$ ,  $d = 4 \text{ mm}$ ,  $G = 77 \text{ GPa}$ , and  $n = 2$ .

$$\phi_A = \frac{Tl}{GJ} \left( 1 + \frac{1}{n^2} + \frac{1}{n^4} \right)$$

Data:  $T = 0.75 \text{ N}\cdot\text{m}$ ,  $l = 0.060 \text{ m}$ ,  $c = \frac{1}{2}d = 0.002 \text{ m}$ ,  $G = 77 \times 10^9 \text{ Pa}$

$n = 2$ ,  $J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.002)^4 = 25.133 \times 10^{-12} \text{ m}^4$

$$\phi_A = \frac{(0.75)(0.060)}{(77 \times 10^9)(25.133 \times 10^{-12})} \left( 1 + \frac{1}{4} + \frac{1}{16} \right) = 30.52 \times 10^{-3} \text{ rad.}$$

$$= 1.749^\circ$$

## PROBLEM 3.45

3.45 The design specifications of a 2-m-long solid circular transmission shaft require that the angle of twist of the shaft not exceed  $3^\circ$  when a torque of  $9 \text{ kN}\cdot\text{m}$  is applied. Determine the required diameter of the shaft, knowing that the shaft is made of (a) a steel with an allowable shearing stress of  $90 \text{ MPa}$  and a modulus of rigidity of  $77 \text{ GPa}$ , (b) a bronze with an allowable shearing stress of  $35 \text{ MPa}$  and a modulus of rigidity of  $42 \text{ GPa}$ .

## SOLUTION

$$\phi = 3^\circ = 52.360 \times 10^{-3} \text{ rad}, \quad T = 9 \times 10^3 \text{ N}\cdot\text{m} \quad L = 2.0 \text{ m}$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi C^4 G} \quad \therefore C^4 = \frac{2TL}{\pi G \phi} \quad \text{based on twist angle}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi C^3} \quad \therefore C^3 = \frac{2T}{\pi \tau} \quad \text{based shearing stress}$$

(a) Steel shaft:  $\tau = 90 \times 10^6 \text{ Pa}, \quad G = 77 \times 10^9 \text{ Pa}$

Based on twist angle  $C^4 = \frac{(2)(9 \times 10^3)(2.0)}{\pi(77 \times 10^9)(52.360 \times 10^{-3})} = 2.842 \times 10^{-6} \text{ m}^4$

$$C = 41.06 \times 10^{-3} \text{ m} = 41.06 \text{ mm} \quad d = 2C = 82.1 \text{ mm}$$

Based on shearing stress  $C^3 = \frac{(2)(9 \times 10^3)}{\pi(90 \times 10^6)} = 63.662 \times 10^{-6} \text{ m}^3$

$$C = 39.93 \times 10^{-3} \text{ m} = 39.93 \text{ mm} \quad d = 2C = 79.9 \text{ mm}$$

Required value of  $d$  is the larger:  $d = 82.1 \text{ mm}$   $\blacktriangleleft$

(b) Bronze shaft:  $\tau = 35 \times 10^6 \text{ Pa}, \quad G = 42 \times 10^9 \text{ Pa}$

Based on twist angle  $C^4 = \frac{(2)(9 \times 10^3)(2.0)}{\pi(42 \times 10^9)(52.360 \times 10^{-3})} = 5.2103 \times 10^{-6} \text{ m}^4$

$$C = 47.78 \times 10^{-3} \text{ m} = 47.78 \text{ mm} \quad d = 2C = 95.6 \text{ mm}$$

Based on shearing stress  $C^3 = \frac{(2)(9 \times 10^3)}{\pi(35 \times 10^6)} = 163.702 \times 10^{-6} \text{ m}^3$

$$C = 54.70 \times 10^{-3} \text{ m} = 54.70 \text{ mm} \quad d = 2C = 109.4 \text{ mm}$$

Required value of  $d$  is the larger  $d = 109.4 \text{ mm}$   $\blacktriangleleft$

## PROBLEM 3.46

3.46 The design specifications of a 4-ft-long solid circular transmission shaft require that the angle of twist of the shaft not exceed  $4^\circ$  when a torque of 6 kip·in. is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 12 ksi and a modulus of rigidity of  $11.2 \times 10^6$  psi.

## SOLUTION

Based on twist angle  $\phi = 4^\circ = 69.81 \times 10^{-3}$  rad.  $L = 4 \text{ ft} = 48 \text{ in.}$

$$T = 6 \text{ kip}\cdot\text{in} = 6000 \text{ lb}\cdot\text{in.}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi Gc^4} \quad c^4 = \frac{2TL}{\pi G\phi} = \frac{(2)(6000)(48)}{\pi(11.2 \times 10^6)(69.81 \times 10^{-3})} = 0.2345 \text{ in}^4$$

$$c = 0.696 \text{ in} \quad d = 2c = 1.392 \text{ in.}$$

Based on shearing stress  $\tau = 12 \text{ ksi} = 12000 \text{ psi}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(6000)}{\pi(12000)} = 0.3183 \text{ in}^3$$

$$c = 0.683 \text{ in} \quad d = 2c = 1.366 \text{ in.}$$

Required diameter is the larger  $d = 1.392 \text{ in.}$   $\blacktriangleleft$

**PROBLEM 3.47**

3.47 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that  $\tau_{max} < 60 \text{ MPa}$  and that the angle  $\phi_D$  through which end D of shaft CD rotates not exceed  $1.5^\circ$ . Knowing that  $G = 77 \text{ GPa}$ , determine the required diameter of the shafts.

**SOLUTION**

$$T_{CB} = T_D = 1000 \text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CB} = \frac{100}{40} (1000) = 2500 \text{ N}\cdot\text{m}$$

For design based on stress, use larger torque  $T_{AB} = 2500 \text{ N}\cdot\text{m}$

$$\tau = \frac{T_C}{J} = \frac{2T}{\pi C^3}$$

$$C^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi (60 \times 10^6)} = 26.526 \times 10^{-6} \text{ m}^3$$

$$C = 29.82 \times 10^{-3} \text{ m} = 29.82 \text{ mm}, \quad d = 2C = 59.6 \text{ mm}$$

Design based on rotation angle  $\phi_D = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

Shaft AB:  $T_{AB} = 2500 \text{ N}\cdot\text{m}$ ,  $L = 0.4 \text{ m}$

$$\phi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$$

Gears  $\left\{ \begin{aligned} \phi_B &= \phi_{AB} = \frac{1000}{GJ} \\ \phi_C &= \frac{r_B}{r_C} \phi_B = \frac{100}{40} \cdot \frac{1000}{GJ} = \frac{2500}{GJ} \end{aligned} \right.$

Shaft CD  $T_{CD} = 1000 \text{ N}\cdot\text{m}$ ,  $L = 0.6 \text{ m}$

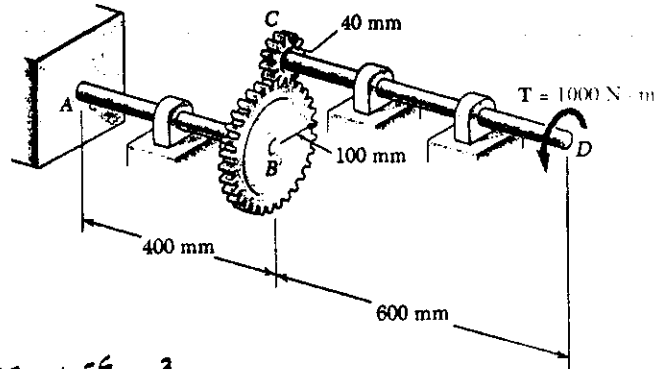
$$\phi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\phi_D = \phi_C + \phi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G \frac{\pi}{2} C^4}$$

$$C^4 = \frac{(2)(3100)}{\pi G \phi_D} = \frac{(2)(3100)}{\pi (77 \times 10^9) (26.18 \times 10^{-3})} = 979.06 \times 10^{-9} \text{ m}^4$$

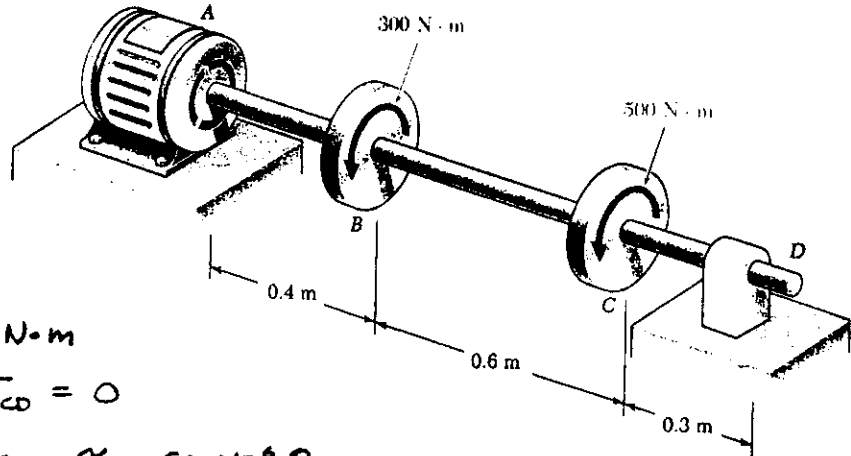
$$C = 31.46 \times 10^{-3} \text{ m} = 31.46 \text{ mm}, \quad d = 2C = 62.9 \text{ mm}$$

Design must use larger value for  $d$   $d = 62.9 \text{ mm}$



PROBLEM 3.48

3.48 The electric motor exerts a torque of 800 N·m on the steel shaft *ABCD* when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from *A* to *D* and that the angle of twist between *A* and *D* not exceed 1.5°. Knowing that  $\tau_{\max} \leq 60$  MPa and  $G = 77$  GPa, determine the minimum diameter shaft that may be used.



SOLUTION

Torques

$$T_{AB} = 300 + 500 = 800 \text{ N}\cdot\text{m}$$

$$T_{BC} = 500 \text{ N}\cdot\text{m}, \quad T_{CD} = 0$$

Design based on stress  $\tau = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(800)}{\pi(60 \times 10^6)} = 8.488 \times 10^{-6} \text{ m}^3$$

$$c = 20.40 \times 10^{-3} \text{ m} = 20.40 \text{ mm}, \quad d = 2c = 40.8 \text{ mm}$$

Design based on deformation  $\phi_{D/A} = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

$$\phi_{D/C} = 0$$

$$\phi_{C/B} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\phi_{B/A} = \frac{T_{AB}L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$$

$$\phi_{D/A} = \phi_{C/B} + \phi_{B/A} = \frac{620}{GJ} = \frac{620}{G \frac{\pi}{2} c^4} = \frac{(2)(620)}{\pi G c^4}$$

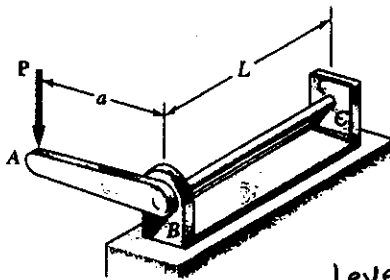
$$c^4 = \frac{(2)(620)}{\pi G \phi_{D/A}} = \frac{(2)(620)}{\pi(77 \times 10^9)(26.18 \times 10^{-3})} = 195.80 \times 10^{-9} \text{ m}^4$$

$$c = 21.04 \times 10^{-3} \text{ m} = 21.04 \text{ mm}, \quad d = 2c = 42.1 \text{ mm}$$

Design must use larger value of *d*

$$d = 42.1 \text{ mm}$$

PROBLEM 3.49



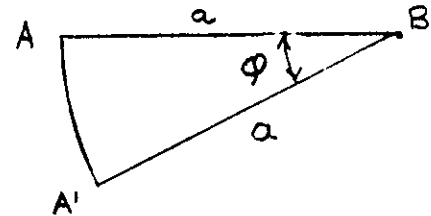
3.49 The solid cylindrical rod  $BC$  is attached to the rigid lever  $AB$  and to the fixed support at  $C$ . The vertical force  $P$  applied at  $A$  causes a small displacement  $\Delta$  at point  $A$ . Show that the corresponding maximum shearing stress in the rod is

$$\tau = \frac{Gd}{2La} \Delta$$

where  $d$  is the diameter of the rod and  $G$  its modulus of rigidity.

SOLUTION

Lever  $AB$  turns through angle  $\phi$  to position  $A'B$  as shown in the auxiliary figure.



Vertical displacement is  $\Delta = a \sin \phi$   
 from which  $\phi = \arcsin \frac{\Delta}{a}$

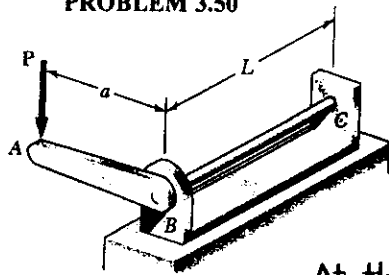
The maximum shearing stress in rod  $BC$  is

$$\tau_{max} = G\gamma_{max} = G \frac{C\phi}{L} = G \frac{d\phi}{2L} = \frac{Gd}{2L} \arcsin \frac{\Delta}{a}$$

For small  $\frac{\Delta}{a}$ ,  $\arcsin \frac{\Delta}{a} \approx \frac{\Delta}{a}$

$$\tau_{max} = \frac{Gd\Delta}{2La}$$

PROBLEM 3.50



3.50 and 3.51 The solid cylindrical rod  $BC$  of length  $L = 24$  in. is attached to the rigid lever  $AB$  of length  $a = 15$  in. and to the support at  $C$ . When a 100-lb force  $P$  is applied at  $A$ , design specifications require that the displacement of  $A$  not exceed 1 in. when a 100-lb force  $P$  is applied at  $A$ . For the material indicated determine the required diameter of the rod.

3.50 Steel:  $\tau_{all} = 15$  ksi,  $G = 11.2 \times 10^6$  psi.

SOLUTION

At the allowable twist angle  $\sin \phi = \frac{\Delta}{a} = \frac{1}{15} = 0.06667$   
 $\phi = 3.8226^\circ = 0.066716$  rad.

$T = Pa \cos \phi = (100)(15) \cos 3.8226^\circ = 1496.7$  lb·in

Based on twist  $\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G C^4} \therefore C^4 = \frac{2TL}{\pi G \phi}$

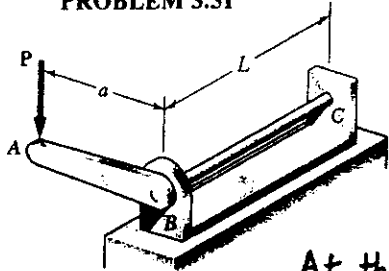
$C^4 = \frac{(2)(1496.7)(24)}{\pi(11.2 \times 10^6)(0.066716)} = 30.603 \times 10^{-3} \text{ in}^4$   $C = 0.418$  in.

Based on stress  $\tau = \frac{Tc}{J} = \frac{2T}{\pi C^3} \therefore C^3 = \frac{2T}{\pi \tau}$  ( $\tau = 15000$  psi)

$C^3 = \frac{2(1496.7)}{\pi(15000)} = 63.522 \times 10^{-3} \text{ in}^3$   $C = 0.399$  in.

Use larger value for design  $C = 0.399$  in  $d = 2C = 0.837$  in. ◀

PROBLEM 3.51



3.50 and 3.51 The solid cylindrical rod  $BC$  of length  $L = 24$  in. is attached to the rigid lever  $AB$  of length  $a = 15$  in. and to the support at  $C$ . When a 100-lb force  $P$  is applied at  $A$ , design specifications require that the displacement of  $A$  not exceed 1 in. when a 100-lb force  $P$  is applied at  $A$ . For the material indicated determine the required diameter of the rod.

3.51 Aluminum:  $\tau_{all} = 10$  ksi,  $G = 3.9 \times 10^6$  psi.

SOLUTION

At the allowable twist angle  $\sin \phi = \frac{\Delta}{a} = \frac{1}{15} = 0.06667$   
 $\phi = 3.8226^\circ = 0.066716$  rad.

$T = Pa \cos \phi = (100)(15) \cos 3.8226^\circ = 1496.7$  lb·in

Based on twist  $\phi = \frac{TL}{GJ} = \frac{2TL}{\pi G C^4} \therefore C^4 = \frac{2TL}{\pi G \phi}$

$C^4 = \frac{(2)(1496.7)(24)}{\pi(3.9 \times 10^6)(0.066716)} = 87.888 \times 10^{-3} \text{ in}^4$   $C = 0.544$  in.

Based on stress  $\tau = \frac{Tc}{J} = \frac{2T}{\pi C^3} \therefore C^3 = \frac{2T}{\pi \tau}$  ( $\tau = 10000$  psi)

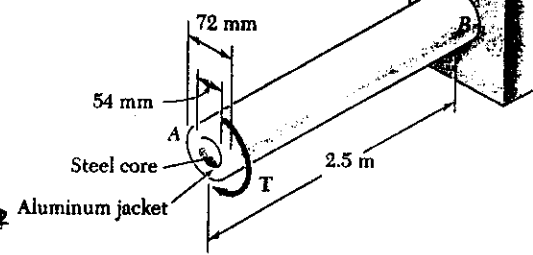
$C^3 = \frac{(2)(1496.7)}{\pi(10000)} = 95.283 \times 10^{-3} \text{ in}^3$   $C = 0.457$  in

Use larger value for design  $C = 0.544$  in  $d = 2C = 1.089$  in. ◀

PROBLEM 3.52

3.52 A 4-kN·m torque  $T$  is applied at end  $A$  of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine (a) the maximum shearing stress in the steel core, (b) the maximum shearing stress in the aluminum jacket, (c) the angle of twist at  $A$ .

SOLUTION



Steel core:  $c_1 = \frac{1}{2} d_1 = 0.027 \text{ m}$

$$J_1 = \frac{\pi}{2} c_1^4 = \frac{\pi}{2} (0.027)^4 = 834.79 \times 10^{-9}$$

$$G_1 J_1 = (77 \times 10^9)(834.79 \times 10^{-9}) = 64.28 \times 10^3 \text{ N}\cdot\text{m}^2$$

Torque carried by steel core  $T_1 = G_1 J_1 \phi / L$

Aluminum jacket:  $c_1 = \frac{1}{2} d_1 = 0.027 \text{ m}$ ,  $c_2 = \frac{1}{2} d_2 = 0.036 \text{ m}$

$$J_2 = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.036^4 - 0.027^4) = 1.80355 \times 10^{-6} \text{ m}^4$$

$$G_2 J_2 = (27 \times 10^9)(1.80355 \times 10^{-6}) = 48.70 \times 10^3 \text{ N}\cdot\text{m}^2$$

Torque carried by aluminum jacket  $T_2 = G_2 J_2 \phi / L$

Total torque  $T = T_1 + T_2 = (G_1 J_1 + G_2 J_2) \phi / L$

$$\frac{\phi}{L} = \frac{T}{G_1 J_1 + G_2 J_2} = \frac{4 \times 10^3}{64.28 \times 10^3 + 48.70 \times 10^3} = 35.406 \times 10^{-3} \text{ rad/m}$$

(a) Maximum shearing stress in steel core

$$\tau = G_1 \gamma = G_1 c_1 \frac{\phi}{L} = (77 \times 10^9)(0.027)(35.406 \times 10^{-3})$$

$$= 73.6 \times 10^6 \text{ Pa}$$

$$73.6 \text{ MPa}$$

(b) Maximum shearing stress in aluminum jacket

$$\tau = G_2 \gamma = G_2 c_2 \frac{\phi}{L} = (27 \times 10^9)(0.036)(35.406 \times 10^{-3})$$

$$= 34.4 \times 10^6 \text{ Pa}$$

$$34.4 \text{ MPa}$$

(c) Angle of twist

$$\phi = L \frac{\phi}{L} = (2.5)(35.406 \times 10^{-3}) = 88.5 \times 10^{-3} \text{ rad}$$

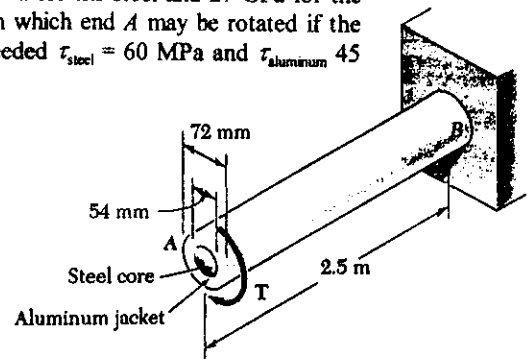
$$= 5.07^\circ$$



PROBLEM 3.53

3.53 The composite shaft shown is to be twisted by applying a torque  $T$  at end  $A$ . Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end  $A$  may be rotated if the following allowable stresses are not to be exceeded  $\tau_{\text{steel}} = 60 \text{ MPa}$  and  $\tau_{\text{aluminum}} = 45 \text{ MPa}$ .

SOLUTION



$$\tau_{\text{max}} = G \gamma_{\text{max}} = G C_{\text{max}} \frac{\phi}{L}$$

$$\frac{\phi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{G C_{\text{max}}} \quad \text{for each material}$$

Steel core:  $\tau_{\text{all}} = 60 \times 10^6 \text{ Pa}$ ,  $C_{\text{max}} = \frac{1}{2}d = 0.027 \text{ m}$ ,  $G = 77 \times 10^9 \text{ Pa}$

$$\frac{\phi_{\text{all}}}{L} = \frac{60 \times 10^6}{(77 \times 10^9)(0.027)} = 28.860 \times 10^{-3} \text{ rad/m}$$

Aluminum jacket:  $\tau_{\text{all}} = 45 \times 10^6 \text{ Pa}$ ,  $C_{\text{max}} = \frac{1}{2}d = 0.036 \text{ m}$ ,  $G = 27 \times 10^9 \text{ Pa}$

$$\frac{\phi_{\text{all}}}{L} = \frac{45 \times 10^6}{(27 \times 10^9)(0.036)} = 46.296 \times 10^{-3} \text{ rad/m}$$

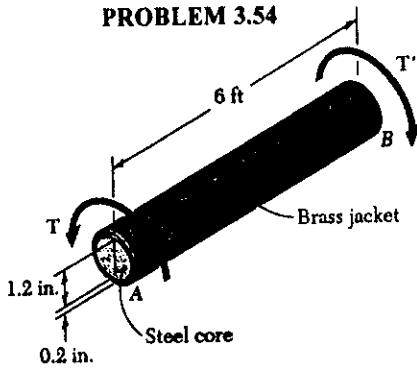
Smaller value governs

$$\frac{\phi_{\text{all}}}{L} = 28.860 \times 10^{-3} \text{ rad/m}$$

Allowable angle of twist

$$\begin{aligned} \phi_{\text{all}} &= L \frac{\phi_{\text{all}}}{L} = (2.5)(28.860 \times 10^{-3}) \\ &= 72.15 \times 10^{-3} \text{ rad} = 4.13^\circ \end{aligned}$$

PROBLEM 3.54



3.54 The composite shaft shown consists of a 0.2-in.-thick brass jacket ( $G = 5.6 \times 10^6$  psi) bonded to a 1.2-in.-diameter steel core ( $G = 11.2 \times 10^6$  psi). Knowing that the shaft is subjected to 5-kip-in. torques, determine (a) the maximum shearing stress in the brass jacket, (b) the maximum shearing stress in the steel core, (c) the angle of twist of end B relative to end A.

SOLUTION

Steel core:  $c_1 = \frac{1}{2}d = 0.6$  in

$$J_1 = \frac{\pi}{2}c_1^4 = \frac{\pi}{2}(0.6)^4 = 0.203575 \text{ in}^4$$

$$G_1J_1 = (11.2 \times 10^6)(0.203575) = 2.2800 \times 10^6 \text{ lb}\cdot\text{in}^2$$

Torque carried by steel core  $T_1 = G_1J_1 \frac{\phi}{L}$

Brass jacket:  $c_2 = c_1 + t = 0.6 + 0.2 = 0.8$  in

$$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.439823 \text{ in}^4$$

$$G_2J_2 = (5.6 \times 10^6)(0.439823) = 2.4630 \times 10^6 \text{ lb}\cdot\text{in}^2$$

Torque carried by brass jacket  $T_2 = G_2J_2 \frac{\phi}{L}$

Total torque  $T = T_1 + T_2 = (G_1J_1 + G_2J_2) \frac{\phi}{L}$

$$\frac{\phi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{5 \times 10^3}{2.2800 \times 10^6 + 2.4630 \times 10^6} = 1.0542 \times 10^{-3} \text{ rad/in}$$

(a) Maximum shearing stress in brass jacket

$$\tau_{\max} = G_2 \gamma_{\max} = G_2 c_2 \frac{\phi}{L} = (5.6 \times 10^6)(0.8)(1.0542 \times 10^{-3})$$

$$= 4.72 \times 10^3 \text{ psi}$$

$$4.72 \text{ ksi} \quad \blacktriangleleft$$

(b) Maximum shearing stress in steel core

$$\tau_{\max} = G_1 \gamma_{\max} = G_1 c_1 \frac{\phi}{L} = (11.2 \times 10^6)(0.6)(1.0542 \times 10^{-3})$$

$$= 7.08 \times 10^3 \text{ psi}$$

$$7.08 \text{ ksi} \quad \blacktriangleleft$$

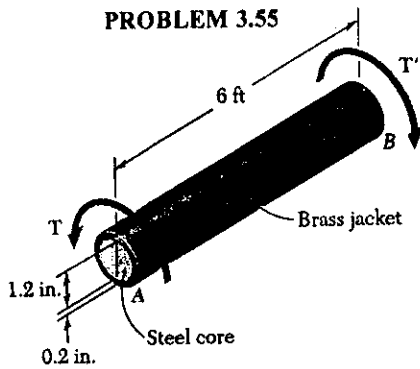
(c) Angle of twist

$$(L = 6\text{ft} = 72 \text{ in})$$

$$\phi = L \frac{\phi}{L} = (72)(1.0542 \times 10^{-3}) = 75.9 \times 10^{-3} \text{ rad}$$

$$= 4.35^\circ \quad \blacktriangleleft$$

PROBLEM 3.55



3.55 The composite shaft shown is to be twisted by applying the torques shown. Knowing that the modulus of rigidity is  $11.2 \times 10^6$  psi for the steel and  $5.6 \times 10^6$  psi for the brass, determine the largest angle of twist of end B relative to end A if the following allowable stresses are not to be exceeded  $\tau_{\text{steel}} = 15$  ksi and  $\tau_{\text{brass}} = 8$  ksi.

SOLUTION

$$\tau_{\text{max}} = G \gamma_{\text{max}} = G C_{\text{max}} \frac{\phi}{L}$$

$$\frac{\phi_{\text{all}}}{L} = \frac{\tau_{\text{all}}}{G C_{\text{max}}} \quad \text{for each material}$$

Steel core:  $\tau_{\text{all}} = 15 \text{ ksi} = 15000 \text{ psi}$ ,  $C_{\text{max}} = \frac{1}{2}d = 0.6 \text{ in}$

$$\frac{\phi_{\text{all}}}{L} = \frac{15000}{(11.2 \times 10^6)(0.6)} = 2.2321 \times 10^{-3} \text{ rad/in}$$

Brass jacket:  $\tau_{\text{all}} = 8 \text{ ksi} = 8000 \text{ psi}$ ,  $C_{\text{max}} = 0.6 + 0.2 = 0.8 \text{ in}$

$$\frac{\phi_{\text{all}}}{L} = \frac{8000}{(5.6 \times 10^6)(0.8)} = 1.7857 \times 10^{-3} \text{ rad/in}$$

Smaller value governs  $\frac{\phi_{\text{all}}}{L} = 1.7857 \times 10^{-3} \text{ rad/in}$

Allowable angle of twist

$$L = 6 \text{ ft} = 72 \text{ in}$$

$$\begin{aligned} \phi_{\text{all}} &= L \frac{\phi_{\text{all}}}{L} = (72)(1.7857 \times 10^{-3}) = 128.57 \times 10^{-3} \text{ rad} \\ &= 7.37^\circ \end{aligned}$$

PROBLEM 3.56

3.56 Two solid steel shafts are fitted with flanges which are then connected by fitted bolts so that there is no relative rotation between the flanges. Knowing that  $G = 77$  GPa, determine the maximum shearing stress in each shaft when a 500 N·m torque is applied to flange B.

SOLUTION

Shaft AB

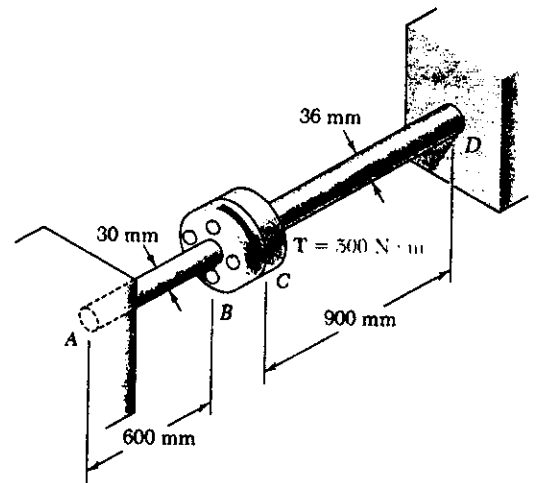
$$T = T_{AB}, L_{AB} = 0.6 \text{ m}, c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}}$$

$$T_{AB} = \frac{G_{AB} J_{AB}}{L_{AB}} \phi_B = \frac{(77 \times 10^9)(79.52 \times 10^{-9})}{0.6} \phi_B$$

$$= 10.205 \times 10^3 \phi_B$$



Shaft CD

$$T = T_{CD}, L_{CD} = 0.9 \text{ m}, c = \frac{1}{2}d = 0.018 \text{ m}, J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$$

$$J_{CD} = 164.896 \times 10^{-9} \text{ m}^4$$

$$T_{CD} = \frac{G_{CD} J_{CD}}{L_{CD}} \phi_C = \frac{(77 \times 10^9)(164.896 \times 10^{-9})}{0.9} \phi_C = 14.108 \times 10^3 \phi_C$$

Matching rotation at the flanges  $\phi_B = \phi_C = \phi$

Total torque on flanges  $T = T_{AB} + T_{CD} = 500 \text{ N·m}$

$$500 = (10.205 \times 10^3 + 14.108 \times 10^3) \phi \quad \therefore \phi = 20.565 \times 10^{-3} \text{ rad}$$

$$T_{AB} = (10.205 \times 10^3)(20.565 \times 10^{-3}) = 209.87 \text{ N·m}$$

$$T_{CD} = (14.108 \times 10^3)(20.565 \times 10^{-3}) = 290.13 \text{ N·m}$$

Maximum shearing stress in AB

$$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(209.87)(0.015)}{79.52 \times 10^{-9}} = 39.59 \times 10^6 \text{ Pa} \quad 39.6 \text{ MPa} \quad \blacktriangleleft$$

Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(290.13)(0.018)}{164.896 \times 10^{-9}} = 31.67 \times 10^6 \text{ Pa} \quad 31.7 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 3.57

3.57 and 3.58 Two solid steel shafts are fitted with flanges which are then connected by bolts as shown. The bolts are slightly undersized and permit a  $1.5^\circ$  rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that  $G = 77 \text{ GPa}$ , determine the maximum shearing stress in each shaft when a  $500 \text{ N}\cdot\text{m}$  torque  $T$  is applied to the flange indicated.

3.57 The torque  $T$  is applied to flange  $B$ .

SOLUTION

Shaft AB

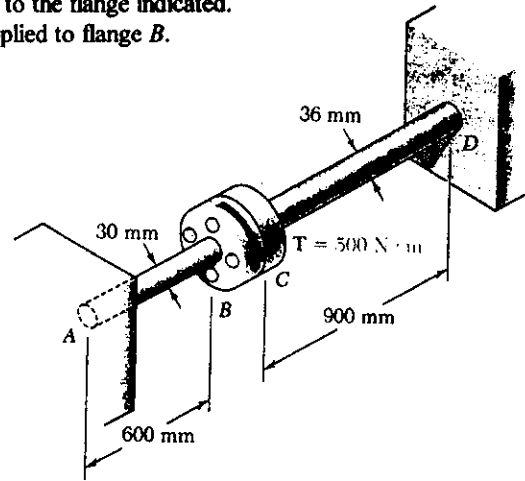
$$T = T_{AB}, \quad L = 0.6 \text{ m}, \quad c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}}$$

$$T_{AB} = \frac{G_{AB} J_{AB}}{L} \phi_B = \frac{(77 \times 10^9)(79.52 \times 10^{-9})}{0.6} \phi_B$$

$$= 10.205 \times 10^3 \phi_B$$



Shaft CD

$$T = T_{CD}, \quad L_{CD} = 0.9 \text{ m}, \quad c = \frac{1}{2}d = 0.018 \text{ m}, \quad J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$$

$$J_{CD} = 164.896 \times 10^{-9} \text{ m}^4$$

$$T_{CD} = \frac{G_{CD} J_{CD}}{L_{CD}} \phi_C = \frac{(77 \times 10^9)(164.896 \times 10^{-9})}{0.9} \phi_C = 14.108 \times 10^3 \phi_C$$

Clearance rotation for flange B  $\phi_B' = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

Torque to remove clearance:  $T_{AB}' = (10.205 \times 10^3)(26.18 \times 10^{-3}) = 267.17 \text{ N}\cdot\text{m}$

Torque  $T''$  to cause additional rotation  $\phi''$ :  $T'' = 500 - 267.17 = 232.83 \text{ N}\cdot\text{m}$

$$T'' = T_{AB}'' + T_{CD}''$$

$$232.83 = (10.205 \times 10^3 + 14.108 \times 10^3) \phi'' \quad \therefore \quad \phi'' = 9.5765 \times 10^{-3} \text{ rad}$$

$$T_{AB}'' = (10.205 \times 10^3)(9.5765 \times 10^{-3}) = 97.73 \text{ N}\cdot\text{m}$$

$$T_{CD}'' = (14.108 \times 10^3)(9.5765 \times 10^{-3}) = 135.10 \text{ N}\cdot\text{m}$$

Maximum shearing stress in AB

$$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(267.17 + 97.73)(0.015)}{79.52 \times 10^{-9}} = 68.8 \times 10^6 \text{ Pa} \quad 68.8 \text{ MPa} \quad \blacktriangleleft$$

Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(135.10)(0.018)}{164.896 \times 10^{-9}} = 14.75 \times 10^6 \text{ Pa} \quad 14.75 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 3.58

3.57 and 3.58 Two solid steel shafts are fitted with flanges which are then connected by bolts as shown. The bolts are slightly undersized and permit a  $1.5^\circ$  rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that  $G = 77 \text{ GPa}$ , determine the maximum shearing stress in each shaft when a  $500 \text{ N}\cdot\text{m}$  torque  $T$  is applied to the flange indicated.

3.58 The torque  $T$  is applied to flange C.

SOLUTION

Shaft AB

$$T = T_{AB}, L_{AB} = 0.6 \text{ m}, c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$\phi_B = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}}$$

$$T_{AB} = \frac{G_{AB} J_{AB}}{L_{AB}} \phi_B = \frac{(77 \times 10^9)(79.52 \times 10^{-9})}{0.6} \phi_B$$

$$= 10.205 \times 10^3 \phi_B$$

Shaft CD

$$T = T_{CD}, L_{CD} = 0.9 \text{ m}, c = \frac{1}{2}d = 0.018 \text{ m}, J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$$

$$J_{CD} = 164.896 \times 10^{-9} \text{ m}^4$$

$$T_{CD} = \frac{G_{CD} J_{CD}}{L_{CD}} \phi_C = \frac{(77 \times 10^9)(164.896 \times 10^{-9})}{0.9} \phi_C = 14.108 \times 10^3 \phi_C$$

Clearance rotation for flange C  $\phi_C' = 1.5^\circ = 26.18 \times 10^{-3} \text{ rad}$

Torque to remove clearance:  $T_{CD}' = (14.108 \times 10^3)(26.18 \times 10^{-3}) = 369.35 \text{ N}\cdot\text{m}$

Torque  $T''$  to cause additional rotation  $\phi''$ :  $T'' = 500 - 369.35 = 130.65 \text{ N}\cdot\text{m}$

$$T'' = T_{AB}'' + T_{CD}''$$

$$130.65 = (10.205 \times 10^3 + 14.108 \times 10^3) \phi'' \quad \phi'' = 5.3737 \times 10^{-3} \text{ rad}$$

$$T_{AB}'' = (10.205 \times 10^3)(5.3737 \times 10^{-3}) = 54.84 \text{ N}\cdot\text{m}$$

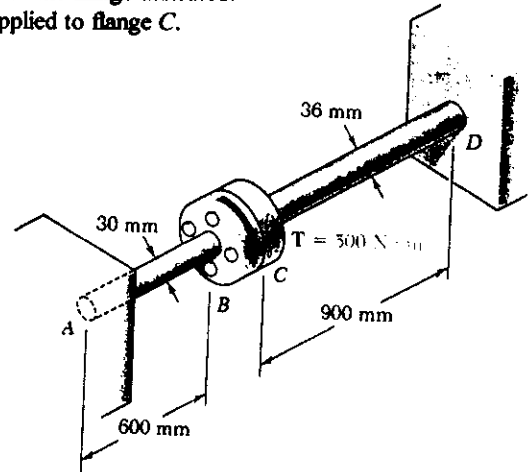
$$T_{CD}'' = (14.108 \times 10^3)(5.3737 \times 10^{-3}) = 75.81 \text{ N}\cdot\text{m}$$

Maximum shearing stress in AB

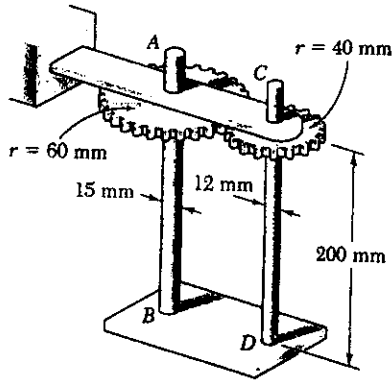
$$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(54.84)(0.015)}{79.52 \times 10^{-9}} = 10.34 \times 10^6 \text{ Pa} = 10.34 \text{ MPa} \leftarrow$$

Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(369.35 + 75.81)(0.018)}{164.896 \times 10^{-9}} = 48.6 \times 10^6 \text{ Pa} = 48.6 \text{ MPa} \leftarrow$$



PROBLEM 3.59



3.59 At a time when rotation is prevented at the lower end of each shaft, a 50-N·m torque is applied to end A of shaft AB. Knowing that  $G = 77 \text{ GPa}$  for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.

SOLUTION

Let  $T_A =$  torque applied at A = 50 N·m

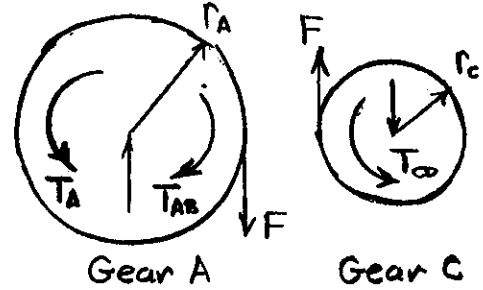
$T_{AB} =$  torque in shaft AB

$T_{CD} =$  torque in shaft CD

Statics

$$T_A - T_{AB} - Fr_A = 0$$

$$T_{CD} - Fr_c = 0$$



$$T_{CD} = \frac{r_c}{r_A} (T_A - T_{AB}) = \frac{2}{3} (T_A - T_{AB})$$

Kinematics:  $r_A \phi_A = r_c \phi_c$        $\phi_A = \frac{r_c}{r_A} \phi_c = \frac{2}{3} \phi_c$

Angles of twist       $\phi_A = \frac{T_{AB} L}{G J_{AB}}$        $\phi_c = \frac{T_{CD} L}{G J_{CD}} = \frac{2}{3} \frac{(T_A - T_{AB}) L}{G J_{CD}}$

$$\frac{T_{AB} L}{G J_{AB}} = \frac{2}{3} \cdot \frac{2}{3} \frac{(T_A - T_{AB}) L}{G J_{CD}}$$

$$\left( \frac{4}{9} + \frac{J_{CD}}{J_{AB}} \right) T_{AB} = \left( \frac{4}{9} + \left( \frac{12}{15} \right)^4 \right) T_{AB} = \frac{4}{9} T_A$$

$$T_{AB} = 0.5204 T_A = (0.5204)(50) = 26.02 \text{ N}\cdot\text{m}$$

$$T_{CD} = \frac{2}{3} (50 - 26.02) = 15.99 \text{ N}\cdot\text{m}$$

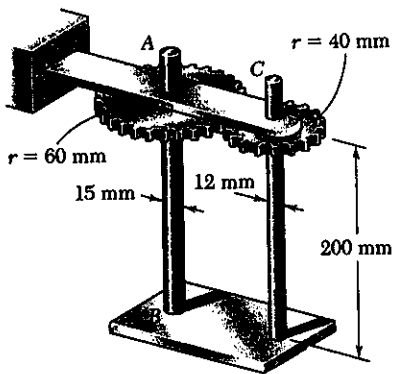
(a) Maximum shearing stress in shaft CD

$$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{2 T_{CD}}{\pi c^3} = \frac{(2)(15.99)}{\pi (0.006)^3} = 47.1 \times 10^6 \text{ Pa} = 47.1 \text{ MPa} \blacktriangleleft$$

(b) Angle of rotation at A

$$\phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{2 T_{AB} L}{\pi G C_{AB}^4} = \frac{(2)(26.02)(0.200)}{\pi (77 \times 10^9)(0.0075)^4} = 13.598 \times 10^{-3} \text{ rad} \\ = 0.779^\circ \blacktriangleleft$$

PROBLEM 3.60



3.59 At a time when rotation is prevented at the lower end of each shaft, a 50-N·m torque is applied to end A of shaft AB. Knowing that  $G = 77$  GPa for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.  
 3.60 Solve Prob. 3.59, assuming that the 50-N·m torque is applied to end C of shaft CD.

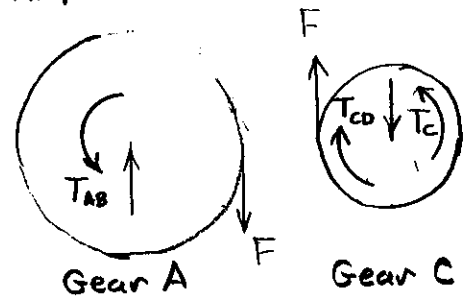
SOLUTION

Let  $T_c$  = torque applied at C = 50 N·m  
 $T_{CD}$  = torque in shaft CD  
 $T_{AB}$  = torque in shaft AB

Statics.

$$T_{AB} - r_A F = 0$$

$$T_c - T_{CD} - r_c F = 0$$



$$T_{AB} = \frac{r_A}{r_c} (T_c - T_{CD}) = \frac{3}{2} (T_c - T_{CD})$$

Kinematics:  $r_A \phi_A = r_c \phi_c$        $\phi_c = \frac{r_A}{r_c} \phi_A = \frac{3}{2} \phi_A$

Angles of twist       $\phi_c = \frac{T_{CD} L}{G J_{CD}}$        $\phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{3}{2} \frac{(T_c - T_{CD}) L}{G J_{AB}}$

$$\frac{T_{CD} L}{G J_{CD}} = \frac{3}{2} \cdot \frac{3}{2} \frac{T_c - T_{CD}}{G J_{AB}}$$

$$\left( \frac{J_{AB}}{J_{CD}} + \frac{9}{4} \right) T_{CD} = \left( \left( \frac{15}{12} \right)^4 + \frac{9}{4} \right) T_{CD} = \frac{9}{4} T_c$$

$$T_{CD} = 0.4796 T_c = (0.4796)(50) = 23.98 \text{ N}\cdot\text{m}$$

$$T_{AB} = \frac{3}{2} (50 - 23.98) = 39.03 \text{ N}\cdot\text{m}$$

(a) Maximum shearing stress in shaft CD

$$\tau_{CD} = \frac{T_{CD} c}{J} = \frac{2 T_{CD}}{\pi c^3} = \frac{(2)(23.98)}{\pi (0.006)^3} = 70.7 \times 10^6 \text{ Pa} = 70.7 \text{ MPa} \blacktriangleleft$$

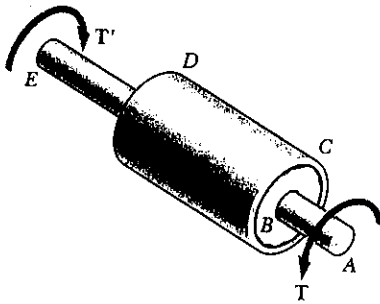
(b) Angle of rotation at A

$$\phi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{2 T_{AB} L}{\pi G c_{AB}^4} = \frac{(2)(39.03)(0.200)}{\pi (77 \times 10^9)(0.0075)^4} = 20.379 \times 10^{-3} \text{ rad} \\ = 1.169^\circ \blacktriangleleft$$



PROBLEM 3.61

3.61 The steel jacket  $CD$  has been attached to the 40-mm-diameter steel shaft  $AE$  by means of rigid flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500-N·m torques are applied as shown, determine the maximum shearing stress in the jacket.



SOLUTION

Solid shaft:  $c = \frac{1}{2}d = 0.020 \text{ m}$

$$J_s = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket:  $c_2 = \frac{1}{2}d = 0.040 \text{ m}$

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.040^4 - 0.036^4) = 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft  $T_s = GJ_s \phi / L$

Torque carried by jacket  $T_J = GJ_J \phi / L$

Total torque  $T = T_s + T_J = (J_s + J_J)G \phi / L \therefore \frac{G\phi}{L} = \frac{T}{J_s + J_J}$

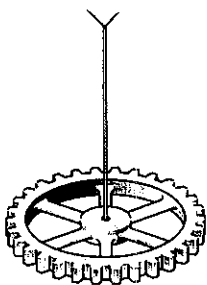
$$T_J = \frac{J_J}{J_s + J_J} T = \frac{(1.3829 \times 10^{-6})(500)}{1.3829 \times 10^{-6} + 251.33 \times 10^{-6}} = 423.1 \text{ N}\cdot\text{m}$$

Maximum shearing stress in jacket

$$\tau = \frac{T_J c}{J_J} = \frac{(423.1)(0.040)}{1.3829 \times 10^{-6}} = 12.24 \times 10^6 \text{ Pa} \quad 12.24 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.62

3.62 The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 6-ft steel wire. Knowing that  $G = 11.2 \times 10^6$  psi, determine the diameter of the wire for which the torsional spring constant will be 4.27 lb·ft/rad.



SOLUTION

Torsion spring constant  $K = 4.27 \text{ lb}\cdot\text{ft}/\text{rad} = 51.24 \text{ lb}\cdot\text{in}/\text{rad}$

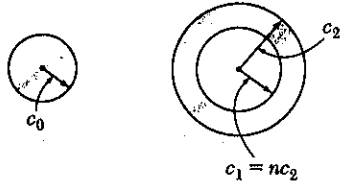
$$K = \frac{I}{\phi} = \frac{I}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(72)(51.24)}{\pi(11.2 \times 10^6)} = 209.7 \times 10^{-6} \text{ in}^4$$

$$c = 0.1203 \text{ in.}$$

$$d = 2c = 0.241 \text{ in.} \blacktriangleleft$$

**PROBLEM 3.63**



3.63 A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by  $n$  the ratio  $c_1/c_2$ , show that the ratio  $T_s/T_h$  of the torque  $T_s$  in the solid shaft to the torque  $T_h$  in the hollow shaft is (a)  $\sqrt{1-n^2}/(1+n^2)$  if the maximum shearing stress is the same in each shaft, (b)  $(1-n^2)/(1+n^2)$  if the angle of twist is the same for each shaft.

**SOLUTION**

For equal weight and length, the areas are equal

$$\pi c_0^2 = \pi(c_2^2 - c_1^2) = \pi c_2^2(1-n^2) \therefore c_0 = c_2 \sqrt{1-n^2}$$

$$J_s = \frac{\pi}{2} c_0^4 = \frac{\pi}{2} c_2^4 (1-n^2)^2 \quad J_h = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} c_2^4 (1-n^4)$$

(a) For equal stresses

$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2} c_2^4 (1-n^2)^2 c_2}{\frac{\pi}{2} c_2^4 (1-n^4) c_2 \sqrt{1-n^2}} = \frac{1-n^2}{(1+n^2)\sqrt{1-n^2}} = \frac{\sqrt{1-n^2}}{1+n^2}$$

(b) For equal angles of twist

$$\phi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2} c_2^4 (1-n^2)^2}{\frac{\pi}{2} c_2^4 (1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2}$$

**PROBLEM 3.64**

3.64 A torque  $T$  is applied as shown to a solid tapered shaft  $AB$ . Show by integration that the angle of twist at  $A$  is

$$\phi = \frac{7TL}{12\pi Gc^4}$$

**SOLUTION**

Introduce coordinate  $y$  as shown.

$$r = \frac{cy}{L}$$

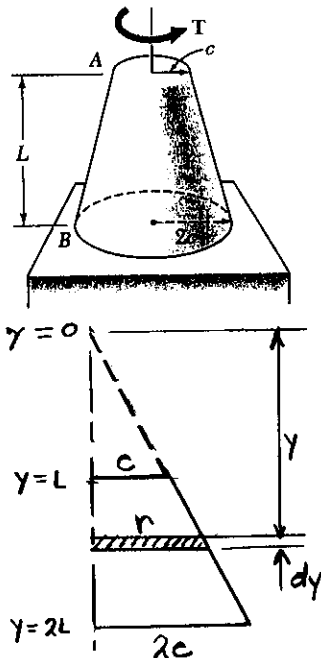
Twist in length  $dy$

$$d\phi = \frac{T dy}{GJ} = \frac{T dy}{G \frac{\pi}{2} r^4} = \frac{2TL^4 dy}{\pi Gc^4 y^4}$$

$$\phi = \int_L^{2L} \frac{2TL^4}{\pi Gc^4} \frac{dy}{y^4} = \frac{2TL^4}{\pi Gc^4} \int_L^{2L} \frac{dy}{y^4}$$

$$= \frac{2TL^4}{\pi Gc^4} \left\{ -\frac{1}{3y^3} \right\}_L^{2L} = \frac{2TL^4}{\pi Gc^4} \left\{ -\frac{1}{24L^3} + \frac{1}{3L^3} \right\}$$

$$= \frac{2TL^4}{\pi Gc^4} \left\{ \frac{7}{24L^3} \right\} = \frac{7TL}{12\pi Gc^4}$$



PROBLEM 3.65

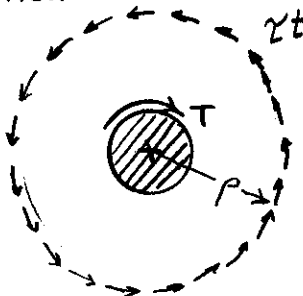
3.65 An annular plate of thickness  $t$  and modulus of rigidity  $G$  is used to connect shaft  $AB$  of radius  $r_1$  to tube  $CD$  of inner radius  $r_2$ . Knowing that a torque  $T$  is applied to end  $A$  of shaft  $AB$  and that end  $D$  of tube  $CD$  is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end  $B$  of the shaft rotates with respect to end  $C$  of the tube is

SOLUTION

Use a free body consisting of shaft  $AB$  and an inner portion of the plate  $BC$ , the outer radius of this portion being  $\rho$

$$\phi_{B/C} = \frac{T}{4\pi Gt} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right)$$

The force per unit length of circumference is  $\tau t$ .



$$\Sigma M = 0$$

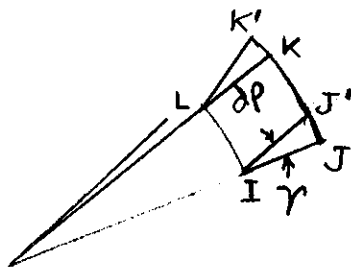
$$\tau t (2\pi \rho) \rho - T = 0$$

$$\tau = \frac{T}{2\pi t \rho^2}$$

(a) Maximum shearing stress occurs at  $\rho = r_1$

$$\tau_{max} = \frac{T}{2\pi t r_1^2} \quad (1)$$

Shearing strain  $\gamma = \frac{\tau}{G} = \frac{T}{2\pi G t \rho^2}$



The relative circumferential displacement in radial length  $dp$  is

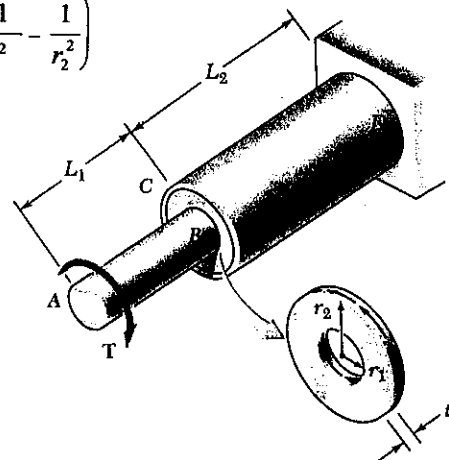
$$ds = \gamma dp = \rho d\phi$$

$$d\phi = \gamma \frac{dp}{\rho}$$

$$d\phi = \frac{T}{2\pi G t \rho^2} \frac{dp}{\rho} = \frac{T}{2\pi G t} \frac{dp}{\rho^3}$$

$$\phi_{B/C} = \int_{r_1}^{r_2} \frac{T}{2\pi G t} \frac{dp}{\rho^3} = \frac{T}{2\pi G t} \int_{r_1}^{r_2} \frac{dp}{\rho^3} = \frac{T}{2\pi G t} \left\{ -\frac{1}{2\rho^2} \right\} \Big|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi G t} \left\{ -\frac{1}{2r_2^2} + \frac{1}{2r_1^2} \right\} = \frac{T}{4\pi G t} \left\{ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right\}$$



PROBLEM 3.66

3.66 An annular aluminum plate ( $G = 27 \text{ GPa}$ ), of thickness  $t = 6 \text{ mm}$ , is used to connect the aluminum shaft  $AB$ , of length  $L_1 = 90 \text{ mm}$  and radius  $r_1 = 30 \text{ mm}$ , to the aluminum tube  $CD$ , of length  $L_2 = 150 \text{ mm}$ , inner radius  $r_2 = 75 \text{ mm}$  and 4 mm thickness. Knowing that a 2500-N·m torque  $T$  is applied to end  $A$  of shaft  $AB$  and that end  $D$  of tube  $CD$  is fixed, determine (a) the maximum shearing stress in the shaft-plate-tube system, (b) the angle through which end  $A$  rotates. (Hint: Use the formula derived in Prob. 3.65 to solve part b.)

SOLUTION

Shaft  $AB$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2T}{\pi r_1^3}$$

$$= \frac{(2)(2500)}{\pi(0.030)^3} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa}$$

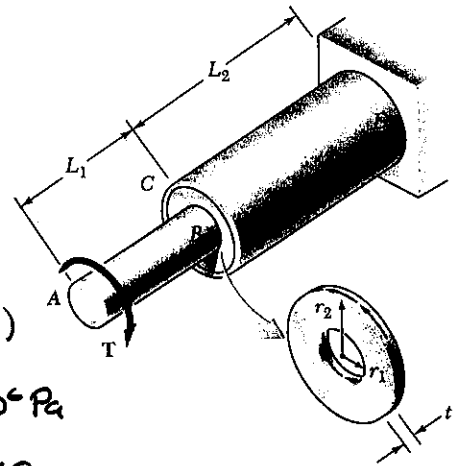


Plate  $BC$  (See PROBLEM 3.65 for derivation)

$$\tau = \frac{T}{2\pi t r_1^2} = \frac{2500}{2\pi(0.006)(0.030)^2} = 73.7 \times 10^6 \text{ Pa} = 73.7 \text{ MPa}$$

Shaft  $CD$   $c_1 = r_2 = 0.075 \text{ m}$ ,  $c_2 = r_2 + t = 0.075 + 0.004 = 0.079 \text{ m}$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.079^4 - 0.075^4) = 11.482 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc_2}{J} = \frac{(2500)(0.079)}{11.482 \times 10^{-6}} = 17.20 \times 10^6 \text{ Pa} = 17.20 \text{ MPa}$$

(a) Largest stress

$$\tau = 73.7 \text{ MPa}$$

Shaft  $AB$   $\phi_{AB} = \frac{T L_{AB}}{GJ} = \frac{2TL_{AB}}{\pi G c^4} = \frac{(2)(2500)(0.090)}{\pi(27 \times 10^9)(0.030)^4} = 6.550 \times 10^{-3} \text{ rad}$

Plate  $BC$  (See PROBLEM 3.65 for derivation)

$$\phi_{BC} = \frac{T}{4\pi G t} \left\{ \frac{1}{r_1^2} - \frac{1}{r_2^2} \right\} = \frac{2500}{4\pi(27 \times 10^9)(0.006)} \left\{ \frac{1}{0.030^2} - \frac{1}{0.075^2} \right\} = 1.146 \times 10^{-3} \text{ rad}$$

Shaft  $CD$   $\phi_{CD} = \frac{T L_{CD}}{GJ} = \frac{(2500)(0.150)}{(27 \times 10^9)(11.482 \times 10^{-6})} = 1.210 \times 10^{-3} \text{ rad}$

Total rotation angle  $\phi = \phi_{AB} + \phi_{BC} + \phi_{CD} = 8.91 \times 10^{-3} \text{ rad} = 0.510^\circ$

**PROBLEM 3.67**

3.67 Using an allowable stress of 55 MPa, design a solid steel shaft to transmit 10 kW at a frequency of 15 Hz.

**SOLUTION**

$$\tau_{all} = 55 \times 10^6 \text{ Pa} \quad P = 10 \times 10^3 \text{ W}, \quad f = 15 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{2\pi(15)} = 106.10 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \therefore c^3 = \frac{2T}{\pi \tau} = \frac{(2)(106.10)}{\pi(55 \times 10^6)} = 1.2281 \times 10^{-6} \text{ m}^3$$

$$c = 10.71 \times 10^{-3} \text{ m} = 10.71 \text{ mm} \quad d = 2c = 21.4 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 3.68**

3.68 Using an allowable stress of 5 ksi, design a solid steel shaft to transmit  $\frac{1}{2}$  hp at a speed of 1725 rpm.

**SOLUTION**

$$\tau_{all} = 5 \text{ ksi} = 5000 \text{ psi} \quad P = \frac{1}{2} \text{ hp} = \frac{1}{2}(6600) = 3300 \text{ lb}\cdot\text{in}/\text{s}$$

$$f = \frac{1725}{60} = 28.75 \text{ Hz} \quad T = \frac{P}{2\pi f} = \frac{3300}{2\pi(28.75)} = 18.268 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \therefore c^3 = \frac{2T}{\pi \tau} = \frac{(2)(18.268)}{\pi(5000)} = 2.3260 \times 10^{-3} \text{ in}^3$$

$$c = 0.1325 \text{ in} \quad d = 2c = 0.265 \text{ in} \quad \blacktriangleleft$$

**PROBLEM 3.69**

3.69 Design a solid steel shaft to transmit 100 hp at a speed of 1200 rpm, if the maximum shearing stress is not to exceed 7500 psi.

**SOLUTION**

$$\tau_{all} = 7500 \text{ psi} \quad P = 100 \text{ hp} = 660 \times 10^3 \text{ lb}\cdot\text{in}/\text{s}$$

$$f = \frac{1200}{60} = 20 \text{ Hz} \quad T = \frac{P}{2\pi f} = \frac{660 \times 10^3}{2\pi(20)} = 5.2521 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \therefore c^3 = \frac{2T}{\pi \tau} = \frac{(2)(5.2521 \times 10^3)}{\pi(7500)} = 0.4458 \text{ in}^3$$

$$c = 0.7639 \text{ in} \quad d = 2c = 1.528 \text{ in.} \quad \blacktriangleleft$$

**PROBLEM 3.70**

3.70 Design a solid steel shaft to transmit 0.375 kW at a frequency of 29 Hz, if the shearing stress in the shaft is not to exceed 35 MPa

**SOLUTION**

$$\tau_{all} = 35 \times 10^6 \text{ Pa} \quad P = 0.375 \times 10^3 \text{ W} \quad f = 29 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{0.375 \times 10^3}{2\pi(29)} = 2.0580 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \therefore c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2.0580)}{\pi(35 \times 10^6)} = 37.43 \times 10^{-9} \text{ m}^3$$

$$c = 3.345 \times 10^{-3} \text{ m} = 3.345 \text{ mm} \quad d = 2c = 6.69 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 3.71**

3.71 A hollow shaft is to transmit 250 kW at a frequency of 30 Hz. Knowing that the shearing stress must not exceed 50 MPa, design a shaft for which the ratio of the inner diameter to the outer diameter is 0.75.

**SOLUTION**

$$\tau_{all} = 50 \times 10^6 \text{ Pa} \quad P = 250 \times 10^3 \text{ W} \quad f = 30 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{250 \times 10^3}{2\pi(30)} = 1326.3 \text{ N}\cdot\text{m}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} \left(1 - \left(\frac{3}{4}\right)^4\right) C_2^4 = 1.0738 C_2^4$$

$$\gamma = \frac{T C_2}{J} = \frac{T C_2}{1.0738 C_2^4} \quad \therefore C_2^3 = \frac{T}{1.0738 \gamma} = \frac{1326.3}{(1.0738)(50 \times 10^6)}$$

$$C_2^3 = 24.70 \times 10^{-6} \text{ m}^3$$

$$C_2 = 29.12 \times 10^{-3} \text{ m} = 29.12 \text{ mm} \quad d_2 = 2C_2 = 58.2 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 3.72**

3.72 One of two hollow drive shafts of an ocean liner is 125 ft long, and its outer and inner diameters are 16 in. and 8 in., respectively. The shaft is made of a steel for which  $\tau_{all} = 8500$  psi and  $G = 11.2 \times 10^6$  psi. Knowing that the maximum speed of rotation of the shaft is 165 rpm, determine (a) the maximum power that can be transmitted by the one shaft to its propeller, (b) the corresponding angle of twist of the shaft.

**SOLUTION**

$$C_2 = \frac{1}{2} d_2 = 8 \text{ in}$$

$$C_1 = \frac{1}{2} d_1 = 4 \text{ in}$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4) = \frac{\pi}{2} (8^4 - 4^4) = 6031.8 \text{ in}^4$$

$$\gamma = \frac{T C_2}{J} \quad \therefore T = \frac{J \gamma}{C_2} = \frac{(6031.8)(8500)}{8} = 6.4088 \times 10^6 \text{ lb}\cdot\text{in}$$

$$f = \frac{165}{60} = 2.75 \text{ Hz}$$

$$(a) P = 2\pi f T = 2\pi(2.75)(6.4088 \times 10^6) = 110.74 \times 10^6 \text{ lb}\cdot\text{in/s}$$

$$\frac{110.74 \times 10^6 \text{ lb}\cdot\text{in/s}}{6600 \text{ lb}\cdot\text{in/s hp}} = 16.78 \times 10^3 \text{ hp.} \quad \blacktriangleleft$$

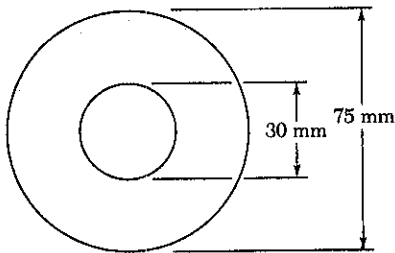
$$L = 125 \text{ ft} = 1500 \text{ in}$$

$$(b) \phi = \frac{TL}{GJ} = \frac{(6.4088 \times 10^6)(1500)}{(11.2 \times 10^6)(6031.8)} = 0.1423 \text{ rad}$$

$$= 8.15^\circ \quad \blacktriangleleft$$

PROBLEM 3.73

3.73 While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is  $2^\circ$  in a 4-m length. Using  $G = 77$  GPa, determine the power being transmitted.



SOLUTION

$$\begin{aligned} \text{Twist angle } \phi &= 2^\circ = 34.907 \times 10^{-3} \text{ rad} \\ c_1 &= \frac{1}{2} d_1 = 0.015 \text{ m}, \quad c_2 = \frac{1}{2} d_2 = 0.0375 \text{ m} \\ J &= \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} (0.0375^4 - 0.015^4) \\ J &= 3.0268 \times 10^{-6} \text{ m}^4, \quad L = 4 \text{ m} \end{aligned}$$

$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ\phi}{L} = \frac{(77 \times 10^9)(3.0268 \times 10^{-6})(34.907 \times 10^{-3})}{4}$$

$$T = 2.0339 \times 10^3 \text{ N}\cdot\text{m} \quad f = 120 \text{ rpm} = \frac{120}{60} \text{ Hz} = 2 \text{ Hz}$$

$$P = (2\pi f)T = 2\pi(2)(2.0339 \times 10^3) = 25.6 \times 10^3 \text{ W} = 25.6 \text{ kW} \quad \blacktriangleleft$$

PROBLEM 3.74

3.74 Determine the required thickness of the 50-mm tubular shaft of Example 3.07, if it is to transmit the same power while rotating at a frequency of 40 Hz.

SOLUTION

$$\text{From Example 3.07} \quad P = 100 \text{ kW} = 100 \times 10^3 \text{ W}$$

$$\tau_{\text{all}} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

$$c_2 = \frac{1}{2} d = 0.025 \text{ m}$$

$$\text{Given } f = 40 \text{ Hz}$$

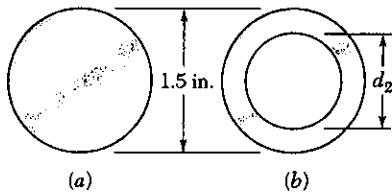
$$T = \frac{P}{2\pi f} = \frac{100 \times 10^3}{2\pi(40)} = 397.89 \text{ N}\cdot\text{m}$$

$$J = \frac{\pi}{2} (c_2^4 - c_1^4) \quad \tau = \frac{TC_2}{J} = \frac{TC_2}{\frac{\pi}{2}(c_2^4 - c_1^4)} \quad \therefore c_1^4 = c_2^4 - \frac{2TC_2}{\pi\tau}$$

$$c_1^4 = c_2^4 - \frac{2TC_2}{\pi\tau} = 0.025^4 - \frac{(2)(397.89)(0.025)}{\pi(60 \times 10^6)} = 285.081 \times 10^{-9} \text{ m}^4$$

$$c_1 = 23.11 \times 10^{-3} \text{ m} \quad t = c_2 - c_1 = 1.89 \times 10^{-3} \text{ m} = 1.89 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 3.75**



3.75 The design of a machine element calls for a 1.5-in.-outer-diameter shaft to transmit 60 hp. (a) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft a. (b) If the speed of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft b for which the maximum shearing stress will be the same in each shaft.

**SOLUTION**

$$P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in}/\text{s}$$

$$f = \frac{720}{60} = 12 \text{ Hz} \quad c = \frac{1}{2}d = 0.75 \text{ in}$$

$$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(12)} = 5.2521 \times 10^3 \text{ lb}\cdot\text{in}$$

$$(a) \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(5.2521 \times 10^3)}{\pi(0.75)^3} = 7.9256 \times 10^3 \text{ psi}$$

7.93 ksi

$$(b) f = \frac{1080}{60} = 18 \text{ Hz} \quad c_2 = \frac{1}{2}d_2 = 0.75 \text{ in}$$

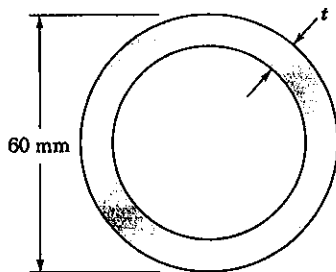
$$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(18)} = 3.5014 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.75^4 - \frac{(2)(3.5014 \times 10^3)(0.75)}{\pi(7.9256 \times 10^3)} = 0.10547 \text{ in}^4$$

$$c_1 = 0.5699 \text{ in} \quad d_1 = 2c_1 = 1.140 \text{ in}$$

**PROBLEM 3.76**



3.76 A steel pipe of 60-mm outer diameter is to be used to transmit a torque of 350 N·m without exceeding an allowable shearing stress of 12 MPa. A series of 60-mm-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 4 mm to 10 mm in 2-mm increments, choose the lightest pipe that can be used.

**SOLUTION**

$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa} \quad c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}$$

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.030^4 - \frac{(2)(350)(0.030)}{\pi(60 \times 10^6)} = 252.96 \times 10^{-9} \text{ m}^4$$

$$c_1 = 22.43 \times 10^{-3} \quad t = c_2 - c_1 = 30 \text{ mm} - 22.43 \text{ mm} = 7.57 \text{ mm}$$

Required thickness  $t > 7.59 \text{ mm}$ . Available size  $t = 8 \text{ mm}$



## PROBLEM 3.77

3.77 A steel drive shaft is 6 ft long and its outer and inner diameters are respectively equal to 2.25 in. and 1.75 in. (a) Knowing that the shaft transmits 240 hp while rotating at 1800 rpm, determine the maximum shearing stress. (b) Using  $G = 11.2 \times 10^6$  psi, determine the corresponding angle of twist of the shaft.

## SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.875 \text{ in}, \quad c_2 = \frac{1}{2}d_2 = 1.125 \text{ in}, \quad L = 6\text{ft} = 72 \text{ in}$$

$$P = 240 \text{ hp} = (240)(6600) = 1.584 \times 10^6 \text{ lb}\cdot\text{in}/\text{s}$$

$$f = \frac{1800}{60} = 30 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{1.584 \times 10^6}{2\pi(30)} = 8.4034 \times 10^3 \text{ lb}\cdot\text{in.}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.125^4 - 0.875^4) = 1.59530 \text{ in}^4$$

$$(a) \quad \tau = \frac{Tc_2}{J} = \frac{(8.4034 \times 10^3)(1.125)}{1.59530} = 5926 \text{ psi} \quad \blacktriangleleft$$

$$(b) \quad \phi = \frac{TL}{GJ} = \frac{(8.4034 \times 10^3)(72)}{(11.2 \times 10^6)(1.59530)} = 33.86 \times 10^{-3} \text{ rad} = 1.940^\circ \quad \blacktriangleleft$$

## PROBLEM 3.78

3.78 Knowing that the allowable shearing stress of the steel to be used is 7500 psi, determine (a) the smallest permissible diameter of a shaft which must transmit 15 hp while rotating at 2000 rpm, (b) the corresponding angle of twist in a 4-ft length of the shaft ( $G = 11.2 \times 10^6$  psi).

## SOLUTION

$$\tau_{\text{all}} = 7500 \text{ psi}, \quad f = \frac{2000}{60} = 33.333 \text{ Hz}$$

$$P = 15 \text{ hp} = (15)(6600) = 99 \times 10^3 \text{ lb}\cdot\text{in}/\text{s}$$

$$T = \frac{P}{2\pi f} = \frac{99 \times 10^3}{2\pi(33.333)} = 472.69 \text{ lb}\cdot\text{in}$$

$$(a) \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore c^3 = \frac{2T}{\pi \tau} = \frac{(2)(472.69)}{\pi(7500)} = 40.123 \times 10^{-3} \text{ in}^3$$

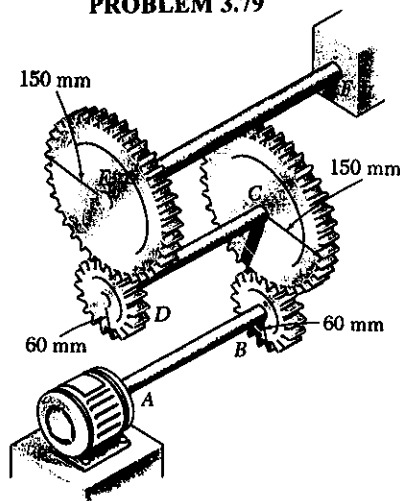
$$c = 0.3425 \text{ in} \quad d = 2c = 0.685 \text{ in.} \quad \blacktriangleleft$$

$$(b) \quad J = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.3425)^4 = 21.516 \times 10^{-3} \text{ in}^4$$

$$L = 4\text{ft} = 48 \text{ in}$$

$$\phi = \frac{TL}{GJ} = \frac{(472.69)(48)}{(11.2 \times 10^6)(21.516 \times 10^{-3})} = 93.89 \times 10^{-3} \text{ rad} \\ = 5.38^\circ \quad \blacktriangleleft$$

PROBLEM 3.79



3.79 Three shafts and four gears are used to form a gear train which will transmit 7.5 kW from the motor at A to a machine tool at F. (Bearing for the shafts are omitted in the sketch.) Knowing that the frequency of the motor is 30 Hz and that the allowable stress for each shaft is 60 MPa, determine the required diameter of each shaft.

SOLUTION

$$P = 7.5 \text{ kW} = 7.5 \times 10^3 \text{ W}$$

$$\tau_{all} = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$$

Shaft AB:  $f_{AB} = 30 \text{ Hz}$

$$T_{AB} = \frac{P}{2\pi f_{AB}} = \frac{7.5 \times 10^3}{2\pi(30)} = 39.789 \text{ N}\cdot\text{m}$$

$$\tau = \frac{T C_{AB}}{J_{AB}} = \frac{2T}{\pi C_{AB}^3} \therefore C_{AB}^3 = \frac{2T}{\pi \tau}$$

$$C_{AB}^3 = \frac{(2)(39.789)}{\pi(60 \times 10^6)} = 422.17 \times 10^{-9} \text{ m}^3$$

$$C_{AB} = 7.50 \times 10^{-3} \text{ m} = 7.50 \text{ mm}$$

$$d_{AB} = 2C_{AB} = 15.00 \text{ mm}$$

Shaft CD:

$$f_{CD} = \frac{r_D}{r_C} f_{AB} = \frac{60}{150} (30) = 12 \text{ Hz}$$

$$T_{CD} = \frac{P}{2\pi f_{CD}} = \frac{7.5 \times 10^3}{2\pi(12)} = 99.472 \text{ N}\cdot\text{m}$$

$$\tau = \frac{T C_{CD}}{J_{CD}} = \frac{2T}{\pi C_{CD}^3} \therefore C_{CD}^3 = \frac{2T_{CD}}{\pi \tau} = \frac{2(99.472)}{\pi(60 \times 10^6)} = 1.05543 \times 10^{-6} \text{ m}^3$$

$$C_{CD} = 10.18 \times 10^{-3} \text{ m} = 10.18 \text{ mm}$$

$$d_{CD} = 2C_{CD} = 20.4 \text{ mm}$$

Shaft EF:

$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150} (12) = 4.8 \text{ Hz}$$

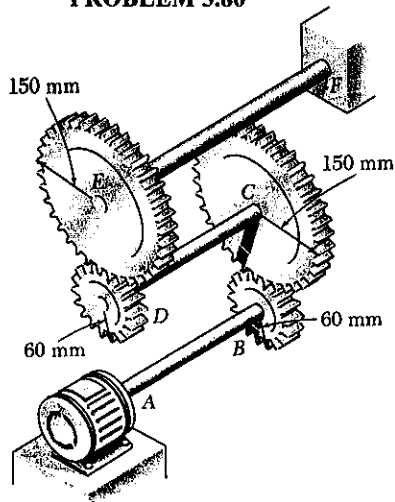
$$T_{EF} = \frac{P}{2\pi f_{EF}} = \frac{7.5 \times 10^3}{2\pi(4.8)} = 248.68 \text{ N}\cdot\text{m}$$

$$\tau = \frac{T C_{EF}}{J_{EF}} = \frac{2T}{\pi C_{EF}^3} \therefore C_{EF}^3 = \frac{(2)(248.68)}{\pi(60 \times 10^6)} = 2.6886 \times 10^{-6} \text{ m}^3$$

$$C_{EF} = 13.82 \times 10^{-3} = 13.82 \text{ mm}$$

$$d_{EF} = 2C_{EF} = 27.6 \text{ mm}$$

**PROBLEM 3.80**



3.80 Three shafts and four gears are used to form a gear train which will transmit power from the motor at *A* to a machine tool at *F*. (Bearing for the shafts are omitted in the sketch.) The diameter of each shaft is as follows:  $d_{AB} = 16$  mm,  $d_{CD} = 20$  mm,  $d_{EF} = 28$  mm. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

**SOLUTION**

$$\tau_{all} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$$

Shaft AB:  $C_{AB} = \frac{1}{2} d_{AB} = 0.008 \text{ m}$

$$\tau = \frac{T C_{AB}}{J_{AB}} = \frac{2T}{\pi C_{AB}^3}$$

$$T_{all} = \frac{\pi}{2} C_{AB}^3 \tau_{all} = \frac{\pi}{2} (0.008)^3 (75 \times 10^6) = 60.319 \text{ N}\cdot\text{m}$$

$$f_{AB} = 24 \text{ Hz}$$

$$P_{all} = 2\pi f_{AB} T_{all} = 2\pi (24)(60.319)$$

$$= 9.10 \times 10^3 \text{ W}$$

Shaft CD:  $C_{CD} = \frac{1}{2} d_{CD} = 0.010 \text{ m}$

$$\tau = \frac{T C_{CD}}{J_{CD}} = \frac{2T}{\pi C_{CD}^3} \quad \therefore \quad T_{all} = \frac{\pi}{2} C_{CD}^3 \tau_{all} = \frac{\pi}{2} (0.010)^3 (75 \times 10^6) = 117.81 \text{ N}\cdot\text{m}$$

$$f_{CD} = \frac{r_B}{r_C} f_{AB} = \frac{60}{150} (24) = 9.6 \text{ Hz}$$

$$P_{all} = 2\pi f_{CD} T_{all} = 2\pi (9.6)(117.81) = 7.11 \times 10^3 \text{ W}$$

Shaft EF:  $C_{EF} = \frac{1}{2} d_{EF} = 0.014 \text{ m}$

$$T_{all} = \frac{\pi}{2} C_{EF}^3 \tau_{all} = \frac{\pi}{2} (0.014)^3 (75 \times 10^6) = 323.27 \text{ N}\cdot\text{m}$$

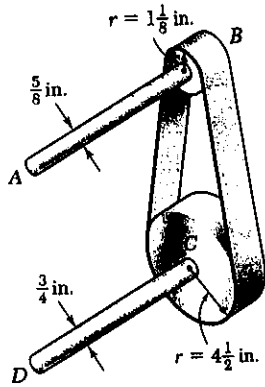
$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150} (9.6) = 3.84 \text{ Hz}$$

$$P_{all} = 2\pi f_{EF} T_{all} = 2\pi (3.84)(323.27) = 7.80 \times 10^3 \text{ W}$$

Maximum allowable power is the smaller

$$P_{all} = 7.11 \times 10^3 \text{ W} = 7.11 \text{ kW}$$

PROBLEM 3.81



3.81 The shaft-disk-belt arrangement shown is used to transmit 3 hp from point A to point D. (a) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft AB. (b) Solve part a, assuming that the diameters of shafts AB and CD are respectively 0.75 in. and 0.625 in.

SOLUTION

$$\tau = 9500 \text{ psi}, \quad P = 3 \text{ hp} = (3)(6600) = 19800 \text{ lb}\cdot\text{in}/\text{s}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad T = \frac{\pi}{2} c^3 \tau$$

Allowable torques

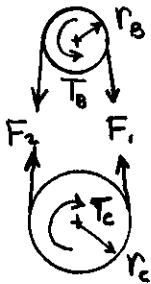
$\frac{5}{8}$  in. diameter shaft

$$c = \frac{5}{16} \text{ in}, \quad T_{all} = \frac{\pi}{2} \left(\frac{5}{16}\right)^3 (9500) = 455.4 \text{ lb}\cdot\text{in}$$

$\frac{3}{4}$  in diameter shaft

$$c = \frac{3}{8} \text{ in}, \quad T_{all} = \frac{\pi}{2} \left(\frac{3}{8}\right)^3 (9500) = 786.9 \text{ lb}\cdot\text{in}$$

Statics:



$$T_B = r_B (F_1 - F_2) \quad T_C = r_C (F_1 - F_2)$$

$$T_B = \frac{r_B}{r_C} T_C = \frac{1.125}{4.5} T_C = 0.25 T_C$$

(a) Allowable torques  $T_{B,all} = 455.4 \text{ lb}\cdot\text{in}, \quad T_{C,all} = 786.9 \text{ lb}\cdot\text{in}$

Assume  $T_C = 786.9 \text{ lb}\cdot\text{in}$ . Then  $T_B = (0.25)(786.9) = 196.73 \text{ lb}\cdot\text{in}$   
 $< 455.4 \text{ lb}\cdot\text{in}$  (okay)

$$P = 2\pi f T \quad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(196.73)} = 16.02 \text{ Hz}$$

(b) Allowable torques  $T_{B,all} = 786.9 \text{ lb}\cdot\text{in}, \quad T_{C,all} = 455.4 \text{ lb}\cdot\text{in}$

Assume  $T_C = 455.4 \text{ lb}\cdot\text{in}$ . Then  $T_B = (0.25)(455.4) = 113.85 \text{ lb}\cdot\text{in}$   
 $< 786.9 \text{ lb}\cdot\text{in}$

$$P = 2\pi f T \quad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(113.85)} = 27.7 \text{ Hz}$$

## PROBLEM 3.82

3.82 A steel shaft must transmit 150 kW at a speed of 360 rpm. Knowing that  $G = 77$  GPa, design a solid shaft so that the maximum stress will not exceed 50 MPa and the angle of twist in a 2.5-m length will not exceed  $3^\circ$ .

## SOLUTION

$$P = 150 \times 10^3 \text{ W} \quad f = \frac{360}{60} = 6 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{150 \times 10^3}{2\pi(6)} = 3.9789 \times 10^3 \text{ N}\cdot\text{m}$$

Design for stress limit  $\tau = 50 \text{ MPa} = 50 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \therefore c^3 = \frac{2T}{\pi \tau} = \frac{(2)(3.9789 \times 10^3)}{\pi(50 \times 10^6)} = 50.661 \times 10^{-6} \text{ m}^3$$

$$c = 37.00 \times 10^{-3} \text{ m}$$

Design for angle of twist limit  $\phi = 3^\circ = 52.36 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ} = \frac{2TL}{\pi Gc^4} \therefore c^4 = \frac{2TL}{\pi G\phi} = \frac{(2)(3.9789 \times 10^3)(2.5)}{\pi(77 \times 10^9)(52.36 \times 10^{-3})} = 1.5707 \times 10^{-6} \text{ m}^4$$

$$c = 35.40 \times 10^{-3} \text{ m}$$

Use larger value  $c = 37.00 \times 10^{-3} \text{ m} = 37.0 \text{ mm}$ ;  $d = 2c = 74.0 \text{ mm}$  ◀

## PROBLEM 3.83

3.83 A steel shaft of 1.5-m length and 48-mm diameter is to be used to transmit 36 kW between a motor and a machine tool. Knowing that  $G = 77$  GPa, determine the lowest speed of rotation of the shaft at which the maximum stress will not exceed 60 MPa and the angle of twist will not exceed  $2.5^\circ$ .

## SOLUTION

$$P = 36 \times 10^3 \text{ W}, \quad c = \frac{1}{2}d = 0.024 \text{ m}, \quad L = 1.5 \text{ m}, \quad G = 77 \times 10^9 \text{ Pa}$$

Torque based on maximum stress  $\tau = 60 \text{ MPa} = 60 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc}{J} \quad T = \frac{J\tau}{c} = \frac{\pi}{2} c^3 \tau = \frac{\pi}{2} (0.024)^3 (60 \times 10^6) = 1.30288 \times 10^3 \text{ N}\cdot\text{m}$$

Torque based on twist angle  $\phi = 2.5^\circ = 43.633 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ} \therefore T = \frac{GJ\phi}{L} = \frac{\pi c^4 G \phi}{2L} = \frac{\pi (0.024)^4 (77 \times 10^9) (43.633 \times 10^{-3})}{(2)(1.5)}$$

$$= 1.16730 \times 10^3 \text{ N}\cdot\text{m}$$

Smaller torque governs  $T = 1.16730 \times 10^3 \text{ N}\cdot\text{m}$

$$P = 2\pi f T \therefore f = \frac{P}{2\pi T} = \frac{36 \times 10^3}{2\pi (1.16730 \times 10^3)} = 4.91 \text{ Hz} \quad \blacktriangleleft$$

**PROBLEM 3.84**

3.84 A 1.5-in.-diameter steel shaft of length 4 ft will be used to transmit 60 hp between a motor and a pump. Knowing that  $G = 11.2 \times 10^6$  psi, determine the lowest speed of rotation at which the shearing stress will not exceed 8500 psi and the angle of twist will not exceed  $2^\circ$ .

**SOLUTION**

$$c = \frac{1}{2}d = 0.75 \text{ in.}, \quad L = 4 \text{ ft} = 48 \text{ in.}$$

Torque based on maximum shearing stress limit  $\tau = 8500 \text{ psi}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore T = \frac{\pi c^3 \tau}{2} = \frac{\pi (0.75)^3 (8500)}{2} = 5.633 \times 10^3 \text{ lb}\cdot\text{in}$$

Torque based on twist angle limit  $\phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ} \quad \therefore T = \frac{GJ\phi}{L} = \frac{\pi c^4 G \phi}{2L} = \frac{\pi (0.75)^4 (11.2 \times 10^6) (34.907 \times 10^{-3})}{(2)(48)}$$

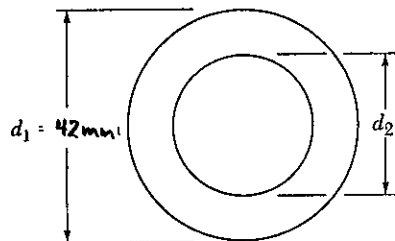
$$= 4.048 \times 10^3 \text{ lb}\cdot\text{in}$$

Smaller torque governs  $T = 4.048 \times 10^3 \text{ lb}\cdot\text{in}$

$$P = 2\pi f T \quad \text{where } P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in/s}$$

$$f = \frac{P}{2\pi T} = \frac{396 \times 10^3}{2\pi (4.048 \times 10^3)} = 15.57 \text{ Hz} = 934 \text{ rpm}$$

**PROBLEM 3.85**



3.85 A 1.6-m-long tubular shaft of 42-mm outer diameter  $d_1$ , having the cross section shown is to be made of a steel for which  $\tau_{\text{all}} = 75 \text{ MPa}$  and  $G = 77 \text{ GPa}$ . Knowing that the angle of twist of the shaft must not exceed  $4^\circ$  when the shaft is subjected to a torque of 900 N·m, determine the largest inner diameter  $d_2$  which can be specified in the design.

**SOLUTION**

$$c_1 = \frac{1}{2}d_1 = 0.021 \text{ m} \quad L = 1.6 \text{ m}$$

Based on stress limit  $\tau = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc_1}{J} \quad \therefore J = \frac{Tc_1}{\tau} = \frac{(900)(0.021)}{75 \times 10^6} = 252 \times 10^{-9} \text{ m}^4$$

Based on angle of twist limit  $\phi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}$

$$\phi = \frac{TL}{GJ} \quad \therefore J = \frac{TL}{G\phi} = \frac{(900)(1.6)}{(77 \times 10^9)(69.813 \times 10^{-3})} = 267.88 \times 10^{-9} \text{ m}^4$$

Larger value for  $J$  governs  $J = 267.88 \times 10^{-9} \text{ m}^4$

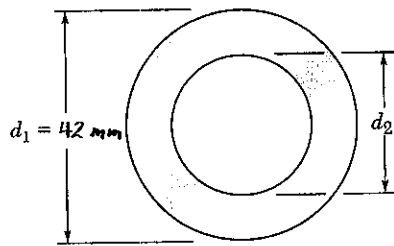
$$J = \frac{\pi}{2} (c_1^4 - c_2^4)$$

$$c_2^4 = c_1^4 - \frac{2J}{\pi} = 0.021^4 - \frac{(2)(267.88 \times 10^{-9})}{\pi} = 23.943 \times 10^{-9} \text{ m}^4$$

$$c_2 = 12.44 \times 10^{-3} \text{ m} = 12.44 \text{ mm}$$

$$d_2 = 2c_2 = 24.9 \text{ mm}$$

PROBLEM 3.86



3.86 A 1.6-m-long tubular steel shaft ( $G = 77 \text{ GPa}$ ) of 42-mm outer diameter  $d_1$  and 30-mm inner diameter  $d_2$  is to transmit 120 kW between a turbine and a generator. Knowing that the allowable shearing stress is 65 MPa and that the angle of twist must not exceed  $3^\circ$ , determine the minimum frequency at which the shaft may rotate.

SOLUTION

$$c_1 = \frac{1}{2}d_1 = 0.021 \text{ m}, \quad c_2 = \frac{1}{2}d_2 = 0.015 \text{ m}$$

$$J = \frac{\pi}{2}(c_1^4 - c_2^4) = \frac{\pi}{2}(0.021^4 - 0.015^4) = 225.97 \times 10^{-9} \text{ m}^4$$

Based on stress limit  $\tau = 65 \text{ MPa} = 65 \times 10^6 \text{ Pa}$

$$\tau = \frac{Tc_1}{J} \therefore T = \frac{J\tau}{c_1} = \frac{(225.97 \times 10^{-9})(65 \times 10^6)}{0.021} = 699.43 \text{ N}\cdot\text{m}$$

Based on angle of twist limit  $\phi = 3^\circ = 52.36 \times 10^{-3} \text{ rad}$

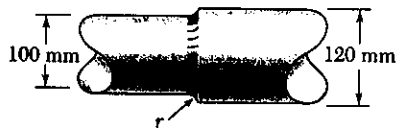
$$\phi = \frac{TL}{GJ} \therefore T = \frac{GJ\phi}{L} = \frac{(77 \times 10^9)(225.97 \times 10^{-9})(52.36 \times 10^{-3})}{1.6} \\ = 569.40 \text{ N}\cdot\text{m}$$

Smaller torque governs  $T = 569.40 \text{ N}\cdot\text{m}$

$$P = 120 \text{ kW} = 120 \times 10^3 \text{ W}$$

$$P = 2\pi fT \therefore f = \frac{P}{2\pi T} = \frac{120 \times 10^3}{2\pi(569.40)} = 33.54 \text{ Hz} \\ = 2012 \text{ rpm}$$

**PROBLEM 3.87**



3.87 The stepped shaft shown rotates at 450 rpm. Knowing that  $r = 10$  mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

**SOLUTION**

$$d = 100 \text{ mm}, \quad D = 120 \text{ mm}, \quad r = 10 \text{ mm}$$

$$\frac{D}{d} = \frac{120}{100} = 1.2, \quad \frac{r}{d} = \frac{10}{100} = 0.10, \quad \text{From Fig. 3.32} \quad K = 1.33$$

For smaller shaft  $c = \frac{1}{2}d = 0.050 \text{ m}$

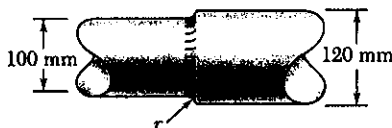
$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.33)} = 6.643 \times 10^3 \text{ N}\cdot\text{m}$$

$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

Power  $P = 2\pi fT = 2\pi (7.5)(6.643 \times 10^3) = 313 \times 10^3 \text{ W} = 313 \text{ kW}$   $\blacktriangleleft$

**PROBLEM 3.88**



3.88 The stepped shaft shown rotates at 450 rpm. Knowing that  $r = 4$  mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

**SOLUTION**

$$d = 100 \text{ mm}, \quad D = 120 \text{ mm}, \quad r = 4 \text{ mm}$$

$$\frac{D}{d} = \frac{120}{100} = 1.2, \quad \frac{r}{d} = \frac{4}{100} = 0.04. \quad \text{From Fig. 3.32} \quad K = 1.55$$

For smaller shaft  $c = \frac{1}{2}d = 0.050 \text{ m}$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

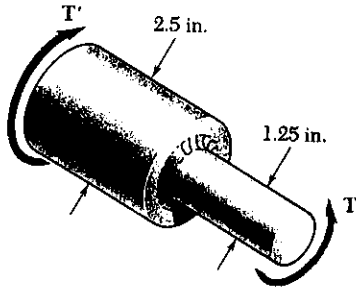
$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.050)^3 (45 \times 10^6)}{(2)(1.55)} = 5.70 \times 10^3 \text{ N}\cdot\text{m}$$

$$f = 450 \text{ rpm} = 7.5 \text{ Hz}$$

Power  $P = 2\pi fT = 2\pi (7.5)(5.70 \times 10^3) = 268 \times 10^3 \text{ W} = 268 \text{ kW}$   $\blacktriangleleft$



**PROBLEM 3.89**



3.89 Knowing that the stepped shaft shown must transmit 60 hp at a speed of 2100 rpm, determine the minimum radius  $r$  of the fillet if an allowable stress of 6000 psi is not to be exceeded.

**SOLUTION**

$$f = \frac{2100}{60} = 35 \text{ Hz}$$

$$P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in}/\text{s}$$

$$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi (35)} = 1.8007 \times 10^3 \text{ lb}\cdot\text{in}$$

For smaller shaft  $c = \frac{1}{2}d = 0.625 \text{ in}$

$$\tau = K \frac{Tc}{J} = \frac{2KT}{\pi c^3}$$

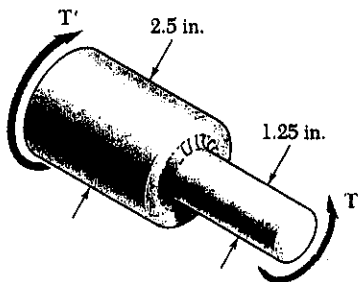
$$K = \frac{\pi c^3 \tau}{2T} = \frac{\pi (0.625)^3 (6000)}{(2)(1.8007 \times 10^3)} = 1.28$$

$$\frac{D}{d} = \frac{2.5}{1.25} = 2$$

From Fig 3.32  $\frac{r}{d} = 0.18$

$$r = 0.18 d = (0.050)(1.25 \text{ in}) = 0.225 \text{ in.}$$

**PROBLEM 3.90**



3.90 The stepped shaft shown must transmit 60 hp. Knowing that the allowable shearing stress in the shaft is 6000 psi and that the radius of the fillet is  $r = 0.25 \text{ in.}$ , determine the smallest permissible speed of the shaft.

**SOLUTION**

$$\frac{r}{d} = \frac{0.25}{1.25} = 0.200, \quad \frac{D}{d} = \frac{2.5}{1.25} = 2.00$$

From Fig. 3.32  $K = 1.26$

For smaller shaft  $c = \frac{1}{2}d = 0.625$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.625)^3 (6000)}{(2)(1.26)} = 1.826 \times 10^3 \text{ lb}\cdot\text{in}$$

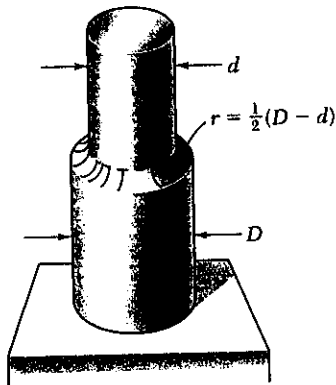
$$P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in}/\text{s}$$

$$P = 2\pi fT \therefore f = \frac{P}{2\pi T} = \frac{396 \times 10^3}{2\pi (1.826 \times 10^3)} = 34.9 \text{ Hz}$$

$$= 2076 \text{ rpm}$$

**PROBLEM 3.91**

3.91 A 25-N·m torque is applied to the stepped shaft shown which has a full quarter-circular fillet. Knowing that  $D = 24$  mm, determine the maximum shearing stress in the shaft when (a)  $d = 20$  mm, (b)  $d = 21.6$  mm.



Full quarter-circular fillet extends to edge of larger shaft

**SOLUTION**

(a)  $\frac{D}{d} = \frac{24}{20} = 1.20$

$r = \frac{1}{2}(D - d) = \frac{1}{2}(24 - 20) = 2$  mm

$\frac{r}{d} = \frac{2}{20} = 0.10$

From Fig 3.32  $K = 1.34$

For smaller shaft  $r = \frac{1}{2}d = 0.010$  m

$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.34)(25)}{\pi(0.010)^3} = 21.3 \times 10^6 \text{ Pa} = 21.6 \text{ MPa} \blacktriangleleft$

(b)  $\frac{D}{d} = \frac{24}{21.6} = 1.111$

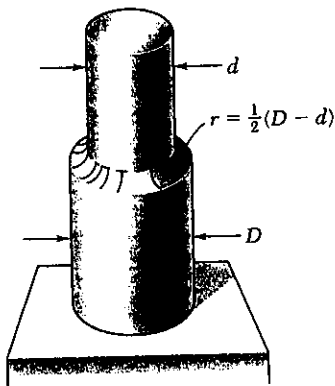
$r = \frac{1}{2}(D - d) = \frac{1}{2}(24 - 21.6) = 1.2$  mm  $\frac{r}{d} = \frac{1.2}{21.6} = 0.0556$

From Fig 3.32  $K = 1.42$ . For smaller shaft  $c = \frac{1}{2}d = 0.0108$  m.

$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.42)(25)}{\pi(0.0108)^3} = 17.9 \times 10^6 \text{ Pa} = 17.9 \text{ MPa} \blacktriangleleft$

**PROBLEM 3.92**

3.92 In the stepped shaft shown, which has a full quarter-circular fillet,  $D = 1.5$  in. and  $d = 1.2$  in. Knowing that the speed of the shaft is 1800 rpm and that the allowable shearing stress is 8000 psi, determine the maximum power that may be transmitted by the shaft.



Full quarter-circular fillet extends to edge of larger shaft

**SOLUTION**

$\frac{D}{d} = \frac{1.5}{1.2} = 1.25$   $r = \frac{1}{2}(D - d) = \frac{1}{2}(1.5 - 1.2) = 0.15$  in

$\frac{r}{d} = \frac{0.15}{1.2} = 0.125$

From Fig. 3.32  $K = 1.31$

For smaller shaft  $c = \frac{1}{2}d = 0.6$  in

$\tau = \frac{KTc}{J}$   $T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$

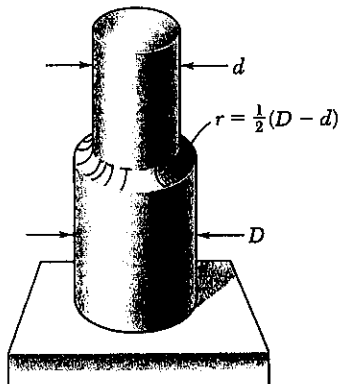
$T = \frac{\pi(0.6)^3(8000)}{(2)(1.31)} = 2072 \text{ lb}\cdot\text{in}$   $f = 1800 \text{ rpm} = 30 \text{ Hz}$

Power  $P = 2\pi f T = 2\pi(30)(2072) = 390.6 \times 10^3 \text{ lb}\cdot\text{in/s}$

$\frac{390.6 \times 10^3 \text{ lb}\cdot\text{in/s}}{6600 \text{ lb}\cdot\text{in/s/hp}} = 59.2 \text{ hp} \blacktriangleleft$

PROBLEM 3.93

3.93 In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 12 ksi. Knowing that  $D = 1.25$  in, determine the largest allowable torque that may be applied to the shaft if (a)  $d = 1.1$  in., (b)  $d = 1.0$  in.



Full quarter-circular fillet extends to edge of larger shaft

SOLUTION

$$\tau_{all} = 12 \text{ ksi} = 12000 \text{ psi}$$

(a)  $D = 1.25$   $d = 1.1$  in  $\frac{D}{d} = \frac{1.25}{1.1} = 1.09$

$$r = \frac{1}{2}(D - d) = \frac{1}{2}(1.25 - 1.1) = 0.075 \text{ in}$$

$$\frac{r}{d} = \frac{0.075}{1.1} = 0.0682$$

From Fig 3.32  $K = 1.40$

For smaller shaft  $c = \frac{1}{2}d = 0.55$  in

$$\tau = \frac{K T c}{J} \therefore T = \frac{J \tau}{K c} = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.55)^3 (12000)}{(2)(1.40)} = 2240 \text{ lb-in}$$

(b)  $D = 1.25$  in.  $d = 1.0$  in  $r = \frac{1}{2}(D - d) = \frac{1}{2}(1.25 - 1.0) = 0.125$  in.

$$\frac{D}{d} = \frac{1.25}{1.0} = 1.25, \frac{r}{d} = \frac{0.125}{1.0} = 0.125 \text{ From Fig 3.32 } K = 1.81$$

For smaller shaft  $c = \frac{1}{2}d = 0.50$  in

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.50)^3 (12000)}{(2)(1.81)} = 179.8 \text{ lb-in}$$

## PROBLEM 3.94

3.94 A 54-mm-diameter solid shaft is made of mild steel which is assumed to be elastoplastic with  $\tau_y = 145$  MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) 4 kN·m, (b) 5 kN·m.

## SOLUTION

$$c = \frac{1}{2}d = 0.027 \text{ m} \quad \tau_y = 145 \times 10^6 \text{ Pa}$$

$$\text{Compute } T_y \quad T_y = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.027)^3 (145 \times 10^6) = 4.488 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \quad T = 4.0 \times 10^3 \text{ N}\cdot\text{m} < T_y \quad \text{elastic} \quad \rho = c = 27 \text{ mm} \quad \blacktriangleleft$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(4.0 \times 10^3)}{\pi (0.027)^3} = 129.4 \times 10^6 \text{ Pa} = 129.4 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad T = 5.0 \times 10^3 \text{ N}\cdot\text{m} > T_y \quad \text{plastic region with elastic core.}$$

$$\text{The maximum shearing stress is } \tau_{\max} = \tau_y = 145 \text{ MPa} \quad \blacktriangleleft$$

$$T = \frac{4}{3} T_y \left(1 - \frac{\rho_y^3}{c^3}\right)$$

$$\frac{\rho_y^3}{c^3} = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(5 \times 10^3)}{4.488 \times 10^3} = 0.6540 \quad \frac{\rho_y}{c} = 0.8680$$

$$\rho_y = 0.8680 c = (0.8680)(0.027) = 0.02344 \text{ m} = 23.4 \text{ mm} \quad \blacktriangleleft$$

## PROBLEM 3.95

3.95 A 1.5-in.-diameter solid shaft is made of mild steel which is assumed to be elastoplastic with  $\tau_y = 21$  ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) 12 kip·in., (b) 18 kip·in.

## SOLUTION

$$c = \frac{1}{2}d = 0.75 \text{ in} \quad \tau_y = 21 \text{ ksi}$$

$$\text{Compute } T_y \quad T_y = \frac{J}{c} \tau_y = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.75)^3 (21) = 13.916 \text{ kip}\cdot\text{in}$$

$$(a) \quad T = 12 \text{ kip}\cdot\text{in} < T_y \quad \text{elastic} \quad \rho = c = 0.75 \text{ in.} \quad \blacktriangleleft$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(12)}{\pi (0.75)^3} = 18.11 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad T = 18 \text{ kip}\cdot\text{in} > T_y \quad \text{plastic region with elastic core}$$

$$\text{The maximum shearing stress is } \tau_{\max} = \tau_y = 21 \text{ ksi} \quad \blacktriangleleft$$

$$T = \frac{4}{3} T_y \left(1 - \frac{\rho_y^3}{c^3}\right)$$

$$\frac{\rho_y^3}{c^3} = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(18)}{13.916} = 0.11957 \quad \frac{\rho_y}{c} = 0.49265$$

$$\rho_y = 0.49265 c = (0.49265)(0.75) = 0.369 \text{ in.} \quad \blacktriangleleft$$

**PROBLEM 3.96**

3.96 A 30-mm-diameter solid rod is made of an elastoplastic material with  $\tau_y = 3.5$  MPa. Knowing that the elastic core of the rod is 25 mm in diameter, determine the magnitude of the torque applied to the rod.

**SOLUTION**

$$\tau_y = 3.5 \times 10^6 \text{ Pa} \quad c = \frac{1}{2}d = 0.015 \text{ m} \quad \rho_y = \frac{1}{2}d_y = 0.0125 \text{ m}$$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi}{2}c^3\tau_y = \frac{\pi}{2}(0.015)^3(3.5 \times 10^6) = 18.555 \text{ N}\cdot\text{m}$$

$$\frac{\rho_y}{c} = \frac{0.0125}{0.015} = 0.83333$$

$$T = \frac{4}{3}T_y\left(1 - \frac{1}{4}\left(\frac{\rho_y}{c}\right)^3\right) = \frac{4}{3}(18.555)\left[1 - \frac{1}{4}(0.83333)^3\right] = 21.2 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 3.97**

3.97 It is observed that a straightened steel paper clip can be twisted through several revolutions by the application of a torque of approximately 0.8 lb·in. Knowing that the diameter of the wire used to form the paper clip is 0.04 in., determine the approximate value of the yield stress of the steel.

**SOLUTION**

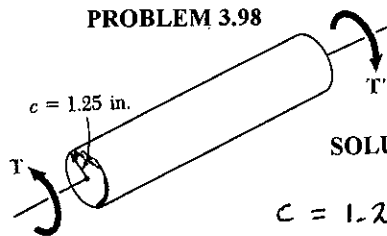
$$c = \frac{1}{2}d = 0.02 \text{ in.} \quad T_p = 0.8 \text{ lb}\cdot\text{in}$$

$$T_p = \frac{4}{3}T_y = \frac{4}{3}\frac{J\tau_y}{c} = \frac{4}{3}\cdot\frac{\pi}{2}c^3\tau_y = \frac{2\pi}{3}c^3\tau_y$$

$$\tau_y = \frac{3T_p}{2\pi c^3} = \frac{(3)(0.8)}{2\pi(0.02)^3} = 47.7 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 3.98**

3.98 The solid circular shaft shown is made of a steel which is assumed to be elastoplastic with  $\tau_y = 21$  ksi. Determine the magnitude  $T$  of the applied torque when the plastic zone is (a) 0.6 in. deep, (b) 1 in. deep.



**SOLUTION**

$$c = 1.25 \text{ in} \quad \tau_y = 21 \text{ ksi}$$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi}{2}c^3\tau_y = \frac{\pi}{2}(1.25)^3(21) = 64.427 \text{ kip}\cdot\text{in}$$

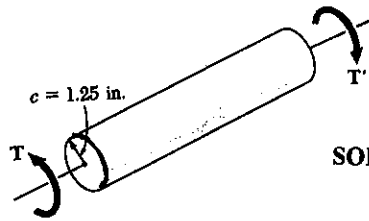
(a)  $t_y = 0.6 \text{ in.}, \rho_y = c - t_y = 1.25 - 0.60 = 0.65 \text{ in.}, \frac{\rho_y}{c} = \frac{0.65}{1.25} = 0.520$

$$T = \frac{4}{3}T_y\left(1 - \frac{1}{4}\left(\frac{\rho_y}{c}\right)^3\right) = \frac{4}{3}(64.427)\left[1 - \frac{1}{4}(0.520)^3\right] = 82.9 \text{ kip}\cdot\text{in} \quad \blacktriangleleft$$

(b)  $t_y = 1.0 \text{ in.}, \rho_y = c - t_y = 1.25 - 1.0 = 0.25 \text{ in.}, \frac{\rho_y}{c} = \frac{0.25}{1.25} = 0.200$

$$T = \frac{4}{3}T_y\left(1 - \frac{1}{4}\left(\frac{\rho_y}{c}\right)^3\right) = \frac{4}{3}(64.427)\left[1 - \frac{1}{4}(0.200)^3\right] = 85.7 \text{ kip}\cdot\text{in} \quad \blacktriangleleft$$

**PROBLEM 3.99**



3.98 The solid circular shaft shown is made of a steel which is assumed to be elastoplastic with  $\tau_Y = 21$  ksi. Determine the magnitude  $T$  of the applied torque when the plastic zone is (a) 0.6 in. deep, (b) 1 in. deep.

3.99 For the shaft and loading of Prob. 3.98, determine the angle of twist in a 4-ft length of shaft.

**SOLUTION**

$$C = 1.25 \text{ in}, \quad \tau_Y = 21 \text{ ksi} = 21 \times 10^3 \text{ psi}, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$L = 4 \text{ ft} = 48 \text{ in}$$

$$\gamma = \frac{\rho r}{L} \quad \gamma_Y = \frac{C \phi_Y}{L} \quad \therefore \phi_Y = \frac{L \gamma_Y}{C} = \frac{L \tau_Y}{C G} = \frac{(48)(21 \times 10^3)}{(1.25)(11.2 \times 10^6)} = 72.00 \times 10^{-3} \text{ rad}$$

$$\frac{\rho_Y}{C} = \frac{\phi_Y}{\phi} \quad \therefore \phi = \frac{C}{\rho_Y} \phi_Y$$

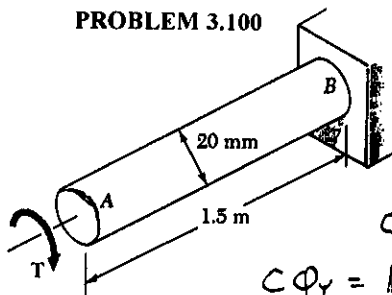
(a)  $t_Y = 0.6 \text{ in} \quad \rho_Y = C - t_Y = 1.25 - 0.6 = 0.65 \quad \frac{\rho_Y}{C} = \frac{0.65}{1.25} = 0.520$

$$\phi = \frac{\phi_Y}{\rho_Y/C} = \frac{72.00 \times 10^{-3}}{0.520} = 138.46 \times 10^{-3} \text{ rad} = 7.93^\circ$$

(b)  $t_Y = 1.0 \text{ in} \quad \rho_Y = C - t_Y = 1.25 - 1.0 = 0.25 \quad \frac{\rho_Y}{C} = \frac{0.25}{1.25} = 0.200$

$$\phi = \frac{\phi_Y}{\rho_Y/C} = \frac{72.00 \times 10^{-3}}{0.200} = 360 \times 10^{-3} \text{ rad} = 20.6^\circ$$

**PROBLEM 3.100**



3.100 A torque  $T$  is applied to the 20-mm-diameter steel rod  $AB$ . Assuming the steel to be elastoplastic with  $G = 77$  GPa and  $\tau_Y = 145$  MPa, determine (a) the torque  $T$  when the angle of twist at  $A$  is  $25^\circ$ , (b) the corresponding diameter of the elastic core of the shaft.

**SOLUTION**

$$C = \frac{1}{2} d = 0.010 \text{ m}, \quad L = 1.5 \text{ m}, \quad G = 77 \times 10^9 \text{ Pa}$$

$$C \phi_Y = L \gamma_Y = \frac{L \tau_Y}{G} \quad \phi_Y = \frac{L \tau_Y}{G C}$$

$$\phi_Y = \frac{(1.5)(145 \times 10^6)}{(77 \times 10^9)(0.010)} = 282.47 \times 10^{-3} \text{ rad}$$

$$T_Y = \frac{J \tau_Y}{C} = \frac{\pi}{2} C^3 \tau_Y = \frac{\pi}{2} (0.010)^3 (145 \times 10^6) = 227.77 \text{ N}\cdot\text{m}$$

$$\phi = 25^\circ = 436.33 \times 10^{-3} \text{ rad} > \phi_Y \quad \frac{\phi_Y}{\phi} = \frac{282.47 \times 10^{-3}}{436.33 \times 10^{-3}} = 0.64737$$

(a)  $T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \left(\frac{\phi_Y}{\phi}\right)^3\right) = \frac{4}{3} (227.77) \left[1 - \frac{1}{4} (0.64737)^3\right] = 283 \text{ N}\cdot\text{m}$

(b)  $\frac{\rho_Y}{C} = \frac{\phi_Y}{\phi} = 0.64737 \quad \rho_Y = 0.64737 C = (0.64737)(0.010)$

$$\rho_Y = 6.4737 \times 10^{-3} \text{ m} = 6.4737 \text{ mm} \quad d_Y = 2\rho_Y = 12.95 \text{ mm}$$

## PROBLEM 3.101

3.101 A 18-mm-diameter solid circular shaft is made of a material which is assumed to be elastoplastic with  $G = 77 \text{ GPa}$  and  $\tau_y = 145 \text{ MPa}$ . For a 1.2-m length, determine the maximum shearing stress and the angle of twist caused by a 200 N·m torque.

## SOLUTION

$$\tau_y = 145 \times 10^6 \text{ Pa}, \quad c = \frac{1}{2}d = 0.009 \text{ m}, \quad L = 1.2 \text{ m}, \quad T = 200 \text{ N}\cdot\text{m}$$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi}{2}c^3\tau_y = \frac{\pi}{2}(0.009)^3(145 \times 10^6) = 166.04 \text{ N}\cdot\text{m}$$

$$T > T_y \quad \text{plastic region with elastic core} \quad \tau_{\max} = \tau_y = 145 \text{ MPa} \quad \blacktriangleleft$$

$$\phi_y = \frac{T_y L}{GJ} = \frac{2T_y L}{\pi C^4 G} = \frac{(2)(166.04)(1.2)}{\pi(0.009)^4(77 \times 10^9)} = 251.08 \times 10^{-3} \text{ rad.}$$

$$T = \frac{4}{3}T_y \left(1 - \frac{1}{4}\frac{\phi^3}{\phi_y^3}\right)$$

$$\left(\frac{\phi_y}{\phi}\right)^3 = 4 - \frac{3T}{T_y} = 4 - \frac{(3)(200)}{166.04} = 0.38641 \quad \frac{\phi_y}{\phi} = 0.72837$$

$$\phi = \frac{\phi_y}{0.72837} = \frac{251.08 \times 10^{-3}}{0.72837} = 344.7 \times 10^{-3} \text{ rad} = 19.75^\circ \quad \blacktriangleleft$$

## PROBLEM 3.102

3.102 A solid circular rod is made of a material which is assumed to be elastoplastic. Denoting by  $\tau_y$  and  $\phi_y$  respectively, the torque and angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (a)  $T = 1.1 T_y$ , (b)  $T = 1.25 T_y$ , (c)  $T = 1.3 T_y$ .

## SOLUTION

$$T = \frac{4}{3}T_y \left(1 - \frac{1}{4}\frac{\phi^3}{\phi_y^3}\right)$$

$$\frac{\phi_y}{\phi} = \sqrt[3]{4 - \frac{3T}{T_y}} \quad \text{or} \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_y}}}$$

$$(a) \quad \frac{T}{T_y} = 1.10 \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - (3)(1.10)}} = 1.126 \quad \phi = 1.126 \phi_y \quad \blacktriangleleft$$

$$(b) \quad \frac{T}{T_y} = 1.25 \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - (3)(1.25)}} = 1.587 \quad \phi = 1.587 \phi_y \quad \blacktriangleleft$$

$$(c) \quad \frac{T}{T_y} = 1.3 \quad \frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - (3)(1.3)}} = 2.15 \quad \phi = 2.15 \phi_y \quad \blacktriangleleft$$

## PROBLEM 3.103

3.103 A 0.75-in.-diameter solid circular shaft is made of a material which is assumed to be elastoplastic with  $G = 11.2 \times 10^6$  psi and  $\tau_y = 21$  ksi. For a 5-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 2-kip-in. torque.

## SOLUTION

$$c = \frac{1}{2}d = 0.375 \text{ in.}, \quad G = 11.2 \times 10^6 \text{ psi}, \quad \tau_y = 21 \text{ ksi} = 21000 \text{ psi}$$

$$L = 5 \text{ ft.} = 60 \text{ in.} \quad T = 2 \text{ kip-in} = 2 \times 10^3 \text{ lb-in}$$

$$T_y = \frac{J\tau_y}{c} = \frac{\pi}{2}c^3\tau_y = \frac{\pi}{2}(0.375)^3(21000) = 1.7395 \times 10^3 \text{ lb-in.}$$

$$T > T_y \quad \text{plastic region with elastic core} \therefore \tau_{\max} = \tau_y = 21 \text{ ksi}$$

$$\tau_y = \frac{c\phi_y}{L} \therefore \phi_y = \frac{L\tau_y}{c} = \frac{L\tau_y}{cG} = \frac{(60)(21000)}{(0.375)(11.2 \times 10^6)} = 300 \times 10^{-3} \text{ rad}$$

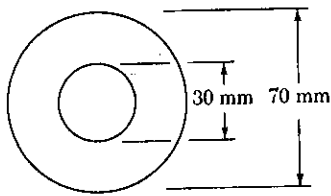
$$T = T_y \left(1 - \frac{\phi^3}{\phi_y^3}\right) =$$

$$\frac{\phi}{\phi_y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_y}}} = \frac{1}{\sqrt[3]{4 - \frac{(3)(2 \times 10^3)}{1.7395 \times 10^3}}} = 1.220$$

$$\phi = 1.220 \phi_y = (1.220)(300 \times 10^{-3}) = 366 \times 10^{-3} \text{ rad} = 21.0^\circ$$



PROBLEM 3.104



3.104 A hollow steel shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with  $\tau_y = 180$  MPa and  $G = 77$  GPa. Determine the applied torque and the corresponding angle of twist (a) at the onset of yield, (b) when the plastic zone is 10 mm deep.

SOLUTION

(a) At the onset of yield, the stress distribution is the elastic distribution with  $\tau_{max} = \tau_y$

$$c_2 = \frac{1}{2}d_2 = 0.035 \text{ m}, \quad c_1 = \frac{1}{2}d_1 = 0.015 \text{ m}.$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.035^4 - 0.015^4) = 2.2777 \times 10^{-6} \text{ m}^4$$

$$\tau_{max} = \tau_y = \frac{T_y c_2}{J} \quad \therefore T_y = \frac{J \tau_y}{c_2} = \frac{(2.2777 \times 10^{-6})(180 \times 10^6)}{0.035} = 11.714 \times 10^3 \text{ N}\cdot\text{m} \\ = 11.71 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi_y = \frac{T_y L}{GJ} = \frac{(11.714 \times 10^3)(0.9)}{(77 \times 10^9)(2.2777 \times 10^{-6})} = 60.11 \times 10^{-3} \text{ rad} = 3.44^\circ \quad \blacktriangleleft$$

(b)  $t = 0.010 \text{ m} \quad p_r = c_2 - t = 0.035 - 0.010 = 0.025 \text{ m}$

$$\gamma = \frac{\rho \phi}{L} = \frac{p_r \phi}{L} = \tau_y = \frac{\tau_y}{G}$$

$$\phi = \frac{\tau_y L}{G p_r} = \frac{(180 \times 10^6)(0.9)}{(77 \times 10^9)(0.025)} = 84.156 \times 10^{-3} \text{ rad} = 4.82^\circ \quad \blacktriangleleft$$

Torque  $T_1$  carried by elastic portion  $c_1 \leq \rho \leq p_r$

$$\tau = \tau_y \text{ at } \rho = p_r. \quad \tau_y = \frac{T_1 p_r}{J_1} \quad \text{where } J_1 = \frac{\pi}{2}(p_r^4 - c_1^4)$$

$$J_1 = \frac{\pi}{2}(0.025^4 - 0.015^4) = 534.07 \times 10^{-9} \text{ m}^4$$

$$T_1 = \frac{J_1 \tau_y}{p_r} = \frac{(534.07 \times 10^{-9})(180 \times 10^6)}{0.025} = 3.845 \times 10^3 \text{ N}\cdot\text{m}$$

Torque  $T_2$  carried by plastic portion

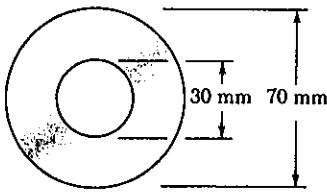
$$T_2 = 2\pi \int_{p_r}^{c_2} \tau_y \rho^2 d\rho = 2\pi \tau_y \left. \frac{\rho^3}{3} \right|_{p_r}^{c_2} = \frac{2\pi}{3} \tau_y (c_2^3 - p_r^3) \\ = \frac{2\pi}{3} (180 \times 10^6)(0.035^3 - 0.025^3) = 10.273 \times 10^3 \text{ N}\cdot\text{m}$$

Total torque

$$T = T_1 + T_2 = 3.845 \times 10^3 + 10.273 \times 10^3 = 14.12 \times 10^3 \text{ N}\cdot\text{m}$$

$$14.12 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 3.105**



**3.105** A hollow steel shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with  $\tau_y = 180$  MPa and  $G = 77$  GPa. Determine (a) the angle of twist at which the section first becomes fully plastic, (b) the corresponding magnitude of the applied torque.

**SOLUTION**

$$c_1 = \frac{1}{2} d_1 = 0.015 \text{ m} \quad c_2 = \frac{1}{2} d_2 = 0.035 \text{ m}$$

(a) For onset of fully plastic yielding,  $p_r = c_1$

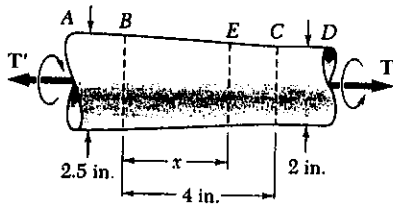
$$\tau = \tau_y \therefore \gamma = \frac{\tau_y}{G} = \frac{p_r \phi}{L} = \frac{c_1 \phi}{L}$$

$$\phi = \frac{L \tau_y}{c_1 G} = \frac{(0.9)(180 \times 10^6)}{(0.015)(77 \times 10^9)} = 140.26 \times 10^{-3} \text{ rad} = 8.04^\circ$$

$$(b) T_p = 2\pi \int_{c_1}^{c_2} \tau_y \rho^2 d\rho = 2\pi \tau_y \left. \frac{\rho^3}{3} \right|_{c_1}^{c_2} = \frac{2\pi}{3} \tau_y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (180 \times 10^6) (0.035^3 - 0.015^3) = 14.89 \times 10^3 \text{ N}\cdot\text{m} = 14.89 \text{ kN}\cdot\text{m}$$

**PROBLEM 3.106**



**3.106** A shaft of mild steel is machined to the shape shown and then twisted by torques of magnitude 45 kip·in. Assuming the steel to be elastoplastic with  $\tau_y = 21$  ksi, determine (a) the thickness of the plastic zone in portion CD of the shaft, (b) the length of the portion BE which remains fully elastic.

**SOLUTION**

(a) In portion CD  $c = \frac{1}{2} d = 1.00$  in

$$T_Y = \frac{J_{CD} \tau_y}{c} = \frac{\pi c^3 \tau_y}{2} = \frac{\pi (1.00)^3 (21)}{2} = 32.987 \text{ kip}\cdot\text{in}$$

$$T = \frac{4}{3} T_Y \left(1 - \frac{p_r^3}{c^3}\right) \therefore \frac{p_r^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(40)}{32.987} = 0.36620$$

$$\frac{p_r}{c} = 0.71283, \quad p_r = (0.71283)(1.00) = 0.713 \text{ in}, \quad t_y = c - p_r = 0.287 \text{ in}$$

(b) For yielding at point E  $\tau = \tau_y, c = c_E, T = 40$  kip·in

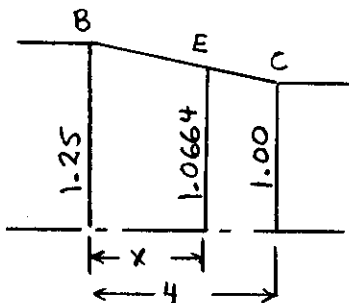
$$\tau_y = \frac{T c_E}{J_E} = \frac{2T}{\pi c_E^3} \therefore c_E^3 = \frac{2T}{\pi \tau_y} = \frac{(2)(40)}{\pi (21)} = 1.2126 \text{ in}^3$$

$$c_E = 1.0664 \text{ in.}$$

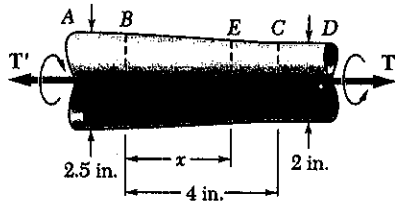
Using proportions from the sketch

$$\frac{1.25 - 1.0664}{1.25 - 1.00} = \frac{x}{4}$$

$$x = 2.94 \text{ in}$$



PROBLEM 3.107



3.107 The magnitude of the torque  $T$  applied to the tapered shaft of Prob. 3.106 is slowly increased. Determine (a) the largest torque which may be applied to the shaft, (b) the length of portion  $BE$  which remains fully elastic.

SOLUTION

(a) The largest torque which may be applied to the shaft makes portion  $CD$  fully plastic.

In portion  $CD$   $c = \frac{1}{2}d = 1.00$  in.

$$T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2} (1.00)^3 (21) = 32.987 \text{ Kip}\cdot\text{in.}$$

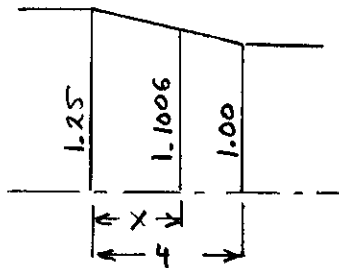
For fully plastic shaft  $p_r = 0$

$$T = \frac{4}{3} T_Y \left(1 - \frac{p_r^3}{c^3}\right) = \frac{4}{3} T_Y = \frac{4}{3} (32.987) = 43.982 \text{ kip}\cdot\text{in} \approx 44.0 \text{ kip}\cdot\text{in}$$

(b) For yielding at point  $E$ ,  $\tau = \tau_Y$ ,  $c = c_E$ ,  $T = 43.982 \text{ kip}\cdot\text{in.}$

$$\tau_Y = \frac{T c_E}{J_E} = \frac{2T}{\pi c_E^3}, \quad c_E^3 = \frac{2T}{\pi \tau_Y} = \frac{(2)(43.982)}{\pi (21)} = 1.33333 \text{ in}^3$$

$$c_E = 1.1006 \text{ in}$$



Using proportions from the sketch

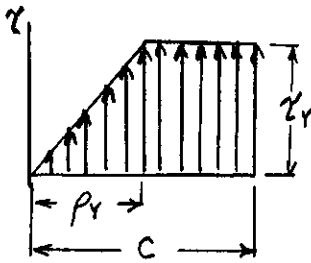
$$\frac{1.25 - 1.1006}{1.25 - 1.00} = \frac{x}{4}$$

$$x = 2.39 \text{ in}$$

PROBLEM 3.108

3.108 Considering the partially plastic shaft of Fig. 3.38c, derive Eq. (3.32) by recalling that the integral in Eq. (3.26) represents the second moment about the  $r$  axis of the area under the  $\tau$ - $\rho$  curve.

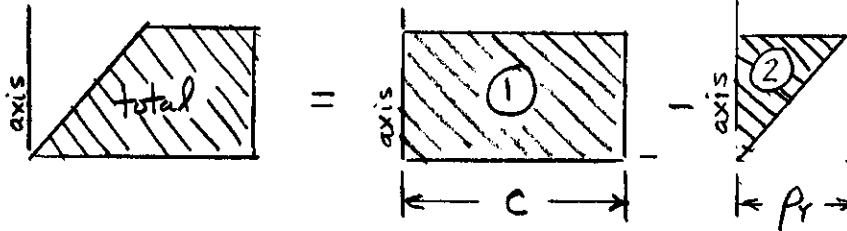
SOLUTION



The stress is that shown on the left.

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi \int \rho^2 dA = 2\pi I$$

where  $dA = \tau d\rho$  and  $I = 2\text{nd moment about the } \tau\text{-axis.}$



$$I = I_1 - I_2$$

$$= \frac{1}{3} \tau_y c^3 - \left\{ \frac{1}{3c} \tau_y \rho_y^3 + \frac{1}{2} \tau_y \rho_y \left( \frac{1}{3} \rho_y \right)^2 \right\}$$

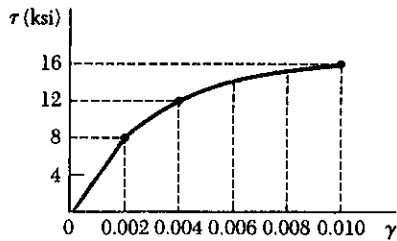
$$= \frac{1}{3} \tau_y c^3 - \frac{1}{12} \tau_y \rho_y^3 = \frac{1}{3} \tau_y c^3 \left( 1 - \frac{1}{4} \frac{\rho_y^3}{c^3} \right)$$

$$T = 2\pi I = \frac{2\pi}{3} \tau_y c^3 \left( 1 - \frac{1}{4} \frac{\rho_y^3}{c^3} \right)$$

Recall that  $T_y = \frac{J \tau_y}{c} = \frac{\pi}{2} c^3 \tau_y \quad \therefore \quad \frac{2\pi}{3} \tau_y c^3 = \frac{4}{3} T_y$

Hence  $T = \frac{4}{3} T_y \left( 1 - \frac{1}{4} \frac{\rho_y^3}{c^3} \right)$

PROBLEM 3.109



3.109 Using the stress-strain diagram shown, determine (a) the torque which causes a maximum shearing stress of 15 ksi in a 0.8-diameter solid rod, (b) the corresponding angle of twist in a 20-in. length of the rod.

SOLUTION

(a)  $\tau_{max} = 15 \text{ ksi}$        $c = \frac{1}{2}d = 0.400 \text{ in}$   
 From the stress-strain diagram  $\gamma_{max} = 0.008$

Let  $z = \frac{r}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz = 2\pi c^3 I$$

where the integral  $I$  is given by  $I = \int_0^1 z^2 \tau dz$

Evaluate  $I$  using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where  $w$  is a weighting factor. Using  $\Delta z = 0.25$ , we get the values given in the table below.

$z$	$\gamma$	$\tau, \text{ksi}$	$z^2 \tau, \text{ksi}$	$w$	$wz^2 \tau, \text{ksi}$
0	0.000	0	0.000	1	0.00
0.25	0.002	8	0.500	4	2.00
0.5	0.004	12	3.000	2	6.00
0.75	0.006	14	7.875	4	31.50
1.0	0.008	15	15.000	1	15.00
					54.50 ← $\sum wz^2 \tau$

$$I = \frac{(0.25)(54.50)}{3} = 4.54 \text{ ksi}$$

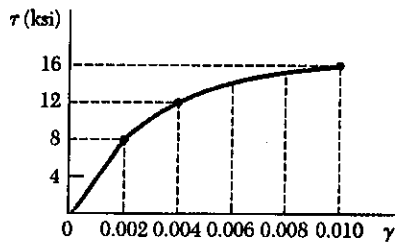
(a)  $T = 2\pi c^3 I = 2\pi (0.400)^3 (4.54) = 1.826 \text{ kip-in}$       ▶

(b)  $\gamma_{max} = \frac{c\phi}{L}$

$$\phi = \frac{L\gamma_m}{c} = \frac{(20)(0.008)}{0.400} = 400 \times 10^{-3} \text{ rad} = 22.9^\circ$$
      ▶

Note: Answers may differ slightly due to differences of opinion in reading the stress-strain curve.

**PROBLEM 3.110**



**3.110** A hollow shaft of outer and inner diameters respectively equal to 0.6 in. and 0.2 in. is fabricated from an aluminum alloy for which the stress-strain diagram is given in the sketch. Determine the torque required to twist a 9-in. length of the shaft through  $10^\circ$ .

**SOLUTION**

$$\phi = 10^\circ = 174.53 \times 10^{-3} \text{ rad}$$

$$C_1 = \frac{1}{2}d_1 = 0.100 \text{ in}, \quad C_2 = \frac{1}{2}d_2 = 0.300 \text{ in.}$$

$$\gamma_{\max} = \frac{C_2 \phi}{L} = \frac{(0.300)(174.53 \times 10^{-3})}{9} = 0.00582$$

$$\gamma_{\min} = \frac{C_1 \phi}{L} = \frac{(0.100)(174.53 \times 10^{-3})}{9} = 0.00194$$

Let  $z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{C_2}$                        $z_1 = \frac{C_1}{C_2} = \frac{1}{3}$

$$T = 2\pi \int_{C_1}^{C_2} \rho^2 \tau d\rho = 2\pi C_2^3 \int_{z_1}^1 z^2 \tau dz = 2\pi C_2^3 I$$

where the integral  $I$  is given by  $I = \int_{1/3}^1 z^2 \tau dz$

Evaluate  $I$  using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where  $w$  is a weighting factor. Using  $\Delta z = \frac{1}{6}$  we get the values given in the table below.

$z$	$\gamma$	$\tau$ , ksi	$z^2 \tau$ , ksi	$w$	$w z^2 \tau$ , ksi
1/3	0.00194	8.0	0.89	1	0.89
1/2	0.00291	10.0	2.50	4	10.00
2/3	0.00383	11.5	5.11	2	10.22
5/6	0.00485	13.0	9.03	4	36.11
1	0.00582	14.0	14.0	1	14.00

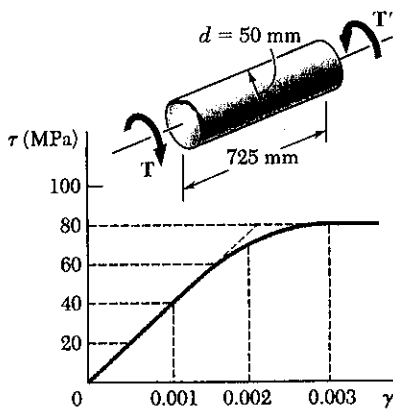
71.22 ←  $\sum w z^2 \tau$

$$I = \frac{(1/6)(71.22)}{3} = 3.96 \text{ ksi}$$

$$T = 2\pi C_2^3 I = 2\pi (0.300)^3 (3.96) = 0.671 \text{ kip-in} = 671 \text{ lb-in} \blacktriangleleft$$

**Note:** Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

**PROBLEM 3.111**



3.111 A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is  $5^\circ$  in a 725-mm length, determine by approximate means the magnitude  $T$  of the torque applied to the shaft.

**SOLUTION**

$$\phi = 5^\circ = 87.266 \times 10^{-3} \text{ rad}$$

$$c = \frac{1}{2}d = 0.025 \text{ m}, \quad L = 0.725 \text{ m}$$

$$\gamma_{\max} = \frac{c\phi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 0.00301$$

$$\text{Let } z = \frac{r}{r_{\max}} = \frac{r}{c}$$

$$T = 2\pi \int_0^c \rho^2 \gamma d\rho = 2\pi c^3 \int_0^1 z^2 \gamma dz = 2\pi c^3 I$$

where the integral  $I$  is given by  $I = \int_0^1 z^2 \gamma dz$

Evaluate  $I$  using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \gamma$$

where  $w$  is a weighting factor. Using  $\Delta z = 0.25$ , we get the values given in the table below.

$z$	$\gamma$	$\gamma, \text{ MPa}$	$z^2 \gamma, \text{ MPa}$	$w$	$wz^2 \gamma, \text{ MPa}$	
0	0	0	0	1	0	
0.25	0.00075	30	1.875	4	7.5	
0.5	0.0015	55	13.75	2	27.5	
0.75	0.00226	75	42.19	4	168.75	
1.0	0.00301	80	80.	1	80.	
					283.75	$\sum wz^2 \gamma$

$$= 283.75 \times 10^6 \text{ Pa}$$

$$I = \frac{(0.25)(283.75 \times 10^6)}{3} = 23.65 \times 10^6 \text{ Pa}$$

$$T = 2\pi c^3 I = 2\pi (0.025)^3 (23.65 \times 10^6) = 2.32 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 2.32 \text{ kN}\cdot\text{m}$$

## PROBLEM 3.112

3.111 A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is  $5^\circ$  in a 725-mm length, determine by approximate means the magnitude  $T$  of the torque applied to the shaft.

3.112 Three points on the nonlinear stress-strain diagram used in Prob. 3.111 are  $(0,0)$ ,  $(0.0015, 55\text{MPa})$ , and  $(0.003, 80\text{MPa})$ . By fitting the polynomial  $\tau = A + B\gamma + C\gamma^2$  through these points the following approximate relation has been obtained.

$$\tau = 46.7 \times 10^9 \gamma - 6.67 \times 10^{12} \gamma^2$$

Solve Prob. 3.111 using the relation, Eq. (3.2) and Eq. (3.26).

## SOLUTION

$$\phi = 5^\circ = 87.266 \times 10^{-3} \text{ rad}, \quad c = \frac{1}{2}d = 0.025 \text{ m}, \quad L = 0.725 \text{ m}$$

$$\gamma_{\max} = \frac{c\phi}{L} = \frac{(0.025)(87.266 \times 10^{-3})}{0.725} = 3.009 \times 10^{-3}$$

$$\text{Let } z = \frac{\gamma}{\gamma_{\max}} = \frac{\rho}{c}$$

$$T = 2\pi \int_0^c \rho^2 \tau d\rho = 2\pi c^3 \int_0^1 z^2 \tau dz$$

The given stress strain curve is

$$\tau = A + B\gamma + C\gamma^2 = A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2$$

$$\begin{aligned} T &= 2\pi c^3 \int_0^1 z^2 (A + B\gamma_{\max} z + C\gamma_{\max}^2 z^2) dz \\ &= 2\pi c^3 \left\{ A \int_0^1 z^2 dz + B\gamma_{\max} \int_0^1 z^3 dz + C\gamma_{\max}^2 \int_0^1 z^4 dz \right\} \\ &= 2\pi c^3 \left\{ \frac{1}{3}A + \frac{1}{4}B\gamma_{\max} + \frac{1}{5}C\gamma_{\max}^2 \right\} \end{aligned}$$

$$\text{Data: } A = 0, \quad B = 46.7 \times 10^9, \quad C = -6.67 \times 10^{12}$$

$$\frac{1}{3}A = 0, \quad \frac{1}{4}B\gamma_{\max} = \frac{1}{4}(46.7 \times 10^9)(3.009 \times 10^{-3}) = 35.13 \times 10^3$$

$$\frac{1}{5}C\gamma_{\max}^2 = -\frac{1}{5}(6.67 \times 10^{12})(3.009 \times 10^{-3})^2 = -12.08 \times 10^3$$

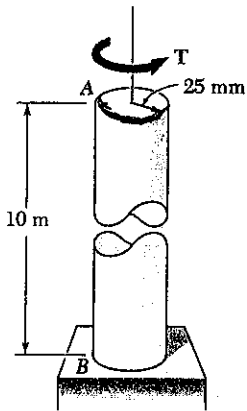
$$T = 2\pi(0.025)^3 \{ 0 + 35.13 \times 10^3 - 12.08 \times 10^3 \} = 2.26 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 2.26 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$



PROBLEM 3.113

3.113 The solid circular drill rod  $AB$  is made of a steel which is assumed to be elastoplastic with  $\tau_y = 160$  MPa and  $G = 77$  GPa. Knowing that a torque  $T = 5$  kN·m is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.



SOLUTION

$$c = 0.025 \text{ m}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.025)^4 = 613.59 \times 10^{-9} \text{ m}^4$$

$$T_Y = \frac{J \tau_y}{c} = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.025)^3 (160 \times 10^6) = 3.927 \times 10^3 \text{ N}\cdot\text{m}$$

Loading:  $T = 5 \times 10^3 \text{ N}\cdot\text{m}$

$$T = \frac{3}{8} T_Y \left(1 - \frac{p_r^3}{c^3}\right)$$

$$\frac{p_r^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(5 \times 10^3)}{3.927 \times 10^3} = 0.18029$$

$$\frac{p_r}{c} = 0.5649, \quad p_r = 0.5649 c = 0.014123 \times 10^{-3} \text{ m} = 14.123 \text{ mm}$$

Unloading:  $\tau' = \frac{T p}{J}$  where  $T = 5 \times 10^3 \text{ kN}\cdot\text{m}$

At  $p = c$   $\tau' = \frac{(5)(0.025)}{613.59 \times 10^{-9}} = 203.72 \times 10^6 \text{ Pa}$

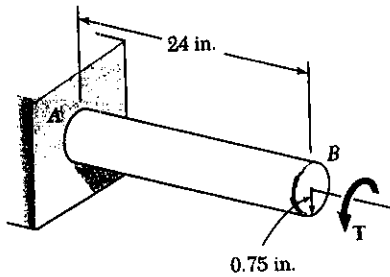
At  $p = p_r$   $\tau' = \frac{(5)(0.014123)}{613.59 \times 10^{-9}} = 115.08 \times 10^6 \text{ Pa}$

Residual:  $\tau_{res} = \tau_{load} - \tau'$

At  $p = c$   $\tau_{res} = 160 \times 10^6 - 203.72 \times 10^6 = -43.72 \times 10^6 \text{ Pa} = -43.7 \text{ MPa}$

At  $p = p_r$   $\tau_{res} = 160 \times 10^6 - 115.08 \times 10^6 = 44.92 \times 10^6 \text{ Pa} = 44.9 \text{ MPa}$

**PROBLEM 3.114**



3.114 The solid circular shaft  $AB$  is made of a steel which is assumed to be elastoplastic with  $G = 11.2 \times 10^6$  psi and  $\tau_r = 21$  ksi. The torque  $T$  is increased until the radius of the elastic core is 0.25 in. Determine the maximum residual shearing stress in the shaft after the torque  $T$  has been removed.

**SOLUTION**

$$c = 0.75 \text{ in.} \quad \rho_r = 0.25 \text{ in.} \quad \frac{\rho_r}{c} = \frac{0.25}{0.75} = \frac{1}{3}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.75)^4 = 0.49701 \text{ in}^4$$

$$T_r = \frac{J \tau_r}{c} = \frac{(0.49701)(21)}{0.75} = 13.916 \text{ kip-in.}$$

At end of loading:  $T = \frac{4}{3} T_r \left(1 - \frac{1}{4} \left(\frac{\rho_r}{c}\right)^3\right) = \frac{4}{3} (13.916) \left[1 - \frac{1}{4} \left(\frac{1}{3}\right)^3\right]$   
 $= 18.383 \text{ kip-in.}$

The stresses are  $\tau_{\text{load}} = 0$  at  $\rho = 0$   
 $\tau_{\text{load}} = \tau_r = 21 \text{ ksi}$  at  $\rho = 0.25 \text{ in}$   
 $\tau_{\text{load}} = \tau_r = 21 \text{ ksi}$  at  $\rho = 0.75 \text{ in.}$

Torque change during unloading  $T = -18.383 \text{ kip-in.}$

Stress changes during unloading  $\tau' = \frac{T \rho}{J}$  (elastic)

At  $\rho = 0$   $\tau' = 0$

At  $\rho = 0.25 \text{ in}$   $\tau' = \frac{(18.383)(0.25)}{0.49701} = 9.25 \text{ ksi}$

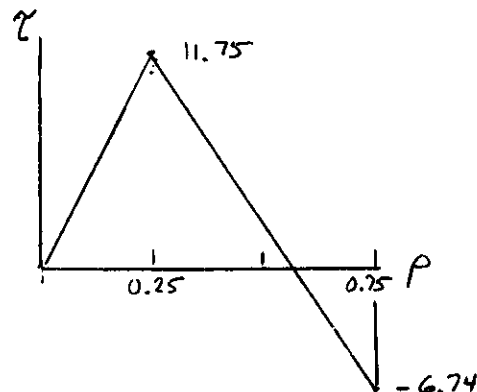
At  $\rho = 0.75 \text{ in}$   $\tau' = \frac{(18.383)(0.75)}{0.49701} = 27.74 \text{ ksi}$

Residual stresses are found by adding  $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$

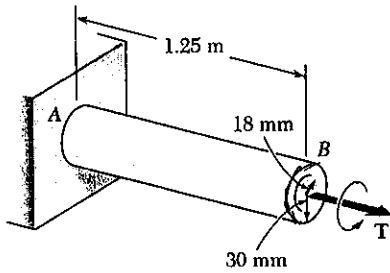
At  $\rho = 0$   $\tau_{\text{res}} = 0$

At  $\rho = 0.25 \text{ in}$   $\tau_{\text{res}} = 21 - 9.75 = 11.75 \text{ ksi}$

At  $\rho = 0.75 \text{ in}$   $\tau_{\text{res}} = 21 - 27.74 = -6.74 \text{ ksi}$



PROBLEM 3.115



3.115 The hollow shaft  $AB$  is made of a steel which is assumed to be elastoplastic with  $\tau_y = 145 \text{ MPa}$  and  $G = 77 \text{ GPa}$ . The magnitude  $T$  of the torque is slowly increased until the plastic zone first reaches the inner surface; the torque is then removed. Determine (a) the maximum residual shearing stress, (b) the permanent angle of twist of the shaft.

SOLUTION

inner radius  $c_1 = 0.018 \text{ m}$ , outer radius  $c_2 = 0.030 \text{ m}$   
 $J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.018^4)$   
 $= 1.10745 \times 10^{-6} \text{ m}^4$

Loading: When  $\rho_r$  reaches inner surface, then  $\tau = \tau_y$

$$T_{load} = 2\pi \int_{c_1}^{c_2} \rho^2 \tau_r d\rho = 2\pi \tau_y \frac{\rho^3}{3} \Big|_{c_1}^{c_2} = \frac{2\pi}{3} \tau_y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (145 \times 10^6) (0.030^3 - 0.018^3) = 6.4285 \times 10^3 \text{ N}\cdot\text{m}$$

$\gamma = \frac{\tau_y}{G}$  at  $\rho = c_1$ , Also  $\gamma = \frac{\rho\phi}{L} \therefore \phi = \frac{L\gamma}{\rho} = \frac{L\tau_y}{c_1 G}$

$$\phi_{load} = \frac{(1.25)(145 \times 10^6)}{(0.018)(77 \times 10^9)} = 130.77 \times 10^{-3} \text{ rad} = 7.493^\circ$$

Unloading:  $T = 6.4285 \times 10^3 \text{ N}\cdot\text{m}$  (elastic)  $\Delta\tau = \frac{\Delta T \rho}{J}$

At  $\rho = c_2$   $\tau' = \frac{(6.4285)(0.030)}{1.10745 \times 10^{-6}} = 174.14 \times 10^6 \text{ Pa}$

At  $\rho = c_1$   $\tau' = \frac{(6.4285)(0.018)}{1.10745 \times 10^{-6}} = 104.49 \times 10^6 \text{ Pa}$

$$\phi' = \frac{(LT)L}{GJ} = \frac{(6.4285)(1.25)}{(77 \times 10^9)(1.10745 \times 10^{-6})} = 94.23 \times 10^{-3} \text{ rad} = -5.399^\circ$$

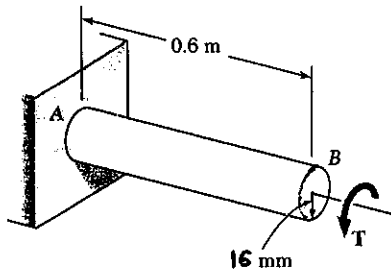
Residual:  $\tau_{res} = \tau_{load} - \tau'$   $\phi_{perm} = \phi_{load} - \phi'$

(a) At  $\rho = c_2$   $\tau_{res} = 145 \times 10^6 - 174.14 \times 10^6 = -29.14 \times 10^6 \text{ Pa}$   
 $= -29.1 \text{ MPa}$

At  $\rho = c_1$   $\tau_{res} = 145 \times 10^6 - 104.49 \times 10^6 = 40.51 \times 10^6 \text{ Pa}$   
 $= 40.5 \text{ MPa}$

(b)  $\phi_{perm} = 130.77 \times 10^{-3} - 94.23 \times 10^{-3} = 36.54 \times 10^{-3} \text{ rad}$   
 $= 2.09^\circ$

PROBLEM 3.116



3.116 The solid shaft shown is made of a steel which is assumed to be elastoplastic with  $\tau_y = 145 \text{ MPa}$  and  $G = 77 \text{ GPa}$ . The torque  $T$  is increased in magnitude until the shaft has been twisted through  $6^\circ$  and then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist of the shaft.

SOLUTION

$$c = 0.016 \text{ m} \quad \phi = 6^\circ = 104.72 \times 10^{-3} \text{ rad}$$

$$\gamma_{\max} = \frac{c\phi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_y = \frac{\tau_y}{G} = \frac{145 \times 10^6}{77 \times 10^9} = 0.0018831$$

$$\frac{\rho_y}{c} = \frac{\gamma_y}{\gamma_{\max}} = \frac{0.0018831}{0.0027925} = 0.67433$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.016)^4 = 102.944 \times 10^{-9} \text{ m}^4$$

$$T_y = \frac{J \tau_y}{c} = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.016)^3 (145 \times 10^6) = 932.93 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \text{At end of loading } T_{\text{load}} &= \frac{4}{3} T_y \left(1 - \frac{1}{4} \left(\frac{\rho_y}{c}\right)^3\right) = \frac{4}{3} (932.93) \left[1 - \frac{1}{4} (0.67433)^3\right] \\ &= 1.14855 \times 10^3 \text{ N}\cdot\text{m} \end{aligned}$$

Unloading: elastic  $T' = -1.14855 \times 10^3 \text{ N}\cdot\text{m}$

$$\text{At } \rho = c \quad \tau' = \frac{T' c}{J} = \frac{(-1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = 178.52 \times 10^6 \text{ Pa}$$

$$\text{At } \rho = \rho_y \quad \tau' = \frac{T' c}{J} \frac{\rho_y}{c} = (-178.52 \times 10^6)(0.67433) = 120.38 \times 10^6 \text{ Pa}$$

$$\phi' = \frac{T' L}{G J} = \frac{(-1.14855 \times 10^3)(0.6)}{(77 \times 10^9)(102.944 \times 10^{-9})} = 86.94 \times 10^{-3} \text{ rad} =$$

Residual:  $\tau_{\text{res}} = \tau_{\text{load}} - \tau'$   $\phi_p = \phi_{\text{load}} - \phi'$

$$\begin{aligned} \text{At } \rho = c \quad \tau_{\text{res}} &= 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6 \text{ Pa} \\ &= -33.5 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{At } \rho = \rho_y \quad \tau_{\text{res}} &= 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6 \text{ Pa} \\ &= 24.6 \text{ MPa} \end{aligned}$$

$$\phi_{\text{perm}} = 104.72 \times 10^{-3} - 86.94 \times 10^{-3} = 17.78 \times 10^{-3} \text{ rad} = 1.019^\circ$$

PROBLEM 3.117

3.113 The solid circular drill rod  $AB$  is made of a steel which is assumed to be elastoplastic with  $\tau_y = 160$  MPa and  $G = 77$  GPa. Knowing that a torque  $T = 5$  kN·m is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

SOLUTION

3.117 In Prob. 3.113, determine the permanent angle of twist of the rod.

From the solution to PROBLEM 3.113

$$c = 0.025 \text{ m}, \quad J = 613.59 \times 10^{-9} \text{ m}^4, \quad \frac{\rho_r}{c} = 0.5649, \quad \rho_r = 0.014123 \text{ m}$$

After loading  $\gamma = \frac{\rho\phi}{L} \therefore \phi = \frac{L\gamma}{\rho} = \frac{L\gamma_r}{\rho_r} = \frac{L\tau_r}{\rho_r G}$

$$\phi_{\text{load}} = \frac{(10)(160 \times 10^6)}{(0.014123)(77 \times 10^9)} = 1.4713 \text{ rad} = 84.30^\circ$$

During unloading  $\phi' = \frac{TL}{GJ}$  (elastic)  $T = 5 \times 10^3 \text{ N}\cdot\text{m}$

$$\phi' = \frac{(5 \times 10^3)(10)}{(77 \times 10^9)(613.59 \times 10^{-9})} = 1.0583 \text{ rad} = 60.64^\circ$$

Permanent twist angle

$$\phi_{\text{perm}} = \phi_{\text{load}} - \phi = 1.4713 - 1.0583 = 0.4130 \text{ rad} = 23.7^\circ \quad \blacktriangleleft$$

PROBLEM 3.118

3.114 The solid circular shaft  $AB$  is made of a steel which is assumed to be elastoplastic with  $G = 11.2 \times 10^6$  psi and  $\tau_y = 21$  ksi. The torque  $T$  is increased until the radius of the elastic core is 0.25 in. Determine the maximum residual shearing stress in the shaft after the torque  $T$  has been removed.

SOLUTION

3.118 In Prob. 3.114, determine the permanent angle of twist of the shaft.

From the solution to PROBLEM 3.114,  $c = 0.75$  in,  $J = 0.49701 \text{ in}^4$

After loading  $T = 18.383 \text{ kip}\cdot\text{in}$ ,  $\rho_r = 0.25$  in,  $\tau_r = \tau_y$

$$\gamma = \frac{\rho\phi}{L} \therefore \phi = \frac{L\gamma}{\rho} = \frac{L\tau_r}{\rho_r G}$$

where  $L = 24$  in,  $\tau_r = 21$  ksi,  $G = 11.2 \times 10^6$  psi =  $11.2 \times 10^3$  psi

$$\phi_{\text{load}} = \frac{(24)(21)}{(0.25)(11.2 \times 10^3)} = 180 \times 10^{-3} \text{ rad} = 10.31^\circ$$

Unloading  $T = 18.383 \text{ kip}\cdot\text{in}$

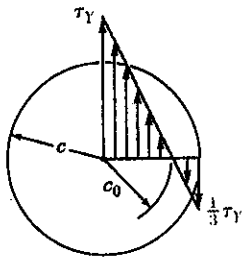
$$\phi' = \frac{TL}{GJ} = \frac{(-18.383)(24)}{(11.2 \times 10^3)(0.49701)} = 79.26 \times 10^{-3} \text{ rad} = 4.54^\circ$$

Permanent angle of twist  $\phi_{\text{perm}} = \phi_{\text{load}} - \phi'$

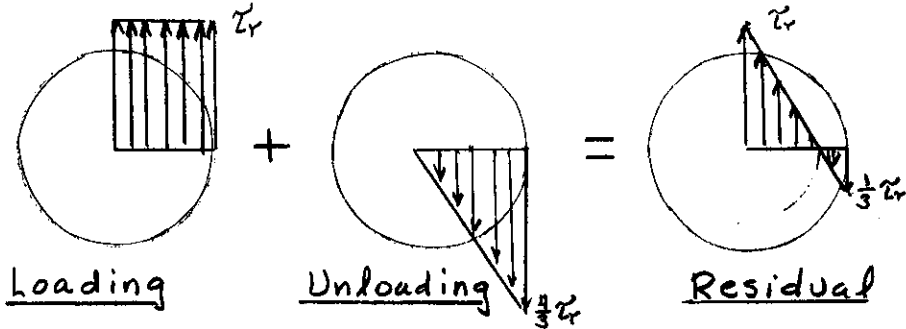
$$\phi_{\text{perm}} = 180 \times 10^{-3} - 79.26 \times 10^{-3} = 100.74 \times 10^{-3} \text{ rad} = 5.77^\circ \quad \blacktriangleleft$$

**PROBLEM 3.119**

3.119 A torque  $T$  applied to a solid rod made of an elastoplastic material is increased until the rod becomes fully plastic and then is removed. (a) Show that the distribution of residual stresses is as represented in the figure. (b) Determine the magnitude of the torque due to the stresses acting on the portion of the rod located within a circle of radius  $c_0$ .



**SOLUTION**



(a)

After loading  $p_r = 0, T_{load} = \frac{4}{3} T_y = \frac{4}{3} \frac{\pi}{2} c^3 \tau_y = \frac{2\pi}{3} c^3 \tau_y$

Unloading  $\tau' = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(T_{load})}{\pi c^3} = \frac{4}{3} \tau_y$  at  $p = c$

$$\tau' = \frac{4}{3} \tau_y \frac{p}{c}$$

Residual  $\tau_{res} = \tau_y - \frac{4}{3} \tau_y \frac{p}{c} = \tau_y \left(1 - \frac{4p}{3c}\right)$

To find  $c_0$  set  $\tau_{res} = 0$  and  $p = c_0$

$$0 = 1 - \frac{4c_0}{3c} \quad \therefore c_0 = \frac{3}{4} c$$

(b)  $T = 2\pi \int_0^{c_0} \rho^2 \tau d\rho = 2\pi \int_0^{\frac{3}{4}c} \rho^2 \tau_y \left(1 - \frac{4}{3} \frac{\rho}{c}\right) d\rho$   
 $= 2\pi \tau_y \left( \frac{\rho^3}{3} - \frac{4}{3} \frac{\rho^4}{4c} \right) \Big|_0^{\frac{3}{4}c} = 2\pi \tau_y c^3 \left\{ \frac{1}{3} \left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right) \frac{1}{4} \left(\frac{3}{4}\right)^4 \right\}$   
 $= 2\pi \tau_y c^3 \left\{ \frac{9}{64} - \frac{27}{256} \right\} = \frac{9\pi}{128} \tau_y c^3 = 0.2209 \tau_y c^3$

## PROBLEM 3.120

3.116 The solid shaft shown is made of a steel which is assumed to be elastoplastic with  $\tau_y = 145 \text{ MPa}$  and  $G = 77 \text{ GPa}$ . The torque  $T$  is increased in magnitude until the shaft has been twisted through  $6^\circ$  and then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist of the shaft.

3.120 After the solid shaft of Prob. 3.116 has been loaded and unloaded as described in that problem, a torque  $T_1$  of sense opposite to the original torque  $T$  is applied to the shaft. Assuming no change in the value of  $\phi_y$ , determine the angle of twist  $\phi_1$  for which yield is initiated in this second loading and compare it with the angle  $\phi_y$  for which the shaft started to yield in the original loading.

## SOLUTION

From the solution to PROBLEM 3.116  $c = 0.016 \text{ m}$ ,  $L = 0.6 \text{ m}$

$$\tau_y = 145 \times 10^6 \text{ Pa}, \quad J = 102.944 \times 10^{-9} \text{ m}^4$$

The residual stress at  $\rho = c$  is  $\tau_{res} = 33.5 \text{ MPa}$

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_y - \tau_{res} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J \tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717 \text{ N}\cdot\text{m}$$

Angle of twist

$$\phi_1 = \frac{T_1 L}{G J} = \frac{(717 \times 10^3)(0.6)}{(77 \times 10^9)(102.944 \times 10^{-9})} = 54.3 \times 10^{-3} \text{ rad}$$

$$= 3.11^\circ$$

## PROBLEM 3.121

3.114 The solid circular shaft  $AB$  is made of a steel which is assumed to be elastoplastic with  $G = 11.2 \times 10^6$  psi and  $\tau_y = 21$  ksi. The torque  $T$  is increased until the radius of the elastic core is 0.25 in. Determine the maximum residual shearing stress in the shaft after the torque  $T$  has been removed.

3.121 After the solid shaft of Prob. 3.114 has been loaded and unloaded as described in that problem, a torque  $T_1$  of sense opposite to the original torque  $T$  is applied to the shaft. Assuming no change in the value of  $\tau_y$ , determine the magnitude  $T_1$  of the torque  $T_1$  required to initiate yield in this second loading and compare it with the magnitude  $T_y$  of the torque  $T$  which caused the shaft to yield in the original loading.

## SOLUTION

From the solution to PROBLEM 3.114  $c = 0.75$  in,  $L = 24$  in.

$$\tau_y = 21 \text{ ksi}, \quad J = 0.49701 \text{ in}^4$$

The residual stress at  $\rho = c$  is  $\tau_{res} = 6.74$  ksi

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_y - \tau_{res} = 21 - 6.74 = 14.26 \text{ ksi}$$

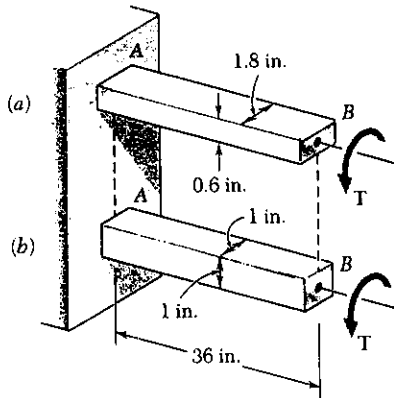
$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J \tau_1}{c} = \frac{(0.49701)(14.26)}{0.75} = 9.45 \text{ kip}\cdot\text{in.} \quad \blacktriangleleft$$

$$\therefore \quad T_y = \frac{J \tau_y}{c} = \frac{(0.49701)(21)}{0.75} = 13.92 \text{ kip}\cdot\text{in.} \quad \blacktriangleleft$$



PROBLEM 3.122

3.122 Knowing that the magnitude of the torque  $T$  is 1800 lb·in., determine for each of the aluminum bars shown the maximum shearing stress and the angle of twist at end  $B$ . Use  $G = 3.9 \times 10^6$  psi.



SOLUTION

$$T = 1800 \text{ lb}\cdot\text{in.}, \quad L = 36 \text{ in.}$$

$$(a) \quad a = 1.8 \text{ in.}, \quad b = 0.6 \text{ in.} \quad \frac{a}{b} = \frac{1.8}{0.6} = 3$$

$$\text{From Table 3.1} \quad C_1 = 0.267, \quad C_2 = 0.263$$

$$\tau_{\max} = \frac{T}{C_1 a b^2} = \frac{1800}{(0.267)(1.8)(0.6)^2}$$

$$= 10.40 \times 10^3 \text{ psi} = 10.40 \text{ ksi}$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{(1800)(36)}{(0.263)(1.8)(0.6)^3(3.9 \times 10^6)} = 162.5 \times 10^{-3} \text{ rad} = 9.31^\circ$$

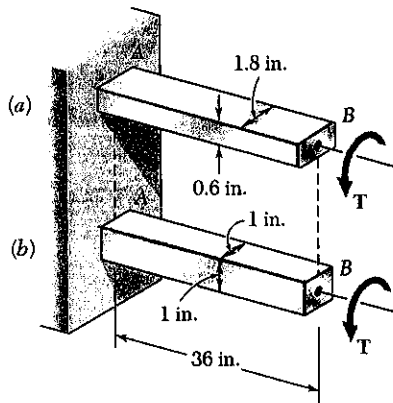
$$(b) \quad a = 1.0 \text{ in.}; \quad b = 1.0 \text{ in.}, \quad \frac{a}{b} = 1.00, \quad C_1 = 0.208, \quad C_2 = 0.1406$$

$$\tau_{\max} = \frac{T}{C_1 a b^2} = \frac{1800}{(0.208)(1.0)(1.0)^2} = 8.65 \times 10^3 \text{ psi} = 8.65 \text{ ksi}$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{(1800)(36)}{(0.1406)(1.0)(1.0)^3(3.9 \times 10^6)} = 118.2 \times 10^{-3} \text{ rad} = 6.77^\circ$$

PROBLEM 3.123

3.123 Using  $\tau_{all} = 10$  ksi, determine for each of the aluminum bars shown the largest torque  $T$  which may be applied and the corresponding angle of twist. Use  $G = 3.9 \times 10^6$  psi.



SOLUTION

$$\tau_{all} = 10 \text{ ksi}, \quad G = 3.9 \times 10^6 \text{ psi} = 3.9 \times 10^3 \text{ ksi}$$

$$(a) \quad a = 1.8 \text{ in.}, \quad b = 0.6 \text{ in.}, \quad \frac{a}{b} = \frac{1.8}{0.6} = 3$$

$$\text{From Table 3.1} \quad C_1 = 0.267, \quad C_2 = 0.263$$

$$\tau_{max} = \frac{T}{C_1 a b^2} \quad \therefore T = C_1 a b^2 \tau_{max}$$

$$T = (0.267)(1.8)(0.6)^2(10) = 1.730 \text{ kip}\cdot\text{in} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{(1.730)(36)}{(0.263)(1.8)(0.6)^3(3.9 \times 10^3)} = 156.2 \times 10^{-3} \text{ rad}$$

$$= 8.95^\circ \quad \blacktriangleleft$$

$$(b) \quad a = 1.0 \text{ in.}, \quad b = 1.0 \text{ in.}, \quad \frac{a}{b} = 1.00 \quad C_1 = 0.208, \quad C_2 = 0.1406$$

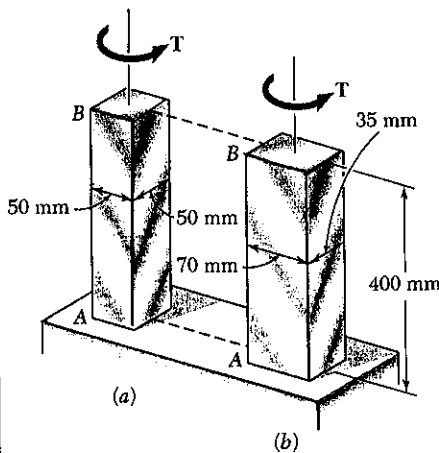
$$T = C_1 a b^2 \tau_{max} = (0.208)(1.0)(1.0)^2(10) = 2.08 \text{ kip}\cdot\text{in} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{(2.08)(36)}{(0.1406)(1.0)(1.0)^3(3.9 \times 10^3)} = 136.6 \times 10^{-3} \text{ rad}$$

$$= 7.82^\circ \quad \blacktriangleleft$$

PROBLEM 3.124

3.124 Knowing that  $T = 800 \text{ N}\cdot\text{m}$ , determine for each of the cold-rolled yellow brass bars shown the maximum shearing stress and the angle of twist at end  $B$ . Use  $G = 39 \text{ GPa}$ .



SOLUTION

$$T = 800 \text{ N}\cdot\text{m} \quad L = 400 \text{ mm} = 0.400 \text{ m}$$

$$G = 39 \times 10^9 \text{ Pa}$$

$$(a) \quad a = 50 \text{ mm} = 0.050 \text{ m}, \quad b = 50 \text{ mm} = 0.050 \text{ m}$$

$$\frac{a}{b} = \frac{50}{50} = 1.00$$

From Table 3.1,  $C_1 = 0.208$ ,  $C_2 = 0.1406$

$$\tau_{\max} = \frac{T}{C_1 a b^2} = \frac{800}{(0.208)(0.050)(0.050)^2} = 30.8 \times 10^6 \text{ Pa} = 30.8 \text{ MPa} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{(800)(0.400)}{(0.1406)(0.05)^3(39 \times 10^9)} = 9.33 \times 10^{-3} \text{ rad} \\ = 0.535^\circ \quad \blacktriangleleft$$

$$(b) \quad a = 70 \text{ mm} = 0.070 \text{ m}, \quad b = 35 \text{ mm} = 0.035 \text{ m}, \quad \frac{a}{b} = \frac{70}{35} = 2.0$$

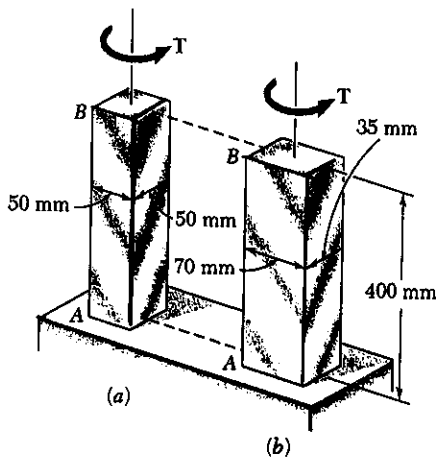
$$C_1 = 0.246, \quad C_2 = 0.229$$

$$\tau_{\max} = \frac{T}{C_1 a b^2} = \frac{800}{(0.246)(0.070)(0.035)^2} = 37.9 \times 10^6 \text{ Pa} = 37.9 \text{ MPa} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{C_2 a b^3 G} = \frac{(800)(0.400)}{(0.229)(0.070)(0.035)^3(39 \times 10^9)} = 11.94 \times 10^{-3} \text{ rad} \\ = 0.684^\circ \quad \blacktriangleleft$$

PROBLEM 3.125

3.125 Using  $\tau_{all} = 50 \text{ MPa}$ , determine for each of the cold-rolled yellow brass bars shown the largest torque  $T$  which may be applied and the corresponding angle of twist. Use  $G = 39 \text{ GPa}$ .



SOLUTION

$$\tau_{all} = 50 \times 10^6 \text{ Pa}, \quad L = 400 \text{ mm} = 0.400 \text{ m}$$

$$G = 39 \times 10^9 \text{ Pa}$$

$$(a) \quad a = 50 \text{ mm} = 0.050 \text{ m}, \quad b = 50 \text{ mm} = 0.050 \text{ m}$$

$$\frac{a}{b} = 1.00$$

$$\text{From Table 3.1} \quad c_1 = 0.208, \quad c_2 = 0.1406$$

$$\tau_{max} = \frac{T}{c_1 a b^2}$$

$$T = c_1 a b^2 \tau_{max} = (0.208)(0.050)(0.050)^2 (50 \times 10^6) = 1300 \text{ N}\cdot\text{m}$$

$$= 1.300 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(1300)(0.400)}{(0.1406)(0.050)(0.050)^3 (39 \times 10^9)} = 15.17 \times 10^{-3} \text{ rad}$$

$$= 0.869^\circ \quad \blacktriangleleft$$

$$(b) \quad a = 70 \text{ mm} = 0.070 \text{ m}, \quad b = 35 \text{ mm} = 0.035 \text{ m}, \quad \frac{a}{b} = \frac{70}{35} = 2.0$$

$$\text{From Table 3.1} \quad c_1 = 0.246, \quad c_2 = 0.229$$

$$T = c_1 a b^2 \tau_{max} = (0.246)(0.070)(0.035)^2 (50 \times 10^6) = 1055 \text{ N}\cdot\text{m}$$

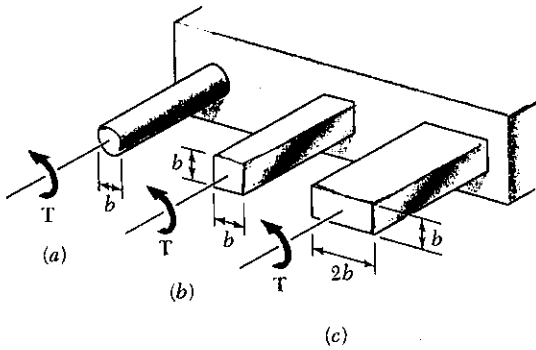
$$= 1.055 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$\phi = \frac{TL}{c_2 a b^3 G} = \frac{(1055)(0.400)}{(0.229)(0.070)(0.035)^3 (39 \times 10^9)} = 15.74 \times 10^{-3} \text{ rad}$$

$$= 0.902^\circ \quad \blacktriangleleft$$

PROBLEM 3.126

3.126 A 2-kip·in. torque  $T$  is applied to each of the steel bars shown. Knowing that  $\tau_{all} = 6$  ksi, determine the required dimension  $b$  for each bar.



SOLUTION

$$T = 2 \text{ kip} \cdot \text{in} \quad \tau_{max} = 6 \text{ ksi}$$

(a) circle:  $c = \frac{1}{2} b$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

$$b^3 = \frac{16T}{\pi \tau_{max}} = \frac{(16)(2)}{\pi(6)} = 1.698 \text{ in}^3$$

$$b = 1.193 \text{ in} \quad \blacktriangleleft$$

(b) square:  $a = b, \frac{a}{b} = 1.0$ . From Table 3.1  $c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3} \therefore b^3 = \frac{T}{c_1 \tau_{max}} = \frac{2}{(0.208)(6)} = 1.603 \text{ in}^3$$

$$b = 1.170 \text{ in} \quad \blacktriangleleft$$

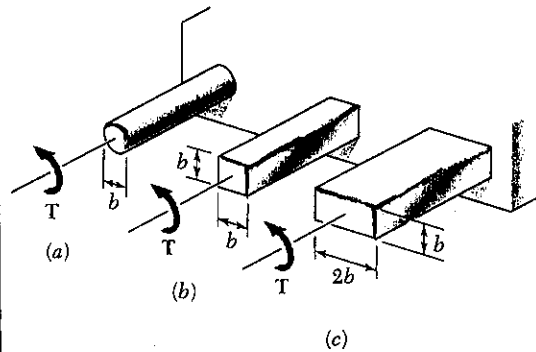
(c) rectangle:  $a = 2b, \frac{a}{b} = 2.0, c_1 = 0.246$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{2c_1 b^3} \therefore b^3 = \frac{T}{2c_1 \tau_{max}} = \frac{2}{(2)(0.246)(6)} = 0.668 \text{ in}^3$$

$$b = 0.878 \text{ in} \quad \blacktriangleleft$$

PROBLEM 3.127

3.127 A 300-N·m torque  $T$  is applied to each of the aluminum bars shown. Knowing that  $\tau_{all} = 60$  MPa, determine the required dimension  $b$  for each bar.



SOLUTION

$$T = 300 \text{ N} \cdot \text{m} \quad \tau_{max} = 60 \times 10^6 \text{ Pa}$$

(a) circle:  $c = \frac{1}{2} b$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$$

$$b^3 = \frac{16T}{\pi \tau_{max}} = \frac{(16)(300)}{\pi(60 \times 10^6)} = 25.46 \times 10^{-6} \text{ m}^3$$

$$b = 29.4 \times 10^{-3} \text{ m} = 29.4 \text{ mm} \quad \blacktriangleleft$$

(b) square:  $a = b, \frac{a}{b} = 1.0$ . From Table 3.1  $c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3} \therefore b^3 = \frac{T}{c_1 \tau_{max}} = \frac{300}{(0.208)(60 \times 10^6)} = 24.04 \times 10^{-6} \text{ m}^3$$

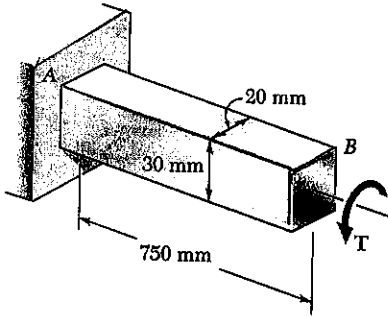
$$b = 28.9 \times 10^{-3} \text{ m} = 28.9 \text{ mm} \quad \blacktriangleleft$$

(c) rectangle:  $a = 2b, \frac{a}{b} = 2.0, c_1 = 0.246$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{2c_1 b^3} \therefore b^3 = \frac{T}{2c_1 \tau_{max}} = \frac{300}{(2)(0.246)(60 \times 10^6)} = 10.16 \times 10^{-6} \text{ m}^3$$

$$b = 21.7 \times 10^{-3} \text{ m} = 21.7 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 3.128**



3.128 The torque  $T$  causes a rotation of  $2^\circ$  at end  $B$  of the stainless steel bar shown. Knowing that  $G = 77$  GPa, determine the maximum shearing stress in the bar.

**SOLUTION**

$$a = 30 \text{ mm} = 0.030 \text{ m}, \quad b = 20 \text{ mm} = 0.020 \text{ m}$$

$$\phi = 2^\circ = 34.907 \times 10^{-3} \text{ rad}$$

$$\phi = \frac{TL}{C_1 a b^3 G} \quad \therefore \quad T = \frac{C_2 a b^3 G \phi}{L}$$

$$\tau_{\max} = \frac{TL}{C_1 a b^2} = \frac{C_2 a b^3 G \phi}{C_1 a b^2 L} = \frac{C_2 b G \phi L}{C_1 L}$$

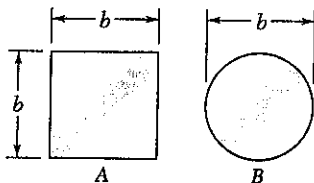
$$\frac{a}{b} = \frac{30}{20} = 1.5$$

From Table 3.1  $C_1 = 0.231$   $C_2 = 0.1958$

$$\tau_{\max} = \frac{(0.1958)(0.020)(77 \times 10^9)(34.907 \times 10^{-3})}{(0.231)(0.750)} = 80.8 \times 10^6 \text{ Pa}$$

$$= 80.8 \text{ MPa}$$

**PROBLEM 3.129**



3.129 Two shafts are made of the same material. The cross section of shaft  $A$  is a square of side  $b$  and that of shaft  $B$  is a circle of diameter  $b$ . Knowing that the shafts are subjected to the same torque, determine the ratio of the maximum shearing stresses occurring in the shafts.

A. square  $\frac{a}{b} = 1$ ,  $C_1 = 0.208$  (Table 3.1)

$$\tau_A = \frac{T}{C_1 a b^2} = \frac{T}{0.208 b^3}$$

B. circle  $c = \frac{1}{2}b$   $\tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$

Ratio  $\frac{\tau_A}{\tau_B} = \frac{1}{0.208} \cdot \frac{\pi}{16} = 0.3005 \pi = 0.944$

**PROBLEM 3.130**

3.130 Determine the largest allowable square cross section of a steel shaft of length 4 m if the maximum shearing stress is not to exceed 100 MPa when the shaft is twisted through one complete revolution. Use  $G = 77$  GPa.

**SOLUTION**

$$\phi = 2\pi \text{ rad}, \quad L = 4 \text{ m}, \quad \tau_{\max} = 100 \times 10^6 \text{ Pa} \quad G = 77 \times 10^9 \text{ Pa}$$

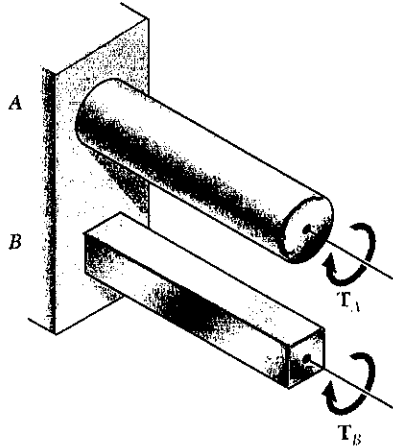
square section:  $C_1 = 0.208$  and  $C_2 = 0.1406$  from Table 3.1

$$\tau_{\max} = \frac{T}{C_1 a b^2} = \frac{T}{C_1 b^3}, \quad \phi = \frac{TL}{C_2 a b^3 G} = \frac{TL}{C_2 b^4 G}$$

$$\frac{\tau_{\max}}{\phi} = \frac{T}{C_1 b^3} \cdot \frac{C_2 b^4 G}{TL} \quad \therefore \quad b = \frac{C_1 \tau_{\max} L}{C_2 G \phi} = \frac{(0.208)(100 \times 10^6)(4.0)}{(0.1406)(77 \times 10^9)(2\pi)}$$

$$= 1.223 \times 10^{-3} \text{ m} = 1.223 \text{ mm}$$

**PROBLEM 3.131**



3.131 Shafts *A* and *B* are made of the same material and have the same cross sectional area, but *A* has a circular cross section and *B* has a square cross section. Determine the ratio of the maximum torques  $T_A$  and  $T_B$  which may be safely applied to *A* and *B*, respectively.

**SOLUTION**

Let  $c$  = radius of circular section *A* and  $b$  = side of square section *B*.

For equal areas  $\pi c^2 = b^2$

$$c = \frac{b}{\sqrt{\pi}}$$

Circle:  $\tau_A = \frac{T_A c}{J} = \frac{2 T_A}{\pi c^3} \therefore T_A = \frac{\pi}{2} c^3 \tau_A$

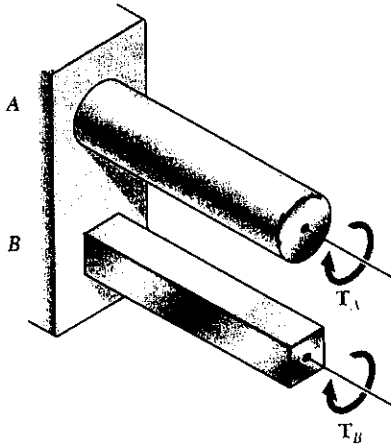
Square:  $C_1 = 0.208$  from Table 3.1

$$\tau_B = \frac{T_B}{C_1 a b^2} = \frac{T_B}{C_1 b^3} \therefore T_B = C_1 b^3 \tau_B$$

$$\text{Ratio: } \frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_B}{C_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_B}{C_1 b^3 \tau_B} = \frac{1}{2 C_1 \sqrt{\pi}} \frac{\tau_A}{\tau_B}$$

For the same stresses  $\tau_B = \tau_A \therefore \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}} = 1.356 \blacktriangleleft$

**PROBLEM 3.132**



3.132 Shafts *A* and *B* are made of the same material and have the same length and cross sectional area, but *A* has a circular cross section and *B* has a square cross section. Determine the ratio of the maximum values if the angles  $\phi_A$  and  $\phi_B$  through which shafts *A* and *B*, respectively, may be twisted.

**SOLUTION**

Let  $c$  = radius of circular section *A* and  $b$  = side of square section *B*.

For equal areas  $\pi c^2 = b^2 \therefore b = \sqrt{\pi} c$

Circle:  $\gamma_{\max} = \frac{\tau_A}{G} = \frac{C \phi_A}{L} \therefore \phi_A = \frac{L \tau_A}{C G}$

Square: Table 3.1  $C_1 = 0.208, C_2 = 0.1406$

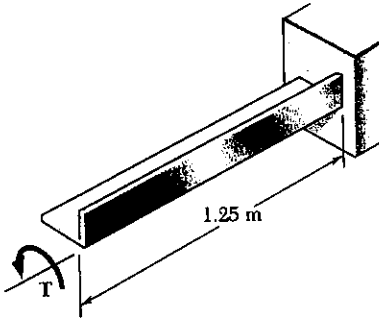
$$\tau_B = \frac{T_B}{C_1 a b^2} = \frac{T_B}{0.208 b^3} \therefore T_B = 0.208 b^3 \tau_B$$

$$\phi_B = \frac{T_B L}{C_2 a b^3 G} = \frac{0.208 b^3 \tau_B L}{0.1406 b^4 G} = \frac{1.4794 L \tau_B}{b G}$$

$$\text{Ratio } \frac{\phi_A}{\phi_B} = \frac{L \tau_A}{C G} \cdot \frac{b G}{1.4794 L \tau_B} = 0.676 \frac{b \tau_A}{C \tau_B} = 0.676 \sqrt{\pi} \frac{\tau_A}{\tau_B}$$

For equal stresses  $\tau_A = \tau_B \quad \frac{\phi_B}{\phi_A} = 0.676 \sqrt{\pi} = 1.198 \blacktriangleleft$

**PROBLEM 3.133**



3.133 A 1.25-m-long steel angle has an L 127 × 76 × 6.4 cross section. From Appendix C we find that the thickness of the section is 6.4 mm and that its area is 1252 mm<sup>2</sup>. Knowing that  $\tau_{all} = 60$  MPa,  $G = 77$  GPa, and ignoring the effect of stress concentrations, determine (a) the largest torque  $T$  which may be applied, (b) the corresponding angle of twist.

**SOLUTION**

$$A = 1252 \text{ mm}^2 \quad b = 6.4 \text{ mm} = 0.0064 \text{ m}$$

$$a = \frac{A}{b} = \frac{1252}{6.4} = 195.6 \text{ mm} = 0.1956 \text{ m}$$

$$\frac{a}{b} = \frac{195.6}{6.4} = 30.56$$

$$c_1 = c_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right) = 0.3265$$

$$\gamma_{max} = \frac{T}{c_1 a b^2} \therefore T = c_1 a b^2 \gamma_{max}$$

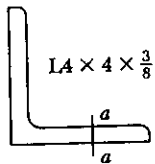
$$(a) \quad T = (0.3265)(0.1956)(0.0064)^2(60 \times 10^6) = 156.95 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 157.0 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$(b) \quad \phi = \frac{TL}{c_2 a b^3 G} = \frac{c_1 a b^2 \gamma_{max} L}{c_2 a b^3 G} = \frac{c_1 \gamma_{max} L}{c_2 b G} = \frac{\gamma_{max} L}{b G}$$

$$\phi = \frac{(60 \times 10^6)(1.25)}{(0.0064)(77 \times 10^9)} = 152.19 \times 10^{-3} \text{ rad} = 8.72^\circ \quad \blacktriangleleft$$

**PROBLEM 3.134**



**SOLUTION**

3.134 A 3000 lb-in. torque is applied to a 6-ft-long steel angle with a L 4 × 4 ×  $\frac{3}{8}$  cross section. From Appendix C we find that the thickness of the section is  $\frac{3}{8}$  in. and that its area is 2.86 in<sup>2</sup>. Knowing that  $G = 11.2 \times 10^6$  psi, determine (a) the maximum shearing stress along line  $a-a$ , (b) the angle of twist.

$$A = 2.86 \text{ in}^2, \quad b = \frac{3}{8} \text{ in} = 0.375 \text{ in}, \quad a = \frac{A}{b} = \frac{2.86}{0.375} = 7.627 \text{ in.}$$

$$\frac{a}{b} = \frac{7.627}{0.375} = 20.34 \quad c_1 = c_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right) = 0.3230$$

$$(a) \quad \gamma_{max} = \frac{T}{c_1 a b^2} = \frac{3000}{(0.3230)(7.627)(0.375)^2} = 8.66 \times 10^3 \text{ psi} = 8.66 \text{ ksi} \quad \blacktriangleleft$$

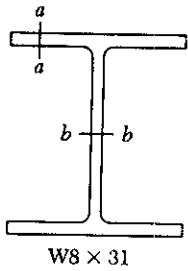
$$(b) \quad \phi = \frac{TL}{c_2 a b^3 G} = \frac{(3000)(72)}{(0.3230)(7.627)(0.375)^3(11.2 \times 10^6)} = 148.45 \times 10^{-3} \text{ rad}$$

$$= 8.51^\circ \quad \blacktriangleleft$$

Note:  $L = 6 \text{ ft.} = 72 \text{ in.}$



PROBLEM 3.135



3.135 An 8-ft-long steel member with a W 8 × 31 cross section is subjected to a 5 kip·in. torque. From Appendix C we find that the thickness of the section is  $\frac{3}{8}$  in. and that its area is  $2.86 \text{ in}^2$ . Knowing that  $G = 11.2 \times 10^6$  psi, determine (a) the maximum shearing stress along line a-a, (b) the maximum shearing stress along line b-b, (c) the angle of twist. (Hint: Consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

Flange:  $a = 7.995 \text{ in}$ ,  $b = 0.435$ ,  $\frac{a}{b} = \frac{7.995}{0.435} = 18.38$

$$C_1 = C_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right) = 0.3219 \quad \phi_F = \frac{T_F L}{C_2 a b^3 G}$$

$$T_F = C_2 a b^3 \frac{G \phi_F}{L} = K_F \frac{G \phi}{L} \quad \text{where } K_F = C_2 a b^3$$

$$K_F = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^3$$

Web:  $a = 8.0 - (2)(0.435) = 7.13 \text{ in}$ ,  $b = 0.285 \text{ in}$ ,  $\frac{a}{b} = \frac{7.13}{0.285} = 25.02$

$$C_1 = C_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right) = 0.3249 \quad \phi_w = \frac{T_w L}{C_2 a b^3 G}$$

$$T_w = C_2 a b^3 \frac{G \phi_w}{L} = K_w \frac{G \phi}{L} \quad \text{where } K_w = C_2 a b^3$$

$$K_w = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^3$$

For matching twist angles  $\phi_F = \phi_w = \phi$

$$\text{Total torque } T = 2T_F + T_w = (2K_F + K_w) \frac{G \phi}{L}$$

$$\frac{G \phi}{L} = \frac{T}{2K_F + K_w}, \quad T_F = \frac{K_F T}{2K_F + K_w}, \quad T_w = \frac{K_w T}{2K_F + K_w}$$

$$T_F = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2221 \text{ lb}\cdot\text{in}; \quad T_w = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb}\cdot\text{in}$$

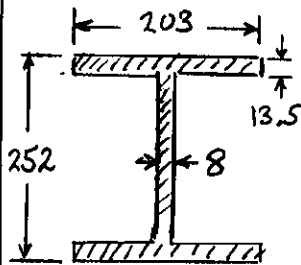
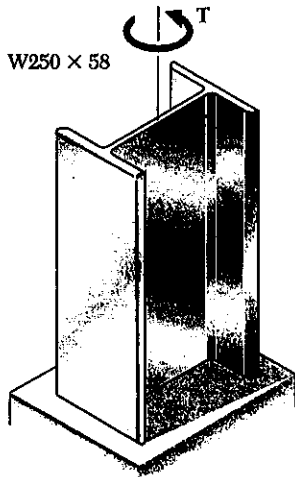
(a)  $\tau_F = \frac{T_F}{C_1 a b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi} = 4.57 \text{ ksi} \quad \blacktriangleleft$

(b)  $\tau_w = \frac{T_w}{C_1 a b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi} = 2.96 \text{ ksi} \quad \blacktriangleleft$

(c)  $\frac{G \phi}{L} = \frac{T}{2K_F + K_w} \quad \therefore \phi = \frac{T L}{G(2K_F + K_w)} \quad \text{where } L = 8 \text{ ft} = 96 \text{ in.}$

$$\phi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.0563]} = 88.6 \times 10^{-3} \text{ rad} = 5.08^\circ \quad \blacktriangleleft$$

PROBLEM 3.136



3.136 A 3-m-long steel member has an W 250 × 58 cross section. Knowing that  $G = 77$  GPa and that the allowable shearing stress is 35 MPa, determine (a) the largest torque  $T$  which may be applied, (b) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.135.)

SOLUTION

Flange:  $a = 203$  mm,  $b = 13.5$  mm,  $\frac{a}{b} = 15.04$

$$C_1 = C_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right) = 0.3194$$

$$\phi_F = \frac{T_F L}{C_2 a b^3 G} \quad \therefore T_F = C_2 a b^3 \frac{G \phi}{L} = K_F \frac{G \phi}{L}$$

$$K_F = (0.3194)(0.203)(0.0135)^3 = 159.53 \times 10^{-9} \text{ m}^4$$

Web:  $a = 252 - (2)(13.5) = 225$  mm,  $b = 8$  mm

$$\frac{a}{b} = 28.13, \quad C_1 = C_2 = \frac{1}{3} \left( 1 - 0.63 \frac{b}{a} \right) = 0.3259$$

$$\phi_w = \frac{T_w L}{C_2 a b^3 G} \quad \therefore T_w = C_2 a b^3 \frac{G \phi}{L} = K_w \frac{G \phi}{L}$$

$$K_w = (0.3259)(0.225)(0.008)^3 = 37.54 \times 10^{-9} \text{ m}^4$$

For matching twist angles  $\phi_F = \phi_w = \phi$

Total torque:  $T = 2T_F + T_w = (2K_F + K_w) \frac{G \phi}{L}$

$$\frac{G \phi}{L} = \frac{T}{2K_F + K_w}, \quad T_F = \frac{K_F T}{2K_F + K_w} \quad \therefore T = \frac{2K_F + K_w}{K_F} T_F$$

$$T_w = \frac{K_w T}{2K_F + K_w} \quad \therefore T = \frac{2K_F + K_w}{K_w} T_w$$

Allowable value for  $T$  based on allowable value for  $T_F$

$$T_F = C_1 a b^2 \tau = (0.3194)(0.203)(0.0135)^2 (35 \times 10^6) = 413.6 \text{ N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + (37.54)}{159.53} (413.6) = 924.5 \text{ N}\cdot\text{m}$$

Allowable value for  $T$  base on allowable value for  $T_w$

$$T_w = C_1 a b^2 \tau = (0.3259)(0.225)(0.008)^2 (35 \times 10^6) = 164.25 \text{ N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + 37.54}{37.54} (164.25) = 1560 \text{ N}\cdot\text{m}$$

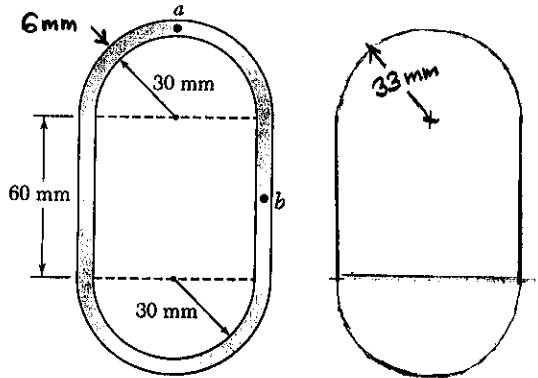
Choose smaller value

$$T = 925 \text{ N}\cdot\text{m}$$

$$\phi = \frac{TL}{(2K_F + K_w)G} = \frac{(924.5)(3.00)}{(356.6 \times 10^{-9})(77 \times 10^9)} = 101.0 \times 10^{-5} \text{ rad} = 5.79^\circ$$

PROBLEM 3.137

3.137 and 3.138 A 750-N·m torque  $T$  is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points  $a$  and  $b$ . Thickness = 6 mm.



SOLUTION

Area bounded by center line

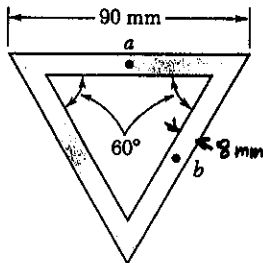
$$Q = 2 \frac{\pi}{2} (33)^2 + (60)(66) = 7381 \text{ mm}^2 = 7381 \times 10^{-6} \text{ m}^2$$

$$t = 0.006 \text{ m at both } a \text{ and } b.$$

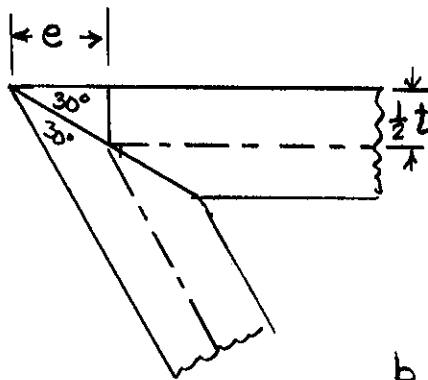
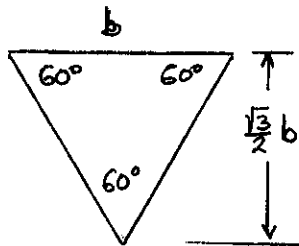
$$\tau = \frac{T}{2tQ} = \frac{750}{(2)(0.006)(7381 \times 10^{-6})} = 8.47 \times 10^6 \text{ Pa} = 8.47 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.138

3.137 and 3.138 A 750-N·m torque  $T$  is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points  $a$  and  $b$ . Thickness = 8 mm.



SOLUTION



Detail of corner

$$\frac{1}{2}t = e \tan 30^\circ$$

$$e = \frac{t}{2 \tan 30^\circ}$$

$$= \frac{8}{2 \tan 30^\circ} = 6.928 \text{ mm}$$

$$b = 90 - 2e = 76.144 \text{ mm}$$

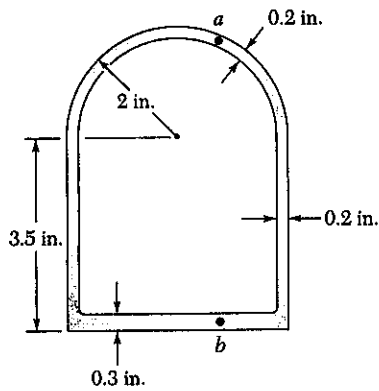
Area bounded by center line

$$Q = \frac{1}{2} b \frac{\sqrt{3}}{2} b = \frac{\sqrt{3}}{4} b^2 = \frac{\sqrt{3}}{4} (76.144)^2 = 2510.6 \text{ mm}^2 = 2510.6 \times 10^{-6} \text{ m}^2$$

$$t = 0.008 \text{ m}$$

$$\tau = \frac{T}{2tQ} = \frac{750}{(2)(0.008)(2510.6 \times 10^{-6})} = 18.67 \times 10^6 \text{ Pa} = 18.67 \text{ MPa} \blacktriangleleft$$

**PROBLEM 3.139**

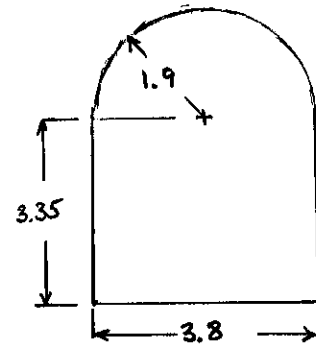


3.139 and 3.140 A 50-kip-in. torque  $T$  is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points  $a$  and  $b$ .

**SOLUTION**

Area bounded by centerline.

$$A = \frac{\pi}{2}(1.9)^2 + (3.35)(3.8) = 18.40 \text{ in}^2$$



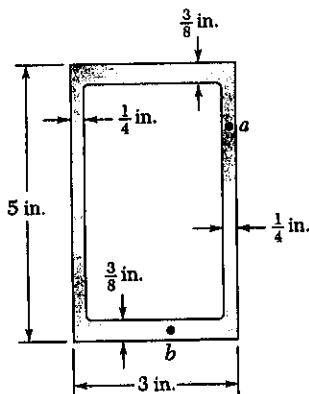
At point  $a$   $t = 0.2 \text{ in}$

$$\tau = \frac{T}{2tA} = \frac{50}{(2)(0.2)(18.40)} = 6.79 \text{ ksi}$$

At point  $b$   $t = 0.3 \text{ in}$

$$\tau = \frac{T}{2tA} = \frac{50}{(2)(0.3)(18.40)} = 4.53 \text{ ksi}$$

**PROBLEM 3.140**

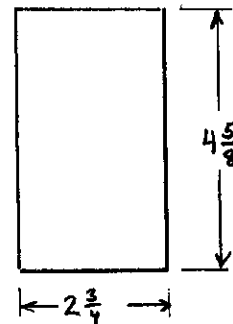


3.139 and 3.140 A 50-kip-in. torque  $T$  is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points  $a$  and  $b$ .

**SOLUTION**

Area bounded by centerline

$$A = (2\frac{3}{4})(4\frac{5}{8}) = 12\frac{23}{32} = 12.719 \text{ in}^2$$



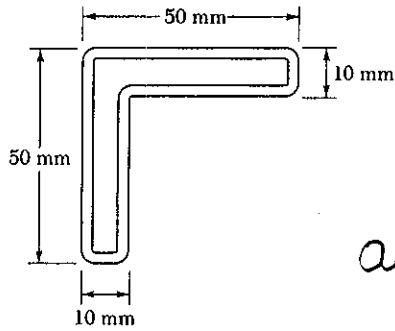
At point  $a$   $t = \frac{1}{4} \text{ in} = 0.25 \text{ in}$

$$\tau = \frac{T}{2tA} = \frac{50}{(2)(0.25)(12.719)} = 7.86 \text{ ksi}$$

At point  $b$   $t = \frac{3}{8} \text{ in} = 0.375 \text{ in}$

$$\tau = \frac{T}{2tA} = \frac{50}{(2)(0.375)(12.719)} = 5.24 \text{ ksi}$$

**PROBLEM 3.141**



**3.141 and 3.142** A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque which may be applied to the member.

**SOLUTION**

Area bounded by centerline

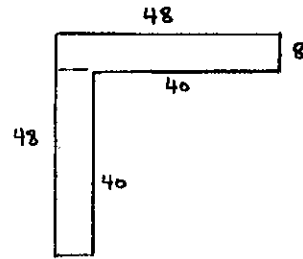
$$A = (48)(8) + (40)(8)$$

$$= 704 \text{ mm}^2 = 704 \times 10^{-6} \text{ m}^2$$

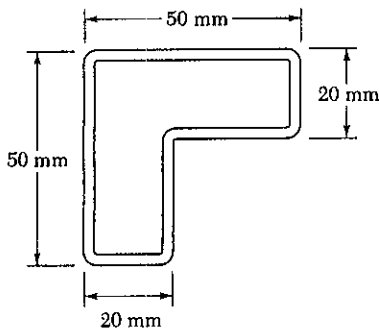
$$t = 0.002 \text{ m}$$

$$\tau = \frac{T}{2tA} \quad \therefore \quad T = 2tA\tau = (2)(0.002)(704 \times 10^{-6})(3 \times 10^6)$$

$$= 8.45 \text{ N}\cdot\text{m}$$



**PROBLEM 3.142**



**3.141 and 3.142** A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque which may be applied to the member.

**SOLUTION**

Area bounded by centerline

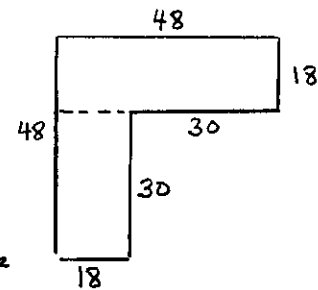
$$A = (48)(18) + (30)(18)$$

$$= 1404 \text{ mm}^2 = 1404 \times 10^{-6} \text{ m}^2$$

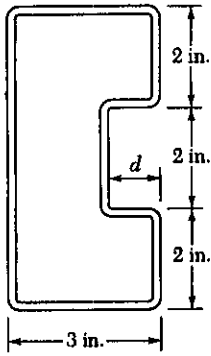
$$t = 0.002 \text{ m}$$

$$\tau = \frac{T}{2tA} \quad \therefore \quad T = 2tA\tau = (2)(0.002)(1404 \times 10^{-6})(3 \times 10^6)$$

$$= 16.85 \text{ N}\cdot\text{m}$$



**PROBLEM 3.143**



3.143 and 3.144 A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb·in. torque will be applied to the member, determine the smallest dimension  $d$  which may be used if the shearing stress is not to exceed 750 psi.

**SOLUTION**

Area bounded by centerline

$$A = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

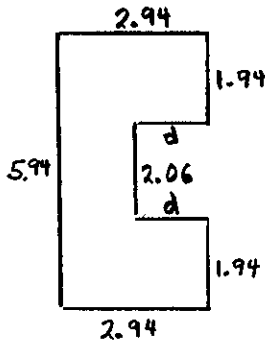
$$t = 0.06 \text{ in.}, \tau = 750 \text{ psi}, T = 1250 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{T}{2tA}$$

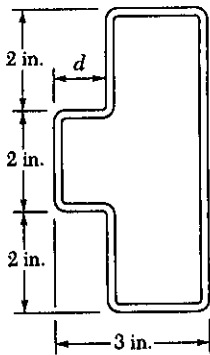
$$A = \frac{T}{2t\tau}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735 \text{ in}$$



**PROBLEM 3.144**



3.143 and 3.144 A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb·in. torque will be applied to the member, determine the smallest dimension  $d$  which may be used if the shearing stress is not to exceed 750 psi.

**SOLUTION**

Area bounded by center

$$A = (5.94)(2.94 - d) + 1.94d = 17.4636 - 4.00d$$

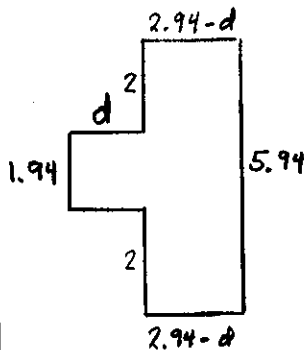
$$t = 0.06 \text{ in.}, \tau = 750 \text{ psi}, T = 1250 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{T}{2tA}$$

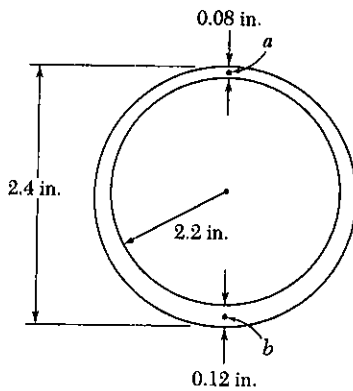
$$A = \frac{T}{2t\tau}$$

$$17.4636 - 4.00d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in.}$$



**PROBLEM 3.145**



3.145 A hollow cylindrical shaft was designed to have a uniform wall thickness of 0.1 in. Defective fabrication, however, resulted in the shaft having the cross section shown. Knowing that a 15-kip·in. torque  $T$  is applied to the shaft, determine the shearing stress at points  $a$  and  $b$ .

**SOLUTION**

Radius of outer circle = 1.2 in

Radius of inner circle = 1.1 in

Mean radius = 1.15 in

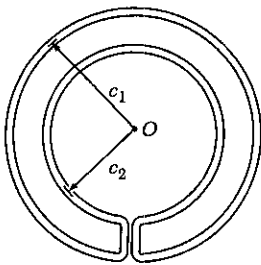
Area bounded by centerline

$$A = \pi r_m^2 = \pi (1.15)^2 = 4.155 \text{ in}^2$$

At point  $a$   $t = 0.08 \text{ in}$   $\tau = \frac{T}{2tA} = \frac{15}{(2)(0.08)(4.155)} = 22.6 \text{ ksi}$   $\blacktriangleleft$

At point  $b$   $t = 0.12 \text{ in}$   $\tau = \frac{T}{2tA} = \frac{15}{(2)(0.12)(4.155)} = 15.04 \text{ ksi}$   $\blacktriangleleft$

**PROBLEM 3.146**



3.146 A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3 mm thickness. The radii  $c_1 = 150 \text{ mm}$  and  $c_2 = 100 \text{ mm}$  are measured to the centerline of the sheet metal. Knowing that a torque of magnitude  $T = 3 \text{ kN}\cdot\text{m}$  is applied to the tube, determine (a) the maximum shearing stress in the tube, (b) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

**SOLUTION**

Area bounded by centerline

$$A = \pi (c_1^2 - c_2^2) = \pi (150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$$

$$= 39.27 \times 10^{-3} \text{ m}^2$$

$t = 0.003 \text{ m}$

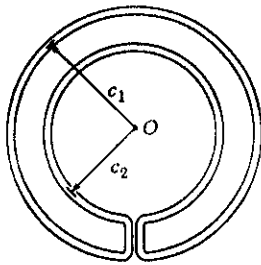
(a)  $\tau = \frac{T}{2tA} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa} = 12.73 \text{ MPa}$   $\blacktriangleleft$

(b)  $T_1 = (2\pi c_1 t \tau c_1) = 2\pi c_1^2 t \tau$

$$= 2\pi (0.150)^2 (0.003) (12.73 \times 10^6) = 5.46 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 5.40 \text{ kN}\cdot\text{m}$$
  $\blacktriangleleft$

PROBLEM 3.147



3.147 A cooling tube having the cross section as shown is formed from a sheet of stainless steel of thickness  $t$ . The radii  $c_1$  and  $c_2$  are measured to the centerline of the sheet metal. Knowing that a torque  $T$  applied to the tube, determine in terms of  $T$ ,  $c_1$ ,  $c_2$ , and  $t$  the maximum shearing stress in the tube.

SOLUTION

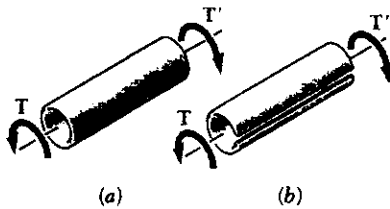
Area bounded by centerline

$$A = \pi (c_1^2 - c_2^2)$$

Shearing stress

$$\tau = \frac{T}{2tA} = \frac{T}{2\pi t(c_1^2 - c_2^2)}$$

PROBLEM 3.148



3.148 Equal torques are applied to thin-walled tubes of the same length  $L$ , same thickness  $t$ , and same radius  $c$ . One of the tubes has been slit lengthwise as shown. Determine (a) the ratio  $\tau_b/\tau_a$  of the maximum shearing stresses in the tubes, (b) the ratio  $\phi_b/\phi_a$  of the angles of twist of the shafts.

SOLUTION

Without slit

Area bounded by centerline:  $A = \pi c^2$

$$\tau_a = \frac{T}{2tA} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t$$

$$\phi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G}$$

With slit:  $a = 2\pi c$ ,  $b = t$ ,  $\frac{a}{b} = \frac{2\pi c}{t} \gg 1$

$$c_1 = c_2 = \frac{1}{3}$$

$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

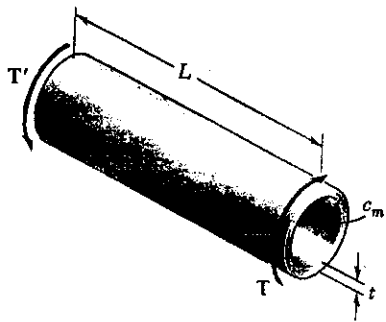
$$\phi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

Stress ratio:  $\frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t}$

Twist ratio:  $\frac{\phi_b}{\phi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \frac{3c^2}{t^2}$



PROBLEM 3.149



3.149 A hollow cylindrical shaft of length  $L$ , mean radius  $c_m$ , and uniform thickness  $t$  is subjected to torques of magnitude  $T$ . Consider, on the one hand, the values of the average shearing stress  $\tau_{ave}$  and the angle of twist  $\phi$  obtained from the elastic torsion formulas developed in Secs. 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled hollow shafts. (a) Show that the relative error introduced by using the thin-wall-shaft formulas rather than the elastic torsion formulas is the same for  $\tau_{ave}$  and  $\phi$  and that the relative error is positive and proportional to the square of the ratio  $t/c_m$ . (b) Compare the percent error corresponding to values of the ratio  $t/c_m$  equal 0.1, 0.2 and 0.4.

SOLUTION

Let  $c_2 =$  outer radius  $= c_m + \frac{1}{2}t$  and  $c_1 =$  inner radius  $= c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(c_2^2 + c_1^2)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}(c_m^2 + c_m t + \frac{1}{4}t^2 + c_m^2 - c_m t + \frac{1}{4}t^2)(2c_m)t$$

$$= 2\pi(c_m^2 + \frac{1}{4}t^2)c_m t$$

$$\tau_m = \frac{T c_m}{J} = \frac{T}{2\pi(c_m^2 + \frac{1}{4}t^2)t}$$

$$\phi_1 = \frac{TL}{JG} = \frac{TL}{2\pi(c_m^2 + \frac{1}{4}t^2)c_m t G}$$

Area bounded by centerline  $Q = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2tQ} = \frac{T}{2\pi c_m^2 t}$$

$$\phi_2 = \frac{TL}{4Q^2 G} \int \frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3 t G}$$

Ratios:  $\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t} \cdot \frac{2\pi(c_m^2 + \frac{1}{4}t^2)t}{T} = 1 + \frac{1}{4} \frac{t^2}{c_m^2}$

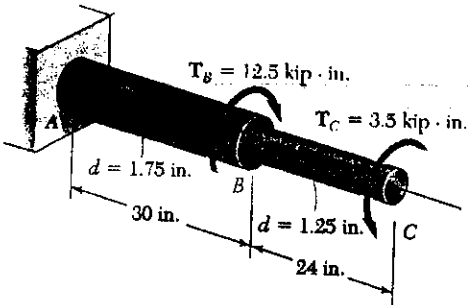
$$\frac{\phi_2}{\phi_1} = \frac{TL}{2\pi c_m^3 t G} \cdot \frac{2\pi(c_m^2 + \frac{1}{4}t^2)c_m t G}{TL} = 1 + \frac{1}{4} \frac{t^2}{c_m^2}$$

$$\frac{\tau_{ave}}{\tau_m} - 1 = \frac{\phi_2}{\phi_1} - 1 = \frac{1}{4} \frac{t^2}{c_m^2}$$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4} \frac{t^2}{c_m^2}$	0.0025	0.01	0.04
	0.25%	1%	4%

PROBLEM 3.150

3.150 For the solid brass shaft shown, determine the maximum shearing stress in (a) portion AB, (b) portion BC.



SOLUTION

AB:  $T = 12.5 - 3.5 = 9 \text{ kip-in}$ ,  $c = \frac{1}{2}d = 0.875 \text{ in}$

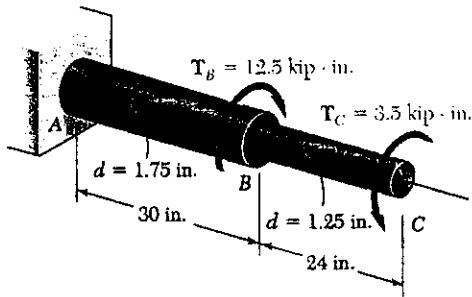
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(9)}{\pi(0.875)^3} = 8.55 \text{ ksi} \quad \blacktriangleleft$$

BC:  $T = 3.5 \text{ kip-in}$ ,  $c = \frac{1}{2}d = 0.625 \text{ in}$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(3.5)}{\pi(0.625)^3} = 9.13 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 3.151

3.151 Knowing that  $G = 5.6 \times 10^6 \text{ psi}$  for the solid brass shaft shown, determine the angle of twist at point C.



SOLUTION

AB:  $T = 12.5 - 3.5 = 9 \text{ kip-in}$ ,  $c = \frac{1}{2}d = 0.875 \text{ in}$

$$G = 5.6 \times 10^6 \text{ psi} = 5.6 \times 10^3 \text{ ksi}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.875)^4 = 0.92077 \text{ in}^4$$

$$\phi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{(9)(30)}{(5.6 \times 10^3)(0.92077)} = 52.36 \times 10^{-3} \text{ rad}$$

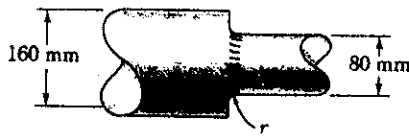
BC:  $T = 3.5 \text{ kip-in}$ ,  $c = \frac{1}{2}d = 0.625 \text{ in}$

$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (0.625)^4 = 0.23968 \text{ in}^4$$

$$\phi_{BC} = \frac{T_{BC} L_{BC}}{G J} = \frac{(3.5)(24)}{(5.6 \times 10^3)(0.23968)} = 62.60 \times 10^{-3} \text{ rad}$$

Net angle of twist  $\phi_{BC} - \phi_{AB} = 10.24 \times 10^{-3} \text{ rad} = 0.587^\circ \quad \blacktriangleleft$

## PROBLEM 3.152



3.152 The stepped shaft shown rotates at 900 rpm. Knowing that  $\tau_{all} = 42$  MPa, determine the maximum power which can be transmitted if the radius  $r$  of the fillet is (a) 12 mm, (b) 20 mm.

## SOLUTION

$$\text{frequency } f = \frac{900 \text{ rpm}}{60 \text{ rpm/Hz}} = 15 \text{ Hz}$$

$$D = 160 \text{ mm}, \quad d = 80 \text{ mm}, \quad \tau_{all} = 42 \text{ MPa} = 42 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d = 40 \text{ mm} = 0.040 \text{ m} \quad \frac{D}{d} = \frac{160}{80} = 2.0$$

$$(a) \quad r = 12 \text{ mm}, \quad \frac{r}{d} = \frac{12}{80} = 0.15 \quad K = 1.33$$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.040)^3 (42 \times 10^6)}{(2)(1.33)} = 3.175 \times 10^3 \text{ N}\cdot\text{m}$$

$$P = 2\pi f T = (2\pi)(15)(3.175 \times 10^3) = 300 \times 10^3 \text{ W} = 300 \text{ kW} \quad \blacktriangleleft$$

$$(b) \quad r = 20 \text{ mm}, \quad \frac{r}{d} = \frac{20}{80} = 0.25 \quad K = 1.20$$

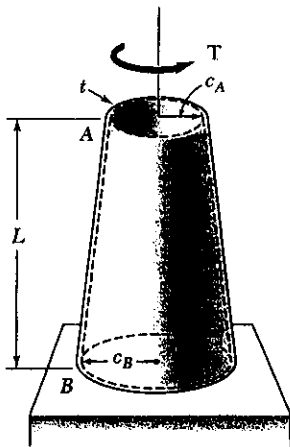
$$T = \frac{\pi c^3 \tau}{2K} = \frac{(\pi)(0.040)^3 (42 \times 10^6)}{(2)(1.20)} = 3.520 \times 10^3 \text{ N}\cdot\text{m}$$

$$P = 2\pi f T = (2\pi)(15)(3.520 \times 10^3) = 332 \times 10^3 \text{ W} = 332 \text{ kW} \quad \blacktriangleleft$$

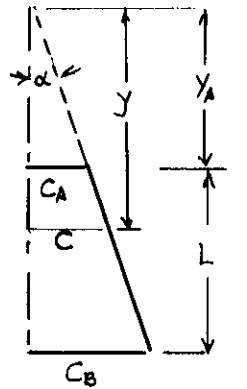
PROBLEM 3.153

3.153 The long, hollow, tapered shaft  $AB$  has a uniform thickness  $t$ . Denoting by  $G$  the modulus of rigidity, shown that the angle of twist at end  $A$  is

$$\phi_A = \frac{TL}{4\pi Gt} \frac{c_A + c_B}{c_A^2 c_B^2}$$



SOLUTION



From geometry

$$\tan \alpha = \frac{c_B - c_A}{L}$$

$$c = y \tan \alpha = \frac{c_B - c_A}{L} y$$

$$y_A = \frac{c_A}{\tan \alpha} = \frac{c_A L}{c_B - c_A}$$

$$y_B = \frac{c_B}{\tan \alpha} = \frac{c_B L}{c_B - c_A}$$

$$J = 2\pi c^3 t = 2\pi \frac{(c_B - c_A)^3}{L^3} y^3 t$$

$$\phi = \int_{y_A}^{y_B} \frac{T dy}{GJ} = \frac{TL^3}{2\pi (c_B - c_A)^3 t G} \int_{y_A}^{y_B} \frac{dy}{y^3} = \frac{TL^3}{2\pi (c_B - c_A)^3 t G} \left( -\frac{1}{2y^2} \Big|_{y_A}^{y_B} \right)$$

$$= \frac{TL^3}{4\pi (c_B - c_A)^3 t G} \left\{ \frac{1}{y_A^2} - \frac{1}{y_B^2} \right\} = \frac{TL^3}{4\pi (c_B - c_A)^3 t G} \left\{ \frac{(c_B - c_A)^2}{L^2 c_A^2} - \frac{(c_B - c_A)^2}{L^2 c_B^2} \right\}$$

$$= \frac{TL}{4\pi (c_B - c_A) t G} \left\{ \frac{1}{c_A^2} - \frac{1}{c_B^2} \right\} = \frac{TL (c_B^2 - c_A^2)}{4\pi (c_B - c_A) t G c_A^2 c_B^2}$$

$$= \frac{TL (c_B + c_A)}{4\pi Gt c_A^2 c_B^2}$$

PROBLEM 3.154

3.154 Two solid steel shafts, each of 30-mm diameter, are connected by the gears shown. Knowing that  $G = 77 \text{ GPa}$ , determine the angle through which end  $A$  rotates when a 200-N·m torque  $T$  is applied at  $A$ .

SOLUTION

Calculation of torques

Circumferential contact force between gears  $B$  and  $D$

$$F = \frac{T_{AB}}{r_B} = \frac{T_{DE}}{r_D}$$

$$T_{DE} = \frac{r_D}{r_B} T_{AB} = \frac{90}{60} (200) = 300 \text{ N}\cdot\text{m}$$

Twist in shaft  $DE$

$$J_{DE} = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$L_{DE} = 0.5 \text{ m}$$

$$\phi_{DE} = \frac{T_{DE} L_{DE}}{G J_{DE}} = \frac{(300)(0.5)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 24.497 \times 10^{-3} \text{ rad.}$$

Rotation at  $D$        $\phi_D = \phi_{DE} = 24.497 \times 10^{-3} \text{ rad}$

Circumferential displacement at gear circles       $\delta = r_D \phi_D = r_B \phi_B$

Rotation at  $B$        $\phi_B = \frac{r_D}{r_B} \phi_D = \frac{90}{60} (24.497 \times 10^{-3}) = 36.745 \times 10^{-3} \text{ rad}$

Twist in shaft  $AB$

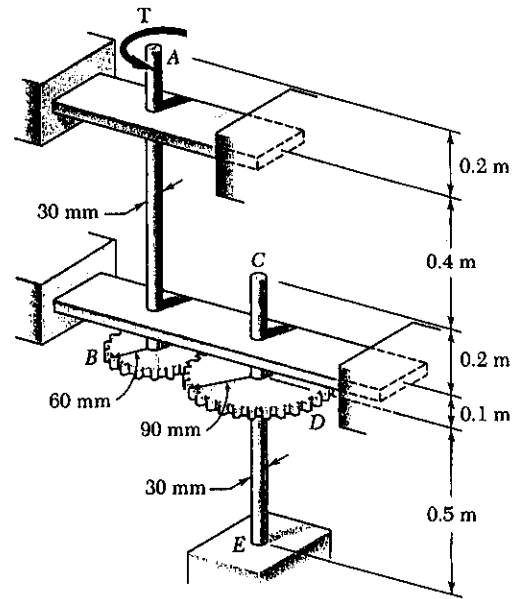
$$L_{AB} = 0.1 + 0.2 + 0.4 + 0.2 = 0.9 \text{ m}, \quad J_{AB} = 79.522 \times 10^{-9} \text{ m}^4$$

$$\phi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{(200)(0.9)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 29.396 \times 10^{-3} \text{ rad}$$

Rotation at  $A$        $\phi_A = \phi_B + \phi_{AB}$

$$= 36.745 \times 10^{-3} + 29.396 \times 10^{-3} \text{ rad} = 66.14 \times 10^{-3} \text{ rad}$$

$$= 3.79^\circ \quad \blacktriangleleft$$



PROBLEM 3.155

3.155 The angle of rotation of end A of the gear-and-shaft system shown must not exceed  $4^\circ$ . Knowing that the shafts are made of a steel for which  $\tau_{all} = 65 \text{ MPa}$  and  $G = 77 \text{ GPa}$ , determine the largest torque T which can be safely applied at end A.

SOLUTION

Calculation of torque ratio

Contact force F

$$F = \frac{T_{AB}}{r_B} = \frac{T_{DE}}{r_D}$$

$$T_{DE} = \frac{r_D}{r_B} T_{AB} = \frac{90}{60} T = 1.5 T$$

Since larger torque occurs in shaft, we find the torque limit based on stress in shaft DE

$$\tau = \frac{T_{DE} C}{J} = \frac{2 T_{DE}}{\pi C^3} = \frac{(2)(1.5) T}{\pi C^3}$$

$$T = \frac{\pi}{(2)(1.5)} C^3 \tau = \frac{\pi}{3} (0.015)^3 (65 \times 10^6) = 229.7 \text{ N}\cdot\text{m}$$

Twist in shaft DE

$$J_{DE} = \frac{\pi}{2} C^4 = \frac{\pi}{2} (0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4, \quad L_{DE} = 0.5 \text{ m}, \quad T_{DE} = 1.5 T$$

$$\phi_{DE} = \frac{T_{DE} L_{DE}}{G J_{DE}} = \frac{(1.5 T)(0.5)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 122.485 \times 10^{-6} T$$

Rotation at D  $\phi_D = 122.485 \times 10^{-6} T$

Circumferential displacement at gear circles  $S = r_D \phi_D = r_B \phi_B$

Rotation at B  $\phi_B = \frac{r_D}{r_B} \phi_D = \frac{90}{60} (122.485 \times 10^{-6} T) = 183.728 \times 10^{-6} T$

Twist in shaft AB:  $L_{AB} = 0.1 + 0.2 + 0.4 + 0.2 = 0.9 \text{ m}$

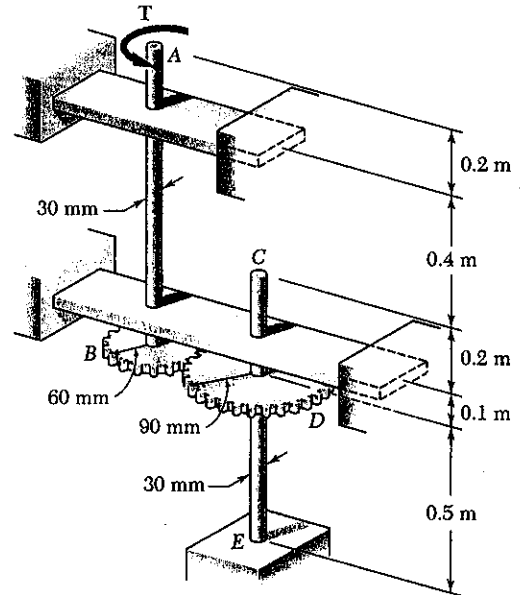
$$\phi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{T(0.9)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 146.982 \times 10^{-6} T$$

Rotation at A  $\phi_A = \phi_B + \phi_{AB} = 183.728 \times 10^{-6} T + 146.982 \times 10^{-6} T = 330.71 \times 10^{-6} T$

Rotation limit  $\phi_A = 4^\circ = 69.813 \times 10^{-3} \text{ rad}$

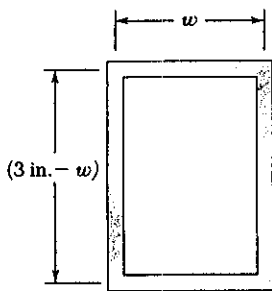
Equating  $330.71 \times 10^{-6} T = 69.813 \times 10^{-3} \therefore T = 211.1 \text{ N}\cdot\text{m}$

Limiting torque is the smaller value  $T = 211 \text{ N}\cdot\text{m}$



**PROBLEM 3.156**

3.156 A sheet metal strip of width 6 in. and 0.12 in. thickness is to be formed into a tube of rectangular cross section. Knowing that  $\tau_{all} = 4$  ksi, determine the largest torque that may be applied to the tube when (a)  $w = 1.5$  in., (b)  $w = 1.2$  in., (c)  $w = 1$  in.



**SOLUTION**

perimeter  $p = 6 \text{ in} = 2w + 2d$   
 depth  $d = \frac{p}{2} - w$

Area bounded by centerline  $A = wd = w(\frac{p}{2} - w)$

$\tau = \frac{T}{2tA} \therefore T = 2tA\tau = 2tw(\frac{p}{2} - w)\tau$

Data:  $t = 0.12 \text{ in}$ ,  $p = 6 \text{ in}$ ,  $\tau = 4 \text{ ksi}$

(a)  $w = 1.5$   $T = (2)(0.12)(1.5)(1.5)(4) = 2.16 \text{ kip}\cdot\text{in}$

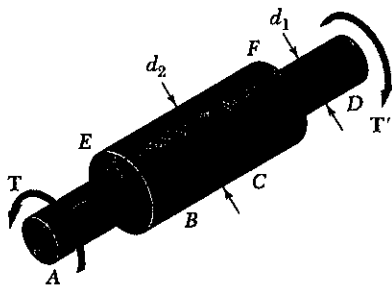
(b)  $w = 1.2$   $T = (2)(0.12)(1.2)(1.3)(4) = 2.07 \text{ kip}\cdot\text{in}$

(c)  $w = 1.0$   $T = (2)(0.12)(1.0)(2.0)(4) = 1.92 \text{ kip}\cdot\text{in}$

The largest allowable torque corresponds to a square section.

**PROBLEM 3.157**

3.157 Two solid brass rods  $AB$  and  $CD$  are brazed to a brass sleeve  $EF$ . Determine the ratio  $d_2/d_1$  for which the same maximum shearing stress occurs in the rods and in the sleeve.



**SOLUTION**

Let  $c_1 = \frac{1}{2}d_1$  and  $c_2 = \frac{1}{2}d_2$

Shaft AB  $\tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$

Sleeve EF  $\tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

For equal stresses  $\frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

$c_2^4 - c_1^4 = c_1^3 c_2$

Let  $x = \frac{c_2}{c_1}$   $x^4 - 1 = x$  or  $x = \sqrt[4]{1+x}$

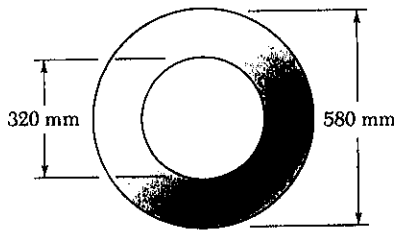
Solve by successive approximations starting with  $x_0 = 1.0$

$x_1 = \sqrt[4]{2} = 1.189$ ,  $x_2 = \sqrt[4]{2.189} = 1.216$   $x_3 = \sqrt[4]{2.216} = 1.220$

$x_4 = \sqrt[4]{2.220} = 1.221$   $x_5 = \sqrt[4]{2.221} = 1.221$  (converged)

$x = 1.221$   $\frac{c_2}{c_1} = \frac{d_2}{d_1} = 1.221$

**PROBLEM 3.158**



**3.158** One of the two hollow steel drive shafts of an ocean liner is 75 m long and has the cross section shown. knowing that  $G = 77$  GPa and that the shaft transmits 44 MW to its propeller when rotating at 144 rpm, determine (a) the maximum shearing stress in the shaft, (b) the angle of twist of the shaft.

**SOLUTION**

$$L = 75 \text{ m}, \quad f = 144 \text{ rpm} = \frac{144}{60} = 2.4 \text{ Hz}$$

$$P = 44 \text{ MW} = 44 \times 10^6 \text{ W}$$

$$P = 2\pi f T \therefore T = \frac{P}{2\pi f} = \frac{44 \times 10^6}{2\pi(2.4)} = 2.9178 \times 10^6 \text{ N}\cdot\text{m}$$

$$c_1 = \frac{d_1}{2} = \frac{320}{2} = 160 \text{ mm} = 0.160 \text{ m}$$

$$c_2 = \frac{d_2}{2} = \frac{580}{2} = 290 \text{ mm} = 0.290 \text{ m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.290^4 - 0.160^4) = 10.08 \times 10^{-3} \text{ m}^4$$

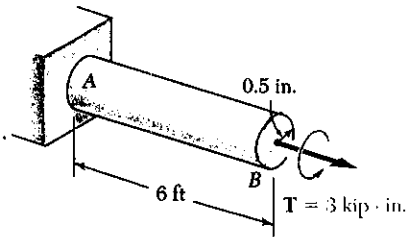
$$(a) \quad \tau = \frac{Tc_2}{J} = \frac{(2.9178 \times 10^6)(0.290)}{10.08 \times 10^{-3}} = 83.9 \times 10^6 \text{ Pa} = 83.9 \text{ MPa}$$

$$(b) \quad \phi = \frac{TL}{GJ} = \frac{(2.9178 \times 10^6)(75)}{(77 \times 10^9)(10.08 \times 10^{-3})} = 281.9 \times 10^{-3} \text{ rad} = 16.15^\circ$$



**PROBLEM 3.159**

3.159 The shaft  $AB$  is made of a material which is elastoplastic with  $\tau_y = 12.5$  ksi and  $G = 4 \times 10^6$  psi. For the loading shown, determine (a) the radius of the elastic core of the shaft, (b) the angle of twist at end  $B$ .



**SOLUTION**

$$c = 0.5 \text{ in}$$

$$T_Y = \frac{J\tau_y}{c} = \frac{\pi}{2} c^3 \tau_y = \frac{\pi}{2} (0.5)^3 (12.5) = 2.454 \text{ kip-in}$$

$T = 3 \text{ kip-in} > T_Y$  plastic region with elastic core

$$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} \frac{\rho_r^3}{c^3}\right) \therefore \frac{\rho_r^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(2.454)}{3} = 0.33307$$

$$\frac{\rho_r}{c} = 0.69318$$

$$\rho_r = (0.69318)(0.5) = 0.347 \text{ in.}$$

$$L = 6 \text{ ft} = 72 \text{ in.}$$

$$G = 4 \times 10^6 \text{ psi} = 4 \times 10^3 \text{ ksi}$$

$$\phi_Y = \frac{T_Y L}{JG} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(2.454)(72)}{\pi (0.5)^4 (4 \times 10^3)} = 0.4499 \text{ rad}$$

$$\frac{\phi_r}{\phi} = \frac{\rho_r}{c} \therefore \phi = \frac{\phi_Y}{\rho_r/c} = \frac{0.4499}{0.69318} = 0.64904 \text{ rad} = 37.2^\circ$$

**PROBLEM 3.160**

3.160 If the 3 kip-in. torque applied to the shaft of Prob. 3.159 is removed, determine (a) the magnitude and location of the maximum residual shearing stress in the shaft, (b) the permanent angle of twist of the shaft.

**SOLUTION**

From the solution of PROBLEM 3.159, at the end of loading  $T = 3$  kip-in

$$\frac{\rho_r}{c} = 0.69318, \quad \phi_{\text{load}} = 0.64904 \text{ rad}$$

Stresses  $\tau = 0$  at  $\rho = 0$ ,  $\tau = 12.5$  ksi at  $\rho = \rho_r$ ,  $\tau = 12.5$  ksi at  $\rho = c$ .

Unloading  $T = -3$  kip-in.  $G = 4 \times 10^6$  psi =  $4 \times 10^3$  ksi

$$\Delta\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(-3)}{\pi (0.5)^3} = -15.28 \text{ ksi at } \rho = c$$

$$\Delta\tau = (0.69318)(-15.28) = 10.59 \text{ ksi at } \rho = \rho_r.$$

$$\Delta\phi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(-3)(72)}{\pi (0.5)^4 (4 \times 10^3)} = -0.55004 \text{ rad.}$$

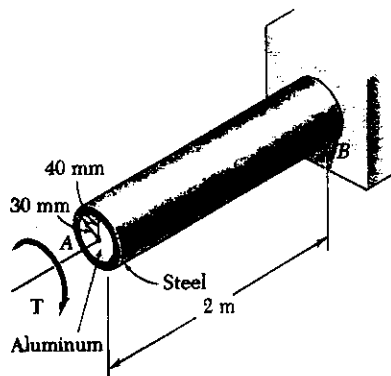
Residual  $\tau_{\text{res}} = \tau_{\text{load}} + \Delta\tau$

At  $\rho = c$   $\tau_{\text{res}} = 12.5 - 15.28 = -2.78$  ksi

At  $\rho = \rho_r$   $\tau_{\text{res}} = 12.5 - 10.59 = 1.91$  ksi

$$\phi_{\text{perm}} = \phi_{\text{load}} + \Delta\phi = 0.64904 - 0.55004 = 0.099 \text{ rad} = 5.67^\circ$$

PROBLEM 3.161



3.161 The composite shaft shown is twisted by applying a torque  $T$  at end  $A$ . Knowing that the maximum shearing stress in the steel shell is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use  $G = 77$  GPa for steel and  $G = 27$  GPa for aluminum.

SOLUTION

Let  $G_1$ ,  $J_1$ , and  $\tau_1$  refer to the aluminum core.  
and  $G_2$ ,  $J_2$ , and  $\tau_2$  refer to the steel shell

At the outer surface on the steel shell

$$\gamma_2 = \frac{C_2 \phi}{L} \therefore \frac{\phi}{L} = \frac{\gamma_2}{C_2} = \frac{\tau_2}{C_2 G_2}$$

At the outer surface of the aluminum core

$$\gamma_1 = \frac{C_1 \phi}{L} \therefore \frac{\phi}{L} = \frac{\gamma_1}{C_1} = \frac{\tau_1}{C_1 G_1}$$

Matching  $\frac{\phi}{L}$  for both components

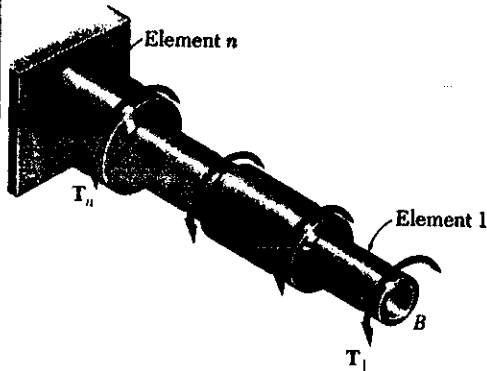
$$\frac{\tau_2}{C_2 G_2} = \frac{\tau_1}{C_1 G_1}$$

Solving for  $\tau_2$

$$\begin{aligned} \tau_2 &= \frac{C_2}{C_1} \cdot \frac{G_2}{G_1} \tau_1 \\ &= \frac{0.030}{0.040} \cdot \frac{27 \times 10^9}{77 \times 10^9} \cdot 150 \times 10^6 \\ &= 39.4 \times 10^6 \text{ Pa} \end{aligned}$$

$$= 39.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.C1



3.C1 Shaft AB consists of  $n$  homogeneous cylindrical elements, which can be solid or hollow. Its end A is fixed, while its end B is free, and it is subjected to the loading shown. The length of element  $i$  is denoted by  $L_i$ , its outer diameter by  $OD_i$ , its inner diameter by  $ID_i$ , its modulus of rigidity by  $G_i$ , and the torque applied to its right end by  $T_i$ , the magnitude  $T_i$  of this torque being assumed to be positive if  $T_i$  is observed as counterclockwise from end B and negative otherwise. (Note that  $ID_i = 0$  if the element is solid.) (a) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (b) Use this program to solve Probs. 3.9, 3.35, 3.37, 3.150, and 3.151.

SOLUTION

FOR EACH CYLINDRICAL ELEMENT, ENTER

$$L_i, OD_i, ID_i, G_i, T_i$$

AND COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

OUTLINE OF PROGRAM

UPDATE TORQUE  $T = T + T_i$

AND COMPUTE

$$\tau_{max,i} = T (OD_i/2) / J_i$$

$$\phi_i = T L_i / G_i J_i$$

ANGLE OF TWIST OF ENTIRE SHAFT, STARTING WITH  $\Theta = 0$ , UPDATE THROUGH  $n^{th}$  ELEMENT

$$\Theta = \Theta + \phi_i$$

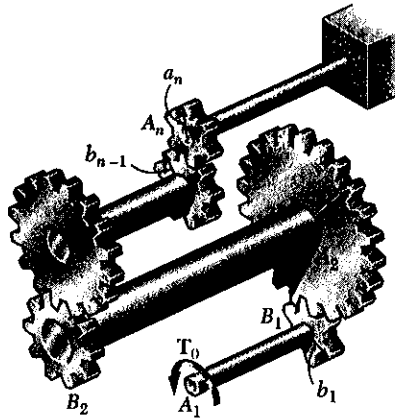
PROGRAM OUTPUT

Problem	3.9 and 3.35		
Element	Maximum Stress (MPa)	Angle of Twist (degrees)	
1.0000	56.5884	2.5265	
2.0000	36.6264	0.8887	
Angle of twist for entire shaft =			3.4152 °

Problem	3.37		
Element	Maximum Stress (MPa)	Angle of Twist (degrees)	
1.0000	33.9531	0.8314	
2.0000	19.6488	0.7413	
Angle of twist for entire shaft =			1.5726 °

Problem	3.150 and 3.151		
Element	Maximum Stress (ksi)	Angle of Twist (degrees)	
1.0000	9.1266	3.5857	
2.0000	-8.5526	-3.0002	
Angle of twist for entire shaft =			0.5855 °

**PROBLEM 3.C2**



**3.C2** The assembly shown consists of  $n$  cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End  $A_1$  of the first shaft is free and is subjected to a torque  $T_0$ , while end  $B_n$  of the last shaft is fixed. The length of shaft  $A_i B_i$  is denoted by  $L_i$ , its outer diameter by  $OD_i$ , its inner diameter by  $ID_i$ , and its modulus of rigidity by  $G_i$ . (Note that  $ID_i = 0$  if the element is solid.) The radius of gear  $A_i$  is denoted by  $a_i$ , and the radius of gear  $B_i$  by  $b_i$ . (a) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end  $A_i$  rotates. (b) Use this program to solve Probs. 3.21, 3.39, 3.41, 3.42, and 3.154.

**SOLUTION**

TORQUE IN SHAFTS. ENTER  $T_L = T_0$

$$T_{L+1} = T_L (A_{L+1} / B_L)$$

FOR EACH SHAFT, ENTER

$L_i \quad OD_i \quad ID_i \quad G_i$

COMPUTE:  $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

$$\tau_{iL} = T_L (OD_i / 2) / J_i$$

$$\phi_{iL} = T_L L_i / G_i J_i$$

ANGLE OF ROTATION AT END  $A_i$

COMPUTE ROTATION AT THE "A" END OF EACH SHAFT

START WITH ANGLE =  $\phi_{iL}$  AND UPDATE

FROM  $i$  TO  $1$ , AND ADD  $\phi_{iL}$

$$\text{ANGLE} = \text{ANGLE}(A_i) / B_{L-1} + \phi_{L-1}$$

PROGRAM OUTPUT

Problem 3.21

Shaft No.	Max.Stress (MPa)	Twist Angle (degrees)
1.0000	68.7420	1.4615
2.0000	72.5013	0.7707
Angle through which A1 rotates = 3.388 °		

Problem 3.39

Shaft No.	Max.Stress (MPa)	Twist Angle (degrees)
1.0000	47.7465	1.7764
2.0000	82.8932	2.0560
Angle through which A1 rotates = 7.945 °		

Problem 3.41

Shaft No.	Max.Stress (ksi)	Twist Angle (degrees)
1.0000	9.0541	1.3587
2.0000	12.0722	1.3175
Angle through which A1 rotates = 3.115 °		

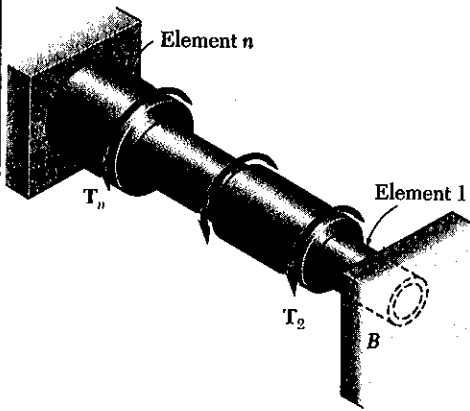
Problem 3.42

Shaft No.	Max.Stress (ksi)	Twist Angle (degrees)
1.0000	9.0541	1.3587
2.0000	6.7906	0.7411
Angle through which A1 rotates = 1.914 °		

Problem 3.154

Shaft No.	Max.Stress (MPa)	Twist Angle (degrees)
1.0000	37.7256	1.6843
2.0000	56.5884	1.4036
Angle through which A1 rotates = 3.790 °		

**PROBLEM 3.C3**



**3.C3** Shaft AB consists of  $n$  homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element  $i$  is denoted by  $L_i$ , its outer diameter by  $OD_i$ , its inner diameter by  $ID_i$ , its modulus of rigidity by  $G_i$ , and the torque applied to its right end by  $T_i$ , the magnitude  $T_i$  of this torque being assumed to be positive if  $T_i$  is observed as counterclockwise from end B and negative otherwise. Note that  $ID_i = 0$  if the element is solid and also that  $T_1 = 0$ . Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (a) to solve Prob. 3.56, (b) to determine the maximum shearing stress in the shaft of Example 3.05.

**SOLUTION** WE CONSIDER THE REACTION AT B AS REDUNDANT AND RELEASE THE SHAFT AT B. COMPUTE  $\Theta_B$  WITH  $T_B = 0$ :

FOR EACH ELEMENT ENTER  $L_i, OD_i, ID_i, G_i, T_i$  (NOTE  $T_1 = T_B = 0$ )  
 COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

UPDATE TORQUE

$$T = T + T_i$$

AND COMPUTE FOR EACH ELEMENT

$$\tau_{iL} = T(OD_i/2)/J_i$$

$$\phi_{iL} = T L_i / G_i J_i$$

COMPUTE  $\Theta_B$ : STARTING WITH  $\Theta = 0$  AND UPDATING THROUGH  $n$  ELEMENTS

$$\Theta_i = \Theta_i + \phi_{iL} \quad ; \quad \Theta_B = \Theta_n$$

COMPUTE  $\Theta_B$  DUE TO UNIT TORQUE AT B

$$\text{UNIT } \tau_{iL} = OD_i / 2 J_i$$

$$\text{UNIT } \phi_{iL} = L_i / G_i J_i$$

FOR  $n$  ELEMENTS:

$$\text{UNIT } \Theta_B(L) = \text{UNIT } \phi_{iL} + \text{UNIT } \tau_{iL}$$

SUPERPOSITION:

FOR TOTAL ANGLE AT B TO BE ZERO.  $\Theta_B + T_B(\text{UNIT } \Theta_B(n)) = 0$

$$T_B = -\Theta_B / (\text{UNIT } \Theta_B(n))$$

THEN  $T_A = \sum T_i(L) + T_B$

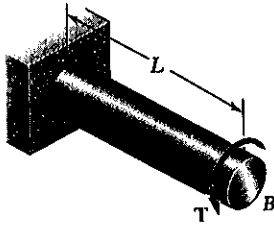
FOR EACH ELEMENT: MAX STRESS: TOTAL  $\tau_{iL} = \tau_{iL} + T_B(\text{UNIT } \tau_{iL})$   
 ANGLE OF TWIST: TOTAL  $\phi_{iL} = \phi_{iL} + T_B(\text{UNIT } \phi_{iL})$

PROGRAM OUTPUT

<u>Problem 3.56</u>		TA =	-0.290 kN*m
		TB =	-0.210 kN*m
Element	tau max (MPa)	Angle of Twist (degrees)	
1	-39.588		-1.178
2	31.670		1.178

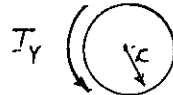
<u>Problem 3.05</u>		TA =	-51.733 lb*ft
		TB =	-38.267 lb*ft

**PROBLEM 3.C4**



**3.C4** The homogeneous, solid cylindrical shaft *AB* has a length *L*, a diameter *d*, a modulus of rigidity *G*, and a yield strength  $\tau_y$ . It is subjected to a torque *T* that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value  $\phi_m$  and then decreased back to zero. (a) Write a computer program that, for each of 16 values of  $\phi_m$  equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value  $T_m$  of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (b) Use this program to obtain approximate answers to Probs. 3.95, 3.113, 3.159, and 3.160.

**SOLUTION**



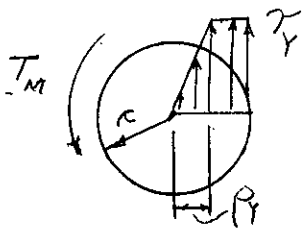
AT ONSET OF YIELD

$$T_Y = \tau_Y \frac{J}{c} = \frac{\pi}{2} \tau_Y c^3$$

$$\phi_Y = \frac{T_Y L}{GJ} = \left( \frac{\tau_Y J}{c} \right) \frac{L}{GJ} = \frac{\tau_Y L}{cG}$$

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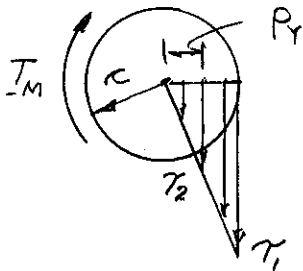
LOADING:  $T_m > T_Y$



$$T_m = \frac{4}{3} T_Y \left[ 1 - \frac{1}{4} \left( \frac{\phi_Y}{\phi_m} \right)^3 \right] \quad \text{EQ. (1)}$$

$$\rho_Y = c \frac{\phi_Y}{\phi_m} \quad \text{EQ. (2)}$$

UNLOADING (ELASTIC)



$$\phi_U = \frac{T_m L}{GJ}$$

$\phi_U =$  ANGLE OF TWIST FOR UNLOADING

$$\tau_1 = \frac{T_m c}{J}$$

$\tau_1 =$  TAU AT  $\rho = c$

$$\tau_2 = \tau_1 \frac{\rho_Y}{c}$$

$\tau_2 =$  TAU AT  $\rho = \rho_Y$

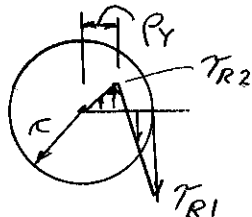
SUPERPOSE LOADING AND UNLOADING

FOR  $\phi = 0$  TO  $\phi = 3\phi_Y$  USING 0.2  $\phi_Y$  INCREMENTS

WHEN  $\phi < \phi_Y$ :  $T_m = \tau_Y \frac{\phi}{\phi_Y}$      $\rho_Y = \frac{1}{2} d$      $\phi_m = \phi_Y \frac{\phi}{\phi_Y}$

WHEN  $\phi > \phi_Y$ :  $T_m$  USE EQ.(1)     $\rho_Y$  USE EQ.(2)

RESIDUAL:  $\phi_p = \phi_m - \phi_U$      $\tau_{R2} = \tau_1 - \tau_Y$      $\tau_{R1} = \tau_2 - \tau_Y$



CONTINUED

PROBLEM 3.C4 - CONTINUED

INTERPOLATE BETWEEN VALUES AT VALUES OF  $T_M$  INDICATED. ↓

Problem 3.95						
PHIM deg	TM kip*in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi
0.000	0.000	0.750	0.000	0.000	0.000	0.000
0.417	2.783	0.750	4.200	0.000	0.000	0.000
0.833	5.567	0.750	8.400	0.000	0.000	0.000
1.250	8.350	0.750	12.600	0.000	0.000	0.000
1.667	11.133	0.750	16.800	0.000	0.000	0.000
2.083	13.916	0.750	21.000	0.000	0.000	0.000
2.500	15.871	0.625	21.000	0.124	1.042	-2.949
2.917	16.865	0.536	21.000	0.392	2.822	-4.449
3.334	17.423	0.469	21.000	0.725	4.568	-5.291
3.750	17.760	0.417	21.000	1.091	6.111	-5.800
4.167	17.975	0.375	21.000	1.476	7.438	-6.125
4.584	18.119	0.341	21.000	1.871	8.572	-6.343
5.000	18.219	0.313	21.000	2.273	9.544	-6.494
5.417	18.291	0.288	21.000	2.679	10.384	-6.602
5.834	18.344	0.268	21.000	3.087	11.114	-6.681
6.250	18.383	0.250	21.000	3.498	11.753	-6.741

←  $T_M = 12 \text{ kip}\cdot\text{in.}$

←  $T_M = 18 \text{ kip}\cdot\text{in.}$

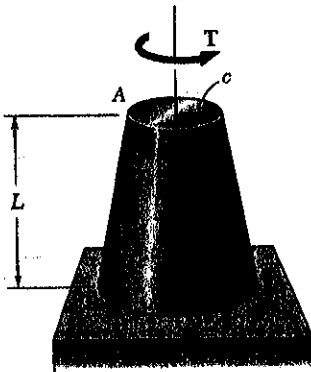
Problem 3.113						
PHIM deg	TM kN*m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa
0.000	0.000	25.000	0.000	0.000	0.000	0.000
9.524	0.785	25.000	32.000	0.000	0.000	0.000
19.049	1.571	25.000	64.000	0.000	0.000	0.000
28.573	2.356	25.000	96.000	0.000	0.000	0.000
38.098	3.142	25.000	128.000	0.000	0.000	0.000
47.622	3.927	25.000	160.000	0.000	0.000	0.000
57.147	4.713	20.833	160.000	2.837	7.942	-22.469
66.671	4.759	17.857	160.000	8.960	21.502	-33.897
76.196	4.916	15.625	160.000	16.575	34.805	-40.313
85.720	5.012	13.889	160.000	24.946	46.562	-44.188
95.245	5.072	12.500	160.000	33.733	56.667	-46.667
104.769	5.113	11.364	160.000	42.764	65.307	-48.325
114.294	5.141	10.417	160.000	51.946	72.719	-49.475
123.818	5.162	9.615	160.000	61.225	79.116	-50.299
133.343	5.176	8.929	160.000	70.569	84.677	-50.904
142.867	5.188	8.333	160.000	79.959	89.547	-51.358

←  $T_M = 5 \text{ kN}\cdot\text{m}$

Problems 3.159 and 3.160						
PHIM deg	TM kip*in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi
0.000	0.000	0.500	0.000	0.000	0.000	0.000
5.157	0.491	0.500	2.500	0.000	0.000	0.000
10.313	0.982	0.500	5.000	0.000	0.000	0.000
15.470	1.473	0.500	7.500	0.000	0.000	0.000
20.626	1.963	0.500	10.000	0.000	0.000	0.000
25.783	2.454	0.500	12.500	0.000	0.000	0.000
30.940	2.799	0.417	12.500	1.536	0.620	-1.755
36.096	2.974	0.357	12.500	4.851	1.680	-2.648
41.253	3.073	0.313	12.500	8.974	2.719	-3.149
46.410	3.132	0.278	12.500	13.506	3.638	-3.452
51.566	3.170	0.250	12.500	18.263	4.427	-3.646
56.723	3.196	0.227	12.500	23.152	5.102	-3.775
61.879	3.213	0.208	12.500	28.124	5.681	-3.865
67.036	3.226	0.192	12.500	33.148	6.181	-3.930
72.193	3.235	0.179	12.500	38.207	6.615	-3.977
77.349	3.242	0.167	12.500	43.290	6.996	-4.012

←  $T_M = 3 \text{ kip}\cdot\text{in.}$

**PROBLEM 3.C5**



**3.C5** The exact expression is given in Prob. 3.64 for the angle of twist of the solid tapered shaft  $AB$  when a torque  $T$  is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by  $n$  cylindrical shafts of equal length and of radius  $r_i = (n + i - \frac{1}{2})(c/n)$ , where  $i = 1, 2, \dots, n$ . Using for  $T, L, G,$  and  $c$  values of your choice, determine the percentage error in the approximate expression when (a)  $n = 4,$  (b)  $n = 8,$  (c)  $n = 20,$  (d)  $n = 100.$

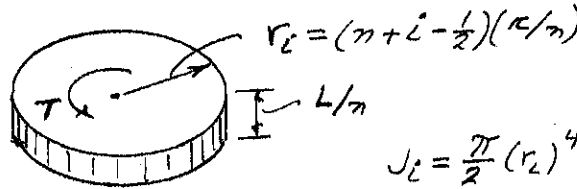
**SOLUTION**

FROM PROB. 3.64 EXACT EXPRESSION:

$$\phi = \frac{7TL}{12\pi Gc^4}$$

OR,  $\phi = \left(\frac{7}{12\pi}\right) \frac{TL}{Gc^4} = 0.18568 \frac{TL}{Gc^4}$

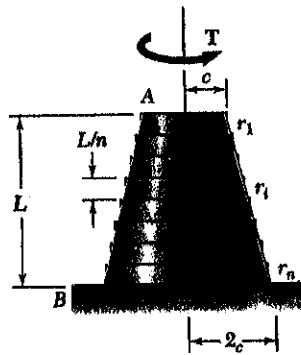
CONSIDER TYPICAL  $i$ th SHAFT



$$r_i = (n + i - \frac{1}{2})(c/n)$$

$$J_i = \frac{\pi}{2} (r_i)^4$$

$$\Delta\phi = \frac{T(L/n)}{GJ_i}$$



ENTER UNIT VALUES OF T, L, G, AND C.  
(NOTE: SPECIFIC VALUES CAN BE ENTERED)

ENTER INITIAL VALUE OF ZERO FOR  $\phi$   
ENTER  $n$  = NUMBER CYLINDRICAL SHAFTS

FOR  $i = 1$  TO  $n$ , UPDATE  $\phi$

$$\phi = \phi + \Delta\phi$$

OUTPUT OF PROGRAM

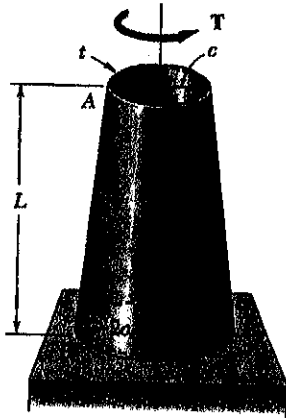
COEFFICIENT of  $TL/Gc^4$

Exact coefficient from Prob. 3.64 is 0.18568  
Number of elemental disks =  $n$

n	approximate	exact	percent error
4	0.17959	0.18568	-3.28185
8	0.18410	0.18568	-0.85311
20	0.18542	0.18568	-0.13810
100	0.18567	0.18568	-0.00554



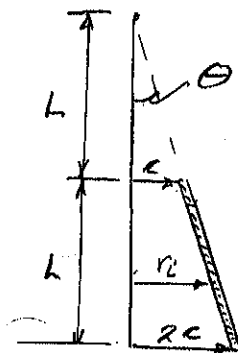
**PROBLEM 3.C6**



**3.C6** A torque  $T$  is applied as shown to the long, hollow, tapered shaft  $AB$  of uniform thickness  $t$ . The exact expression for the angle of twist of the shaft can be obtained from the expression given in Prob. 3.153. Derive an approximate expression for the angle of twist by replacing the tapered shaft by  $n$  cylindrical rings of equal length and of radius  $r_i = (n + i - \frac{1}{2})(c/n)$ , where  $i = 1, 2, \dots, n$ . Using for  $T, L, G, c$  and  $t$  values of your choice, determine the percentage error in the approximate expression when (a)  $n = 4$ , (b)  $n = 8$ , (c)  $n = 20$ , (d)  $n = 100$ .

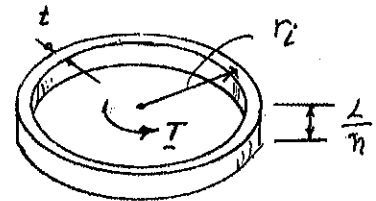
**SOLUTION**

SINCE THE SHAFT IS LONG  $c \ll L$ , THE ANGLE  $\theta$  IS SMALL AND WE CAN USE  $t$  AS THE THICKNESS OF THE  $n$  CYLINDRICAL RINGS.



FOR  $c \ll L$

$$\theta = \tan \theta = \frac{2c - c}{L} = \frac{c}{L}$$



$$r_i = (n + i - \frac{1}{2}) \left( \frac{c}{n} \right)$$

$$J_i \approx (\text{AREA}) r_i^2 = (2\pi r_i t) r_i^2 = 2\pi t r_i^3$$

$$\Delta \phi = \frac{T (L/n)}{G J_i}$$

ENTER UNIT VALUES FOR  $T, L, G, t$ , AND  $n$

(NOTE: SPECIFIC VALUES CAN BE ENTERED IF DESIRED)

ENTER INITIAL VALUE OF ZERO FOR  $\phi$

ENTER  $n$  = NUMBER OF CYLINDRICAL RINGS

FOR  $i = 1$  TO  $n$ , UPDATE  $\phi$

$$\phi = \phi + \Delta \phi$$

OUTPUT OF PROGRAM

COEFFICIENT of  $TL/Gtc^3$

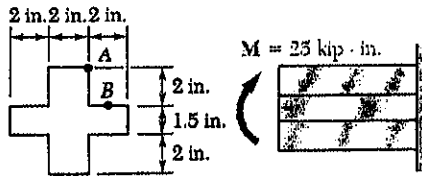
Exact coefficient from Prob. 3.153 is 0.05968  
Number of elemental disks =  $n$

$n$	approximate	exact	percent error
4	0.058559	0.059683	-1.883078
8	0.059394	0.059683	-0.483688
20	0.059637	0.059683	-0.078022
100	0.059681	0.059683	-0.003127

# CHAPTER 4

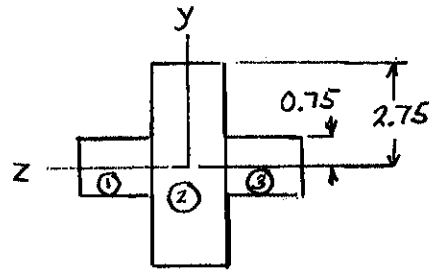
[www.konkur.in](http://www.konkur.in)

**PROBLEM 4.1**



4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.

**SOLUTION**



For rectangle  $I = \frac{1}{12}bh^3$

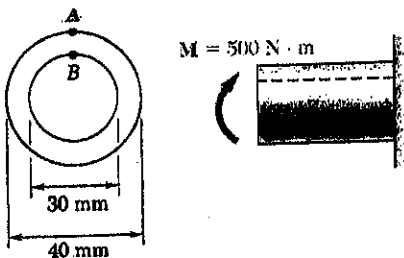
For cross sectional area

$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(a)  $y_A = 2.75 \text{ in}$        $\sigma_A = -\frac{My_A}{I} = -\frac{(25)(2.75)}{28.854} = -2.38 \text{ ksi}$        $\blacktriangleleft$

(b)  $y_B = 0.75 \text{ in}$        $\sigma_B = -\frac{My_B}{I} = -\frac{(25)(0.75)}{28.854} = -0.650 \text{ ksi}$        $\blacktriangleleft$

**PROBLEM 4.2**



4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.

**SOLUTION**

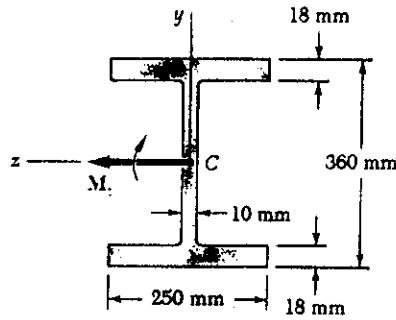
$r_i = \frac{1}{2}d_i = 15 \text{ mm}$        $r_o = \frac{1}{2}d_o = 20 \text{ mm}$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(20^4 - 15^4) = 85.903 \times 10^3 \text{ mm}^4 = 85.903 \times 10^{-9} \text{ m}^4$$

(a)  $y_A = 20 \text{ mm} = 0.020 \text{ m}$        $\sigma_A = -\frac{My_A}{I} = -\frac{(500)(0.020)}{85.903 \times 10^{-9}} = -116.4 \times 10^6 \text{ Pa} = -116.4 \text{ MPa}$        $\blacktriangleleft$

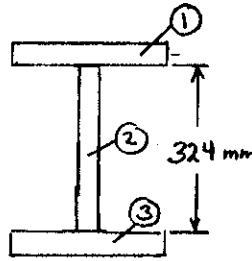
(b)  $y_B = 15 \text{ mm} = 0.015 \text{ m}$        $\sigma_B = -\frac{My_B}{I} = -\frac{(500)(0.015)}{85.903 \times 10^{-9}} = -87.3 \times 10^6 \text{ Pa} = -87.3 \text{ MPa}$        $\blacktriangleleft$

PROBLEM 4.3



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which  $\sigma_y = 345 \text{ MPa}$  and  $\sigma_t = 450 \text{ MPa}$ . Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the  $z$  axis. Neglect the effect of fillets.

SOLUTION



$$\begin{aligned}
 I_1 &= \frac{1}{12} b h^3 + A d^2 \\
 &= \frac{1}{12} (250)(18^3) \\
 &\quad + (250)(18)(171)^2 \\
 &= 131.706 \times 10^6 \text{ mm}^4
 \end{aligned}$$

$$I_2 = \frac{1}{12} (10)(324)^3 = 28.344 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 131.706 \times 10^6 \text{ mm}^4$$

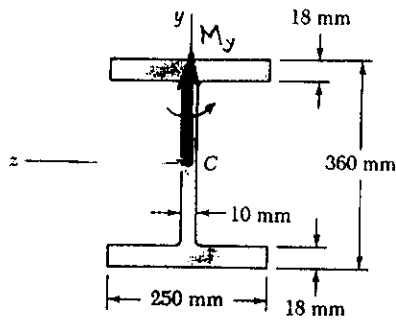
$$I = I_1 + I_2 + I_3 = 291.76 \times 10^6 \text{ mm}^4 = 291.76 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{M c}{I} \quad \text{where} \quad c = \frac{360}{2} = 180 \text{ mm} = 0.180 \text{ m}$$

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}$$

$$\begin{aligned}
 M_{all} &= \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(291.76 \times 10^{-6})}{0.180} = 243 \times 10^3 \text{ N}\cdot\text{m} \\
 &= 243 \text{ kN}\cdot\text{m}
 \end{aligned}$$

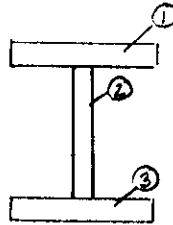
**PROBLEM 4.4**



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which  $\sigma_y = 345 \text{ MPa}$  and  $\sigma_t = 450 \text{ MPa}$ . Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

4.4 Solve Prob. 4.3, assuming that is bent about the y axis.

**SOLUTION**



$$I_1 = \frac{1}{12} (18)(250)^3 = 23.438 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (324)(10)^3 = 27 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 23.438 \text{ mm}^4$$

$$I_y = I_1 + I_2 + I_3 = 46.903 \times 10^6 \text{ mm}^4 = 46.903 \times 10^{-6} \text{ m}^4$$

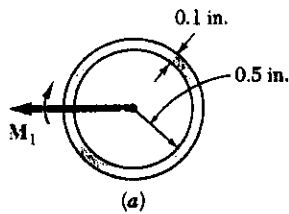
$$c = \frac{250}{2} \text{ mm} = 125 \text{ mm} = 0.125 \text{ m}$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}$$

$$\sigma = \frac{Mc}{I} \quad M_y = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(46.903 \times 10^{-6})}{0.125} = 56.3 \times 10^3 \text{ N}\cdot\text{m} = 56.3 \text{ kN}\cdot\text{m}$$

**PROBLEM 4.5**

4.5 Using an allowable stress of 16 ksi, determine the largest that can be applied to each pipe.



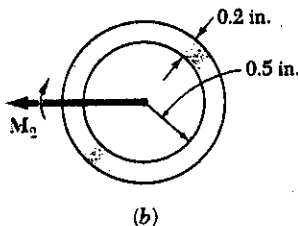
**SOLUTION**

$$(a) \quad I = \frac{\pi}{4} (r_o^4 - r_i^4) = \frac{\pi}{4} (0.6^4 - 0.5^4) = 52.7 \times 10^{-3} \text{ in}^4$$

$$c = 0.6 \text{ in}$$

$$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(16)(52.7 \times 10^{-3})}{0.6}$$

$$= 1.405 \text{ kip}\cdot\text{in.}$$



$$(b) \quad I = \frac{\pi}{4} (0.7^4 - 0.5^4) = 139.49 \times 10^{-3} \text{ in}^4$$

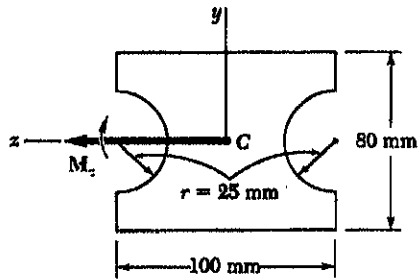
$$c = 0.7 \text{ in}$$

$$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(16)(139.49 \times 10^{-3})}{0.7}$$

$$= 3.19 \text{ kip}\cdot\text{in.}$$

PROBLEM 4.6

4.6 A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple  $M_z$  that can be applied to the bar.



SOLUTION

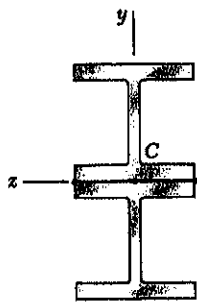
$$\begin{aligned}
 I &= I_{\text{rect}} - I_{\text{circle}} \\
 &= \frac{1}{12} b h^3 - \frac{\pi}{4} r^4 \\
 &= \frac{1}{12} (100)(80)^3 - \frac{\pi}{4} (25)^4 = 3.9599 \times 10^6 \text{ mm}^4 \\
 &= 3.9599 \times 10^{-6} \text{ m}^4
 \end{aligned}$$

$$c = \frac{80}{2} = 40 \text{ mm} = 0.040 \text{ m}$$

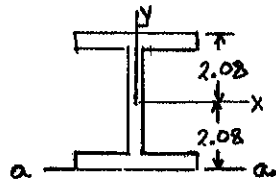
$$\begin{aligned}
 \sigma &= \frac{M c}{I} & M &= \frac{\sigma I}{c} = \frac{(24 \times 10^6)(3.9599 \times 10^{-6})}{0.040} = 2.38 \times 10^3 \text{ N}\cdot\text{m} \\
 & & &= 2.38 \text{ kN}\cdot\text{m}
 \end{aligned}$$

**PROBLEM 4.7**

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used  $\sigma_y = 36$  ksi and  $\sigma_u = 58$  ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



**SOLUTION**



Properties of W 4 × 13 rolled section  
See Appendix B

Area = 3.83 in<sup>2</sup>      Depth = 4.16 in  
I<sub>x</sub> = 11.3 in<sup>4</sup>

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections  $I_2 = 2I_a = 55.74 \text{ in}^4$

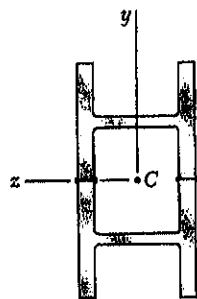
c = depth = 4.16 in

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \quad \sigma = \frac{Mc}{I}$$

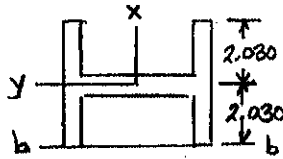
$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(55.74)}{4.16} = 259 \text{ kip}\cdot\text{in.}$$

**PROBLEM 4.8**

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used  $\sigma_y = 36$  ksi and  $\sigma_u = 58$  ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



**SOLUTION**



Properties of W 4 × 13 rolled section  
See Appendix B

Area = 3.83 in<sup>2</sup>      Width = 4.060 in  
I<sub>y</sub> = 3.86 in<sup>4</sup>

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections  $I_2 = 2I_b = 39.286 \text{ in}^4$

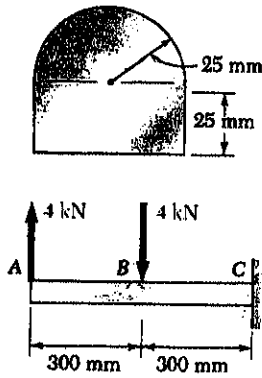
c = width = 4.060 in

$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \quad \sigma = \frac{Mc}{I}$$

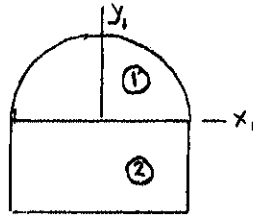
$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip}\cdot\text{in.}$$

PROBLEM 4.9

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



SOLUTION



$$A_1 = \frac{\pi}{2} r^2 = \frac{\pi}{2} (25)^2 = 981.7 \text{ mm}^2$$

$$\bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2$$

$$\bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250} = -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1 \bar{y}_1^2 = \frac{\pi}{8} r^4 - A_1 \bar{y}_1^2 = \frac{\pi}{8} (25)^4 - (981.7)(10.610)^2 = 42.886 \times 10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = 42.866 \times 10^6 + (981.7)(12.944)^2 = 207.35 \times 10^6 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12} b h^3 = \frac{1}{12} (50)(25)^3 = 65.104 \times 10^6 \text{ mm}^4$$

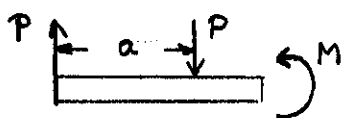
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2 d_2^2 = 65.104 \times 10^6 + (1250)(10.166)^2 = 194.288 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16 \times 10^6 \text{ mm}^4 = 401.16 \times 10^{-9} \text{ m}^4$$

$$y_{\text{top}} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{\text{bot}} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$



$$M - Pa = 0 \quad M = Pa = (4 \times 10^3)(300 \times 10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{top}} = -\frac{M y_{\text{top}}}{I} = -\frac{(1200)(0.027334)}{401.16 \times 10^{-9}} = -81.76 \times 10^6 \text{ Pa} = -81.8 \text{ MPa}$$

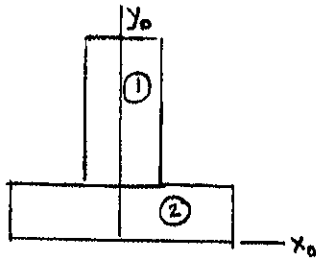
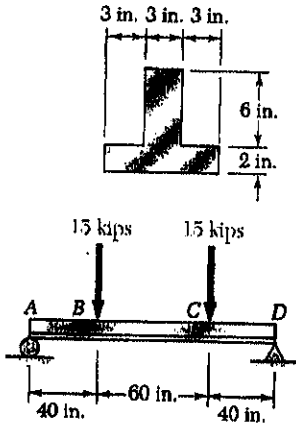
$$\sigma_{\text{bot}} = -\frac{M y_{\text{bot}}}{I} = -\frac{(1200)(-0.022666)}{401.16 \times 10^{-9}} = 67.80 \times 10^6 \text{ Pa} = 67.8 \text{ MPa}$$



**PROBLEM 4.10**

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

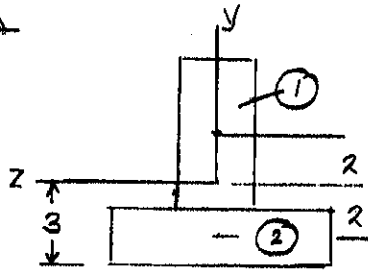
**SOLUTION**



	A	$\bar{y}_0$	$A\bar{y}_0$
①	18	5	90
②	18	1	18
$\Sigma$	36		108

$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in}$$

Neutral axis lies 3 in. above the base.



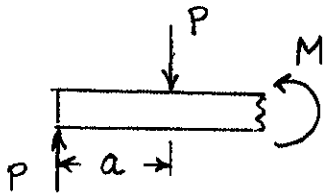
$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{\text{top}} = 5 \text{ in}$$

$$y_{\text{bot}} = -3 \text{ in}$$



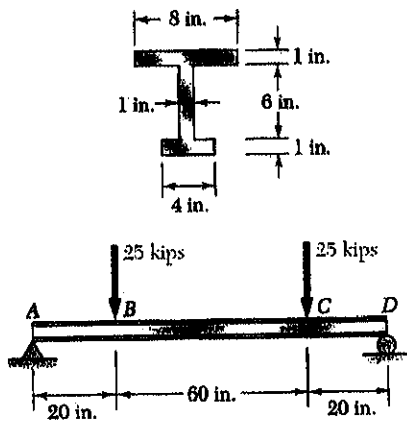
$$M - Pa = 0$$

$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

$$\sigma_{\text{top}} = -\frac{My_{\text{top}}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi}$$

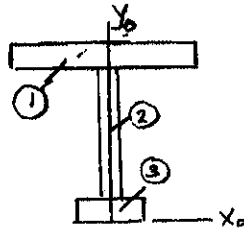
$$\sigma_{\text{bot}} = -\frac{My_{\text{bot}}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi}$$

**PROBLEM 4.11**



4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

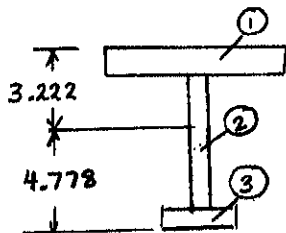
**SOLUTION**



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
$\Sigma$	18		86

$$\bar{Y}_0 = \frac{86}{18} = 4.778 \text{ in}$$

Neutral axis lies 4.778 in above the base.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (8)(1)^3 + (8)(2.722)^2 = 59.94 \text{ in}^4$$

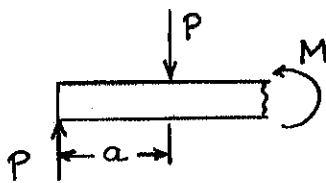
$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12} (4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4$$

$$y_{\text{top}} = 3.222 \text{ in}$$

$$y_{\text{bot}} = -4.778 \text{ in}$$



$$M - Pa = 0$$

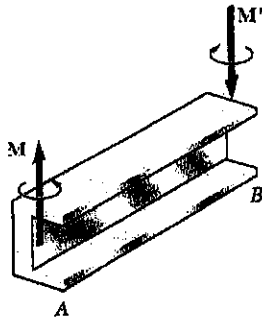
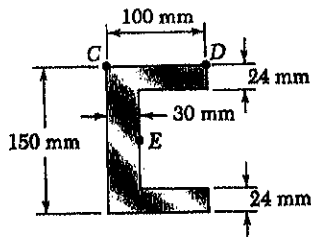
$$M = Pa = (25)(20) = 500 \text{ kip}\cdot\text{in.}$$

$$\sigma_{\text{top}} = - \frac{M y_{\text{top}}}{I} = - \frac{(500)(3.222)}{155.16} = -10.38 \text{ ksi}$$

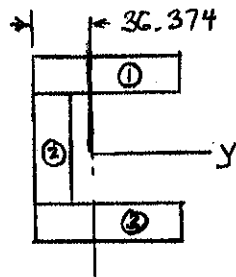
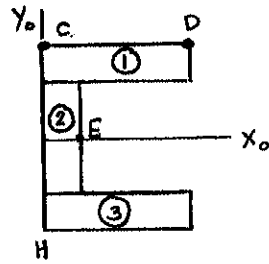
$$\sigma_{\text{bot}} = - \frac{M y_{\text{bot}}}{I} = - \frac{(500)(-4.778)}{155.16} = 15.40 \text{ ksi}$$

**PROBLEM 4.12**

4.12 Two equal and opposite couples of magnitude of  $M = 15 \text{ kN}\cdot\text{m}$  are applied to the channel-shaped beam  $AB$ . Observing that the couples cause the beam to bend in a horizontal plane, determine the stress (a) at point  $C$ , (b) at point  $D$ , (c) at point  $E$ .



**SOLUTION**



	$A_i, \text{mm}^2$	$\bar{X}_i, \text{mm}$	$A\bar{X}_i, \text{mm}^3$
①	2400	50	$120 \times 10^3$
②	3060	15	$45.9 \times 10^3$
③	2400	50	$120 \times 10^3$
$\Sigma$	7860		$285.9 \times 10^3$

$$\bar{X} = \frac{285.9 \times 10^3}{7860} = 36.374 \text{ mm}$$

$$y_c = -36.374 \text{ mm} = -0.036374 \text{ m}$$

$$y_D = 100 - 36.374 = 63.626 \text{ mm} = 0.063626 \text{ m}$$

$$y_E = 30 - 36.374 = -6.374 \text{ mm} = -0.006374 \text{ m}$$

$$d_1 = 50 - 36.374 = 13.626 \text{ mm}$$

$$d_2 = 36.374 - 15 = 21.374 \text{ mm}$$

$$d_3 = d_1$$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (24)(100)^3 + (2400)(13.626)^2 = 2.4456 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (102)(30)^3 + (3060)(21.374)^2 = 1.6275 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 6.5187 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N}\cdot\text{m}$$

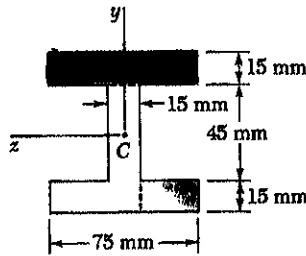
(a) Point C:  $\sigma_c = -\frac{My_c}{I} = -\frac{(15 \times 10^3)(-0.036374)}{6.5187 \times 10^{-6}} = 83.7 \times 10^6 \text{ Pa} = 83.7 \text{ MPa}$

(b) Point D:  $\sigma_D = -\frac{My_D}{I} = -\frac{(15 \times 10^3)(0.063626)}{6.5187 \times 10^{-6}} = -146.4 \times 10^6 \text{ Pa} = -146.4 \text{ MPa}$

(c) Point E:  $\sigma_E = -\frac{My_E}{I} = -\frac{(15 \times 10^3)(-0.006374)}{6.5187 \times 10^{-6}} = 14.67 \times 10^6 \text{ Pa} = 14.67 \text{ MPa}$

PROBLEM 4.13

4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is  $8 \text{ kN} \cdot \text{m}$ , determine the total force acting on the top flange.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where  $y$  is a coordinate with its origin on the neutral axis and  $I$  is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element  $dA$  the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

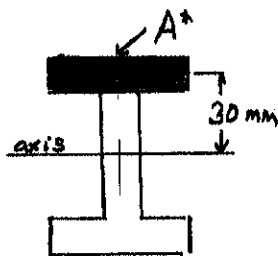
where  $\bar{y}^*$  is the centroidal coordinate of the shaded portion and  $A^*$  is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (75)(15)^3 + (75)(15)(30)^2 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (15)(45)^3 = 0.1139 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 2.1811 \times 10^6 \text{ mm}^4 = 2.1811 \times 10^{-6} \text{ m}^4$$



$$A^* = (75)(15) = 1125 \text{ mm}^2 = 1125 \times 10^{-6} \text{ m}^2$$

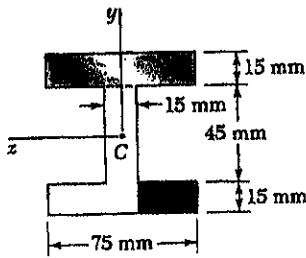
$$\bar{y}^* = 30 \text{ mm} = 0.030 \text{ m}$$

$$F = -\frac{M \bar{y}^* A^*}{I} = -\frac{(8 \times 10^3)(0.030)(1125 \times 10^{-6})}{2.1811 \times 10^{-6}}$$

$$= -123.8 \times 10^3 \text{ N} = -123.8 \text{ kN}$$

PROBLEM 4.14

4.14 Knowing that a beam of the cross section shown is bent about a vertical axis and that the bending moment is  $4 \text{ kN} \cdot \text{m}$ , determine the total force acting on the shaded portion of the lower flange.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula:

$$\sigma_x = -\frac{My}{I}$$

where  $y$  is a coordinate with its origin on the neutral axis and  $I$  is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element  $dA$  the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

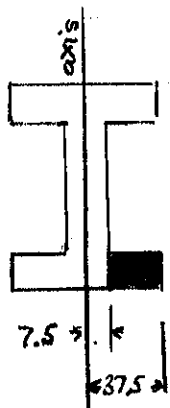
where  $\bar{y}^*$  is the centroidal coordinate of the shaded portion and  $A^*$  is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 = \frac{1}{12} (15)(75)^3 = 0.52734 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (45)(15)^3 = 0.01256 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 0.5273 \times 10^6$$

$$I = I_1 + I_2 + I_3 = 1.0672 \times 10^6 \text{ mm}^4 = 1.0672 \times 10^{-6} \text{ m}^4$$



$$A^* = (37.5 - 7.5)(15) = 450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$$

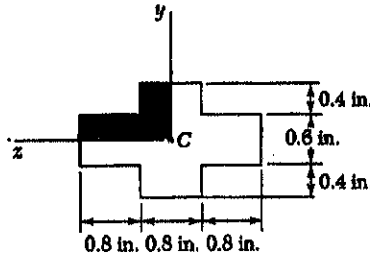
$$\bar{y}^* = \frac{1}{2}(37.5 + 7.5) = 22.5 \text{ mm} = 0.0225 \text{ m}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(4 \times 10^3)(0.0225)(450 \times 10^{-6})}{1.0672 \times 10^{-6}}$$

$$= 37.9 \times 10^3 \text{ N} = 37.9 \text{ kN}$$

PROBLEM 4.15

4.15 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 3.5 kip·in., determine the total force acting on the shaded portion of the beam.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where  $y$  is a coordinate with its origin on the neutral axis and  $I$  is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element  $dA$  the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

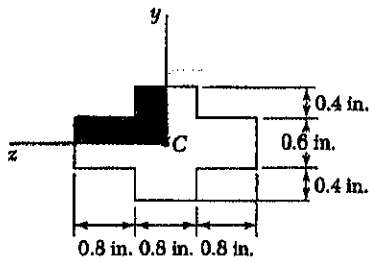
where  $\bar{y}^*$  is the centroidal coordinate of the shaded portion and  $A^*$  is its area.

$$\begin{aligned} I &= I_1 + I_2 + I_3 = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 \\ &= \frac{1}{12} (0.8)(0.6)^3 + \frac{1}{12} (0.8)(1.4)^3 + \frac{1}{12} (0.8)(0.6)^3 = 0.21173 \text{ in}^4 \end{aligned}$$

(b)

-\*A\*    =    A    =    A

PROBLEM 4.16



4.15 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 3.5 kip·in., determine the total force acting on the shaded portion of the beam.

4.16 Solve Prob. 4.15, assuming that the beam is bent about a vertical axis and that the bending moment is 6 kip·in.

SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where  $y$  is a coordinate with its origin on the neutral axis and  $I$  is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element  $dA$  the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

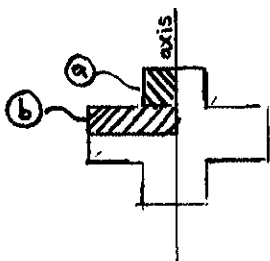
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where  $\bar{y}^*$  is the centroidal coordinate of the shaded portion and  $A^*$  is its area.

$$I = I_1 + I_2 + I_3 = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3$$

$$= \frac{1}{12} (0.4)(0.8)^3 + \frac{1}{12} (0.6)(2.4)^3 + \frac{1}{12} (0.4)(0.8)^3 = 0.7253 \text{ in}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$

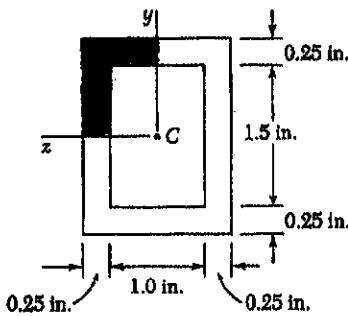
$$= (0.2)(0.4)(0.4) + (0.6)(0.3)(1.2)$$

$$= 0.248 \text{ in}^3$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(6)(0.248)}{0.7253} = 2.05 \text{ kips} \quad \blacktriangleleft$$

PROBLEM 4.17

4.17 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kip·in., determine the total force acting on the shaded portion of the beam.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

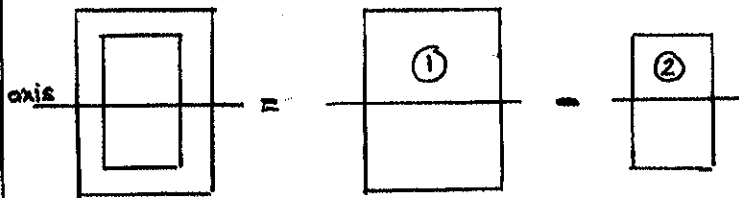
where  $y$  is a coordinate with its origin on the neutral axis and  $I$  is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element  $dA$  the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

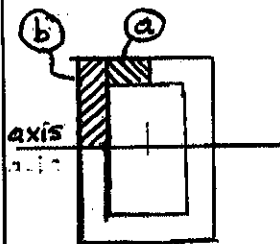
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where  $\bar{y}^*$  is the centroidal coordinate of the shaded portion and  $A^*$  is its area.



$$\begin{aligned} I &= I_1 - I_2 \\ &= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (1.5)(2.0)^3 - \frac{1}{12} (1.0)(1.5)^3 \\ &= 0.71875 \text{ in}^4 \end{aligned}$$

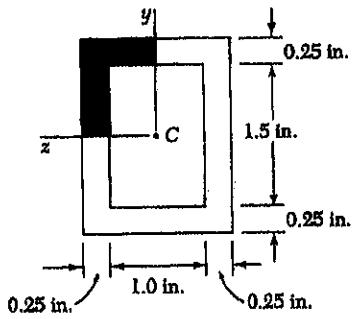


$$\begin{aligned} \bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\ &= (0.25)(0.25)(0.25) + (0.5)(0.25)(1.0) = 0.23438 \text{ in}^3 \end{aligned}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(6)(0.23438)}{0.71875} = 1.957 \text{ kips}$$



**PROBLEM 4.18**



4.17 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kip·in., determine the total force acting on the shaded portion of the beam.

4.18 Solve Prob. 4.17, assuming that the beam is bent about a vertical axis and that the bending moment is 6 kip·in.

**SOLUTION**

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

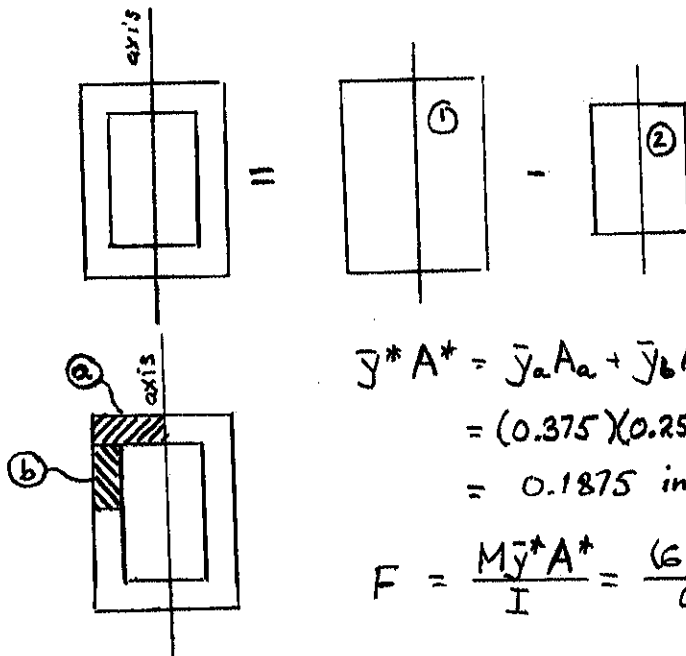
where  $y$  is a coordinate with its origin on the neutral axis and  $I$  is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element  $dA$  the force is

$$dF = \sigma_x dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I} \int y dA = -\frac{M}{I} \bar{y}^* A^*$$

where  $\bar{y}^*$  is the centroidal coordinate of the shaded portion and  $A^*$  is its area.



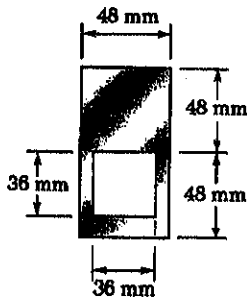
$$\begin{aligned} I &= I_1 - I_2 \\ &= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3 \\ &= \frac{1}{12} (2)(1.5)^3 - \frac{1}{12} (1.5)(1.0)^3 \\ &= 0.4375 \text{ in}^4 \end{aligned}$$

$$\begin{aligned} \bar{y}^* A^* &= \bar{y}_a A_a + \bar{y}_b A_b \\ &= (0.375)(0.25)(0.75) + (0.625)(0.75)(0.25) \\ &= 0.1875 \text{ in}^3 \end{aligned}$$

$$F = \frac{M \bar{y}^* A^*}{I} = \frac{(6)(0.1875)}{0.4375} = 2.57 \text{ kips.}$$

PROBLEM 4.19

4.19 and 4.20 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.



SOLUTION

	$A, \text{mm}^2$	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$
① solid rectangle	4608	48	221184
② square cutout	-1296	30	-38880
$\Sigma$	3312		182304

$$\bar{Y} = \frac{182304}{3312} = 55.04 \text{ mm}$$

Neutral axis lies 55.04 mm above bottom.



$$y_{\text{top}} = 96 - 55.04 = 40.96 \text{ mm} = 0.04096 \text{ m}$$

$$y_{\text{bot}} = -55.04 \text{ mm} = -0.05504 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (48)(96)^3 + (48)(96)(7.04)^2 = 3.7673 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (36)(36)^3 + (36)(36)(25.04)^2 = 0.9526 \times 10^6 \text{ mm}^4$$

$$I = I_1 - I_2 = 2.8147 \times 10^6 \text{ mm}^4 = 2.8147 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left| \frac{My}{I} \right| \therefore M = + \left| \frac{\sigma I}{y} \right|$$

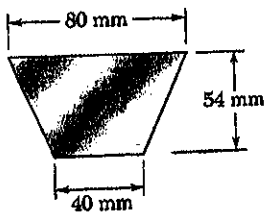
$$\text{Top: tension side} \quad M = \frac{(120 \times 10^6)(2.8147 \times 10^{-6})}{0.04096} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Bottom: compression} \quad M = \frac{(150 \times 10^6)(2.8147 \times 10^{-6})}{0.05504} = 7.67 \times 10^3 \text{ N}\cdot\text{m}$$

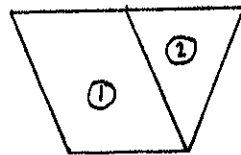
$$M_{\text{all}} \text{ is the smaller value} \quad M = 7.67 \times 10^3 \text{ N}\cdot\text{m} = 7.67 \text{ kN}\cdot\text{m}$$

PROBLEM 4.20

4.19 and 4.20 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.



SOLUTION



	$A, \text{mm}^2$	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	2160	27	58320
②	1080	36	38880
$\Sigma$	3240		97200

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$$

$$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = b_2 h_2^3 + A_2 d_2^2 = \frac{1}{36} (40)(54)^3 + \frac{1}{2} (40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{My}{I} \right|$$

$$|M| = \left| \frac{\sigma I}{y} \right|$$

top: tension side

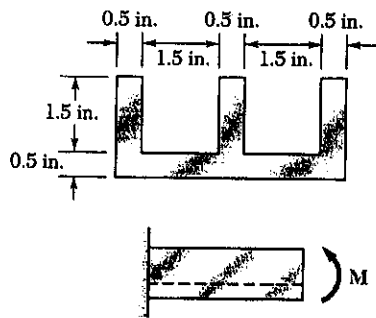
$$M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$$

bottom: compression

$$M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$$

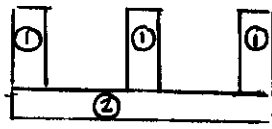
Choose the smaller as  $M_{all}$   $M_{all} = 3.7908 \times 10^3 \text{ N}\cdot\text{m} = 3.79 \text{ kN}\cdot\text{m}$  ◀

**PROBLEM 4.21**



**4.21** Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple  $M$  that can be applied.

**SOLUTION**



	A	$\bar{y}_o$	$A\bar{y}_o$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{\text{top}} = 2.0 - 0.75 = 1.25 \text{ in}, \quad y_{\text{bot}} = -0.75 \text{ in}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

$$|\sigma| = \left| \frac{My}{I} \right| \quad M = \left| \frac{\sigma I}{y} \right|$$

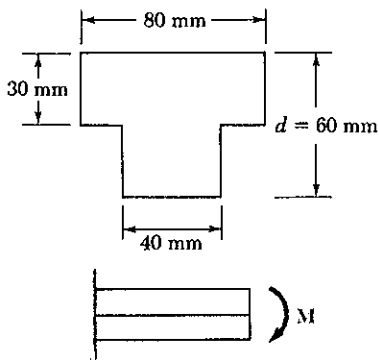
Top: compression  $M = \frac{(16)(1.59375)}{1.25} = 20.4 \text{ kip}\cdot\text{in}$

Bottom: tension  $M = \frac{(12)(1.59375)}{0.75} = 25.5 \text{ kip}\cdot\text{in}$

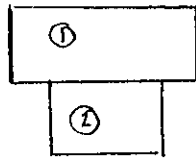
Choose the smaller as  $M_{\text{all}}$   $M_{\text{all}} = 20.4 \text{ kip}\cdot\text{in}$

**PROBLEM 4.22**

4.22 The beam shown is made of a nylon for which the allowable stress 24 MPa in tension and 30 MPa in compression. Determine the largest couple  $M$  that can be applied to the beam.



**SOLUTION**



	$A, \text{mm}^2$	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	2400	45	108000
②	1200	15	18000
$\Sigma$	3600		126000

$$\bar{Y}_o = \frac{126000}{3600} = 35 \text{ mm}$$

The neutral axis lies 35 mm above the bottom.

$$y_{\text{top}} = 60 - 35 = 25 \text{ mm} = 0.025 \text{ m}, \quad y_{\text{bot}} = -35 \text{ mm} = -0.035 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (80)(30)^3 + (2400)(10)^2 = 420 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (40)(30)^3 + (1200)(20)^2 = 570 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 990 \times 10^3 \text{ mm}^4 = 990 \times 10^{-9} \text{ m}^4$$

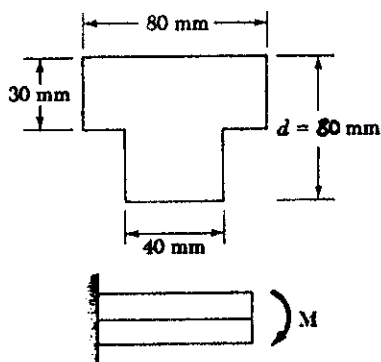
$$|G| = \left| \frac{M y}{I} \right| \quad M = \left| \frac{G I}{y} \right|$$

Top: tension side  $M = \frac{(24 \times 10^6)(990 \times 10^{-9})}{0.025} = 950 \text{ N}\cdot\text{m}$

Bottom: compression  $M = \frac{(30 \times 10^6)(990 \times 10^{-9})}{0.035} = 849 \text{ N}\cdot\text{m}$

Choose smaller value  $M = 849 \text{ N}\cdot\text{m}$

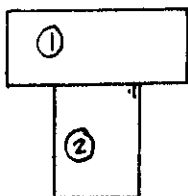
**PROBLEM 4.23**



4.22 The beam shown is made of a nylon for which the allowable stress 24 MPa in tension and 30 MPa in compression. Determine the largest couple  $M$  that can be applied to the beam.

4.23 Solve Prob. 4.22, assuming that  $d = 80$  mm.

**SOLUTION**



	$A, \text{mm}^2$	$\bar{y}_o, \text{mm}$	$A\bar{y}_o, \text{mm}^3$
①	2400	65	156000
②	2000	25	50000
$\Sigma$	4400		206000

$$\bar{Y}_o = \frac{206000}{4400} = 46.82 \text{ mm}$$

The neutral axis lies 46.82 mm above the bottom.

$$y_{\text{top}} = 80 - 46.82 = 33.18 \text{ mm} = 0.03318 \text{ m}$$

$$y_{\text{bot}} = -46.82 \text{ mm} = -0.04682 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (80)(30)^3 + (2400)(18.18)^2 = 0.97323 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12} (40)(80)^3 + (2000)(21.82)^2 = 1.36889 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.342 \times 10^6 \text{ mm}^4 = 2.342 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left| \frac{M y}{I} \right| \quad M = \left| \frac{\sigma I}{y} \right|$$

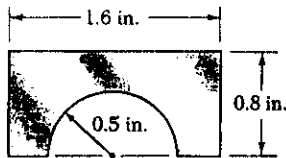
Top: tension side  $M = \frac{(24 \times 10^6)(2.342 \times 10^{-6})}{0.03318} = 1.694 \times 10^3 \text{ N}\cdot\text{m}$

Bottom: compression  $M = \frac{(30 \times 10^6)(2.342 \times 10^{-6})}{0.04682} = 1.501 \times 10^3 \text{ N}\cdot\text{m}$

Choose smaller value  $M = 1.501 \times 10^3 \text{ N}\cdot\text{m} = 1.501 \text{ kN}\cdot\text{m}$

PROBLEM 4.24

4.24 Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple  $M$  that can be applied.



SOLUTION

① = rectangle      ② = semi-circular cutout

$$A_1 = (1.6)(0.8) = 1.28 \text{ in}^2$$

$$A_2 = \frac{\pi}{2}(0.5)^2 = 0.3927 \text{ in}^2$$

$$A = 1.28 - 0.3927 = 0.8873 \text{ in}^2$$

$$\bar{y}_1 = 0.4 \text{ in}$$

$$\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.5)}{3\pi} = 0.2122 \text{ in}$$

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{(1.28)(0.4) - (0.3927)(0.2122)}{0.8873} = 0.4831 \text{ in.}$$

Neutral axis lies 0.4831 in above the bottom

Moment of inertia about the base

$$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(1.6)(0.8)^3 - \frac{\pi}{8}(0.5)^4 = 0.24852 \text{ in}^4$$

Centroidal moment of inertia

$$\begin{aligned} \bar{I} &= I_b - A\bar{y}^2 = 0.24852 - (0.8873)(0.4831)^2 \\ &= 0.04144 \text{ in}^4 \end{aligned}$$

$$y_{\text{top}} = 0.8 - 0.4831 = 0.3169 \text{ in}, \quad y_{\text{bot}} = -0.4831 \text{ in}$$

$$|\sigma| = \left| \frac{My}{I} \right| \quad M = \left| \frac{\sigma I}{y} \right|$$

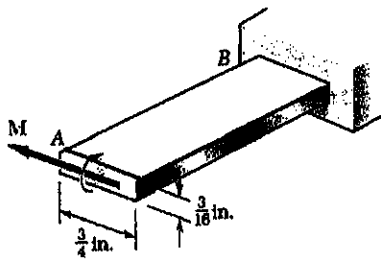
$$\text{Top: tension side} \quad M = \frac{(12)(0.04144)}{0.3169} = 1.569 \text{ kip}\cdot\text{in}$$

$$\text{Bottom: compression} \quad M = \frac{(16)(0.04144)}{0.4831} = 1.372 \text{ kip}\cdot\text{in}$$

Choose the smaller value

$$M = 1.372 \text{ kip}\cdot\text{in.}$$

PROBLEM 4.25



4.25 Knowing that  $\sigma_{in} = 24$  ksi for the steel strip AB, determine (a) the largest couple M that can be applied, (b) the corresponding radius of curvature. Use  $E = 29 \times 10^6$  psi.

SOLUTION

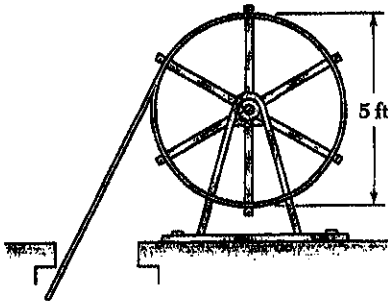
$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{16}\right)\left(\frac{3}{16}\right)^3 = 412.0 \times 10^{-6} \text{ in}^4$$

$$\sigma = \frac{Mc}{I} \quad c = \frac{1}{2}\left(\frac{3}{16}\right) = 0.09375 \text{ in}$$

$$(a) \quad M = \frac{\sigma I}{c} = \frac{(24 \times 10^3)(412.0 \times 10^{-6})}{0.09375} = 105.5 \text{ lb}\cdot\text{in}$$

$$(b) \quad \frac{c}{\rho} = \frac{\sigma_{max}}{E} \quad \rho = \frac{Ec}{\sigma_{max}} = \frac{(29 \times 10^6)(0.09375)}{24 \times 10^3} = 113.3 \text{ in}$$

PROBLEM 4.26



4.26 Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a rod, when the rod, which was initially straight, is wrapped on a spool, (b) the corresponding bending moment in the rod. Use  $E = 29 \times 10^6$  psi.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15 \text{ in}$$

$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6} \text{ in}^4$$

$$D = 5 \text{ ft} = 60 \text{ in} \quad \rho = \frac{1}{2}D = 30 \text{ in}$$

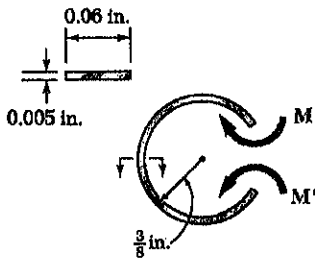
$$c = r = 0.15 \text{ in}$$

$$(a) \quad \sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3 \text{ psi} = 145 \text{ ksi}$$

$$(b) \quad M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30} = 384 \text{ lb}\cdot\text{in}$$



**PROBLEM 4.27**



4.27 It is observed that a thin steel strip of 0.06-in. width can be bent into a circle of  $\frac{3}{8}$ -in. diameter without any resulting permanent deformation. Knowing that  $E = 29 \times 10^6$  psi, determine (a) the maximum stress in the bent strip, (b) the magnitude of the couples required to bend the strip.

**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.06)(0.005)^3 = 625 \times 10^{-12} \text{ in}^4$$

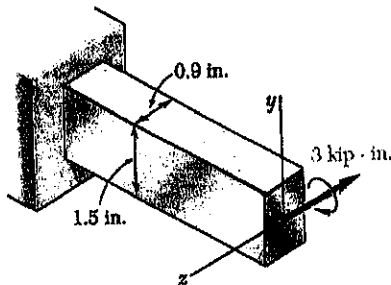
$$\rho = \frac{1}{2} D = \frac{1}{2} \left(\frac{3}{8}\right) = 0.375 \text{ in}$$

$$c = \frac{1}{2} h = 0.0025 \text{ in}$$

$$(a) \quad \sigma_{max} = \frac{E c}{\rho} = \frac{(29 \times 10^6)(0.0025)}{0.375} = 193.3 \times 10^3 \text{ psi} = 193.3 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad M = \frac{E I}{\rho} = \frac{(29 \times 10^6)(625 \times 10^{-12})}{0.375} = 0.0483 \text{ lb. in.} \quad \blacktriangleleft$$

**PROBLEM 4.28**



4.28 A 3 kip-in. couple is applied to the steel bar shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the bar. (b) Solve part a, assuming that the couple is applied about the y axis. Use  $E = 29 \times 10^6$  psi.

**SOLUTION**

(a) Bending about z-axis.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.9)(1.5)^3 = 0.25313 \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2} (1.5) = 0.75 \text{ in}$$

$$\sigma = \frac{M c}{I} = \frac{(3 \times 10^3)(0.75)}{0.25313} = 8.89 \times 10^3 \text{ psi} = 8.89 \text{ ksi} \quad \blacktriangleleft$$

$$\frac{1}{\rho} = \frac{M}{E I} = \frac{3 \times 10^3}{(29 \times 10^6)(0.25313)} = 409 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 2450 \text{ in} = 204 \text{ ft} \quad \blacktriangleleft$$

(b) Bending about y-axis

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (1.5)(0.9)^3 = 0.091125 \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2} (0.9) = 0.45 \text{ in}$$

$$\sigma = \frac{M c}{I} = \frac{(3 \times 10^3)(0.45)}{0.091125} = 14.81 \times 10^3 \text{ psi} = 14.81 \text{ ksi} \quad \blacktriangleleft$$

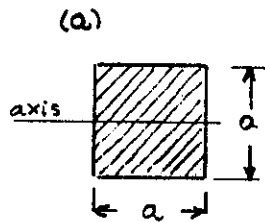
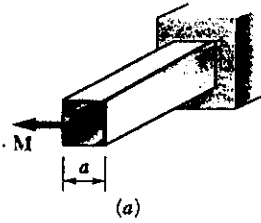
$$\frac{1}{\rho} = \frac{M}{E I} = \frac{3 \times 10^3}{(29 \times 10^6)(0.091125)} = 1.135 \times 10^{-5} \text{ in}^{-1}$$

$$\rho = 881 \text{ in} = 73.4 \text{ ft.} \quad \blacktriangleleft$$

PROBLEM 4.29

4.29 A couple of magnitude  $M$  is applied to a square bar of side  $a$ . For each of the orientations shown, determine the maximum stress and the curvature of the bar.

SOLUTION

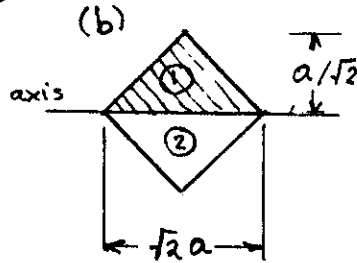
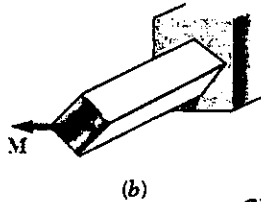


$$I = \frac{1}{12} bh^3 = \frac{1}{12} a a^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M \frac{a}{2}}{\frac{a^4}{12}} = \frac{6M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}} = \frac{12M}{Ea^4}$$



For one triangle the moment of inertia about its base is

$$I_1 = \frac{1}{12} bh^3 = \frac{1}{12} (\sqrt{2}a) \left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

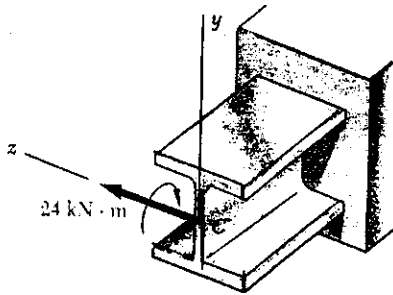
$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M \frac{a}{\sqrt{2}}}{\frac{a^4}{12}} = \frac{6\sqrt{2}M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}} = \frac{12M}{Ea^4}$$

PROBLEM 4.30



4.30 A 24 kN·m couple is applied to the W200 × 46.1 rolled-steel beam shown. (a) Assuming that the couple is applied about the z axis as shown, determine the maximum stress and the radius of curvature of the beam. (b) Solve part a, assuming that the couple is applied about the y axis. Use  $E = 200$  GPa.

SOLUTION

For W 200 × 46.1 rolled steel section

$$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$$

$$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

(a)  $M_z = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 379 \text{ m}$$

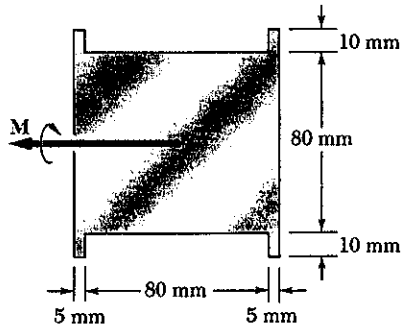
(b)  $M_y = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 127.5 \text{ m}$$

PROBLEM 4.31



4.31 (a) Using an allowable stress of 120 MPa, determine the largest couple  $M$  that can be applied to a beam of the cross section shown. (b) Solve part  $a$ , assuming that the cross section of the beam is an 80-mm square.

SOLUTION

(a)  $I = I_1 + 4I_2$ , where  $I_1$  is the moment of inertia of an 80-mm square and  $I_2$  is the moment of inertia of one of the 4 protruding ears.

$$I_1 = \frac{1}{12} b h^3 = \frac{1}{12} (80)(80)^3 = 3.4133 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 + A d^2 = \frac{1}{12} (5)(10)^3 + (5)(10)(45)^2 = 101.667 \times 10^3 \text{ mm}^4$$

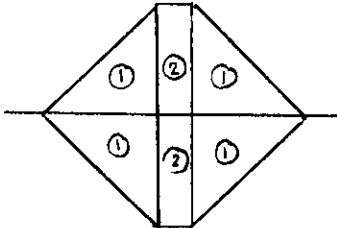
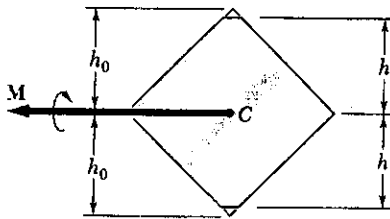
$$I = I_1 + 4I_2 = 3.82 \times 10^6 \text{ mm}^4 = 3.82 \times 10^{-6} \text{ m}^4, \quad c = 50 \text{ mm} = 0.050 \text{ m}$$

$$\sigma = \frac{M c}{I} \therefore M = \frac{\sigma I}{c} = \frac{(120 \times 10^6)(3.82 \times 10^{-6})}{0.050} = 9.168 \times 10^3 \text{ N}\cdot\text{m} = 9.17 \text{ kN}\cdot\text{m}$$

(b) Without the ears  $I = I_1 = 3.4133 \times 10^{-6} \text{ m}^4$ ,  $c = 40 \text{ mm} = 0.040 \text{ m}$

$$M_i = \frac{\sigma I}{c} = \frac{(120 \times 10^6)(3.4133 \times 10^{-6})}{0.040} = 10.24 \times 10^3 \text{ N}\cdot\text{m} = 10.24 \text{ kN}\cdot\text{m}$$

PROBLEM 4.32



4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple  $M$ . Considering the case where  $h = 0.9h_0$ , express the maximum stress in the bar in the form  $\sigma_m = k\sigma_0$ , where  $\sigma_0$  is the maximum stress that would have occurred if the original square bar had been bent by the same couple  $M$ , and determine the value of  $k$ .

SOLUTION

$$I = 4I_1 + 2I_2$$

$$= (4)\left(\frac{1}{12}\right)h h^3 + (2)\left(\frac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \frac{1}{3}h^4 + \frac{4}{3}h_0 h^3 - \frac{4}{3}h h^3 = \frac{4}{3}h_0 h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0 h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

For the original square  $h = h_0, c = h_0$

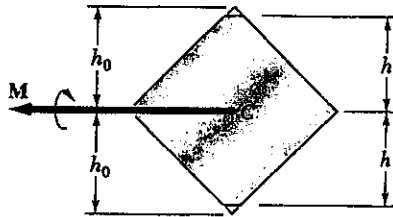
$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9h_0^2)} = 0.950$$

$$\sigma = 0.950 \sigma_0$$

$$k = 0.950$$

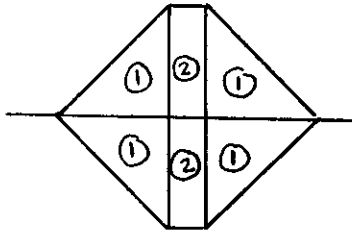
**PROBLEM 4.33**



4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple  $M$ . Considering the case where  $h = 0.9h_0$ , express the maximum stress in the bar in the form  $\sigma_m = k\sigma_0$ , where  $\sigma_0$  is the maximum stress that would have occurred if the original square bar had been bent by the same couple  $M$ , and determine the value of  $k$ .

4.33 In Prob. 4.32, determine (a) the value of  $h$  for which the maximum stress  $\sigma_m$  is as small as possible. (b) the corresponding value of  $k$ .

**SOLUTION**



$$I = 4I_1 + 2I_2$$

$$= (4 \times \frac{1}{12})h^3 + (2)(\frac{1}{3})(2h_0 - 2h)h^3$$

$$= \frac{1}{3}h^4 - \frac{4}{3}h_0h^3 + \frac{4}{3}h^3 = \frac{4}{3}h_0h^3 - h^4$$

$$c = h \quad \frac{I}{c} = \frac{4}{3}h_0h^2 - h^3$$

$$\frac{I}{c} \text{ is maximum at } \frac{d}{dh}[\frac{4}{3}h_0h^2 - h^3] = 0$$

$$\frac{8}{3}h_0h - 3h^2 = 0 \quad h = \frac{8}{9}h_0$$

$$\frac{I}{c} = \frac{4}{3}h_0(\frac{8}{9}h_0)^2 - (\frac{8}{9}h_0)^3 = \frac{256}{729}h_0^3$$

$$\sigma = \frac{Mc}{I} = \frac{729}{256} \frac{M}{h_0^3}$$

For the original square  $h = h_0$

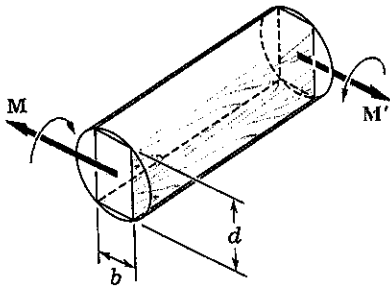
$$c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256} \cdot \frac{1}{3} = \frac{729}{768} = 0.949$$

$$k = 0.949$$

PROBLEM 4.34



4.34 A couple  $M$  will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio  $d/b$ , for which (a) the maximum stress  $\sigma_m$  will be as small as possible, (b) the radius of curvature of the beam will be maximum.

SOLUTION

Let  $D$  be the diameter of the log.

$$D^2 = b^2 + d^2 \quad d^2 = D^2 - b^2$$

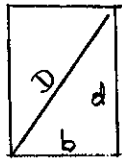
$$I = \frac{1}{12} b d^3 \quad c = \frac{1}{2} d \quad \frac{I}{c} = \frac{1}{6} b d^2$$

(a)  $\sigma_m$  is minimum when  $\frac{I}{c}$  is maximum

$$\frac{I}{c} = \frac{1}{6} b (D^2 - b^2) = \frac{1}{6} D^2 b - \frac{1}{6} b^3$$

$$\frac{d}{db} \left( \frac{I}{c} \right) = \frac{1}{6} D^2 - \frac{2}{6} b^2 = 0 \quad b = \frac{1}{\sqrt{3}} D$$

$$d = \sqrt{D^2 - \frac{1}{3} D^2} = \sqrt{\frac{2}{3}} D \quad \frac{d}{b} = \sqrt{2}$$



$$\rho = \frac{EI}{M} \quad \rho \text{ is maximum when } I \text{ is maximum.}$$

$\frac{1}{12} b d^3$  is maximum or  $b^2 d^6$  is maximum

$(D^2 - d^2) d^6$  is maximum.

$$6D^2 d^5 - 8d^7 = 0 \quad d = \frac{\sqrt{3}}{2} D$$

$$b = \sqrt{D^2 - \frac{3}{4} D^2} = \frac{1}{2} D \quad \frac{d}{b} = \sqrt{3}$$

**PROBLEM 4.35**

4.35 For the bar and loading of Example 4.01, determine (a) the radius of curvature  $\rho$ , (b) the radius of curvature  $\rho'$  of a transverse cross section, (c) the angle between the sides of the bar which were originally vertical. Use  $E = 29 \times 10^6$  psi and  $\nu = 0.29$ .

**SOLUTION**

From Example 4.01  $M = 30 \text{ kip}\cdot\text{in}$ ,  $I = 1.042 \text{ in}^4$

$$(a) \frac{1}{\rho} = \frac{M}{EI} = \frac{(30 \times 10^3)}{(29 \times 10^6)(1.042)} = 993 \times 10^{-6} \text{ in}^{-1} \quad \rho = 1007 \text{ in.} \quad \blacktriangleleft$$

$$(b) \epsilon' = \nu \epsilon = \frac{\nu C}{\rho} = \nu \frac{C}{\rho'}$$

$$\frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(993 \times 10^{-6}) \text{ in}^{-1} = 288 \text{ in}^{-1} \quad \rho' = 3470 \text{ in.} \quad \blacktriangleleft$$

$$(c) \theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230 \times 10^{-6} \text{ rad} = 0.01320^\circ \quad \blacktriangleleft$$

**PROBLEM 4.36**

4.36 For the aluminum bar and loading of Sample Prob. 4.1, determine (a) the radius of curvature  $\rho'$  of a transverse cross section, (b) the angle between the sides of the bar which were originally vertical. Use  $E = 10.6 \times 10^6$  psi and  $\nu = 0.33$ .

**SOLUTION**

From Sample Problem 4.1  $I = 12.97 \text{ in}^4$   $M = 103.8 \text{ kip}\cdot\text{in}$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{103.8 \times 10^3}{(10.6 \times 10^6)(12.97)} = 755 \times 10^{-6} \text{ in}^{-1}$$

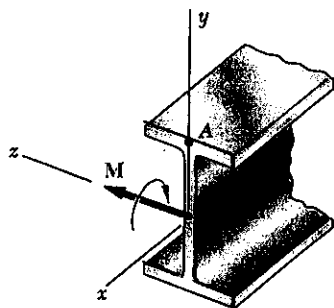
$$(a) \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.33)(755 \times 10^{-6}) = 249 \times 10^{-6} \text{ in}^{-1}$$

$$\rho' = 4010 \text{ in} = 334 \text{ ft.} \quad \blacktriangleleft$$

$$(b) \theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{3.25}{4010} = 810 \times 10^{-6} \text{ rad} = 0.0464^\circ \quad \blacktriangleleft$$

**PROBLEM 4.37**

4.37 A W 200  $\times$  31.3 rolled-steel beam is subjected to a couple  $M$  of moment 45 kN $\cdot$ m. Knowing that  $E = 200 \text{ GPa}$ ,  $\nu = 0.29$ , determine (a) the radius of curvature  $\rho$ , (b) the radius of curvature  $\rho'$  of a transverse cross section.



**SOLUTION**

For W 200  $\times$  31.3 rolled steel section

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

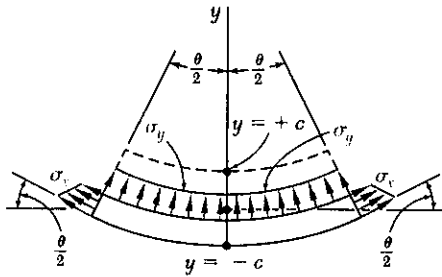
$$(a) \frac{1}{\rho} = \frac{M}{EI} = \frac{45 \times 10^3}{(200 \times 10^9)(31.4 \times 10^{-6})} = 7.17 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 139.6 \text{ m} \quad \blacktriangleleft$$

$$(b) \frac{1}{\rho'} = \nu \frac{1}{\rho} = (0.29)(7.17 \times 10^{-3}) = 2.07 \times 10^{-3} \text{ m}^{-1} \quad \rho' = 481 \text{ m} \quad \blacktriangleleft$$



**PROBLEM 4.38**



4.38 It was assumed in Sec. 4.3 that the normal stresses  $\sigma_x$  in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (a) derive an approximate expression for  $\sigma_y$  as a function of  $y$ , (b) show that  $(\sigma_y)_{\max} \approx -(c/2\rho)(\sigma_x)_{\max}$  and, thus, that  $\sigma_y$  can be neglected in all practical situations. (Hint: Consider the free-body diagram of the portion of beam located below the surface of ordinate  $y$  and assume the distribution of the stress  $\sigma_x$  is still linear.)

**SOLUTION**

Denote the width of the beam by  $b$  and the length by  $L$ .

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with  $\cos \frac{\theta}{2} \approx 1$

$$\sum F_y = 0 \quad \sigma_y b L + 2 \int_{-c}^y \sigma_x b dy \sin \frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L} \sin \frac{\theta}{2} \int_{-c}^y \sigma_x dy \approx -\frac{\theta}{L} \int_{-c}^y \sigma_x dy = -\frac{1}{\rho} \int_{-c}^y \sigma_x dy$$

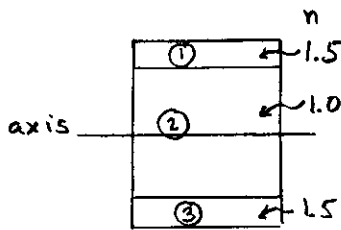
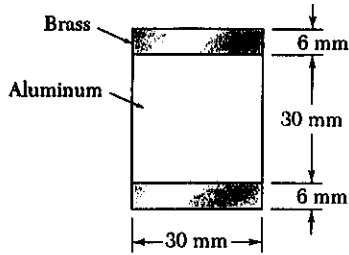
But  $\sigma_x = -(\sigma_x)_{\max} \frac{y}{c}$

$$\sigma_y = \frac{(\sigma_x)_{\max}}{\rho c} \int_{-c}^y y dy = \frac{(\sigma_x)_{\max}}{\rho c} \left. \frac{y^2}{2} \right|_{-c}^y = \frac{(\sigma_x)_{\max}}{2\rho c} (y^2 - c^2)$$

The maximum value  $\sigma_y$  occurs at  $y = 0$

$$(\sigma_y)_{\max} = -\frac{(\sigma_x)_{\max} c^2}{2\rho c} = -\frac{(\sigma_x)_{\max} c}{2\rho}$$

**PROBLEM 4.39**



**4.39 and 4.40** Two brass strips are securely bonded to an aluminum bar of  $30 \times 30$ -mm square cross section. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.

Modulus of elasticity:  
Allowable stress:

**Aluminum**  
70 GPa  
100 MPa

**Brass**  
105 GPa  
160 MPa

**SOLUTION**

Use aluminum as the reference material

$n = 1.0$  in aluminum

$n = E_b / E_a = 105 / 70 = 1.5$  in brass

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.5}{12} (30)(6)^3 + (1.5)(30)(6)(18)^2 = 88.29 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4, \quad I_3 = I_1 = 88.29 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 244.08 \times 10^3 \text{ mm}^4 = 244.08 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{n M y}{I} \right| \quad M = \frac{\sigma I}{n y}$$

Aluminum:  $n = 1.0$ ,  $y = 15 \text{ mm} = 0.015 \text{ m}$ ,  $\sigma = 100 \times 10^6 \text{ Pa}$

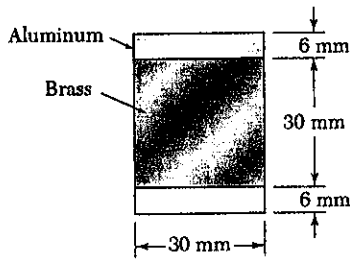
$$M = \frac{(100 \times 10^6)(244.08 \times 10^{-9})}{(1.0)(0.015)} = 1.627 \times 10^3 \text{ N}\cdot\text{m}$$

Brass:  $n = 1.5$ ,  $y = 21 \text{ mm} = 0.021 \text{ m}$ ,  $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(244.08 \times 10^{-9})}{(1.5)(0.021)} = 1.240 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value  $M = 1.240 \times 10^3 \text{ N}\cdot\text{m} = 1.240 \text{ kN}\cdot\text{m}$  ←

PROBLEM 4.40



4.39 and 4.40 Two ~~aluminum~~ strips are securely bonded to an ~~aluminum~~ brass bar of  $30 \times 30$ -mm square cross section. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.

Modulus of elasticity:  
Allowable stress:

Aluminum	Brass
70 GPa	105 GPa
100 MPa	160 MPa

SOLUTION

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = E_b/E_a = 105/70 = 1.5 \text{ in brass}$$

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.0}{12} (30)(6)^3 + (1.0)(30)(6)(18)^2 = 58.86 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12} (30)(30)^3 = 101.25 \times 10^3 \text{ mm}^4, \quad I_3 = I_1 = 58.86 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 218.97 \times 10^3 \text{ mm}^4 = 218.97 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{n M y}{I} \right| \therefore M = \frac{\sigma I}{n y}$$

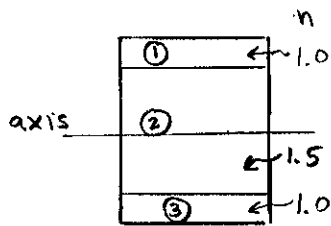
Aluminum:  $n = 1.0$ ,  $y = 21 \text{ mm} = 0.021 \text{ m}$ ,  $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(218.97 \times 10^{-9})}{(1.0)(0.021)} = 1.043 \times 10^3 \text{ N}\cdot\text{m}$$

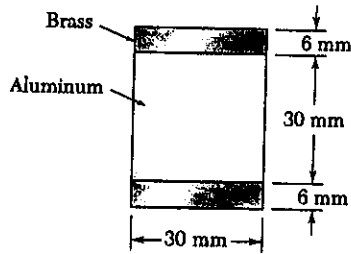
Brass:  $n = 1.5$ ,  $y = 15 \text{ mm} = 0.015 \text{ m}$ ,  $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(218.97 \times 10^{-9})}{(1.5)(0.015)} = 1.557 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value  $M = 1.043 \times 10^3 \text{ N}\cdot\text{m} = 1.043 \text{ kN}\cdot\text{m}$



PROBLEM 4.41



4.41 and 4.42 For the composite bar indicated, determine the permissible bending moment when the bar is bent about a vertical axis.

4.41 Bar of Prob. 4.39

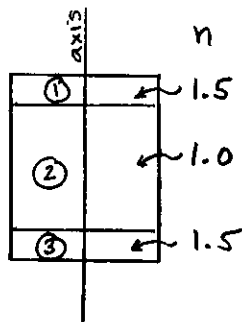
SOLUTION

Use aluminum as reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = E_b / E_a = 105 / 70 = 1.5 \text{ in brass}$$

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.5}{12} (6)(30)^3 = 20.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3$$

$$= \frac{1.0}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 20.25 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 108 \times 10^3 \text{ mm}^4 = 108 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \therefore M = \frac{\sigma I}{ny}$$

Aluminum:  $n = 1.0$ ,  $y = 15 \text{ mm} = 0.015 \text{ m}$ ,  $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(108 \times 10^{-9})}{(1.0)(0.015)} = 720 \text{ N}\cdot\text{m}$$

Brass:  $n = 1.5$ ,  $y = 15 \text{ mm} = 0.015 \text{ m}$ ,  $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(108 \times 10^{-9})}{(1.5)(0.015)} = 768 \text{ N}\cdot\text{m}$$

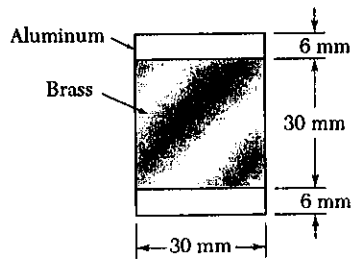
Choose the smaller value

$$M = 720 \text{ N}\cdot\text{m}$$

**PROBLEM 4.42**

4.41 and 4.42 For the composite bar indicated, determine the permissible bending moment when the bar is bent about a vertical axis.

4.42 Bar of Prob. 4.40



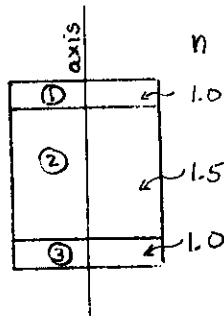
**SOLUTION**

Use aluminum as reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = E_b/E_a = 105/70 = 1.5 \text{ in brass}$$

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.0}{12} (6)(30)^3 = 13.5 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12} (30)(30)^3 = 101.25 \text{ mm}^4$$

$$I_3 = I_1 = 13.5 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 128.25 \times 10^3 \text{ mm}^4 = 128.25 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad M = \frac{\sigma I}{ny}$$

Aluminum:  $n = 1.0$ ,  $y = 15 \text{ mm} = 0.015 \text{ m}$ ,  $\sigma = 100 \times 10^6 \text{ Pa}$

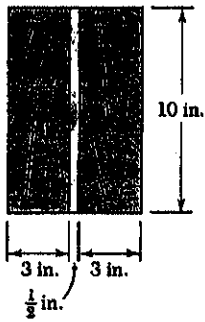
$$M = \frac{(100 \times 10^6)(128.25 \times 10^{-9})}{(1.0)(0.015)} = 855 \text{ N}\cdot\text{m}$$

Brass:  $n = 1.5$ ,  $y = 15 \text{ mm} = 0.015 \text{ m}$ ,  $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(128.25 \times 10^{-9})}{(1.5)(0.015)} = 912 \text{ N}\cdot\text{m}$$

Choose the smaller value  $M = 855 \text{ N}\cdot\text{m}$

**PROBLEM 4.43**



4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

Modulus of elasticity:  
Allowable stress:

Wood  
 $2 \times 10^6$  psi  
2000 psi

Steel  
 $30 \times 10^6$  psi  
22 ksi

**SOLUTION**

Use wood as the reference material

$$n = 1.0 \text{ in wood}$$

$$n = E_s / E_w = 30 / 2 = 15 \text{ in steel}$$

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12} (3)(10)^3 = 250 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{15}{12} \left(\frac{1}{8}\right)(10)^3 = 625 \text{ in}^4$$

$$I_3 = I_1 = 250 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 1125 \text{ in}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \therefore M = \frac{\sigma I}{ny}$$

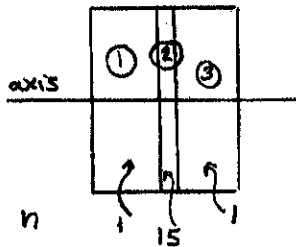
Wood:  $n = 1.0$ ,  $y = 5 \text{ in}$ ,  $\sigma = 2000 \text{ psi}$

$$M = \frac{(2000)(1125)}{(1.0)(5)} = 450 \times 10^3 \text{ lb}\cdot\text{in}$$

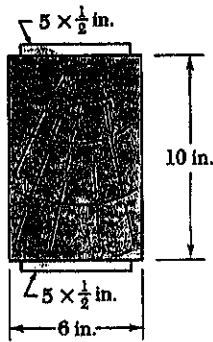
Steel:  $n = 15$ ,  $y = 5 \text{ in}$ ,  $\sigma = 22 \text{ ksi} = 22 \times 10^3 \text{ psi}$

$$M = \frac{(22 \times 10^3)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value  $M = 330 \times 10^3 \text{ lb}\cdot\text{in} = 330 \text{ kip}\cdot\text{in}$



**PROBLEM 4.44**



**SOLUTION**

4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

Modulus of elasticity:  
Allowable stress:

Wood  
 $2 \times 10^6$  psi  
2000 psi

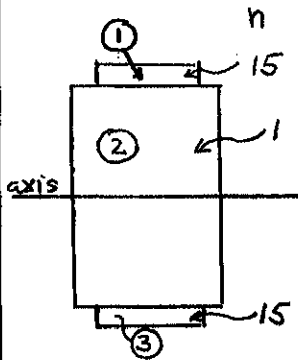
Steel  
 $30 \times 10^6$  psi  
22 ksi

Use wood as the reference material

$$n = 1.0 \text{ in wood}$$

$$n = E_s / E_w = 30 / 2 = 15 \text{ in steel}$$

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{15}{12} (5) \left(\frac{1}{2}\right)^3 + (15)(5) \left(\frac{1}{2}\right) (5.25)^2 = 1034.4 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12} (6) (10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 1034.4 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2569 \text{ in}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood:  $n = 1.0$ ,  $y = 5 \text{ in}$ ,  $\sigma = 2000 \text{ psi}$

$$M = \frac{(2000)(2569)}{(1.0)(5)} = 1.028 \times 10^6 \text{ lb}\cdot\text{in}$$

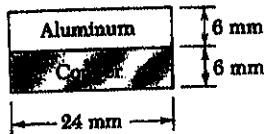
Steel  $n = 15$ ,  $y = 5.5 \text{ in}$ ,  $\sigma = 22 \text{ ksi} = 22 \times 10^3 \text{ psi}$

$$M = \frac{(22 \times 10^3)(2569)}{(15)(5.5)} = 685 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value

$$M = 685 \times 10^3 \text{ lb}\cdot\text{in} = 685 \text{ kip}\cdot\text{in}$$

PROBLEM 4.45



4.45 and 4.46 A copper strip ( $E_c = 105$  GPa) and an aluminum strip ( $E_a = 75$  GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment  $35$  N·m, determine the maximum stress in (a) the aluminum strip, (b) the copper strip.

SOLUTION

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = E_c/E_a = 105/75 = 1.4 \text{ in copper}$$



Transformed section

	$A, \text{mm}^2$	$nA, \text{mm}^2$	$\bar{y}_0, \text{mm}$	$nA\bar{y}_0, \text{mm}^3$
①	144	144	9	1296
②	144	201.6	3	604.8
$\Sigma$		345.6		1900.8

$$\bar{y}_0 = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies  $5.50$  mm above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12} (24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

(a) Aluminum  $n = 1.0$   $y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$

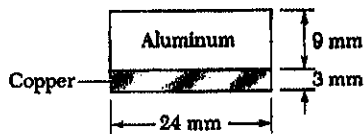
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa} \quad \blacktriangleleft$$

(b) Copper  $n = 1.4$   $y = -5.5 \text{ mm} = -0.0055 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa} \quad \blacktriangleleft$$



**PROBLEM 4.46**



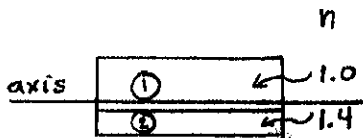
4.45 and 4.46 A copper strip ( $E_c = 105$  GPa) and an aluminum strip ( $E_a = 75$  GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N·m, determine the maximum stress in (a) the aluminum strip, (b) the copper strip.

**SOLUTION**

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = E_c / E_a = 105 / 75 = 1.4 \text{ in copper}$$



Transformed section

	$A, \text{mm}^2$	$nA, \text{mm}^2$	$\bar{y}_o, \text{mm}$	$nA\bar{y}_o, \text{mm}^3$
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
$\Sigma$		316.8		1771.2

$$\bar{y}_o = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom

$$I_1 = \frac{n_1 b_1 h_1^3}{12} + n_1 A_1 d_1^2 = \frac{1.0 (24) (9)^3}{12} + (1.0) (24) (9) (1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2 b_2 h_2^3}{12} + n_2 A_2 d_2^2 = \frac{1.4 (24) (3)^3}{12} + (1.4) (24) (3) (4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4839 \text{ mm}^4 = 4.008 \times 10^{-9} \text{ m}^4$$

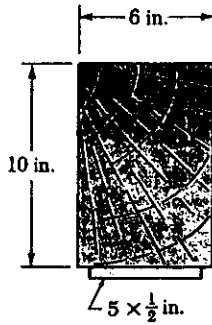
(a) Aluminum:  $n = 1.0$   $y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa} \blacktriangleleft$$

(b) Copper:  $n = 1.4$   $y = -5.5909 \text{ mm} = -0.0055909 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008 \times 10^{-9}} = 68.4 \times 10^6 \text{ Pa} = 68.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.47



4.47 and 4.48 A 6 × 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is  $1.5 \times 10^6$  psi for the wood and  $30 \times 10^6$  psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip·in., determine the maximum stress in (a) the wood, (b) the steel.

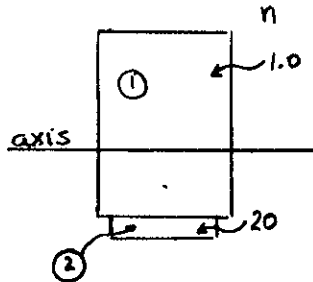
SOLUTION

Use wood as the reference material

$$n = 1.0 \text{ in wood}$$

$$n = E_s/E_w = 30/1.5 = 20 \text{ for steel}$$

Transformed section



	A	nA	$\bar{y}_0$	$nA\bar{y}_0$
①	60	60	5.5	330
②	2.5	50	0.25	12.5
$\Sigma$		110		342.5

$$\bar{Y}_0 = \frac{342.5}{110} = 3.114 \text{ in}$$

The neutral axis lies 3.114 in about the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (6)(10)^3 + (1.0)(60)(2.386)^2 = 841.6 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{20}{12} (5)\left(\frac{1}{2}\right)^3 + (20)(2.5)(2.864)^2 = 411.2 \text{ in}^4$$

$$I = I_1 + I_2 = 1252.8 \text{ in}^4$$

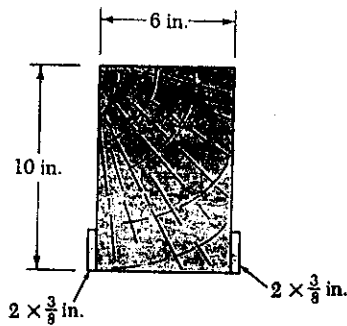
(a) Wood:  $n = 1.0$   $y = 10.5 - 3.114 = 7.386 \text{ in}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(200)(7.386)}{1252.8} = -1.179 \text{ ksi}$$

(b) Steel:  $n = 20$   $y = -3.114 \text{ in}$

$$\sigma = -\frac{nMy}{I} = -\frac{(20)(200)(-3.114)}{1252.8} = 9.94 \text{ ksi}$$

PROBLEM 4.48



4.47 and 4.48 A 6 x 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is  $1.5 \times 10^6$  psi for the wood and  $30 \times 10^6$  psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip-in., determine the maximum stress in (a) the wood, (b) the steel.

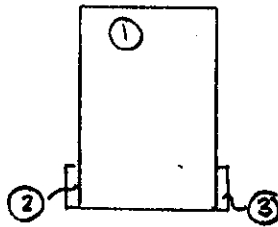
SOLUTION

Use wood as the reference material

$$n = 1.0 \text{ in wood}$$

$$n = E_s / E_w = 30 / 1.5 = 20 \text{ in steel}$$

Transformed section



	A	nA	$\bar{y}_0$	$nA\bar{y}_0$
①	60	60	5	300
②	0.75	15	1	15
③	0.75	15	1	15
$\Sigma$		90		330

$$\bar{y}_0 = \frac{330}{90} = 3.667 \text{ in}$$

The neutral axis lies 3.667 in. above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12} (6)(10)^3 + (60)(1.333)^2 = 606.7 \text{ in}^4$$

$$I_2 = I_3 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{20}{12} \left(\frac{3}{8}\right)(2)^3 + (15)(2.667)^2 = 111.7 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 830 \text{ in}^4$$

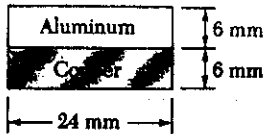
(a) Wood:  $n = 1.0$ ,  $y = 10 - 3.667 = 6.333 \text{ in}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(200)(6.333)}{830} = -1.526 \text{ ksi}$$

(b) Steel:  $n = 20$ ,  $y = -3.667 \text{ in}$

$$\sigma = -\frac{nMy}{I} = -\frac{(20)(200)(-3.667)}{830} = 17.67 \text{ ksi}$$

**PROBLEM 4.49**



**4.49 and 4.50** For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35N·m.

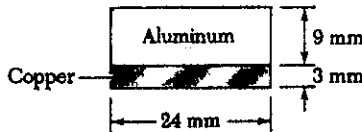
4.49 Bar of Prob. 4.45

**SOLUTION**

See solution to PROBLEM 4.45 for the calculation of I

$$\frac{1}{\rho} = \frac{M}{E_c I} = \frac{35}{(75 \times 10^9)(4.0608 \times 10^{-9})} = 0.1149 \text{ m}^{-1}, \quad \rho = 8.70 \text{ m} \quad \blacktriangleleft$$

**PROBLEM 4.50**



**4.49 and 4.50** For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35N·m.

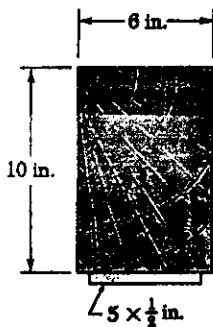
4.50 Bar of Prob. 4.46

**SOLUTION**

See solution to PROBLEM 4.46 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_c I} = \frac{35}{(75 \times 10^9)(4.008 \times 10^{-9})} = 0.1164 \text{ m}^{-1}, \quad \rho = 8.59 \text{ m} \quad \blacktriangleleft$$

**PROBLEM 4.51**



**4.51 and 4.52** For the composite beam indicated, determine the radius of curvature caused by the couple of moment 200 kip·in.

4.51 Beam of Prob. 4.47

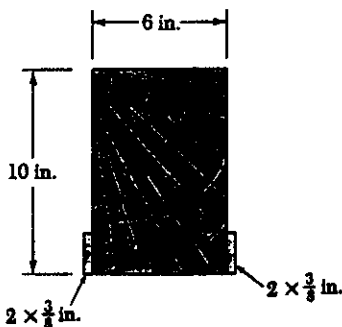
**SOLUTION**

See solution to PROBLEM 4.47 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_w I} = \frac{200 \times 10^3}{(1.5 \times 10^6)(1252.8)} = 106.4 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 9396 \text{ in} = 783 \text{ ft.} \quad \blacktriangleleft$$

**PROBLEM 4.52**



**4.51 and 4.52** For the composite beam indicated, determine the radius of curvature caused by the couple of moment 200 kip·in.

4.52 Beam of Prob. 4.48

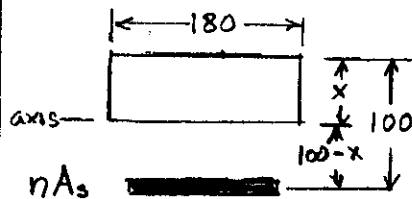
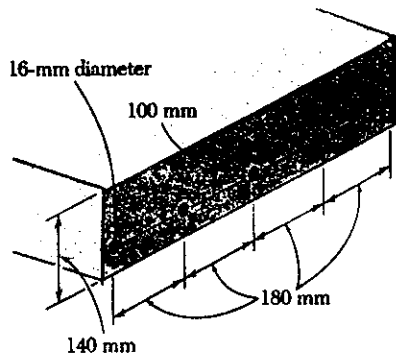
**SOLUTION**

See solution to PROBLEM 4.48 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_w I} = \frac{200 \times 10^3}{(1.5 \times 10^6)(830)} = 160.6 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 6225 \text{ in} = 519 \text{ ft.} \quad \blacktriangleleft$$

PROBLEM 4.53



4.53 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPa for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPa for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$

Consider a section 180 mm wide with one steel rod.

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3 \text{ mm}^2$$

Locate the neutral axis

$$180 \times \frac{x}{2} - (100 - x)(2.0106 \times 10^3) = 0$$

$$90x^2 + 2.0106 \times 10^3 x - 201.06 \times 10^3 = 0$$

Solving for x

$$x = \frac{-2.0106 \times 10^3 + \sqrt{(2.0106 \times 10^3)^2 + (4)(90)(201.06 \times 10^3)}}{(2)(90)}$$

$$x = 37.397 \text{ mm}, \quad 100 - x = 62.603 \text{ mm}$$

$$I = \frac{1}{3}(180)x^3 + (2.0106 \times 10^3)(100 - x)^2$$

$$= \frac{1}{3}(180)(37.397)^3 + (2.0106 \times 10^3)(62.603)^2$$

$$= 11.018 \times 10^6 \text{ mm}^4 = 11.018 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete:  $n = 1$ ,  $y = 37.397 \text{ mm} = 0.037397 \text{ m}$ ,  $\sigma = 9 \times 10^6 \text{ Pa}$

$$M = \frac{(9 \times 10^6)(11.018 \times 10^{-6})}{(1.0)(0.037397)} = 2.6516 \times 10^3 \text{ N}\cdot\text{m}$$

Steel:  $n = 10$ ,  $y = 62.603 \text{ mm} = 0.062603 \text{ m}$ ,  $\sigma = 120 \times 10^6 \text{ Pa}$

$$M = \frac{(120 \times 10^6)(11.018 \times 10^{-6})}{(10)(0.062603)} = 2.1120 \times 10^3 \text{ N}\cdot\text{m}$$

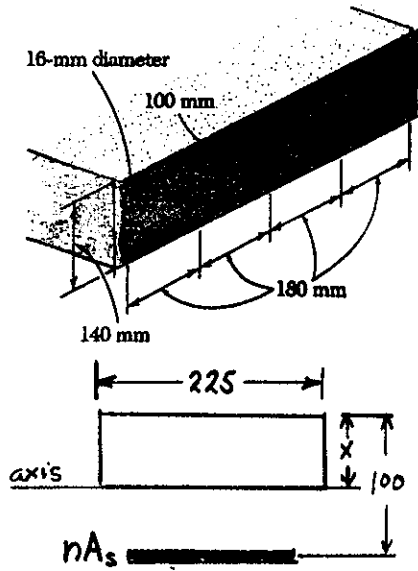
Choose the smaller value  $M = 2.1120 \times 10^3 \text{ N}\cdot\text{m}$

The above is the allowable positive moment for a 180 mm wide section.

For a 1 m = 1000 mm width, multiply by  $\frac{1000}{180} = 5.556$

$$M = (5.556)(2.1120 \times 10^3) = 11.73 \times 10^3 \text{ Nm} = 11.73 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

PROBLEM 4.54



4.53 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPa for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPa for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

4.54 Solve Prob. 4.53, assuming that the spacing of the 16-mm-diameter rods is increased to 225 mm on centers.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$

Consider a section 225 mm wide with one steel rod.

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3 \text{ mm}^2$$

Locate the neutral axis

$$225 \times \frac{x}{2} - (100 - x)(2.0106 \times 10^3) = 0$$

$$112.5x^2 + 2.0106x - 201.06 \times 10^3 = 0$$

Solving for x 
$$x = \frac{-2.0106 \times 10^3 + \sqrt{(2.0106 \times 10^3)^2 + (4)(112.5)(201.06 \times 10^3)}}{(2)(112.5)}$$

$$x = 34.273 \text{ mm}$$

$$100 - x = 65.727$$

$$\begin{aligned} I &= \frac{1}{3}(225)x^3 + 2.0106 \times 10^3 (100 - x)^2 \\ &= \frac{1}{3}(225)(34.273)^3 + (2.0106 \times 10^3)(65.727)^2 \\ &= 11.705 \times 10^6 \text{ mm}^4 = 11.705 \times 10^{-6} \text{ m}^4 \end{aligned}$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \therefore M = \frac{\sigma I}{ny}$$

Concrete:  $n = 1$ ,  $y = 34.273 \text{ mm} = 0.034273 \text{ m}$ ,  $\sigma = 9 \times 10^6 \text{ Pa}$

$$M = \frac{(9 \times 10^6)(11.705 \times 10^{-6})}{(1)(0.034273)} = 3.0738 \times 10^3 \text{ N}\cdot\text{m}$$

Steel:  $n = 10$ ,  $y = 65.727 \text{ mm} = 0.065727 \text{ m}$ ,  $\sigma = 120 \times 10^6 \text{ Pa}$

$$M = \frac{(120 \times 10^6)(11.705 \times 10^{-6})}{(10)(0.065727)} = 2.1370 \times 10^3 \text{ N}\cdot\text{m}$$

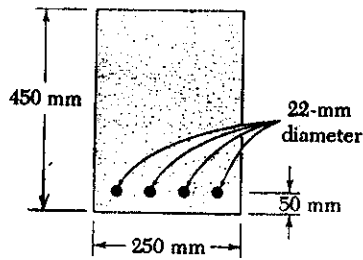
Choose the smaller value  $M = 2.1370 \times 10^3 \text{ N}\cdot\text{m}$

The above is the allowable positive moment for a 225 mm wide section.

For a 1 m = 1000 mm section, multiply by  $\frac{1000}{225} = 4.4444$

$$M = (4.4444)(2.1370 \times 10^3) = 9.50 \times 10^3 \text{ N}\cdot\text{m} = 9.50 \text{ kN}\cdot\text{m}$$

PROBLEM 4.55



4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN-m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4) \left( \frac{\pi}{4} \right) (22)^2 = 1.5205 \times 10^3 \text{ mm}^2$$

$$nA_s = 12.164 \times 10^3 \text{ mm}^2$$

Locate the neutral axis

$$250 \times \frac{x}{2} - (12.164 \times 10^3)(400 - x) = 0$$

$$125x^2 + 12.164 \times 10^3 x - 4.8657 \times 10^6 = 0$$

Solving for x

$$x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(4.8657 \times 10^6)}}{(2)(125)}$$

$$x = 154.55 \text{ mm}, \quad 400 - x = 245.45 \text{ mm}$$

$$\begin{aligned} I &= \frac{1}{3} 250 x^3 + (12.164 \times 10^3)(400 - x)^2 \\ &= \frac{1}{3} (250)(154.55)^3 + (12.164 \times 10^3)(245.45)^2 \\ &= 1.0404 \times 10^9 \text{ mm}^4 = 1.0404 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\sigma = -\frac{nMy}{I}$$

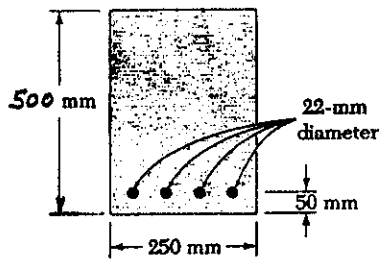
(a) Steel:  $y = -245.45 \text{ mm} = -0.24545 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.24545)}{1.0404 \times 10^{-3}} = 330 \times 10^6 \text{ Pa} = 330 \text{ MPa} \quad \blacktriangleleft$$

(b) Concrete:  $y = 154.55 \text{ mm} = 0.15455 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.15455)}{1.0404 \times 10^{-3}} = -26.0 \times 10^6 \text{ Pa} = -26.0 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 4.56



4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

4.56 Solve Prob. 4.55 assuming that the 450-mm depth of the beam is increased to 500 mm.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \left( \frac{\pi}{4} d^2 \right) = (4) \left( \frac{\pi}{4} \right) (22)^2 = 1.5205 \times 10^3 \text{ mm}^2$$

$$nA_s = 12.164 \times 10^3 \text{ mm}^2$$

locate the neutral axis

$$250 \times \frac{x}{2} - (12.164 \times 10^3)(450 - x) = 0$$

$$125x^2 + 12.164 \times 10^3 x - 5.4738 \times 10^6 = 0$$

Solving for x

$$x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(5.4738 \times 10^6)}}{(2)(125)}$$

$$x = 166.19 \text{ mm}, \quad 450 - x = 283.81 \text{ mm}$$

$$\begin{aligned} I &= \frac{1}{3}(250)x^3 + (12.164 \times 10^3)(450 - x)^2 \\ &= \frac{1}{3}(250)(166.19)^3 + (12.164 \times 10^3)(283.81)^2 \\ &= 1.3623 \times 10^9 \text{ mm}^4 = 1.3623 \times 10^{-3} \text{ m}^4 \end{aligned}$$

$$\sigma = -\frac{nMy}{I}$$

(a) Steel:  $y = -283.81 \text{ mm} = -0.28381 \text{ m}$

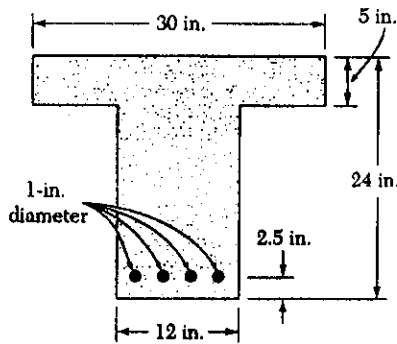
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.28381)}{1.3623 \times 10^{-3}} = 292 \times 10^6 \text{ Pa} = 292 \text{ MPa} \blacktriangleleft$$

(b) Concrete:  $y = 166.19 \text{ mm} = 0.16619 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.16619)}{1.3623 \times 10^{-3}} = -21.3 \times 10^6 \text{ Pa} = -21.3 \text{ MPa} \blacktriangleleft$$



PROBLEM 4.57



4.57 Knowing that the bending moment in the reinforced concrete beam shown is +150 kip-ft and that the modulus of elasticity is  $3.75 \times 10^6$  psi for the concrete and  $30 \times 10^6$  psi for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3.75 \times 10^6} = 8.0$$

$$A_s = 4 \left( \frac{\pi}{4} d^2 \right) = 4 \left( \frac{\pi}{4} \right) (1)^2 = 3.1416 \text{ in}^2$$

$$n A_s = 25.133 \text{ in}^2$$

Locate the neutral axis

$$(30)(5)(x + 2.5) + 12 \times \frac{x}{2} - (25.133)(16.5 - x) = 0$$

$$150x + 375 + 6x^2 - 414.69 + 25.133x = 0$$

$$6x^2 + 175.133x - 39.69 = 0$$

Solve for  $x$  
$$x = \frac{-175.133 + \sqrt{(175.133)^2 + (4)(6)(39.69)}}{(2)(6)} = 0.225 \text{ in.}$$

$$16.5 - x = 16.275 \text{ in.}$$

$$I_1 = \frac{1}{12} b h^3 + A_1 d_1^2 = \frac{1}{12} (30)(5)^3 + (30)(5)(2.225)^2 = 1426.3 \text{ in}^4$$

$$I_2 = \frac{1}{3} b_2 x^3 = \frac{1}{3} (12)(0.225)^3 = 0.1 \text{ in}^4$$

$$I_3 = n A_s d_3^2 = (25.133)(16.275)^2 = 6657.1 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 8083.5 \text{ in}^4$$

$$\sigma = -\frac{n M y}{I} \quad \text{where} \quad M = 150 \text{ kip}\cdot\text{ft} = 1800 \text{ kip}\cdot\text{in.}$$

(a) Steel  $n = 8.0$ ,  $y = -16.275 \text{ in}$

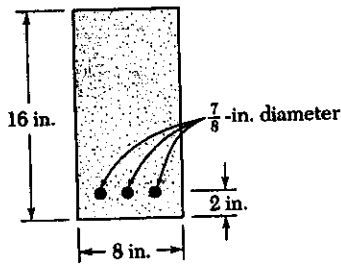
$$\sigma = -\frac{(8.0)(1800)(-16.275)}{8083.5} = 29.0 \text{ ksi}$$

(b) Concrete  $n = 1.0$ ,  $y = 5.225 \text{ in}$

$$\sigma = -\frac{(1.0)(1800)(5.225)}{8083.5} = -1.163 \text{ ksi}$$

**PROBLEM 4.58**

4.58 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is  $3 \times 10^6$  psi for the concrete and  $30 \times 10^6$  psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.



**SOLUTION**

$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3 \times 10^6} = 10$$

$$A_s = 3 \left( \frac{\pi}{4} d^2 \right) = 3 \left( \frac{\pi}{4} \right) \left( \frac{7}{8} \right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$

Locate neutral axis.

$$8 \cdot \frac{x}{2} - (18.040)(14-x) = 0$$

$$4x^2 + 18.040x - 252.56 = 0$$

Solve for  $x$  
$$x = \frac{-18.040 + \sqrt{18.040^2 + (4)(4)(252.56)}}{(2)(4)} = 6.005 \text{ in.}$$

$$14 - x = 7.995 \text{ in}$$

$$I = \frac{1}{3} 8 x^3 + nA_s (14-x)^2 = \frac{1}{3} (8)(6.005)^3 + (18.040)(7.995)^2 = 1730.4 \text{ in}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \therefore M = \frac{\sigma I}{ny}$$

Concrete:  $n = 1.0$ ,  $|y| = 6.005 \text{ in}$ ,  $|\sigma| = 1350 \text{ psi}$

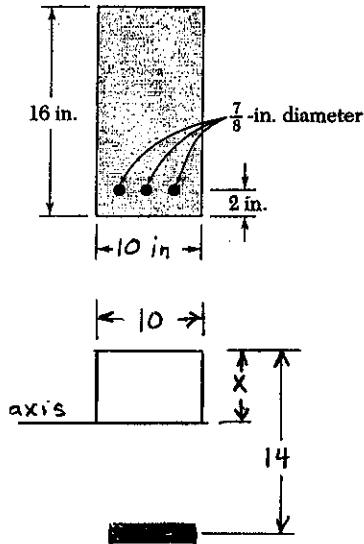
$$M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 389 \times 10^3 \text{ lb}\cdot\text{in} = 389 \text{ kip}\cdot\text{in}$$

Steel:  $n = 10$ ,  $|y| = 7.995$ ,  $\sigma = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(1730.5)}{(10)(7.995)} = 433 \times 10^3 \text{ lb}\cdot\text{in} = 433 \text{ kip}\cdot\text{in}$$

Choose the smaller value  $M = 389 \text{ kip}\cdot\text{in} = 32.4 \text{ kip}\cdot\text{ft}$

PROBLEM 4.59



4.58 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is  $3 \times 10^6$  psi for the concrete and  $30 \times 10^6$  psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

4.59 Solve Prob. 4.58, assuming that the width of the concrete beam is increased to 10 in.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3 \times 10^6} = 10$$

$$A_s = 3 \frac{\pi}{4} d^2 = 3 \left( \frac{\pi}{4} \right) \left( \frac{7}{8} \right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$

locate the neutral axis

$$10 \times \frac{x}{2} - (18.040)(14 - x) = 0$$

$$5x^2 + 18.040x - 252.56 = 0$$

Solve for  $x$

$$x = \frac{-18.040 + \sqrt{(18.040)^2 + (4)(5)(252.56)}}{(2)(5)} = 5.529 \text{ in}$$

$$14 - x = 8.471 \text{ in}$$

$$I = \frac{1}{3}(10)x^3 + nA_s(14 - x)^2 = \frac{1}{3}(10)(5.529)^3 + (18.040)(8.471)^2 = 1857.9 \text{ in}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \therefore M = \frac{\sigma I}{ny}$$

Concrete:  $n = 1.0$   $|y| = 5.529 \text{ in}$   $|\sigma| = 1350 \text{ psi}$

$$M = \frac{(1350)(1857.9)}{(1.0)(5.529)} = 453.6 \times 10^3 \text{ lb}\cdot\text{in}$$

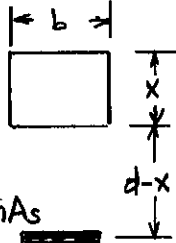
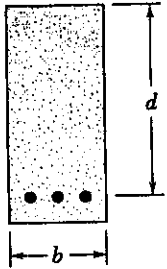
Steel:  $n = 10$   $|y| = 8.471 \text{ in}$   $|\sigma| = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(1857.9)}{(10)(8.471)} = 438.6 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value

$$M = 438.6 \times 10^3 \text{ lb}\cdot\text{in} \\ = 438.6 \text{ kip}\cdot\text{in} \\ = 36.6 \text{ kip}\cdot\text{ft}$$

**PROBLEM 4.60**



4.60 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses  $\sigma_s$  and  $\sigma_c$ . Show that to achieve a balanced design the distance  $x$  from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \frac{\sigma_s E_c}{\sigma_c E_s}}$$

where  $E_c$  and  $E_s$  are the moduli of elasticity of concrete and steel, respectively, and  $d$  is the distance from the top of the beam to the reinforcing steel.

**SOLUTION**

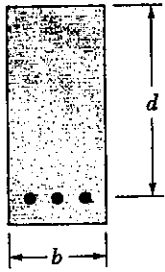
$$\sigma_s = \frac{n M (d-x)}{I} \qquad \sigma_c = \frac{M x}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \frac{E_c \sigma_s}{E_s \sigma_c}}$$

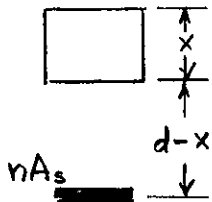
PROBLEM 4.61



SOLUTION

4.60 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses  $\sigma_s$  and  $\sigma_c$ .

4.61 For the concrete beam shown, the modulus of elasticity is  $3.5 \times 10^6$  psi for the concrete and  $29 \times 10^6$  psi for the steel. Knowing that  $b = 8$  in. and  $d = 22$  in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (a) the required area  $A_s$  of the steel reinforcement if the design of the beam is to be balanced, (b) the largest allowable bending moment. (See Prob. 4.60 for definition of a balanced beam.)



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.5 \times 10^6} = 8.2857$$

$$\sigma_s = \frac{n M (d-x)}{I} \quad \sigma_c = \frac{M x}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n \frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n} \frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857} \cdot \frac{20 \times 10^3}{1800} = 2.3410$$

$$x = 0.42717 d = (0.42717)(22) = 9.398 \text{ in}$$

$$d-x = 22 - 9.398 = 12.602 \text{ in}$$

Locate neutral axis

$$b x \frac{x}{2} - n A_s (d-x) = 0$$

$$(a) \quad A_s = \frac{b x^2}{2n(d-x)} = \frac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2$$

$$I = \frac{1}{3} b x^3 + n A_s (d-x)^2 = \frac{1}{3} (8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2 = 6665.6 \text{ in}^4$$

$$\sigma = \frac{n M y}{I} \quad M = \frac{\sigma I}{n y}$$

Concrete:  $n = 1.0 \quad y = 9.398 \text{ in} \quad \sigma = 1800 \text{ psi}$

$$M = \frac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277 \times 10^6 \text{ lb}\cdot\text{in}$$

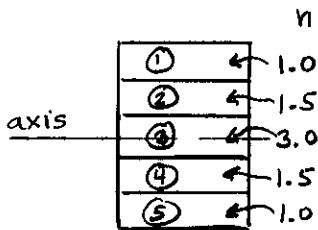
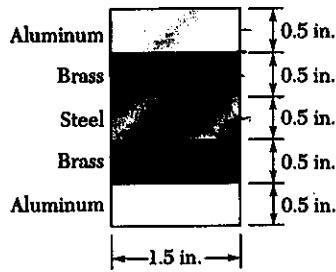
Steel:  $n = 8.2857 \quad |y| = 12.602 \text{ in} \quad \sigma = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(6665.6)}{(8.2857)(12.602)} = 1.277 \times 10^6 \text{ lb}\cdot\text{in}$$

Note that both values are the same for balanced design

$$M = 1.277 \times 10^3 \text{ kip}\cdot\text{in} = 106.4 \text{ kip}\cdot\text{ft}$$

**PROBLEM 4.62**



4.62 and 4.63 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30 × 10<sup>6</sup> psi for the steel, 15 × 10<sup>6</sup> psi for the brass, and 10 × 10<sup>6</sup> psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 12 kip-in., determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.

**SOLUTION**

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum.}$$

For the transformed section

$$I_1 = \frac{n_1 b_1 h_1^3}{12} + n_1 A_1 d_1^2 = \frac{1}{12} (1.5) (0.5)^3 + (0.75) (1.0)^2 = 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2 b_2 h_2^3}{12} + n_2 A_2 d_2^2 = \frac{1.5}{12} (1.5) (0.5)^3 + (1.5) (0.75) (0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3 b_3 h_3^3}{12} = \frac{3.0}{12} (1.5) (0.5)^3 = 0.0469 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in}^4, \quad I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_1^5 I_i = 2.1875 \text{ in}^4$$

(a) Aluminum:  $\sigma = \frac{nMy}{I} = \frac{(1.0)(12)(1.25)}{2.1875} = 6.86 \text{ ksi}$

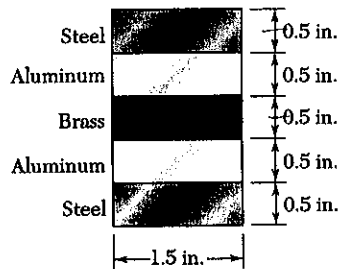
Brass:  $\sigma = \frac{nMy}{I} = \frac{(1.5)(12)(0.75)}{2.1875} = 6.17 \text{ ksi}$

Steel:  $\sigma = \frac{nMy}{I} = \frac{(3.0)(12)(0.25)}{2.1875} = 4.11 \text{ ksi}$

(b)  $\frac{1}{\rho} = \frac{M}{E_a I} = \frac{12 \times 10^3}{(10 \times 10^6)(2.1875)} = 548.57 \times 10^{-6} \text{ in}^{-1}$

$$\rho = 1823 \text{ in} = 151.9 \text{ ft.}$$

**PROBLEM 4.63**



4.62 and 4.63 Five metal strips, each of  $0.5 \times 1.5$ -in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is  $30 \times 10^6$  psi for the steel,  $15 \times 10^6$  psi for the brass, and  $10 \times 10^6$  psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 12 kip-in., determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.

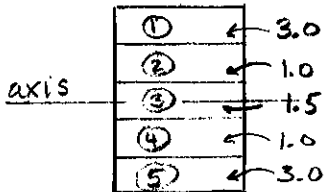
**SOLUTION**

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum}$$



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12} (1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2 = 2.2969 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12} (1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12} (1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4, \quad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_1^5 I_i = 5.0234 \text{ in}^4$$

(a) Steel:  $\sigma = \frac{nMy}{I} = \frac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$

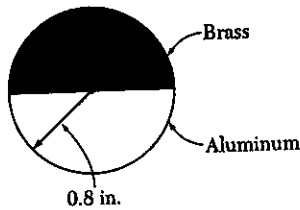
Aluminum:  $\sigma = \frac{nMy}{I} = \frac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$

Brass:  $\sigma = \frac{nMy}{I} = \frac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$

(b)  $\frac{1}{\rho} = \frac{M}{E_a I} = \frac{12 \times 10^3}{(10 \times 10^6)(5.0234)} = 238.89 \times 10^{-6} \text{ in}^{-1}$

$$\rho = 4186 \text{ in.} = 349 \text{ ft}$$

PROBLEM 4.64



4.64 The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is  $15 \times 10^6$  psi for the brass and  $10 \times 10^6$  psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 8 kip-in., determine the maximum stress (a) in the brass, (b) in the aluminum.

SOLUTION

For each semicircle  $r = 0.8$  in,

$$A = \frac{\pi}{2} r^2 = 1.00531 \text{ in}^2, \quad \bar{y}_o = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953 \text{ in}$$

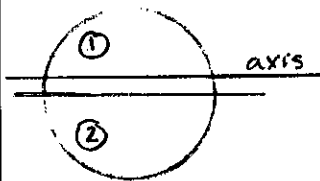
$$I_{\text{base}} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$$

$$\begin{aligned} \bar{I} &= I_{\text{base}} - A\bar{y}_o^2 = 0.160850 - (1.00531)(0.33953)^2 \\ &= 0.044957 \text{ in}^4 \end{aligned}$$

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$



Locate neutral axis

	$A, \text{in}^2$	$nA, \text{in}^2$	$\bar{y}_o, \text{in}$	$nA\bar{y}_o, \text{in}^3$
①	1.00531	1.50796	0.33953	0.51200
②	1.0053	1.00531	-0.33953	-0.34133
$\Sigma$		2.51327		0.17067

$$\bar{y}_o = \frac{0.17067}{2.51327} = 0.06791 \text{ in}$$

The neutral axis lies 0.06791 in above the material interface.

$$d_1 = 0.33953 - 0.06791 = 0.27162 \text{ in}, \quad d_2 = 0.33953 + 0.06791 = 0.40744 \text{ in}$$

$$I_1 = n_1 \bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$$

$$I_2 = n_2 \bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$$

$$I = I_1 + I_2 = 0.39054 \text{ in}^4$$

(a) Brass:  $n = 1.5, \quad y = 0.8 - 0.06791 = 0.73209 \text{ in}$

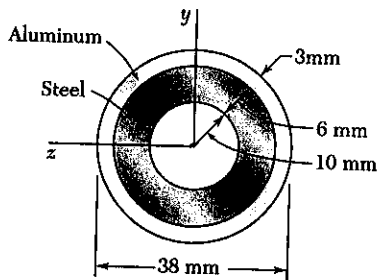
$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054} = -22.5 \text{ ksi} \quad \blacktriangleleft$$

(b) Aluminum:  $n = 1.0 \quad y = -0.8 - 0.06791 = -0.86791 \text{ in}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054} = 17.78 \text{ ksi} \quad \blacktriangleright$$



**PROBLEM 4.65**



4.65 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couples of moment 500 N·m, determine the maximum stress (a) in the aluminum, (b) in the steel.

**SOLUTION**

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel}$$

$$\text{Steel: } I_1 = n_1 \frac{\pi}{4} (r_o^4 - r_i^4) = (3.0) \frac{\pi}{4} (16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$$

$$\text{Aluminum: } I_2 = n_2 \frac{\pi}{4} (r_o^4 - r_i^4) = (1.0) \frac{\pi}{4} (19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$$

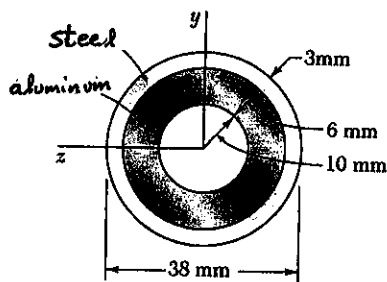
(a) Aluminum:  $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa} \quad \blacktriangleleft$$

(b) Steel:  $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 4.66**



4.65 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couples of moment 500 N·m, determine the maximum stress (a) in the aluminum, (b) in the steel.

4.66 Solve Prob. 4.65, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

**SOLUTION**

Use aluminum as the reference material

$$n = 1.0 \text{ in aluminum}$$

$$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0 \text{ in steel.}$$

$$\text{Steel: } I_1 = n_1 \frac{\pi}{4} (r_o^4 - r_i^4) = (3.0) \frac{\pi}{4} (19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$$

$$\text{Aluminum: } I_2 = n_2 \frac{\pi}{4} (r_o^4 - r_i^4) = (1.0) \frac{\pi}{4} (16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$$

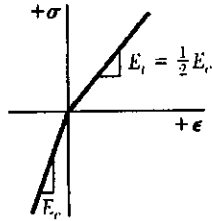
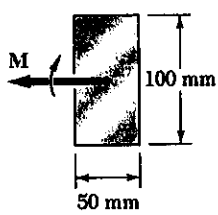
(a) Aluminum:  $c = 16 \text{ mm} = 0.016 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa} \quad \blacktriangleleft$$

(b) Steel:  $c = 19 \text{ mm} = 0.019 \text{ m}$

$$\sigma = \frac{nMc}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 4.67



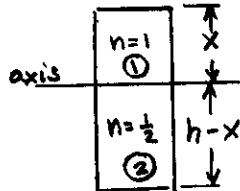
4.67 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one half of its value in compression. For a bending moment  $M = 600 \text{ N}\cdot\text{m}$ , determine the maximum (a) tensile stress, (b) compressive stress.

SOLUTION

$n = \frac{1}{2}$  on the tension side of neutral axis

$n = 1$  on the compression side

Locate neutral axis.



$$n_1 b x \frac{x}{2} - n_2 b (h-x) \frac{h-x}{2} = 0$$

$$\frac{1}{2} b x^2 - \frac{1}{4} b (h-x)^2 = 0$$

$$x^2 = \frac{1}{2} (h-x)^2 \quad x = \frac{1}{\sqrt{2}} (h-x)$$

$$x = \frac{1}{\sqrt{2}+1} h = 0.41421 h = 41.421 \text{ mm}$$

$$h-x = 58.579 \text{ mm}$$

$$I_1 = n_1 \frac{1}{3} b x^3 = (1) \left(\frac{1}{3}\right) (50) (41.421)^3 = 1.1844 \times 10^6 \text{ mm}^4$$

$$I_2 = n_2 \frac{1}{3} b (h-x)^3 = \left(\frac{1}{2}\right) \left(\frac{1}{3}\right) (50) (58.579)^3 = 1.6751 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595 \times 10^6 \text{ mm}^4 = 2.8595 \times 10^{-6} \text{ m}^4$$

(a) tensile stress:  $n = \frac{1}{2}$ ,  $y = -58.579 \text{ mm} = -0.058579 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595 \times 10^{-6}} = 6.15 \times 10^6 \text{ Pa} = 6.15 \text{ MPa}$$

(b) compressive stress:  $n = 1$ ,  $y = 41.421 \text{ mm} = 0.041421 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595 \times 10^{-6}} = -8.69 \times 10^6 \text{ Pa} = -8.69 \text{ MPa}$$

PROBLEM 4.68

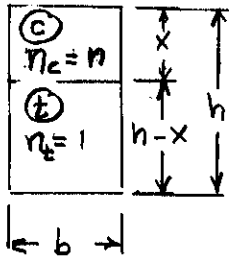
\*4.68 A rectangular beam is made of a material for which the modulus of elasticity is  $E_t$  in tension and  $E_c$  in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION



Use  $E_t$  as the reference modulus.

Then  $E_c = n E_t$

Locate neutral axis

$$n b x \frac{x}{2} - b (h-x) \frac{h-x}{2} = 0$$

$$n x^2 - (h-x)^2 = 0 \quad \sqrt{n} x = (h-x)$$

$$x = \frac{h}{\sqrt{n} + 1} \quad h-x = \frac{\sqrt{n} h}{\sqrt{n} + 1}$$

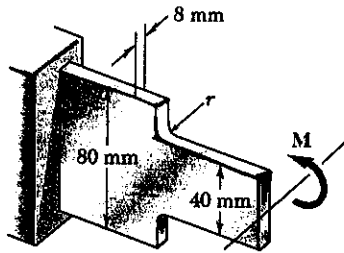
$$\begin{aligned} I_{trans} &= \frac{n}{3} b x^3 + \frac{1}{3} b (h-x)^3 = \left[ \frac{n}{3} \left( \frac{1}{\sqrt{n} + 1} \right)^3 + \left( \frac{\sqrt{n}}{\sqrt{n} + 1} \right)^3 \right] b h^3 \\ &= \frac{1}{3} \frac{n + n^{3/2}}{(\sqrt{n} + 1)^3} b h^3 = \frac{1}{3} \frac{n(1 + \sqrt{n})}{(\sqrt{n} + 1)^3} b h^3 = \frac{1}{3} \cdot \frac{n}{(\sqrt{n} + 1)^2} b h^3 \end{aligned}$$

$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \quad \text{where } I = \frac{1}{12} b h^3$$

$$E_r I = E_t I_{trans}$$

$$\begin{aligned} E_r &= \frac{E_t I_{trans}}{I} = \frac{12}{b h^3} \cdot E_t \cdot \frac{n}{3(\sqrt{n} + 1)^2} b h^3 \\ &= \frac{4 E_t E_c / E_t}{(\sqrt{E_c / E_t} + 1)^2} = \frac{4 E_t E_c}{(\sqrt{E_c} + \sqrt{E_t})^2} \end{aligned}$$

**PROBLEM 4.69**



4.69 Knowing that  $M = 250 \text{ N}\cdot\text{m}$ , determine the maximum stress in the beam shown when the radius  $r$  of the fillets is (a) 4 mm, (b) 8 mm.

**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

(a)  $\frac{r}{d} = \frac{4 \text{ mm}}{40 \text{ mm}} = 0.10$

From Fig. 4.31  $K = 1.87$

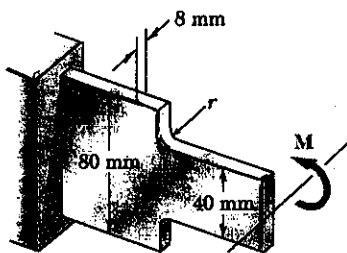
$$\sigma_{\max} = K \frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667 \times 10^{-9}} = 219 \times 10^6 \text{ Pa} = 219 \text{ MPa}$$

(b)  $\frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.20$

From Fig. 4.31  $K = 1.50$

$$\sigma_{\max} = K \frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667 \times 10^{-9}} = 176 \times 10^6 \text{ Pa} = 176 \text{ MPa}$$

**PROBLEM 4.70**



4.70 Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment  $M$  when the radius  $r$  of the fillets is (a) 8 mm, (b) 12 mm.

**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (8)(40)^3 = 42.667 \times 10^3 \text{ mm}^4 = 42.667 \times 10^{-9} \text{ m}^4$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

$$\frac{D}{d} = \frac{80 \text{ mm}}{40 \text{ mm}} = 2.00$$

(a)  $\frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.2$

From Fig. 4.31  $K = 1.50$

$$\sigma_{\max} = K \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{\max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$$

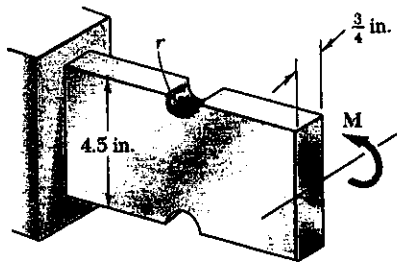
$$= 128 \text{ N}\cdot\text{m}$$

(b)  $\frac{r}{d} = \frac{12 \text{ mm}}{40 \text{ mm}} = 0.30$

From Fig. 4.31  $K = 1.35$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)} = 142 \text{ N}\cdot\text{m}$$

PROBLEM 4.71



4.71 Semicircular grooves of radius  $r$  must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment that can be applied to the member when the radius  $r$  of the semicircular grooves is

- (a)  $\frac{3}{8}$  in., (b)  $\frac{3}{4}$  in.

SOLUTION

(a)  $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$  in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32  $K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296$  in<sup>4</sup>,  $c = \frac{1}{2}d = 1.875$  in

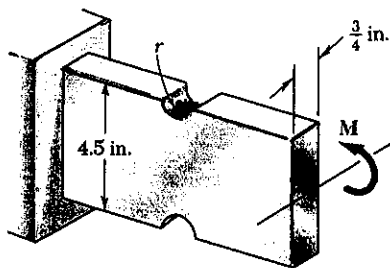
$\sigma = K \frac{Mc}{I} \therefore M = \frac{\sigma I}{Kc} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79$  kip-in

(b)  $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.0, \quad \frac{D}{d} = \frac{4.5}{3.0} = 1.5, \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

From Fig. 4.32  $K = 1.61, \quad I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.0)^3 = 1.6875$  in<sup>4</sup>

$c = \frac{1}{2}d = 1.5$  in.  $M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59$  kip-in.

PROBLEM 4.72



4.72 Semicircular grooves of radius  $r$  must be milled as shown in the sides of a steel member. Knowing that  $M = 4$  kip-in., determine the maximum stress in the member when (a)  $r = \frac{3}{8}$  in., (b)  $r = \frac{3}{4}$  in.

SOLUTION

(a)  $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$  in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32  $K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296$  in<sup>4</sup>,  $c = \frac{1}{2}d = 1.875$  in.

$\sigma = K \frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71$  ksi

(b)  $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.00$  in.,  $\frac{D}{d} = \frac{4.5}{3.00} = 1.50, \quad \frac{r}{d} = \frac{0.75}{3.00} = 0.25$

From Fig 4.32  $K = 1.61$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.00)^3 = 1.6875$  in<sup>4</sup>,  $c = \frac{1}{2}d = 1.5$  in

$\sigma = K \frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72$  ksi

**PROBLEM 4.73**

**SOLUTION**

For both configurations

$$D = 150 \text{ mm}, d = 100 \text{ mm},$$

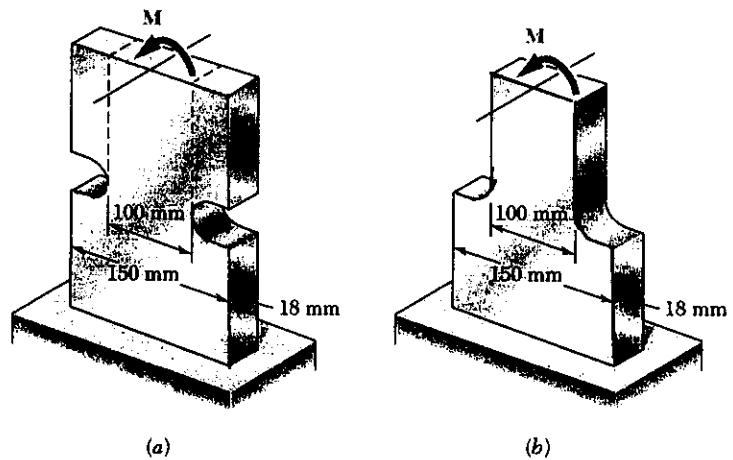
$$r = 15 \text{ mm}.$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives  $K_a = 1.92$ .

For configuration (b) Fig 4.31 gives  $K_b = 1.57$ .



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.050 \text{ m}$$

$$(a) \quad \sigma = \frac{K M c}{I} \quad \therefore \quad M = \frac{\sigma I}{K c} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3 \text{ N}\cdot\text{m} = 1.25 \text{ kN}\cdot\text{m}$$

$$(b) \quad M = \frac{\sigma I}{K c} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3 \text{ N}\cdot\text{m} = 1.53 \text{ kN}\cdot\text{m}$$

PROBLEM 4.74

4.74 A couple of moment  $M = 2 \text{ kN}\cdot\text{m}$  is to be applied to the end of a steel bar. Determine the maximum stress in the bar (a) if the bar is designed with grooves having semicircular portions of radius  $r = 10 \text{ mm}$ , as shown in Fig. a, (b) if the bar is redesigned by removing the material above the grooves as shown in Fig. b.

SOLUTION

For both configurations

$$D = 150 \text{ mm}, \quad d = 100 \text{ mm}$$

$$r = 10 \text{ mm}$$

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (a),

Fig 4.32 give  $K_a = 2.21$

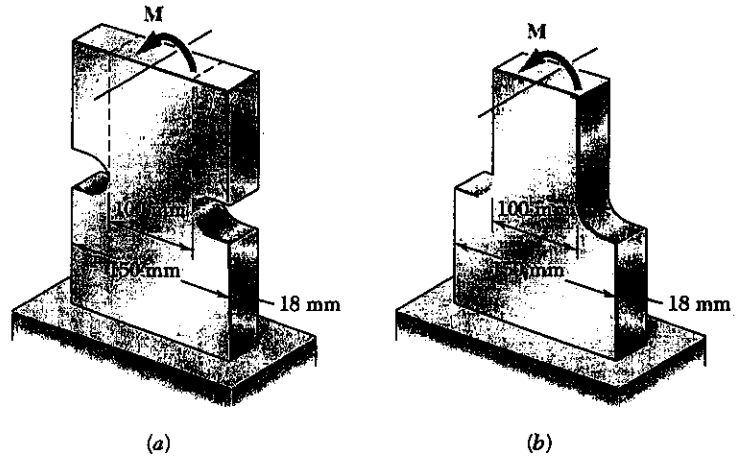
For configuration (b), Fig. 4.31 gives  $K_b = 1.79$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

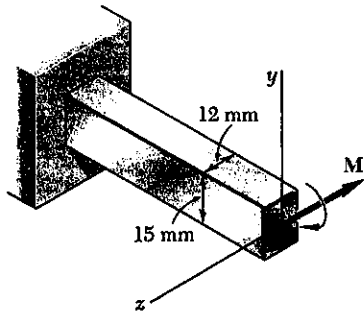
$$c = \frac{1}{2} d = 50 \text{ mm} = 0.05 \text{ m}$$

$$(a) \quad \sigma = \frac{K M c}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad \sigma = \frac{K M c}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa} \quad \blacktriangleleft$$



**PROBLEM 4.75**



4.75 A bar of rectangular cross section, made of a steel assumed to be elastoplastic with  $\sigma_Y = 320$  MPa, is subjected to a couple  $M$  parallel to the  $z$  axis. Determine the moment  $M$  of the couple for which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 5 mm thick.

**SOLUTION**

(a)  $I = \frac{1}{12} b h^3 = \frac{1}{12} (12)(15)^3 = 3.375 \times 10^3 \text{ mm}^4 = 3.375 \times 10^{-9} \text{ m}^4$

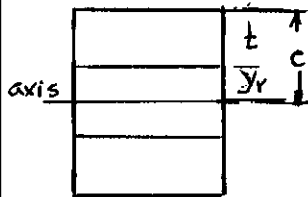
$c = \frac{1}{2} h = 7.5 \text{ mm} = 0.0075 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(320 \times 10^6)(3.375 \times 10^{-9})}{0.0075} = 144 \text{ N}\cdot\text{m}$

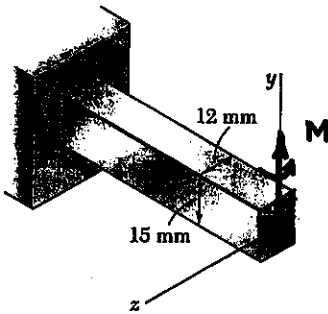
(b)  $t = 5 \text{ mm}$      $y_Y = c - t = 7.5 - 5 \text{ mm} = 2.5 \text{ mm} = 0.0025 \text{ m}$

$M = \frac{3}{2} M_Y \left[ 1 - \frac{1}{3} \left( \frac{y_Y}{c} \right)^2 \right]$

$= \frac{3}{2} (144) \left[ 1 - \frac{1}{3} \left( \frac{2.5}{7.5} \right)^2 \right] = 208 \text{ N}\cdot\text{m}$



**PROBLEM 4.76**



4.75 A bar of rectangular cross section, made of a steel assumed to be elastoplastic with  $\sigma_Y = 320$  MPa, is subjected to a couple  $M$  parallel to the  $z$  axis. Determine the moment  $M$  of the couple for which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 5 mm thick.

4.76 Solve Prob. 4.75, assuming that the couple  $M$  is parallel to the  $y$  axis.

**SOLUTION**

(a)  $I = \frac{1}{12} b h^3 = \frac{1}{12} (15)(12)^3 = 2.16 \times 10^3 \text{ mm}^4 = 2.16 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2} h = 6 \text{ mm} = 0.006 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(320 \times 10^6)(2.16 \times 10^{-9})}{0.006} = 115.2 \text{ N}\cdot\text{m}$

(b)  $t = 5 \text{ mm}$      $z_Y = c - t = 6 - 5 = 1 \text{ mm}$

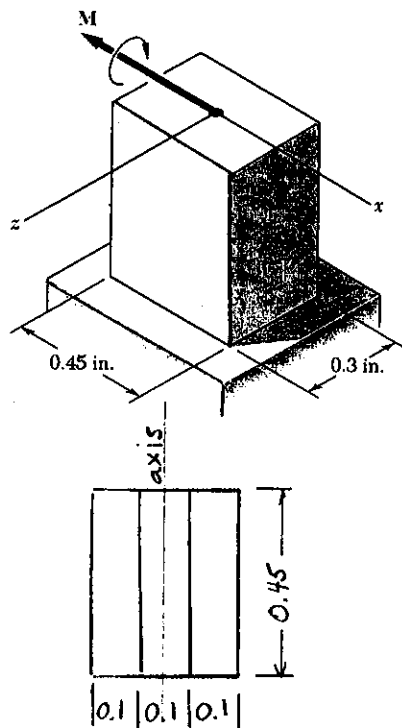
$M = \frac{3}{2} M_Y \left[ 1 - \frac{1}{3} \left( \frac{z_Y}{c} \right)^2 \right]$

$= \frac{3}{2} (115.2) \left[ 1 - \frac{1}{3} \left( \frac{1}{6} \right)^2 \right] = 171.2 \text{ N}\cdot\text{m}$





**PROBLEM 4.77**



4.77 The prismatic bar shown, made of a steel assumed to be elastoplastic with  $\sigma_y = 42$  ksi, is subjected to a couple  $M$  parallel to the  $x$  axis. Determine the moment  $M$  of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 0.1 in. thick.

**SOLUTION**

$$(a) I = \frac{1}{12} b h^3 = \frac{1}{12} (0.45)(0.3)^3 = 1.0125 \times 10^{-3} \text{ in}^4$$

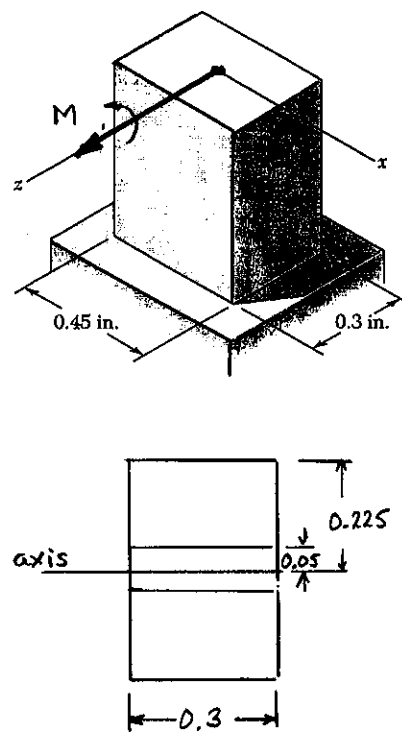
$$c = \frac{1}{2} h = 0.15 \text{ in}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(42)(1.0125 \times 10^{-3})}{0.15} = 0.2835 \text{ kip}\cdot\text{in} \\ = 283.5 \text{ lb}\cdot\text{in} \quad \blacktriangleleft$$

$$(b) z_y = \frac{1}{2} t_e = \frac{1}{2} (0.1) = 0.05 \text{ in}$$

$$M_p = \frac{3}{2} M_y \left[ 1 - \frac{1}{3} \left( \frac{z_y}{c} \right)^2 \right] \\ = \frac{3}{2} (283.5) \left[ 1 - \frac{1}{3} \left( \frac{0.05}{0.15} \right)^2 \right] = 409.5 \text{ lb}\cdot\text{in} \quad \blacktriangleleft$$

**PROBLEM 4.78**



4.77 The prismatic bar shown, made of a steel assumed to be elastoplastic with  $\sigma_y = 42$  ksi, is subjected to a couple  $M$  parallel to the  $x$  axis. Determine the moment  $M$  of the couple for which (a) yield first occurs, (b) the elastic core of the bar is 0.1 in. thick.

4.78 Solve Prob. 4.77, assuming that the couple  $M$  is parallel to the  $z$  axis.

**SOLUTION**

$$(a) I = \frac{1}{12} b h^3 = \frac{1}{12} (0.3)(0.45)^3 = 2.2781 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = 0.225 \text{ in}$$

$$M_y = \frac{\sigma_y I}{c} = \frac{(42)(2.2781 \times 10^{-3})}{0.225} = 0.425 \text{ kip}\cdot\text{in} \\ = 425 \text{ lb}\cdot\text{in} \quad \blacktriangleleft$$

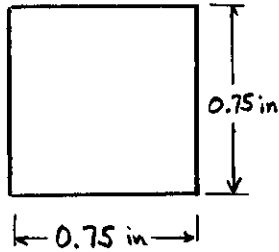
$$(b) x_y = \frac{1}{2} t_e = \frac{1}{2} (0.1) = 0.05 \text{ in}$$

$$M_p = \frac{3}{2} M_y \left[ 1 - \frac{1}{3} \left( \frac{x_y}{c} \right)^2 \right] \\ = \frac{3}{2} (425) \left[ 1 - \frac{1}{3} \left( \frac{0.05}{0.225} \right)^2 \right] = 627 \text{ lb}\cdot\text{in} \quad \blacktriangleleft$$

**PROBLEM 4.79**

4.79 A solid square rod of side 0.75 in. is made of a steel that is assumed to be elastoplastic with  $E = 29 \times 10^6$  psi and  $\sigma_Y = 40$  ksi. Determine the maximum stress and the radius of curvature caused by a 4 kip-in. couple applied and maintained about an axis parallel to a side of the cross section.

**SOLUTION**



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.75)(0.75)^3 = 0.026367 \text{ in}^4$$

$$c = \frac{1}{2} h = 0.375 \text{ in}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(40)(0.026367)}{0.375} = 2.8125 \text{ kip-in.}$$

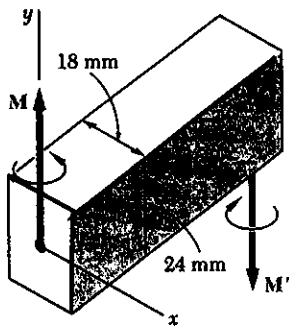
$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{y_r^2}{c^2}\right) \text{ or } \frac{y_r}{c} = \sqrt{3 - 2 \frac{M}{M_Y}}$$

$$\frac{y_r}{c} = \sqrt{3 - 2 \frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(4)}{2.8125}} = 0.39441$$

$$\frac{\sigma}{\rho_r} = \epsilon_r = \frac{\sigma_r}{E} \quad \therefore \rho_r = \frac{E c}{\sigma_r} = \frac{(29 \times 10^6)(0.375)}{40 \times 10^3} = 271.88 \text{ in}$$

$$\frac{\rho}{\rho_r} = \frac{y_r}{c} \quad \therefore \rho = \rho_r \frac{y_r}{c} = (271.88)(0.39441) = 107.2 \text{ in} \\ = 8.94 \text{ ft}$$

**PROBLEM 4.80**



4.80 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_Y = 280$  MPa. Knowing that couples  $M$  and  $M'$  of moment 525 N-m are applied and maintained about axes parallel to the  $y$  axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (24)(18)^3 = 11.664 \times 10^3 \text{ mm}^4 = 11.664 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} h = 9 \text{ mm} = 0.009 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280 \times 10^6)(11.664 \times 10^{-9})}{0.009} = 362.88 \text{ N-m}$$

$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3} \frac{y_r^2}{c^2}\right) \text{ or } \frac{y_r}{c} = \sqrt{3 - 2 \frac{M}{M_Y}}$$

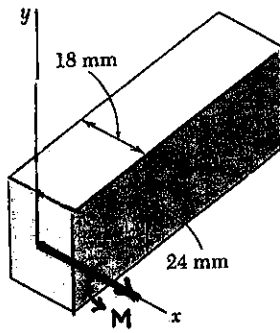
$$\frac{y_r}{c} = \sqrt{3 - \frac{(2)(525)}{362.88}} = 0.32632, \quad y_r = 0.32632 c = 2.9368 \text{ mm}$$

(a)

$$t_{\text{core}} = 2y_r = 5.87 \text{ mm}$$

(b)  $\epsilon_r = \frac{y_r}{\rho} = \frac{\sigma_r}{E} \quad \therefore \rho = \frac{E y_r}{\sigma_r} = \frac{(200 \times 10^9)(2.9368 \times 10^{-3})}{280 \times 10^6} = 2.09 \text{ m}$

**PROBLEM 4.81**



4.80 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with  $E = 200 \text{ GPa}$  and  $\sigma_r = 280 \text{ MPa}$ . Knowing that couples  $M$  and  $M'$  of moment  $525 \text{ N}\cdot\text{m}$  are applied and maintained about axes parallel to the  $y$  axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

4.81 Solve Prob. 4.80, assuming that the couples  $M$  and  $M'$  are applied and maintained about axes parallel to the  $x$  axis.

**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (18)(24)^3 = 20.736 \times 10^3 \text{ mm}^4 = 20.736 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} h = 12 \text{ mm} = 0.012 \text{ m}$$

$$M_y = \frac{\sigma_r I}{c} = \frac{(280 \times 10^6)(20.736 \times 10^{-9})}{0.012} = 483.84 \text{ N}\cdot\text{m}$$

$$M = \frac{3}{2} M_y \left(1 - \frac{1}{3} \frac{y_r^2}{c^2}\right) \quad \text{or} \quad \frac{y_r}{c} = \sqrt{3 - 2 \frac{M}{M_y}}$$

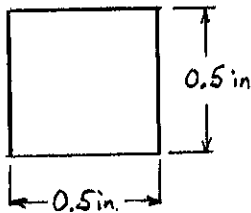
$$\frac{y_r}{c} = \sqrt{3 - \frac{(2)(525)}{483.84}} = 0.91097, \quad y_r = 0.91097 c = 10.932 \text{ mm}$$

(a)

$$t_{\text{core}} = 2y_r = 21.9 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad \epsilon_r = \frac{y_r}{\rho} = \frac{\sigma_r}{E} \quad \therefore \quad \rho = \frac{E y_r}{\sigma_r} = \frac{(200 \times 10^9)(10.932 \times 10^{-3})}{280 \times 10^6} = 7.81 \text{ m} \quad \blacktriangleleft$$

**PROBLEM 4.82**



**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.5)(0.5)^3 = 5.2083 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = 0.25 \text{ in.}$$

$$M_y = \frac{\sigma_r I}{c} = \frac{(42 \times 10^3)(5.2083 \times 10^{-3})}{0.25} = 875 \text{ lb}\cdot\text{in.}$$

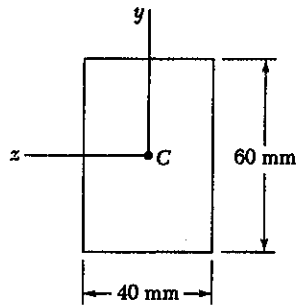
$$\epsilon_r = \frac{c}{\rho_r} = \frac{\sigma_r}{E} \quad \therefore \quad \rho_r = \frac{E c}{\sigma_r} = \frac{(29 \times 10^6)(0.25)}{42 \times 10^3} = 172.62 \text{ in.}$$

$$M = \frac{3}{2} M_y \left[1 - \frac{1}{3} \left(\frac{\rho}{\rho_r}\right)^2\right]$$

$$(a) \quad \rho = 5 \text{ ft.} = 60 \text{ in.} \quad M = \frac{3}{2} (875) \left[1 - \frac{1}{3} \left(\frac{60}{172.62}\right)^2\right] = 1260 \text{ lb}\cdot\text{in.} \quad \blacktriangleleft$$

$$(b) \quad \rho = 2 \text{ ft.} = 24 \text{ in.} \quad M = \frac{3}{2} (875) \left[1 - \frac{1}{3} \left(\frac{24}{172.62}\right)^2\right] = 1304 \text{ lb}\cdot\text{in.} \quad \blacktriangleleft$$

PROBLEM 4.83



4.83 and 4.84 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with  $E = 200$  GPa and  $\sigma_y = 240$  MPa. For bending about the  $z$  axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 20 mm thick.

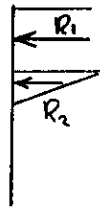
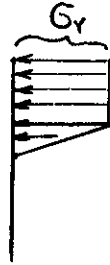
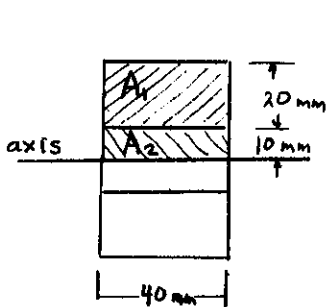
SOLUTION

$$(a) \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (40)(60)^3 = 720 \times 10^3 \text{ mm}^4 = 720 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$$

$$M_Y = \frac{\sigma_y I}{c} = \frac{(240 \times 10^6)(720 \times 10^{-9})}{0.030} = 5.76 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 5.76 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$



$$R_1 = \sigma_y A_1 = (240 \times 10^6)(0.040)(0.020)$$

$$= 192 \times 10^3 \text{ N}$$

$$y_1 = 10 \text{ mm} + 10 \text{ mm} = 0.020 \text{ m}$$

$$R_2 = \frac{1}{2} \sigma_y A_2 = \left(\frac{1}{2}\right)(240 \times 10^6)(0.040)(0.010)$$

$$= 48 \times 10^3 \text{ N}$$

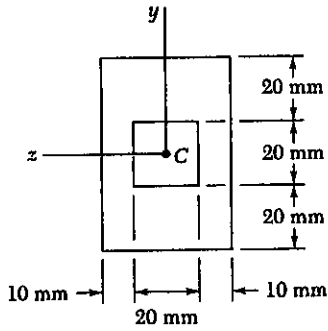
$$y_2 = \frac{2}{3}(10 \text{ mm}) = 6.667 \text{ mm} = 0.006667 \text{ m}$$

$$(b) \quad M = 2(R_1 y_1 + R_2 y_2) = 2[(192 \times 10^3)(0.020) + (48 \times 10^3)(0.006667)]$$

$$= 8.32 \times 10^3 \text{ N}\cdot\text{m} = 8.32 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 4.84**

4.83 and 4.84 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with  $E = 200 \text{ GPa}$  and  $\sigma_y = 240 \text{ MPa}$ . For bending about the  $z$  axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 20 mm thick.

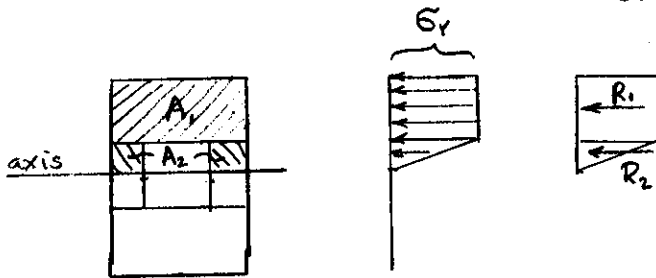


**SOLUTION**

$$\begin{aligned} \text{(a) } I_{\text{rect}} &= \frac{1}{12} b h^3 = \frac{1}{12} (40)(60)^3 = 720 \times 10^3 \text{ mm}^4 \\ I_{\text{cutout}} &= \frac{1}{12} b h^3 = \frac{1}{12} (20)(20)^3 = 13.33 \times 10^3 \text{ mm}^4 \\ I &= 720 \times 10^3 - 13.33 \times 10^3 = 706.67 \times 10^3 \text{ mm}^4 \\ &= 706.67 \times 10^{-9} \text{ m}^4 \end{aligned}$$

$$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$$

$$\begin{aligned} M_Y &= \frac{\sigma_y I}{c} = \frac{(240 \times 10^6)(706.67 \times 10^{-9})}{0.030} \\ &= 5.6533 \times 10^3 \text{ N}\cdot\text{m} = 5.65 \text{ kN}\cdot\text{m} \end{aligned}$$



$$R_1 = \sigma_y A_1 = (240 \times 10^6)(0.040)(0.020) = 192 \times 10^3 \text{ N}$$

$$y_1 = 10 \text{ mm} + 10 \text{ mm} = 20 \text{ mm} = 0.020 \text{ m}$$

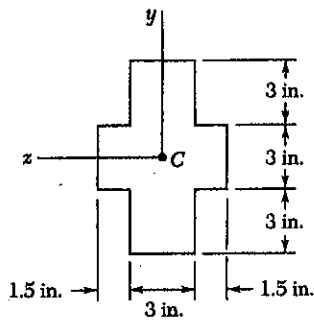
$$R_2 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2} (240 \times 10^6)(0.020)(0.010) = 24 \times 10^3 \text{ N}$$

$$y_2 = \frac{2}{3} (10 \text{ mm}) = 6.667 \text{ mm} = 0.006667 \text{ m}$$

$$\begin{aligned} \text{(b) } M &= 2 (R_1 y_1 + R_2 y_2) \\ &= 2 [(192 \times 10^3)(0.020) + (24 \times 10^3)(0.006667)] \\ &= 8.00 \times 10^3 \text{ N}\cdot\text{m} = 8.00 \text{ kN}\cdot\text{m} \end{aligned}$$

**PROBLEM 4.85**

4.85 and 4.86 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with  $E = 29 \times 10^6$  psi and  $\sigma_Y = 42$  ksi. For bending about the  $z$  axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.



**SOLUTION**

$$(a) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$$

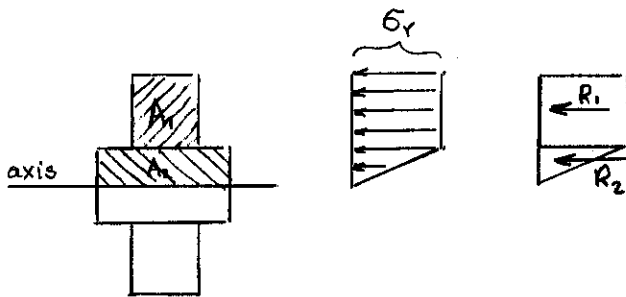
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (6)(3)^3 = 13.5 \text{ in}^4$$

$$I_3 = I_1 = 87.75 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$$

$$c = 4.5 \text{ in.}$$

$$M_y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759 \text{ kip}\cdot\text{in}$$



$$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378 \text{ kip}$$

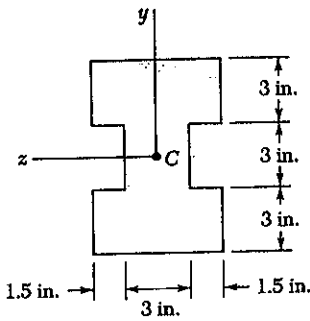
$$y_1 = 1.5 + 1.5 = 3.0 \text{ in}$$

$$R_2 = \frac{1}{2} \sigma_Y A_2 = \frac{1}{2} (42)(6)(1.5) = 189 \text{ kip}$$

$$y_2 = \frac{2}{3} (1.5) = 1.0 \text{ in.}$$

$$(b) M = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (189)(1.0)] = 2646 \text{ kip}\cdot\text{in}$$

**PROBLEM 4.86**



4.85 and 4.86 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with  $E = 29 \times 10^6$  psi and  $\sigma_r = 42$  ksi. For bending about the  $z$  axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.

**SOLUTION**

$$(a) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$$

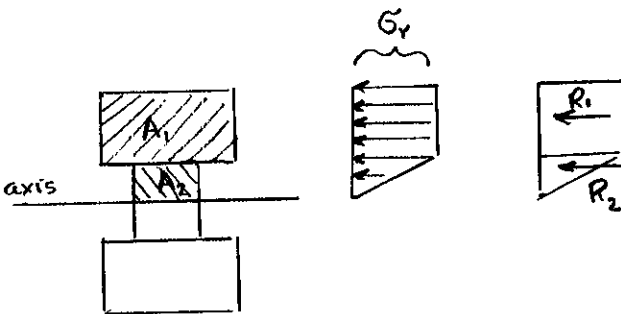
$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (3)(3)^3 = 6.75 \text{ in}^4$$

$$I_3 = I_1 = 175.5 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$$

$$c = 4.5 \text{ in}$$

$$M_y = \frac{\sigma_r I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip}\cdot\text{in}$$



$$R_1 = \sigma_r A_1 = (42)(6)(3) = 756 \text{ kip}$$

$$y_1 = 1.5 + 1.5 = 3 \text{ in}$$

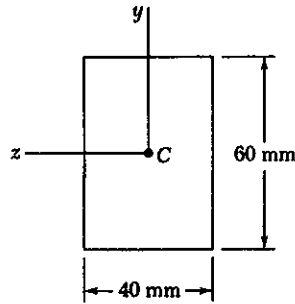
$$R_2 = \frac{1}{2} \sigma_r A_2 = \frac{1}{2} (42)(3)(1.5) = 94.5 \text{ kip}$$

$$y_2 = \frac{2}{3} (1.5) = 1.0 \text{ in}$$

$$(b) M = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3) + (94.5)(1.0)] = 4725 \text{ kip}\cdot\text{in}$$

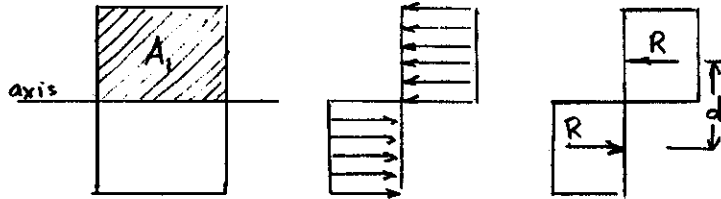
**PROBLEM 4.87**

4.87 through 4.90 For the bar indicated, determine (a) the fully plastic moment  $M_p$ ,  
 (b) the shape factor of the cross section.  
 4.87 Bar of Prob. 4.83



**SOLUTION**

From PROBLEM 4.83  $E = 200 \text{ GPa}$  and  $\sigma_Y = 240 \text{ MPa}$ .



$$A_1 = (40)(30) = 1200 \text{ mm}^2$$

$$= 1200 \times 10^{-6} \text{ m}^2$$

$$R = \sigma_Y A_1$$

$$= (240 \times 10^6)(1200 \times 10^{-6})$$

$$= 288 \times 10^3 \text{ N}$$

$$d = 30 \text{ mm} = 0.030 \text{ m}$$

$$(a) \quad M_p = R d = (288 \times 10^3)(0.030) = 8.64 \times 10^3 \text{ N}\cdot\text{m} = 8.64 \text{ kN}\cdot\text{m}$$

$$(b) \quad I = \frac{1}{12} b h^3 = \frac{1}{12} (40)(60)^3 = 720 \times 10^3 \text{ mm}^4 = 720 \times 10^{-9} \text{ m}^4$$

$$c = 30 \text{ mm} = 0.030 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(720 \times 10^{-9})}{0.030} = 5.76 \text{ kN}\cdot\text{m}$$

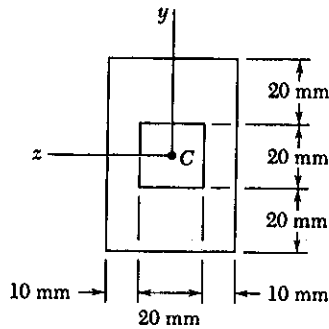
$$k = \frac{M_p}{M_Y} = \frac{8.64}{5.76} = 1.5$$



PROBLEM 4.88

4.87 through 4.90 For the bar indicated, determine (a) the fully plastic moment  $M_p$ ,  
(b) the shape factor of the cross section.

4.88 Bar of Prob. 4.84



SOLUTION

From PROBLEM 4.84  $E = 200 \text{ GPa}$  and  $\sigma_Y = 240 \text{ MPa}$ .

$$R_1 = \sigma_Y A_1$$

$$= (240 \times 10^6)(0.040)(0.020)$$

$$= 192 \times 10^3 \text{ N}$$

$$y_1 = 10 \text{ mm} + 10 \text{ mm} = 20 \text{ mm}$$

$$= 0.020 \text{ m}$$

$$R_2 = \sigma_Y A_2$$

$$= (240 \times 10^6)(0.020)(0.010)$$

$$= 48 \times 10^3 \text{ N}$$

$$y_2 = \frac{1}{2}(10) = 5 \text{ mm} = 0.005 \text{ m}$$

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(192 \times 10^3)(0.020) + (48 \times 10^3)(0.005)]$$

$$= 8.16 \times 10^3 \text{ N}\cdot\text{m} = 8.16 \text{ kN}\cdot\text{m}$$

(b)  $I_{\text{rect}} = \frac{1}{12} b h^3 = \frac{1}{12} (40)(60)^3 = 720 \times 10^3 \text{ mm}^4$

$$I_{\text{cutout}} = \frac{1}{12} b h^3 = \frac{1}{12} (20)(20)^3 = 13.33 \times 10^3 \text{ mm}^4$$

$$I = I_{\text{rect}} - I_{\text{cutout}} = 720 \times 10^3 - 13.33 \times 10^3 = 706.67 \times 10^3 \text{ mm}^4$$

$$= 706.67 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$$

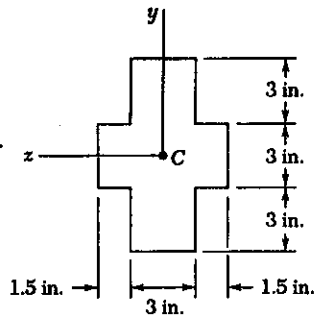
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(706.67 \times 10^{-9})}{0.030} = 5.6533 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{8.16}{5.6533} = 1.443$$

PROBLEM 4.89

4.87 through 4.90 For the bar indicated, determine (a) the fully plastic moment  $M_p$ ,  
(b) the shape factor of the cross section.

4.89 Bar of Prob. 4.85



SOLUTION

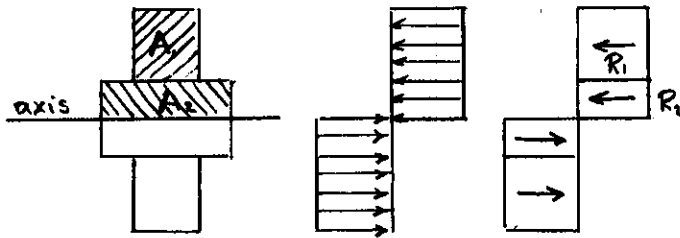
From PROBLEM 4.85  $E = 29 \times 10^6$  psi and  $\sigma_Y = 42$  ksi.

$$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378 \text{ kip}$$

$$y_1 = 1.5 + 1.5 = 3.0 \text{ in}$$

$$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378 \text{ kip}$$

$$y_2 = \frac{1}{2}(1.5) = 0.75 \text{ in}$$



$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)] = 2835 \text{ kip}\cdot\text{in}$$

$$(b) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (6)(3)^3 = 13.5 \text{ in}^4$$

$$I_3 = I_1 = 87.75 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$$

$$c = 4.5 \text{ in}$$

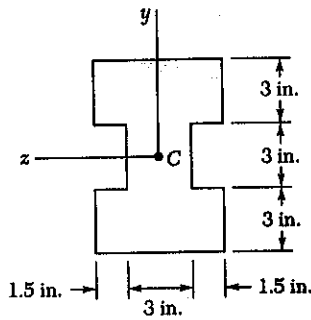
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3 \text{ kip}\cdot\text{in}$$

$$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3} = 1.611$$

PROBLEM 4.90

4.87 through 4.90 For the bar indicated, determine (a) the fully plastic moment  $M_p$ ,  
(b) the shape factor of the cross section.

4.90 Bar of Prob. 4.86



SOLUTION

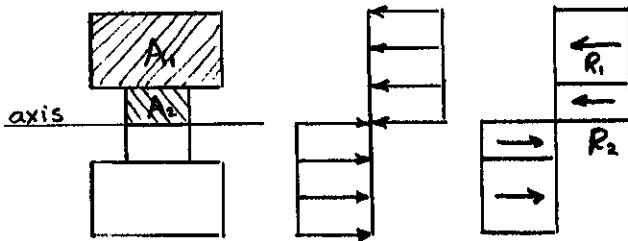
From PROBLEM 4.86  $E = 29 \times 10^6$  and  $\sigma_Y = 42$  ksi.

$$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756 \text{ kip}$$

$$y_1 = 1.5 + 1.5 = 3.0 \text{ in}$$

$$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189 \text{ k}\cdot\text{p}$$

$$y_2 = \frac{1}{2}(1.5) = 0.75 \text{ in.}$$



$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)] = 4819.5 \text{ kip}\cdot\text{in}$$

$$(b) I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12} (6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (3)(3)^3 = 6.75 \text{ in}^4$$

$$I_3 = I_1 = 175.5 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$$

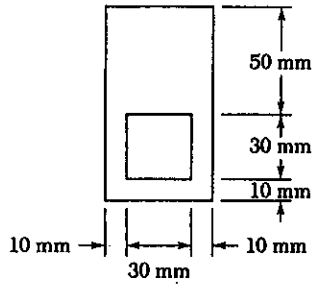
$$c = 4.5 \text{ in}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip}\cdot\text{in}$$

$$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339} = 1.443$$

PROBLEM 4.91

4.91 and 4.92 Determine the plastic moment  $M_p$  of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

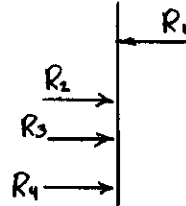
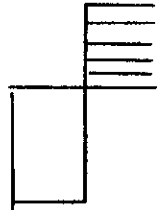
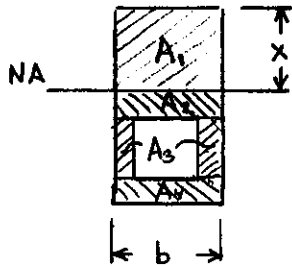


SOLUTION

Total area  $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$\frac{1}{2} A = 1800 \text{ mm}^2$

$x = \frac{\frac{1}{2} A}{b} = \frac{1800}{50} = 36 \text{ mm}$



$A_1 = (50)(36) = 1800 \text{ mm}^2, \bar{y}_1 = 18 \text{ mm}$

$A_1 \bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \bar{y}_2 = 7 \text{ mm}$

$A_2 \bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \bar{y}_3 = 29 \text{ mm}$

$A_3 \bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$

$A_4 = (50)(10) = 500 \text{ mm}^2, \bar{y}_4 = 49 \text{ mm}$

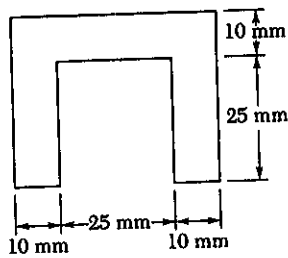
$A_4 \bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$

$A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$

$M_p = \sigma_y \sum A_i \bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N}\cdot\text{m}$   
 $= 19.01 \text{ kN}\cdot\text{m}$

**PROBLEM 4.92**

**4.91 and 4.92** Determine the plastic moment  $M_p$  of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

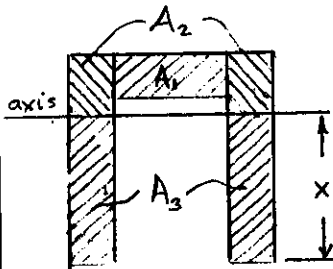


**SOLUTION**

$$\text{Total area } A = (25)(10) + (2)(10)(25) = 950 \text{ mm}^2$$

$$\frac{1}{2}A = 475 \text{ mm}^2$$

$$x = \frac{\frac{1}{2}A}{2b} = \frac{475}{20} = 23.75 \text{ mm} = 0.02375 \text{ m}$$



$$R_1 = \sigma_y A_1 = (240 \times 10^6)(0.025)(0.010) = 60 \times 10^3 \text{ N}$$

$$\bar{y}_1 = 30 - 23.75 = 6.25 \text{ mm} = 0.00625 \text{ m}$$

[www.konkur.in](http://www.konkur.in)

$$R_2 = \sigma_y A_2 = (240 \times 10^6)(0.020)(0.01125) = 54 \times 10^3 \text{ N}$$

$$\bar{y}_2 = \frac{1}{2}(0.01125) = 0.005625 \text{ m}$$

$$R_3 = \sigma_y A_3 = (240 \times 10^6)(0.020)(0.02375) = 114 \times 10^3 \text{ N}$$

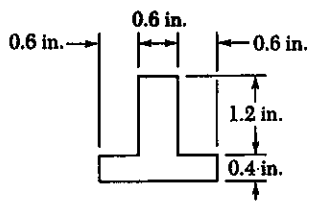
$$\bar{y}_3 = \frac{1}{2}x = 0.011875 \text{ m}$$

$$M_p = R_1 \bar{y}_1 + R_2 \bar{y}_2 + R_3 \bar{y}_3$$

$$= (60 \times 10^3)(0.00625) + (54 \times 10^3)(0.005625) + (114 \times 10^3)(0.011875)$$

$$= 2.0325 \times 10^3 \text{ N}\cdot\text{m} = 2.03 \text{ kN}\cdot\text{m}$$

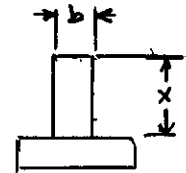
**PROBLEM 4.93**



**SOLUTION**

$$\begin{aligned} \text{Total area } A &= (1.8)(0.4) + (0.6)(1.2) = 1.44 \text{ in}^2 \\ \frac{1}{2}A &= 0.72 \text{ in}^2 \\ x &= \frac{\frac{1}{2}A}{b} = \frac{0.72}{0.6} = 1.2 \text{ in.} \end{aligned}$$

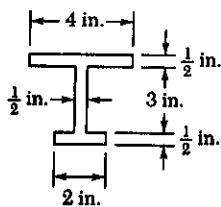
Neutral axis lies 1.2 in below the top



$$\begin{aligned} A_1 &= \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_1 = \frac{1}{2}(1.2) = 0.6 \text{ in} \\ A_2 &= \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_2 = \frac{1}{2}(0.4) = 0.2 \text{ in} \end{aligned}$$

$$\begin{aligned} M_p &= \sigma_r (A_1 \bar{y}_1 + A_2 \bar{y}_2) \\ &= (36) [(0.72)(0.6) + (0.72)(0.2)] \\ &= 20.7 \text{ kip}\cdot\text{in} \end{aligned}$$

**PROBLEM 4.94**



**SOLUTION**

$$\text{Total area: } A = (4)(\frac{1}{2}) + (\frac{1}{2})(3) + (2)(\frac{1}{2}) = 4.5 \text{ in}^2$$

$$\frac{1}{2}A = 2.25 \text{ in}^2$$

$$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75,$$

$$A_1 \bar{y}_1 = 1.50 \text{ in}^3$$

$$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25,$$

$$A_2 \bar{y}_2 = 0.0625 \text{ in}^3$$

$$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25,$$

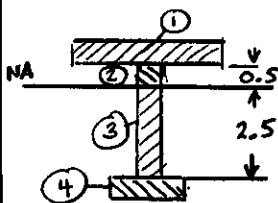
$$A_3 \bar{y}_3 = 1.5625 \text{ in}^3$$

$$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75,$$

$$A_4 \bar{y}_4 = 2.75 \text{ in}^3$$

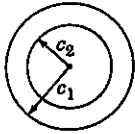
$$M_p = \sigma_r (A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4)$$

$$= (36) (1.50 + 0.0625 + 1.5625 + 2.75) = 211.5 \text{ kip}\cdot\text{in}$$

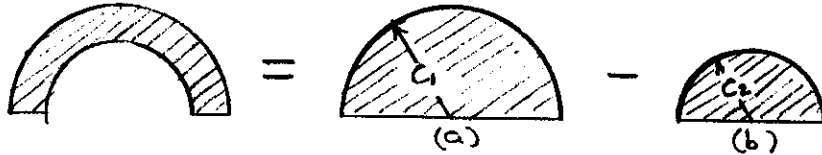
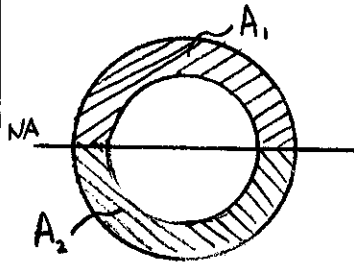


PROBLEM 4.95

4.95 A thick-walled pipe of the cross section shown is made of a steel that is assumed to be elastoplastic with a yield strength  $\sigma_Y$ . Derive an expression for the plastic moment  $M_p$  of the pipe in terms of  $c_1$ ,  $c_2$ , and  $\sigma_Y$ .



SOLUTION



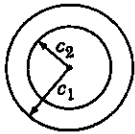
$$\begin{aligned} A_1 \bar{y}_1 &= A_2 \bar{y}_a - A_b \bar{y}_b \\ &= \left(\frac{\pi}{2} c_1^2\right) \left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2} c_2^2\right) \left(\frac{4c_2}{3\pi}\right) \\ &= \frac{2}{3} (c_1^3 - c_2^3) \end{aligned}$$

$$A_2 \bar{y}_2 = A_1 \bar{y}_1 = \frac{2}{3} (c_1^3 - c_2^3)$$

$$M_p = \sigma_Y (A_1 \bar{y}_1 + A_2 \bar{y}_2) = \frac{4}{3} \sigma_Y (c_1^3 - c_2^3) \blacktriangleleft$$

PROBLEM 4.96

4.96. Determine the plastic moment  $M_p$  of a thick-walled pipe of the cross section shown, knowing that  $c_1 = 60$  mm,  $c_2 = 40$  mm, and  $\sigma_Y = 240$  MPa.



SOLUTION

See the solution to PROBLEM 4.95 for derivation of the following expression for  $M_p$ .

$$M_p = \frac{4}{3} \sigma_Y (c_1^3 - c_2^3)$$

Data:  $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$

$$c_1 = 60 \text{ mm} = 0.060 \text{ m}$$

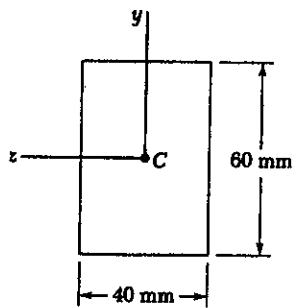
$$c_2 = 40 \text{ mm} = 0.040 \text{ m}$$

$$\begin{aligned} M_p &= \frac{4}{3} (240 \times 10^6) (0.060^3 - 0.040^3) = 48.64 \times 10^3 \text{ N}\cdot\text{m} \\ &= 48.6 \text{ kN}\cdot\text{m} \blacktriangleleft \end{aligned}$$

**PROBLEM 4.97**

4.97 and 4.98 For the beam indicated a couple of moment equal to the fully plastic moment  $M_p$  is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at  $y = 30$  mm.

4.97 Beam of Prob. 4.83

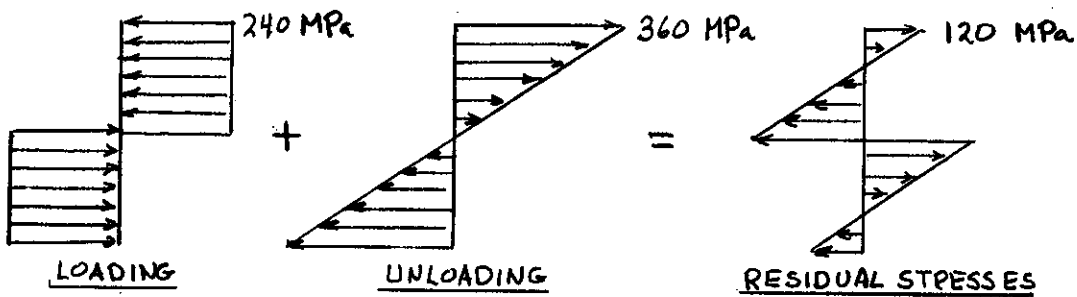


**SOLUTION**

$$M_p = 8.64 \text{ kN}\cdot\text{m} \quad (\text{See SOLUTION to PROBLEM 4.87})$$

$$I = 720 \times 10^{-9} \text{ m}^4, \quad c = 0.030 \text{ m}$$

$$\sigma' = \frac{M_{max} Y}{I} = \frac{M_p c}{I} \quad \text{at } y = c = 30 \text{ mm.}$$



$$\sigma' = \frac{(8.64 \times 10^3)(0.030)}{720 \times 10^{-9}} = 360 \times 10^6 \text{ Pa}$$

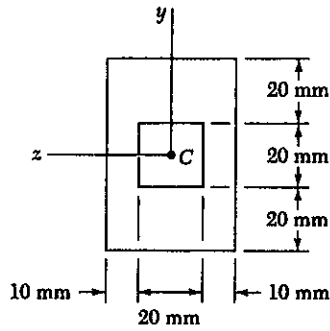
$$\sigma_{res} = \sigma' - \sigma_y = 360 \times 10^6 - 240 \times 10^6 = 120 \times 10^6 \text{ Pa} = 120 \text{ MPa}$$



**PROBLEM 4.98**

4.97 and 4.98 For the beam indicated a couple of moment equal to the fully plastic moment  $M_p$  is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at  $y = 30$  mm.

4.98 Beam of Prob. 4.84



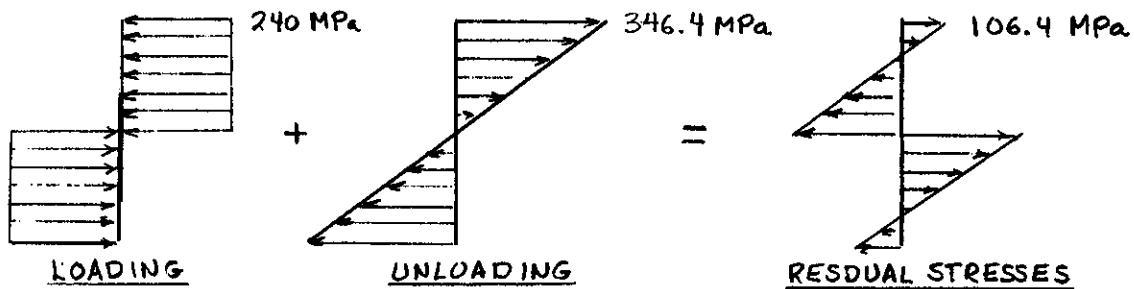
**SOLUTION**

$$M_p = 8.16 \text{ kN}\cdot\text{m} \quad (\text{See SOLUTION to PROBLEM 4.88})$$

$$I = 706.67 \times 10^{-7} \text{ m}^4, \quad c = 0.030 \text{ m}$$

$$\sigma' = \frac{M_{max} Y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(8.16 \times 10^3)(0.030)}{706.67 \times 10^{-7}} = 346.4 \times 10^6 \text{ Pa}$$

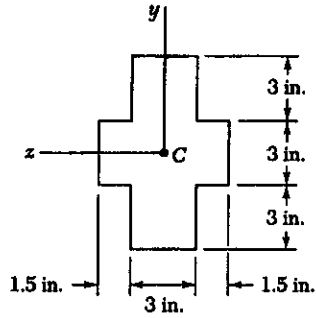


$$\sigma_{res} = \sigma' - \sigma_y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6 \text{ Pa} = 106.4 \text{ MPa}$$

**PROBLEM 4.99**

**4.99 and 4.100** For the beam indicated a couple of moment equal to the fully plastic moment  $M_p$  is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at  $y = 4.5$  in.

**4.99** Beam of Prob. 4.85



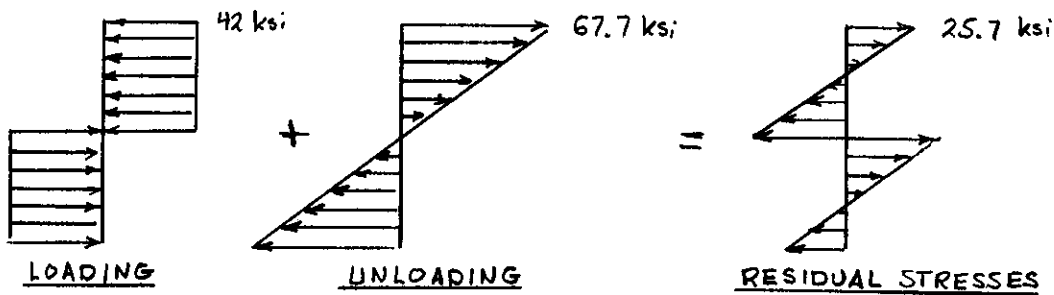
**SOLUTION**

$$M_p = 2835 \text{ kip}\cdot\text{in} \quad (\text{See SOLUTION to PROBLEM 4.89})$$

$$I = 188.5 \text{ in}^4, \quad c = 4.5 \text{ in}$$

$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(2835)(4.5)}{188.5} = 67.7 \text{ ksi}$$

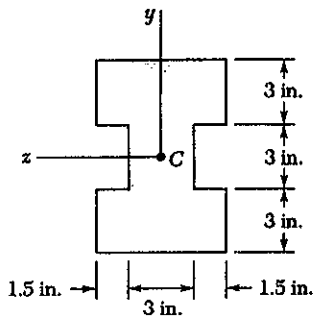


$$\sigma_{res} = \sigma' - \sigma_y = 67.7 - 42 = 25.7 \text{ ksi}$$

**PROBLEM 4.100**

4.99 and 4.100 For the beam indicated a couple of moment equal to the fully plastic moment  $M_p$  is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at  $y = 4.5$  in.

4.100 Beam of Prob. 4.86



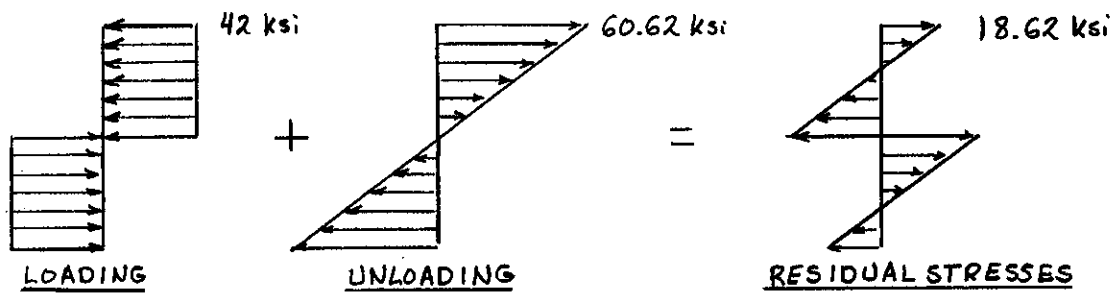
**SOLUTION**

$$M_p = 4819.5 \text{ kip}\cdot\text{in} \quad (\text{See SOLUTION to PROBLEM 4.90})$$

$$I = 357.75 \text{ in}^4, \quad c = 4.5 \text{ in.}$$

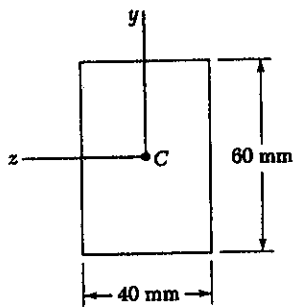
$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(4819.5)(4.5)}{357.75} = 60.62 \text{ ksi}$$



$$\sigma_{res} = \sigma' - \sigma_y = 60.62 - 42 = 18.62 \text{ ksi}$$

**PROBLEM 4.101**



**4.101 and 4.102** A bending couple is applied to the bar indicated, causing plastic zones 20-mm thick to develop at the top and bottom of the bar. After the couple has been removed, determine (a) the residual stress at  $y = 30$  mm, (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the bar.

4.101 Bar of Prob. 4.83

**SOLUTION**

See SOLUTION to PROBLEM 4.83 for bending couple and stress distribution during loading.

$M = 8.32 \text{ kN}$

$y_r = 10 \text{ mm} = 0.010 \text{ m}$

$E = 200 \text{ GPa}$

$\sigma_r = 240 \text{ MPa}$

$I = 720 \times 10^{-9} \text{ m}^4$

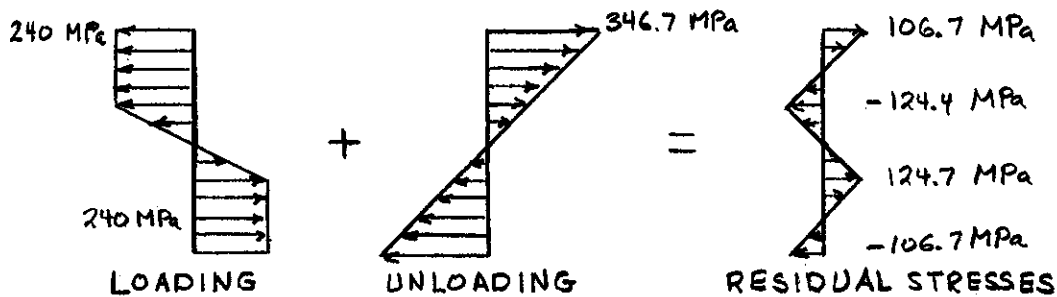
$c = 0.030 \text{ m}$

(a)  $\sigma' = \frac{Mc}{I} = \frac{(8.32 \times 10^3)(0.030)}{720 \times 10^{-9}} = 346.7 \times 10^6 \text{ Pa} = 346.7 \text{ MPa}$

$\sigma'' = \frac{My_r}{I} = \frac{(8.32 \times 10^3)(0.010)}{720 \times 10^{-9}} = 115.6 \times 10^6 \text{ Pa} = 115.6 \text{ MPa}$

At  $y = c$   $\sigma_{res} = \sigma' - \sigma_r = 346.7 - 240 = 106.7 \text{ MPa}$

At  $y = y_r$   $\sigma_{res} = \sigma'' - \sigma_r = 115.6 - 240 = -124.4 \text{ MPa}$



(b)  $\sigma_{res} = 0 \quad \therefore \frac{My_0}{I} - \sigma_r = 0$

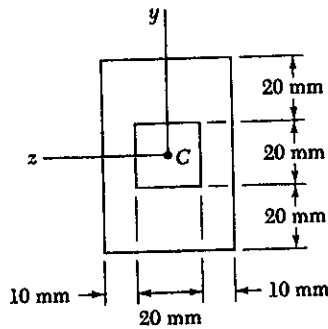
$y_0 = \frac{I \sigma_r}{M} = \frac{(720 \times 10^{-9})(240 \times 10^6)}{8.32 \times 10^3} = 20.77 \times 10^{-3} \text{ m} = 20.77 \text{ mm}$

ans.  $y_0 = -20.77 \text{ mm}, 0, 20.77 \text{ mm}$

(c) At  $y = y_r$ ,  $\sigma_{res} = -124.4 \times 10^6 \text{ Pa}$

$\sigma = -\frac{Ey}{\rho} \quad \therefore \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.010)}{-124.4 \times 10^6} = 16.08 \text{ m}$

**PROBLEM 4.102**



4.101 and 4.102 A bending couple is applied to the bar indicated, causing plastic zones 20-mm thick to develop at the top and bottom of the bar. After the couple has been removed, determine (a) the residual stress at  $y = 30$  mm, (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the bar.

4.102 Bar of Prob. 4.84

**SOLUTION**

See SOLUTION to PROBLEM 4.84 for bending couple and stress distribution during loading.

$$M = 8.00 \text{ kN}\cdot\text{m} \quad y_r = 10 \text{ mm} = 0.010 \text{ m}$$

$$E = 200 \text{ GPa} \quad \sigma_y = 240 \text{ MPa}$$

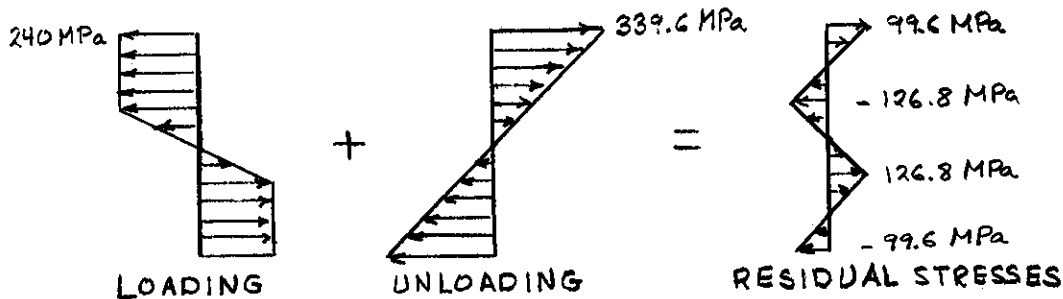
$$I = 706.67 \times 10^{-9} \text{ m}^4 \quad c = 0.030 \text{ m}$$

$$(a) \quad \sigma' = \frac{Mc}{I} = \frac{(8.00 \times 10^3)(0.030)}{706.67 \times 10^{-9}} = 339.6 \times 10^6 \text{ Pa} = 339.6 \text{ MPa}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(8.00 \times 10^3)(0.010)}{706.67 \times 10^{-9}} = 113.2 \times 10^6 \text{ Pa} = 113.2 \text{ MPa}$$

$$\text{At } y = c \quad \sigma_{res} = \sigma' - \sigma_y = 339.6 - 240 = 99.6 \text{ MPa}$$

$$\text{At } y = y_r \quad \sigma_{res} = \sigma'' - \sigma_y = 113.2 - 240 = -126.8 \text{ MPa}$$



$$(b) \quad \sigma_{res} = 0 \quad \therefore \quad \frac{My_0}{I} - \sigma_y = 0$$

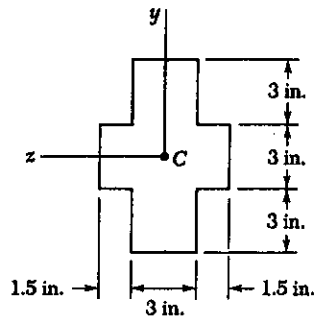
$$y_0 = \frac{I\sigma_y}{M} = \frac{(706.67 \times 10^{-9})(240 \times 10^6)}{8.00 \times 10^3} = 21.2 \times 10^{-3} \text{ m} = 21.2 \text{ mm}$$

$$\text{ans. } y_0 = -21.2 \text{ mm}, 0, 21.2 \text{ mm}$$

$$(c) \quad \text{At } y = y_r \quad \sigma_{res} = -126.8 \times 10^6 \text{ Pa}$$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200 \times 10^9)(0.010)}{126.8 \times 10^6} = 15.77 \text{ m}$$

**PROBLEM 4.103**



**4.103 and 4.104** A bending couple is applied to the bar indicated, causing plastic zones 3-in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (a) the residual stress at  $y = 4.5$  in., (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the bar.

**4.103** Bar of Prob. 4.85

**SOLUTION**

See SOLUTION to PROBLEM 4.85 for bending couple and stress distribution during loading

$$M = 2646 \text{ kip}\cdot\text{in}$$

$$y_r = 1.5 \text{ in.}$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$\sigma_r = 42 \text{ ksi}$$

$$I = 188.5 \text{ in}^4$$

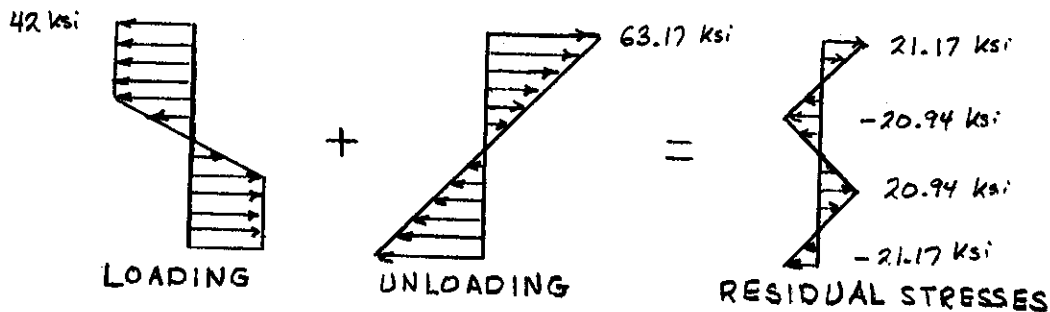
$$c = 4.5 \text{ in.}$$

$$(a) \quad \sigma' = \frac{Mc}{I} = \frac{(2646)(4.5)}{188.5} = 63.17 \text{ ksi}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(2646)(1.5)}{188.5} = 21.06 \text{ ksi}$$

$$\text{At } y = c \quad \sigma_{res} = \sigma' - \sigma_r = 63.17 - 42 = 21.17 \text{ ksi}$$

$$\text{At } y = y_r \quad \sigma_{res} = \sigma'' - \sigma_r = 21.06 - 42 = -20.94 \text{ ksi}$$



$$(b) \quad \sigma_{res} = 0 \quad \therefore \frac{My_0}{I} = \sigma_r$$

$$y_0 = \frac{I\sigma_r}{M} = \frac{(188.5)(42)}{2646} = 2.992 \text{ in}$$

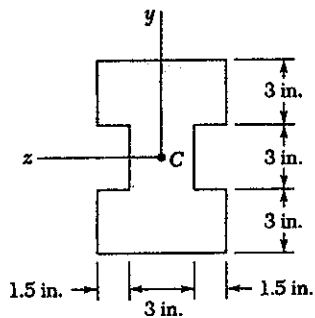
$$\text{ans. } y_0 = -2.992 \text{ in., } 0, 2.992 \text{ in}$$

$$(c) \quad \text{At } y = y_r, \quad \sigma_{res} = -20.94 \text{ ksi}$$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.5)}{20.94} = -2077 \text{ in}$$

$$= 173.1 \text{ ft}$$

PROBLEM 4.104



4.103 and 4.104 A bending couple is applied to the bar indicated, causing plastic zones 3-in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (a) the residual stress at  $y = 4.5$  in., (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the bar.

4.104 Bar of Prob. 4.86

SOLUTION

See SOLUTION to PROBLEM 4.86 for bending couple and stress distribution

$$M = 4725 \text{ kip}\cdot\text{in}$$

$$y_r = 1.5 \text{ in.}$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi} \quad \sigma_r = 42 \text{ ksi}$$

$$I = 357.75 \text{ in}^4$$

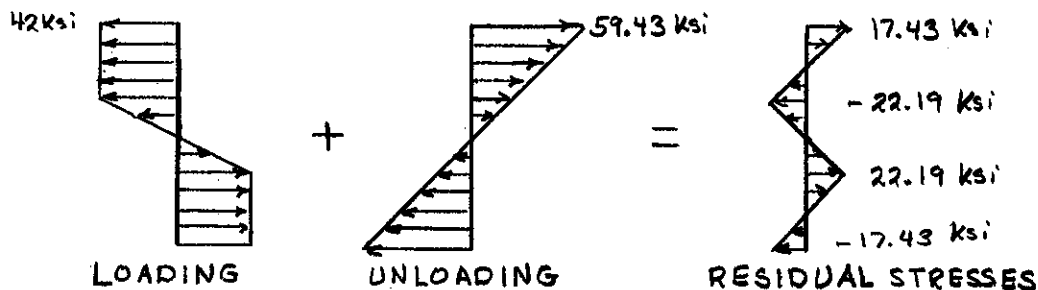
$$c = 4.5 \text{ in.}$$

$$(a) \quad \sigma' = \frac{Mc}{I} = \frac{(4725)(4.5)}{357.75} = 59.43 \text{ ksi}$$

$$\sigma'' = \frac{My_r}{I} = \frac{(4725)(1.5)}{357.75} = 19.81 \text{ ksi}$$

$$\text{At } y = c \quad \sigma_{res} = \sigma' - \sigma_r = 59.43 - 42 = 17.43 \text{ ksi}$$

$$\text{At } y = y_r \quad \sigma_{res} = \sigma'' - \sigma_r = 19.81 - 42 = -22.19 \text{ ksi}$$



$$(b) \quad \sigma_{res} = 0 \quad \therefore \frac{My_o}{I} - \sigma_r = 0$$

$$y_o = \frac{I \sigma_r}{M} = \frac{(357.75)(42)}{4725} = 3.18 \text{ in}$$

$$\text{ans. } y_o = -3.18 \text{ in, } 0, 3.18 \text{ in.}$$

$$(c) \quad \text{At } y = y_r, \quad \sigma_{res} = -22.19 \text{ ksi}$$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(29 \times 10^3)(1.5)}{22.19} = 1960 \text{ in} \\ = 163.4 \text{ ft.}$$

PROBLEM 4.105

\*4.105 A rectangular bar that is straight and unstressed is bent into an arc of circle of radius  $\rho$  by two couples of moment  $M$ . After the couples are removed, it is observed that the radius of curvature of the bar is  $\rho_R$ . Denoting by  $\rho_Y$  the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation

$$\frac{1}{\rho_R} = \frac{1}{\rho} \left\{ 1 - \frac{3}{2} \frac{\rho}{\rho_Y} \left[ 1 - \frac{1}{3} \left( \frac{\rho}{\rho_Y} \right)^2 \right] \right\}$$

SOLUTION

$$\frac{1}{\rho} = \frac{M_Y}{EI}, \quad M = \frac{3}{2} M_Y \left( 1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \quad \text{Let } m \text{ denote } \frac{M}{M_Y}$$

$$m = \frac{M}{M_Y} = \frac{3}{2} \left( 1 - \frac{\rho^2}{\rho_Y^2} \right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2m$$

$$\begin{aligned} \frac{1}{\rho_R} &= \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} \\ &= \frac{1}{\rho} \left\{ 1 - \frac{\rho}{\rho_Y} m \right\} = \frac{1}{\rho} \left\{ 1 - \frac{3}{2} \frac{\rho}{\rho_Y} \left( 1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \right\} \end{aligned}$$

PROBLEM 4.106

4.106 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by  $M_Y$  and  $\rho_Y$ , respectively, the bending moment and radius of curvature at the onset of yield, determine (a) the radius of curvature when a couple of moment  $M = 1.25M_Y$  is applied to the bar, (b) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.105.

SOLUTION

$$(a) \quad \frac{1}{\rho_Y} = \frac{M_Y}{EI}, \quad M = \frac{3}{2} M_Y \left( 1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \quad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2} \left( 1 - \frac{1}{3} \frac{\rho^2}{\rho_Y^2} \right) \quad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

$$\rho = 0.70711 \rho_Y$$

$$(b) \quad \frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711 \rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y} \quad \therefore \quad \rho_R = 6.09 \rho_Y$$



PROBLEM 4.107

4.106 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by  $M_y$  and  $\rho_r$ , respectively, the bending moment and radius of curvature at the onset of yield, determine (a) the radius of curvature when a couple of moment  $M = 1.25M_y$  is applied to the bar, (b) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.105.

4.107 Solve Prob. 4.106, assuming that the moment of the couple applied to the bar is  $1.40M_y$ .

SOLUTION

$$(a) \frac{1}{\rho_r} = \frac{M_y}{EI}, \quad M = \frac{3}{2} M_y \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_r^2}\right) \quad \text{Let } m = \frac{M}{M_y} = 1.40$$

$$m = \frac{M}{M_y} = \frac{3}{2} \left(1 - \frac{1}{3} \frac{\rho^2}{\rho_r^2}\right) \quad \frac{\rho}{\rho_r} = \sqrt{3 - 2m} = 0.44721$$

$$\rho = 0.44721 \rho_r$$

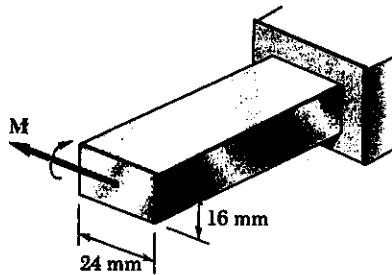
$$(b) \frac{1}{\rho_r} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_r} = \frac{1}{0.44721 \rho_r} - \frac{1.40}{\rho_r}$$

$$\frac{0.83607}{\rho_r}$$

$$\therefore \rho_r = 1.196 \rho_r$$

**PROBLEM 4.108**

4.108 The prismatic bar shown is made of a steel that is assumed to be elastoplastic and for which  $E = 200 \text{ GPa}$ . Knowing that the radius of curvature of the bar is  $2.4 \text{ m}$  when a couple of moment  $M = 420 \text{ N}\cdot\text{m}$  is applied as shown, determine (a) the yield strength  $\sigma_Y$  of the steel, (b) the thickness of the elastic core of the bar.



**SOLUTION**

$$\begin{aligned} M &= \frac{3}{2} M_Y \left( 1 - \frac{1}{3} \frac{\rho^2}{c^2} \right) \\ &= \frac{3}{2} \frac{\sigma_Y I}{c} \left( 1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right) \\ &= \frac{3}{2} \frac{\sigma_Y b (2c)^3}{12 c} \left( 1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right) \\ &= \sigma_Y b c^2 \left( 1 - \frac{1}{3} \frac{\rho^2 \sigma_Y^2}{E^2 c^2} \right) \end{aligned}$$

(a)  $bc^2 \sigma_Y \left( 1 - \frac{\rho^2 \sigma_Y^2}{3E^2 c^2} \right) = M$       Cubic equation for  $\sigma_Y$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $M = 420 \text{ N}\cdot\text{m}$        $\rho = 2.4 \text{ m}$

$b = 24 \text{ mm} = 0.024 \text{ m}$        $c = \frac{1}{2} h = 8 \text{ mm} = 0.008 \text{ m}$

$(1.536 \times 10^{-6}) \sigma_Y [1 - 750 \times 10^{-21} \sigma_Y^2] = 420$

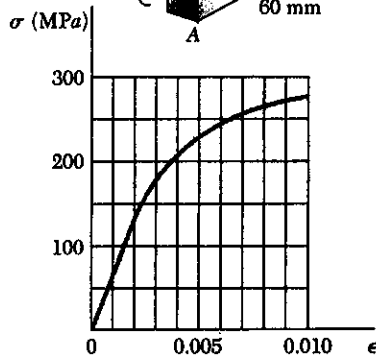
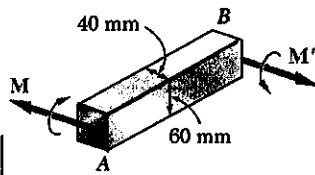
$\sigma_Y [1 - 750 \times 10^{-21} \sigma_Y^2] = 273.44 \times 10^6$

Solving by trial       $\sigma_Y = 292 \times 10^6 \text{ Pa} = 292 \text{ MPa}$        $\blacktriangleleft$

(b)  $y_r = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3} \text{ m} = 3.504 \text{ mm}$

thickness of elastic core =  $2y_r = 7.01 \text{ mm}$        $\blacktriangleleft$

**PROBLEM 4.109**



4.109 The prismatic bar  $AB$  is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the  $\sigma$ - $\epsilon$  diagram is the same in compression as in tension, determine (a) the radius of curvature of the bar when the maximum stress is 250 MPa, (b) the corresponding value of the bending moment. (Hint: For part b, plot  $\sigma$  versus  $y$  and use an approximate method of integration.)

**SOLUTION**

(a)  $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\epsilon_m = 0.0064$  from curve

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$\frac{1}{\rho} = \frac{\epsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$

$\rho = 4.69 \text{ m}$

(b) Strain distribution  $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$  where  $U = \frac{y}{c}$

Bending couple

$M = -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 U |\sigma| dU = 2bc^2 J$

where the integral  $J$  is given by  $\int_0^1 U |\sigma| dU$

Evaluate  $J$  using a method of numerical integration. If Simpson's rule is used, the integration formula is

$J = \frac{\Delta U}{3} \sum w U |\sigma|$

where  $w$  is a weighting factor, Using  $\Delta U = 0.25$  we get the values given in the table below:

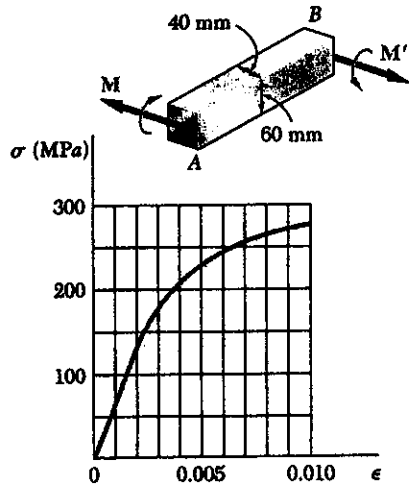
$U$	$ E $	$ \sigma , (\text{MPa})$	$U \sigma , (\text{MPa})$	$w$	$wU \sigma , (\text{MPa})$
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\sum wU \sigma $

$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$

$M = (2)(0.040)(0.030)^2 (101.25 \times 10^6) = 7.29 \times 10^3 \text{ N}\cdot\text{m} = 7.29 \text{ kN}\cdot\text{m}$

**PROBLEM 4.110**

4.110 For the bar of Prob. 4.109, determine (a) the maximum stress when the radius of curvature of the bar is 3 m, (b) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)



**SOLUTION**

(a)  $\rho = 3 \text{ m}$  ,  $c = 0.030 \text{ m} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$\epsilon_m = \frac{c}{\rho} = \frac{0.030}{3} = 0.010$

From curve  $\sigma_m = 275 \text{ MPa}$

(b) Strain distribution  $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$  where  $U = \frac{y}{c}$

Bending couple

$M = -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 U |\sigma| dU = 2bc^2 J$

where the integral J is given by  $\int_0^1 U |\sigma| dU$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$J = \frac{\Delta U}{3} \sum w U |\sigma|$

where w is a weighting factor, Using  $\Delta U = 0.25$  we get the values given in the table below:

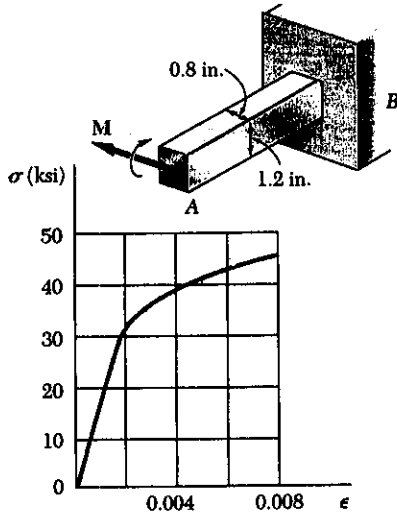
U	U	\sigma  (MPa)	U \sigma  (MPa)	w	wU \sigma  (MPa)
0	0	0	0	1	0
0.25	0.0025	160	40	4	160
0.5	0.0050	254	127	2	254
0.75	0.0075	266	199.5	4	798
1.00	0.0100	275	275	1	275
					1487

$J = \frac{(0.25)(1487)}{3} = 123.9 \text{ MPa} = 123.9 \times 10^6 \text{ Pa}$

$M = (2)(0.040)(0.030)^2 (123.9 \times 10^6) = 8.92 \times 10^3 \text{ N}\cdot\text{m} = 8.92 \text{ kN}\cdot\text{m}$

**PROBLEM 4.111**

4.111 The prismatic bar  $AB$  is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the  $\sigma$ - $\epsilon$  diagram is the same in compression as in tension, determine (a) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (b) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)



**SOLUTION**

(a)  $\rho = 100$  in.,  $b = 0.8$  in.,  $c = 0.6$  in.

$$\epsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve  $\sigma_m = 43$  ksi

(b) Strain distribution  $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$  where  $U = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 U |\sigma| dU = 2bc^2 J$$

where the integral  $J$  is given by  $\int_0^1 U |\sigma| dU$

Evaluate  $J$  using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta U}{3} \sum w U |\sigma|$$

where  $w$  is a weighting factor, Using  $\Delta U = 0.25$  we get the values given in the table below:

$U$	$ U $	$ \sigma , \text{ksi}$	$U  \sigma , \text{ksi}$	$w$	$w U  \sigma , \text{ksi}$
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43

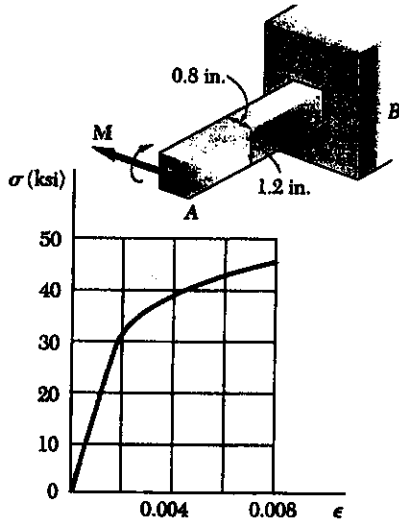
224 ←  $\sum w U |\sigma|$

$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67) = 10.75 \text{ kip}\cdot\text{in.}$$

**PROBLEM 4.112**

4.112 For the bar of Prob. 4.111, determine (a) the radius of curvature of the bar when the maximum stress is 45 ksi, (b) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)



**SOLUTION**

(a)  $b = 0.8 \text{ in}$      $c = 0.6 \text{ in}$

$\sigma_m = 45 \text{ ksi}$

From the curve  $\epsilon_m = 0.008$

$\frac{1}{\rho} = \frac{\epsilon_m}{c} = \frac{0.008}{0.6} = 0.013333 \text{ in}^{-1}$

$\rho = 75 \text{ in.}$

(b) Strain distribution     $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$     where  $U = \frac{y}{c}$

Bending couple

$M = -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 U |\sigma| du = 2bc^2 J$

where the integral  $J$  is given by  $\int_0^1 U |\sigma| du$

Evaluate  $J$  using a method of numerical integration. If Simpson's rule is used, the integration formula is

$J = \frac{\Delta U}{3} \sum w U |\sigma|$

where  $w$  is a weighting factor, Using  $\Delta U = 0.25$  we get the values given in the table below:

$U$	$ E $	$ \sigma , \text{ksi}$	$U \sigma , \text{ksi}$	$w$	$wU \sigma , \text{ksi}$	
0	0	0	0	1	0	
0.25	0.002	32	8	4	32	
0.5	0.004	38	19	2	38	
0.75	0.006	43	32.25	4	129	
1.0	0.008	45	45	5	45	
					244	$\leftarrow \sum wU \sigma $

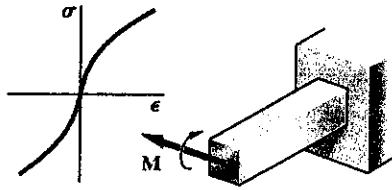
$J = \frac{(0.25)(244)}{3} = 20.33 \text{ ksi}$

$M = (2)(0.8)(0.6)^2(20.33) = 11.7 \text{ kip-in}$

PROBLEM 4.113

4.113 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation,  $\epsilon = k\sigma^n$  for  $\sigma > 0$ , and  $\epsilon = -k|\sigma|^n$  for  $\sigma < 0$ . If a couple  $M$  is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1 + 2n}{3n} \frac{Mc}{I}$$



SOLUTION

Strain distribution  $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m u$  where  $u = \frac{y}{c}$

Bending couple

$$\begin{aligned} M &= -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 \frac{y}{c} |\sigma| \frac{dy}{c} \\ &= 2bc^2 \int_0^1 u |\sigma| du \end{aligned}$$

For  $\epsilon = K\sigma^n$ ,  $\epsilon_m = K\sigma_m^n$

$$\frac{\epsilon}{\epsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \quad \therefore |\sigma| = \sigma_m u^{1/n}$$

$$\begin{aligned} \text{Then } M &= 2bc^2 \int_0^1 u \sigma_m u^{1/n} du = 2bc^2 \sigma_m \int_0^1 u^{1+1/n} du \\ &= 2bc^2 \sigma_m \left. \frac{u^{2+1/n}}{2+1/n} \right|_0^1 = \frac{2n}{2n+1} bc^2 \sigma_m \end{aligned}$$

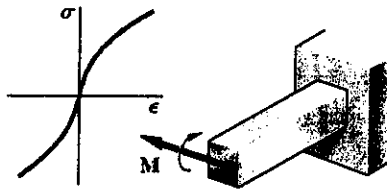
$$\sigma_m = \frac{2n+1}{2} \frac{M}{bc^2}$$

Recall  $\frac{I}{C} = \frac{1}{12} \frac{b(2c)^3}{c} = \frac{2}{3} bc^2 \quad \therefore \frac{1}{bc^2} = \frac{2}{3} \frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n} \frac{Mc}{I}$$

PROBLEM 4.114

4.114 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation  $\epsilon = k\sigma^2$ . If a couple  $M$  is applied to the bar, show that the maximum stress is



$$\sigma_m = \frac{7}{9} \frac{Mc}{I}$$

SOLUTION

Strain distribution  $\epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m U$  where  $U = \frac{y}{c}$

Bending couple

$$\begin{aligned} M &= -\int_{-c}^c y \sigma b dy = 2b \int_0^c y |\sigma| dy = 2bc^2 \int_0^1 \frac{y}{c} |\sigma| \frac{dy}{c} \\ &= 2bc^2 \int_0^1 U |\sigma| du \end{aligned}$$

For  $\epsilon = K\sigma^n$ ,  $\epsilon_m = K\sigma_m^n$

$$\frac{\epsilon}{\epsilon_m} = U = \left(\frac{\sigma}{\sigma_m}\right)^n \quad \therefore |\sigma| = \sigma_m U^{\frac{1}{n}}$$

$$\begin{aligned} \text{Then } M &= 2bc^2 \int_0^1 U \sigma_m U^{\frac{1}{n}} du = 2bc^2 \sigma_m \int_0^1 U^{1+\frac{1}{n}} du \\ &= 2bc^2 \sigma_m \left. \frac{U^{2+\frac{1}{n}}}{2+\frac{1}{n}} \right|_0^1 = \frac{2n}{2n+1} bc^2 \sigma_m \end{aligned}$$

$$\sigma_m = \frac{2n+1}{2} \frac{M}{bc^2}$$

Recall  $\frac{I}{c} = \frac{1}{12} \frac{b(2c)^3}{c} = \frac{2}{3} bc^2 \quad \therefore \frac{1}{bc^2} = \frac{2}{3} \frac{c}{I}$

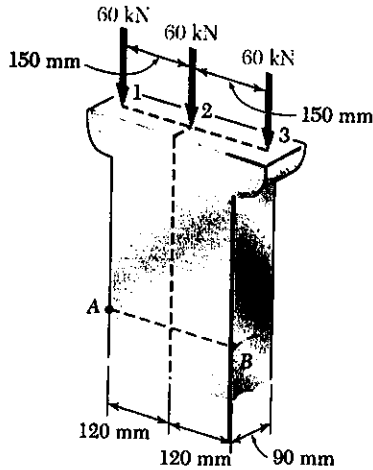
$$\sigma_m = \frac{2n+1}{3n} \frac{Mc}{I}$$

With  $n = 3$   $\sigma_m = \frac{(2)(3)+1}{(3)(3)} \frac{Mc}{I} = \frac{7}{9} \frac{Mc}{I}$



**PROBLEM 4.115**

4.115 Determine the stress at points A and B  
60-kN loads are applied at points 1 and 2 only



**SOLUTION**

(a) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-6} \text{ m}^2$$

$$\text{At A and B } \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-6}} = -8.33 \times 10^6 \text{ Pa} = -8.33 \text{ MPa} \blacktriangleleft$$

(b) Eccentric loading

$$P = 120 \text{ kN} = 120 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

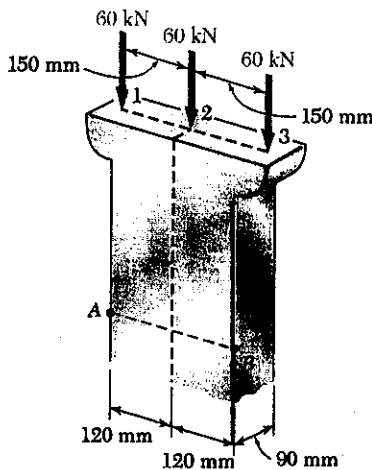
$$c = 120 \text{ mm} = 0.120 \text{ m}$$

$$\text{At A } \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-6}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -15.97 \times 10^6 \text{ Pa} = -15.97 \text{ MPa} \blacktriangleleft$$

$$\text{At B } \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-6}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 4.86 \times 10^6 \text{ Pa} = 4.86 \text{ MPa} \blacktriangleleft$$

**PROBLEM 4.116**

4.116 Determine the stress at points A and B, (a) for the loading shown, (b) if the 60-kN loads are applied at points 2 and 3 are removed.



**SOLUTION**

(a) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-6} \text{ m}^2$$

$$\text{At A and B } \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-6}} = -8.33 \times 10^6 \text{ Pa} = -8.33 \text{ MPa} \blacktriangleleft$$

(b) Eccentric loading

$$P = 120 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N}\cdot\text{m}$$

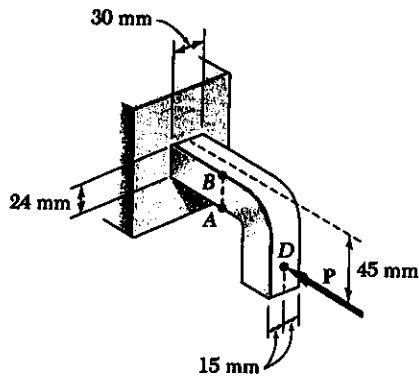
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

$$\text{At A } \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60 \times 10^3}{21.6 \times 10^{-6}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -13.19 \times 10^6 \text{ Pa} = -13.19 \text{ MPa} \blacktriangleleft$$

$$\text{At B } \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60 \times 10^3}{21.6 \times 10^{-6}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 7.64 \times 10^6 \text{ Pa} = 7.64 \text{ MPa} \blacktriangleleft$$

**PROBLEM 4.117**



4.117 Knowing that the magnitude of the horizontal force  $P$  is 8-kN, determine the stress at (a) point  $A$ , (b) point  $B$ .

**SOLUTION**

$$A = (30)(24) = 720 \text{ mm}^2 = 720 \times 10^{-6} \text{ m}^2$$

$$e = 45 - 12 = 33 \text{ mm} = 0.033 \text{ m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3 \text{ mm}^4 = 34.56 \times 10^{-9} \text{ m}^4$$

$$c = 24 \text{ mm} = 0.12 \text{ m} \quad P = 8 \times 10^3 \text{ N}$$

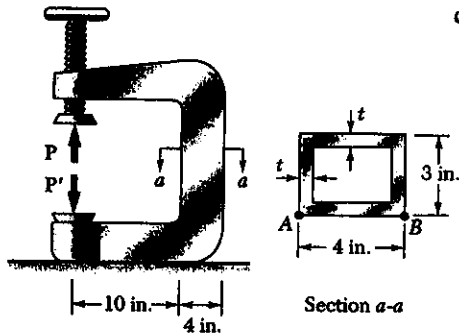
$$M = Pe = (8 \times 10^3)(0.033) = 264 \text{ N}\cdot\text{m}$$

$$\text{At } A \quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.12)}{34.56 \times 10^{-9}}$$

$$= -102.8 \times 10^6 \text{ Pa} = -102.8 \text{ MPa} \quad \blacktriangleleft$$

$$\text{At } B \quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.12)}{34.56 \times 10^{-9}} = 80.6 \times 10^6 \text{ Pa} = 80.6 \text{ MPa} \quad \blacktriangleright$$

**PROBLEM 4.118**



4.118 The vertical portion of the press shown consists of a rectangular tube having a wall thickness  $t = \frac{1}{2}$  in. Knowing that the press has been tightened until  $P = 6$  kips, determine the stress (a) at point  $A$ , (b) at point  $B$ .

**SOLUTION**

$$t = \frac{1}{2} \text{ in.} \quad P = 6 \text{ kips}$$

$$A = (3)(4) - (2)(3) = 6 \text{ in}^2$$

$$I = \frac{1}{12}(3)(4)^3 - \frac{1}{12}(2)(3)^3 = 11.5 \text{ in}^4$$

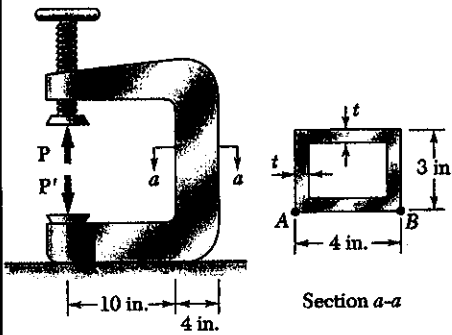
$$c = 2 \text{ in.}, \quad e = 10 + 2 = 12 \text{ in.}$$

$$M = Pe = (6)(12) = 72 \text{ kip}\cdot\text{in.}$$

$$(a) \quad \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{6}{6} + \frac{(72)(2)}{11.5} = 13.52 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{6}{6} - \frac{(72)(2)}{11.5} = -11.52 \text{ ksi} \quad \blacktriangleright$$

**PROBLEM 4.119**



4.118 The vertical portion of the press shown consists of a rectangular tube having a wall thickness  $t = \frac{1}{2}$  in. Knowing that the press has been tightened until  $P = 6$  kips, determine the stress (a) at point A, (b) at point B.

4.119 Solve Prob. 4.118, assuming that the wall thickness of the vertical portion of the press is  $t = \frac{3}{8}$  in.

**SOLUTION**

Rectangular cutout is  $2\frac{1}{4}$  in.  $\times$   $3\frac{1}{4}$  in.

$$A = (3)(4) - (2.25)(3.25) = 4.6875 \text{ in}^2$$

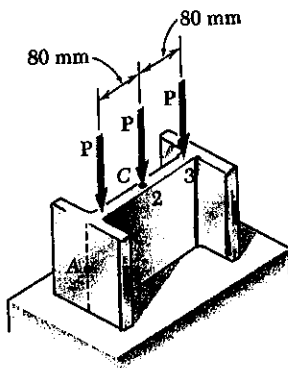
$$I = \frac{1}{12}(3)(4)^3 - \frac{1}{12}(2.25)(3.25)^3 = 9.5635 \text{ in}^4$$

$$c = 2 \text{ in.}, \quad e = 10 + 2 = 12 \text{ in.}, \quad M = Pe = (6)(12) = 72 \text{ kip}\cdot\text{in}$$

$$(a) \quad \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{6}{4.6875} + \frac{(72)(2)}{9.5635} = 16.34 \text{ ksi}$$

$$(b) \quad \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{6}{4.6875} - \frac{(72)(2)}{9.5635} = -13.78 \text{ ksi}$$

**PROBLEM 4.120**



4.120 As many as three axial loads each of magnitude  $P = 50$  kN can be applied to the end of a ~~W 100 x 21~~ rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 1 and 2 only.

W 200  $\times$  31.3

**SOLUTION**

For W 200  $\times$  31.3 rolled steel shape

$$A = 4000 \text{ mm}^2 = 4.000 \times 10^{-3} \text{ m}^2$$

$$c = \frac{1}{2}d = \frac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$$

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

(a) Centric load

$$3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{4.0 \times 10^{-3}} = -37.5 \times 10^6 \text{ Pa} = -37.5 \text{ MPa}$$

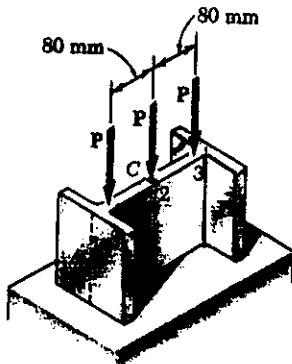
(b) Eccentric loading  $e = 80 \text{ mm} = 0.080 \text{ m}$

$$2P = 50 + 50 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_A = -\frac{2P}{A} - \frac{Mc}{I} = -\frac{100 \times 10^3}{4.0 \times 10^{-3}} - \frac{(4.0 \times 10^3)(0.105)}{31.4 \times 10^{-6}} = -38.4 \times 10^6 \text{ Pa} = -38.4 \text{ MPa}$$

**PROBLEM 4.121**



4.121 As many as three axial loads, each of magnitude  $P = 50 \text{ kN}$ , can be applied to the end of a ~~W 8 x 21~~ rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 2 and 3 only.

**SOLUTION**

W 200 x 31.3

For a W 200 x 31.3 rolled steel shape

$$A = 4000 \text{ mm}^2 = 4.00 \times 10^{-3} \text{ m}^2$$

$$c = \frac{1}{2}d = \frac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$$

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

(a) Centric loading

$$3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{4.0 \times 10^{-3}} = -37.5 \times 10^6 \text{ Pa} = -37.5 \text{ MPa}$$

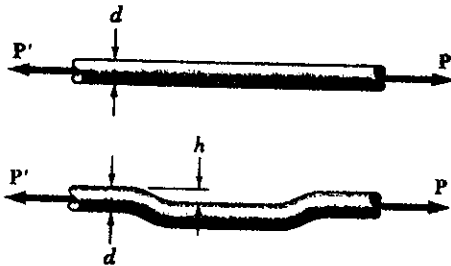
(b) Eccentric loading

$$e = 80 \text{ mm} = 0.080 \text{ m}$$

$$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_A = -\frac{2P}{A} + \frac{Mc}{I} = -\frac{100 \times 10^3}{4.0 \times 10^{-3}} + \frac{(4.0 \times 10^3)(0.105)}{31.4 \times 10^{-6}} = -11.62 \times 10^6 \text{ Pa} = -11.62 \text{ MPa}$$

**PROBLEM 4.122**



4.122 An offset  $h$  must be introduced into a solid circular rod of diameter  $d$ . Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

**SOLUTION**

For centric loading  $\sigma_c = \frac{P}{A}$

For eccentric loading  $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given  $\sigma_e = 4\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{Phc}{I} = 3\frac{P}{A} \quad \therefore h = \frac{3I}{cA} = \frac{(3)(\frac{\pi}{4}d^4)}{(\frac{d}{2})(\frac{\pi}{4}d^2)} = \frac{3}{8}d = 0.375d$$

**PROBLEM 4.123**

4.123 An offset  $h$  must be introduced into a metal tube of 18-mm outer diameter and 2-mm wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

**SOLUTION**

$$d_i = d_o - 2t = 18 - (2)(2) = 14 \text{ mm} \qquad c = \frac{1}{2}d_o = 9 \text{ mm}$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(18^2 - 14^2) = 100.53 \text{ mm}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(18^4 - 14^4) = 3.2673 \times 10^3 \text{ mm}^4$$

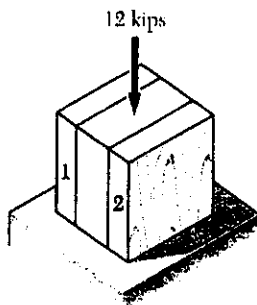
For centric loading  $\sigma_c = \frac{P}{A}$ ; For eccentric loading  $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given  $\sigma_e = 4\sigma_c \therefore \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A} \therefore \frac{Phc}{I} = 3\frac{P}{A}$

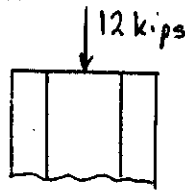
$$h = \frac{3I}{cA} = \frac{(3)(3.2673 \times 10^3)}{(9)(100.53)} = 10.83 \text{ mm}$$

**PROBLEM 4.124**

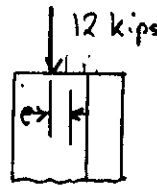
4.124 A short column is made by nailing two 1 × 4-in. planks to a 2 × 4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown at the center of the top section of the timber if (a) the column is as described, (b) plank 1 is removed, (c) both planks are removed.



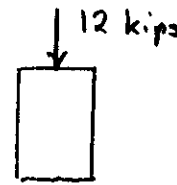
**SOLUTION**



(a)



(b)



(c)

(a) Centric loading: 4 in × 4 in cross section  $A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{16} = -0.75 \text{ ksi}$$

(b) Eccentric loading: 4 in × 3 in cross section  $A = (4)(3) = 12 \text{ in}^2$

$$c = (\frac{1}{2})(3) = 1.5 \text{ in} \qquad e = 1.5 - 1.0 = 0.5 \text{ in}$$

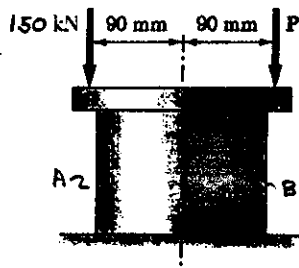
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(3)^3 = 9 \text{ in}^4$$

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} = -2.00 \text{ ksi}$$

(c) Centric loading: 4 in. × 2 in cross section  $A = (4)(2) = 8 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{8} = -1.50 \text{ ksi}$$

PROBLEM 4.125



4.125 The two forces shown are applied to a rigid plate supported by a steel pipe of 140-mm outer diameter and 120-mm inner diameter. Knowing that the allowable compressive stress is 100 MPa, determine the range of allowable values of  $P$ .

SOLUTION

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(140^2 - 120^2) = 4.084 \times 10^3 \text{ mm}^2 = 4.084 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(140^4 - 120^4) = 8.679 \times 10^6 \text{ mm}^4 = 8.679 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}d_o = 70 \text{ mm} = 0.070 \text{ m}$$

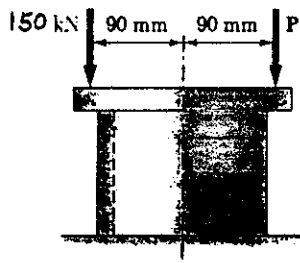
$$F = 150 \times 10^3 + P, \quad M = (0.090)(150 \times 10^3) - (0.090)P = 13.5 \times 10^3 - 0.09P$$

$$\begin{aligned} \text{At A} \quad \sigma_A &= -\frac{F}{A} - \frac{Mc}{I} = -\frac{(150 \times 10^3) + P}{4.084 \times 10^{-3}} - \frac{(13.5 \times 10^3 - 0.09P)(0.070)}{8.679 \times 10^{-6}} \\ &= -145.61 \times 10^6 + 481.03P = -100 \times 10^6 \quad \therefore P = 94.8 \times 10^3 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{At B} \quad \sigma_B &= -\frac{F}{A} + \frac{Mc}{I} = -\frac{(150 \times 10^3) + P}{4.084 \times 10^{-3}} + \frac{(13.5 \times 10^3 - 0.09P)(0.070)}{8.679 \times 10^{-6}} \\ &= 72.155 \times 10^6 - 970.75P = -100 \times 10^6 \quad \therefore P = 177.3 \times 10^3 \text{ N} \end{aligned}$$

$$94.8 \text{ kN} < P < 177.3 \text{ kN}$$

PROBLEM 4.126



4.126 The two forces shown are applied to a rigid plate supported by a steel pipe of 140-mm outer diameter and 120-mm inner diameter. Determine the range of allowable values of  $P$  for which all stresses in the pipe are compressive and less than 100 MPa.

SOLUTION

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(140^2 - 120^2) = 4.084 \times 10^3 \text{ mm}^2 = 4.084 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(140^4 - 120^4) = 8.679 \times 10^6 \text{ mm}^4 = 8.679 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}d_o = 70 \text{ mm} = 0.070 \text{ m}$$

$$F = 150 \times 10^3 + P, \quad M = (0.090)(150 \times 10^3) - 0.090P = 13.5 \times 10^3 - 0.09P$$

$$\begin{aligned} \text{At A} \quad \sigma_A &= -\frac{F}{A} - \frac{Mc}{I} = -\frac{(150 \times 10^3) + P}{4.084 \times 10^{-3}} - \frac{(13.5 \times 10^3 - 0.09P)(0.070)}{8.679 \times 10^{-6}} \\ &= -145.61 \times 10^6 + 481.03P = -100 \times 10^6 \quad \therefore P = 94.8 \times 10^3 \text{ N} \end{aligned}$$

$$\sigma_A = -145.61 \times 10^6 + 481.03P = 0 \quad P = 303 \times 10^3 \text{ N}$$

Based on stress limits at A  $94.8 \text{ kN} \leq P \leq 303 \text{ kN}$

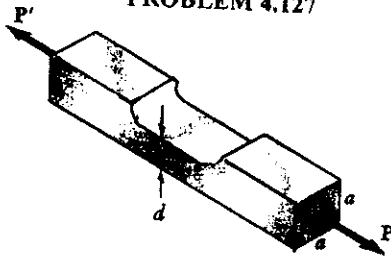
$$\begin{aligned} \text{At B} \quad \sigma_B &= -\frac{F}{A} + \frac{Mc}{I} = -\frac{(150 \times 10^3) + P}{4.084 \times 10^{-3}} + \frac{(13.5 \times 10^3 - 0.09P)(0.070)}{8.679 \times 10^{-6}} \\ &= 72.155 \times 10^6 - 970.75P = -100 \times 10^6 \quad P = 177.3 \times 10^3 \text{ N} \end{aligned}$$

$$\sigma_B = 72.155 \times 10^6 - 970.75P = 0 \quad P = 74.3 \times 10^3 \text{ N}$$

Based on stress limits at B  $74.3 \text{ kN} \leq P \leq 177.3 \text{ kN}$

Based on both limits  $94.8 \text{ kN} \leq P \leq 177.3 \text{ kN}$

PROBLEM 4.127



4.127 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that  $a = 1.2$  in.,  $d = 0.8$  in., and  $\sigma_{all} = 8$  ksi, determine the largest magnitude  $P$  of the forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12} ad^3, \quad c = \frac{1}{2} d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

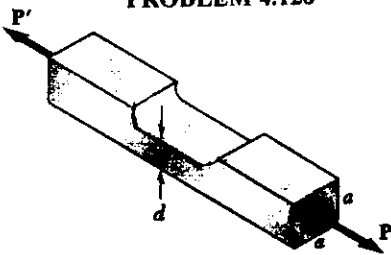
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP$$

$$K = \frac{1}{ad} + \frac{3(a-d)}{ad^2} = \frac{1}{(1.2)(0.8)} + \frac{(3)(1.2-0.8)}{(1.2)(0.8)^2} = 2.604 \text{ in}^{-1}$$

$$P = \frac{\sigma}{K} = \frac{8}{2.604} = 3.07 \text{ kips}$$

PROBLEM 4.128



4.128 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude  $P = 4$  kips are applied at the centers of the ends of the bar. Knowing that  $a = 1.2$  in. and  $\sigma_{all} = 8$  ksi, determine the smallest allowable depth  $d$  of the milled portion of the rod.

SOLUTION

$$A = ad, \quad I = \frac{1}{12} ad^3, \quad c = \frac{1}{2} d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

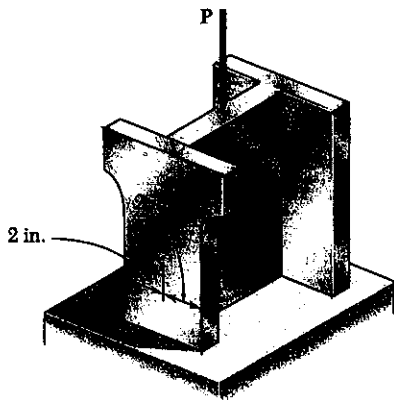
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P \frac{1}{2}(a-d) \frac{1}{2} d}{\frac{1}{12} ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \quad \text{or} \quad \sigma d^2 + \frac{2P}{a} d - 3P = 0$$

$$\text{Solving for } d \quad d = \frac{1}{2\sigma} \left\{ \sqrt{\left(\frac{2P}{a}\right)^2 + (12P\sigma)} - \frac{2P}{a} \right\}$$

$$d = \frac{1}{(2)(8)} \left\{ \sqrt{\left[\frac{(2)(4)}{1.2}\right]^2 + (12)(4)(8)} - \frac{(2)(4)}{1.2} \right\} = 0.877 \text{ in.}$$

PROBLEM 4.129



4.129 Three steel plates, each of 1 × 6-in. cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 1-in. strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 15 ksi, determine the largest force P, (a) which could be applied to the original column, (b) which can be applied to the modified column.

SOLUTION

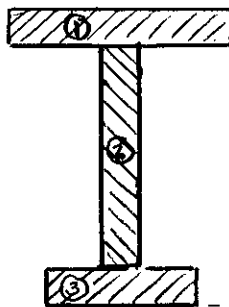
(a) Centric loading

$$A = (3)(1)(6) = 18 \text{ in}^2$$

$$\sigma = -\frac{P}{A} \therefore P = \sigma A = (15)(18) = 270 \text{ kips}$$

(b) Eccentric loading

Reduced cross section



	$A_i, \text{in}^2$	$y_{o_i}, \text{in.}$	$A_i \bar{y}_{o_i}, \text{in}^3$
①	6	3.5	21.0
②	6	0	0
③	4	-3.5	-14.0
$\Sigma$	16		7.0

$$\begin{aligned} \bar{y}_o &= \frac{\Sigma A_i \bar{y}_{o_i}}{\Sigma A_i} \\ &= \frac{7.0}{16} \\ &= 0.4375 \text{ in} \end{aligned}$$

The centroid lies 0.4375 in from the midpoint of the web.

$$I_1 = \frac{1}{12}(6)(1)^3 + (6)(3.0625)^2 = 56.773 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(6)^3 + (6)(0.4375)^2 = 19.148 \text{ in}^4$$

$$I_3 = \frac{1}{12}(4)(1)^3 + (4)(3.9375)^2 = 62.349 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 138.27 \text{ in}^4, \quad c = 4.4375 \text{ in}$$

$$M = Pe \text{ where } e = 0.4375 \text{ in}$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} + \frac{Pec}{I} = -K P$$

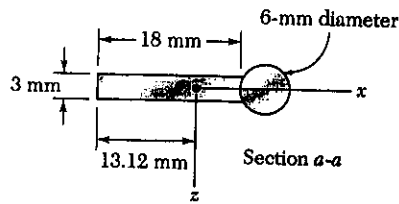
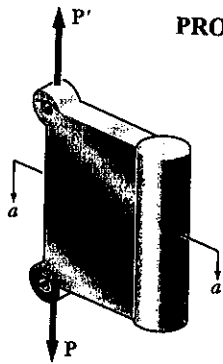
$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{16} + \frac{(0.4375)(4.4375)}{138.27} = 0.076541 \text{ in}^{-2}$$

$$P = -\frac{\sigma}{K} = \frac{-15}{0.076541} = 196.0 \text{ kips}$$

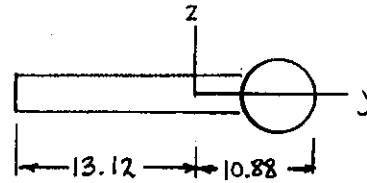


PROBLEM 4.130

4.130 A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (a) the largest force  $P$  that can be applied to the element, (b) the corresponding location of the neutral axis. Given: Centroid of the cross section is at  $C$  and  $I_z = 4195 \text{ mm}^4$ .



SOLUTION



$$(a) \quad A = (3)(18) + \frac{\pi}{4}(6)^2 = 82.27 \text{ mm}^2 = 82.27 \times 10^{-6} \text{ m}^2$$

$$I = 4195 \text{ mm}^4 = 4195 \times 10^{-12} \text{ m}^4$$

$$e = 13.12 \text{ mm} = 0.01312 \text{ m}$$

Based on tensile stress at  $y = -13.12 \text{ mm} = -0.01312 \text{ m}$

$$\sigma = \frac{P}{A} + \frac{Pec}{I} = \left(\frac{1}{A} + \frac{ec}{I}\right)P = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{82.27 \times 10^{-6}} + \frac{(0.01312)(0.01312)}{4195 \times 10^{-12}} = 53.188 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{135 \times 10^6}{53.188 \times 10^3} = 2.538 \times 10^3 \text{ N} = 2.54 \text{ kN}$$

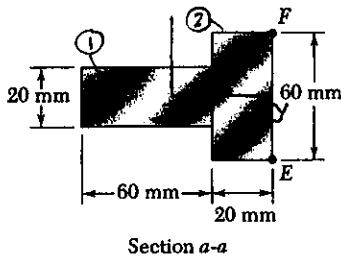
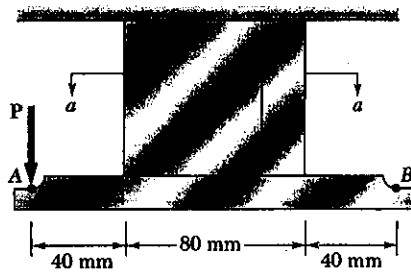
(b) Location neutral axis.  $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \quad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{4195 \times 10^{-12}}{(82.27 \times 10^{-6})(0.01312)} = 3.89 \times 10^{-3} \text{ m} = 3.89 \text{ mm}$$

The neutral axis lies 3.89 mm to the right of the centroid or 17.01 mm to the right of the line of action of the loads.

**PROBLEM 4.131**



4.131 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (a) the largest vertical force **P** that can be applied at point **A**, (b) the corresponding location of the neutral axis of section *a-a*.

**SOLUTION**

Locate centroid

	$A, \text{mm}^2$	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$
①	1200	30	$36 \times 10^3$
②	1200	70	$84 \times 10^3$
$\Sigma$	2400		$120 \times 10^3$

$$\begin{aligned} \bar{y}_0 &= \frac{\Sigma A\bar{y}_0}{\Sigma A} \\ &= \frac{120 \times 10^3}{2400} \\ &= 50 \text{ mm} \end{aligned}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple  $M = Pe$   
 $e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$

$$I_1 = \frac{1}{12} (20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \quad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge:  $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = K P$$

$$K = \frac{1}{A} - \frac{e y}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(0.090)(-0.050)}{1.360 \times 10^{-6}} = 3.7255 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{3.7255 \times 10^3} = 40.3 \times 10^3 \text{ N} = 40.3 \text{ kN}$$

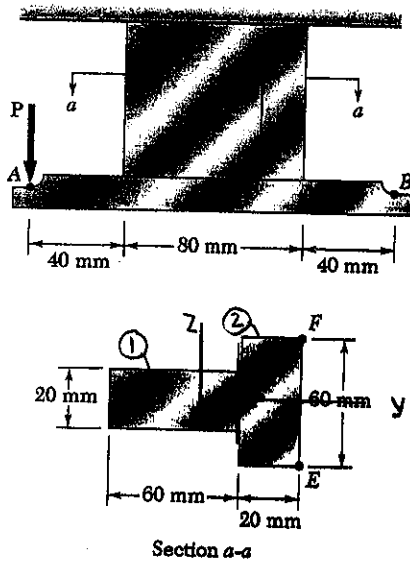
(b) Location of neutral axis:  $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = 0 \quad \frac{e y}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(0.090)} = 6.30 \times 10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left edge

PROBLEM 4.132



4.131 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (a) the largest vertical force *P* that can be applied at point *A*, (b) the corresponding location of the neutral axis of section *a-a*.

4.132 Solve Prob. 4.131, assuming that the vertical force *P* is applied at point *B*.

SOLUTION

Locate centroid

	$A, \text{mm}^2$	$\bar{y}_0, \text{mm}$	$A\bar{y}_0, \text{mm}^3$
①	1200	30	$36 \times 10^3$
②	1200	70	$84 \times 10^3$
$\Sigma$	2400		$120 \times 10^3$

$$\bar{y}_0 = \frac{\Sigma A\bar{y}_0}{\Sigma A} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple  $M = Pe$   
 $e = 50 - 120 = -70 \text{ mm} = -0.070 \text{ m}$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \quad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section:  $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = K_L P$$

$$K_L = \frac{1}{A} - \frac{e y}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(-0.050)}{1.360 \times 10^{-6}} = -2.1569 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_L} = \frac{-150 \times 10^6}{-2.1569 \times 10^3} = 69.6 \times 10^3 \text{ N}$$

Based on stress at right edge of section:  $y = 30 \text{ mm} = 0.030 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = K_R P$$

$$K_R = \frac{1}{A} - \frac{e y}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(-0.070)(0.030)}{1.360 \times 10^{-6}} = 1.9608 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{1.9608 \times 10^3} = 76.5 \times 10^3 \text{ N}$$

Choose the smaller value  $P = 69.6 \times 10^3 \text{ N} = 69.6 \text{ kN}$

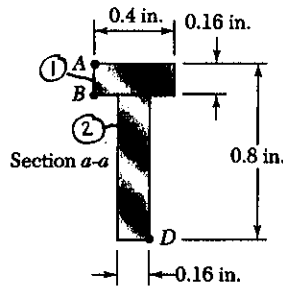
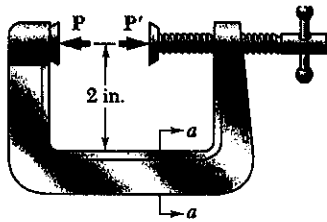
(b) Location of neutral axis:  $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pe y}{I} = 0 \quad \frac{e y}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(-0.070)} = -8.10 \times 10^{-3} \text{ m} = -8.10 \text{ mm}$$

Neutral axis lies  $50 - 8.10 = 41.9 \text{ mm}$  from left edge.

PROBLEM 4.133



4.133 Knowing that the clamp shown has been tightened until  $P = 75$  lb, determine in section  $a-a$  (a) the stress at point  $A$ , (b) the stress at point  $D$ , (c) the location of the neutral axis.

SOLUTION

Locate centroid

Part	$A, \text{in}^2$	$\bar{y}_0, \text{in}$	$A\bar{y}_0, \text{in}^3$
①	0.064	0.72	0.04608
②	0.1024	0.32	0.03277
$\Sigma$	0.1664		0.07885

$$\bar{y}_0 = \frac{\Sigma A\bar{y}_0}{\Sigma A} = \frac{0.07885}{0.1664} = 0.4739 \text{ in.}$$

The centroid lies 0.4739 in. above point  $D$ .

Bending couple  $M = Pe$

$$e = -(2 + 0.8 - 0.4739) = -2.3261 \text{ in}$$

$$I_1 = \frac{1}{12}(0.4)(0.16)^3 + (0.064)(0.72 - 0.4739)^2 = 4.013 \times 10^{-3} \text{ in}^4$$

$$I_2 = \frac{1}{12}(0.16)(0.64)^3 + (0.1024)(0.4739 - 0.32)^2 = 5.921 \times 10^{-3} \text{ in}^4$$

$$I = I_1 + I_2 = 9.934 \times 10^{-3} \text{ in}^4$$

(a) Stress at point  $A$ :  $y = 0.8 - 0.4739 = 0.3261 \text{ in}$

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pe y}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(0.3261)}{9.934 \times 10^{-3}} = 6.18 \times 10^3 \text{ psi} = 6.18 \text{ ksi}$$

(b) Stress at point  $D$ :  $y = -0.4739 \text{ in.} = 0.1661 \text{ in}$

$$\sigma_D = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pe y}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(-0.4739)}{9.934 \times 10^{-3}} = -7.87 \times 10^3 \text{ psi} = -7.87 \text{ ksi}$$

(c) Location of neutral axis  $\sigma = 0$

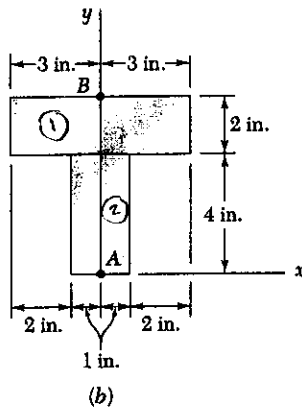
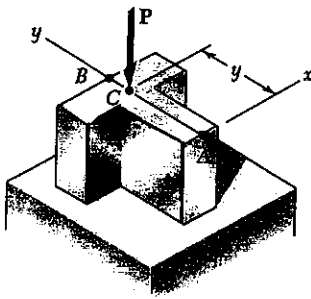
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pe y}{I} = 0 \quad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{9.934 \times 10^{-3}}{(0.1664)(-2.3261)} = -0.0257 \text{ in}$$

The neutral axis lies  $0.4739 - 0.0257 = 0.448 \text{ in.}$  above point  $D$ .

PROBLEM 4.134

4.134 A vertical force  $P$  of magnitude 20 kips is applied at a point  $C$  located on the line of symmetry of the cross section of a short column. Knowing that  $y = 5$  in., determine (a) the stress at point  $A$ , (b) the stress at point  $B$ , (c) the location of the neutral axis.



SOLUTION

Locate centroid

Part	$A_i, \text{in}^2$	$\bar{y}_i, \text{in}$	$A_i \bar{y}_i, \text{in}^3$
①	12	5	60
②	8	2	16
$\Sigma$	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load  $e = 5 - 3.8 = 1.2 \text{ in.}$

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at  $A$   $c_A = 3.8 \text{ in}$

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{(20)(1.2)(3.8)}{57.867} = 0.576 \text{ ksi}$$

(b) Stress at  $B$   $c_B = 6 - 3.8 = 2.2 \text{ in}$

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = -\frac{20}{20} - \frac{(20)(1.2)(2.2)}{57.867} = -1.912 \text{ ksi}$$

(c) Location of neutral axis:  $\sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \frac{ea}{I} = \frac{1}{A}$$

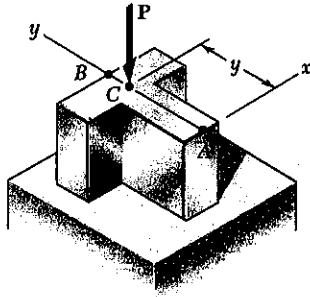
$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in}$$

Neutral axis lies 2.411 in. below centroid or  $3.8 - 2.411 = 1.389$  in above point  $A$ .

Answer 1.389 in from point  $A$

PROBLEM 4.135

4.135 A vertical force  $P$  is applied at a point  $C$  located on the line of symmetry of the cross section of a short column. Determine the range of values of  $y$  for which tensile stresses do not occur in the column.

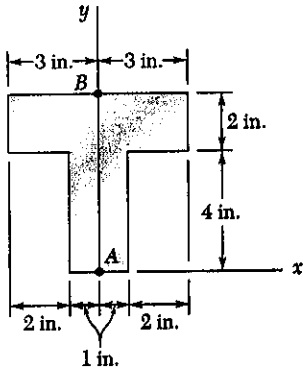


SOLUTION

Locate centroid

	$A_i$ in <sup>2</sup>	$\bar{y}_i$ in	$A_i \bar{y}_i$ in <sup>3</sup>
①	12	5	60
②	8	2	16
$\Sigma$	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$



Eccentricity of load  $e = y - 3.8$  in.

$$y = e + 3.8 \text{ in}$$

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero.  $c_A = 3.8$  in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \quad \therefore \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in} \quad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero.  $c_B = 6 - 3.8 = 2.2$  in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \quad \therefore \frac{ec_B}{I} = -\frac{1}{A}$$

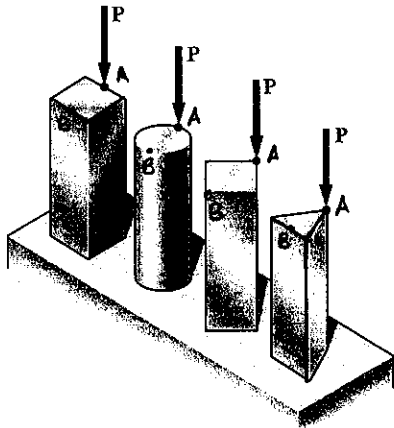
$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer:  $2.485 \text{ in} < y < 4.561 \text{ in.}$

PROBLEM 4.136

4.136 The four bars shown have the same cross-sectional area. For the given loadings, show that (a) the maximum compressive stresses are in the ratio 4:5:7:9, (b) the maximum tensile stresses are in the ratio 2:3:5:3. (Note: the cross section of the triangular bar is an equilateral triangle.)

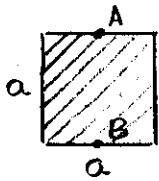


SOLUTION

Stresses

$$\begin{aligned} \text{At A} \quad \sigma_A &= -\frac{P}{A} - \frac{Pec_A}{I} \\ &= -\frac{P}{A} \left( 1 + \frac{Aec_A}{I} \right) \end{aligned}$$

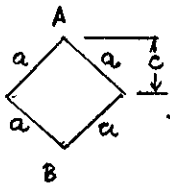
$$\begin{aligned} \text{At B} \quad \sigma_B &= -\frac{P}{A} + \frac{Pec_B}{I} \\ &= \frac{P}{A} \left( \frac{Aec_B}{I} - 1 \right) \end{aligned}$$



$$\left\{ \begin{aligned} A_1 &= a^2, \quad I_1 = \frac{1}{12} a^4, \quad c_A = c_B = \frac{1}{2} a, \quad e = \frac{1}{2} a \\ \sigma_A &= -\frac{P}{A} \left( 1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12} a^4} \right) = -4 \frac{P}{A_1} \\ \sigma_B &= \frac{P}{A} \left( \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12} a^4} - 1 \right) = 2 \frac{P}{A_1} \end{aligned} \right.$$



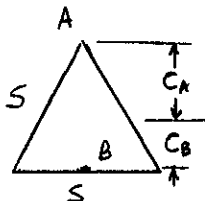
$$\left\{ \begin{aligned} A_2 &= \pi c^2 = \frac{\pi}{4} a^2 \quad \therefore c = \frac{a}{2}, \quad I_2 = \frac{\pi}{4} c^4 \\ \sigma_A &= -\frac{P}{A_2} \left( 1 + \frac{(\frac{\pi}{4} c^2)(c)(c)}{\frac{\pi}{4} c^4} \right) = -5 \frac{P}{A_2} \\ \sigma_B &= \frac{P}{A_2} \left( \frac{(\frac{\pi}{4} c^2)(c)(c)}{\frac{\pi}{4} c^4} - 1 \right) = 3 \frac{P}{A_2} \end{aligned} \right.$$



$$\left\{ \begin{aligned} A_3 &= a^2 \quad c = \frac{\sqrt{2}}{2} a \quad I_3 = \frac{1}{12} a^4 \quad e = c \\ \sigma_A &= -\frac{P}{A_3} \left( 1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12} a^4} \right) = -7 \frac{P}{A_3} \\ \sigma_B &= \frac{P}{A_3} \left( \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12} a^4} - 1 \right) = 5 \frac{P}{A_3} \end{aligned} \right.$$

$$A_4 = \frac{1}{2}(s)(\frac{\sqrt{3}}{2}s) = \frac{\sqrt{3}}{4} s^2 \quad I_4 = \frac{1}{36} s(\frac{\sqrt{3}}{2}s)^3 = \frac{\sqrt{3}}{96} s^4$$

$$c_A = \frac{2}{3} \frac{\sqrt{3}}{2} s = \frac{s}{\sqrt{3}} = e \quad c_B = \frac{s}{2\sqrt{3}}$$

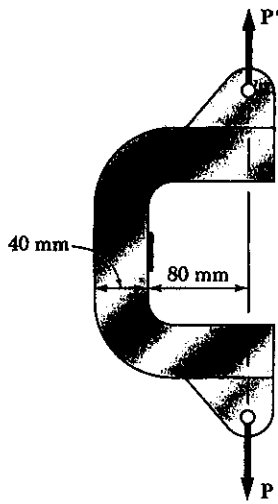


$$\sigma_A = -\frac{P}{A_4} \left( 1 + \frac{(\frac{\sqrt{3}}{4} s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96} s^4} \right) = -9 \frac{P}{A_4}$$

$$\sigma_B = \frac{P}{A_4} \left( \frac{(\frac{\sqrt{3}}{4} s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96} s^4} - 1 \right) = 3 \frac{P}{A_4}$$

PROBLEM 4.137

4.137 The C-shaped steel bar is used as a dynamometer to determine the magnitude  $P$  of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be  $450 \mu$ , determine the magnitude  $P$  of the forces. Use  $E = 200 \text{ GPa}$ .



SOLUTION

At the strain gage location

$$\sigma = E \epsilon = (200 \times 10^9)(450 \times 10^{-6}) = 90 \times 10^6$$

$$A = (40)(40) = 1600 \text{ mm}^2 = 1600 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213.33 \times 10^3 \text{ mm}^4 = 213.33 \times 10^{-9} \text{ m}^4$$

$$e = 80 + 20 = 100 \text{ mm} = 0.100 \text{ m}$$

$$c = 20 \text{ mm} = 0.020 \text{ m}$$

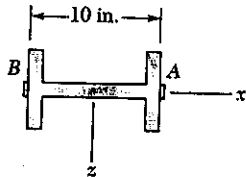
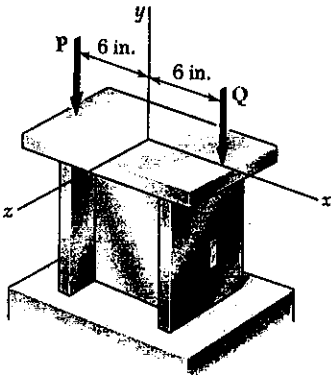
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600 \times 10^{-6}} + \frac{(0.100)(0.020)}{213.33 \times 10^{-9}} = 10.00 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90 \times 10^6}{10.00 \times 10^3} = 9.00 \times 10^3 \text{ N} = 9.00 \text{ kN}$$



PROBLEM 4.138



$A = 10.0 \text{ in}^2$   
 $I_z = 273 \text{ in}^4$

4.138 A short length of a rolled-steel column supports a rigid plate on which two loads  $P$  and  $Q$  are applied as shown. The strains at two points  $A$  and  $B$  on the center lines of the outer faces of the flanges have been measured and found to be  $\epsilon_A = -400 \times 10^{-6} \text{ in./in.}$   $\epsilon_B = -300 \times 10^{-6} \text{ in./in.}$  Knowing that  $E = 29 \times 10^6 \text{ psi}$ , determine the magnitude of each load.

SOLUTION

Stresses at  $A$  and  $B$  from strain gages

$$\sigma_A = E \epsilon_A = (29 \times 10^6)(-400 \times 10^{-6}) = -11.6 \times 10^3 \text{ psi}$$

$$\sigma_B = E \epsilon_B = (29 \times 10^6)(-300 \times 10^{-6}) = -8.7 \times 10^3 \text{ psi}$$

Centric force  $F = P + Q$

Bending couple  $M = 6P - 6Q$

$c = 5 \text{ in.}$

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6 \times 10^3 = +0.00989 P - 0.20989 Q \quad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

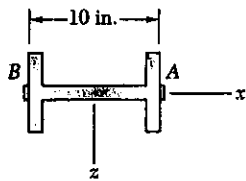
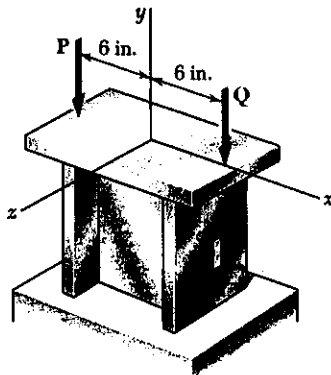
$$-8.7 \times 10^3 = -0.20989 P + 0.00989 Q \quad (2)$$

Solving (1) and (2) simultaneously

$$P = 44.2 \times 10^3 \text{ lb} = 44.2 \text{ kips}$$

$$Q = 57.3 \times 10^3 \text{ lb} = 57.3 \text{ kips}$$

PROBLEM 4.139



$A = 10.0 \text{ in}^2$   
 $I_z = 273 \text{ in}^4$

4.138 A short length of a rolled-steel column supports a rigid plate on which two loads  $P$  and  $Q$  are applied as shown. The strains at two points  $A$  and  $B$  on the center lines of the outer faces of the flanges have been measured and found to be  $\epsilon_A = -400 \times 10^{-6} \text{ in./in.}$   $\epsilon_B = -300 \times 10^{-6} \text{ in./in.}$  Knowing that  $E = 29 \times 10^6 \text{ psi}$ , determine the magnitude of each load.

4.139 Solve Prob. 4.138, assuming that the measured strains are  $\epsilon_A = -350 \times 10^{-6} \text{ in./in.}$   $\epsilon_B = -50 \times 10^{-6} \text{ in./in.}$

SOLUTION

Stresses at  $A$  and  $B$  from strain gages

$$\sigma_A = E \epsilon_A = (29 \times 10^6)(-350 \times 10^{-6}) = -10.15 \times 10^3 \text{ psi}$$

$$\sigma_B = E \epsilon_B = (29 \times 10^6)(-50 \times 10^{-6}) = -1.45 \times 10^3 \text{ psi}$$

Centric force  $F = P + Q$

Bending couple  $M = 6P - 6Q$

$c = 5 \text{ in}$

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-10.15 \times 10^3 = 0.00989 P - 0.20989 Q \tag{1}$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

$$-1.45 \times 10^3 = -0.20989 P + 0.00989 Q \tag{2}$$

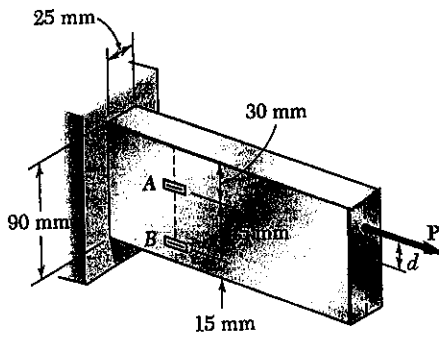
Solving (1) and (2) simultaneously

$$P = 9.21 \times 10^3 \text{ lb} = 9.21 \text{ kips}$$

$$Q = 48.8 \times 10^3 \text{ lb} = 48.8 \text{ kips}$$

**PROBLEM 4.140**

4.140 An eccentric axial force  $P$  is applied as shown to a steel bar of  $25 \times 90$ -mm cross section. The strains at  $A$  and  $B$  have been measured and found to be  $\epsilon_A = +350 \mu$  and  $\epsilon_B = -70 \mu$ . Knowing that  $E = 200$  GPa, determine (a) the distance  $d$ , (b) the magnitude of the force  $P$ .



**SOLUTION**

$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm} \quad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \quad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

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Stresses from strain gages at  $A$  and  $B$

$$\sigma_A = E\epsilon_A = (200 \times 10^9)(350 \times 10^{-6}) = 70 \times 10^6 \text{ Pa}$$

$$\sigma_B = E\epsilon_B = (200 \times 10^9)(-70 \times 10^{-6}) = -14 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting  $\sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(84 \times 10^6)}{0.045} = -2835 \text{ N}\cdot\text{m}$$

Multiplying (2) by  $y_A$  and (1) by  $y_B$  and subtracting

$$y_A \sigma_B - y_B \sigma_A = (y_A - y_B) \frac{P}{A}$$

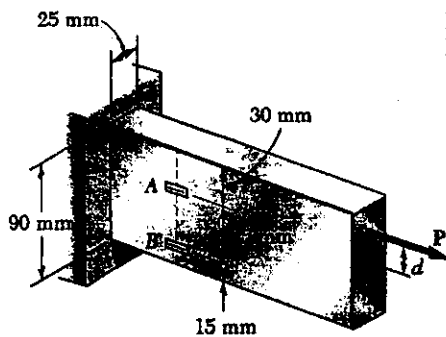
$$P = \frac{A(y_A \sigma_B - y_B \sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(-14 \times 10^6) - (-0.030)(70 \times 10^6)]}{0.045}$$

$$= 94.5 \times 10^3 \text{ N}$$

(a)  $M = -Pd \therefore d = -\frac{M}{P} = -\frac{-2835}{94.5 \times 10^3} = 0.030 \text{ m} = 30 \text{ mm} \quad \blacktriangleleft$

(b)  $P = 94.5 \text{ kN} \quad \blacktriangleleft$

**PROBLEM 4.141**



**4.140** An eccentric axial force  $P$  is applied as shown to a steel bar of  $25 \times 90$ -mm cross section. The strains at  $A$  and  $B$  have been measured and found to be  
 $\epsilon_A = +350 \mu$        $\epsilon_B = -70 \mu$   
 Knowing that  $E = 200$  GPa, determine (a) the distance  $d$ , (b) the magnitude of the force  $P$ .

**4.141** Solve Prob. 4.140, assuming that the measured strains are  
 $\epsilon_A = +600 \mu$        $\epsilon_B = +420 \mu$

**SOLUTION**

$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm} \quad c = \frac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25 \times 10^3 \text{ mm}^2 = 2.25 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875 \times 10^6 \text{ mm}^4 = 1.51875 \times 10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \quad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at  $A$  and  $B$

$$\sigma_A = E\epsilon_A = (200 \times 10^9)(600 \times 10^{-6}) = 120 \times 10^6 \text{ Pa}$$

$$\sigma_B = E\epsilon_B = (200 \times 10^9)(420 \times 10^{-6}) = 84 \times 10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \tag{1}$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \tag{2}$$

Subtracting  $\sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875 \times 10^{-6})(36 \times 10^6)}{0.045} = -1215 \text{ N}\cdot\text{m}$$

Multiplying (2) by  $y_A$  and (1) by  $y_B$  and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25 \times 10^{-3})[(0.015)(84 \times 10^6) - (-0.030)(120 \times 10^6)]}{0.045}$$

$$= 243 \times 10^3 \text{ N}$$

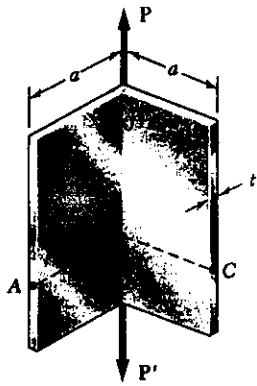
$$M = -Pd$$

$$(a) \quad \therefore d = -\frac{M}{P} = -\frac{-1215}{243 \times 10^3} = 5 \times 10^{-3} \text{ m} = 5 \text{ mm}$$

$$(b) \quad P = 243 \text{ kN}$$

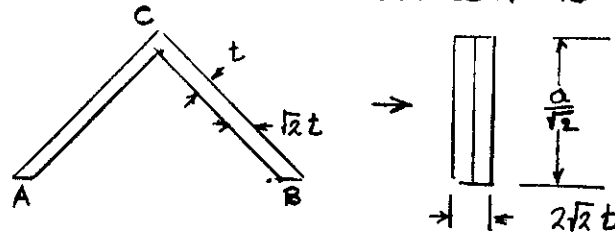
PROBLEM 4.142

4.142 The shape shown was formed by bending a thin steel plate. Assuming that the thickness  $t$  is small compared to the length  $a$  of a side of the shape, determine the stress (a) at A, (b) at B, (c) at C.



SOLUTION

Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3$$

$$= \frac{1}{12}2a^3$$

Area  $A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at$

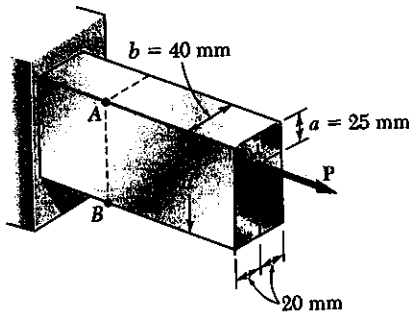
$$c = \frac{a}{2\sqrt{2}}$$

$$(a) \quad \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}2a^3} = -\frac{P}{2at}$$

$$(b) \quad \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right)}{\frac{1}{12}2a^3} = \frac{2P}{at}$$

$$(c) \quad \sigma_c = \sigma_A = -\frac{P}{2at}$$

**PROBLEM 4.143**



4.143 The eccentric axial force  $P$  acts at point  $D$ , which must be located 25 mm below the top surface of the steel bar shown. For  $P = 60$  kN, determine (a) the depth  $d$  of the bar for which the tensile stress at point  $A$  is maximum, (b) the corresponding stress at point  $A$ .

**SOLUTION**

$$A = bd \quad I = \frac{1}{12} bd^3$$

$$c = \frac{1}{2}d \quad e = \frac{1}{2}d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

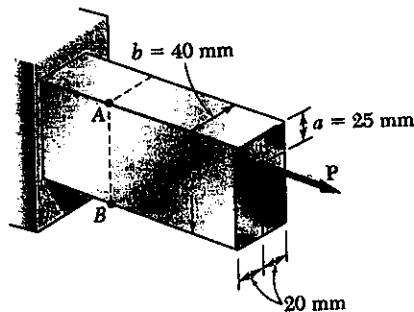
$$\sigma_A = \frac{P}{b} \left\{ \frac{1}{d} + \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3} \right\} = \frac{P}{b} \left\{ \frac{4}{d} - \frac{6a}{d^2} \right\}$$

(a) Depth  $d$  for maximum  $\sigma_A$ . Differentiate with respect to  $d$ .

$$\frac{d\sigma_A}{dd} = \frac{P}{b} \left\{ -\frac{4}{d^2} + \frac{12a}{d^3} \right\} = 0 \quad d = 3a = 75 \text{ mm}$$

(b)  $\sigma_A = \frac{60 \times 10^3}{40 \times 10^{-3}} \left\{ \frac{4}{75 \times 10^{-3}} - \frac{(6)(25 \times 10^{-3})}{(75 \times 10^{-3})^2} \right\} = 40 \times 10^6 \text{ Pa} = 40 \text{ MPa}$

**PROBLEM 4.144**



4.143 The eccentric axial force  $P$  acts at point  $D$ , which must be located 25 mm below the top surface of the steel bar shown. For  $P = 60$  kN, determine (a) the depth  $d$  of the bar for which the tensile stress at point  $A$  is maximum, (b) the corresponding stress at point  $A$ .

4.144 For the bar and loading of Prob. 4.143, determine (a) the depth  $d$  of the bar for which the compressive stress at point  $B$  is maximum, (b) the corresponding stress at point  $B$ .

**SOLUTION**

$$A = bd \quad I = \frac{1}{12} bd^3$$

$$c = \frac{1}{2}d \quad e = \frac{1}{2}d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

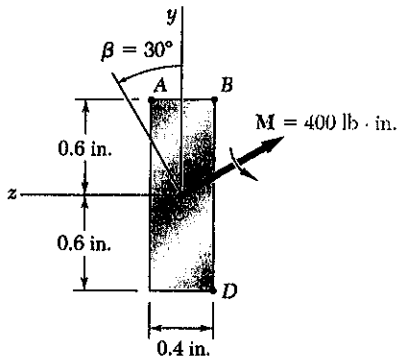
$$\sigma_B = \frac{P}{b} \left\{ \frac{1}{d} - \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3} \right\} = \frac{P}{b} \left\{ -\frac{2}{d} + \frac{6a}{d^2} \right\}$$

(a) Depth  $d$  for maximum  $\sigma_B$ : Differentiate with respect to  $d$

$$\frac{d\sigma_B}{dd} = \frac{P}{b} \left\{ \frac{2}{d^2} + \frac{12a}{d^3} \right\} = 0 \quad d = 6a = 150 \text{ mm}$$

(b)  $\sigma_B = \frac{60 \times 10^3}{40 \times 10^{-3}} \left\{ -\frac{2}{150 \times 10^{-3}} + \frac{(6)(25 \times 10^{-3})}{(150 \times 10^{-3})^2} \right\} = -10 \times 10^6 \text{ Pa} = -10 \text{ MPa}$

PROBLEM 4.145



4.145 through 4.147 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point  $A$ , (b) point  $B$ , (c) point  $D$ .

SOLUTION

$$I_z = \frac{1}{12} (0.4)(1.2)^3 = 57.6 \times 10^{-6} \text{ in}^4$$

$$I_y = \frac{1}{12} (1.2)(0.4)^3 = 6.40 \times 10^{-6} \text{ in}^4$$

$$y_A = y_B = -y_D = 0.6 \text{ in}$$

$$z_A = -z_B = -z_D = \left(\frac{1}{2}\right)(0.4) = 0.2 \text{ in}$$

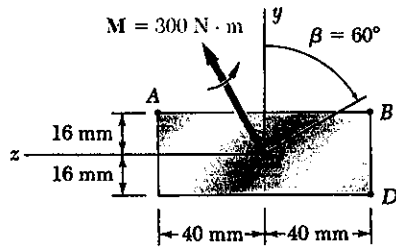
$$M_y = 400 \cos 60^\circ = 200 \text{ lb}\cdot\text{in} \quad , \quad M_z = -400 \sin 60^\circ = -346.41 \text{ lb}\cdot\text{in}$$

$$\begin{aligned} \text{(a)} \quad \sigma_A &= -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41)(0.6)}{57.6 \times 10^{-6}} + \frac{(200)(0.2)}{6.40 \times 10^{-6}} \\ &= 9.86 \times 10^3 \text{ psi} = 9.86 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \sigma_B &= -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6 \times 10^{-6}} + \frac{(200)(0.2)}{6.4 \times 10^{-6}} \\ &= -2.64 \times 10^3 \text{ psi} = -2.64 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \sigma_D &= -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6 \times 10^{-6}} + \frac{(200)(-0.2)}{6.40 \times 10^{-6}} \\ &= -9.86 \times 10^3 \text{ psi} = -9.86 \text{ ksi} \end{aligned}$$

PROBLEM 4.146



4.145 through 4.147 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point  $A$ , (b) point  $B$ , (c) point  $D$ .

SOLUTION

$$I_z = \frac{1}{12} (80)(32)^3 = 218.45 \times 10^3 \text{ mm}^4 = 218.45 \times 10^{-9} \text{ m}^4$$

$$I_y = \frac{1}{12} (32)(80)^3 = 1.36533 \times 10^6 \text{ mm}^4 = 1.36533 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 16 \text{ mm}$$

$$z_A = -z_B = -z_D = 40 \text{ mm}$$

$$M_y = 300 \cos 30^\circ = 259.81 \text{ N}\cdot\text{m}, \quad M_z = 300 \sin 30^\circ = 150 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16 \times 10^{-3})}{218.45 \times 10^{-9}} + \frac{(259.81)(40 \times 10^{-3})}{1.36533 \times 10^{-6}}$$

$$= -3.37 \times 10^6 \text{ Pa} = -3.37 \text{ MPa}$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16 \times 10^{-3})}{218.45 \times 10^{-9}} + \frac{(259.81)(-40 \times 10^{-3})}{1.36533 \times 10^{-6}}$$

$$= -18.60 \times 10^6 \text{ Pa} = -18.60 \text{ MPa}$$

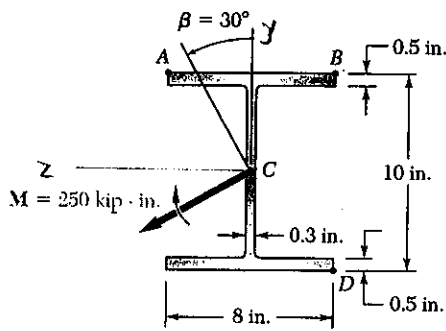
$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16 \times 10^{-3})}{218.45 \times 10^{-9}} + \frac{(259.81)(-40 \times 10^{-3})}{1.36533 \times 10^{-6}}$$

$$= 3.37 \times 10^6 \text{ Pa} = 3.37 \text{ MPa}$$



PROBLEM 4.147

4.145 through 4.147 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



SOLUTION

$$\text{Flange: } I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2$$

$$= 90.333 \text{ in}^4$$

$$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$$

$$\text{Web: } I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$$

$$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$$

$$\text{Total: } I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$$

$$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$$

$$y_A = y_B = -y_D = 5 \text{ in.} \quad ; \quad z_A = -z_B = -z_D = 4 \text{ in.}$$

$$M_z = 250 \cos 30^\circ = 216.51 \text{ kip-in}$$

$$M_y = -250 \sin 30^\circ = -125 \text{ kip-in}$$

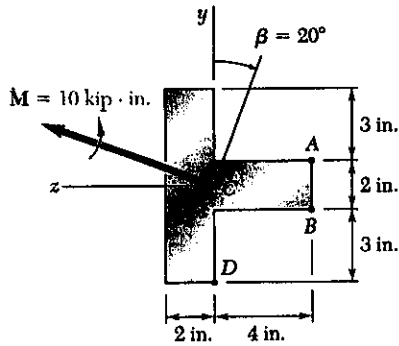
$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(4)}{42.687} = -17.16 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(-4)}{42.687} = 6.27 \text{ ksi} \quad \blacktriangleright$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-5)}{198.89} + \frac{(-125)(-4)}{42.687} = 17.16 \text{ ksi} \quad \blacktriangleright$$

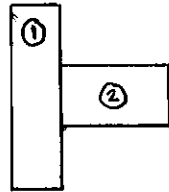
PROBLEM 4.148

4.148 through 4.150 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.



SOLUTION

Locate centroid



	$A_j, \text{in}^2$	$\bar{z}_j, \text{in}$	$A\bar{z}_j, \text{in}^3$
①	16	-1	-16
②	8	2	16
$\Sigma$	24		0

The centroid lies at point C

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in}, \quad y_D = -4 \text{ in}$$

$$z_A = z_B = -4 \text{ in}, \quad z_D = 0$$

$$M_z = 10 \cos 20^\circ = 9.3969 \text{ kip-in}$$

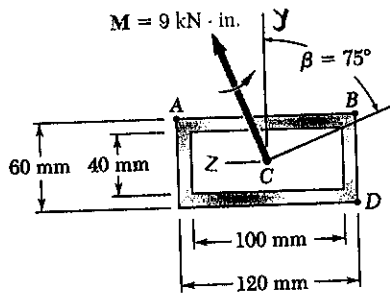
$$M_y = 10 \sin 20^\circ = 3.4202 \text{ kip-in}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(-4)}{64} = 0.321 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64} = -0.107 \text{ ksi} \quad \blacktriangleleft$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64} = 0.427 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 4.149



4.148 through 4.150 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point  $A$ , (b) point  $B$ , (c) point  $D$ .

SOLUTION

$$I_z = \frac{1}{12}(120)(60)^3 - \frac{1}{12}(100)(40)^3 = 1.62667 \times 10^6 \text{ mm}^4$$

$$= 1.62667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(60)(120)^3 - \frac{1}{12}(40)(100)^3 = 5.3067 \times 10^6 \text{ mm}^4$$

$$= 5.3067 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 30 \text{ mm}$$

$$z_A = -z_B = -z_D = 60 \text{ mm}$$

$$M_z = (9 \times 10^3) \sin 15^\circ = 2.3294 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (9 \times 10^3) \cos 15^\circ = 8.6933 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(2.3294 \times 10^3)(30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$

$$= 55.3 \times 10^6 \text{ Pa} = 55.3 \text{ MPa}$$

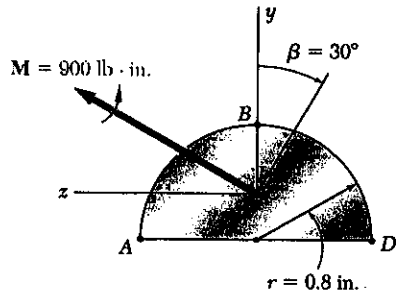
$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(2.3294 \times 10^3)(30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(-60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$

$$= -141.2 \times 10^6 \text{ Pa} = -141.2 \text{ MPa}$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.3294 \times 10^3)(-30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(-60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$

$$= -55.3 \times 10^6 \text{ Pa} = -55.3 \text{ MPa}$$

PROBLEM 4.150



4.148 through 4.150 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Determine the stress at (a) point  $A$ , (b) point  $B$ , (c) point  $D$ .

SOLUTION

$$I_z = \frac{\pi}{8} r^4 - \left( \frac{\pi}{2} r^2 \right) \left( \frac{4r}{3\pi} \right)^2 = \left( \frac{\pi}{8} - \frac{8}{9\pi} \right) r^4$$

$$= (0.109757)(0.8)^4 = 44.956 \times 10^{-3} \text{ in}^4$$

$$I_y = \frac{\pi}{8} r^4 = \frac{\pi}{8} (0.8)^4 = 160.85 \times 10^{-3} \text{ in}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(0.8)}{3\pi} = -0.33953 \text{ in.}$$

$$y_B = 0.8 - 0.33953 = 0.46047 \text{ in.}$$

$$z_A = -z_D = 0.8 \text{ in.}, \quad z_B = 0$$

$$M_y = 900 \sin 30^\circ = 450 \text{ lb}\cdot\text{in}$$

$$M_z = 900 \cos 30^\circ = 779.42 \text{ lb}\cdot\text{in.}$$

$$(a) \quad \sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(779.42)(-0.33953)}{44.956 \times 10^{-3}} + \frac{(450)(0.8)}{160.85 \times 10^{-3}} = 8.12 \times 10^3 \text{ psi}$$

$$= 8.12 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad \sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(779.42)(0.46047)}{44.956 \times 10^{-3}} + \frac{(450)(0)}{160.85 \times 10^{-3}} = -7.98 \times 10^3 \text{ psi}$$

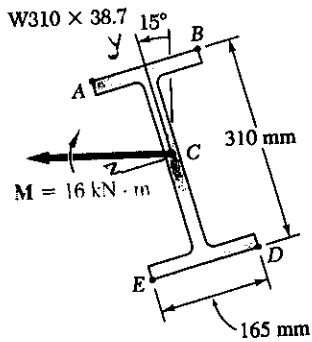
$$= -7.98 \text{ ksi} \quad \blacktriangleleft$$

$$(c) \quad \sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(779.42)(-0.33953)}{44.956 \times 10^{-3}} + \frac{(450)(-0.8)}{160.85 \times 10^{-3}} = 3.65 \times 10^3 \text{ psi}$$

$$= 3.65 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 4.151

4.151 through 4.153 The couple  $M$  acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.



SOLUTION

For W 310 x 38.7 rolled steel shape

$$I_z = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$$

$$I_y = 7.27 \times 10^6 \text{ mm}^4 = 7.27 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = \left(\frac{1}{2}\right)(310) = 155 \text{ mm}$$

$$z_A = z_E = -z_B = -z_D = \left(\frac{1}{2}\right)(165) = 82.5 \text{ mm}$$

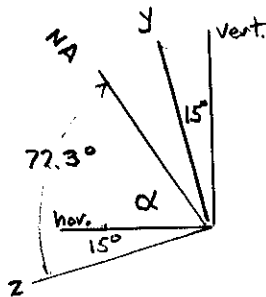
$$M_z = (16 \times 10^3) \cos 15^\circ = 15.455 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (16 \times 10^3) \sin 15^\circ = 4.1411 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15^\circ = 3.1365$$

$$\phi = 72.3^\circ$$

$$\alpha = 72.3 - 15 = 57.3^\circ$$



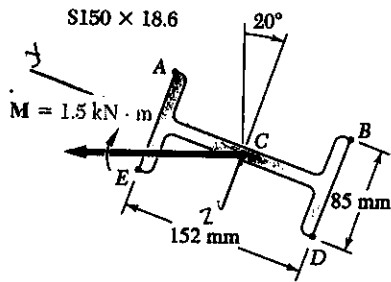
(b) Maximum tensile stress occurs at point E

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6 \text{ Pa} = 75.1 \text{ MPa}$$

PROBLEM 4.152

4.151 through 4.153 The couple  $M$  acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.



SOLUTION

For S 150 × 18.6 rolled steel shape

$$I_z = 9.11 \times 10^6 \text{ mm}^4 = 9.11 \times 10^{-6} \text{ m}^4$$

$$I_y = 0.782 \times 10^6 \text{ mm}^4 = 0.782 \times 10^{-6} \text{ m}^4$$

$$z_E = -z_A = -z_B = z_D = \left(\frac{1}{2}\right)(85) = 42.5 \text{ mm}$$

$$y_A = y_B = -y_D = -y_E = \left(\frac{1}{2}\right)(152) = 76 \text{ mm}$$

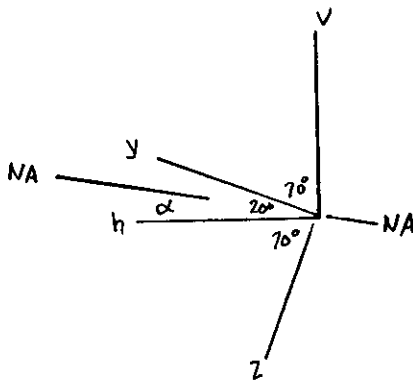
$$M_z = (1.5 \times 10^3) \sin 20^\circ = 0.51303 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (1.5 \times 10^3) \cos 20^\circ = 1.4095 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{9.11 \times 10^{-6}}{0.782 \times 10^{-6}} \tan (90^\circ - 20^\circ) = 32.007$$

$$\phi = 88.21$$

$$\alpha = 88.21 - 70^\circ = 18.21^\circ$$



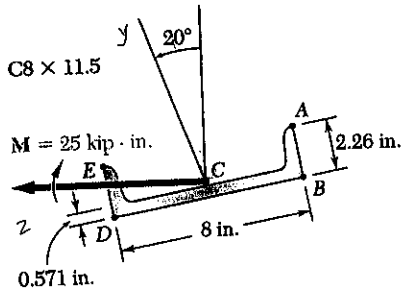
(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(0.51303 \times 10^3)(-76 \times 10^{-3})}{9.11 \times 10^{-6}} + \frac{(1.4095 \times 10^3)(42.5 \times 10^{-3})}{0.782 \times 10^{-6}}$$

$$= 80.9 \times 10^6 \text{ Pa} = 80.9 \text{ MPa}$$

PROBLEM 4.153

4.151 through 4.153 The couple  $M$  acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.



SOLUTION

For C8 x 11.5 rolled steel shape

$$I_z = 1.32 \text{ in}^4, \quad I_y = 32.6 \text{ in}^4$$

$$z_E = z_D = 0.4 \text{ in}, \quad z_B = z_A = -0.4 \text{ in}$$

$$y_D = y_B = -0.571 \text{ in.}$$

$$y_E = y_A = 2.26 - 0.571 = 1.689 \text{ in.}$$

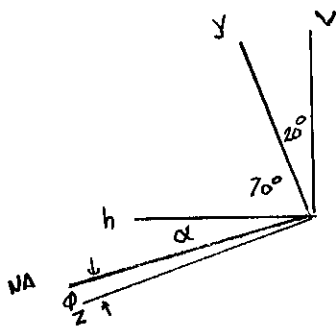
$$M_y = 25 \sin 20^\circ = 8.5505 \text{ kip}\cdot\text{in}$$

$$M_z = 25 \cos 20^\circ = 23.492 \text{ kip}\cdot\text{in.}$$

$$(a) \tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{1.32}{32.6} \tan 20^\circ = 0.014737$$

$$\phi = 0.844^\circ$$

$$\alpha = 20 - 0.844 = 19.16^\circ$$

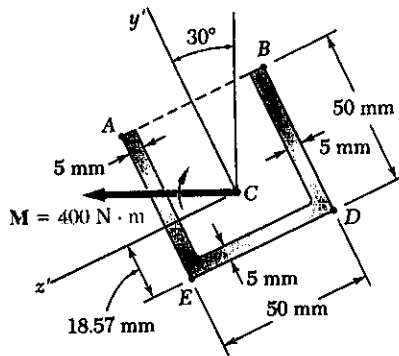


(b) Maximum tensile stress occurs at point D

$$\begin{aligned} \sigma_D &= -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(23.492)(-0.571)}{1.32} + \frac{(8.5505)(4)}{32.6} \\ &= 10.162 + 1.049 = 11.21 \text{ ksi} \end{aligned}$$

**PROBLEM 4.154**

4.154 through 4.156 The couple  $M$  acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.



$$I_y = 281 \times 10^3 \text{ mm}^4$$

$$I_z = 176.9 \times 10^3 \text{ mm}^4$$

**SOLUTION**

$$I_{z'} = 176.9 \times 10^3 \text{ mm}^4 = 176.9 \times 10^{-9} \text{ m}^4$$

$$I_{y'} = 281 \times 10^3 \text{ mm}^4 = 281 \times 10^{-9} \text{ m}^4$$

$$y_E' = -18.57 \text{ mm}, \quad z_E = 25 \text{ mm}$$

$$M_{z'} = 400 \cos 30^\circ = 346.41 \text{ N}\cdot\text{m}$$

$$M_{y'} = 400 \sin 30^\circ = 200 \text{ N}\cdot\text{m}$$

$$(a) \quad \tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30^\circ = 0.36346$$

$$\phi = 19.97^\circ$$

$$\alpha = 30^\circ - 19.97^\circ = 10.03^\circ$$

(b) Maximum tensile stress occurs at point E

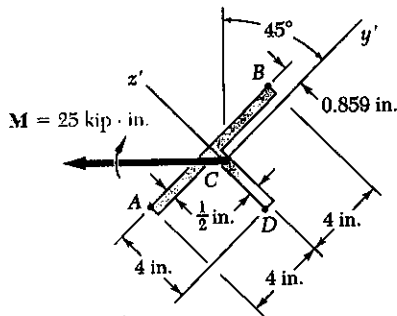
$$\sigma_D = -\frac{M_{z'} y_E'}{I_{z'}} + \frac{M_{y'} z_E}{I_{y'}} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6 \text{ Pa}$$

$$= 54.2 \text{ MPa}$$



PROBLEM 4.155



$I_y = 6.74 \text{ in}^4$   
 $I_z = 21.4 \text{ in}^4$

4.154 through 4.156 The couple  $M$  acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.

SOLUTION

$I_z = 21.4 \text{ in}^4, \quad I_{y'} = 6.74 \text{ in}^4$

$z'_A = z'_B = 0.859 \text{ in}$

$z_0 = -4 + 0.859 \text{ in} = -3.141 \text{ in}$

$y_A = -4 \text{ in}, \quad y_B = 4 \text{ in}, \quad y_0 = -0.25 \text{ in}$

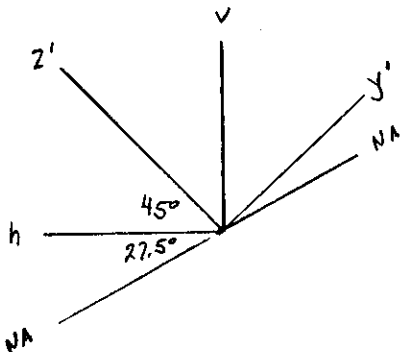
$M_{y'} = -25 \sin 45^\circ = -17.678 \text{ kip}\cdot\text{in}$

$M_{z'} = 25 \cos 45^\circ = 17.678 \text{ kip}\cdot\text{in}$

(a)  $\tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{21.4}{6.74} \tan (-45^\circ) = 3.1751$

$\phi = -72.5^\circ$

$\alpha = 72.5^\circ - 45^\circ = 27.5^\circ$

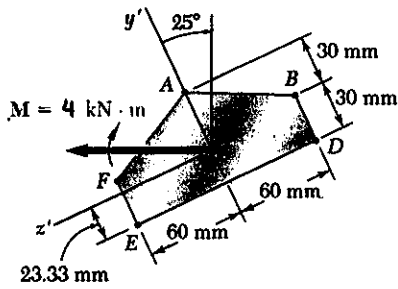


(b) Maximum tensile stress occurs at point D.

$\sigma_D = -\frac{M_{z'} y_D}{I_z} + \frac{M_{y'} z_D}{I_{y'}} = -\frac{(-17.678)(-0.25)}{21.4} + \frac{(-17.678)(-3.141)}{6.74}$

$= 0.2065 + 8.238 = 8.44 \text{ ksi}$

PROBLEM 4.156



4.154 through 4.156 The couple  $M$  acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.

SOLUTION

$$I_z = \frac{1}{36} (120)(30)^3 + \left(\frac{1}{2}\right)(120)(30)(40 - 23.33)^2 + \frac{1}{12} (120)(30)^3 + (120)(30)(23.33 - 15)^2 = 1.11 \times 10^6 \text{ mm}^4 = 1.11 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \left\{ \frac{1}{12} (30)(60)^3 + \frac{1}{3} (30)(60)^3 \right\} = 5.40 \times 10^6 \text{ mm}^4 = 5.40 \times 10^{-6} \text{ m}^4$$

$$y_E = -23.33 \text{ mm}$$

$$z_E = 60 \text{ mm}$$

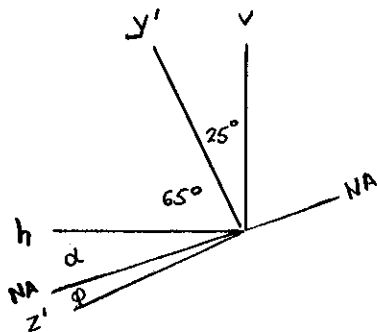
$$M_{z'} = (4 \times 10^3) \cos 25^\circ = 3.6252 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_{y'} = (4 \times 10^3) \sin 25^\circ = 1.6905 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{1.11 \times 10^{-6}}{5.40 \times 10^{-6}} \tan 25^\circ = 0.095822$$

$$\phi = 5.475^\circ$$

$$\alpha = 25^\circ - 5.475^\circ = 19.52^\circ$$



(b) Maximum tensile stress occurs at point E

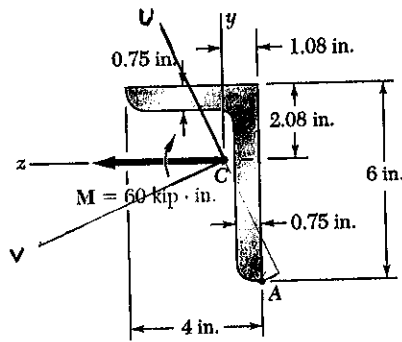
$$\sigma_E = -\frac{M_{z'} y_E}{I_z} + \frac{M_{y'} z_E}{I_y} = -\frac{(3.6252 \times 10^3)(-23.33 \times 10^{-3})}{1.11 \times 10^{-6}} + \frac{(1.6905 \times 10^3)(60 \times 10^{-3})}{5.40 \times 10^{-6}}$$

$$= 76.195 \times 10^6 + 18.783 \times 10^6 = 95.0 \times 10^6 \text{ Pa}$$

$$= 95.0 \text{ MPa}$$

PROBLEM 4.157

\*4.157 and 4.158 The couple  $M$  acts in a vertical plane and is applied to a beam of the cross section shown. Determine the stress at point  $A$ .



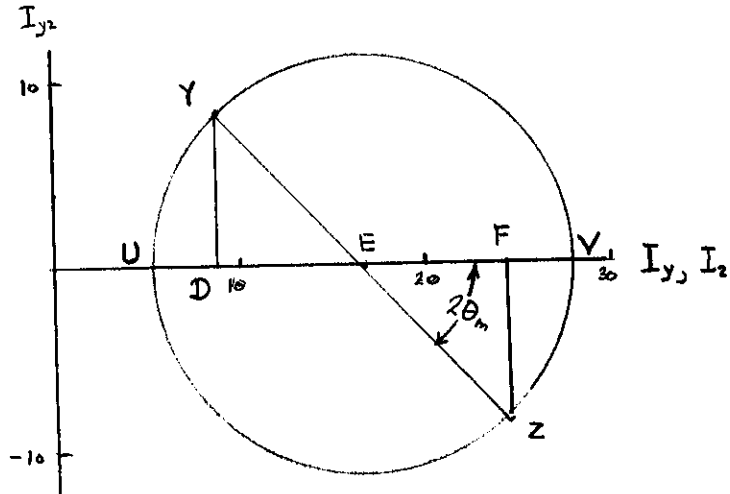
$I_y = 8.7 \text{ in}^4$   
 $I_z = 24.5 \text{ in}^4$   
 $I_{yz} = +8.3 \text{ in}^4$

$Y (8.7, 8.3) \text{ in}^4$   
 $Z (24.5, -8.3) \text{ in}^4$   
 $E (16.6, 0) \text{ in}^4$

$EF = 7.9 \text{ in}^4$   
 $FZ = 8.3 \text{ in}^4$

SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia



$R = \sqrt{7.9^2 + 8.3^2} = 11.46 \text{ in}^4$       $\tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$

$\theta_m = 23.2^\circ$       $I_u = 16.6 - 11.46 = 5.14 \text{ in}^4$ ,      $I_v = 16.6 + 11.46 = 28.06 \text{ in}^4$

$M_u = M \sin \theta_m = (60) \sin 23.2^\circ = 23.64 \text{ kip}\cdot\text{in}$

$M_v = M \cos \theta_m = (60) \cos 23.2^\circ = 55.15 \text{ kip}\cdot\text{in}$

$U_A = y_A \cos \theta_m + z_A \sin \theta_m = -3.92 \cos 23.2^\circ + 1.08 \sin 23.2^\circ = -4.03 \text{ in}$

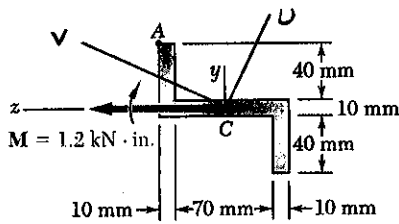
$V_A = z_A \cos \theta_m - y_A \sin \theta_m = -1.08 \cos 23.2^\circ + 3.92 \sin 23.2^\circ = 0.552 \text{ in}$

$\sigma_A = -\frac{M_v U_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$

$= 10.46 \text{ ksi}$

**PROBLEM 4.158**

\*4.157 and 4.158 The couple  $M$  acts in a vertical plane and is applied to a beam of the cross section shown. Determine the stress at point  $A$ .



$$I_y = 1.894 \times 10^6 \text{ mm}^4$$

$$I_z = 0.614 \times 10^6 \text{ mm}^4$$

$$I_{yz} = +0.800 \times 10^6 \text{ mm}^4$$

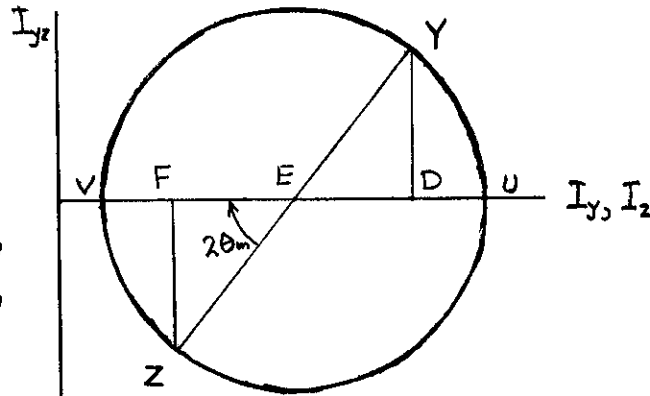
$$Y (1.894, 0.800) \times 10^6 \text{ mm}^4$$

$$Z (0.614, 0.800) \times 10^6 \text{ mm}^4$$

$$E (1.254, 0) \times 10^6 \text{ mm}^4$$

**SOLUTION**

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$$R = \sqrt{EF^2 + FZ^2} = \sqrt{0.640^2 + 0.800^2} \times 10^6 = 1.0245 \times 10^6 \text{ mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6 \text{ mm}^4 = 0.2295 \times 10^6 \text{ mm}^4 = 0.2295 \times 10^{-6} \text{ m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6 \text{ mm}^4 = 2.2785 \times 10^6 \text{ mm}^4 = 2.2785 \times 10^{-6} \text{ m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \quad \theta_m = 25.67^\circ$$

$$M_v = M \cos \theta_m = (1.2 \times 10^3) \cos 25.67^\circ = 1.0816 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_u = -M \sin \theta_m = -(1.2 \times 10^3) \sin 25.67^\circ = -0.5198 \times 10^3 \text{ N}\cdot\text{m}$$

$$U_A = y_A \cos \theta_m - z_A \sin \theta_m = 45 \cos 25.67^\circ - 45 \sin 25.67^\circ = 21.07 \text{ mm}$$

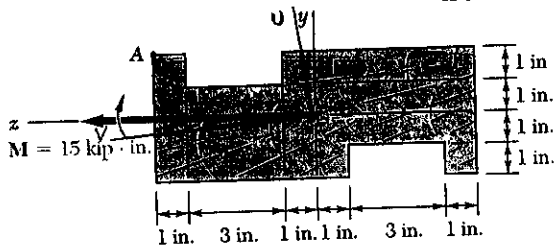
$$V_A = z_A \cos \theta_m + y_A \sin \theta_m = 45 \cos 25.67^\circ + 45 \sin 25.67^\circ = 60.05 \text{ mm}$$

$$\sigma_A = -\frac{M_v U_A}{I_v} + \frac{M_u V_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6 \text{ Pa} = 113.0 \text{ MPa}$$

PROBLEM 4.159

\*4.159 A 4 × 10-in. timber has been trimmed to form a beam of the cross section shown. Knowing that the couple M acts in a vertical plane, determine the stress at point A.



$$I_y = 291 \text{ in}^4$$

$$I_z = 39.3 \text{ in}^4$$

$$I_{yz} = -22.5 \text{ in}^4$$

$$Y (291, -22.5) \text{ in}^4$$

$$Z (39.3, 22.5) \text{ in}^4$$

$$E (165.15, 0) \text{ in}^4$$

$$\tan 2\theta_m = \frac{FZ}{EF} = \frac{22.5}{125.85}$$

$$= 0.17878$$

$$\theta_m = 5.07^\circ$$

$$R = \sqrt{EF^2 + FZ^2} = \sqrt{125.85^2 + 22.5^2} = 127.85 \text{ in}^4$$

$$I_v = 165.15 - 127.85 = 37.30 \text{ in}^4$$

$$I_u = 165.15 + 127.85 = 293.0 \text{ in}^4$$

$$u_A = y_A \cos \theta_m + z_A \sin \theta_m = 2 \cos 5.07^\circ + 5 \sin 5.07^\circ = 2.434 \text{ in}$$

$$v_A = z_A \cos \theta_m - y_A \sin \theta_m = 5 \cos 5.07^\circ - 2 \sin 5.07^\circ = 4.804 \text{ in}$$

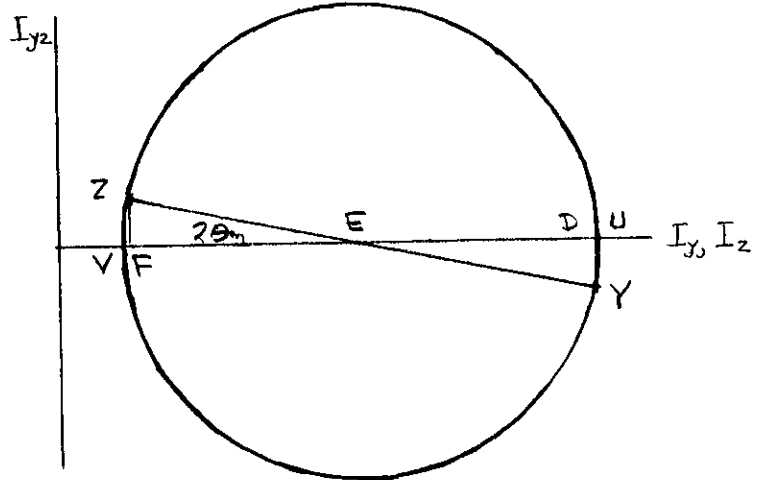
$$M_v = 15 \cos 5.07^\circ = 14.94 \text{ kip}\cdot\text{in}$$

$$M_u = 15 \sin 5.07^\circ = 1.326 \text{ kip}\cdot\text{in}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(14.94)(2.434)}{37.30} + \frac{(1.326)(4.804)}{293.0} = -0.953 \text{ ksi} \leftarrow$$

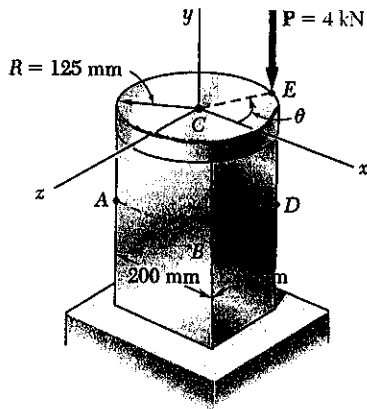
SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.

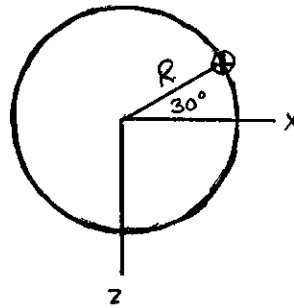


**PROBLEM 4.160**

4.160 A rigid plate of 125-mm diameter is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force  $P$  is applied at  $E$  with  $\theta = 30^\circ$ , determine (a) the stress at point  $A$ , (b) the stress at point  $B$ , (c) the point where the neutral axis intersects line  $ABD$ .



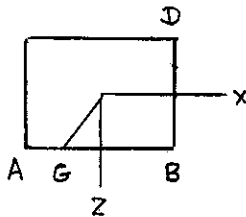
**SOLUTION**



$$P = 4 \times 10^3 \text{ N (compression)}$$

$$M_x = -PR \sin 30^\circ = -(4 \times 10^3)(125 \times 10^{-3}) \sin 30^\circ = -250 \text{ N}\cdot\text{m}$$

$$M_z = -PR \cos 30^\circ = -(4 \times 10^3)(125 \times 10^{-3}) \cos 30^\circ = -433 \text{ N}\cdot\text{m}$$



$$I_x = \frac{1}{12} (200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12} (150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm}$$

$$z_A = z_B = 75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

$$(a) \sigma_A = -\frac{P}{A} + \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(-100 \times 10^{-3})}{100 \times 10^{-6}} = 633 \times 10^3 \text{ Pa} = 633 \text{ kPa}$$

$$(b) \sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4 \times 10^3}{30 \times 10^{-3}} - \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} + \frac{(-433)(100 \times 10^{-3})}{100 \times 10^{-6}} = -233 \times 10^3 \text{ Pa} = -233 \text{ kPa}$$

(c) Let  $G$  be the point on  $AB$  where neutral axis intersects.

$$\sigma_G = 0 \quad z_G = 75 \text{ mm} \quad x_G = ?$$

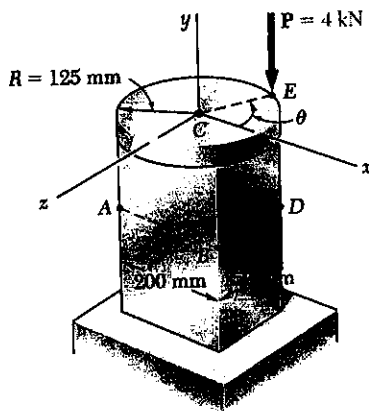
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z} \left\{ \frac{P}{A} + \frac{M_x z_G}{I_x} \right\} = \frac{100 \times 10^{-6}}{-433} \left\{ \frac{4 \times 10^3}{30 \times 10^{-3}} + \frac{(-250)(75 \times 10^{-3})}{56.25 \times 10^{-6}} \right\}$$

$$= -46.2 \times 10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point  $G$  lies 46.2 mm from point  $A$

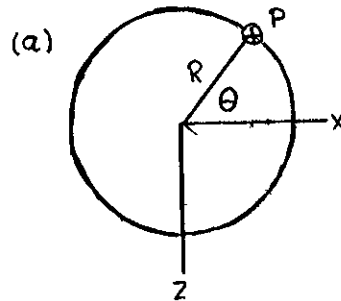
**PROBLEM 4.161**



**4.160** A rigid plate of 125-mm diameter is attached to a solid 150 x 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force  $P$  is applied at  $E$  with  $\theta = 30^\circ$ , determine (a) the stress at point  $A$ , (b) the stress at point  $B$ , (c) the point where the neutral axis intersects line  $ABD$ .

**4.161** In Prob. 4.160, determine (a) the value of  $\theta$  for which the stress at  $D$  reaches its largest value, (b) the corresponding values of the stress at  $A$ ,  $B$ ,  $C$ , and  $D$ .

**SOLUTION**

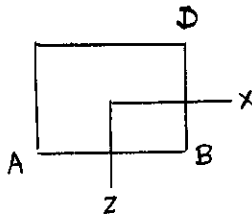


$$P = 4 \times 10^3 \text{ N}$$

$$PR = (4 \times 10^3)(125 \times 10^{-3}) = 500 \text{ N}\cdot\text{m}$$

$$M_x = -PR \sin \theta = -500 \sin \theta$$

$$M_z = -PR \cos \theta = -500 \cos \theta$$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25 \times 10^6 \text{ mm}^4 = 56.25 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100 \times 10^6 \text{ mm}^4 = 100 \times 10^{-6} \text{ m}^4$$

$$x_D = 100 \text{ mm}$$

$$z_D = -75 \text{ mm}$$

$$A = (200)(150) = 30 \times 10^3 \text{ mm}^2 = 30 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P \left\{ \frac{1}{A} - \frac{Rz \sin \theta}{I_x} + \frac{Rx \cos \theta}{I_z} \right\}$$

For  $\sigma$  to be a maximum  $\frac{d\sigma}{d\theta} = 0$  with  $z = z_D$ ,  $x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P \left\{ 0 + \frac{Rz_D \cos \theta}{I_x} + \frac{Rx_D \sin \theta}{I_z} \right\} = 0$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100 \times 10^{-6})(-75 \times 10^{-3})}{(56.25 \times 10^{-6})(100 \times 10^{-3})} = \frac{4}{3}$$

$$\sin \theta = 0.8, \quad \cos \theta = 0.6, \quad \theta = 53.1^\circ$$

$$(b) \quad \sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4 \times 10^3}{30 \times 10^3} + \frac{(500)(0.8)(15 \times 10^{-3})}{56.25 \times 10^{-6}} - \frac{(500)(0.6)(100 \times 10^{-3})}{100 \times 10^{-6}}$$

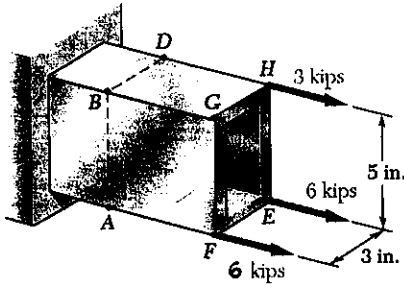
$$= (-0.13333 + 0.53333 + 0.300) \times 10^6 \text{ Pa} = 0.700 \times 10^6 \text{ Pa} = 700 \text{ kPa}$$

$$\sigma_B = (-0.13333 + 0.53333 - 0.300) \times 10^6 \text{ Pa} = 0.100 \times 10^6 \text{ Pa} = 100 \text{ kPa}$$

$$\sigma_C = (-0.13333 + 0 + 0) \times 10^6 \text{ Pa} = -133.3 \text{ kPa}$$

$$\sigma_D = (-0.13333 - 0.53333 - 0.300) \times 10^6 \text{ Pa} = -0.967 \times 10^6 \text{ Pa} = -967 \text{ kPa}$$

PROBLEM 4.162



4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.

SOLUTION

Add y- and z- axes as shown.

$$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$$

$$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$$

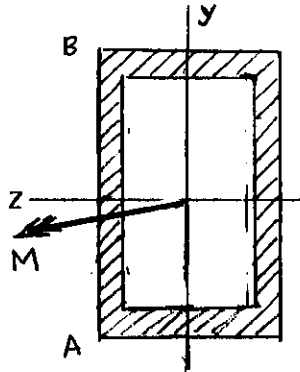
$$A = (3)(5) - (2)(4) = 7.0 \text{ in}^2$$

Resultant force and bending couples

$$P = 3 + 6 + 6 = 15 \text{ kips}$$

$$M_z = -(2.5)(3) + (2.5)(6) + (2.5)(6) = 22.5 \text{ kip-in.}$$

$$M_y = -(1.5)(3) - (1.5)(6) + (1.5)(6) = -4.5 \text{ kip-in.}$$



$$(a) \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{15}{7} - \frac{(22.5)(-2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = 4.09 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{15}{7} - \frac{(22.5)(2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = -1.376 \text{ ksi} \quad \blacktriangleleft$$

(b) Let point H be the point where the neutral axis intersects AB.

$$z_H = 1.5, \quad y_H = ?, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

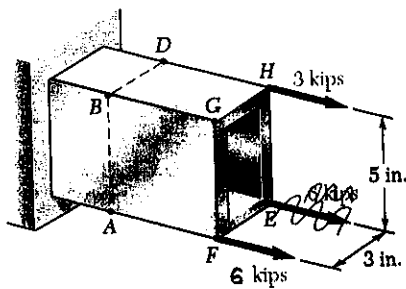
$$y_H = \frac{I_z}{M_z} \left( \frac{P}{A} + \frac{M_y z_H}{I_y} \right) = \frac{20.583}{22.5} \left\{ \frac{15}{7} + \frac{(-4.5)(1.5)}{8.5833} \right\} = 1.241 \text{ in.}$$

$$2.5 + 1.241 = 3.741 \text{ in.}$$

Answer: 3.741 in. above point A. \blacktriangleleft



**PROBLEM 4.163**



4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine (a) the stress at points A and B, (b) the point where the neutral axis intersects line ABD.

4.163 Solve Prob. 4.162, assuming that the 6-kip force at point E is removed.

**SOLUTION**

Add y- and z-axes as shown.

$$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$$

$$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$$

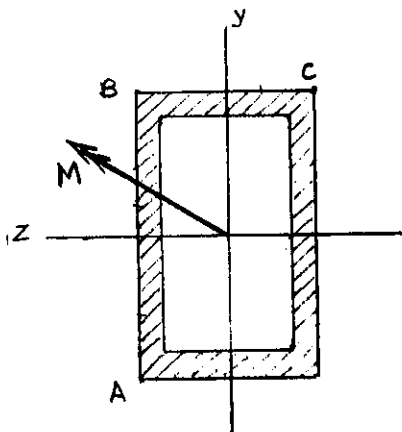
$$A = (3)(5) - (2)(4) = 7 \text{ in}^2$$

Resultant force and couples

$$P = 3 + 6 = 9 \text{ kips}$$

$$M_z = -(2.5)(3) + (2.5)(6) = 7.5 \text{ kip}\cdot\text{in.}$$

$$M_y = -(1.5)(3) + (1.5)(6) = 4.5 \text{ kip}\cdot\text{in.}$$



$$(a) \quad \sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{9}{7} - \frac{(7.5)(-2.5)}{20.583} + \frac{(4.5)(1.5)}{8.5833} = 2.98 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{9}{7} - \frac{(7.5)(2.5)}{20.583} + \frac{(4.5)(1.5)}{8.5833} = 1.161 \text{ ksi} \quad \blacktriangleleft$$

(b) Let point K be the point where the neutral axis intersects BC.

$$y_K = 2.5 \text{ in.}, \quad z_K = ?, \quad \sigma_K = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_K}{I_z} + \frac{M_y z_K}{I_y}$$

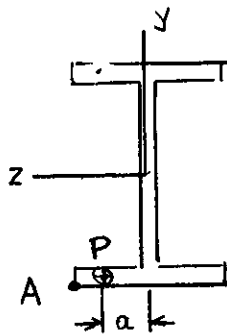
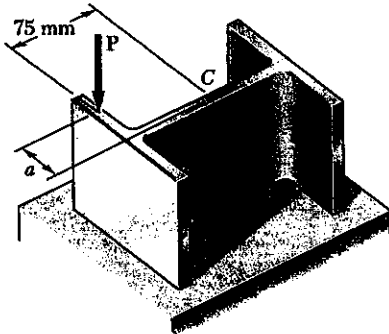
$$z_K = \frac{I_y}{M_y} \left( \frac{M_z y_K}{I_z} - \frac{P}{A} \right) = \frac{8.5833}{4.5} \left\{ \frac{(7.5)(2.5)}{20.583} - \frac{9}{7} \right\} = -0.715 \text{ in.}$$

$$1.5 + 0.715 = 2.215 \text{ in}$$

Answer: 2.215 in. to the right of point B.  $\blacktriangleleft$

PROBLEM 4.164

4.164 An axial load  $P$  of magnitude 50 kN is applied as shown to a short section of a W 150 × 24 rolled-steel member. Determine the largest distance  $a$  for which the maximum compressive stress does not exceed 90 MPa.



SOLUTION

Add  $y$ - and  $z$ - axes.

For W 150 × 24 rolled-steel section

$$A = 3060 \text{ mm}^2 = 3060 \times 10^{-6} \text{ m}^2$$

$$I_z = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$$

$$I_y = 1.83 \times 10^6 \text{ mm}^4 = 1.83 \times 10^{-6} \text{ m}^4$$

[www.konkur.in](http://www.konkur.in)  $d = 150 \text{ mm}$ ,  $b_f = 102 \text{ mm}$

$$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}.$$

$$P = 50 \times 10^3 \text{ N}$$

$$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = -Pa$$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$\sigma_A = -90 \times 10^6 \text{ Pa}$$

$$M_y = \frac{I_y}{z_A} \left\{ \frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A \right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}} \left\{ \frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6) \right\}$$

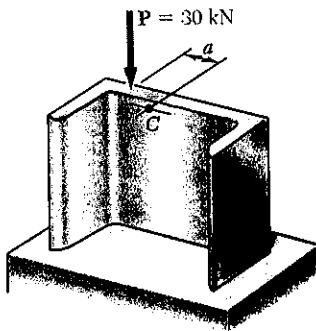
$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}} \left\{ +22.388 + 16.340 - 90 \right\} \times 10^6$$

$$= -1.8398 \times 10^3 \text{ N}\cdot\text{m}$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3} \text{ m} = 36.8 \text{ mm}$$

PROBLEM 4.165

4.165 An axial load  $P$  of magnitude 30 kN is applied as shown to a short section of a C 150 × 12.2 rolled-steel channel. Determine the largest distance  $a$  for which the maximum compressive stress is 60 MPa.



SOLUTION

Add  $y$ - and  $z$ - axes as shown

For C 150 × 12.2 rolled steel section

$$\begin{aligned} A &= 1540 \text{ mm}^2 = 1540 \times 10^{-6} \text{ m}^2 \\ d &= 152 \text{ mm} \\ b_f &= 48 \text{ mm} \\ t_w &= 5.1 \text{ mm} \\ I_z &= 5.35 \times 10^6 \text{ mm}^4 = 5.35 \times 10^{-6} \text{ m}^4 \\ I_y &= 0.276 \times 10^6 \text{ mm}^4 = 0.276 \times 10^{-6} \text{ m}^4 \\ \bar{x} &= 12.7 \text{ mm} \end{aligned}$$

Line of action of force  $P$

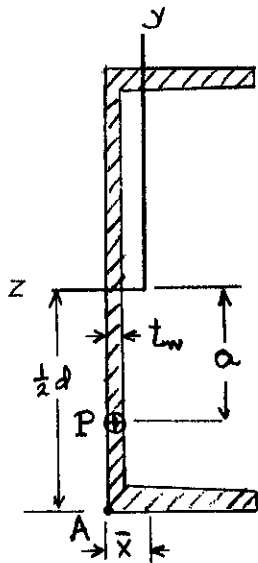
$$y_p = -a \quad z_p = \bar{x} - \frac{1}{2} t_w = 10.15 \text{ mm}$$

$$P = 30 \times 10^3 \text{ N}$$

$$M_y = -Pz_p = -(30 \times 10^3)(10.15 \times 10^{-3}) = -304.5 \text{ N}\cdot\text{m}$$

$$M_z = -Pa \quad \sigma_A = -60 \times 10^6 \text{ Pa}$$

$$y_A = -\frac{1}{2}d = -76 \text{ mm} \quad z_A = \bar{x} = 12.7 \text{ mm}$$



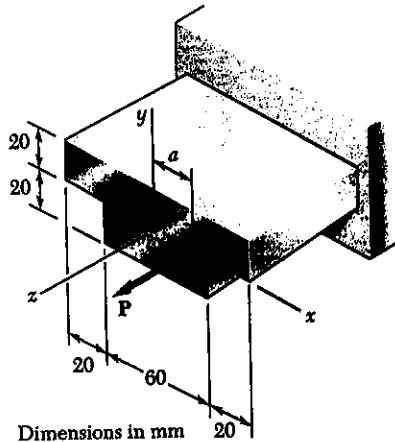
$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$\begin{aligned} M_z &= \frac{I_z}{y_A} \left\{ \frac{M_y z_A}{I_y} + \frac{P}{A} - \sigma_A \right\} \\ &= \frac{5.35 \times 10^{-6}}{-76 \times 10^{-3}} \left\{ \frac{(-304.5)(12.7 \times 10^{-3})}{0.276 \times 10^{-6}} + \frac{30 \times 10^3}{1540 \times 10^{-6}} + 60 \times 10^6 \right\} \\ &= -\frac{5.35 \times 10^{-6}}{76 \times 10^{-3}} \left\{ -14.011 - 19.481 + 60 \right\} \times 10^6 = -1.866 \times 10^3 \text{ N}\cdot\text{m} \end{aligned}$$

$$a = -\frac{M_z}{P} = -\frac{(-1.866 \times 10^3)}{30 \times 10^3} = 62.2 \times 10^{-3} \text{ m} = 62.2 \text{ mm}$$

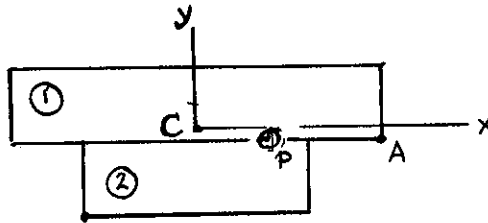
PROBLEM 4.166

4.166 A horizontal load  $P$  is applied to the beam shown. Knowing that  $a = 20$  mm and that the tensile stress in the beam is not to exceed 75 MPa, determine the largest permissible load  $P$ .



SOLUTION

Locate the centroid



	$A, \text{mm}^2$	$\bar{Y}, \text{mm}$	$A\bar{Y}, \text{mm}^3$
①	2000	10	$20 \times 10^3$
②	1200	-10	$-12 \times 10^3$
$\Sigma$	3200		$8 \times 10^3$

$$\bar{Y} = \frac{\Sigma A\bar{Y}}{\Sigma A}$$

$$= \frac{8 \times 10^3}{3200}$$

$$= 2.5 \text{ mm}$$

Move coordinate origin to the centroid.

Coordinates of load point:  $x_p = a$ ,  $y_p = -2.5$  mm

Bending couples  $M_x = y_p P$   $M_y = -aP$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4$$

$$= 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_y Y}{I_x} - \frac{M_x X}{I_y} = P \left\{ \frac{1}{A} + \frac{y_p Y}{I_x} + \frac{aX}{I_y} \right\} = KP$$

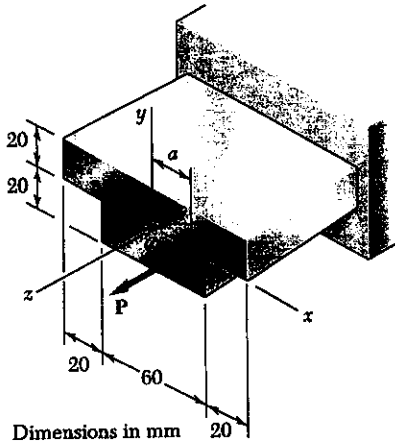
For point A  $K_A = \frac{1}{3200 \times 10^{-6}} + \frac{(-2.5 \times 10^{-3})(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} + \frac{(20 \times 10^{-3})(50 \times 10^{-3})}{2.0267 \times 10^{-6}}$

$$= 821.28 \text{ m}^{-2}$$

$$P = \frac{\sigma_A}{K_A} = \frac{75 \times 10^6}{821.28} = 91.3 \times 10^3 \text{ N} = 91.3 \text{ kN}$$

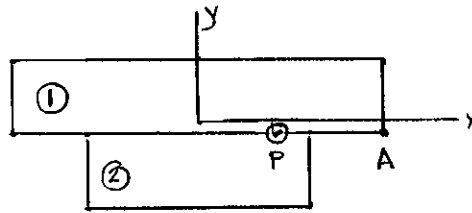
**PROBLEM 4.167**

4.167 A horizontal load  $P$  of magnitude 100 kN is applied to the beam shown. Determine the largest distance  $a$  for which the maximum tensile stress in the beam does not exceed 75 MPa.



**SOLUTION**

Locate the centroid



	$A, \text{mm}^2$	$\bar{y}_2, \text{mm}$	$A\bar{y}_2, \text{mm}^3$
①	2000	10	$20 \times 10^3$
②	1200	-10	$-12 \times 10^3$
$\Sigma$	3200		$8 \times 10^3$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{8 \times 10^3}{3200}$$

$$= 2.5 \text{ mm}$$

Move coordinate origin to the centroid

Coordinates of load point:  $x_p = a, y_p = -2.5 \text{ mm}$

Bending couples  $M_x = y_p P$        $M_y = -aP$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4$$

$$= 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

$$\sigma_A = 75 \times 10^6 \text{ Pa}, \quad P = 100 \times 10^3 \text{ N}$$

$$M_y = \frac{I_y}{x} \left\{ \frac{P}{A} + \frac{M_x y}{I_x} - \sigma \right\}$$

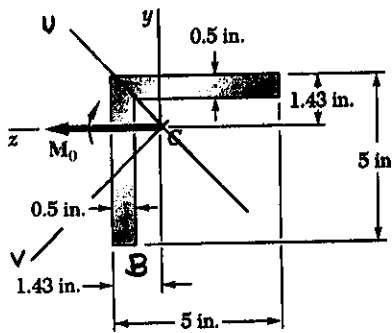
For point A  $x = 50 \text{ mm}, y = -2.5 \text{ mm}$

$$M_y = \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}} \left\{ \frac{100 \times 10^3}{3200 \times 10^{-6}} + \frac{(-2.5)(100 \times 10^3)(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} - 75 \times 10^6 \right\}$$

$$= \frac{2.0267 \times 10^{-6}}{50 \times 10^{-3}} \left\{ 31.25 + 1.537 - 75 \right\} \times 10^6 = -1.7111 \times 10^3 \text{ N}\cdot\text{m}$$

$$a = -\frac{M_y}{P} = -\frac{(-1.7111 \times 10^3)}{100 \times 10^3} = 17.11 \times 10^{-3} \text{ m} = 17.11 \text{ mm}$$

PROBLEM 4.168



4.168 A beam having the cross section shown is subjected to a couple  $M_0$  which acts in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress in the beam is not to exceed 12 ksi. Given:  $I_y = I_z = 11.3 \text{ in}^4$ ,  $A = 4.75 \text{ in}^2$ ,  $k_{\min} = 0.983 \text{ in}$ . (Hint: By reason of symmetry, the principal axes form an angle of  $45^\circ$  with the coordinate axes. Use the relations  $I_{\min} = Ak_{\min}^2$  and  $I_{\min} + I_{\max} = I_y + I_z$ .)

SOLUTION

$$M_u = M_0 \sin 45^\circ = 0.70711 M_0$$

$$M_v = M_0 \cos 45^\circ = 0.7071 M_0$$

$$I_{\min} = Ak_{\min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_B = y_B \cos 45^\circ + z_B \sin 45^\circ = -3.57 \cos 45^\circ + 0.93 \sin 45^\circ = -1.866 \text{ in}$$

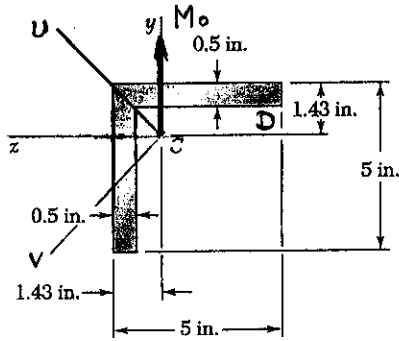
$$v_B = z_B \cos 45^\circ - y_B \sin 45^\circ = -0.93 \cos 45^\circ - (-3.57) \sin 45^\circ = 3.182 \text{ in}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -0.70711 M_0 \left[ -\frac{u_B}{I_{\min}} + \frac{v_B}{I_{\max}} \right]$$

$$= 0.70711 M_0 \left[ -\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_0$$

$$M_0 = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip}\cdot\text{in}$$

PROBLEM 4.169



4.168 A beam having the cross section shown is subjected to a couple  $M_0$  which acts in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress in the beam is not to exceed 12 ksi. Given:  $I_y = I_z = 11.3 \text{ in}^4$ ,  $A = 4.75 \text{ in}^2$ ,  $k_{\min} = 0.983 \text{ in}$ . (Hint: By reason of symmetry, the principal axes form an angle of  $45^\circ$  with the coordinate axes. Use the relations  $I_{\min} = Ak_{\min}^2$  and  $I_{\min} + I_{\max} = I_x + I_y$ .)

4.169 Solve Prob. 4.168, assuming that the couple  $M_0$  acts in a horizontal plane.

SOLUTION

$$M_U = M_0 \cos 45^\circ = 0.70711 M_0$$

$$M_V = -M_0 \sin 45^\circ = -0.70711 M_0$$

$$I_{\min} = A k_{\min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{\max} = I_y + I_z - I_{\min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$U_D = y_D \cos 45^\circ + z_D \sin 45^\circ = 0.93 \cos 45^\circ + (-3.57 \sin 45^\circ) = -1.866 \text{ in}$$

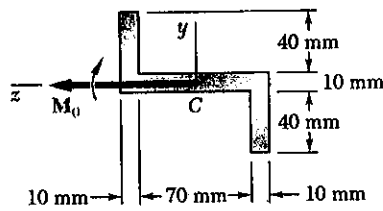
$$V_D = z_D \cos 45^\circ - y_D \sin 45^\circ = (-3.57) \cos 45^\circ - (0.93) \sin 45^\circ = 3.182 \text{ in}$$

$$\begin{aligned} \sigma_D &= -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = 0.70711 M_0 \left[ -\frac{U_D}{I_{\min}} + \frac{V_D}{I_{\max}} \right] \\ &= 0.70711 M_0 \left[ -\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124 M_0 \end{aligned}$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip}\cdot\text{in}$$

**PROBLEM 4.170**

4.170 The Z section shown is subjected to a couple  $M_0$  acting in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress is not to exceed 80 MPa. Given:  $I_{max} = 2.28 \times 10^6 \text{ mm}^4$ ,  $I_{min} = 0.23 \times 10^6 \text{ mm}^4$ , principal axes  $25.7^\circ$  and  $64.3^\circ$ .



**SOLUTION**

$$I_v = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$

$$M_v = M_0 \cos 64.3^\circ$$

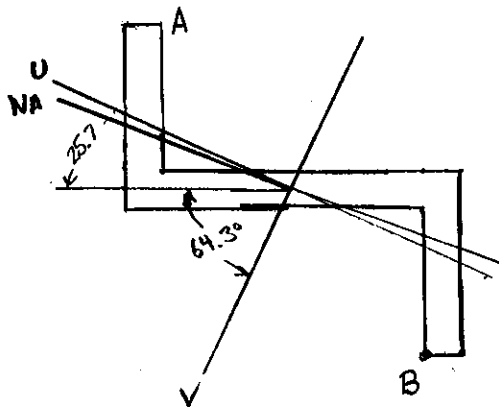
$$M_u = M_0 \sin 64.3^\circ$$

$$\theta = 64.3^\circ$$

$$\tan \phi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3^\circ = 20.597$$

$$\phi = 87.22^\circ$$



Points A and B are farthest from the neutral axis.

$$U_B = y_B \cos 64.3^\circ + z_B \sin 64.3^\circ = (-45) \cos 64.3^\circ + (-35) \sin 64.3^\circ$$

$$= -51.05 \text{ mm}$$

$$V_B = z_B \cos 64.3^\circ - y_B \sin 64.3^\circ = (-35) \cos 64.3^\circ - (-45) \sin 64.3^\circ$$

$$= +25.37 \text{ mm}$$

$$\sigma_B = -\frac{M_v U_B}{I_v} + \frac{M_u V_B}{I_u}$$

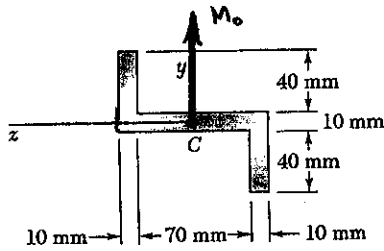
$$80 \times 10^6 = -\frac{(M_0 \cos 64.3^\circ)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3^\circ)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$= 93.81 \times 10^3 M_0$$

$$M_0 = \frac{80 \times 10^6}{109.1 \times 10^3} = 733 \text{ N}\cdot\text{m}$$



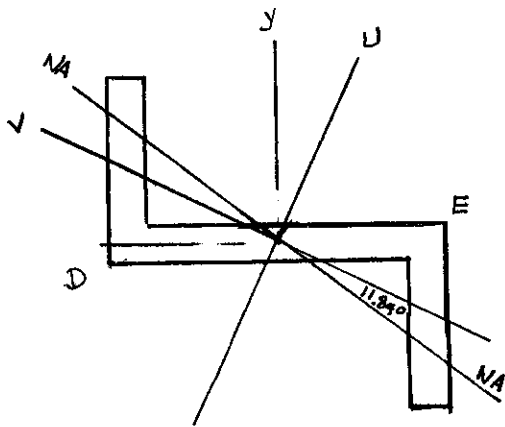
PROBLEM 4.171



4.170 The Z section shown is subjected to a couple  $M_0$  acting in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress is not to exceed 80 MPa. Given:  $I_{max} = 2.28 \times 10^6 \text{ mm}^4$ ,  $I_{min} = 0.23 \times 10^6 \text{ mm}^4$ , principal axes  $25.7^\circ$  and  $64.3^\circ$ .

4.171 Solve Prob. 4.170, assuming that the couple  $M_0$  acts in a horizontal plane

SOLUTION



$$I_V = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^6 \text{ m}^4$$

$$I_U = I_{max} = 2.23 \times 10^6 \text{ mm}^4 = 2.23 \times 10^6 \text{ m}^4$$

$$M_V = M_0 \cos 64.3^\circ$$

$$M_U = M_0 \sin 64.3^\circ$$

$$\theta = 64.3^\circ$$

$$\tan \phi = \frac{I_V}{I_U} \tan \theta$$

$$= \frac{0.23 \times 10^6}{2.28 \times 10^6} \tan 64.3^\circ = 0.20961$$

$$\phi = 11.84^\circ$$

Points D and E are farthest from the neutral axis.

$$U_D = y_D \cos 25.7^\circ - z_D \sin 25.7^\circ = (-5) \cos 25.7^\circ - 45 \sin 25.7^\circ$$

$$= -24.02 \text{ mm}$$

$$V_D = z_D \cos 25.7^\circ + y_D \sin 25.7^\circ = 45 \cos 25.7^\circ + (-5) \sin 25.7^\circ$$

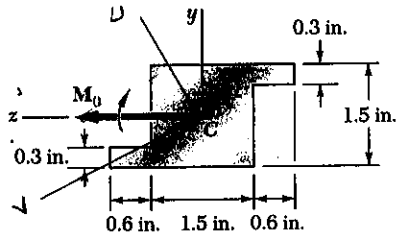
$$= 38.38 \text{ mm}$$

$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = -\frac{(M_0 \cos 64.3^\circ)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}} + \frac{(M_0 \sin 64.3^\circ)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$$80 \times 10^6 = 60.48 \times 10^3 M_0$$

$$M_0 = 1.323 \times 10^3 \text{ N}\cdot\text{m} = 1.323 \text{ kN}\cdot\text{m}$$

**PROBLEM 4.172**



4.172 An extruded aluminum member having the cross section shown is subjected to a couple  $M_0$  acting in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress is not to exceed 12 ksi. Given:  $I_{max} = 0.957 \text{ in}^4$ ,  $I_{min} = 0.427 \text{ in}^4$ , principal axes  $29.4^\circ$  and  $60.6^\circ$ .

**SOLUTION**

$$I_u = I_{max} = 0.957 \text{ in}^4$$

$$I_v = I_{min} = 0.427 \text{ in}^4$$

$$M_u = M_0 \sin 29.4^\circ, \quad M_v = M_0 \cos 29.4^\circ$$

$$\theta = 29.4^\circ$$

$$\tan \phi = \frac{I_v}{I_u} \tan \theta = \frac{0.427}{0.957} \tan 29.4^\circ$$

$$= 0.2514 \quad \phi = 14.11^\circ$$

Point A is farthest from the neutral axis.

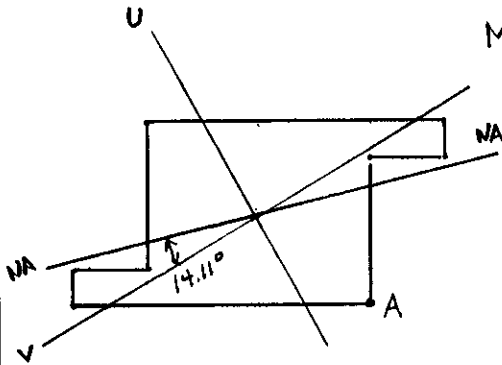
$$y_A = -0.75 \text{ in}, \quad z_A = -0.75 \text{ in}$$

$$u_A = y_A \cos 29.4^\circ + z_A \sin 29.4^\circ = -1.0216 \text{ in}$$

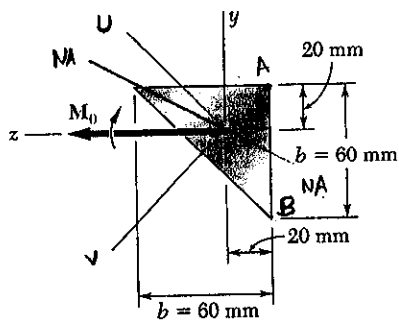
$$v_A = z_A \cos 29.4^\circ - y_A \sin 29.4^\circ = -0.2852 \text{ in}$$

$$\begin{aligned} \sigma_A &= -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(M_0 \cos 29.4^\circ)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4^\circ)(-0.2852)}{0.957} \\ &= 1.9381 M_0 \end{aligned}$$

$$M_0 = \frac{\sigma_A}{1.9381} = \frac{12}{1.9381} = 6.19 \text{ ksi}$$



PROBLEM 4.173



4.173 A beam having the cross section shown is subjected to a couple  $M_0$  acting in a vertical plane. Determine the largest permissible value of the moment  $M_0$  of the couple if the maximum stress is not to exceed 100 MPa. Given:  $I_y = I_z = b^4/36$  and  $I_{yz} = b^4/72$ .

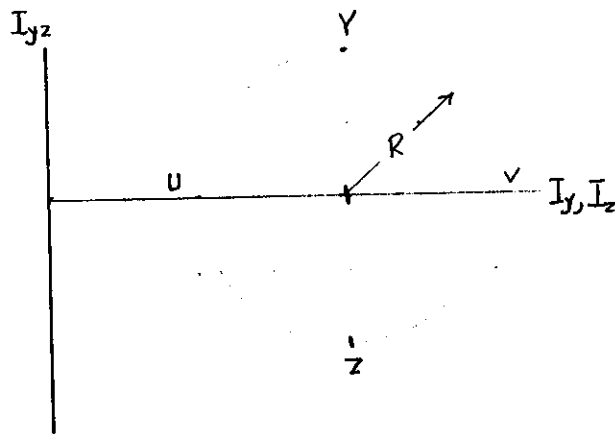
SOLUTION

$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia.



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R = 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R = 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_0 \sin 45^\circ = 0.70711 M_0, \quad M_v = M_0 \cos 45^\circ = 0.70711 M_0$$

$$\theta = 45^\circ \quad \tan \phi = \frac{I_v}{I_u} \tan \theta = \frac{0.540 \times 10^6}{0.180 \times 10^6} \tan 45^\circ = 3$$

$$\phi = 71.56^\circ$$

Point A:  $u_A = 0, \quad v_A = -20\sqrt{2} \text{ mm}$

$$\sigma_A = -\frac{M_u u_A}{I_u} + \frac{M_v v_A}{I_v} = 0 + \frac{(0.70711 M_0)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -111.11 \times 10^3 M_0$$

$$M_0 = \frac{\sigma_A}{-111.11 \times 10^3} = -\frac{100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$

Point B:  $u_B = -\frac{60}{\sqrt{2}} \text{ mm}, \quad v_B = \frac{20}{\sqrt{2}} \text{ mm}$

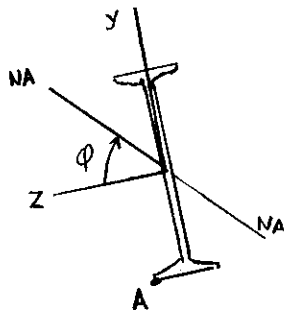
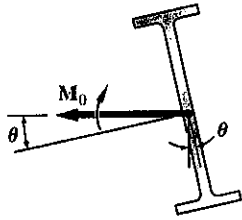
$$\sigma_B = -\frac{M_u u_B}{I_u} + \frac{M_v v_B}{I_v} = -\frac{(0.70711 M_0)(-\frac{60}{\sqrt{2}} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.70711 M_0)(\frac{20}{\sqrt{2}} \times 10^{-3})}{0.180 \times 10^{-6}}$$

$$= 111.11 \times 10^3 M_0$$

$$M_0 = \frac{\sigma_B}{111.11 \times 10^3} = \frac{100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$

PROBLEM 4.174

4.174 A couple  $M_0$  acting in a vertical plane is applied to a W 12 × 16 rolled-steel beam, whose web forms an angle  $\theta$  with the vertical. Denoting by  $\sigma_0$  the maximum stress in the beam when  $\theta = 0$ , determine the angle of inclination  $\theta$  of the beam for which the maximum stress is  $2\sigma_0$ .



SOLUTION

For W 12 × 16 rolled steel section

$$I_z = 103 \text{ in}^4 \quad I_y = 2.82 \text{ in}^4$$

$$d = 11.99 \text{ in} \quad b_f = 3.990 \text{ in.}$$

$$y_A = -\frac{d}{2} \quad z_A = \frac{b_f}{2}$$

$$\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{103}{2.82} \tan \theta = 36.52 \tan \theta$$

Point A is farthest from the neutral axis.

$$M_y = M_0 \sin \theta \quad M_z = M_0 \cos \theta$$

$$\begin{aligned} \sigma_A &= -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z} \cos \theta + \frac{M_0 b_f}{2I_y} \sin \theta \\ &= \frac{M_0 d}{2I_z} \left( 1 + \frac{I_z b_f}{I_y d} \tan \theta \right) \end{aligned}$$

For  $\theta = 0$

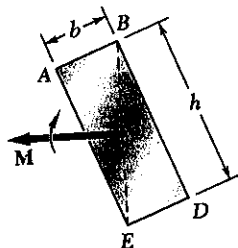
$$\sigma_0 = \frac{M_0 d}{2I_z}$$

$$\sigma_A = \sigma_0 \left( 1 + \frac{I_z b_f}{I_y d} \tan \theta \right) = 2\sigma_0$$

$$\tan \theta = \frac{I_y d}{I_z b_f} = \frac{(2.82)(11.99)}{(103)(3.990)} = 0.08273 \quad \theta = 4.70^\circ$$

PROBLEM 4.175

4.175 Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of the rectangular cross section, the neutral axis will lie along the other diagonal.



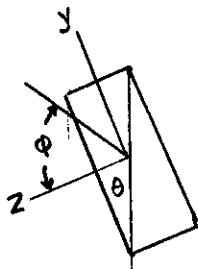
SOLUTION

$$\tan \theta = \frac{b}{h}$$

$$M_z = M \cos \theta, \quad M_y = M \sin \theta$$

$$I_z = \frac{1}{12} b h^3 \quad I_y = \frac{1}{12} h b^3$$

$$\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{\frac{1}{12} b h^3}{\frac{1}{12} h b^3} \cdot \frac{b}{h} = \frac{h}{b}$$



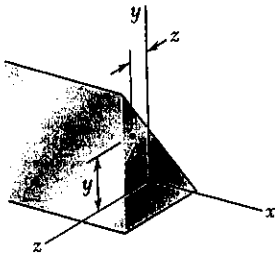
Thus neutral axis passes through corner A

PROBLEM 4.176

4.176 A beam of unsymmetric cross section is subjected to a couple  $M_z$  acting in the vertical  $xy$  plane. Show that the stress at point  $A$ , of coordinates  $y$  and  $z$ , is

$$\sigma_A = - \frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where  $I_y$ ,  $I_z$ , and  $I_{yz}$  denote the moments and product of inertia of the cross section with respect to centroidal axes, and  $M_z$  the moment of the couple.



SOLUTION

The stress  $\sigma_A$  varies linearly with the coordinates  $y$  and  $z$ . Since the axial force is zero, the  $y$ - and  $z$ -axes are centroidal axes.

$$\sigma_A = C_1 y + C_2 z \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.}$$

$$M_y = \int z \sigma_A dA = C_1 \int y z dA + C_2 \int z^2 dA$$

$$= I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = - \frac{I_{yz}}{I_y} C_1$$

$$M_z = - \int y \sigma_A dz = - C_1 \int y^2 dA + C_2 \int y z dA$$

$$= - I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

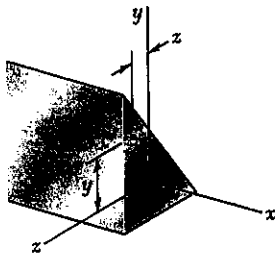
$$I_y M_z = - (I_y I_z - I_{yz}^2) C_1$$

$$C_1 = - \frac{I_y M_z}{I_y I_z - I_{yz}^2}$$

$$C_2 = + \frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = - \frac{I_y z - I_{yz} y}{I_y I_z - I_{yz}^2} M_z$$

PROBLEM 4.177



4.177 A beam of unsymmetric cross section is subjected to a couple  $M_0$  acting in the horizontal  $xz$  plane. Show that the stress at point  $A$  is

$$\sigma_A = -\frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where  $I_y$ ,  $I_z$ , and  $I_{yz}$  denote the moments and product of inertia of the cross section with respect to centroidal axes, and  $M_y$  the moment of the couple.

SOLUTION

The stress  $\sigma_A$  varies linearly with the coordinates  $y$  and  $z$ . Since the axial force is zero, the  $y$ - and  $z$ -axes are centroidal axes.

$$\sigma_A = C_1 y + C_2 z \quad \text{where } C_1 \text{ and } C_2 \text{ are constants.}$$

$$\begin{aligned} M_z &= -\int y \sigma_A dA = -C_1 \int y^2 dA - C_2 \int yz dA \\ &= -I_z C_1 - I_{yz} C_2 = 0 \\ C_1 &= -\frac{I_{yz}}{I_z} C_2 \end{aligned}$$

$$\begin{aligned} M_y &= \int z \sigma_A dA = C_1 \int yz dA + C_2 \int z^2 dA \\ &= I_{yz} C_1 + I_y C_2 \\ &\quad - I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2 \end{aligned}$$

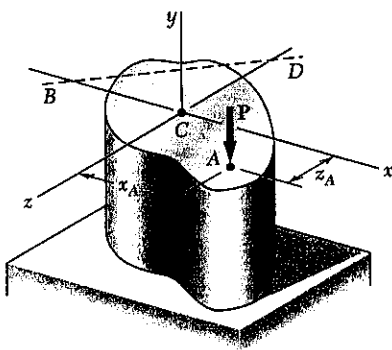
$$I_z M_y = (I_y I_z - I_{yz}^2) C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2}$$

$$C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z y - I_{yz} z}{I_y I_z - I_{yz}^2} M_y$$

PROBLEM 4.178



4.178 (a) Show that, if a vertical force  $P$  is applied at point  $A$  of the section shown, the equation of the neutral axis  $BD$  is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where  $k_z$  and  $k_x$  denote the radius of gyration of the cross section with respect to the  $z$  axis and the  $x$  axis, respectively. (b) Further show that, if a vertical force  $Q$  is applied at any point located on line  $BD$ , the stress at point  $A$  will be zero.

SOLUTION

Definitions  $k_x^2 = \frac{I_x}{A}$ ,  $k_z^2 = \frac{I_z}{A}$

(a)  $M_x = Pz_A$ ,  $M_z = -Px_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{Px_A x_E}{Ak_z^2} - \frac{Pz_A z_E}{Ak_x^2}$$

$$= -\frac{P}{A} \left[ 1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E \right] = 0 \text{ if } E \text{ lies on neutral axis.}$$

$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \quad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1 \quad \blacktriangleleft$$

(b)  $M_x = Pz_E$ ,  $M_z = -Px_E$

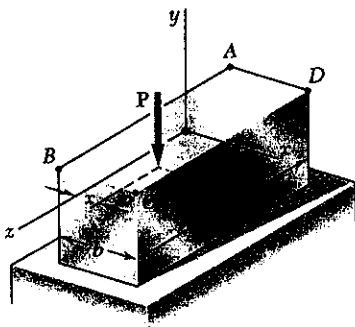
$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{Px_E x_A}{Ak_z^2} - \frac{Pz_E z_A}{Ak_x^2}$$

$$= 0 \text{ by equation from Part (a)} \quad \blacktriangleleft$$

PROBLEM 4.179

4.179 (a) Show that the stress at corner A of the prismatic member shown in Fig. (a) will be zero if the vertical force P is applied at a point located on the line

$$\frac{x}{b/6} + \frac{z}{h/6} = 1$$



(b) Further show that, if no tensile stress is to occur in the member, the force P must be applied at a point located within the area bounded by the line found in part a and the three similar lines corresponding to the condition of zero stress at B, C, and D, respectively. This area, shown in Fig.(b), is known as the kern of the cross section.

SOLUTION

$$I_z = \frac{1}{12} h b^3 \quad I_x = \frac{1}{12} b h^3 \quad A = bh$$

$$z_A = -\frac{h}{2} \quad x_A = -\frac{b}{2}$$

Let P be the load point

$$M_z = -P x_p \quad M_x = P z_p$$

$$\begin{aligned} \sigma_A &= -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} \\ &= -\frac{P}{bh} + \frac{(-P x_p)(-\frac{b}{2})}{\frac{1}{12} h b^3} - \frac{P z_p(-\frac{h}{2})}{\frac{1}{12} b h^3} \\ &= -\frac{P}{bh} \left[ 1 - \frac{x_p}{b/6} - \frac{z_p}{h/6} \right] \end{aligned}$$

For  $\sigma_A = 0$

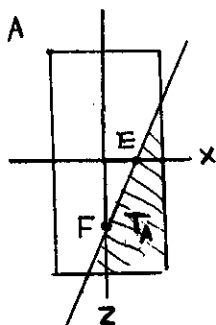
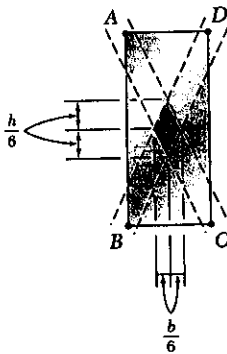
$$1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \quad \frac{x}{b/6} + \frac{z}{h/6} = 1$$

At point E  $z = 0 \quad \therefore x_E = b/6$

At point F  $x = 0 \quad \therefore z_F = h/6$

If the line of action  $(x_p, z_p)$  lies within the portion marked  $T_A$ , a tensile will occur at corner A.

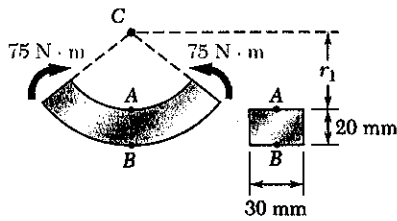
By considering  $\sigma_B = 0$ ,  $\sigma_C = 0$ , and  $\sigma_D = 0$ , the other portions producing tensile stresses are identified.





PROBLEM 4.180

4.180 For the curved bar and loading shown, determine the stress at point A when (a)  $r_1 = 30$  mm, (b)  $r_1 = 50$  mm.



SOLUTION

(a)  $r_1 = 30$  mm      $r_2 = 30 + 20 = 50$  mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{50}{30}} = 39.1523 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm}$$

$$e = \bar{r} - R = 0.8477 \text{ mm}$$

$$A = (20)(30) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

$$y_A = 39.1523 - 30 = 9.1523 \text{ mm}$$

$$\sigma_A = -\frac{My}{Aer} = -\frac{(75)(9.1523 \times 10^{-3})}{(600 \times 10^{-6})(0.8477 \times 10^{-3})(30 \times 10^{-3})} = -45.0 \times 10^6 \text{ Pa} = -45.0 \text{ MPa}$$

(b)  $r_1 = 50$  mm,      $r_2 = 50 + 20 = 70$  mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{70}{50}} = 59.44027$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 60 \text{ mm}$$

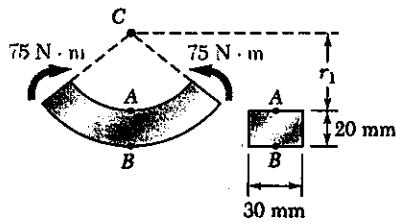
$$e = \bar{r} - R = 0.55973 \text{ mm}$$

$$y_A = 59.44027 - 50 = 9.44027 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_2} = -\frac{(75)(9.44027)}{(600 \times 10^{-6})(0.55973 \times 10^{-3})(50 \times 10^{-3})} = -42.2 \times 10^6 \text{ Pa} = -42.2 \text{ MPa}$$

PROBLEM 4.181

4.181 For the curved bar and loading shown, determine the stress at points A and B when  $r_1 = 40$  mm.



SOLUTION

$$h = 20 \text{ mm} \quad r_1 = 40 \text{ mm} \quad r_2 = 40 + 20 = 60 \text{ mm}$$

$$A = (30)(20) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{60}{40}} = 49.3261 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 50 \text{ mm}$$

$$e = \bar{r} - R = 0.6739 \text{ mm}$$

$$y_A = 49.3261 - 40 = 9.3261 \text{ mm} \quad r_A = 40 \text{ mm}$$

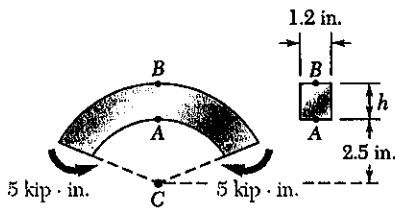
$$\sigma_A = - \frac{My_A}{Aer_A} = - \frac{(75)(9.3261 \times 10^{-3})}{(600 \times 10^{-6})(0.6739 \times 10^{-3})(40 \times 10^{-3})} = -43.2 \times 10^6 \text{ Pa} \\ = -43.2 \text{ MPa} \quad \leftarrow$$

$$y_B = 49.3261 - 60 = -10.6739 \text{ mm}$$

$$\sigma_B = - \frac{My_B}{Aer_B} = - \frac{(75)(-10.6739 \times 10^{-3})}{(600 \times 10^{-6})(0.6739 \times 10^{-3})(60 \times 10^{-3})} = 33.0 \times 10^6 \text{ Pa} \\ = 33.0 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 4.182

4.182 For the curved bar and loading shown, determine the stress at point A when (a)  $h = 2.5$  in., (b)  $h = 3$  in.



SOLUTION

(a)  $h = 2.5$  in,  $r_1 = 2.5$  in.  $r_2 = 5$  in.

$A = (1.2)(2.5) = 3.00$  in<sup>2</sup>,  $M = 5$  kip·in.

$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{5}{2.5}} = 3.6067$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 3.75$

$e = \bar{r} - R = 0.1433$  in.

$y_A = 3.6067 - 2.5 = 1.1067$  in  $r_A = 2.5$  in.

$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(5)(1.1067)}{(3.00)(0.1433)(2.5)} = -5.15$  ksi

(b)  $h = 3$  in,  $r_1 = 2.5$  in  $r_2 = 5.5$  in.  $A = (1.2)(3) = 3.6$  in<sup>2</sup>

$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{3}{\ln \frac{5.5}{2.5}} = 3.8049$  in.

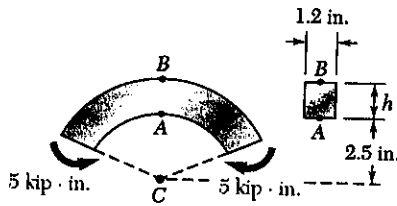
$\bar{r} = \frac{1}{2}(r_1 + r_2) = 4.0000$  in,  $e = \bar{r} - R = 0.1951$  in.

$y_A = 3.8049 - 2.5 = 1.3049$  in  $r_A = 2.5$  in

$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(5)(1.3049)}{(3.6)(0.1951)(2.5)} = -3.72$  ksi

PROBLEM 4.183

4.183 For the curved bar and loading shown, determine the stress at points *A* and *B* when  $h = 2.75$  in.



SOLUTION

$$h = 2.75 \text{ in.} \quad r_1 = 2.5 \text{ in.}, \quad r_2 = 5.25 \text{ in.}$$

$$A = (1.2)(2.75) = 3.30 \text{ in}^2, \quad M = 5 \text{ kip} \cdot \text{in.}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.75}{\ln \frac{5.25}{2.5}} = 3.7065 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 3.875 \text{ in.} \quad e = \bar{r} - R = 0.1685 \text{ in.}$$

$$y_A = 3.7065 - 2.5 = 1.2065 \text{ in.} \quad r_A = 2.5 \text{ in.}$$

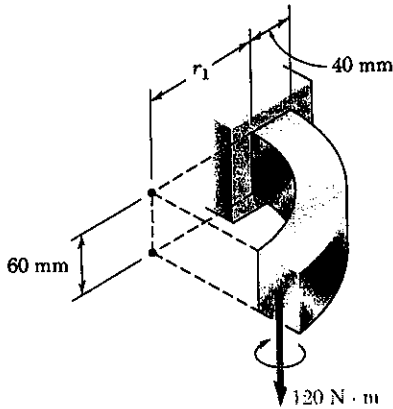
$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(5)(1.2065)}{(3.30)(0.1685)(2.5)} = -4.34 \text{ ksi}$$

$$y_B = 3.7065 - 5.25 = -1.5435 \text{ in.} \quad r_B = 5.25 \text{ in.}$$

$$\sigma_B = -\frac{My_B}{Aer_B} = -\frac{(5)(-1.5435)}{(3.30)(0.1685)(5.25)} = 2.64 \text{ ksi}$$

PROBLEM 4.184

4.184 The curved bar shown has a cross section of  $40 \times 60$  mm and an inner radius  $r_1 = 15$  mm. For the loading shown determine the largest tensile and compressive stresses.



SOLUTION

$$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55 \text{ mm}$$

$$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{55}{15}} = 30.786 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$$

$$e = \bar{r} - R = 4.214 \text{ mm} \quad \sigma = -\frac{My}{Aer}$$

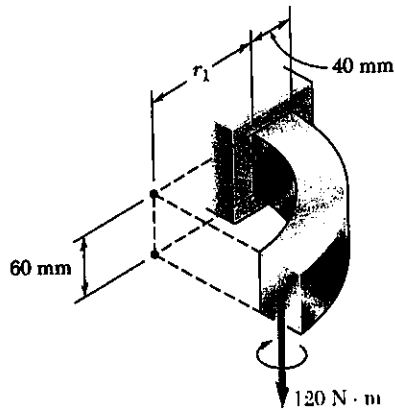
$$\text{At } r = 15 \text{ mm} \quad y = 30.786 - 15 = 15.786 \text{ mm}$$

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^6 \text{ Pa} = -12.49 \text{ MPa}$$

$$\text{At } r = 55 \text{ mm} \quad y = 30.786 - 55 = -24.214 \text{ mm}$$

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^6 \text{ Pa} = 5.22 \text{ MPa}$$

PROBLEM 4.185



4.185 For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (a)  $r_1 = 20$  mm, (b)  $r_1 = 200$  mm, (c)  $r_1 = 2$  m.

SOLUTION

$$h = 40 \text{ mm}, \quad A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$M = 120 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(40)^3 = 0.32 \times 10^6 \text{ mm}^4 = 0.32 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}h = 20 \text{ mm}$$

Assuming that the bar is straight

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20 \times 10^{-3})}{0.32 \times 10^{-6}} = 7.5 \times 10^6 \text{ Pa} = 7.5 \text{ MPa}$$

(a)  $r_1 = 20$  mm       $r_2 = 60$  mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{60}{20}} = 36.4096 \text{ mm} \quad r_1 - R = -16.4096 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm} \quad e = \bar{r} - R = 3.5904 \text{ mm}$$

$$\sigma_a = \frac{M(r_1 - R)}{Aer} = \frac{(120)(-16.4096 \times 10^{-3})}{(2400 \times 10^{-6})(3.5904 \times 10^{-3})(20 \times 10^{-3})} = -11.426 \times 10^6 \text{ Pa} = -11.426 \text{ MPa}$$

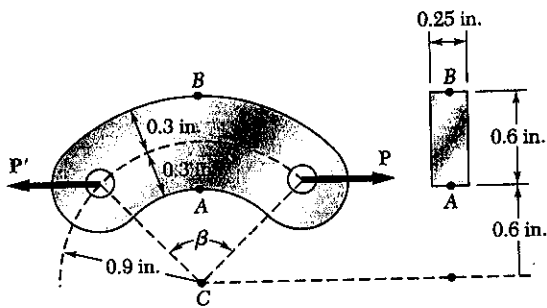
$$\% \text{ error} = \frac{-11.426 - (-7.5)}{-11.426} \times 100\% = -34.4\%$$

For parts (b) and (c) we get the values in the table below:

	$r_1, \text{mm}$	$r_2, \text{mm}$	$R, \text{mm}$	$\bar{r}, \text{mm}$	$e, \text{mm}$	$\sigma, \text{MPa}$	% error
(a)	20	60	36.4096	40	3.5904	-11.426	-34.4%
(b)	200	240	219.3920	220	0.6074	-7.982	6.0%
(c)	2000	2040	2019.9340	2020	0.0660	-7.546	0.6%

PROBLEM 4.186

4.186 Steel links having the cross section shown are available with different central angles  $\beta$ . Knowing that the allowable stress is 15 ksi, determine the largest force  $P$  that can be applied to a link for which  $\beta = 90^\circ$ .



SOLUTION

Reduce section force to a force-couple system at  $G$ , the centroid of the cross section  $AB$ .

$$a = \bar{r} (1 - \cos \frac{\beta}{2})$$

The bending couple is  $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \quad \text{Also } e = \bar{r} - R$$

At point  $A$  the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Ae r_1} = \frac{P}{A} + \frac{Pa y_A}{Ae r_1} = \frac{P}{A} \left( 1 + \frac{a y_A}{e r_1} \right) = K \frac{P}{A}$$

where  $K = 1 + \frac{a y_A}{e r_1}$  and  $y_A = R - r_1$

$$P = \frac{A \sigma_A}{K}$$

Data:  $\bar{r} = 0.9$  in,  $r_1 = 0.6$  in,  $r_2 = 1.2$  in,  $h = 0.6$  in,  $b = 0.25$  in.

$$A = (0.25)(0.6) = 0.15 \text{ in}^2, \quad R = \frac{0.6}{\ln \frac{1.2}{0.6}} = 0.86562 \text{ in}$$

$$e = 0.9 - 0.86562 = 0.03438 \text{ in}, \quad y_A = 0.86562 - 0.6 = 0.26562 \text{ in}$$

$$a = 0.9 (1 - \cos 45^\circ) = 0.26360 \text{ in}$$

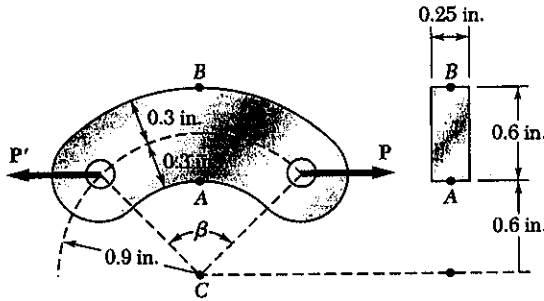
$$K = 1 + \frac{(0.26360)(0.26562)}{(0.03438)(0.6)} = 4.3943$$

$$P = \frac{(0.15)(15)}{4.3943} = 0.512 \text{ kips} = 512 \text{ lb}$$

PROBLEM 4.187

4.186 Steel links having the cross section shown are available with different central angles  $\beta$ . Knowing that the allowable stress is 15 ksi, determine the largest force  $P$  that can be applied to a link for which  $\beta = 90^\circ$ .

4.187 Solve Prob. 4.186, assuming that  $\beta = 60^\circ$ .



SOLUTION

Reduce section force to a force-couple system at  $G$ , the centroid of the cross section  $AB$ .

$$a = \bar{r} \left( 1 - \cos \frac{\beta}{2} \right)$$

The bending couple is  $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \quad \text{Also } e = \bar{r} - R$$

At point  $A$  the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pa y_A}{Aer_1} = \frac{P}{A} \left( 1 + \frac{a y_A}{er_1} \right) = K \frac{P}{A}$$

where  $K = 1 + \frac{a y_A}{er_1}$  and  $y_A = R - r_1$

$$P = \frac{A \sigma_A}{K}$$

Data:  $\bar{r} = 0.9 \text{ in}$ ,  $r_1 = 0.6 \text{ in}$ ,  $r_2 = 1.2 \text{ in}$ ,  $h = 0.6 \text{ in}$ ,  $b = 0.25$

$$A = (0.25)(0.6) = 0.15 \text{ in}^2, \quad R = \frac{0.6}{\ln \frac{1.2}{0.6}} = 0.86562 \text{ in}$$

$$e = 0.9 - 0.86562 = 0.03438 \text{ in}, \quad y_A = 0.86562 - 0.6 = 0.26562$$

$$a = 0.9 (1 - \cos 30^\circ) = 0.12058 \text{ in}$$

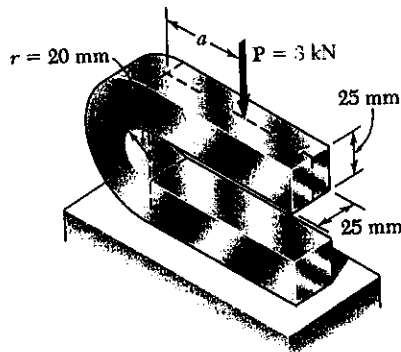
$$K = 1 + \frac{(0.12058)(0.26562)}{(0.03438)(0.6)} = 2.5526$$

$$P = \frac{(0.15)(15)}{2.5526} = 0.881 \text{ kips} = 881 \text{ lb}$$



**PROBLEM 4.188**

4.188 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance  $a = 60$  mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.



**SOLUTION**

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \quad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\begin{aligned} \sigma_A &= -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a + \bar{r}) y_A}{A e r_1} \\ &= -K \frac{P}{A} \quad \text{with } y_A = R - r_1 \\ \text{Thus, } K &= 1 + \frac{(a + \bar{r})(R - r_1)}{e r_1} \end{aligned}$$

Data:  $h = 25$  mm,  $r_1 = 20$  mm,  $r_2 = 45$  mm,  $\bar{r} = 32.5$  mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 92.5 \text{ mm}, \quad R - r_1 = 10.8288 \text{ mm}$$

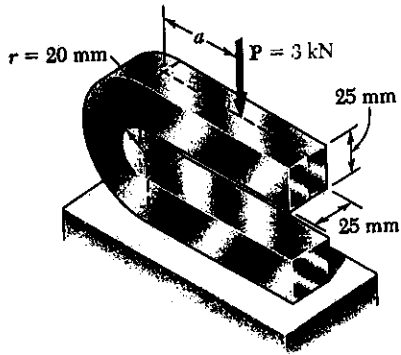
$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

$$P = 3 \times 10^3 \text{ N}$$

$$\begin{aligned} \sigma_A &= -\frac{KP}{A} = -\frac{(30.968)(3 \times 10^3)}{625 \times 10^{-6}} = -148.6 \times 10^6 \text{ Pa} \\ &= -148.6 \text{ MPa} \end{aligned}$$

PROBLEM 4.189

4.189 Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance  $a$  from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neut axis for bending couple only lies

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \quad \text{Also } e = \bar{r} - R$$

The maximum compressive stress occ at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a + \bar{r})y_A}{Aer_1}$$

$$= -K \frac{P}{A} \quad \text{with } y_A = R - r_1$$

$$\text{Thus, } K = 1 + \frac{(a + \bar{r})(R - r_1)}{er_1}$$

Data:  $h = 25 \text{ mm}$ ,  $r_1 = 20 \text{ mm}$ ,  $r_2 = 45 \text{ mm}$ ,  $\bar{r} = 32.5 \text{ mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$R - r_1 = 10.8288 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N} \cdot \text{m} \quad \sigma_A = -150 \times 10^6 \text{ Pa}$$

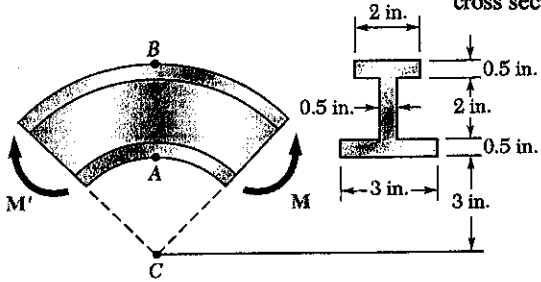
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)er_1}{R-r_1} = \frac{(31.25-1)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5 = 60.9 \text{ mm}$$

PROBLEM 4.190

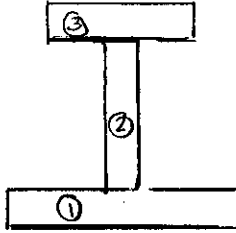
4.190 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance  $e$  between the neutral axis and the centroid of the cross section.



SOLUTION

$$R = \frac{\sum A}{\sum S \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A \bar{r}_i}{\sum A}$$



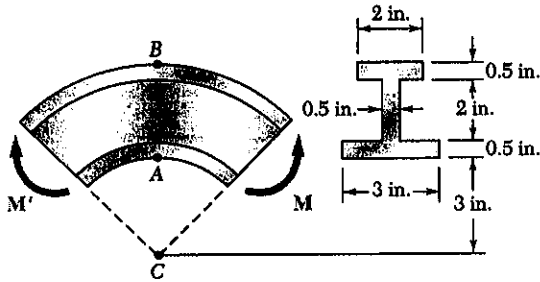
r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A \bar{r}$
3							
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5							
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5							
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6							
6	$\Sigma$			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.}$$

PROBLEM 4.191

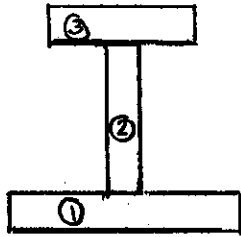
4.191 Three plates are welded together to form the curved beam shown. For  $M = 8$  kip-in., determine the stress at (a) point A, (b) point B, (c) the centroid of the cross section.



SOLUTION

$$R = \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A \bar{r}_i}{\sum A}$$



r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A \bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	$\Sigma$			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \quad M = -8 \text{ kip-in.}$$

(a)  $y_A = R - r_1 = 4.05812 - 3 = 1.05812 \text{ in.}$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} = 3.06 \text{ ksi}$$

(b)  $y_B = R - r_2 = 4.05812 - 6 = -1.94188 \text{ in.}$

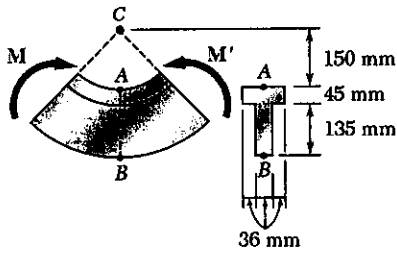
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} = -2.81 \text{ ksi}$$

(c)  $y_C = R - \bar{r} = -e =$

$$\sigma_C = -\frac{M y_C}{A e \bar{r}} = -\frac{M e}{A e \bar{r}} = -\frac{M}{A \bar{r}} = -\frac{-8}{(3.5)(4.32143)} = 0.529 \text{ ksi}$$

PROBLEM 4.192

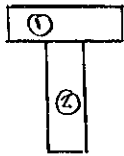
4.192 and 4.193 Knowing that  $M = 20 \text{ kN}\cdot\text{m}$ , determine the stress at (a) point A, (b) point B.



SOLUTION

$$R = \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{2+i}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{2+i}}{r_i}}$$

$$\bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$



$r, \text{mm}$	Part	$b, \text{mm}$	$h, \text{mm}$	$A, \text{mm}^2$	$b_i \ln \frac{r_{2+i}}{r_i}, \text{mm}$	$\bar{r}_i, \text{mm}$	$A \bar{r}_i, \text{mm}^3$
150							
	①	108	45	4860	28.3353	172.5	$838.35 \times 10^3$
195							
	②	36	135	4860	18.9394	262.5	$1275.75 \times 10^3$
330							
	$\Sigma$			9720	47.2747		$2114.1 \times 10^3$

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \quad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm} \quad M = 20 \times 10^3 \text{ N}\cdot\text{m}$$

(a)  $y_A = R - r_1 = 205.606 - 150 = 55.606 \text{ mm}$

$$\sigma_A = - \frac{M y_A}{A e r_1} = - \frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa} = -64.1 \text{ MPa}$$

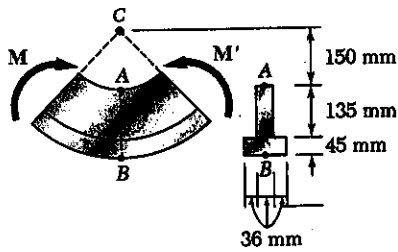
(b)  $y_B = R - r_2 = 205.606 - 330 = -124.394 \text{ mm}$

$$\sigma_B = - \frac{M y_B}{A e r_2} = - \frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa} = 65.2 \text{ MPa}$$

PROBLEM 4.193

4.192 and 4.193 Knowing that  $M = 20 \text{ kN}\cdot\text{m}$ , determine the stress at (a) point A, (b) point B.



SOLUTION

$$R = \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{\sum b_i h_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\sum A_i}{\sum b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\sum A_i \bar{r}_i}{\sum A_i}$$

$r$	$b_i, \text{mm}$	$h_i, \text{mm}$	$A_i, \text{mm}^2$	$b_i \ln \frac{r_{i+1}}{r_i}, \text{mm}$	$\bar{r}_i, \text{mm}$	$A \bar{r}_i, \text{mm}^3$
150						
①	36	135	4860	23.1067	217.5	$1.05705 \times 10^6$
285						
②	108	45	4860	15.8332	307.5	$1.49445 \times 10^6$
330						
$\Sigma$			9720	38.9399		$2.5515 \times 10^6$

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \quad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \quad M = 20 \times 10^3 \text{ N}\cdot\text{m}$$

(a)  $y_A = R - r_1 = 249.615 - 150 = 99.615 \text{ mm}$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(99.615 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -106.1 \times 10^6 \text{ Pa} = -106.1 \text{ MPa}$$

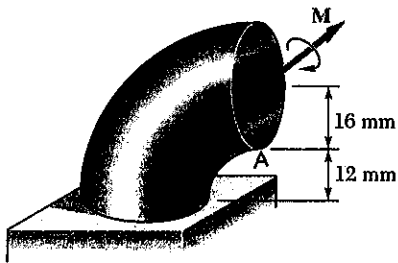
(b)  $y_B = R - r_2 = 249.615 - 330 = -80.385 \text{ mm}$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-80.385 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 38.9 \times 10^6 \text{ Pa} = 38.9 \text{ MPa}$$

PROBLEM 4.194

4.194 The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple  $M$  that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.



SOLUTION

$$c = 16 \text{ mm} \quad \bar{r} = 12 + 16 = 28 \text{ mm}$$

$$R = \frac{1}{2} [\bar{r} + \sqrt{\bar{r}^2 - c^2}]$$

$$= \frac{1}{2} [28 + \sqrt{28^2 - 16^2}] = 25.4891 \text{ mm}$$

$$e = \bar{r} - R = 28 - 25.4891 = 2.5109 \text{ mm.}$$

$\sigma_{\max}$  occurs at A is given by  $|\sigma_{\max}| = \left| \frac{M y_A}{A e r_i} \right|$  from which

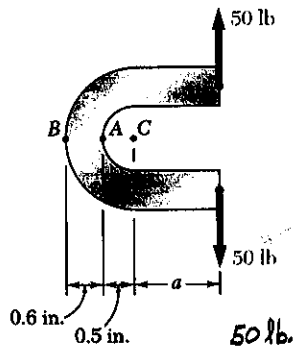
$$M = \frac{A e r_i |\sigma_{\max}|}{y_A} \quad \text{Also } A = \pi c^2 = \pi (16)^2 = 804.25 \text{ mm}^2$$

Data:  $y_A = R - r_i = 25.4891 - 12 = 13.4891 \text{ mm}$

$$M = \frac{(804.25 \times 10^{-6})(2.5109 \times 10^{-3})(12 \times 10^{-3})(60 \times 10^6)}{13.4891 \times 10^{-3}} = 107.8 \text{ N}\cdot\text{m}$$

PROBLEM 4.195

4.195 The bar shown has a circular cross section of 0.6-in. diameter. Knowing that  $a = 1.2$  in., determine the stress at (a) point A, (b) point B.



SOLUTION

$$c = \frac{1}{2}d = 0.3 \text{ in.} \quad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2} \left[ \bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[ 0.8 + \sqrt{0.8^2 - 0.3^2} \right]$$

$$= 0.77081 \text{ in}$$

$$e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi (0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r}) = -50(1.2 + 0.8) = -100 \text{ lb}\cdot\text{in}$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919 \text{ in.}$$

$$(a) \quad \sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{50}{0.28274} - \frac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3 \text{ psi}$$

$$= 6.74 \text{ ksi} \quad \blacktriangleleft$$

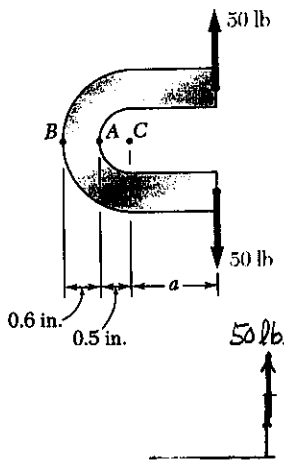
$$(b) \quad \sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2} = \frac{50}{0.28274} - \frac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3 \text{ psi}$$

$$= -3.45 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 4.196

4.196 The bar shown has a circular cross section of 0.6-in. diameter. Knowing that the allowable tensile stress is 8 ksi, determine the largest permissible distance  $a$  from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.



SOLUTION

$$c = \frac{1}{2}d = 0.3 \text{ in.}, \quad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in.}$$

$$R = \frac{1}{2} \left[ \bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[ 0.8 + \sqrt{0.8^2 - 0.3^2} \right]$$

$$= 0.77081 \text{ in.} \quad e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi (0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r})$$

$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a + \bar{r})y_A}{Aer_1} = \frac{P}{A} \left[ 1 + \frac{(a + \bar{r})y_A}{er_1} \right]$$

$$= \frac{KP}{A} \quad \text{where } K = 1 + \frac{(a + \bar{r})y_A}{er_1}$$

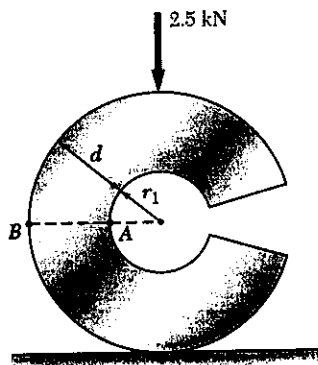
$$K = \frac{\sigma_A A}{P} = \frac{(8 \times 10^3)(0.28274)}{50} = 45.238$$

$$a + \bar{r} = \frac{(K-1)er_1}{y_A} = \frac{(44.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in.}$$

$$a = 2.384 - 0.8 = 1.584 \text{ in.}$$

PROBLEM 4.197

4.197 The split ring shown has an inner radius  $r_1 = 20$  mm and a circular cross section of diameter  $d = 32$  mm. For the loading shown, determine the stress at (a) point A, (b) point B.



SOLUTION

$$c = \frac{1}{2}d = 16 \text{ mm} \quad r_1 = 20 \text{ mm}; \quad r_2 = r_1 + d = 52 \text{ mm}$$

$$\bar{r} = r_1 + c = 36 \text{ mm}$$

$$R = \frac{1}{2} \left[ \bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[ 36 + \sqrt{36^2 - 16^2} \right]$$

$$= 34.1245 \text{ mm}$$

$$e = \bar{r} - R = 1.8755 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N}$$

$$M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90 \text{ N}\cdot\text{m}$$

(a) Point A:  $y_A = R - r_1 = 34.1245 - 20 = 14.1245 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -45.2 \times 10^6 \text{ Pa} = -45.2 \text{ MPa}$$

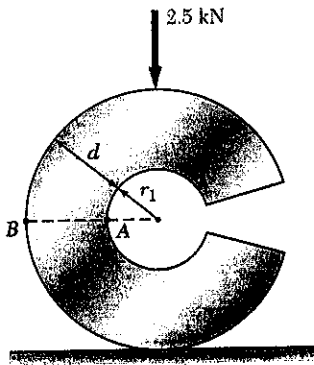
(b) Point B:  $y_B = R - r_2 = 34.1245 - 52 = -17.8755 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$$= 17.40 \times 10^6 \text{ Pa} = 17.40 \text{ MPa}$$

PROBLEM 4.198

4.198 The split ring shown has an inner radius  $r_1 = 16$  mm and a circular cross section of diameter  $d = 32$  mm. For the loading shown, determine the stress at (a) point A, (b) point B.



SOLUTION

$$c = \frac{1}{2}d = 16 \text{ mm}, \quad r_1 = 16 \text{ mm}, \quad r_2 = r_1 + d = 48 \text{ mm}$$

$$\bar{r} = r_1 + c = 32 \text{ mm}$$

$$R = \frac{1}{2} \left[ \bar{r} + \sqrt{\bar{r}^2 - c^2} \right] = \frac{1}{2} \left[ 32 + \sqrt{32^2 - 16^2} \right]$$

$$= 29.8564 \text{ mm}$$

$$e = \bar{r} - R = 2.1436 \text{ mm}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N}$$

$$M = P \bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80 \text{ N}\cdot\text{m}$$

(a) Point A:  $y_A = R - r_1 = 29.8564 - 16 = 13.8564 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})}$$

$$= -43.3 \times 10^6 \text{ Pa} = -43.3 \text{ MPa}$$

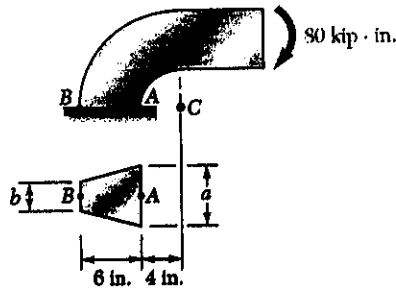
(b) Point B:  $y_B = R - r_2 = 29.8564 - 48 = -18.1436 \text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{M y_B}{A e r_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})}$$

$$= 14.43 \times 10^6 \text{ Pa} = 14.43 \text{ MPa}$$

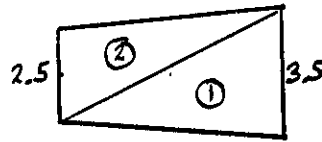
PROBLEM 4.199

4.199 Knowing that the machine component shown has a trapezoidal cross section with  $a = 3.5$  in. and  $b = 2.5$  in., determine the stress at (a) point A, (b) point B.



SOLUTION

Locate centroid



	$A, \text{in}^2$	$\bar{r}, \text{in}$	$A\bar{r}, \text{in}^3$
①	10.5	6	63
②	7.5	8	60
$\Sigma$	18		123

$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2 (3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)] \ln \frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in}$$

$$e = \bar{r} - R = 0.4452 \text{ in} \quad M = 80 \text{ kip}\cdot\text{in.}$$

(a)  $y_A = R - r_1 = 6.3878 - 4 = 2.3878 \text{ in.}$

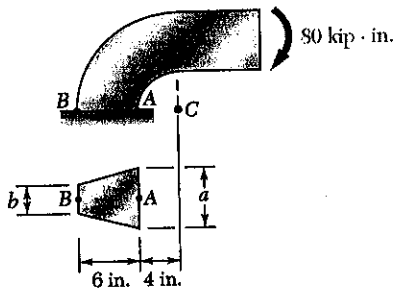
$$\sigma_A = - \frac{M y_A}{A e r_1} = - \frac{(80)(2.3878)}{(18)(0.4452)(4)} = - 5.96 \text{ ksi}$$

(b)  $y_B = R - r_2 = 6.3878 - 10 = - 3.6122 \text{ in.}$

$$\sigma_B = - \frac{M y_B}{A e r_2} = - \frac{(80)(-3.6122)}{(18)(0.4452)(10)} = 3.61 \text{ ksi}$$

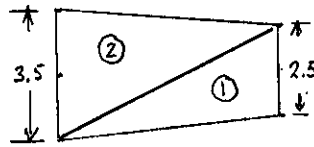
**PROBLEM 4.200**

4.200 Knowing that the machine component shown has a trapezoidal cross section with  $a = 2.5$  in. and  $b = 3.5$  in., determine the stress at (a) point A, (b) point B.



**SOLUTION**

locate centroid



	$A_j, \text{in}^2$	$\bar{r}_j, \text{in.}$	$A\bar{r}_j, \text{in}^3$
①	7.5	6	45
②	10.5	8	84
$\Sigma$	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5 + 3.5)}{[(2.5)(10) - (3.5)(4)] \ln \frac{10}{4} - (6)(2.5 - 3.5)} = 6.7168 \text{ in.}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \quad M = 80 \text{ kip-in.}$$

(a)  $y_A = R - r_1 = 2.7168 \text{ in.}$

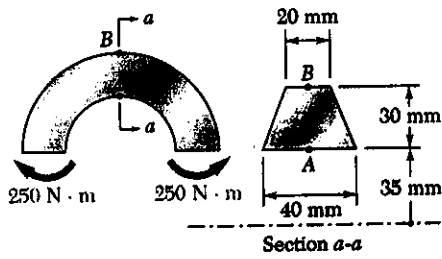
$$\sigma_A = - \frac{M y_A}{A e r_1} = - \frac{(80)(2.7168)}{(18)(0.4499)(4)} = - 6.71 \text{ ksi}$$

(b)  $y_B = R - r_2 = -3.2832 \text{ in.}$

$$\sigma_B = - \frac{M y_B}{A e r_2} = - \frac{(80)(-3.2832)}{(18)(0.4499)(10)} = 3.24 \text{ ksi}$$

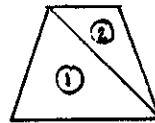
PROBLEM 4.201

4.201 For the curved beam and loading shown, determine the stress at (a) point A, (b) point B.



SOLUTION

locate centroid.



	$A, \text{mm}^2$	$\bar{r}, \text{mm}$	$A\bar{r}, \text{mm}^3$
①	600	45	$27 \times 10^3$
②	300	55	$16.5 \times 10^3$
$\Sigma$	900		$43.5 \times 10^3$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1) \ln \frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)] \ln \frac{65}{35} - (30)(40 - 20)}$$

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm}$$

$$= 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm}$$

$$M = -250 \text{ N}\cdot\text{m}$$

(a)  $y_A = R - r_1 = 11.8608 \text{ mm}$

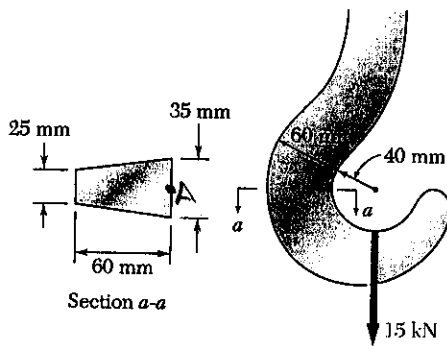
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa} = 63.9 \text{ MPa} \leftarrow$$

(b)  $y_B = R - r_2 = -18.1392 \text{ mm}$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa} = -52.6 \text{ MPa} \leftarrow$$

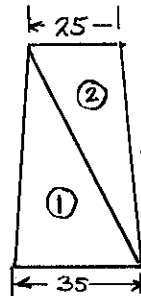
PROBLEM 4.202

4.202 For the crane hook shown, determine the largest tensile stress in section  $a-a$ .



SOLUTION

Locate centroid.



	$A, \text{mm}^2$	$\bar{r}, \text{mm}$	$A\bar{r}, \text{mm}^3$
①	1050	60	$63 \times 10^3$
②	750	80	$60 \times 10^3$
$\Sigma$	1800		$103 \times 10^3$

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm.}$$

Force-couple system at centroid:  $P = 15 \times 10^3 \text{ N}$

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2 (35 + 25)}{[(35)(100) - (25)(40)] \ln \frac{100}{40} - (60)(35 + 25)} = 63.878 \text{ mm.}$$

$$e = \bar{r} - R = 4.452 \text{ mm.}$$

Maximum tensile stress occurs at point A

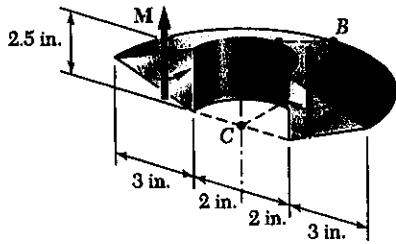
$$y_A = R - r_1 = 23.878 \text{ mm.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

$$= 84.7 \times 10^6 \text{ Pa} = 84.7 \text{ MPa}$$

PROBLEM 4.203

4.203 and 4.204 Knowing that  $M = 5 \text{ kip}\cdot\text{in.}$ , determine the stress at (a) point A, (b) point B.



SOLUTION

$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 1 = 3.00000 \text{ in}$$

$$b_1 = 2.5 \text{ in.}, r_1 = 2 \text{ in.}, b_2 = 0, r_2 = 5 \text{ in.}$$

Use formula for trapezoid

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1r_2 - b_2r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5 + 0)}{[(2.5)(5) - (0)(2)] \ln \frac{5}{2} - (3)(2.5 - 0)} = 2.84548 \text{ in.}$$

$$e = \bar{r} - R = 0.15452 \text{ in.} \quad M = 5 \text{ kip}\cdot\text{in.}$$

(a)  $y_A = R - r_1 = 0.84548 \text{ in.}$

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)} = -3.65 \text{ ksi}$$

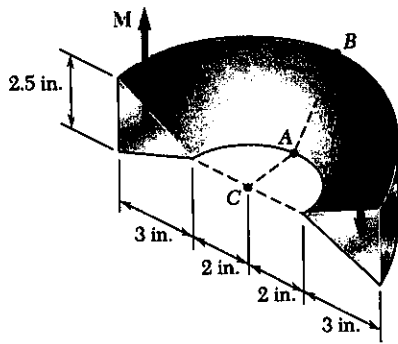
(b)  $y_B = R - r_2 = -2.15452 \text{ in.}$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)} = 3.72 \text{ ksi}$$



PROBLEM 4.204

4.203 and 4.204 Knowing that  $M = 5 \text{ kip}\cdot\text{in.}$ , determine the stress at (a) point A, (b) point B.



SOLUTION

$$A = \frac{1}{2} (2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 2 = 4.00000 \text{ in}$$

$$b_1 = 0, \quad r_1 = 2 \text{ in.}, \quad b_2 = 2.5 \text{ in.}, \quad r_2 = 5 \text{ in.}$$

Use formula for trapezoid.

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2 (0 + 2.5)}{[(0)(5) - (2.5)(2)] \ln \frac{5}{2} - (3)(0 - 2.5)} = 3.85466 \text{ in.}$$

$$e = \bar{r} - R = 0.14534 \text{ in.} \quad M = 5 \text{ kip}\cdot\text{in.}$$

(a)  $y_A = R - r_1 = 1.85466 \text{ in.}$

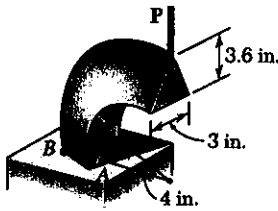
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} = -8.51 \text{ ksi}$$

(b)  $y_B = R - r_2 = -1.14534 \text{ in.}$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} = 2.10 \text{ ksi}$$

PROBLEM 4.205

4.205 Knowing that  $P = 3.5$  kips, determine the stress at (a) point A, (b) point B.



SOLUTION

$$b = 3 \text{ in.}, \quad h = 3.6 \text{ in.}, \quad r_1 = 4 \text{ in.}$$

$$A = \frac{1}{2}bh = \frac{1}{2}(3)(3.6) = 5.4 \text{ in}^2$$

$$r_2 = r_1 + h = 7.6 \text{ in.} \quad \bar{r} = r_1 + \frac{1}{3}h = 5.2 \text{ in.}$$

Reduce section forces to a force-couple system at the centroid

$$P = 3.5 \text{ kips} \quad M = P\bar{r} = (3.5)(5.2) = 18.2 \text{ kip-in}$$

For a triangular section  $R = \frac{1}{3}h$

$$e = \frac{\frac{1}{2} \ln \frac{r_2}{r_1} - 1}{\frac{7.6}{3.6} \ln \frac{7.6}{4} - 1} = \frac{(0.5)(3.6)}{\frac{7.6}{3.6} \ln \frac{7.6}{4} - 1} = 5.07007 \text{ in}$$

$$e = \bar{r} - R = 0.12993 \text{ in.}$$

$$(a) \quad y_A = R - r_1 = 1.07007 \text{ in.}$$

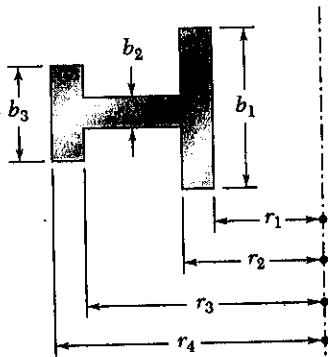
$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{3.5}{5.4} - \frac{(18.2)(1.07007)}{(5.4)(0.12993)(4)} = -7.59 \text{ ksi}$$

$$(b) \quad y_B = R - r_2 = -2.52993 \text{ in.}$$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{3.5}{5.4} - \frac{(18.2)(-2.52993)}{(5.4)(0.12993)(7.6)} = 7.99 \text{ ksi}$$

PROBLEM 4.206

4.206 Show that if the cross section of a curved beam consists of two or more rectangles, the radius  $R$  of the neutral surface can be expressed as



$$R = \frac{A}{\ln \left[ \left( \frac{r_2}{r_1} \right)^{b_1} \left( \frac{r_3}{r_2} \right)^{b_2} \left( \frac{r_4}{r_3} \right)^{b_3} \right]}$$

where  $A$  is the total area of the cross section.

SOLUTION

$$\begin{aligned} R &= \frac{\sum A}{\sum \int \frac{1}{r} dA} = \frac{A}{\sum b_i \ln \frac{r_{i+1}}{r_i}} \\ &= \frac{A}{\sum \ln \left( \frac{r_{i+1}}{r_i} \right)^{b_i}} = \frac{A}{\ln \left[ \left( \frac{r_2}{r_1} \right)^{b_1} \left( \frac{r_3}{r_2} \right)^{b_2} \left( \frac{r_4}{r_3} \right)^{b_3} \right]} \end{aligned}$$

Note that for each rectangle  $\int \frac{1}{r} dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$   
 $= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$

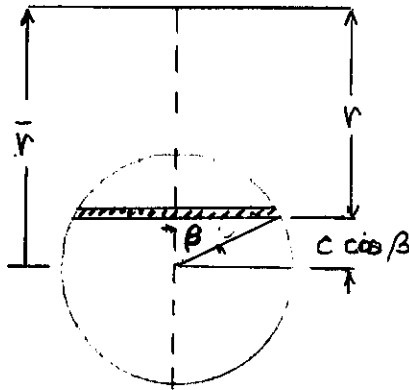
PROBLEM 4.207

4.207 through 4.209 Using Eq. (4.66), derive the expression for  $R$  given in Fig. 4.79 for

\*4.207 A circular cross section

SOLUTION

Use polar coordinate  $\beta$  as shown.



width  $w = 2c \sin \beta$

$r = \bar{r} - c \cos \beta$

$dr = -c \sin \beta d\beta$

$dA = w dr = -2c^2 \sin^2 \beta d\beta$

$\int \frac{dA}{r} = \int_0^\pi \frac{2c^2 \sin^2 \beta}{\bar{r} - c \cos \beta} d\beta$

$\int \frac{dA}{r} = \int_0^\pi \frac{c^2(1 - \cos^2 \beta)}{\bar{r} - c \cos \beta} d\beta = 2 \int_0^\pi \frac{\bar{r}^2 - c^2 \cos^2 \beta - (\bar{r}^2 - c^2)}{\bar{r} - c \cos \beta} d\beta$

$= 2 \int_0^\pi (\bar{r} + c \cos \beta) d\beta - 2(\bar{r}^2 - c^2) \int_0^\pi \frac{dr}{\bar{r} - c \cos \beta}$

$= 2\bar{r} \beta \Big|_0^\pi + 2c \sin \beta \Big|_0^\pi$

$- 2(\bar{r}^2 - c^2) \frac{2}{\sqrt{\bar{r}^2 - c^2}} \tan^{-1} \frac{\sqrt{\bar{r}^2 - c^2} \tan \frac{1}{2} \beta}{\bar{r} + c} \Big|_0^\pi$

$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2} \cdot \left(\frac{\pi}{2} - 0\right)$

$2\pi \bar{r} - 2\pi \sqrt{\bar{r}^2 - c^2}$

$A = \pi c^2$

$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi \bar{r} - 2\pi \sqrt{\bar{r}^2 - c^2}}$

$= \frac{1}{2} \frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}} \cdot \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$

$= \frac{1}{2} \frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2} \frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$

$= \frac{1}{2} (\bar{r} + \sqrt{\bar{r}^2 - c^2})$

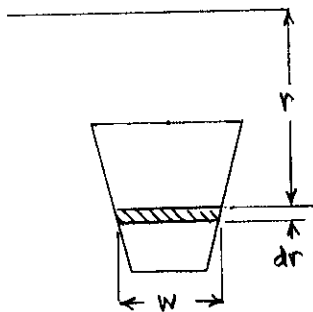
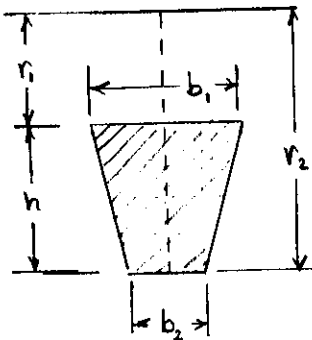
PROBLEM 4.208

4.207 through 4.209 Using Eq. (4.66), derive the expression for  $R$  given in Fig. 4.79 for

4.208 A trapezoidal section

SOLUTION

The section width  $w$  varies linearly with  $r$



$$w = C_0 + C_1 r$$

$$w = b_1 \text{ at } r = r_1 \text{ and } w = b_2 \text{ at } r = r_2$$

$$b_1 = C_0 + C_1 r_1$$

$$b_2 = C_0 + C_1 r_2$$

$$b_1 - b_2 = C_1 (r_1 - r_2) = -C_1 h$$

$$C_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) C_0 = h C_0$$

$$C_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$

$$\begin{aligned} \int \frac{dA}{r} &= \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{C_0 + C_1 r}{r} dr \\ &= C_0 \ln r \Big|_{r_1}^{r_2} + C_1 r \Big|_{r_1}^{r_2} \\ &= C_0 \ln \frac{r_2}{r_1} + C_1 (r_2 - r_1) \\ &= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h \\ &= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2) \end{aligned}$$

$$A = \frac{1}{2} (b_1 + b_2) h$$

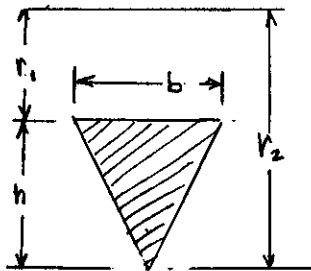
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h (b_1 - b_2)}$$

PROBLEM 4.209

4.207 through 4.209 Using Eq. (4.66), derive the expression for  $R$  given in Fig. 4.79 for

4.209 A triangular cross section

SOLUTION



The section width  $w$  varies linearly with  $r$

$$w = c_0 + c_1 r$$

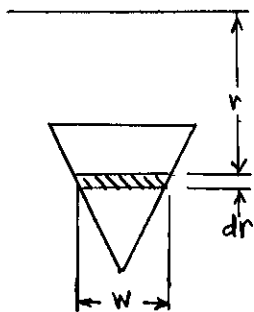
$$w = b \text{ at } r = r_1 \text{ and } w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1 (r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \text{ and } c_0 = -c_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r} dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1 (r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b \left( \frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)$$

$$A = \frac{1}{2} b h$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b \left( \frac{r_2}{h} \ln \frac{r_2}{r_1} - 1 \right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

PROBLEM 4.210

\*4.210 For a curved bar of rectangular cross section subjected to a bending couple  $M$ , show that the radial stress at the neutral surface is

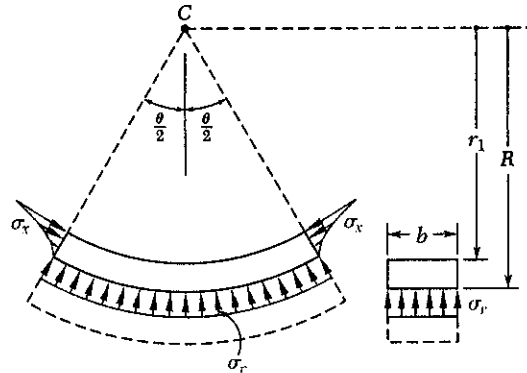
$$\sigma_r = \frac{M}{Ae} \left( 1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right)$$

SOLUTION

At radial distance  $r$

$$\begin{aligned} \sigma_a &= \frac{M(r-R)}{Aer} \\ &= \frac{M}{Ae} - \frac{MR}{Aer} \end{aligned}$$

and compute the value of  $\sigma_r$  for the curved bar of Examples 4.10 and 4.11. (Hint: consider the free-body diagram of the portion of the beam located above the neutral surface.)

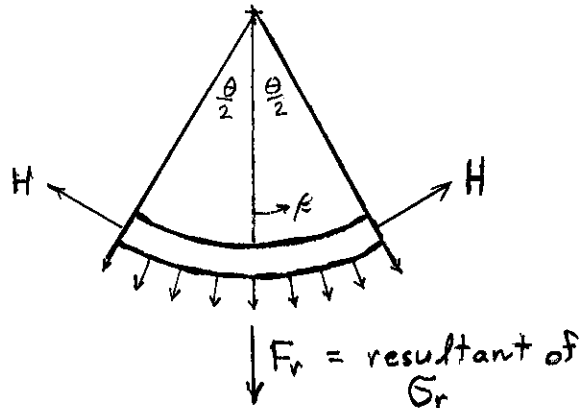


For portion above the neutral axis, the resultant force is

$$\begin{aligned} H &= \int \sigma_a dA = \int_{r_1}^R \sigma_a b dr \\ &= \frac{Mb}{Ae} \int_{r_1}^R dr - \frac{MRb}{Ae} \int_{r_1}^R \frac{dr}{r} \\ &= \frac{Mb}{Ae} (R - r_1) - \frac{MRb}{Ae} \ln \frac{R}{r_1} = \frac{MbR}{Ae} \left( 1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right) \end{aligned}$$

Resultant of  $\sigma_r$

$$\begin{aligned} F_r &= \int \sigma_r \cos \beta dA \\ &= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \sigma_r \cos \beta b R d\beta \\ &= \sigma_r b R \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos \beta d\beta \\ &= \sigma_r b R \sin \beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \\ &= 2 \sigma_r b R \sin \frac{\theta}{2} \end{aligned}$$



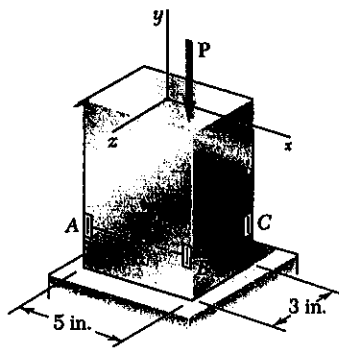
For equilibrium

$$F_r - 2H \sin \frac{\theta}{2} = 0$$

$$2 \sigma_r b R \sin \frac{\theta}{2} - 2 \frac{MbR}{Ae} \left( 1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right) = 0$$

$$\sigma_r = \frac{M}{Ae} \left( 1 - \frac{r_1}{R} - \ln \frac{R}{r_1} \right)$$

PROBLEM 4.211



4.211 A single vertical force  $P$  is applied to a short steel post as shown. Gages located at  $A$ ,  $B$ , and  $C$  indicate the following strains:

$$\epsilon_A = -500 \mu \quad \epsilon_B = -1000 \mu \quad \epsilon_C = -200 \mu$$

Knowing that  $E = 29 \times 10^6$  psi, determine (a) the magnitude of  $P$ , (b) the line of action of  $P$ , (c) the corresponding strain at the hidden edge of the post, where  $x = -2.5$  in. and  $z = -1.5$  in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in}^4$$

$$A = (5)(3) = 15 \text{ in}^2$$

$$M_x = Pz \quad M_z = -Px$$

$$x_A = -2.5 \text{ in}, \quad x_B = 2.5 \text{ in}, \quad x_C = 2.5 \text{ in}, \quad x_D = -2.5 \text{ in}$$

$$z_A = 1.5 \text{ in}, \quad z_B = 1.5 \text{ in}, \quad z_C = -1.5 \text{ in}, \quad z_D = -1.5 \text{ in}$$

$$\sigma_A = E\epsilon_A = (29 \times 10^6)(-500 \times 10^{-6}) = -14500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\epsilon_B = (29 \times 10^6)(-1000 \times 10^{-6}) = -29000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\epsilon_C = (29 \times 10^6)(-200 \times 10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} + \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667 P - 0.13333 M_x - 0.08 M_z \quad (1)$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667 P - 0.13333 M_x + 0.08 M_z \quad (2)$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667 P + 0.13333 M_x + 0.08 M_z \quad (3)$$

Substituting the values for  $\sigma_A$ ,  $\sigma_B$ , and  $\sigma_C$  into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87 \text{ kip}\cdot\text{in}, \quad M_z = -90.625 \text{ kip}\cdot\text{in}, \quad P = 152.25 \text{ kips} \quad \blackrightarrow$$

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25} = 0.595 \text{ in.} \quad \blackrightarrow$$

$$z = \frac{M_x}{P} = \frac{87}{152.25} = 0.571 \text{ in.} \quad \blackrightarrow$$

$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667 P + 0.13333 M_x - 0.08 M_z$$

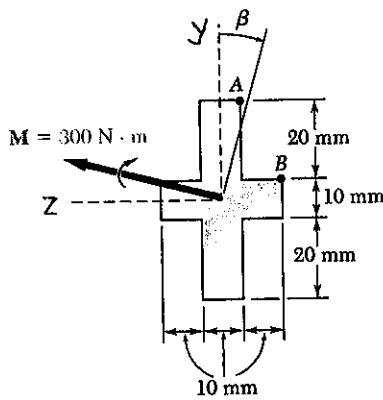
$$= -(0.06667)(152.25) + (0.13333)(87) + (0.08)(-90.625)$$

$$= 8.70 \text{ ksi} \quad \blackleftarrow$$



PROBLEM 4.212

4.212 The couple  $M$ , which acts in a vertical plane ( $\beta=0$ ), is applied to an aluminum beam of the cross section shown. Determine (a) the stress at point  $A$ , (b) the stress at point  $B$ , (c) the radius of curvature of the beam. Use  $E = 72 \text{ GPa}$ .



SOLUTION

Label axes  $y$  and  $z$  as shown on the sketch.

$$I_z = \frac{1}{12}(10)(50)^3 + 2 \cdot \frac{1}{12}(10)(10)^3$$

$$= 0.105833 \times 10^{-6} \text{ m}^4 = 0.105833 \times 10^{-6} \text{ m}^4$$

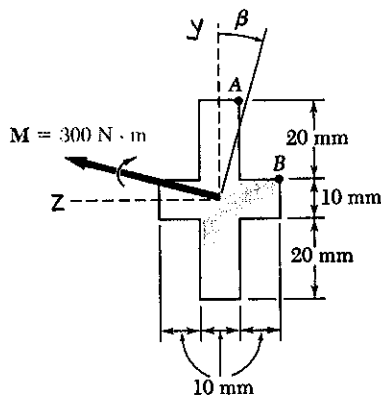
$$M_z = 300 \text{ N}\cdot\text{m} \quad M_y = 0$$

(a)  $y_A = 25 \text{ mm}$   $\sigma_A = -\frac{M_z y_A}{I_z} = -\frac{(300)(25 \times 10^{-3})}{0.105833 \times 10^{-6}} = -70.9 \times 10^6 \text{ Pa} = -70.9 \text{ MPa}$   $\blacktriangleleft$

(b)  $y_B = 5 \text{ mm}$   $\sigma_B = -\frac{M_z y_B}{I_z} = -\frac{(300)(5 \times 10^{-3})}{0.105833 \times 10^{-6}} = -14.17 \times 10^6 \text{ Pa} = -14.17 \text{ MPa}$   $\blacktriangleleft$

(c)  $\frac{1}{\rho} = \frac{M_z}{EI_z} \therefore \rho = \frac{EI_z}{M_z} = \frac{(72 \times 10^9)(0.105833 \times 10^{-6})}{300} = 25.4 \text{ m}$   $\blacktriangleleft$

**PROBLEM 4.213**



4.213 The couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta = 15^\circ$  with the vertical. Determine (a) the stress at point  $A$ , (b) the stress at point  $B$ , (c) the angle that the neutral axis forms with the horizontal.

**SOLUTION**

Label axes  $y$  and  $z$  as shown on the sketch.

$$I_z = \frac{1}{12}(10)(50)^3 + 2 \cdot \frac{1}{12}(10)(10)^3$$

$$= 0.105833 \times 10^6 \text{ mm}^4 = 0.105833 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(10)(30)^3 + 2 \cdot \frac{1}{12}(20)(10)^3$$

$$= 0.025833 \times 10^6 \text{ mm}^4 = 0.025833 \times 10^{-6} \text{ m}^4$$

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For  $\beta = 15^\circ$        $M_z = 300 \cos 15^\circ = 289.78 \text{ N}\cdot\text{m}$

$M_y = 300 \sin 15^\circ = 77.65 \text{ N}\cdot\text{m}$

(a)  $y_A = 25 \text{ mm}$ ,  $z_A = -5 \text{ mm}$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(289.78)(25 \times 10^{-3})}{0.105833 \times 10^{-6}} + \frac{(77.65)(-5 \times 10^{-3})}{0.025833 \times 10^{-6}}$$

$$= -83.5 \times 10^6 \text{ Pa} = -83.5 \text{ MPa}$$

(b)  $y_B = 5 \text{ mm}$ ,  $z_B = -15 \text{ mm}$

$$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(289.78)(5 \times 10^{-3})}{0.105833 \times 10^{-6}} + \frac{(77.65)(-15 \times 10^{-3})}{0.025833 \times 10^{-6}}$$

$$= -58.8 \times 10^6 \text{ Pa} = -58.8 \text{ MPa}$$

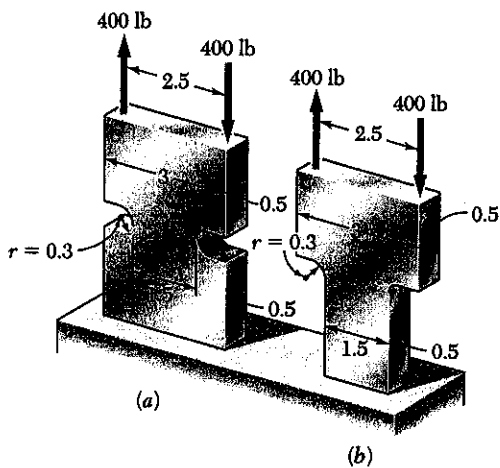
(c)  $\tan \phi = \frac{I_z}{I_y} \tan \theta = \frac{0.105833 \times 10^{-6}}{0.025833 \times 10^{-6}} \tan 15^\circ = 1.0977$

$\phi = 47.7^\circ$

PROBLEM 4.214

4.214 Determine the maximum stress in each of the two machine elements shown.

SOLUTION



All dimensions in inches

For each case  $M = (400)(2.5) = 1000 \text{ lb}\cdot\text{in}$

At the minimum section

$$I = \frac{1}{12} (0.5)(1.5)^3 = 0.140625 \text{ in}^4$$

$$c = 0.75 \text{ in.}$$

(a)  $D/d = 3/1.5 = 2$

$$r/d = 0.3/1.5 = 0.2$$

From Fig 4.32  $K = 1.75$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3 \text{ psi} = 9.33 \text{ ksi.}$$

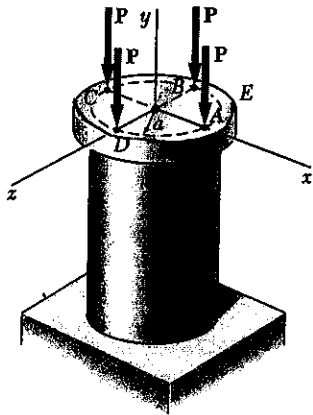
(b)  $D/d = 3/1.5 = 2$        $r/d = 0.3/1.5 = 0.2$

From Fig 4.31  $K = 1.50$

$$\sigma_{\max} = \frac{KMc}{I} = \frac{(1.50)(1000)(0.75)}{0.140625} = 8.00 \times 10^3 \text{ psi} = 8.00 \text{ ksi}$$

PROBLEM 4.215

4.215 The four forces shown are applied to a rigid plate supported by a solid steel post of radius  $a$ . Determine the maximum stress in the post when (a) all four forces are applied, (b) the force at  $D$  is removed, (c) the forces at  $C$  and  $D$  are removed.



SOLUTION

For a solid circular section of radius  $a$

$$A = \pi a^2 \quad I = \frac{\pi}{4} a^4$$

(a) Centric force  $F = 4P$ ,  $M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2} = -1.273 P/a^2 \quad \blacktriangleleft$$

(b) Force at  $D$  is removed.

$$F = 3P, \quad M_x = -Pa, \quad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^4} = -\frac{7P}{\pi a^2} = -2.228 P/a^2 \quad \blacktriangleleft$$

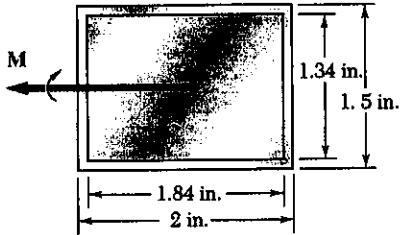
(c) Forces at  $C$  and  $D$  are removed

$$F = 2P \quad M_x = -Pa, \quad M_z = -Pa$$

Resultant bending couple  $M = \sqrt{M_x^2 + M_z^2} = \sqrt{2} Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2} Pa a}{\frac{\pi}{4} a^4} = -\frac{2+4\sqrt{2}}{\pi} \frac{P}{a^2} = -2.437 P/a^2 \quad \blacktriangleleft$$

PROBLEM 4.216



4.216 In order to increase corrosion resistance, a 0.08-in.-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is  $29 \times 10^6$  psi for steel and  $10.4 \times 10^6$  psi for aluminum. For a bending moment of 12 kip-in., determine (a) the maximum stress in the steel, (b) the maximum stress in the aluminum, (c) the radius of curvature of the bar.

SOLUTION

Use steel as the reference material

$$n_{\text{steel}} = 1 \quad n_{\text{alum}} = \frac{E_a}{E_s} = \frac{10.4}{29} = 0.3586$$

$$I_{\text{trans}} = I_{\text{steel}} + n_{\text{alum}} I_{\text{alum}}$$

$$= \frac{1}{12} (1.84)(1.34)^3 + 0.3586 \cdot \frac{1}{12} [(2)(1.5)^3 - (1.84)(1.34)^3] = 0.43835 \text{ in}^4$$

(a)  $y_s = \frac{1.34}{2} = 0.67 \text{ in}$

$$\sigma_s = \frac{M y_s}{I_{\text{trans}}} = \frac{(12)(0.67)}{0.43835} = 18.35 \text{ ksi}$$

(b)  $y_a = \frac{1.5}{2} = 0.75 \text{ in}$

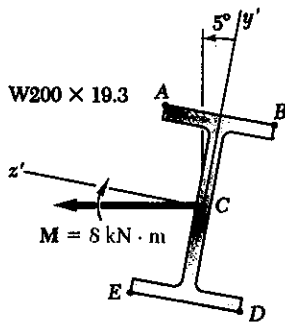
$$\sigma_a = n_a \frac{M y_a}{I} = 0.3586 \frac{(12)(0.75)}{0.43835} = 7.36 \text{ ksi}$$

(c)  $\frac{1}{\rho} = \frac{M}{E_{\text{ref}} I_{\text{trans}}} = \frac{12 \times 10^3}{(29 \times 10^6)(0.43835)} = 944 \times 10^{-6} \text{ in}^{-1}$

$$\rho = 1059 \text{ in} = 88.3 \text{ ft}$$

PROBLEM 4.217

4.217 A couple  $M$  of moment  $8 \text{ kN}\cdot\text{m}$  acting in a vertical plane is applied to a  $W 200 \times 19.3$  rolled-steel beam as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum stress in the beam.



SOLUTION

For  $W 200 \times 19.3$  rolled steel section

$$I_{z'} = 16.6 \times 10^6 \text{ mm}^4 = 16.6 \times 10^{-6} \text{ m}^4$$

$$I_{y'} = 1.15 \times 10^6 \text{ mm}^4 = 1.15 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = \frac{203}{2} = 101.5 \text{ mm}$$

$$z_A = -z_B = -z_D = z_E = \frac{102}{2} = 51 \text{ mm}$$

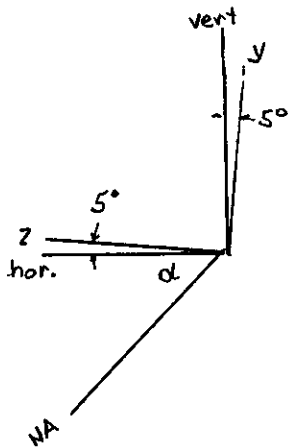
$$M_z = (8 \times 10^3) \cos 5^\circ = 7.9696 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = -(8 \times 10^3) \sin 5^\circ = -0.6972 \times 10^3 \text{ N}\cdot\text{m}$$

$$(a) \tan \phi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{16.6 \times 10^{-6}}{1.15 \times 10^{-6}} \tan (-5^\circ) = -1.2629$$

$$\phi = -51.6^\circ$$

$$\alpha = 51.6^\circ - 5^\circ = 46.6^\circ$$



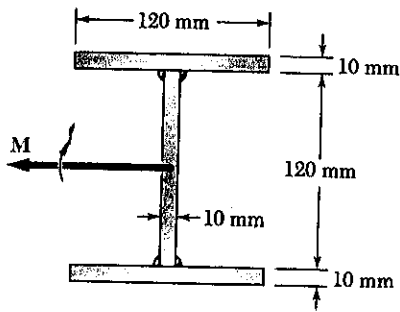
(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_{z'}} + \frac{M_y z_D}{I_{y'}} = -\frac{(7.9696 \times 10^3)(-101.5 \times 10^{-3})}{16.6 \times 10^{-6}} + \frac{(0.6972 \times 10^3)(51 \times 10^{-3})}{1.15 \times 10^{-6}}$$

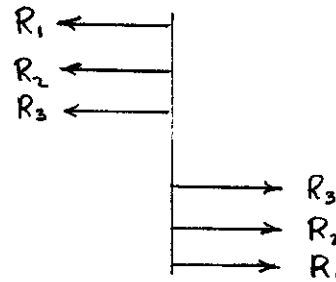
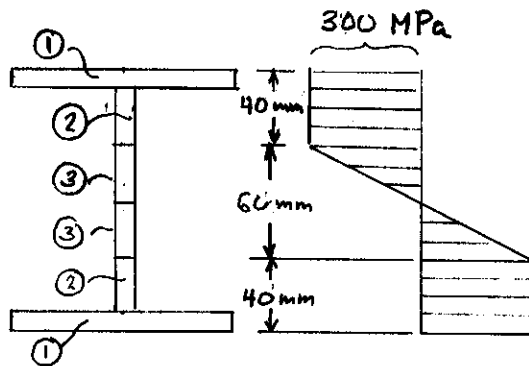
$$= 79.6 \times 10^6 \text{ Pa} = 79.6 \text{ MPa}$$

PROBLEM 4.218

4.218 Three  $120 \times 10$ -mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with  $E = 200$  GPa and  $\sigma_y = 300$  MPa, determine (a) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (b) the corresponding radius of curvature of the beam.



SOLUTION



$$A_1 = (120)(10) = 1200 \text{ mm}^2$$

$$R_1 = \sigma_y A_1 = (300 \times 10^6)(1200 \times 10^{-6}) = 360 \times 10^3 \text{ N}$$

$$A_2 = (30)(10) = 300 \text{ mm}^2$$

$$R_2 = \sigma_y A_2 = (300 \times 10^6)(300 \times 10^{-6}) = 90 \times 10^3 \text{ N}$$

$$A_3 = (30)(10) = 300 \text{ mm}^2$$

$$R_3 = \frac{1}{2} \sigma_y A_2 = \frac{1}{2} (300 \times 10^6)(300 \times 10^{-6}) = 45 \times 10^3 \text{ N}$$

$$y_1 = 65 \text{ mm} = 65 \times 10^{-3} \text{ m}$$

$$y_2 = 45 \text{ mm} = 45 \times 10^{-3} \text{ m}$$

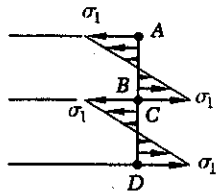
$$y_3 = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$(a) \quad M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$$

$$= 56.7 \times 10^3 \text{ N}\cdot\text{m} = 56.7 \text{ kN}\cdot\text{m}$$

$$(b) \quad \frac{y}{\rho} = \frac{\sigma_y}{E} \quad \rho = \frac{E y}{\sigma_y} = \frac{(200 \times 10^9)(30 \times 10^{-3})}{300 \times 10^6} = 20 \text{ m}$$

PROBLEM 4.219



4.219 Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by  $\sigma_1$  the maximum stress and by  $\rho_1$  the radius of curvature of each strip while the couples were applied, determine (a) the final stresses at points A, B, C, and D, (b) the final radius of curvature

SOLUTION

Let  $b$  = width and  $t$  = thickness of one strip.

Loading one strip.  $M = M_1$ ,

$$I_1 = \frac{1}{12} b t^3, \quad c = \frac{1}{2} t$$

$$\sigma_1 = \frac{M_1 c}{I_1} = \frac{6 M_1}{b t^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{E I_1} = \frac{12 M_1}{E t^3}$$

After  $M_1$  is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is  $2M_1$ .

After gluing, this couple is removed.

$$M' = 2M_1, \quad I' = \frac{1}{12} b (2t)^3 = \frac{2}{3} b t^3$$

$c = t$ . The stresses removed are

$$\sigma' = -\frac{M' y}{I'} = -\frac{2M_1 y}{\frac{2}{3} b t^3} = -\frac{3M_1 y}{b t^2}$$

$$\sigma'_A = -\frac{3M_1}{b t^2} = -\frac{1}{2} \sigma_1, \quad \sigma'_B = \sigma'_C = 0, \quad \sigma'_D = \frac{3M_1}{b t^2} = \frac{1}{2} \sigma_1$$

(a) Final stresses:  $\sigma_A = -\sigma_1 - (-\frac{1}{2} \sigma_1) = -\frac{1}{2} \sigma_1$

$$\sigma_B = \sigma_1$$

$$\sigma_C = -\sigma_1$$

$$\sigma_D = \sigma_1 - \frac{1}{2} \sigma_1 = \frac{1}{2} \sigma_1$$

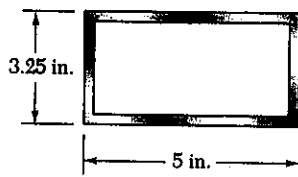
$$\frac{1}{\rho'} = \frac{M'}{E I'} = \frac{2M_1}{E \frac{2}{3} b t^3} = \frac{3M_1}{E t^3} = \frac{1}{4} \frac{1}{\rho_1}$$

(b) Final radius  $\frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4\rho_1} = \frac{3}{4} \frac{1}{\rho_1}$

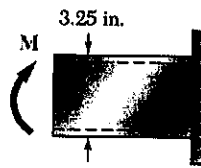
$$\rho = \frac{4}{3} \rho_1$$



**PROBLEM 4.220**



4.220 Knowing that the hollow beam shown has a uniform wall thickness of 0.25 in. determine (a) the largest couple that can be applied without exceeding the allowable stress of 20 ksi, (b) the corresponding radius of curvature of the beam.



**SOLUTION**

$$E = 10.6 \times 10^6 \text{ psi}$$

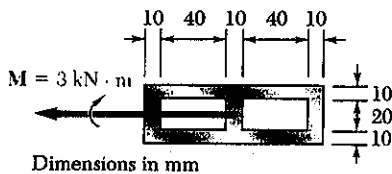
$$I = \frac{1}{12} b_o h^3 - \frac{1}{12} b_i h_i^3 = \frac{1}{12} (5)(3.25)^3 - \frac{1}{12} (4.5)(2.75)^3 = 6.5046 \text{ in}^4$$

$$c = \frac{3.25}{2} = 1.625 \text{ in.}$$

$$(a) \sigma_{max} = \frac{Mc}{I} \therefore M = \frac{\sigma_{max} I}{c} = \frac{(20)(6.5046)}{1.625} = 80.1 \text{ kip}\cdot\text{in.}$$

$$(b) \epsilon_{max} = \frac{c}{\rho} = \frac{\sigma_{max}}{E} \therefore \rho = \frac{Ec}{\sigma_{max}} = \frac{(10.6 \times 10^6)(1.625)}{20 \times 10^3} = 861 \text{ in} = 71.8 \text{ ft.}$$

**PROBLEM 4.221**



4.221 A beam of the cross section shown is extruded from an aluminum alloy for which  $E = 72 \text{ GPa}$ . Knowing that the couple shown acts in a vertical plane, determine (a) the maximum stress in the beam, (b) the corresponding radius of curvature.

**SOLUTION**

For outer rectangle:  $b = 110 \text{ mm}, h = 40 \text{ mm}$

$$I_1 = \frac{1}{12} bh^3 = \frac{1}{12} (110)(40)^3 = 0.58667 \times 10^6 \text{ mm}^4$$

For one cutout rectangle:  $b = 40 \text{ mm}, h = 20 \text{ mm}$

$$I_2 = \frac{1}{12} bh^3 = \frac{1}{12} (40)(20)^3 = 0.02667 \times 10^6 \text{ mm}^4$$

$$I = I_1 - 2I_2 = 0.53333 \times 10^6 \text{ mm}^4 = 0.53333 \times 10^{-6} \text{ m}^4$$

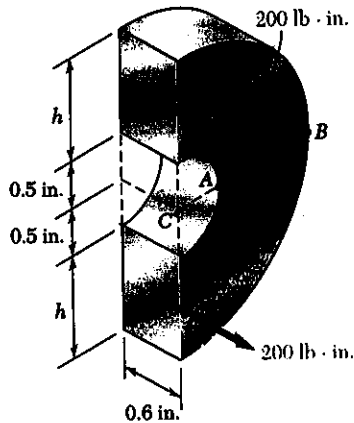
$$M = 3 \times 10^3 \text{ N}\cdot\text{m} \quad c = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$(a) \sigma = \frac{Mc}{I} = \frac{(3 \times 10^3)(20 \times 10^{-3})}{0.53333 \times 10^{-6}} = 112.5 \times 10^6 \text{ Pa} = 112.5 \text{ MPa}$$

$$(b) \frac{1}{\rho} = \frac{M}{EI} \therefore \rho = \frac{EI}{M} = \frac{(72 \times 10^9)(0.53333 \times 10^{-6})}{3 \times 10^3} = 12.80 \text{ m}$$

PROBLEM 4.222

4.222 For the machine element and loading shown, determine the stress at point A, knowing that (a)  $h = 0.9$  in., (b)  $h = 1.5$  in.



SOLUTION

(a)  $h = 0.9$  in,  $r_1 = 0.5$  in,  $r_2 = 1.4$  in

$$A = (0.6)(0.9) = 0.54 \text{ in}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{0.9}{\ln \frac{1.4}{0.5}} = 0.87411 \text{ in.}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 0.95 \text{ in}$$

$$e = \bar{r} - R = 0.07589 \text{ in}$$

$$M = -200 \text{ lb}\cdot\text{in}, \quad y_A = R - r_1 = 0.37411$$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-200)(0.37411)}{(0.54)(0.07589)(0.5)} = \frac{3.65 \times 10^3 \text{ psi}}{= 3.65 \text{ ksi}}$$

(b)  $h = 1.5$  in,  $r_1 = 0.5$  in,  $r_2 = 2.0$  in.

$$A = (0.6)(1.5) = 0.90 \text{ in}^2$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{1.5}{\ln \frac{2.0}{0.5}} = 1.08202 \text{ in}$$

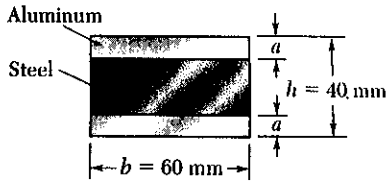
$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 1.25 \text{ in.}$$

$$e = \bar{r} - R = 0.16798 \text{ in.}$$

$$M = -200 \text{ lb}\cdot\text{in.} \quad y_A = R - r_1 = 0.58202 \text{ in}$$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-200)(0.58202)}{(0.90)(0.16798)(0.5)} = \frac{1.540 \times 10^3 \text{ psi}}{= 1.540 \text{ ksi}}$$

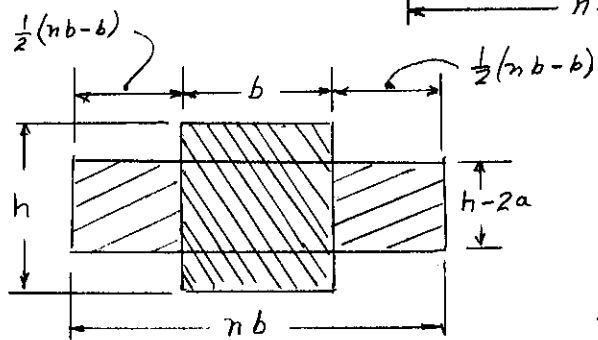
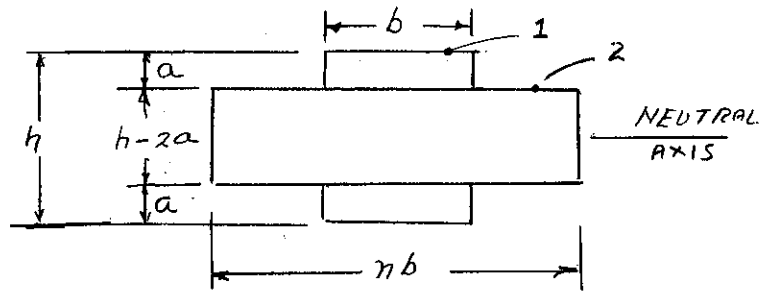
**PROBLEM 4.C1**



**4.C1** Two aluminum strips and a steel strip are to be bonded together to form a composite member of width  $b = 60$  mm and depth  $h = 40$  mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that  $M = 1500$  N · m, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of  $a$  from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (a) the largest stress that can occur in the steel, (b) the corresponding value of  $a$ .

**SOLUTION**

TRANSFORMED SECTION (ALL STEEL)  $\eta = \frac{E_{STEEL}}{E_{ALUM}}$



$$\bar{I} = \frac{1}{12} b h^3 + \frac{1}{12} \left[ 2 \left( \frac{1}{2} \right) (nb - b) \right] (h - 2a)^3$$

AT POINT 1:  $\tau_{ALUM} = \frac{M \left( \frac{h}{2} \right)}{\bar{I}}$

AT POINT 2:  $\tau_{STEEL} = \eta \frac{M \left( \frac{h}{2} - a \right)}{\bar{I}}$

FOR  $a = 0$  TO 20 mm USING 2-mm INTERVALS COMPUTE:  $\eta, \bar{I}, \tau_{ALUM}, \tau_{STEEL}$ .

$b = 60$  mm     $h = 40$  mm     $M = 1500$  N.m  
Moduli of elasticity: Steel = 200 GPa    Aluminum = 75 GPa

PROGRAM OUTPUT

a mm	I m <sup>4</sup> /10 <sup>6</sup>	sigma aluminum MPa	sigma steel MPa
0.000	0.8533	35.156	93.750
2.000	0.7088	42.325	101.580
4.000	0.5931	50.585	107.914
6.000	0.5029	59.650	111.347
8.000	0.4352	68.934	110.294
10.000	0.3867	77.586	103.448
12.000	0.3541	84.714	90.361
14.000	0.3344	89.713	71.770
16.000	0.3243	92.516	49.342
18.000	0.3205	93.594	24.958
20.000	0.3200	93.750	0.000

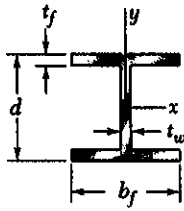
Find 'a' for max steel stress  
and the corresponding aluminum stress

6.600	0.4804	62.447	111.572083
6.610	0.4800	62.494	111.572159
6.620	0.4797	62.540	111.572113

Max Steel Stress = 111.6 MPa occurs when  $a = 6.61$  mm  
Corresponding Aluminum stress = 62.5 MPa

**PROBLEM 4.C2**

**4.C2** A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength  $\sigma_y$  and a modulus of elasticity  $E$ , is bent about the  $x$  axis. (a) Denoting by  $y_y$  the half thickness of the elastic core, write a computer program to calculate the bending moment  $M$  and the radius of curvature  $\rho$  for values of  $y_y$  from  $\frac{1}{2}d$  to  $\frac{1}{6}d$  using decrements equal to  $\frac{1}{2}t_f$ . Neglect the effect of fillets. (b) Use this program to solve Prob. 4.218.



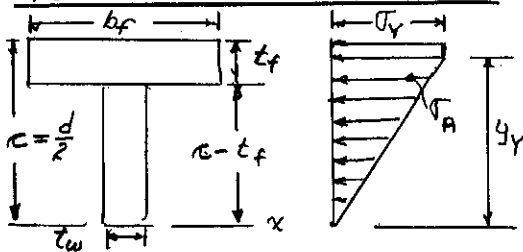
**SOLUTION**

COMPUTE MOMENT OF INERTIA  $I_x$

$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12} (b_f - t_w) (d - 2t_f)^3$$

MAXIMUM ELASTIC MOMENT:  $M_Y = \sigma_Y \frac{I_x}{(d/2)}$

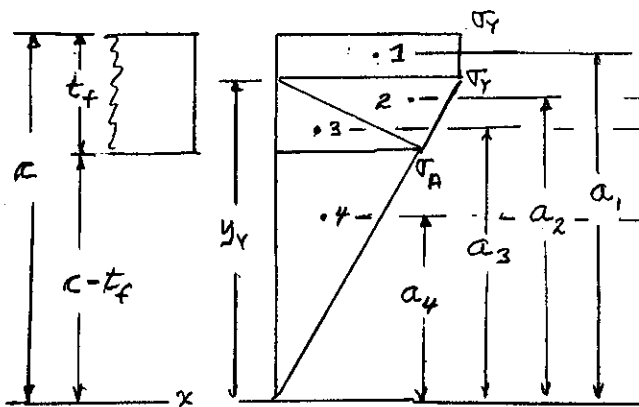
FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION)  $c = \frac{d}{2}$



STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2) - t_f}{y_y} \sigma_Y$$

DETAIL OF STRESS DIAGRAM



RESULTANT FORCES

$$R_1 = \sigma_Y [b_f (c - \sigma_Y)]$$

$$R_2 = \frac{1}{2} \sigma_Y b_f [y_y - (c - t_f)]$$

$$R_3 = \frac{1}{2} \sigma_A b_f [y_y - (c - t_f)]$$

$$R_4 = \frac{1}{2} \sigma_A t_w (c - t_f)$$

$$a_1 = \frac{1}{2} (c + y_y)$$

$$a_2 = y_y - \frac{1}{3} [y_y - (c - t_f)]$$

$$a_3 = y_y - \frac{2}{3} [y_y - (c - t_f)]$$

$$a_4 = \frac{2}{3} (c - t_f)$$

BENDING MOMENT

$$M = 2 \sum_{n=1}^4 R_n a_n$$

RADIUS OF CURVATURE

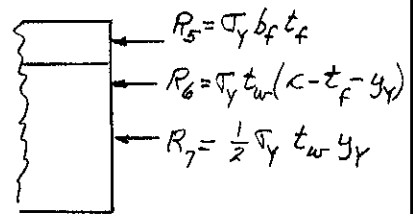
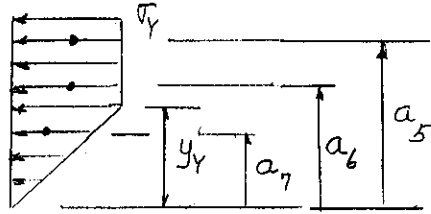
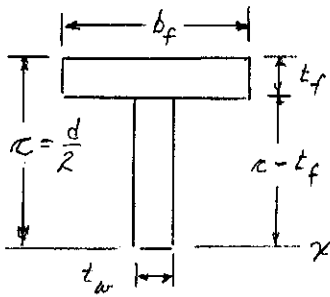
$$y_y = \epsilon_Y \rho = \frac{\sigma_Y}{E} \rho ; \rho = \frac{y_y E}{\sigma_Y}$$

CONTINUED

**PROBLEM 4.C2 - CONTINUED**

FOR YIELDING IN THE WEB

(CONSIDER UPPER HALF OF CROSS SECTION)



$$a_5 = c - \frac{1}{2} t_f$$

$$a_6 = \frac{1}{2} [y_Y + (c - t_f)]$$

$$a_7 = \frac{2}{3} y_Y$$

BENDING MOMENT

$$M = 2 \sum_{n=5}^7 R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = E_y \rho = \frac{\sigma_Y}{E} \rho \quad \rho = \frac{y_Y E}{\sigma_Y}$$

PROGRAM: KEY IN EXPRESSIONS FOR  $a_n$  AND  $R_n$  FOR  $n=1$  TO  $7$ .

FOR  $y_Y = c$  TO  $(c - t_f)$  AT  $-t_f/2$  DECREMENTS

COMPUTE  $M = 2 \sum R_n a_n$  FOR  $n=1$  TO  $4$  AND  $\rho = \frac{y_Y E}{\sigma_Y}$ , THEN PRINT

FOR  $y_Y = (c - t_w)$  TO  $c/3$  AT  $-t_f/2$  DECREMENTS

COMPUTE  $M = 2 \sum R_n a_n$  FOR  $n=5$  TO  $7$  AND  $\rho = \frac{y_Y E}{\sigma_Y}$ , THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

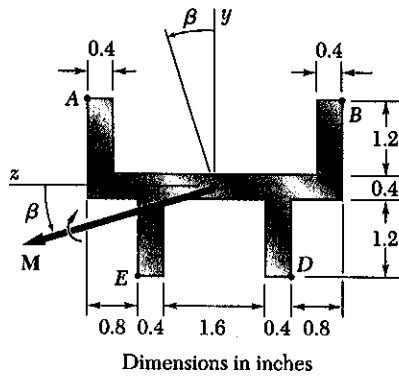
PROGRAM OUTPUT

For a beam of Prob 4.218  
 Depth  $d = 140.00$  mm      Width of flange  $b_f = 120.00$  mm  
 Thickness of flange  $t_f = 10.00$  mm      Thickness of web  $t_w = 10.00$  mm

$I = 0.000011600$  m to the 4th  
 Yield strength of Steel  $\sigma_{Y} = 300$  MPa  
 Yield Moment  $M_Y = 49.71$  kip.in.

y <sub>Y</sub> (mm)	M (kN.m)	rho (m)
For yielding still in the flange.		
70.000	49.71	46.67
65.000	52.59	43.33
60.000	54.00	40.00
For yielding in the web		
60.000	54.00	40.00
55.000	54.58	36.67
50.000	55.10	33.33
45.000	55.58	30.00
40.000	56.00	26.67
35.000	56.38	23.33
30.000	56.70	20.00
25.000	56.97	16.67

**PROBLEM 4.C3**



**4.C3** An 8 kip · in. couple  $M$  is applied to a beam of the cross section shown in a plane forming an angle  $\beta$  with the vertical. Noting that the centroid of the cross section is located at  $C$  and that the  $y$  and  $z$  axes are principal axes, write a computer program to calculate the stress at  $A$ ,  $B$ ,  $C$ , and  $D$  for values of  $\beta$  from  $0$  to  $180^\circ$  using  $10^\circ$  increments. (Given:  $I_y = 6.23 \text{ in}^4$  and  $I_z = 1.481 \text{ in}^4$ .)

**SOLUTION**

INPUT COORDINATES OF A, B, C, D

$$\begin{aligned} z_A = z(1) &= 2 & y_A = y(1) &= 1.4 \\ z_B = z(2) &= -2 & y_B = y(2) &= 1.4 \\ z_C = z(3) &= -1 & y_C = y(3) &= -1.4 \\ z_D = z(4) &= 1 & y_D = y(4) &= -1.4 \end{aligned}$$

COMPONENTS OF M.

$$M_y = -M \sin \beta \qquad M_z = M \cos \beta$$

Eq 4.55 page 273:  $\sigma(n) = -\frac{M_z y(n)}{I_z} + \frac{M_y z(n)}{I_y}$

```
PROGRAM: FOR beta = 0 TO 180 USING 10 INCREMENTS,
FOR n = 1 TO 4 USING UNIT INCREMENTS,
EVALUATE EQ 4.55 AND PRINT STRESS
RETURN
RETURN
```

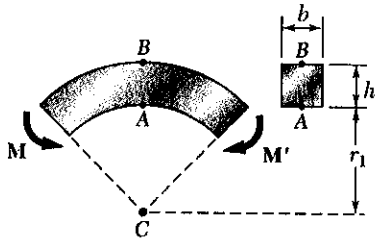
PROGRAM OUTPUT

Moment of couple  $M = 8.00 \text{ kip}\cdot\text{in.}$   
 Moments of inertia:  $I_y = 6.23 \text{ in}^4$        $I_z = 1.481 \text{ in}^4$

Coordinates of points A, B, D, and E  
 Point A:  $z(1) = 2$ ;       $y(1) = 1.4$   
 Point B:  $z(2) = -2$ ;       $y(2) = 1.4$   
 Point D:  $z(3) = -1$ ;       $y(3) = -1.4$   
 Point E:  $z(4) = 1$ ;       $y(4) = -1.4$

- - - Stress at Points - - -				
beta °	A ksi	B ksi	D ksi	E ksi
0	-7.565	-7.565	7.565	7.565
10	-7.896	-7.004	7.673	7.227
20	-7.987	-6.230	7.548	6.669
30	-7.836	-5.267	7.193	5.909
40	-7.446	-4.144	6.621	4.970
50	-6.830	-2.895	5.846	3.879
60	-6.007	-1.558	4.895	2.670
70	-5.001	-0.174	3.794	1.381
80	-3.843	1.216	2.578	0.049
90	-2.569	2.569	1.284	-1.284
100	-1.216	3.843	-0.049	-2.578
110	0.174	5.001	-1.381	-3.794
120	1.558	6.007	-2.670	-4.895
130	2.895	6.830	-3.879	-5.846
140	4.144	7.446	-4.970	-6.621
150	5.267	7.836	-5.909	-7.193
160	6.230	7.987	-6.669	-7.548
170	7.004	7.896	-7.227	-7.673
180	7.565	7.565	-7.565	-7.565

**PROBLEM 4.C4**



**4.C4** Couples of moment  $M = 2 \text{ kN} \cdot \text{m}$  are applied as shown to a curved bar having a rectangular cross section with  $h = 100 \text{ mm}$  and  $b = 25 \text{ mm}$ . Write a computer program and use it to calculate the stresses at points A and B for values of the ratio  $r_1/h$  from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio  $r_1/h$  for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a straight bar of the same cross section.

**SOLUTION** INPUT:  $h = 100 \text{ mm}$ ,  $b = 25 \text{ mm}$ ,  $M = 2.2 \text{ kN} \cdot \text{m}$

FOR STRAIGHT BAR:  $\sigma_{\text{STRAIGHT}} = \frac{M}{S} = \frac{6M}{h^2 b} = 48 \text{ MPa}$

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$r_2 = h + r_1$  ;  $R = h / \ln(r_2/r_1)$  ;  $\bar{r} = r_1 + r_2$  ;  $e = \bar{r} - R$  ;  $A = bh = 2500$  (I)

STRESSES:  $\sigma_A = \sigma_1 = M(r_1 - R) / (Ae r_1)$  ;  $\sigma_B = \sigma_2 = M(r_2 - R) / (Ae r_2)$  (II)

SINCE  $h = 100 \text{ mm}$ , FOR  $r_1/h = 10$ ,  $r_1 = 1000 \text{ mm}$ . ALSO  $r_1/h = 10$ ,  $r_2 = 1100$

PROGRAM: FOR  $r_1 = 1000$  TO  $100$  AT  $-100$  DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE  $r_2, R, \bar{r}, e, \sigma_1$  AND  $\sigma_2$

ALSO EVALUATE:  $\text{RATIO} = \sigma_1 / \sigma_{\text{STRAIGHT}}$

RETURN AND REPEAT FOR  $r_1 = 100$  TO  $10$  AT  $-10$  DECREMENT

PROGRAM OUTPUT

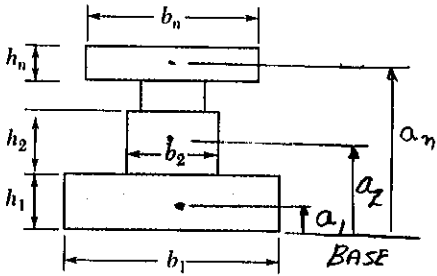
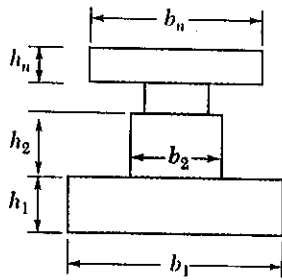
M = Bending Moment = 2. kN.m    h = 100.000 in.    A = 2500.00 mm<sup>2</sup>  
 Stress in straight beam = 48.00 MPa

r1 mm	rbar mm	R mm	e mm	sigma1 MPa	sigma2 MPa	r1/h	ratio
1000	1050	1049	0.794	-49.57	46.51	10.000	-1.033
900	950	949	0.878	-49.74	46.36	9.000	-1.036
800	850	849	0.981	-49.95	46.18	8.000	-1.041
700	750	749	1.112	-50.22	45.95	7.000	-1.046
600	650	649	1.284	-50.59	45.64	6.000	-1.054
500	550	548	1.518	-51.08	45.24	5.000	-1.064
400	450	448	1.858	-51.82	44.66	4.000	-1.080
300	350	348	2.394	-53.03	43.77	3.000	-1.105
200	250	247	3.370	-55.35	42.24	2.000	-1.153
100	150	144	5.730	-61.80	38.90	1.000	-1.288
=====							
100	150	144	5.730	-61.80	38.90	1.000	-1.288
90	140	134	6.170	-63.15	38.33	0.900	-1.316
80	130	123	6.685	-64.80	37.69	0.800	-1.350
70	120	113	7.299	-66.86	36.94	0.700	-1.393
60	110	102	8.045	-69.53	36.07	0.600	-1.449
50	100	91	8.976	-73.13	35.04	0.500	-1.523
40	90	80	10.176	-78.27	33.79	0.400	-1.631
30	80	68	11.803	-86.30	32.22	0.300	-1.798
20	70	56	14.189	-100.95	30.16	0.200	-2.103
10	60	42	18.297	-138.62	27.15	0.100	-2.888

Find r1/h for (sigma max)/(sigma straight) = 1.5  
 52.70 103    94    8.703    -72.036    35.34    0.527    -1.501  
 52.80 103    94    8.693    -71.998    35.35    0.528    -1.500  
 52.90 103    94    8.683    -71.959    35.36    0.529    -1.499  
 Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

[ Note: The desired ratio r1/h is valid for any beam having a rectangular cross section. ]

**PROBLEM 4.C5**



**4.C5** The couple  $M$  is applied to a beam of the cross section shown. (a) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (b) Use this program to solve Probs. 4.1, 4.10, and 4.11.

**SOLUTION**

INPUT: BENDING MOMENT  $M$



FOR  $n=1$  TO  $n$ : ENTER  $b_n$  AND  $h_n$   
 $\Delta \text{AREA} = b_n h_n$  (PRINT)

$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$   
 [ MOMENT OF RECTANGLE ABOUT BASE ]

$\Delta m = (\Delta \text{AREA}) a_n$   
 [ FOR WHOLE CROSS SECTION ]

$m = m + \Delta m$  ;  $\text{AREA} = \text{AREA} + \Delta \text{AREA}$

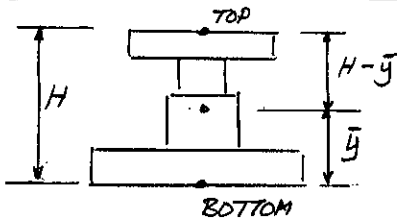
LOCATION OF CENTROID ABOVE BASE

$\bar{y} = m / \text{AREA}$  (PRINT)

MOMENT OF INERTIA ABOUT HORIZONTAL CENTROIDAL AXIS

FOR  $n=1$  TO  $n$ :  $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$   
 $\Delta I = b_n h_n^3 / 12 + (b_n h_n) (\bar{y} - a_n)^2$   
 $I = I + \Delta I$  (PRINT)

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR  $n=1$  TO  $n$   
 $H = H + h_n$

STRESS AT TOP

$$M_{\text{TOP}} = -M \frac{H - \bar{y}}{I}$$
 (PRINT)

STRESS AT BOTTOM

$$M_{\text{BOTTOM}} = M \frac{\bar{y}}{I}$$
 (PRINT)

SEE NEXT PAGE FOR PRINT OUTS FOR PROBLEMS 4.1, 4.10, 4.11

CONTINUED



**PROBLEM 4.C5 - CONTINUED**

Problem 4.1

Summary of Cross Section Dimensions

Width (in.)	Height (in.)
2.00	2.00
6.00	1.50
2.00	2.00

Bending Moment = 25.000 kip.in.  
Centroid is 2.750 in. above lower edge  
Centroidal Moment of Inertia is 28.854 in<sup>4</sup>

Stress at top of beam = -2.383 ksi  
Stress at bottom of beam = 2.383 ksi

Problem 4.10

Summary of Cross Section Dimensions

Width (in.)	Height (in.)
9.00	2.00
3.00	6.00

Bending Moment = 600.000 kip.in.  
Centroid is 3.000 in. above lower edge  
Centroidal Moment of Inertia is 204.000 in<sup>4</sup>

Stress at top of beam = -14.706 ksi  
Stress at bottom of beam = 8.824 ksi

Problem 4.11

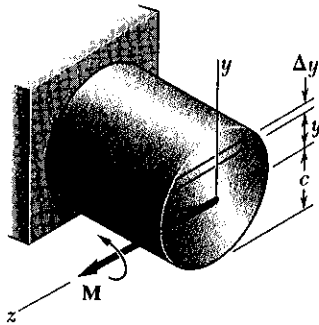
Summary of Cross Section Dimensions

Width (in.)	Height (in.)
4.00	1.00
1.00	6.00
8.00	1.00

Bending Moment = 500.000 kip.in.  
Centroid is 4.778 in. above lower edge  
Centroidal Moment of Inertia is 155.111 in<sup>4</sup>

Stress at top of beam = -10.387 ksi  
Stress at bottom of beam = 15.401 ksi

**PROBLEM 4.C6**



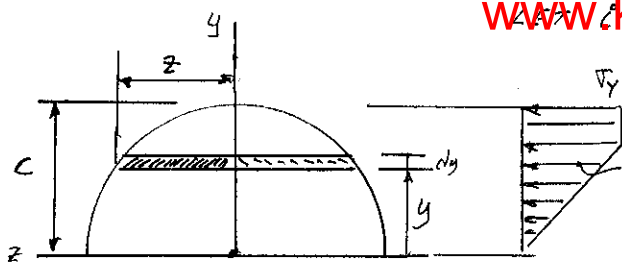
**4.C6** A solid rod of radius  $c = 1.2$  in. is made of a steel that is assumed to be elastoplastic with  $E = 29,000$  ksi and  $\sigma_y = 42$  ksi. The rod is subjected to a couple of moment  $M$  that increases from zero to the maximum elastic moment  $M_y$  and then to the plastic moment  $M_p$ . Denoting by  $y_y$  the half thickness of the elastic core, write a computer program and use it to calculate the bending moment  $M$  and the radius of curvature  $\rho$  for values of  $y_y$  from 1.2 in. to 0 using 0.2-in. decrements. (Hint: Divide the cross section into 80 horizontal elements of 0.03-in. height.)

**SOLUTION**

$$M_y = \sigma_y \frac{\pi}{4} c^3 = (42 \text{ ksi}) \frac{\pi}{4} (1.2 \text{ in.})^3 = 57 \text{ kip}\cdot\text{in.}$$

$$M_p = \sigma_y \frac{4}{3} c^3 = (42 \text{ ksi}) \frac{4}{3} (1.2 \text{ in.})^3 = 96.8 \text{ kip}\cdot\text{in.}$$

CONSIDER TOP HALF OF ROD  
 NUMBER OF ELEMENTS IN TOP HALF



$\Delta h = \text{HEIGHT OF EACH ELEMENT} : \Delta h = \frac{c}{L}$

$\sigma_E = \text{STRESS ON ELEMENT}$

FOR  $\eta = 0$  TO  $L-1$  STEP 1

$y = \eta (\Delta h)$

$z = [c^2 - \{(\eta + 0.5)\Delta h\}^2]^{.5}$

←  $z$  AT MIDGEAT OF ELEMENT

IF  $y \geq y_y$  GO TO 100

$\sigma_E = \sigma_y \frac{(\eta + 0.5)\Delta h}{y_y}$

← STRESS IN ELASTIC CORE

GO TO 200

$\sigma_E = \sigma_y$

← STRESS IN PLASTIC ZONE

100

200

$\Delta \text{AREA} = 2z(\Delta h)$

$\Delta \text{FORCE} = \sigma_E (\Delta \text{AREA})$

$\Delta \text{MOMENT} = \Delta \text{FORCE} (\eta + 0.5) \Delta h$

$M = M + \Delta \text{MOMENT}$

$\rho = y_y E / \sigma_y$

PRINT  $y_y, M, \text{ AND } \rho.$

NEXT

REPEAT

FOR

$y_y = 1.2 \text{ in.}$

TO

$y_y = 0$

AT  $-0.2$ -in.

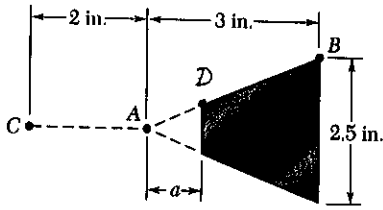
DECREMENTS

PROGRAM OUTPUT

Radius of rod = 1.2 in.  
 Yield point of steel = 42 ksi  
 Yield moment = 57.0 kip·in      Plastic moment = 96.8 kip·in.  
 Number of elements in half of the rod = 40

For $y_y = 1.20$ in.	$M = 57.1$ kip·in.	Radius of curvature = 828.57 in.
For $y_y = 1.00$ in.	$M = 67.2$ kip·in.	Radius of curvature = 690.48 in.
For $y_y = 0.80$ in.	$M = 76.9$ kip·in.	Radius of curvature = 552.38 in.
For $y_y = 0.60$ in.	$M = 85.2$ kip·in.	Radius of curvature = 414.29 in.
For $y_y = 0.40$ in.	$M = 91.6$ kip·in.	Radius of curvature = 276.19 in.
For $y_y = 0.20$ in.	$M = 95.5$ kip·in.	Radius of curvature = 138.10 in.
For $y_y = 0.00$ in.	$M = \text{infinite}$	Radius of curvature = zero

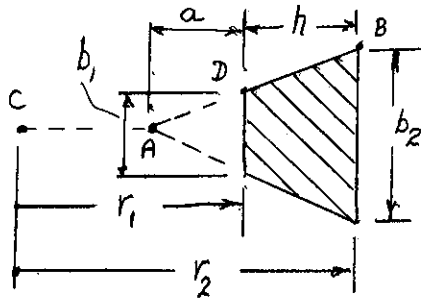
**PROBLEM 4.C7**



4.C7 The machine element of Prob. 4.204 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width  $a$  will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of  $a$  from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance  $a$  for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

**SOLUTION** SEE FIG 4.79 PAGE 289

$$M = 5 \text{ kip}\cdot\text{in.} \quad r_2 = 5 \text{ in.} \quad b_2 = 2.5 \text{ in}$$



FOR  $a$  FROM 0 TO 1 IN. AT 0.1 INTERVALS  
 $h = 3 - a$

$$r_1 = 2 + a$$

$$b_1 = b_2 (a / (h + a))$$

$$\text{AREA} = (b_1 + b_2)(h/2)$$

$$\bar{x} = a \left[ \frac{1}{2} b_1 h (h/3) + \frac{1}{2} b_2 h (2h/3) \right] / \text{AREA}$$

$$\bar{r} = r_2 - (h - \bar{x})$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$e = \bar{r} - R$$

$$\sigma_D = M(r_1 - R) / [\text{AREA}(e)(r_1)]$$

$$\sigma_B = M(r_2 - R) / [\text{AREA}(e)(r_2)]$$

PRINT AND RETURN

PROGRAM OUTPUT

a	R	sigmaD	sigmaB	b1	rbar	e
in.	in.	ksi	ksi			
0.00	3.855	-8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	-7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	-7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	-6.9260	2.2438	0.25	4.02	0.134
0.40	3.904	-6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	-6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	-6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	-6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	-6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	-6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	-6.9647	3.4992	0.83	4.17	0.078

Determination of the maximum compressive stress that is as small as possible

a	R	sigmaD	sigmaB	b1	rbar	e
in.	in.	ksi	ksi			
0.620	3.961	-6.51198	2.6425	0.52	4.07	0.109
0.625	3.963	-6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	-6.51188	2.6591	0.52	4.07	0.109

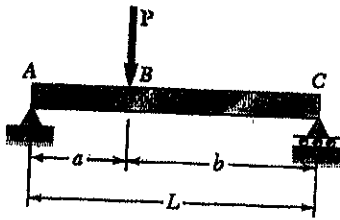
ANSWER: When  $a = 0.625$  in. the compressive stress is 6.51 ksi

# CHAPTER 5

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**PROBLEM 5.1**

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



**SOLUTION**

Reactions

$$\sum M_c = 0 \quad LA - bP = 0 \quad A = \frac{Pb}{L}$$

$$\sum M_a = 0 \quad LC - aP = 0 \quad C = \frac{Pa}{L}$$

From A to B  $0 < x < a$

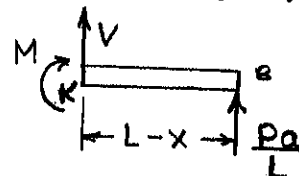
$$\uparrow \sum F_y = 0 \quad \frac{Pb}{L} - V = 0$$

$$V = \frac{Pb}{L}$$

$$\curvearrowright \sum M_J = 0 \quad M - \frac{Pb}{L}x = 0$$

$$M = \frac{Pbx}{L}$$

From B to C  $a < x < L$

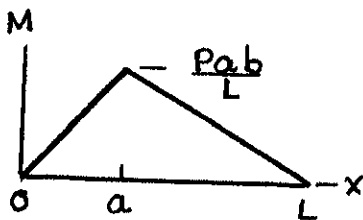
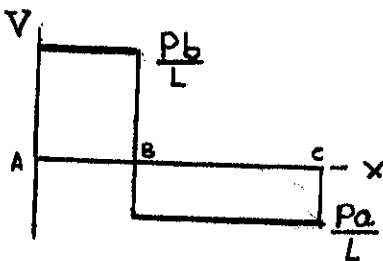
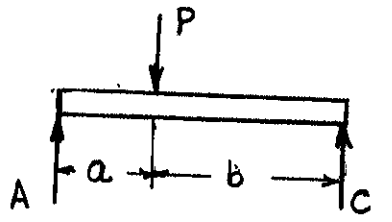


$$\uparrow \sum F_y = 0 \quad V + \frac{Pa}{L} = 0$$

$$V = -\frac{Pa}{L}$$

$$\curvearrowright \sum M_K = 0 \quad -M + \frac{Pa}{L}(L-x) = 0$$

$$M = \frac{Pa(L-x)}{L}$$

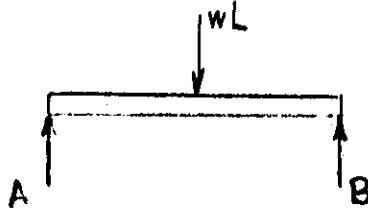
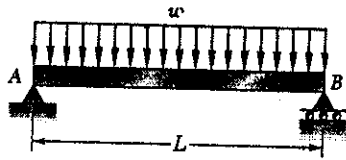


At section B

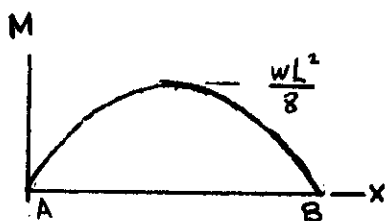
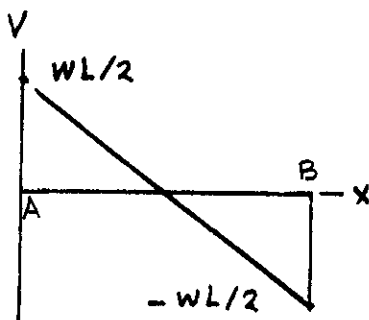
$$M = \frac{Pab}{L}$$

PROBLEM 5.2

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



Free body diagram for determining reactions



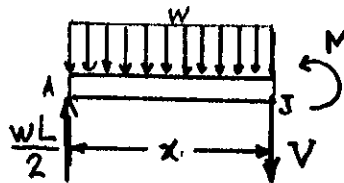
SOLUTION

Reactions

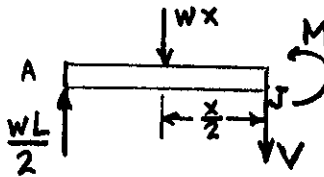
$$\sum M_B = 0 \quad -AL + wL \cdot \frac{L}{2} = 0 \quad A = \frac{wL}{2}$$

$$\sum M_A = 0 \quad BL - wL \cdot \frac{L}{2} = 0 \quad B = \frac{wL}{2}$$

Over whole beam  $0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.

$$\sum F_y = 0 \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right)$$

$$\sum M_J = 0 \quad -\frac{wL}{2}x + wx \cdot \frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

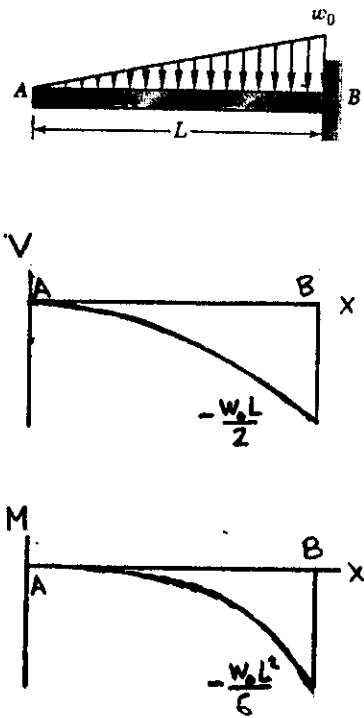
$$= \frac{w}{2}x(L - x)$$

Maximum bending moment occurs at  $x = \frac{L}{2}$ .

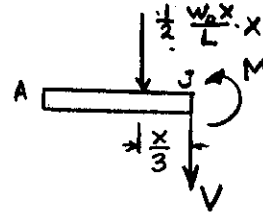
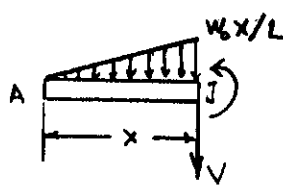
$$M_{max} = \frac{wL^2}{8}$$

PROBLEM 5.3

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION



$$+\uparrow \Sigma F_y = 0$$

$$-\frac{1}{2} \frac{w_0 x}{L} \cdot x - V = 0$$

$$V = -\frac{w_0 x^2}{2L}$$

$$\circlearrowleft \Sigma M_J = 0$$

$$\frac{1}{2} \frac{w_0 x}{L} \cdot x \cdot \frac{x}{3} + M = 0$$

$$M = -\frac{w_0 x^3}{6L}$$

At  $x = L$

$$V = -\frac{w_0 L}{2}$$

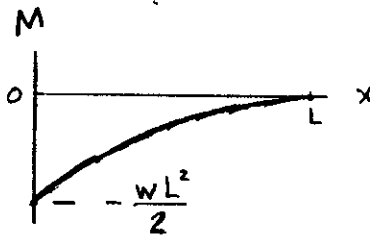
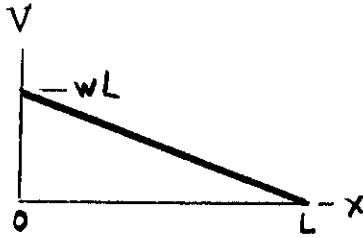
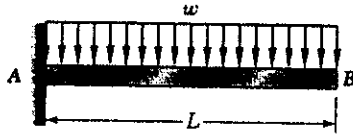
$$|V|_{\max} = \frac{w_0 L}{2}$$

$$M = -\frac{w_0 L^2}{6}$$

$$|M|_{\max} = \frac{w_0 L^2}{6}$$

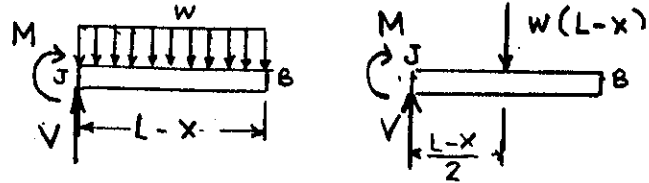
PROBLEM 5.4

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

Use portion to the right of the section as the free body.



Replace distributed load by equivalent concentrated load.

$$\uparrow \sum F_y = 0 \quad V - w(L-x) = 0$$

$$V = w(L-x) \quad \blacktriangleleft$$

$$\curvearrowright \sum M_J = 0 \quad -M - w(L-x)\left(\frac{L-x}{2}\right) = 0$$

$$M = -\frac{w}{2}(L-x)^2 \quad \blacktriangleleft$$

Largest negative bending moment occurs at  $x = 0$ .

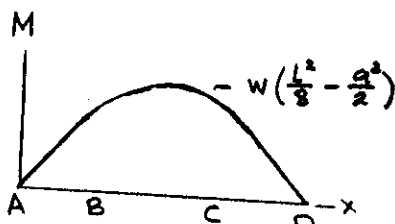
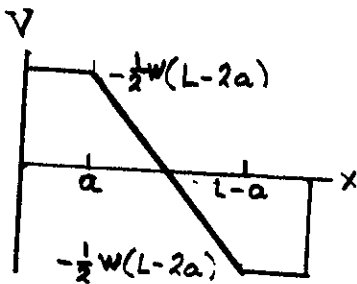
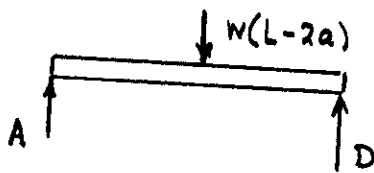
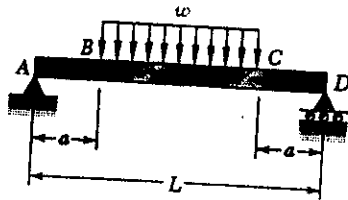
$$M_{\min} = -\frac{wL^2}{2} \quad \blacktriangleleft$$

Thus,  $|M|_{\max} = \frac{wL^2}{2} \quad \blacktriangleleft$



**PROBLEM 5.5**

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



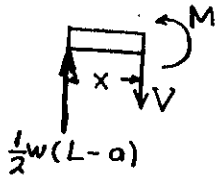
**SOLUTION**

Calculate reactions after replacing distributed load by an equivalent concentrated load.

Reactions are

$$A = D = \frac{1}{2} w(L-2a)$$

From A to B  $0 < x < a$



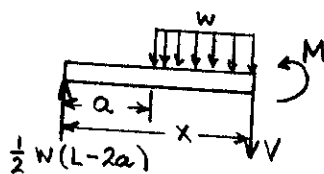
$$\uparrow \sum F_y = 0 \quad \frac{1}{2} w(L-2a) - V = 0$$

$$V = \frac{1}{2} w(L-2a) \quad \leftarrow$$

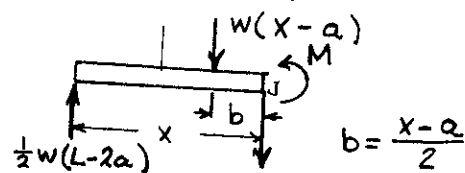
$$\curvearrow \sum M = 0 \quad -\frac{1}{2} w(L-2a)x + M = 0$$

$$M = \frac{1}{2} w(L-2a)x \quad \leftarrow$$

From B to C



$a < x < L-a$



Place section cut at x. Replace distributed load by equiv. conc. load.

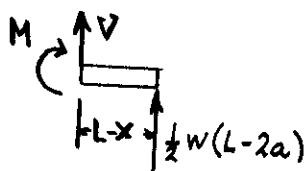
$$\uparrow \sum F_y = 0 \quad \frac{1}{2} w(L-2a) - w(x-a) - V = 0 \quad V = w\left(\frac{L}{2} - x\right) \quad \leftarrow$$

$$\curvearrow \sum M_s = 0 \quad -\frac{1}{2} w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$$

$$M = \frac{1}{2} w \left[ (L-2a)x - (x-a)^2 \right] \quad \leftarrow$$

From C to D

$L-a < x < L$



$$\uparrow \sum F_y = 0 \quad V + \frac{1}{2} w(L-2a) = 0$$

$$V = -\frac{w}{2}(L-2a)$$

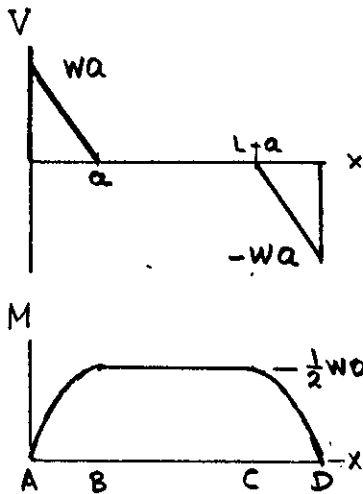
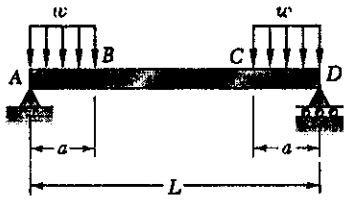
$$\curvearrow \sum M_T = 0 \quad -M + \frac{1}{2} w(L-2a)(L-x) = 0$$

$$M = \frac{1}{2} w(L-2a)(L-x) \quad \leftarrow$$

At  $x = \frac{L}{2}$   $M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right) \quad \leftarrow$

PROBLEM 5.6

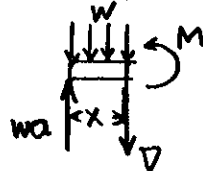
5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



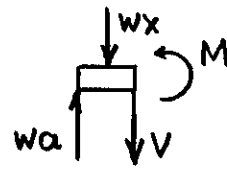
SOLUTION

Reactions:  $A = D = wa$

From A to B  $0 < x < a$



$$+\uparrow \Sigma F_y = 0$$



$$wa - wx - V = 0$$

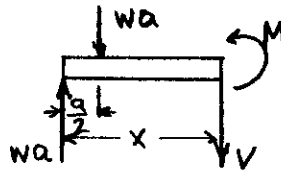
$$V = w(a - x)$$

$$\curvearrowright \Sigma M_J = 0$$

$$-wax + (wx)\frac{x}{2} + M = 0$$

$$M = w\left(ax - \frac{x^2}{2}\right)$$

From B to C  $a < x < L - a$



$$\Sigma F_y = 0$$

$$wa - wa - V = 0$$

$$V = 0$$

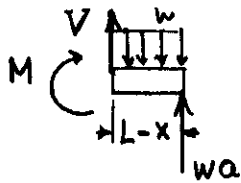
$$\curvearrowright \Sigma M_J = 0$$

$$-wax + wa\left(x - \frac{a}{2}\right) + M = 0$$

$$M = \frac{1}{2}wa^2$$

From C to D

$$L - a < x < L$$



$$+\uparrow \Sigma F_y = 0$$

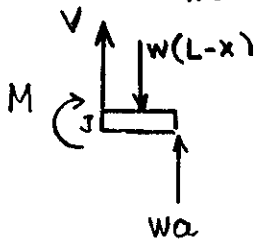
$$V - w(L - x) + wa = 0$$

$$V = w(L - x - a)$$

$$\curvearrowright \Sigma M_J = 0$$

$$-M - w(L - x)\left(\frac{L - x}{2}\right) + wa(L - x) = 0$$

$$M = w\left[a(L - x) - \frac{1}{2}(L - x)^2\right]$$



**PROBLEM 5.7**

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point *A*.)

**SOLUTION**

See **PROBLEM 5.1**

**PROBLEM 5.8**

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point *A*.)

**SOLUTION**

See **PROBLEM 5.2**

**PROBLEM 5.9**

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point *A*.)

**SOLUTION**

See **PROBLEM 5.3**

**PROBLEM 5.10**

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point *A*.)

**SOLUTION**

See **PROBLEM 5.4**

**PROBLEM 5.11**

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point *A*.)

**SOLUTION**

See **PROBLEM 5.5**

**PROBLEM 5.12**

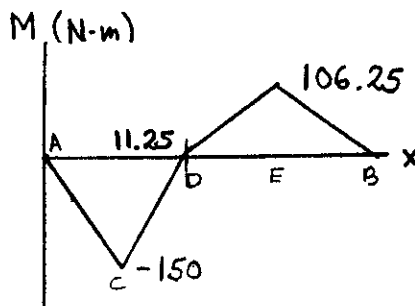
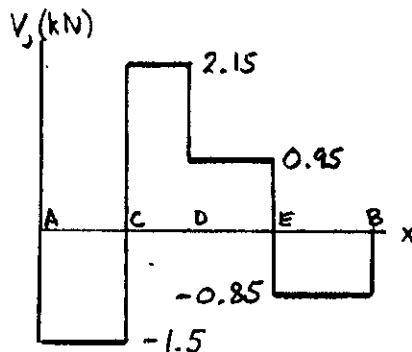
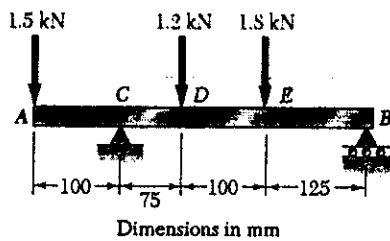
5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point *A*.)

**SOLUTION**

See **PROBLEM 5.6**

**PROBLEM 5.13**

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the



**SOLUTION**

Calculate reactions

$$\sum M_B = 0$$

$$(400)(1.5) - 300C + (225)(1.2) + (125)(1.8) = 0$$

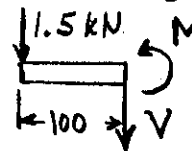
$$C = 3.65 \text{ kN}$$

$$\sum M_C = 0$$

$$B = 0.85 \text{ kN}$$

At A  $V = -1.5 \text{ kN}$ ,  $M = 0$

At C-

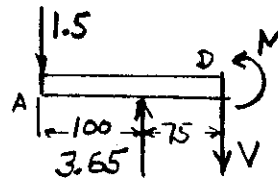


$$\sum F_y = 0 \quad -1.5 - V = 0 \quad V = -1.5 \text{ kN}$$

$$\sum M_C = 0 \quad (100)(1.5) + M = 0$$

$$M = -150 \text{ N}\cdot\text{m} \quad \blacktriangleleft \quad (b)$$

At D-

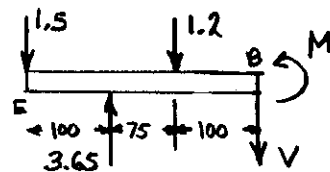


$$\sum F_y = 0 \quad -1.5 + 3.65 - V = 0, \quad V = 2.15 \text{ kN} \quad \blacktriangleleft$$

(a)

$$\sum M_D = 0, \quad (175)(1.5) - (75)(3.65) + M = 0 \quad M = 11.25 \text{ N}\cdot\text{m}$$

At E-



$$\sum F_y = 0$$

$$-1.5 + 3.65 - 1.2 - V = 0$$

$$V = 0.95 \text{ kN}$$

$$\sum M_E = 0 \quad (275)(1.5) - (175)(3.65) + (100)(1.2) + M = 0$$

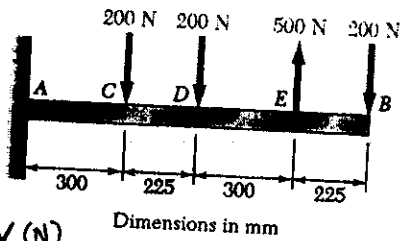
$$M = 106.25 \text{ N}\cdot\text{m}$$

At B  $V = -B = -0.85 \text{ kN}$

$$M = 0$$

**PROBLEM 5.14**

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

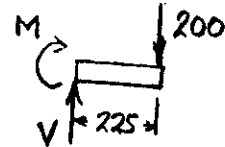


**SOLUTION**

At B

$$V = 200 \text{ N}, M = 0$$

At E<sup>+</sup>



$$+\uparrow \sum F_y = 0$$

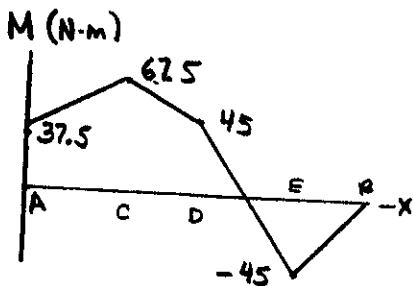
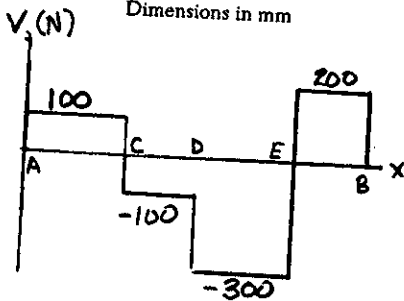
$$V - 200 = 0$$

$$V = 200 \text{ N}$$

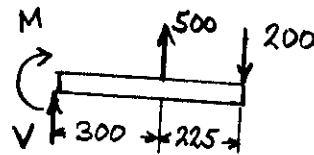
$$\circlearrowleft \sum M_E = 0$$

$$-M - (0.225)(200) = 0$$

$$M = -45 \text{ N}\cdot\text{m}$$



At D<sup>+</sup>



$$+\uparrow \sum F_y = 0$$

$$V + 500 - 200 = 0$$

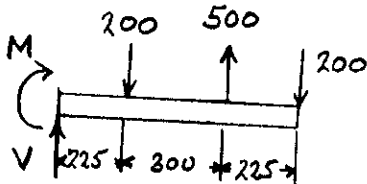
$$V = -300 \text{ N}$$

$$\circlearrowleft \sum M_D = 0$$

$$-M + (0.3)(500) - (0.525)(200) = 0$$

$$M = 45 \text{ N}\cdot\text{m}$$

At C<sup>+</sup>



$$+\uparrow \sum F_y = 0$$

$$V - 200 + 500 - 200 = 0$$

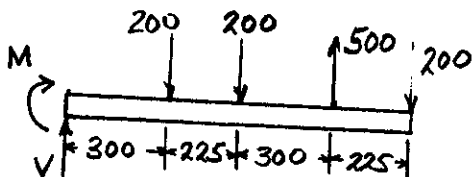
$$V = -100 \text{ N}$$

$$+\circlearrowleft \sum M_C = 0$$

$$-M - (0.225)(200) + (0.525)(500) - (0.75)(200) = 0$$

$$M = 67.5 \text{ N}\cdot\text{m}$$

At A



$$+\uparrow \sum F_y = 0$$

$$V - 200 - 200 + 500 - 200 = 0$$

$$V = 100 \text{ N}$$

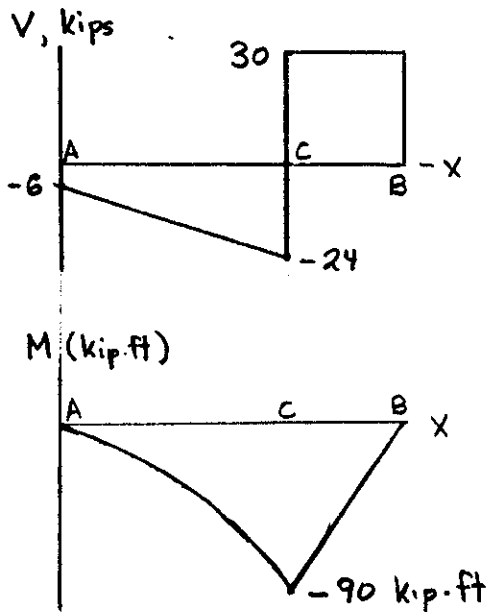
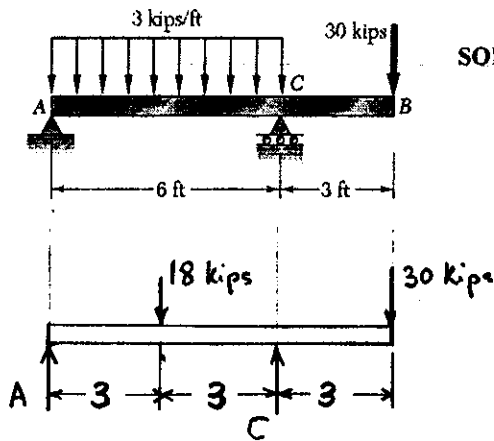
$$+\circlearrowleft \sum M_A = 0$$

$$-M - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$$

$$M = 37.5 \text{ N}\cdot\text{m}$$

PROBLEM 5.15

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



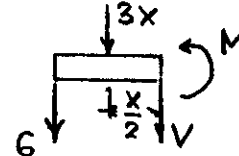
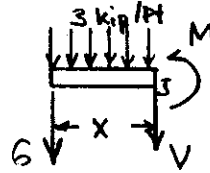
SOLUTION

Reactions

$$\begin{aligned} \sum M_C = 0 & \quad -6A + (3)(18) - (3)(30) = 0 \\ & \quad A = -6 \text{ kips} \quad \text{ie. } 6 \text{ kips } \downarrow \\ \sum M_A = 0 & \quad 6C - (3)(18) - (9)(30) = 0 \\ & \quad C = 54 \text{ kips } \uparrow \end{aligned}$$

A to C

$$0 < x < 6 \text{ ft.}$$

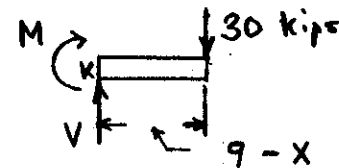


$$\begin{aligned} +\uparrow \sum F_y = 0 & \quad -6 - 3x - V = 0 \\ & \quad V = -6 - 3x \text{ kips.} \end{aligned}$$

$$\begin{aligned} \sum M_j = 0 & \quad -6x - (3x)\left(\frac{x}{2}\right) - M = 0 \\ & \quad M = -6x - 1.5x^2 \text{ kip}\cdot\text{ft} \end{aligned}$$

C to B

$$6 \text{ ft} < x < 9 \text{ ft}$$



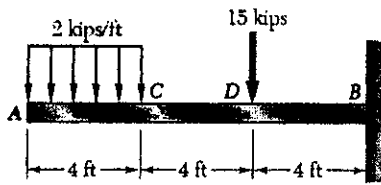
$$\begin{aligned} +\uparrow \sum F_y = 0 & \quad V - 30 = 0 \\ & \quad V = 30 \text{ kips} \quad \blacktriangleleft \quad (a) \end{aligned}$$

$$\begin{aligned} \sum M_k = 0 & \quad -M - (9-x)(30) = 0 \\ & \quad M = 30x - 270 \text{ kip}\cdot\text{ft} \end{aligned}$$

$$(b) \quad |M|_{\max} = 90 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

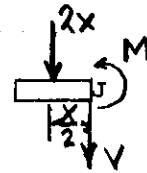
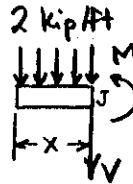
PROBLEM 5.16

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



SOLUTION

A to C  
 $0 < x < 4 \text{ ft}$

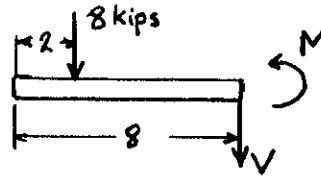


$$+\uparrow \Sigma F_y = 0 \quad -V - 2x = 0 \quad V = -2x \text{ kips}$$

$$\curvearrowright \Sigma M_J = 0 \quad M + (2x)\left(\frac{x}{2}\right) = 0 \quad M = -x \text{ kip}\cdot\text{ft}$$

At C  $V = -8 \text{ kips} \quad M = -16 \text{ kip}\cdot\text{ft}$

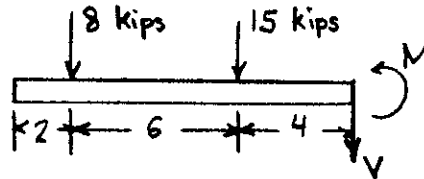
At D<sup>-</sup>



$$+\uparrow \Sigma F_y = 0 \quad -8 - V = 0 \quad V = -8 \text{ kips}$$

$$\curvearrowright \Sigma M_D = 0 \quad (6)(8) - M = 0 \quad M = -48 \text{ kip}\cdot\text{ft}$$

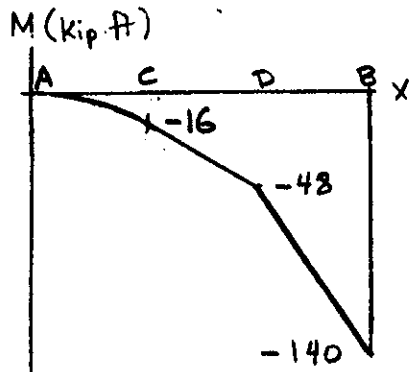
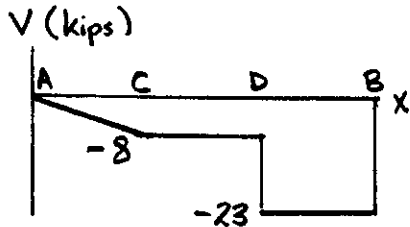
At B<sup>-</sup>



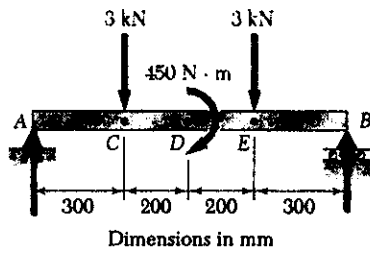
$$+\uparrow \Sigma F_y = 0 \quad -8 - 15 - V = 0 \quad V = -23 \text{ kips} \quad \blacktriangle (a)$$

$$\curvearrowright \Sigma M_B = 0 \quad -(10)(8) - (4)(15) - M = 0$$

$$M = -140 \text{ kip}\cdot\text{ft} \quad \blacktriangle (b)$$



**PROBLEM 5.17**

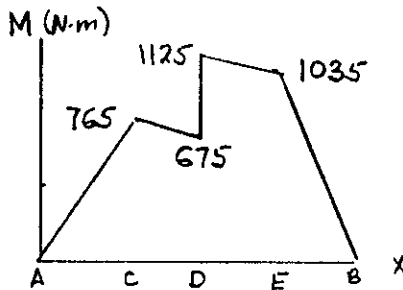
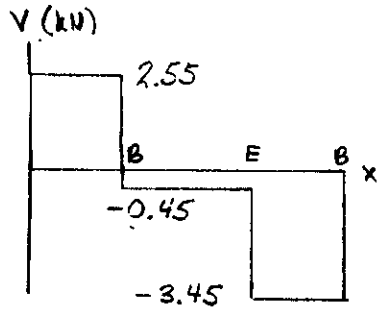


5.17 and 5.18 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

**SOLUTION**

$$\begin{aligned} \sum M_B = 0 & \quad (700)(3) - 450 + (300)(3) - 1000A = 0 \\ & \quad A = 2.55 \text{ kN } \uparrow \end{aligned}$$

$$\begin{aligned} \sum M_A = 0 & \quad -(300)(3) - 450 - (700)(3) + 1000B = 0 \\ & \quad B = 3.45 \text{ kN } \uparrow \end{aligned}$$



At A  $V = 2.55 \text{ kN}$   $M = 0$

A to C  $V = 2.55 \text{ kN}$

At C  $\sum M_c = 0$   
 $-(300)(2.55) + M = 0$   
 $M = 765 \text{ N}\cdot\text{m}$

C to E  $V = -0.45 \text{ N}\cdot\text{m}$

At D<sup>-</sup>  $\sum M_D = 0$   
 $-(500)(2.55) + (200)(3) + M = 0$   
 $M = 675 \text{ N}\cdot\text{m}$

At D<sup>+</sup>  $\sum M_D = 0$   
 $-(500)(2.55) + (200)(3) - 450 + M = 0$   
 $M = 1125 \text{ N}\cdot\text{m}$

E to B  $V = -3.45 \text{ kN}$

At E  $\sum M_E = 0$   
 $-M + (300)(3.45) = 0$   
 $M = 1035 \text{ N}\cdot\text{m}$

At B  $V = 3.45 \text{ N}\cdot\text{m}$ ,  $M = 0$

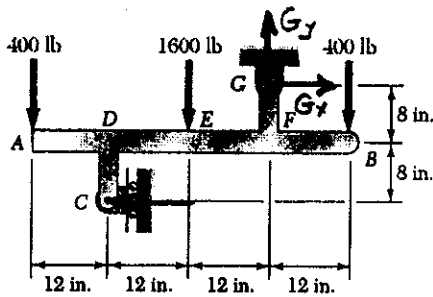
Maximum  $|V| = 3.45 \text{ kN}$   $\blacktriangleleft$

Maximum  $|M| = 1125 \text{ N}\cdot\text{m}$   $\blacktriangleleft$



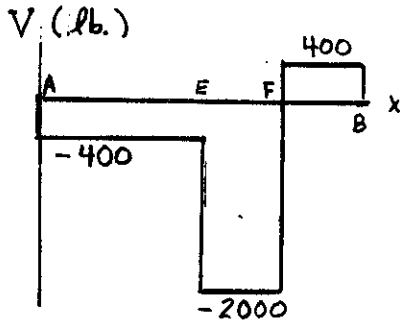
**PROBLEM 5.18**

5.17 and 5.18 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

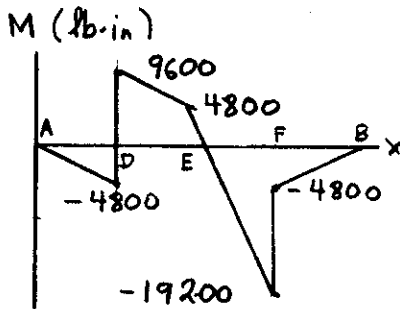


**SOLUTION**

$$\begin{aligned} \sum M_o &= 0 \\ -16C + (36)(400) + (12)(1600) - (12)(400) &= 0 \\ C &= 1800 \text{ lb.} \\ \sum F_x &= 0 \quad -C + G_x = 0 \quad G_x = 1800 \text{ lb.} \\ \sum F_y &= 0 \quad -400 - 1600 + G_y - 400 = 0 \\ G_y &= 2400 \text{ lb.} \end{aligned}$$



A to E  $V = -400 \text{ lb.}$   
 E to F  $V = -2000 \text{ lb.}$   
 F to B  $V = 400 \text{ lb.}$



At A and B  $M = 0$

At D<sup>-</sup>

$$\begin{aligned} \sum M_o &= 0 \\ (12)(400) + M &= 0 \\ M &= -4800 \text{ lb.in.} \end{aligned}$$

At D<sup>+</sup>

$$\begin{aligned} \sum M_o &= 0 \\ (12)(400) - (8)(1800) + M &= 0 \\ M &= 9600 \text{ lb.in.} \end{aligned}$$

(a) Maximum  $|V| = 2000 \text{ lb}$   $\blacktriangleleft$

(b) Maximum  $|M| = 19200 \text{ lb.in.}$   $\blacktriangleleft$

At E

$$\begin{aligned} \sum M_E &= 0 \\ (24)(400) - (8)(1800) + M &= 0 \\ M &= 4800 \text{ lb.in.} \end{aligned}$$

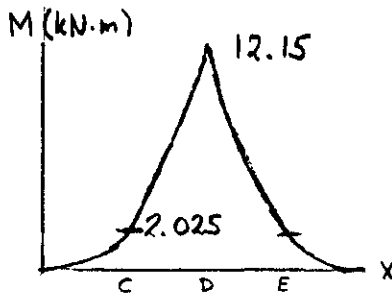
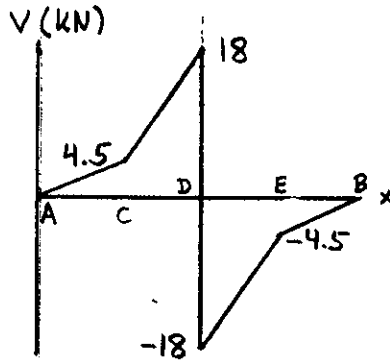
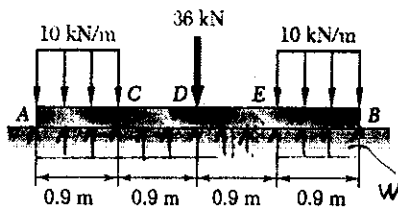
At F<sup>-</sup>

$$\begin{aligned} \sum M_F &= 0 \\ -M - (8)(1800) - (12)(400) &= 0 \\ M &= -19200 \text{ lb.in.} \end{aligned}$$

At F<sup>+</sup>

$$\begin{aligned} \sum M_F &= 0 \\ -M - (12)(400) &= 0 \\ M &= -4800 \text{ lb.in.} \end{aligned}$$

**PROBLEM 5.19**



(a) Maximum  $|V|$   
= 18 kN

(b) Maximum  $|M|$   
= 12.15 kN-m

5.19 and 5.20 Assuming the upward reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum value (a) of the shear, (b) of the bending moment.

**SOLUTION**

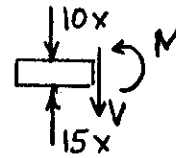
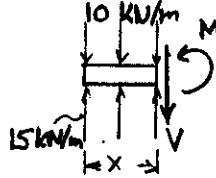
Over whole beam  $+\uparrow \Sigma F_y = 0$

$$3.6w - (0.9)(10) - 36 - (0.9)(10) = 0$$

$$w = 15 \text{ kN/m}$$

A to C

$$0 < x < 0.9 \text{ m}$$



$$+\uparrow \Sigma F_y = 0$$

$$15x - 10x - V = 0$$

$$V = 5x$$

$$\circlearrowleft \Sigma M_J = 0$$

$$-(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0$$

$$M = 2.5x^2$$

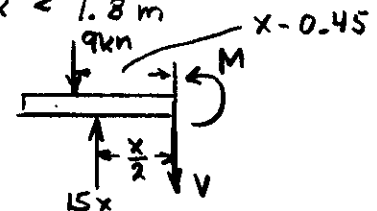
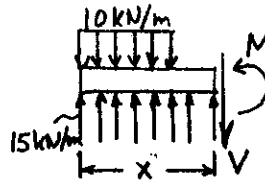
At  $x = C$

$$V = 4.5 \text{ kN}$$

$$M = 2.025 \text{ kN-m}$$

C to D

$$0.9 \text{ m} < x < 1.8 \text{ m}$$



$$+\uparrow \Sigma F_y = 0$$

$$15x - 9 - V = 0$$

$$V = 15x - 9$$

$$\circlearrowleft \Sigma M_J = 0$$

$$-(15x)\left(\frac{x}{2}\right) + 9(x - 0.45) + M = 0$$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

At  $D^-$

$$V = 18 \text{ kN}$$

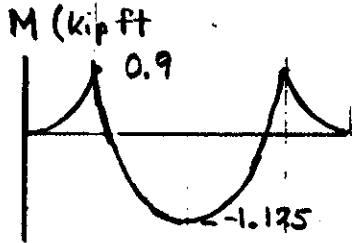
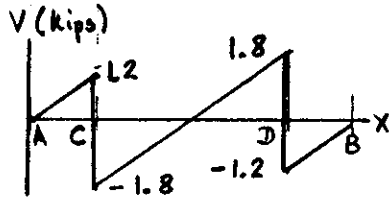
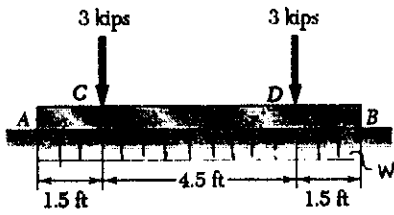
$$M = 12.15 \text{ kN-m}$$

D to B

Use symmetry to calculate the shear and bending moment.

PROBLEM 5.20

5.19 and 5.20 Assuming the upward reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam  $AB$  and determine the maximum value (a) of the shear, (b) of the bending moment.

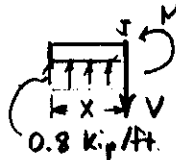


SOLUTION

Over the whole beam

$$\begin{aligned} \uparrow \sum F_y = 0 \quad & 7.5w - 3 - 3 = 0 \\ & w = 0.8 \text{ kip/ft} \end{aligned}$$

A to C  $0 < x < 1.5 \text{ ft}$

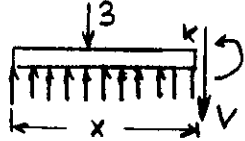


$$\begin{aligned} \uparrow \sum F_y = 0 \quad & 0.8x - V = 0 \\ & V = 0.8x \end{aligned}$$

$$\begin{aligned} \circlearrowleft \sum M_y = 0 \\ & -(0.8x)(\frac{x}{2}) + M = 0 \\ & M = 0.4x^2 \end{aligned}$$

At  $C^-$   $V = 1.2 \text{ kips}$ ,  $M = 0.9 \text{ kip}\cdot\text{ft}$

C to D  $1.5 \text{ ft} < x < 6 \text{ ft}$



$$\begin{aligned} \uparrow \sum F_y = 0 \\ & 0.8x - 3 - V = 0 \\ & V = 0.8x - 3 \end{aligned}$$

$$\begin{aligned} \circlearrowleft \sum M_K = 0 \quad & -(0.8x)(\frac{x}{2}) + 3(x - 1.5) + M = 0 \\ & M = 0.4x^2 - 3x + 4.5 \end{aligned}$$

At the center of the beam  $x = 3.75 \text{ ft}$

$$V = 0, \quad M = -1.125 \text{ kip}\cdot\text{ft}$$

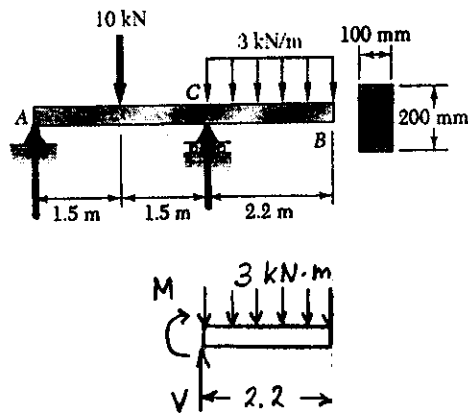
At  $C^+$   $V = -1.8 \text{ kip}$   $M = 0.9 \text{ kip}\cdot\text{ft}$

(a) Maximum  $|V| = 1.8 \text{ kips}$  ▶

(b) Maximum  $|M| = 1.125 \text{ kip}\cdot\text{ft}$  ▶

**PROBLEM 5.21**

5.21 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



**SOLUTION**

Using CB as a free body

$$\sum M_C = 0$$

$$-M + (2.2)(3 \times 10^3)(1.1) = 0$$

$$M = 7.26 \times 10^3 \text{ N}\cdot\text{m}$$

Section modulus for rectangle

$$S = \frac{1}{6} b h^2$$

$$= \frac{1}{6} (100)(200)^2 = 666.7 \times 10^3 \text{ mm}^3$$

$$= 666.7 \times 10^{-6} \text{ m}^3$$

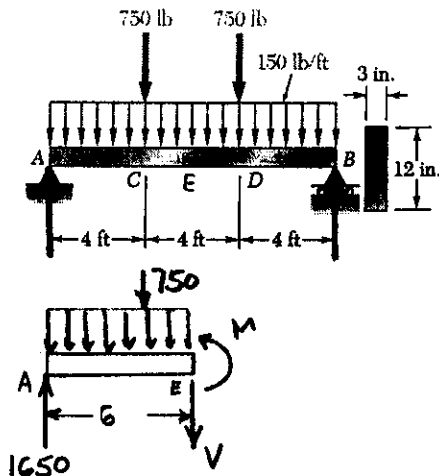
Normal stress

$$\sigma = \frac{M}{S} = \frac{7.26 \times 10^3}{666.7 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$$

$$\sigma = 10.89 \text{ MPa}$$

**PROBLEM 5.22**

5.22 For the beam and loading shown, determine the maximum normal stress on a transverse section at the center of the beam



**SOLUTION**

Reactions:  $C = A$  by symmetry

$$+\uparrow \sum F_y = 0 \quad A + C - (2)(750) - (12)(150) = 0$$

$$A = C = 1650 \text{ lb.}$$

Use left half of beam as free body

$$\sum M_E = 0$$

$$-(1650)(6) + (750)(2) + (150)(6)(3) + M = 0$$

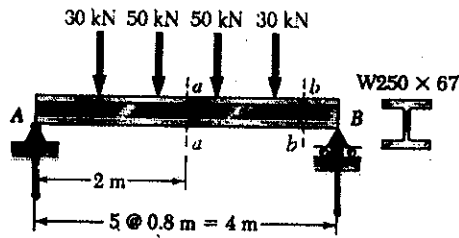
$$M = 5700 \text{ lb}\cdot\text{ft} = 68.4 \times 10^3 \text{ lb}\cdot\text{in.}$$

Section modulus  $S = \frac{1}{6} b h^2 = (\frac{1}{6})(3)(12)^2 = 72 \text{ in}^3$

Normal stress  $\sigma = \frac{M}{S} = \frac{68.4 \times 10^3}{72} = 950 \text{ psi}$

**PROBLEM 5.23**

5.23 For the beam and loading shown, determine the maximum normal stress on section a-a.



**SOLUTION**

Reactions: By symmetry  $A = B$

$$+\uparrow \Sigma F_y = 0 \quad A = B = 80 \text{ kN}$$

Using left half of beam as free body

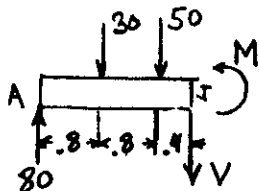
$$\circlearrowleft \Sigma M_I = 0$$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

$$M = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$$

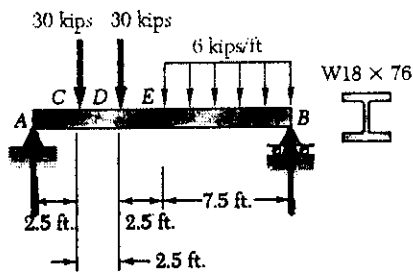
$$\text{For } W250 \times 67 \quad S = 809 \times 10^3 \text{ mm}^2 = 809 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{104 \times 10^3}{809 \times 10^{-6}} = 128.6 \times 10^6 \text{ Pa} = 128.6 \text{ MPa}$$



**PROBLEM 5.24**

5.24 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



**SOLUTION**

Use entire beam as free body

$$\circlearrowleft \Sigma M_B = 0$$

$$-15A + (12.5)(30) + (10)(30) + (6)(7.5)(3.75) = 0$$

$$A = 56.25 \text{ kips}$$

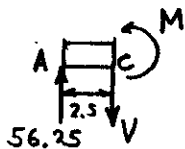
Use portion AC as free body

$$\circlearrowleft \Sigma M_C = 0 \quad -(56.25)(2.5) + M = 0$$

$$\text{Bending moment at C } M = 140.625 \text{ kip}\cdot\text{ft} = 1687.5 \text{ kip}\cdot\text{in.}$$

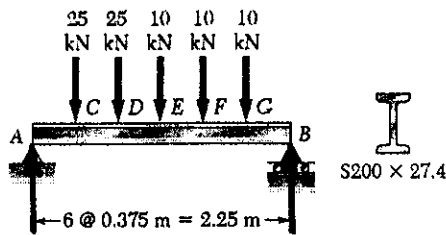
$$\text{For } W18 \times 76 \quad S = 146 \text{ in}^3$$

$$\text{Normal stress } \sigma = \frac{M}{S} = \frac{1687.5}{146} = 11.56 \text{ ksi}$$



**PROBLEM 5.25**

5.25 and 5.26 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



**SOLUTION**

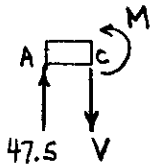
Use entire beam as free body

$$\sum M_B = 0$$

$$2.25 A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$$

$$A = 47.5 \text{ kN}$$

Use portion AC as free body



$$-(0.375)(47.5) + M = 0 \quad M = 17.8125 \text{ kN}\cdot\text{m}$$

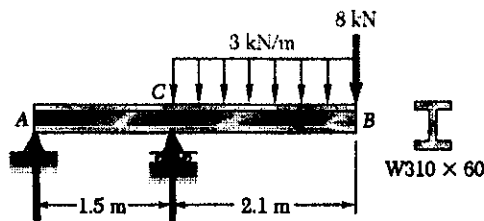
For S 200 x 27.4

$$S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$$

Normal stress  $\sigma = \frac{M}{S} = \frac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa} = 75.8 \text{ MPa}$

**PROBLEM 5.26**

5.25 and 5.26 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



**SOLUTION**

Use portion CB as free body.

$$\sum M_C = 0$$

$$-M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

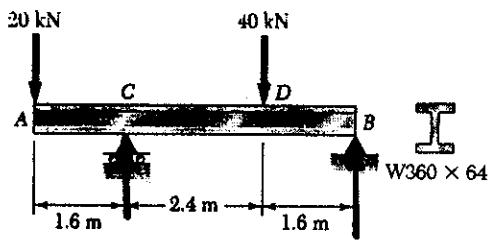
$$M = -23.415 \text{ kN}\cdot\text{m}$$

For W 310 x 60  $S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{23.415 \times 10^3}{851 \times 10^{-6}} = 27.5 \times 10^6 \text{ Pa} = 27.5 \text{ MPa}$

PROBLEM 5.27

5.27 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



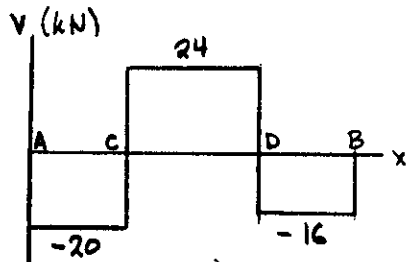
SOLUTION

$$\begin{aligned} \sum M_C = 0 \\ (1.6)(20) - (2.4)(40) + (4.0)B = 0 \\ B = 16 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 \\ (5.6)(20) - (4.0)C + (1.6)(40) = 0 \\ C = 44 \text{ kN} \end{aligned}$$

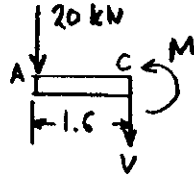
Shear

$$\begin{aligned} A \text{ to } C^- & V = -20 \text{ kN} \\ C^+ \text{ to } D^- & V = 24 \text{ kN} \\ D^+ \text{ to } B & V = -16 \text{ kN} \end{aligned}$$



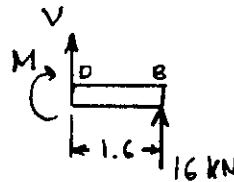
Bending moment

At C

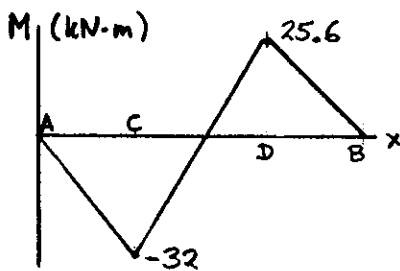


$$\begin{aligned} \sum M_C = 0 \\ (1.6)(20) + M = 0 \\ M = -32 \text{ kN}\cdot\text{m} \end{aligned}$$

At D



$$\begin{aligned} \sum M_D = 0 \\ -M + (1.6)(16) = 0 \\ M = 25.6 \text{ kN}\cdot\text{m} \end{aligned}$$



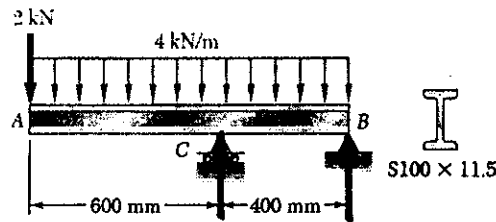
$$\max |M| = 32 \text{ kN}\cdot\text{m} = 32 \times 10^3 \text{ N}\cdot\text{m}$$

For rolled steel section W 360 x 64  $S = 1030 \times 10^3 \text{ mm}^3 = 1030 \times 10^{-6} \text{ m}^3$

Normal stress 
$$\sigma = \frac{|M|}{S} = \frac{32 \times 10^3}{1030 \times 10^{-6}} = 31.1 \times 10^6 \text{ Pa} = 31.1 \text{ MPa}$$

PROBLEM 5.28

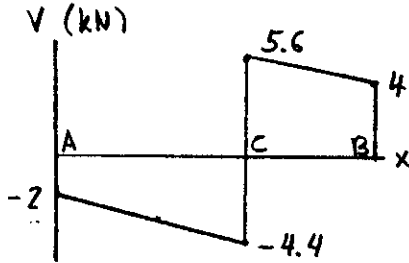
5.28 and 5.29 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

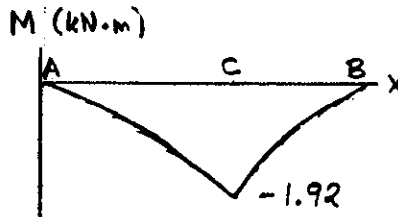
$$\begin{aligned} \sum M_C = 0 \\ (0.6)(2) + (0.1)(4) + (0.4)B = 0 \\ B = -4 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 \\ (1.0)(2) + (0.5)(4) - (0.4)C = 0 \\ C = 10 \text{ kN} \end{aligned}$$



A to C  $0 < x < 0.6 \text{ m}$

$$\begin{aligned} \sum F_y = 0 \\ -2 - 4x - V = 0 \\ V = -2 - 4x \end{aligned}$$



$$\begin{aligned} \sum M_x = 0 \\ 2x + (4x)\left(\frac{x}{2}\right) + M = 0 \\ M = -2x^2 - 2x \end{aligned}$$

At C  $M = -1.92 \text{ kN}\cdot\text{m}$

C to B  $0.6 \text{ m} < x < 1.0 \text{ m}$

$$\begin{aligned} \sum F_y = 0 \quad -2 - 4x + 10 - V = 0 \\ V = 8 - 4x \\ \sum M_x = 0 \quad -2x + (4x)\left(\frac{x}{2}\right) - (10)(x - 0.6) + M = 0 \end{aligned}$$

$$M = -2x^2 + 8x - 6$$

$$\max |M| = 1.92 \text{ kN}\cdot\text{m} = 1.92 \times 10^3 \text{ N}\cdot\text{m}$$

For rolled steel section S 100 x 11.5

$$\begin{aligned} S &= 49.6 \times 10^3 \text{ mm}^3 \\ &= 49.6 \times 10^{-6} \text{ m}^3 \end{aligned}$$

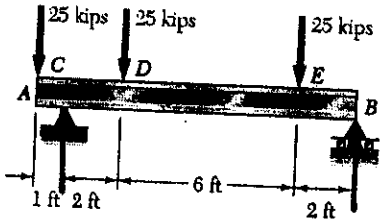
Maximum normal stress

$$\begin{aligned} \sigma &= \frac{|M|}{S} = \frac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} \\ &= 38.7 \text{ MPa} \end{aligned}$$



PROBLEM 5.29

5.28 and 5.29 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$$\begin{aligned} \sum M_B = 0 \\ (11)(25) - 10C + (8)(25) + (2)(25) = 0 \\ C = 52.5 \text{ kips} \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 \\ (1)(25) - (2)(25) - (8)(25) + 10B = 0 \\ B = 22.5 \text{ kips} \end{aligned}$$

Shear

- A to C<sup>-</sup>      V = -25 kips
- C<sup>+</sup> to D<sup>-</sup>    V = 27.5 kips
- D<sup>+</sup> to E<sup>-</sup>    V = 2.5 kips
- E<sup>+</sup> to B       V = -22.5 kips

Bending moments

At C

$$\begin{aligned} \sum M_C = 0 \\ (1)(25) + M = 0 \\ M = -25 \text{ kip}\cdot\text{ft} \end{aligned}$$

At D

$$\begin{aligned} \sum M_D = 0 \\ (3)(25) - (2)(52.5) + M = 0 \\ M = 30 \text{ kip}\cdot\text{ft} \end{aligned}$$

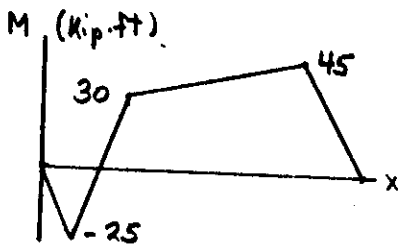
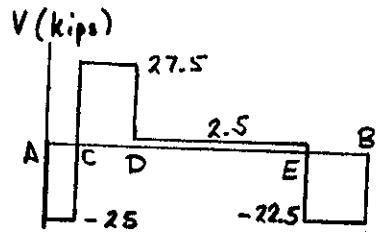
At E

$$\begin{aligned} \sum M_E = 0 \\ -M + (2)(22.5) = 0 \\ M = 45 \text{ kip}\cdot\text{ft} \end{aligned}$$

max |M| = 45 kip·ft = 540 kip·in

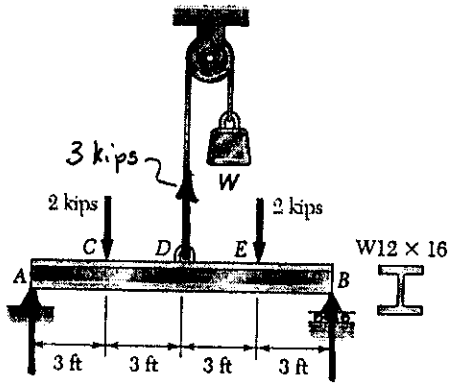
For S12 x 35 rolled steel section      S = 38.2 in<sup>3</sup>

Normal stress       $\sigma = \frac{M}{S} = \frac{540}{38.2} = 14.14 \text{ ksi}$



PROBLEM 5.30

5.30 Knowing that  $W = 3$  kips, draw the shear and bending-moment diagrams for beam  $AB$  and determine the maximum normal stress due to bending.



SOLUTION

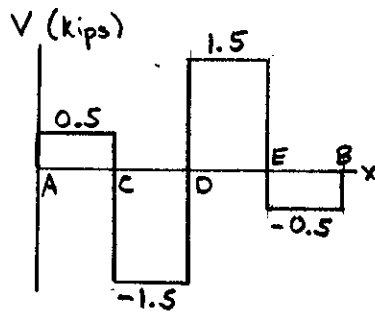
By symmetry  $A = B$

$$+\uparrow \sum F_y = 0 \quad A - 2 + 3 - 2 + B = 0$$

$$A = B = 0.5 \text{ kip}$$

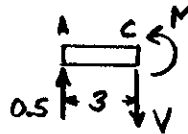
Shear

A to C <sup>-</sup>	$V = 0.5$ kips
C <sup>+</sup> to D <sup>-</sup>	$V = -1.5$ kips
D <sup>+</sup> to E <sup>-</sup>	$V = 1.5$ kips
E <sup>+</sup> to B	$V = -0.5$ kips



Bending moment

At C

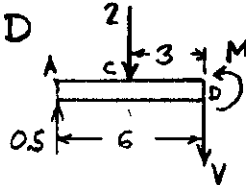


$$\sum M_c = 0$$

$$-(3)(0.5) + M = 0$$

$$M = 1.5 \text{ kip}\cdot\text{ft}$$

At D

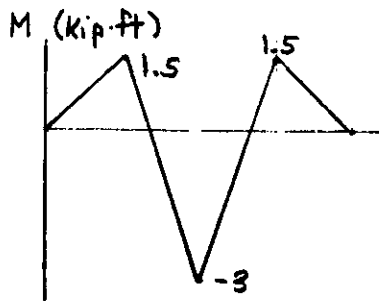


$$\sum M_D = 0$$

$$-(6)(0.5) + (3)(2) + M = 0$$

$$M = -3 \text{ kip}\cdot\text{ft}$$

At E  $M = 1.5 \text{ kip}\cdot\text{ft}$  by symmetry



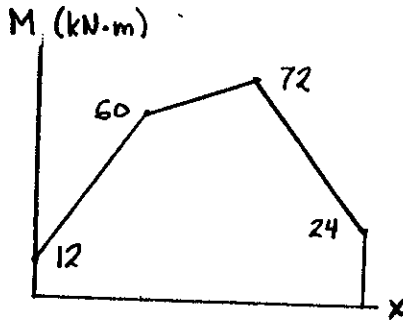
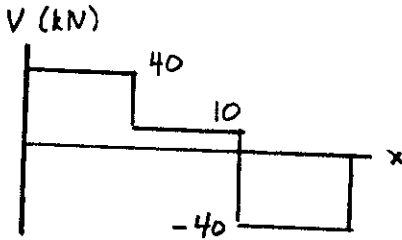
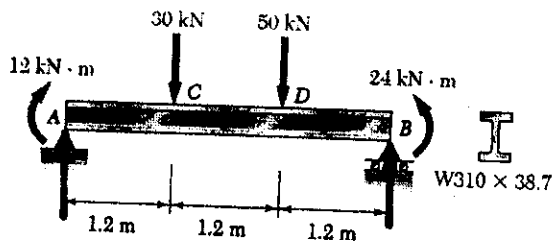
$$\max |M| = 3 \text{ kip}\cdot\text{ft} = 36 \text{ kip}\cdot\text{in}$$

For rolled steel section  $W 12 \times 16$   $S = 17.1 \text{ in}^3$

$$\text{Normal stress} \quad \sigma = \frac{|M|}{S} = \frac{36}{17.1} = 2.11 \text{ ksi}$$

**PROBLEM 5.31**

5.31 and 5.32 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



**SOLUTION**

$$\sum M_B = 0$$

$$-12 - 3.6A + (2.4)(30) + (1.2)(50) + 24 = 0$$

$$A = 40 \text{ kN}$$

$$\sum M_A = 0$$

$$-12 - (1.2)(30) - (2.4)(50) + 24 + 3.6B = 0$$

$$B = 40 \text{ kN}$$

Shear

$$A \text{ to } C^- \quad V = 40 \text{ kN}$$

$$C^+ \text{ to } D^- \quad V = 10 \text{ kN}$$

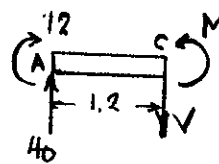
$$D^+ \text{ to } B \quad V = -50 \text{ kN}$$

Bending moment

$$\text{At } A \quad M = 12 \text{ kN}\cdot\text{m}$$

$$\text{At } B \quad M = 24 \text{ kN}\cdot\text{m}$$

At C

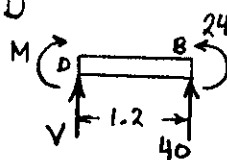


$$\sum M_C = 0$$

$$-12 - (1.2)(40) + M = 0$$

$$M = 60 \text{ kN}\cdot\text{m}$$

At D



$$\sum M_D = 0$$

$$-M + 24 + (1.2)(40) = 0$$

$$M = 72 \text{ kN}\cdot\text{m}$$

$$\max |M| = 72 \text{ kN}\cdot\text{m} = 72 \times 10^3 \text{ N}\cdot\text{m}$$

For rolled steel section W 310 x 38.7

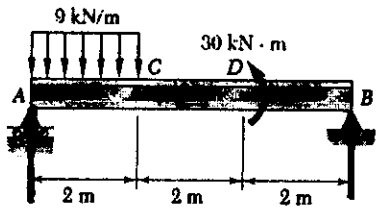
$$S = 549 \times 10^3 \text{ mm}^3$$

$$= 549 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{72 \times 10^3}{549 \times 10^{-6}} = 131.1 \times 10^6 \text{ Pa} = 131.1 \text{ MPa}$$

PROBLEM 5.32

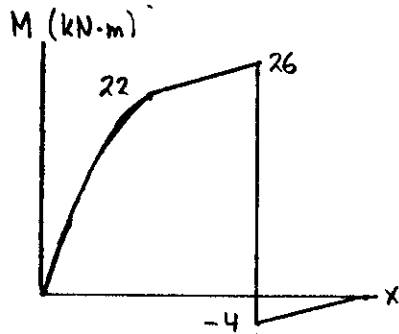
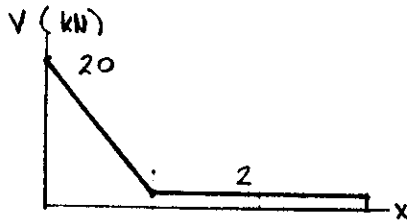
5.31 and 5.32 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending



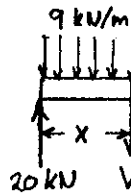
SOLUTION

$$\begin{aligned} \uparrow \sum M_B = 0 \\ -6A + (2)(9)(5) + 30 = 0 \\ A = 20 \text{ kN} \end{aligned}$$

$$\begin{aligned} \uparrow \sum M_A = 0 \\ -(2)(9)(1) + 30 + 6B = 0 \\ B = -2 \text{ kN i.e. } 2 \text{ kN } \downarrow \end{aligned}$$



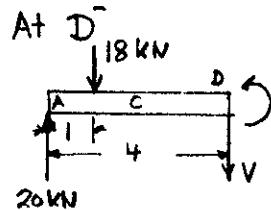
A to C  $0 < x < 2 \text{ m}$



$$\begin{aligned} \uparrow \sum F_y = 0 \quad 20 - 9x - V = 0 \\ V = 20 - 9x \end{aligned}$$

$$\begin{aligned} \curvearrowright \sum M_x = 0 \\ -20x + (9x)\frac{x}{2} + M = 0 \\ M = 20x - 4.5x^2 \end{aligned}$$

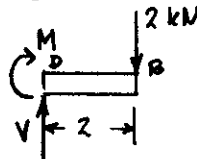
At C  $V = 2 \text{ kN} \quad M = 22 \text{ kN}\cdot\text{m}$



$$\begin{aligned} \uparrow \sum F_y = 0 \\ 20 - 18 - V = 0 \\ V = 2 \text{ kN} \end{aligned}$$

$$\begin{aligned} \curvearrowright \sum M_D = 0 \\ -(4)(20) + (3)(18) + M = 0 \\ M = 26 \text{ kN}\cdot\text{m} \end{aligned}$$

At D<sup>+</sup>



$$\begin{aligned} \uparrow \sum F_y = 0 \quad V - 2 = 0 \\ V = 2 \text{ kN} \end{aligned}$$

$$\begin{aligned} \curvearrowright \sum M_D = 0 \\ -M - (2)(2) = 0 \\ M = -4 \text{ kN}\cdot\text{m} \end{aligned}$$

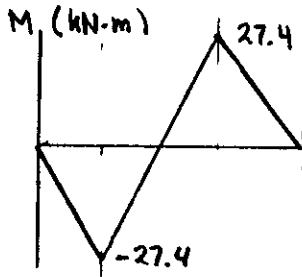
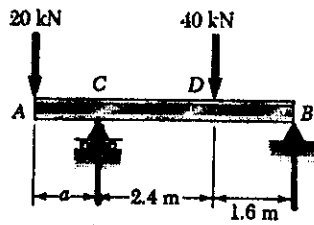
$$\max |M| = 26 \text{ kN}\cdot\text{m} = 26 \times 10^3 \text{ N}\cdot\text{m}$$

For rolled steel section W 200 x 22.5

$$\begin{aligned} S &= 194 \times 10^3 \text{ mm}^3 \\ &= 194 \times 10^{-6} \text{ m}^3 \end{aligned}$$

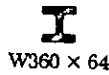
$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa} \quad \leftarrow$$

PROBLEM 5.33



5.33 Determine (a) the distance  $a$  for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending.  
(Hint: Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION



Reaction at B  $\sum \Sigma M_c = 0$   
 $20a - (2.4)(40) + (4.0)B = 0$   
 $B = 24 - 5a$

Bending moment at D

$\sum \Sigma M_D = 0$   
 $-M + 1.6B = 0$   
 $M_D = 1.6B = 38.4 - 8a$

Bending moment at C

$\sum \Sigma M_c = 0$   
 $20a + M = 0$   
 $M_c = -20a$

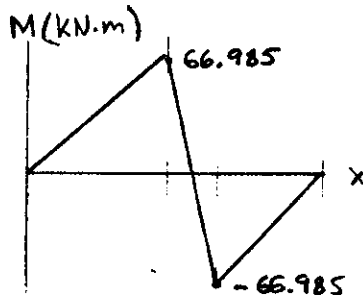
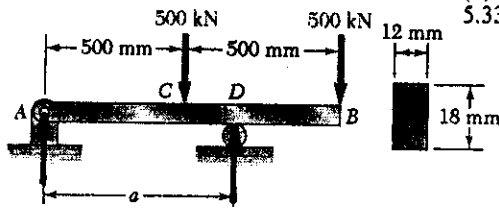
Equate  $-M_c = M_D$   
 $20a = 38.4 - 8a$   
 $a = 1.3714 \text{ m}$

$M_c = -27.429 \text{ kN}\cdot\text{m}$        $M_D = 27.429 \text{ kN}\cdot\text{m}$

For W 360 x 64 rolled steel section  $S = 1030 \times 10^3 \text{ mm}^3$   
 $= 1030 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{27.429 \times 10^3}{1030 \times 10^{-6}} = 26.6 \times 10^6 \text{ Pa} = 26.6 \text{ MPa}$

PROBLEM 5.34



5.34 For the beam and loading shown, determine (a) the distance  $a$  for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33.)

SOLUTION

Reaction at A  $\circlearrowleft \sum M_b = 0$

$$-Aa + (500)(a - 0.5) - 500(1 - a) = 0$$

$$Aa = 1000a - 750$$

$$A = 1000 - \frac{750}{a}$$

Bending moment at C  $\circlearrowleft \sum M_c = 0$

$$-(0.5)(1000 - \frac{750}{a}) + M_c = 0$$

$$M_c = 500 - \frac{375}{a}$$

Bending moment at D  $\circlearrowleft \sum M_b = 0$

$$-M_b - (500)(1 - a) = 0$$

$$M_b = -500(1 - a)$$

Equate  $-M_b = M_c$

$$500(1 - a) = 500 - \frac{375}{a}$$

$$a = 0.86603 \text{ m} = 866.03 \text{ mm} \quad \blacktriangleleft$$

$A = 133.98 \text{ kN}$        $M_c = 66.985 \text{ kN}\cdot\text{m}$        $M_b = -66.985 \text{ kN}\cdot\text{m}$

For rectangular cross section  $S = \frac{1}{6}bh^3 = \frac{1}{6}(12)(18)^3 = 11.664 \times 10^3 \text{ mm}^3$

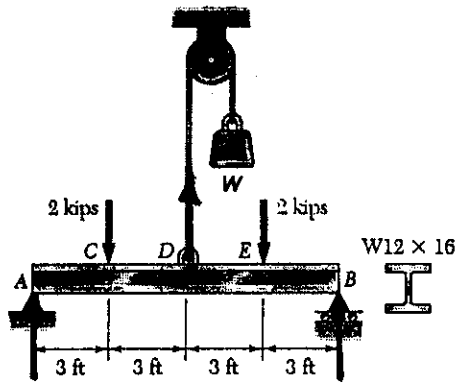
$$= 11.664 \times 10^{-6} \text{ m}^3$$

Normal stress  $\sigma = \frac{M|}{S} = \frac{66.985 \times 10^3}{11.664 \times 10^{-6}} = 5.74 \times 10^6 \text{ Pa}$

$$= 5.74 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 5.35

5.35 Determine (a) the magnitude of the counterweight  $W$  for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33.)



SOLUTION

By symmetry  $A = B$

$$+\uparrow \Sigma F_y = 0 \quad A - 2 + W - 2 + B = 0$$

$$A = B = 2 - \frac{W}{2}$$

Bending moment at C  $\odot \Sigma M_c = 0$

$$-(3)(2 - \frac{W}{2}) + M_c = 0$$

$$M_c = 6 - 1.5W$$

Bending moment at D  $\odot \Sigma M_D = 0$

$$-(6)(2 - \frac{W}{2}) + (3)(2) + M_D = 0$$

$$M_D = 6 - 3W$$

Equate  $-M_D = M_c$

$$3W - 6 = 6 - 1.5W$$

$$W = 2.667 \text{ kips}$$

$$M_c = 2.0 \text{ kip ft}$$

$$M_D = -2.0 \text{ kip ft}$$

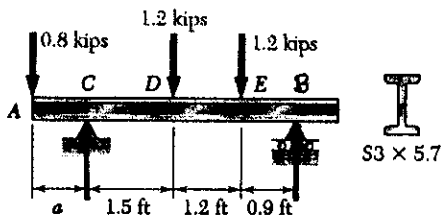
$$\max |M| = 2.0 \text{ kip ft} = 24 \text{ kip-in}$$

For W 12 x 16 rolled steel section  $S = 17.1 \text{ in}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{24}{17.1} = 1.404 \text{ ksi}$

**PROBLEM 5.36**

5.36 For the beam and loading shown, determine (a) the distance  $a$  for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33).

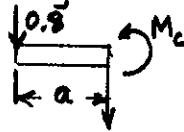


**SOLUTION**

$$\begin{aligned} \sum M_C = 0 \\ 0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0 \\ B = 1.4 - 0.22222a \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 \\ (0.8)(3.6+a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0 \\ C = 1.8 + 0.22222a \end{aligned}$$

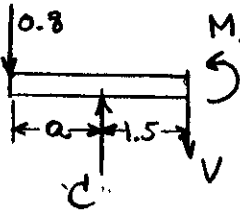
Bending moment at C



$$\sum M_C = 0$$

$$\begin{aligned} M_C + (0.8)(a) = 0 \\ M_C = -0.8a \end{aligned}$$

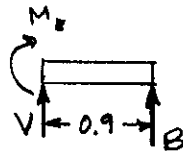
Bending moment at D



$$\sum M_D = 0$$

$$\begin{aligned} M_D + (0.8)(a+1.5) - 1.5C = 0 \\ M_D = 1.5 - 0.46667a \end{aligned}$$

Bending moment at E



$$\sum M_E = 0$$

$$\begin{aligned} -M_E + 0.9B = 0 \\ M_E = 1.26 - 0.2a \end{aligned}$$

Assume  $-M_C = M_E$        $0.8a = 1.26 - 0.2a$        $a = 1.26 \text{ ft}$  ←

$M_C = -1.008 \text{ kip}\cdot\text{ft}$        $M_E = 1.008 \text{ kip}\cdot\text{ft}$        $M_D = 0.912 \text{ kip}\cdot\text{ft}$

$\max |M| = 1.008 \text{ kip}\cdot\text{ft} = 12.096 \text{ kip}\cdot\text{in}$

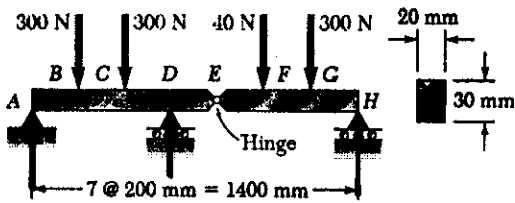
For rolled steel section S 3 x 5.7       $S = 1.68 \text{ in}^3$

Normal stress       $\sigma = \frac{|M|}{S} = \frac{12.096}{1.68} = 7.20 \text{ ksi}$  ←



PROBLEM 5.37

5.37 and 5.38 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending



SOLUTION

Free body EFGH  
Note that  $M_E = 0$  due to hinge.

$$\sum M_E = 0$$

$$0.6 H - (0.2)(40) - (0.4)(300) = 0$$

$$H = 213.33 \text{ N}$$

$$\sum F_y = 0 \quad V_E - 40 - 300 + 213.33 = 0$$

$$V_E = 126.67 \text{ N}$$

Shear: E to F  $V = 126.67 \text{ N}$

F to G  $V = 86.67 \text{ N}$

G to H  $V = -213.33 \text{ N}$

Bending moment at F

$$\sum M_F = 0$$

$$M_F - (0.2)(126.67) = 0$$

$$M_F = 25.33 \text{ N}\cdot\text{m}$$

Bending moment at G

$$\sum M_G = 0$$

$$-M_G + (0.2)(213.33) = 0$$

$$M_G = 42.67 \text{ N}\cdot\text{m}$$

Free body ABCDE

$$\sum M_B = 0 \quad -0.6 A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$$

$$A = 257.78 \text{ N}$$

$$\sum M_A = 0 \quad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6 D = 0$$

$$D = 468.89 \text{ N}$$

Bending moment at B

$$\sum M_B = 0$$

$$-(0.2)(257.78) + M_B = 0$$

$$M_B = 51.56 \text{ N}\cdot\text{m}$$

Bending moment at C

$$\sum M_C = 0$$

$$-(0.4)(257.78) + (0.2)(300) + M_C = 0$$

$$M_C = 43.11 \text{ N}\cdot\text{m}$$

Bending moment at D

$$\sum M_D = 0$$

$$-M_D - (0.2)(213.33) = 0$$

$$M_D = -25.33 \text{ N}\cdot\text{m}$$

$$\max |M| = 51.56 \text{ N}\cdot\text{m}$$

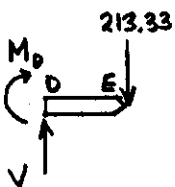
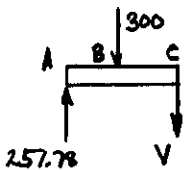
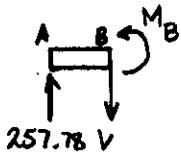
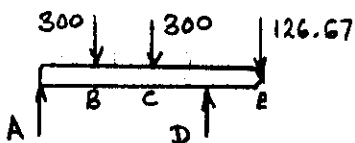
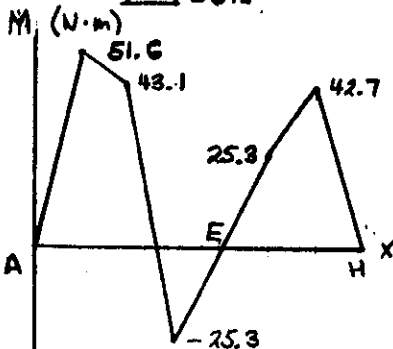
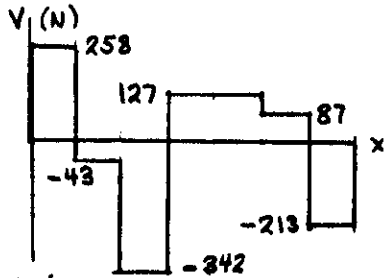
$$S = \frac{1}{6} b h^2 = \frac{1}{6} (20)(30)^2$$

$$= 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$$

Normal stress

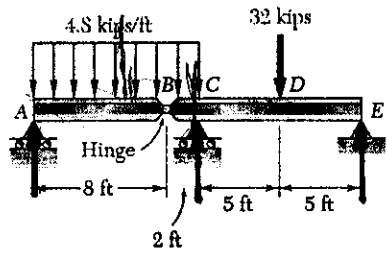
$$\sigma = \frac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa}$$

$$= 17.19 \text{ MPa}$$



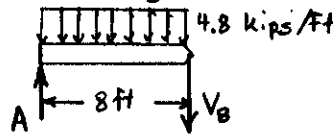
PROBLEM 5.38

5.37 and 5.38 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending

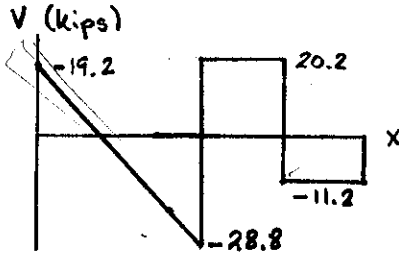


SOLUTION

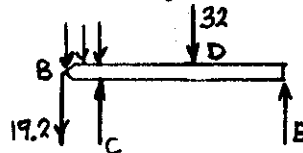
Free body AB



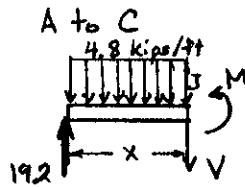
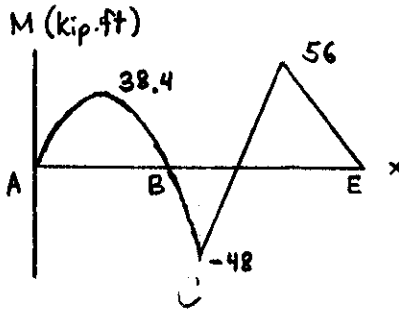
$$\begin{aligned} \sum M_B = 0 \\ (4.8)(8)(4) - 8A = 0 \\ A = 19.2 \text{ kips} \\ \sum M_A = 0 \\ -(4.8)(8)(4) - 8V_B = 0 \\ V_B = -19.2 \text{ kips} \end{aligned}$$



Free body BCDE



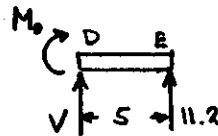
$$\begin{aligned} \sum M_E = 0 \\ (19.2)(12) + (4.8)(2)(11) - 10C + (32)(5) = 0 \\ C = 49.2 \text{ kips} \\ \sum M_C = 0 \\ (19.2)(2) + (4.8)(2)(1) - (32)(5) + 10E = 0 \\ E = 11.2 \text{ kips} \end{aligned}$$



$$\begin{aligned} 0 < x < 10 \text{ ft.} \\ \sum F_y = 0 \\ 19.2 - 4.8x - V = 0 \\ V = 19.2 - 4.8x \text{ kips.} \\ \sum M_f = 0 \\ -19.2x + (4.8x)\left(\frac{x}{2}\right) + M = 0 \\ M = 19.2x - 2.4x^2 \text{ kip-ft} \end{aligned}$$

At C  $x = 10$   $V = 19.2 - (4.8)(10) = -28.8$  kips  
 At C  $x = 10$   $M_C = (19.2)(10) - (2.4)(10)^2 = -48$  kip-ft  
 C to D  $V = 19.2 - (4.8)(10) + 49.2 = 20.8$  kips.  
 D to E  $V = -11.2$  kips

Bending moment at D



$$\begin{aligned} \sum M_D = 0 \\ -M_D + (11.2)(5) = 0 \\ M_D = 56 \text{ kip-ft} \end{aligned}$$

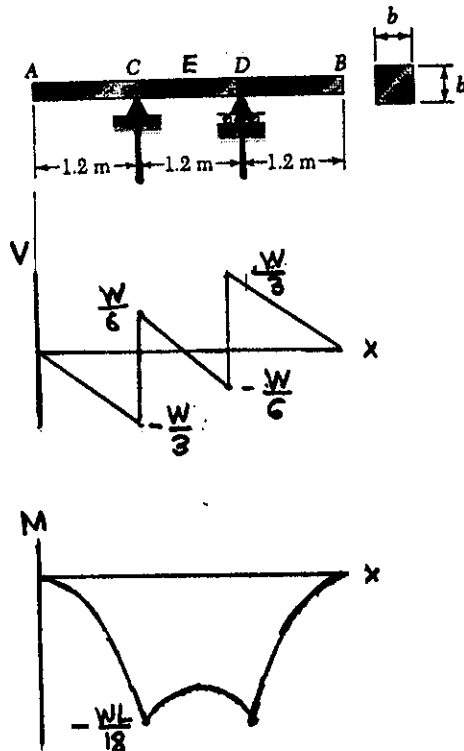
max |M| = 56 kip-ft = 672 kip-in

For W12 x 40 rolled steel section  $S = 51.9 \text{ in}^3$

Normal stress  $\sigma = \frac{M}{S} = \frac{672}{51.9} = 12.95 \text{ ksi}$

PROBLEM 5.39

5.39 A solid steel bar has a square cross section of side  $b$  and is supported as shown. Knowing that for steel  $\rho = 7860 \text{ kg/m}^3$ , determine the dimension  $b$  of the bar for which the maximum normal stress due to bending is (a) 10 MPa, (b) 50 MPa.



SOLUTION Weight density  $\gamma = \rho g$

Let  $L =$  total length of beam

$$W = \gamma V = AL\rho g = b^2 L\rho g$$

Reactions at C and D  $C = D = \frac{W}{2}$

Bending moment at C

$$\begin{aligned} \sum M_C = 0 \\ \left(\frac{1}{3}\right)\left(\frac{W}{3}\right) + M = 0 \\ M = -\frac{WL}{18} \end{aligned}$$

Bending moment at center of beam

$$\begin{aligned} \sum M_E = 0 \\ \left(\frac{1}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{1}{6}\right)\left(\frac{W}{2}\right) + M = 0 \\ M = -\frac{WL}{24} \end{aligned}$$

$$\max |M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

For a square section  $S = \frac{1}{6} b^3$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{b^2 L^2 \rho g / 18}{b^3 / 6} = \frac{L^2 \rho g}{3b}$$

Solve for  $b$   $b = \frac{L^2 \rho g}{3\sigma}$

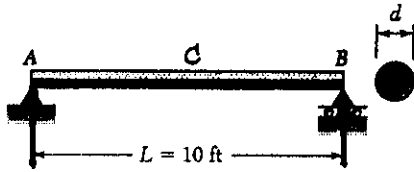
Data:  $L = 3.6 \text{ m}$   $\rho = 7860 \text{ kg/m}^3$   $g = 9.81 \text{ m/s}^2$   
 (a)  $\sigma = 10 \times 10^6 \text{ Pa}$  (b)  $\sigma = 50 \times 10^6 \text{ Pa}$

(a)  $b = \frac{(3.6)^2 (7860) (9.81)}{(3)(10 \times 10^6)} = 33.3 \times 10^{-3} \text{ m} = 33.3 \text{ mm}$

(b)  $b = \frac{(3.6)^2 (7860) (9.81)}{(3)(50 \times 10^6)} = 6.66 \times 10^{-3} \text{ m} = 6.66 \text{ mm}$

PROBLEM 5.40

5.40 A solid steel rod of diameter  $d$  is supported as shown. Knowing that for steel  $\gamma = 490 \text{ lb/ft}^3$ , determine the smallest diameter  $d$  which can be used if the normal stress due to bending is not to exceed 4 ksi.



SOLUTION

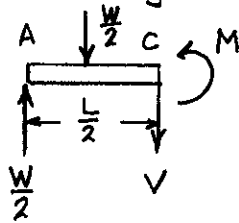
Let  $W = \text{total weight}$

$$W = \gamma V = AL\gamma = \frac{\pi}{4} d^2 L \gamma$$

Reaction at A

$$A = \frac{1}{2} W$$

Bending moment at center of beam



$$\sum M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section ( $c = \frac{1}{2}d$ )

$$I = \frac{\pi}{4} c^4, \quad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for  $d$   $d = \frac{L^2 \gamma}{\sigma}$

Data:  $L = 10 \text{ ft} = (12)(10) = 120 \text{ in}$

$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

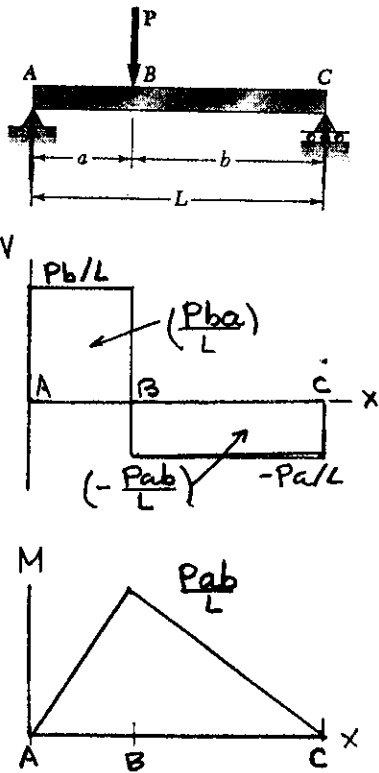
$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

$$d = \frac{(120)^2 (0.28356)}{4000} = 1.021 \text{ in.}$$

PROBLEM 5.41

5.41 Using the methods of Sec. 5.3, solve Prob. 5.1.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

$$\begin{aligned} \textcircled{D} \sum M_C = 0 \quad & LA - bP = 0 \quad & A = \frac{Pb}{L} \\ \textcircled{D} \sum M_A = 0 \quad & LC - aP = 0 \quad & C = \frac{Pa}{L} \end{aligned}$$

At A<sup>+</sup>  $V = A = \frac{Pb}{L}$   $M = 0$

A to B<sup>-</sup>  $0 < x < a$

$$w = 0 \quad \int_0^x w dx = 0$$

$$V_B - V_A = 0 \quad V_B = \frac{Pb}{L}$$

$$M_B - M_A = \int_0^a V dx = \int_0^a \frac{Pb}{L} dx = \frac{Pba}{L}$$

$$M_B = \frac{Pba}{L}$$

At B<sup>+</sup>  $V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$

B<sup>+</sup> to C  $a < x < L$

$$w = 0 \quad \int_a^x w dx = 0$$

$$V_C - V_B = 0 \quad V = -\frac{Pa}{L}$$

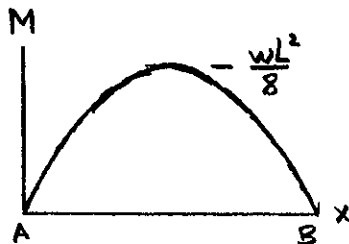
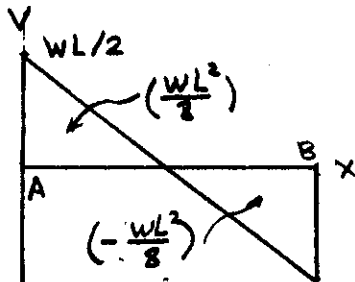
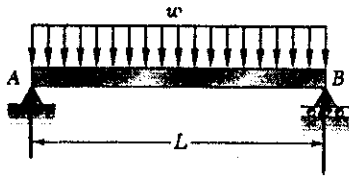
$$M_C - M_B = \int_a^L V dx = -\frac{Pa}{L}(L-a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

PROBLEM 5.42

5.42 Using the methods of Sec. 5.3, solve Prob. 5.2.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

$$\ominus \sum M_B = 0 \quad -AL + wL \cdot \frac{L}{2} = 0 \quad A = \frac{wL}{2}$$

$$\ominus \sum M_A = 0 \quad BL - wL \cdot \frac{L}{2} = 0 \quad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = - \int_0^x w \, dx = -wx$$

$$V = V_A - wx = A - wx = \frac{wL}{2} - wx$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V \, dx = \int_0^x \left( \frac{wL}{2} - wx \right) dx$$

$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2} (Lx - x^2)$$

Maximum M occurs at  $x = \frac{L}{2}$  where

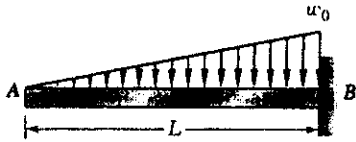
$$V = \frac{dM}{dx} = 0$$

$$\text{Max } M = \frac{wL^2}{8}$$

PROBLEM 5.43

5.43 Using the methods of Sec. 5.3, solve Prob. 5.3.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \quad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

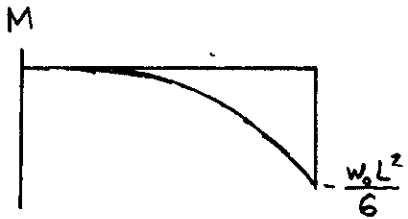
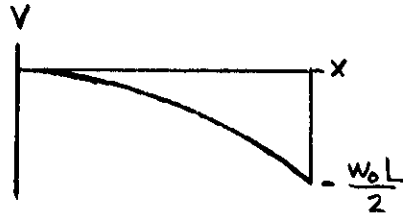
$$V - V_A = -\int_0^x \frac{w_0 x}{L} dx = -\frac{w_0 x^2}{2L}$$

$$V = -\frac{w_0 x^2}{2L}$$

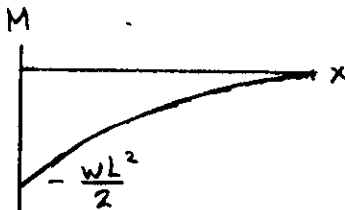
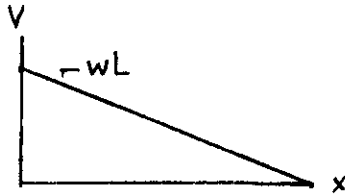
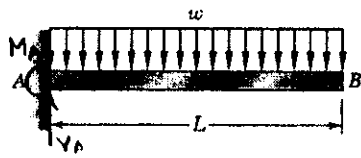
$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V dx = -\int_0^x \frac{w_0 x^2}{2L} dx$$

$$= -\frac{w_0 x^3}{6L}$$



PROBLEM 5.44



5.44 Using the methods of Sec. 5.3, solve Prob. 5.4.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

$$\uparrow \sum F_y = 0 \quad V_A - wL = 0 \quad V_A = wL$$

$$\circlearrowleft \sum M_A = 0 \quad -M - (wL)(\frac{L}{2}) = 0 \quad M_A = -\frac{wL^2}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w dx = -wx$$

$$V = wL - wx$$

$$\frac{dM}{dx} = V = wL - wx$$

$$M - M_A = \int_0^x (wL - wx) dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

$$\max |V| = wL$$

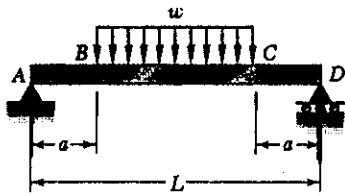
$$\max |M| = \frac{wL^2}{2}$$



PROBLEM 5.45

5.45 Using the methods of Sec. 5.3, solve Prob. 5.5.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

Reactions  $A = D = \frac{1}{2} w(L - 2a)$

At A  $V_A = A = \frac{1}{2} w(L - 2a)$ ,  $M_A = 0$

A to B  $0 < x < a$   $w = 0$

$$V_B - V_A = -\int_0^a w dx = 0$$

$$V_B = V_A = \frac{1}{2} w(L - 2a)$$

$$M_B - M_A = \int_0^a V dx = \int_0^a \frac{1}{2} w(L - 2a) dx$$

$$M_B = \frac{1}{2} w(L - 2a)a$$

B to C  $a < x < L - a$   $w = w$

$$V - V_B = -\int_a^x w dx = -w(x - a)$$

$$V = \frac{1}{2} w(L - 2a) - w(x - a) = \frac{1}{2} w(L - 2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2} w(L - 2x)$$

$$M - M_B = \int_a^x V dx = \frac{1}{2} w(Lx - x^2) \Big|_a^x$$

$$= \frac{1}{2} w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2} w(L - 2a)a + \frac{1}{2} w(Lx - x^2 - La + a^2)$$

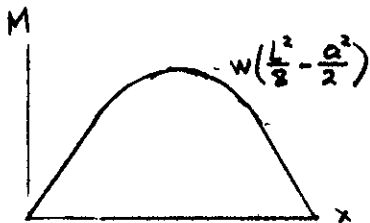
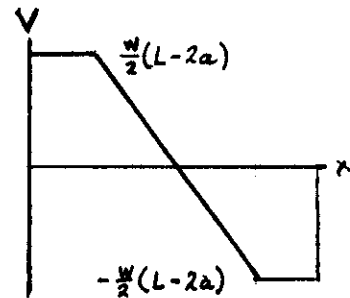
$$= \frac{1}{2} w(Lx - x^2 - a^2)$$

At C  $x = L - a$   $V_c = -\frac{1}{2} w(L - 2a)$   $M_c = \frac{1}{2} w(L - 2a)a$

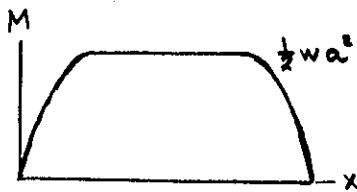
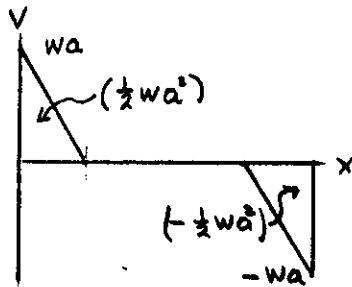
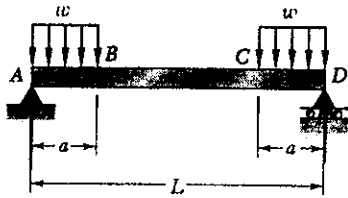
C to D  $V = V_c = -\frac{1}{2} w(L - 2a)$

$$M_D = 0$$

At  $x = \frac{L}{2}$   $M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$



PROBLEM 5.46



5.46 Using the methods of Sec. 5.3, solve Prob. 5.6.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

Reactions  $A = D = wa$

A to B  $0 < x < a$   $w = w$

$V_A = A = wa$ ,  $M_A = 0$

$V - V_A = -\int_0^x w dx = -wx$

$V = w(a - x)$   $V_B = 0$

$\frac{dM}{dx} = V = wa - wx$

$M - M_A = \int_0^x V dx = \int_0^x (wa - wx) dx$   
 $= wax - \frac{1}{2}wx^2$

$M_B = \frac{1}{2}wa^2$  at  $x = a$ .

B to C  $a < x < L - a$   $V = 0$

$\frac{dM}{dx} = V = 0$

$M - M_B = \int_a^x V dx = 0$

$M = M_B = \frac{1}{2}wa^2$

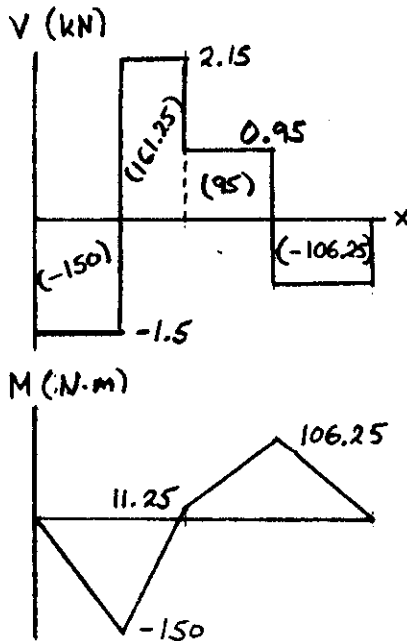
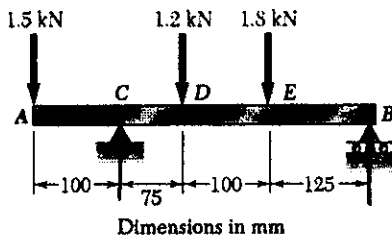
C to D  $V - V_C = -\int_{L-a}^x w dx = -w[x - (L-a)]$

$V = -w[x - (L-a)]$

$M - M_C = \int_{L-a}^x V dx = \int_{L-a}^x -w[x - (L-a)] dx$   
 $= -w \left[ \frac{x^2}{2} - (L-a)x \right] \Big|_{L-a}^x$   
 $= -w \left[ \frac{x^2}{2} - (L-a)x - \frac{(L-a)^2}{2} + (L-a)^2 \right]$   
 $= -w \left[ \frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2} \right]$

$M = \frac{1}{2}wa^2 - w \left[ \frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2} \right]$

**PROBLEM 5.47**



5.47 Using the methods of Sec. 5.3, solve Prob. 5.13.

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

**SOLUTION**

$$\sum \circlearrowleft M_B = 0$$

$$(400)(1.5) - 300C + (225)(1.2) + (125)(1.8) = 0$$

$$C = 3.65 \text{ kN}$$

$$\sum \circlearrowright M_C = 0 \quad B = 0.85 \text{ kN}$$

Shear:

A to C  $V = -1.5 \text{ kN}$

C to D  $V = -1.5 + 3.65 = 2.15 \text{ kN}$

D to E  $V = 2.15 - 1.2 = 0.95 \text{ kN}$

E to B  $V = 0.95 - 1.8 = -0.85 \text{ kN}$

Areas of shear diagram

A to C  $\int V dx = (-1.5)(100) = -150 \text{ N}\cdot\text{m}$

C to D  $\int V dx = (2.15)(75) = 161.25 \text{ N}\cdot\text{m}$

D to E  $\int V dx = (0.95)(100) = 95 \text{ N}\cdot\text{m}$

E to B  $\int V dx = (-0.85)(125) = -106.25 \text{ N}\cdot\text{m}$

Bending moments

$$M_A = 0$$

$$M_C = M_A + \int_A^C V dx = 0 - 150 = -150 \text{ N}\cdot\text{m}$$

$$M_D = M_C + \int_C^D V dx = -150 + 161.25 = 11.25 \text{ N}\cdot\text{m}$$

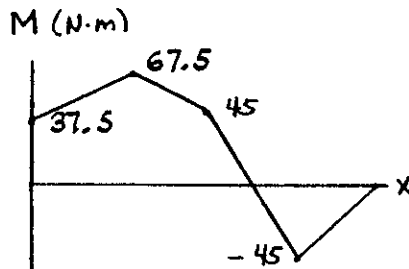
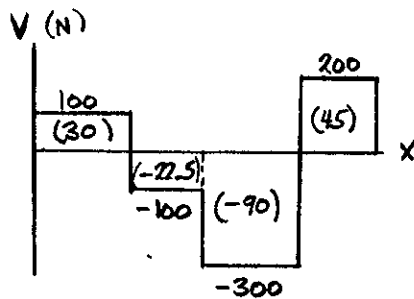
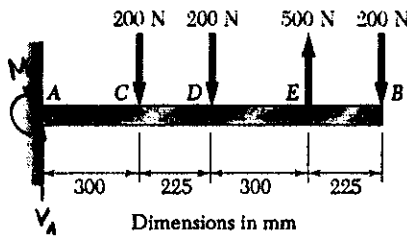
$$M_E = M_D + \int_D^E V dx = 11.25 + 95 = 106.25 \text{ N}\cdot\text{m}$$

$$M_B = M_E + \int_E^B V dx = 106.25 - 106.25 = 0$$

Maximum  $|V| = 2.15 \text{ kN}$   $\blacktriangleleft$

Maximum  $|M| = 150 \text{ N}\cdot\text{m}$   $\blacktriangleleft$

**PROBLEM 5.48**



5.48 Using the methods of Sec. 5.3, solve Prob. 5.14.

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

**SOLUTION**

$$\begin{aligned} \sum M_A &= 0 \\ -M_A - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) &= 0 \\ M_A &= 37.5 \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ V_A - 200 - 200 + 500 - 200 &= 0 \\ V_A &= 100 \text{ N} \end{aligned}$$

**Shear**

A to C	$V = 100 \text{ N}$
C to D	$V = 100 - 200 = -100 \text{ N}$
D to E	$V = -100 - 200 = -300 \text{ N}$
E to B	$V = -300 + 500 = 200 \text{ N}$

**Areas under shear diagram**

A to C	$\int V dx = (100)(0.3) = 30 \text{ N}\cdot\text{m}$
C to D	$\int V dx = (-100)(0.225) = -22.5 \text{ N}\cdot\text{m}$
D to E	$\int V dx = (-300)(0.3) = -90 \text{ N}\cdot\text{m}$
E to B	$\int V dx = (200)(0.225) = 45 \text{ N}\cdot\text{m}$

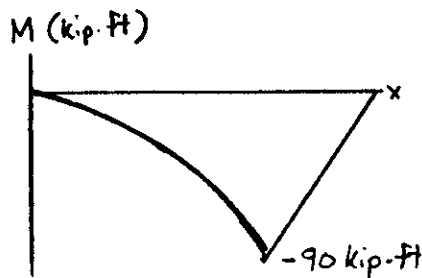
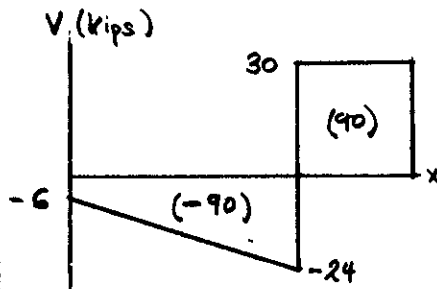
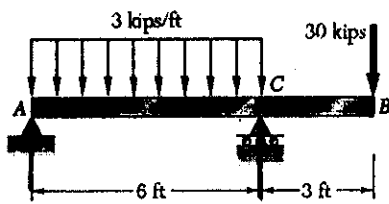
**Bending moments**

$$\begin{aligned} M_A &= 37.5 \text{ N}\cdot\text{m} \\ M_C &= M_A + \int_A^C V dx = 37.5 + 30 = 67.5 \text{ N}\cdot\text{m} \\ M_D &= M_C + \int_C^D V dx = 67.5 - 22.5 = 45 \text{ N}\cdot\text{m} \\ M_E &= M_D + \int_D^E V dx = 45 - 90 = -45 \text{ N}\cdot\text{m} \\ M_B &= M_E + \int_E^B V dx = -45 + 45 = 0 \end{aligned}$$

Maximum  $|V| = 300 \text{ N}$  ▶

Maximum  $|M| = 67.5 \text{ N}\cdot\text{m}$  ▶

PROBLEM 5.49



5.49 Using the methods of Sec. 5.3, solve Prob. 5.15.

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

SOLUTION

$$+\circlearrowleft \sum M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$$

$$A = -6 \text{ kips} \quad \text{i.e. } 6 \text{ kips } \downarrow$$

$$+\circlearrowleft \sum M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$$

$$C = 54 \text{ kips } \uparrow$$

Shear

$$V_A = -6 \text{ kips}$$

$$A \text{ to } C \quad 0 < x < 6 \text{ ft.} \quad w = -3 \text{ kips/ft}$$

$$V_B - V_A = -\int_0^6 w \, dx = -\int_0^6 3 \, dx = -18 \text{ kips}$$

$$V_B = -6 - 18 = -24 \text{ kips}$$

$$C \text{ to } B \quad V = -24 + 54 = 30 \text{ kips.}$$

Areas under shear diagram

$$A \text{ to } C \quad \int V \, dx = \left(\frac{1}{2}\right)(-6-24)(6) = -90 \text{ kip-ft.}$$

$$C \text{ to } B \quad \int V \, dx = (3)(30) = 90 \text{ kip-ft}$$

Bending moments

$$M_A = 0$$

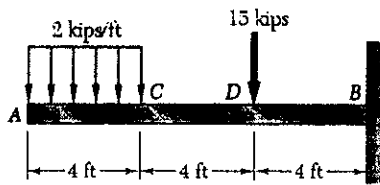
$$M_C = M_A + \int V \, dx = 0 - 90 = -90 \text{ kip-ft}$$

$$M_B = M_C + \int V \, dx = -90 + 90 = 0$$

$$\text{Maximum } |V| = 30 \text{ kips} \quad \blacktriangle$$

$$\text{Maximum } |M| = 90 \text{ kip-ft} \quad \blacktriangle$$

**PROBLEM 5.50**



5.50 Using the methods of Sec. 5.3, solve Prob. 5.16.

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

**SOLUTION**

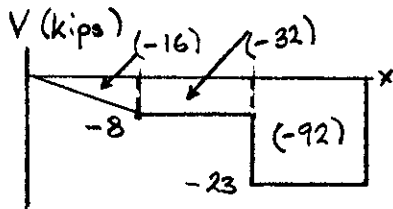
**Shear**

$$V_A = 0$$

$$V_B = V_A - \int_A^B w dx = 0 - (4)(2) = -8 \text{ kips.}$$

$$C \text{ to } D \quad V = -8 \text{ kips}$$

$$D \text{ to } B \quad V = -8 - 15 = -23 \text{ kips}$$



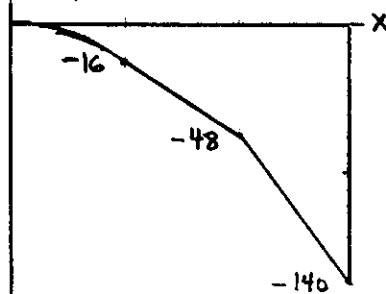
**Areas under shear diagram**

$$A \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(4)(-8) = -16 \text{ kip}\cdot\text{ft}$$

$$C \text{ to } D \quad \int V dx = (4)(-8) = -32 \text{ kip}\cdot\text{ft}$$

$$D \text{ to } B \quad \int V dx = (4)(-23) = -92 \text{ kip}\cdot\text{ft}$$

**M (kip·ft)**



**Bending moments**

$$M_A = 0$$

$$M_C = M_A + \int V dx = 0 - 16 = -16 \text{ kip}\cdot\text{ft}$$

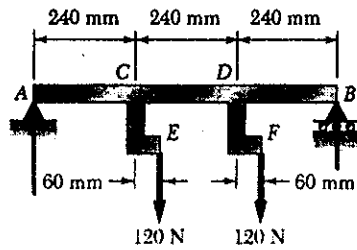
$$M_D = M_C + \int V dx = -16 - 32 = -48 \text{ kip}\cdot\text{ft}$$

$$M_B = M_D + \int V dx = -48 - 92 = -140 \text{ kip}\cdot\text{ft}$$

$$\text{Maximum } |V| = 23 \text{ kips} \quad \blacktriangleleft$$

$$\text{Maximum } |M| = 140 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

**PROBLEM 5.51**



5.51 and 5.52 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

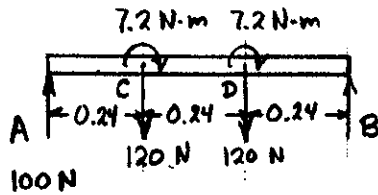
**SOLUTION**

$$\begin{aligned} +\curvearrowright \sum M_A = 0 & \quad -0.72 A + (0.48)(120) + (0.24)(120) \\ & \quad - 7.2 - 7.2 = 0 \end{aligned}$$

$$A = 100 \text{ N}$$

$$\begin{aligned} \uparrow \sum M_B = 0 & \quad -(0.24)(120) - (0.48)(120) - 7.2 \\ & \quad - 7.2 + 0.72 B = 0 \end{aligned}$$

$$B = 140 \text{ N}$$



**Shear**

A to C  $V = 100 \text{ N}$

C to D  $V = 100 - 120 = -20 \text{ N}$

D to B  $V = -20 - 120 = -140 \text{ N}$

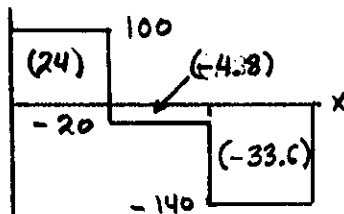
**Areas under shear diagram**

A to C  $\int V dx = (0.24)(100) = 24 \text{ N}\cdot\text{m}$

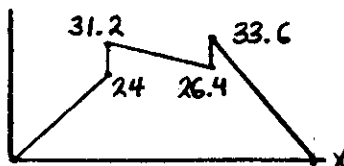
C to D  $\int V dx = (0.24)(-20) = -4.8 \text{ N}\cdot\text{m}$

D to B  $\int V dx = (0.24)(-140) = -33.6 \text{ N}\cdot\text{m}$

V (N)



M (N·m)



**Bending moments**

$$M_A = 0$$

$$M_C^- = 0 + 24 = 24 \text{ N}\cdot\text{m}$$

$$M_C^+ = 24 + 7.2 = 31.2 \text{ N}\cdot\text{m}$$

$$M_D^- = 31.2 - 4.8 = 26.4 \text{ N}\cdot\text{m}$$

$$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N}\cdot\text{m}$$

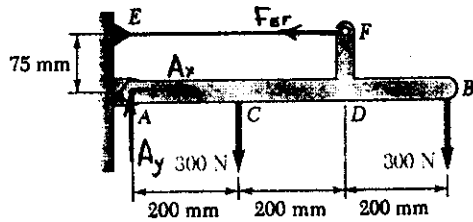
$$M_B = 33.6 - 33.6 = 0$$

Maximum  $|V| = 140 \text{ N}$  ▶

Maximum  $|M| = 33.6 \text{ N}\cdot\text{m}$  ▶

PROBLEM 5.52

5.51 and 5.52 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



SOLUTION

$$\curvearrowright \sum M_A = 0$$

$$0.075 F_{EF} - (0.2)(300) - (0.6)(300) = 0$$

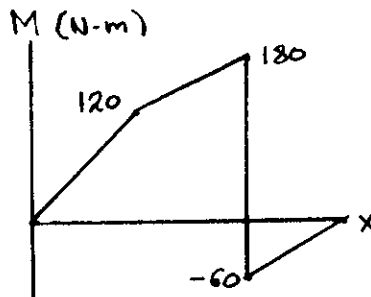
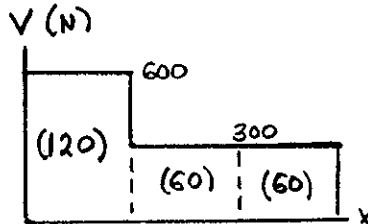
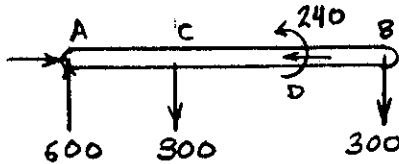
$$F_{EF} = 3.2 \times 10^3 \text{ N}$$

$$\rightarrow \sum F_x = 0 \quad A_x - F_{EF} = 0 \quad A_x = 3.2 \times 10^3 \text{ N}$$

$$\uparrow \sum F_y = 0 \quad A_y - 300 - 300 = 0$$

$$A_y = 600 \text{ N}$$

Couple at D  $M_D = (0.075)(3.2 \times 10^3)$   
 $= 240 \text{ N}\cdot\text{m}$



Shear

A to C  $V = 600 \text{ N}$

C to B  $V = 600 - 300 = 300 \text{ N}$

Areas under shear diagram

A to C  $\int V dx = (0.2)(600) = 120 \text{ N}\cdot\text{m}$

C to D  $\int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$

D to B  $\int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$

Bending moments

$$M_A = 0$$

$$M_C = 0 + 120 = 120 \text{ N}\cdot\text{m}$$

$$M_D^- = 120 + 60 = 180 \text{ N}\cdot\text{m}$$

$$M_D^+ = 180 - 240 = -60 \text{ N}\cdot\text{m}$$

$$M_B = -60 + 60 = 0$$

Maximum  $|V| = 600 \text{ N}$

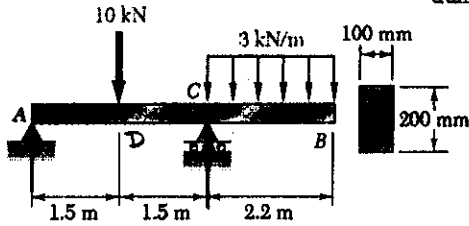
Maximum  $|M| = 180 \text{ N}\cdot\text{m}$



PROBLEM 5.53

5.53 Using the methods of Sec. 5.3, solve Prob. 5.21.

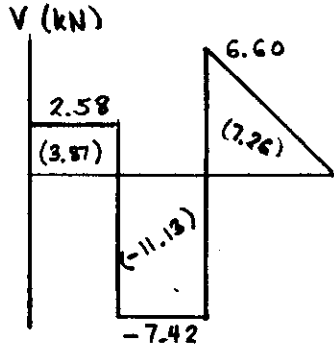
5.21 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



SOLUTION

$$\begin{aligned} \sum M_C = 0 \\ -3A + (1.5)(10) - (1.1)(2.2)(3) = 0 \\ A = 2.58 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_A = 0 \\ -(1.5)(10) + 3C - (4.1)(2.2)(3) = 0 \\ C = 14.02 \text{ kN} \end{aligned}$$

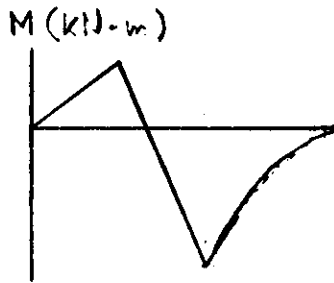


Shear

$$\begin{aligned} \text{A to D}^- \quad V &= 2.58 \text{ kN} \\ \text{D}^+ \text{ to C}^- \quad V &= 2.58 - 10 = -7.42 \text{ kN} \\ \text{C}^+ \quad V &= -7.42 + 14.02 = 6.60 \text{ kN} \\ \text{B} \quad V &= 6.60 - (2.2)(3) = 0 \end{aligned}$$

Areas under shear diagram

$$\begin{aligned} \text{A to D} \quad \int V dx &= (1.5)(2.58) = 3.87 \text{ kN}\cdot\text{m} \\ \text{D to C} \quad \int V dx &= (1.5)(-7.42) = -11.13 \text{ kN}\cdot\text{m} \\ \text{C to B} \quad \int V dx &= \left(\frac{1}{2}\right)(2.2)(6.60) = 7.26 \text{ kN}\cdot\text{m} \end{aligned}$$



Bending moments

$$\begin{aligned} M_A &= 0 \\ M_D &= 0 + 3.87 = 3.87 \text{ kN}\cdot\text{m} \\ M_C &= 3.87 - 11.13 = -7.26 \text{ kN}\cdot\text{m} \\ M_B &= 7.26 - 7.26 = 0 \end{aligned}$$

$$|M_C| = 7.26 \text{ kN}\cdot\text{m} = 7.26 \times 10^3 \text{ N}\cdot\text{m}$$

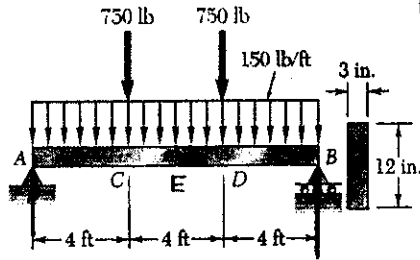
For rectangular cross section  $S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(100)(200)^2$   
 $= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{|M_C|}{S} = \frac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6 \text{ Pa}$   
 $= 10.89 \text{ MPa}$

PROBLEM 5.54

5.54 Using the methods of Sec. 5.3, solve Prob. 5.22.

5.22 For the beam and loading shown, determine the maximum normal stress on a transverse section at the center of the beam



SOLUTION

Reactions:  $C = A$  by symmetry

$$+\uparrow \sum F_y = 0 \quad A + C - (2)(750) - (12)(150) = 0$$

$$A = C = 1650 \text{ lb.}$$

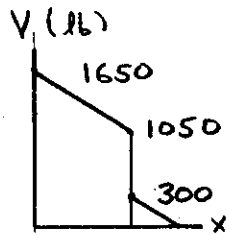
Shear:

$$V_A = 1650 \text{ lb.}$$

$$V_C^- = 1650 - (4)(150) = 1050 \text{ lb}$$

$$V_C^+ = 1050 - 750 = 300 \text{ lb}$$

$$V_E = 300 - (2)(150) = 0$$

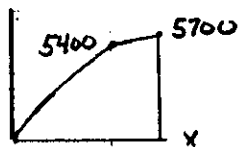


Areas under shear diagram

$$A \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(1650 + 1050)(4) = 5400 \text{ lb}\cdot\text{ft}$$

$$C \text{ to } E \quad \int V dx = \left(\frac{1}{2}\right)(300)(2) = 300 \text{ lb}\cdot\text{ft.}$$

M (lb·ft)



Bending moments

$$M_A = 0$$

$$M_C = 0 + 5400 = 5400 \text{ lb}\cdot\text{ft}$$

$$M_E = 5400 + 300 = 5700 \text{ lb}\cdot\text{ft.}$$

$$M_E = 5700 \text{ lb}\cdot\text{ft} = 68.4 \times 10^3 \text{ lb}\cdot\text{in.}$$

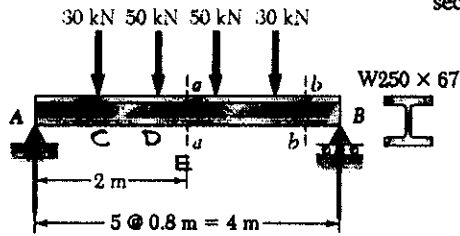
For rectangular cross section  $S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(3)(12)^2 = 72 \text{ in}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{68.4 \times 10^3}{72} = 950 \text{ psi}$

**PROBLEM 5.55**

5.55 Using the methods of Sec. 5.3, solve Prob. 5.23.

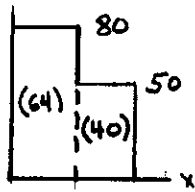
5.23 For the beam and loading shown, determine the maximum normal stress on section a-a.



**SOLUTION**

Reactions: By symmetry  $A = B$   
 $\uparrow \Sigma F_y = 0 \quad A = B = 80 \text{ kN}$

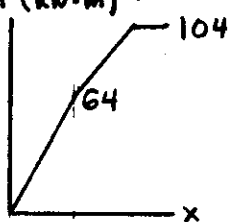
$V \text{ (kN)}$



Shear

A to C  $V = 80 \text{ kN}$   
 C to D  $V = 80 - 30 = 50 \text{ kN}$   
 D to E  $V = 50 - 50 = 0$

$M \text{ (kN}\cdot\text{m)}$



Areas under shear diagram

A to C  $\int V dx = (80)(0.8) = 64 \text{ kN}\cdot\text{m}$   
 C to D  $\int V dx = (50)(0.8) = 40 \text{ kN}\cdot\text{m}$   
 D to E  $\int V dx = 0$

Bending moments

$M_A = 0$   
 $M_C = 0 + 64 = 64 \text{ kN}\cdot\text{m}$   
 $M_D = 64 + 40 = 104 \text{ kN}\cdot\text{m}$   
 $M_E = 104 + 0 = 104 \text{ kN}\cdot\text{m}$

$M_E = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$

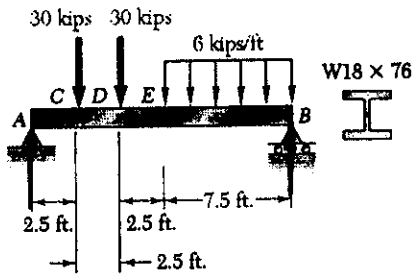
For W 250 x 67  $S = 809 \times 10^3 \text{ mm}^3 = 809 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{Ml}{S} = \frac{104 \times 10^3}{809 \times 10^{-6}} = 128.6 \times 10^6 \text{ Pa} = 128.6 \text{ MPa}$

**PROBLEM 5.56**

5.56 Using the methods of Sec. 5.3, solve Prob. 5.24.

5.24 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



**SOLUTION**

$$+\circlearrowleft \sum M_B = 0$$

$$-15A + (12.5)(30) + (10)(30) + (6)(7.5)(3.75) = 0$$

$$A = 56.25 \text{ kips}$$

Shear A to C  $V = 56.25 \text{ kips}$

Area under shear curve A to C.  $\int V dx = (56.25)(2.5) = 140.625 \text{ kip}\cdot\text{ft}$

$$M_A = 0$$

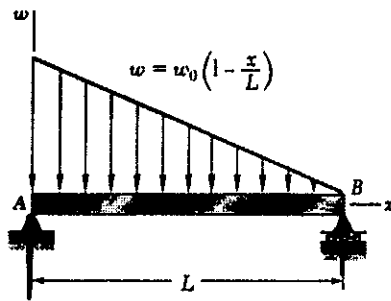
$$M_C = 0 + 140.625 = 140.625 \text{ kip}\cdot\text{ft} = 1687.5 \text{ kip}\cdot\text{in}$$

For W 18 x 76 rolled steel section  $S = 146 \text{ in}^3$

Normal stress  $\sigma = \frac{M}{S} = \frac{1687.5}{146} = 11.56 \text{ ksi}$

PROBLEM 5.57

5.57 and 5.58 Determine (a) the equations of the shear and bending-moment curves for the given beam and loading, (b) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$w = w_0 \left(1 - \frac{x}{L}\right)$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{w_0 x}{L}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = L$$

$$0 = -\frac{w_0 L^2}{2} + \frac{w_0 L^2}{6} + C_1 L \therefore C_1 = \frac{w_0 L}{3}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L x}{3}$$

$$M \text{ is maximum where } \frac{dM}{dx} = V = 0$$

$$0 = -w_0 x_m + \frac{w_0 x_m^2}{2L} + \frac{w_0 L}{3}$$

$$\frac{1}{2} x_m^2 - L x_m + \frac{1}{3} L^2 = 0$$

$$x_m = \frac{L \pm \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)}}{2(\frac{1}{2})}$$

$$= (1 \pm \frac{\sqrt{3}}{3}) L$$

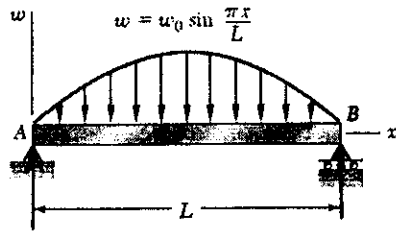
$$= 1.57735 L, 0.42265 L$$

$$M_{max} = \frac{-w_0 (0.42265 L)^2}{2} + \frac{w_0 (0.42265 L)^3}{6L} + \frac{w_0 L (0.42265 L)}{3}$$

$$= 0.06415 w_0 L^2$$

PROBLEM 5.58

5.57 and 5.58 Determine (a) the equations of the shear and bending-moment curves for the given beam and loading, (b) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = 0 \text{ at } x = L$$

$$0 = 0 + C_1 L + 0$$

$$C_1 = 0$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

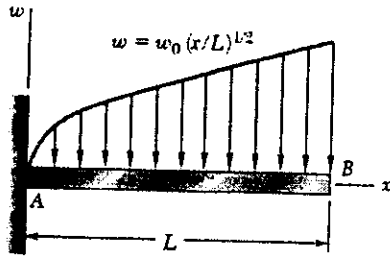
$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \text{ at } x = \frac{L}{2}$$

$$M_{max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2} = \frac{w_0 L^2}{\pi^2}$$

**PROBLEM 5.59**

5.59 Determine (a) the equations of the shear and bending-moment curves for the given beam and loading, (b) the maximum absolute value of the bending moment in the beam.



**SOLUTION**

$$\frac{dV}{dx} = -w = -w_0 \left(\frac{x}{L}\right)^{1/2} = -\frac{w_0 x^{1/2}}{L^{1/2}}$$

$$V = -\frac{2}{3} \frac{w_0 x^{3/2}}{L^{1/2}} + C_1$$

$$V = 0 \text{ at } x = L$$

$$0 = -\frac{2}{3} w_0 L + C_1$$

$$C_1 = \frac{2}{3} w_0 L$$

$$V = \frac{2}{3} w_0 L - \frac{2}{3} \frac{w_0 x^{3/2}}{L^{1/2}}$$

$$\frac{dM}{dx} = V$$

$$M = C_2 + \frac{2}{3} w_0 L x - \frac{2}{3} \cdot \frac{2}{5} \frac{w_0 x^{5/2}}{L^{1/2}}$$

$$M = 0 \text{ at } x = L$$

$$0 = C_2 + \frac{2}{3} w_0 L^2 - \frac{4}{15} w_0 L^2$$

$$C_2 = -\frac{2}{5} w_0 L^2$$

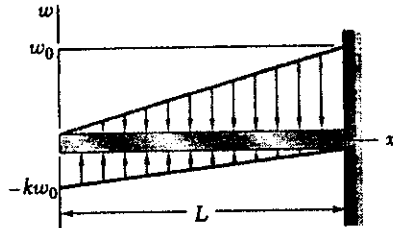
$$M = \frac{2}{3} w_0 L x - \frac{4}{15} \frac{w_0 x^{5/2}}{L^{1/2}} - \frac{2}{5} w_0 L^2$$

$$|M|_{\max} \text{ occurs at } x = 0$$

$$|M|_{\max} = \frac{2}{5} w_0 L^2$$

PROBLEM 5.60

5.60 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (a)  $k = 1$ , (b)  $k = 0.5$ .



SOLUTION

$$w = \frac{w_0 x}{L} - \frac{k w_0 (L-x)}{L} = (1+k) \frac{w_0 x}{L} - k w_0$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k) \frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k) \frac{w_0 x^2}{2L} + C_1$$

$$V = 0 \text{ at } x = 0$$

$$C_1 = 0$$

$$\frac{dM}{dx} = V = k w_0 x - (1+k) \frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k) \frac{w_0 x^3}{6L} + C_2$$

$$M = 0 \text{ at } x = 0$$

$$C_2 = 0$$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

(a)  $k = 1$

$$V = w_0 x - \frac{w_0 x^2}{L}$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L}$$

Maximum  $M$  occurs at  $x = L$

$$|M|_{\max} = \frac{w_0 L^2}{6}$$

(b)  $k = \frac{1}{2}$

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L}$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L}$$

$$V = 0 \text{ at } x = \frac{2}{3} L$$

$$\text{At } x = \frac{2}{3} L \quad M = \frac{w_0 (\frac{2}{3} L)^2}{4} - \frac{w_0 (\frac{2}{3} L)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704 w_0 L^2$$

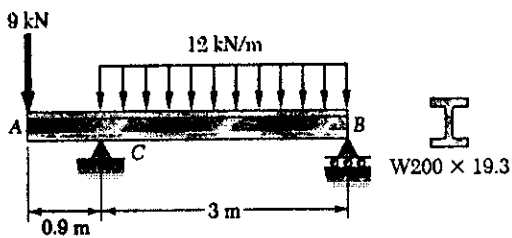
$$\text{At } x = L \quad M = 0$$

$$|M|_{\max} = \frac{w_0 L^2}{27}$$



**PROBLEM 5.61**

5.61 and 5.62 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



**SOLUTION**

$$+\circlearrowleft \sum M_C = 0$$

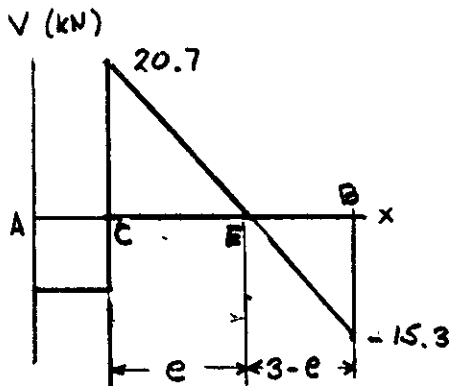
$$(0.9)(9) - (1.5)(3)(12) + 3B = 0$$

$$B = 15.3 \text{ kN}$$

$$+\circlearrowleft \sum M_B = 0$$

$$(3.9)(9) - 3C + (1.5)(3)(12) = 0$$

$$C = 29.7 \text{ kN}$$



Shear: A to C  $V = -9 \text{ kN}$

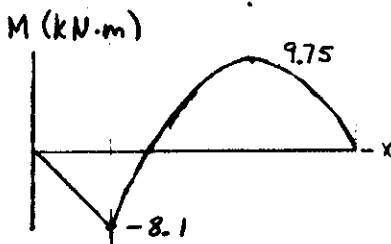
$$C^+ \quad V = -9 + 29.7 = 20.7 \text{ kN}$$

$$B \quad V = 20.7 - (3)(12) = -15.3 \text{ kN}$$

Locate point E where  $V = 0$

$$\frac{e}{20.7} = \frac{3-e}{15.3} \quad 36e = (20.7)(3)$$

$$e = 1.725 \text{ ft} \quad 3 - e = 1.275 \text{ ft}$$



Areas under shear diagram

$$A \text{ to } C \quad \int V dx = (0.9)(9) = 8.1 \text{ kN}\cdot\text{m}$$

$$C \text{ to } E \quad \int V dx = \left(\frac{1}{2}\right)(1.725)(20.7) = 17.85375 \text{ kN}\cdot\text{m}$$

$$E \text{ to } B \quad \int V dx = \left(\frac{1}{2}\right)(1.275)(-15.3) = -9.75375 \text{ kN}\cdot\text{m}$$

Bending moments

$$M_A = 0$$

$$M_B = 0 - 8.1 = -8.1 \text{ kN}\cdot\text{m}$$

$$M_E = -8.1 + 17.85375 = 9.75375 \text{ kN}\cdot\text{m}$$

$$M_B = 9.75375 - 9.75375 = 0$$

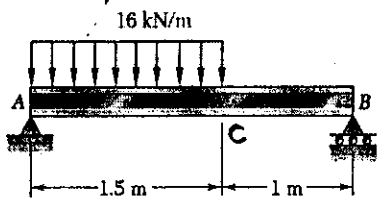
Maximum  $|M| = 9.75375 \times 10^3 \text{ N}\cdot\text{m}$  at point E

For W 200 x 19.3 rolled steel section  $S = 164 \times 10^3 \text{ mm}^3 = 164 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{9.75375 \times 10^3}{164 \times 10^{-6}} = 59.5 \times 10^6 \text{ Pa} = 59.5 \text{ MPa}$$

PROBLEM 5.62

5.61 and 5.62 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$$+\circlearrowleft \sum M_B = 0$$

$$-2.5 A + (1.75)(1.5)(16) = 0$$

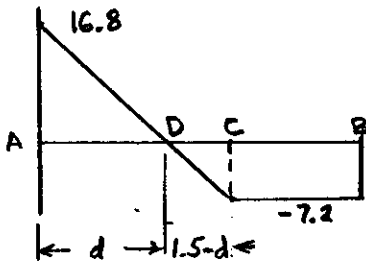
$$A = 16.8 \text{ kN}$$

$$+\circlearrowright \sum M_A = 0$$

$$-(0.75)(1.5)(16) + 2.5 B = 0$$

$$B = 7.2 \text{ kN}$$

V (kN)



Shear:

$$V_A = 16.8 \text{ kN}$$

$$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$$

$$V_B = -7.2 \text{ kN}$$

Locate point D where  $V = 0$

$$\frac{d}{16.8} = \frac{1.5-d}{7.2} \quad 24d = 25.2$$

$$d = 1.05 \text{ m} \quad 1.5 - d = 0.45 \text{ m}$$

Areas under shear diagram

$$\text{A to D} \quad \int V dx = \left(\frac{1}{2}\right)(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$$

$$\text{D to C} \quad \int V dx = \left(\frac{1}{2}\right)(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$$

$$\text{C to B} \quad \int V dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$$

Bending moments

$$M_A = 0$$

$$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$$

$$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$$

$$M_B = 7.2 - 7.2 = 0$$

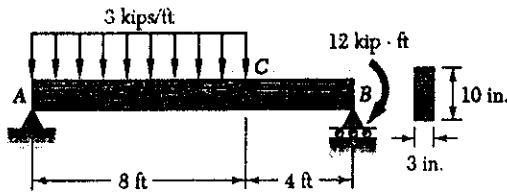
$$\text{Maximum } |M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{For S } 150 \times 18.6 \text{ rolled steel section } S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa} = 73.5 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 5.63

5.63 and 5.64 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$$\begin{aligned} +\circlearrowleft \sum M_B &= 0 \\ -12A + (8)(3)(3) - 12 &= 0 \\ A &= 15 \text{ kips} \end{aligned}$$

$$\begin{aligned} \uparrow \sum M_A &= 0 \\ -(4)(3)(3) + 12B - 12 &= 0 \\ B &= 9 \text{ kips} \end{aligned}$$

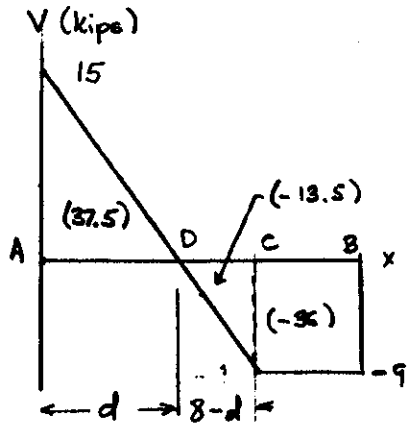
Shear:  $V_A = 15$  kips

$$V_C = 15 - (8)(3) = -9 \text{ kips}$$

C to B  $V = -9$  kips.

Locate point D where  $V = 0$

$$\begin{aligned} \frac{d}{15} &= \frac{8-d}{9} & 24d &= 120 \\ d &= 5 \text{ ft} & 8-d &= 3 \text{ ft} \end{aligned}$$

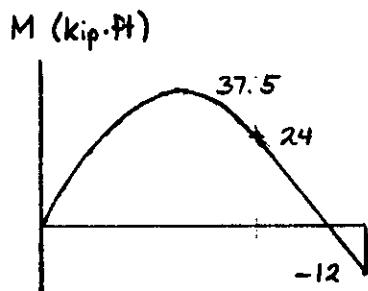


Areas under shear diagram

$$A \text{ to } D \quad \int V dx = \left(\frac{1}{2}\right)(5)(15) = 37.5 \text{ kip}\cdot\text{ft}$$

$$D \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(3)(-9) = -13.5 \text{ kip}\cdot\text{ft}$$

$$C \text{ to } B \quad \int V dx = (4)(-9) = -36 \text{ kip}\cdot\text{ft}$$



Bending moments:  $M_A = 0$

$$M_D = 0 + 37.5 = 37.5 \text{ kip}\cdot\text{ft}$$

$$M_C = 37.5 - 13.5 = 24 \text{ kip}\cdot\text{ft}$$

$$M_B = 24 - 36 = -12 \text{ kip}\cdot\text{ft}$$

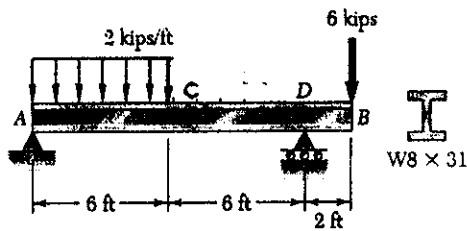
$$\text{Maximum } |M| = 37.5 \text{ kip}\cdot\text{ft} = 450 \text{ kip}\cdot\text{in}$$

$$\text{For rectangular cross section } S = \frac{1}{6}bh^2 = \left(\frac{1}{6}\right)(3)(10)^2 = 50 \text{ in}^3$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{450}{50} = 9 \text{ ksi}$$

PROBLEM 5.64

5.63 and 5.64 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$$+\circlearrowleft M_D = 0$$

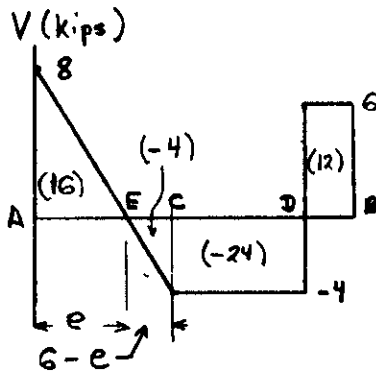
$$-12A + (9)(6)(2) - (2)(6) = 0$$

$$A = 8 \text{ kips}$$

$$+\circlearrowleft M_A = 0$$

$$-(3)(6)(2) + 12D - (14)(6) = 0$$

$$D = 10 \text{ kips}$$

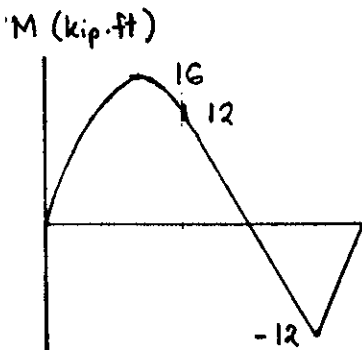


Shear:  $V_A = 8 \text{ kips}$   
 $V_C = 8 - (6)(2) = -4 \text{ kips}$   
 C to D  $V = -4 \text{ kips}$   
 D to B  $V = -4 + 10 = 6 \text{ kips}$

Locate point E where  $V = 0$

$$\frac{e}{8} = \frac{6-e}{4} \quad 12e = 48$$

$$e = 4 \text{ ft} \quad 6-e = 3 \text{ ft}$$



Areas under shear diagram

A to E  $\int V dx = (\frac{1}{2})(4)(8) = 16 \text{ kip}\cdot\text{ft}$   
 E to C  $\int V dx = (\frac{1}{2})(2)(-4) = -4 \text{ kip}\cdot\text{ft}$   
 C to D  $\int V dx = (6)(-4) = -24 \text{ kip}\cdot\text{ft}$   
 D to B  $\int V dx = (2)(6) = 12 \text{ kip}\cdot\text{ft}$

Bending moments:  $M_A = 0$   
 $M_E = 0 + 16 = 16 \text{ kip}\cdot\text{ft}$   
 $M_C = 16 - 4 = 12 \text{ kip}\cdot\text{ft}$   
 $M_D = 12 - 24 = -12 \text{ kip}\cdot\text{ft}$   
 $M_B = -12 + 12 = 0$

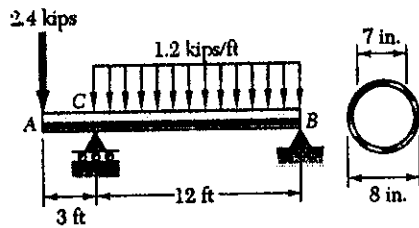
Maximum  $|M| = 16 \text{ kip}\cdot\text{ft} = 192 \text{ kip}\cdot\text{in}$

For W 8x31 rolled steel section  $S = 27.5 \text{ in}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{192}{27.5} = 6.98 \text{ ksi}$

PROBLEM 5.65

5.65 and 5.66 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$$+\circlearrowleft \sum M_C = 0$$

$$(3)(2.4) - (6)(12)(1.2) + 12 B = 0$$

$$B = 6.6 \text{ kips}$$

$$+\circlearrowleft \sum M_B = 0$$

$$(15)(2.4) + (6)(12)(1.2) - 12 C = 0$$

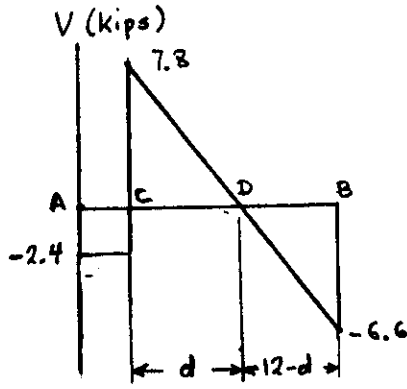
$$C = 10.2 \text{ kips}$$

Shear: A to C  $V = -2.4 \text{ kips}$   
 C<sup>+</sup>  $V = -2.4 + 10.2 = 7.8 \text{ kips}$   
 B  $V_B = 7.8 - (12)(1.2) = -6.6 \text{ kips}$

locate point D where  $V = 0$

$$\frac{d}{7.8} = \frac{12-d}{6.6} \quad 14.4 d = 93.6$$

$$d = 6.5 \text{ ft.} \quad 12-d = 5.5 \text{ ft.}$$



Areas under shear diagram

A to C  $\int V dx = (3)(-2.4) = -7.2 \text{ kip}\cdot\text{ft}$   
 C to D  $\int V dx = (\frac{1}{2})(6.5)(7.8) = 25.35 \text{ kip}\cdot\text{ft}$   
 D to B  $\int V dx = (\frac{1}{2})(5.5)(-6.6) = -18.15 \text{ kip}\cdot\text{ft}$

Bending moments  $M_A = 0$

$$M_C = 0 - 7.2 = -7.2 \text{ kip}\cdot\text{ft.}$$

$$M_D = -7.2 + 25.35 = 18.15 \text{ kip}\cdot\text{ft}$$

$$M_B = 18.15 - 18.15 = 0$$

Maximum  $|M| = 18.15 \text{ kip}\cdot\text{ft} = 217.8 \text{ kip}\cdot\text{in.}$

For pipe  $C_o = \frac{d_o}{2} = \frac{8}{2} = 4 \text{ in.}$   $C_i = \frac{d_i}{2} = \frac{7}{2} = 3.5 \text{ in.}$

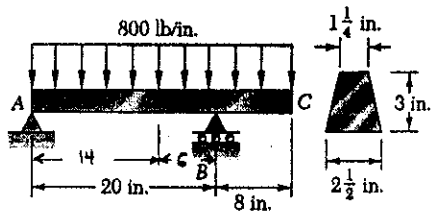
$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}(4^4 - 3.5^4) = 82.20 \text{ in}^4$$

$$S = \frac{I}{C_o} = \frac{82.20}{4} = 20.80 \text{ in}^3$$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{217.8}{20.80} = 10.47 \text{ ksi}$

PROBLEM 5.66

5.65 and 5.66 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



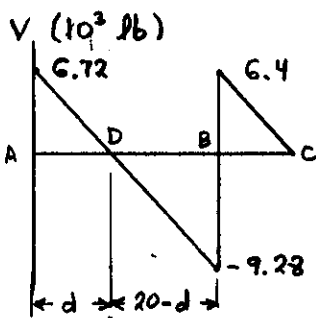
SOLUTION

$$\sum M_B = 0 \quad -20A + (6)(28)(800) = 0$$

$$A = 6.72 \times 10^3 \text{ lb.}$$

$$\sum M_A = 0 \quad 20B - (14)(28)(800) = 0$$

$$B = 15.68 \times 10^3 \text{ lb.}$$



Shear:  $V_A = 6.72 \times 10^3 \text{ lb.}$

$$B^- \quad V_B^- = 6.72 \times 10^3 - (20)(800) = -9.28 \times 10^3 \text{ lb.}$$

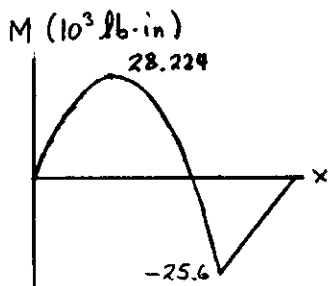
$$B^+ \quad V_B^+ = -9.28 \times 10^3 + (15.68 \times 10^3) = 6.4 \times 10^3 \text{ lb.}$$

$$C \quad V_C = 6.4 \times 10^3 - (8)(800) = 0$$

locate point D where  $V = 0$

$$\frac{d}{6.72} = \frac{20-d}{9.28} \quad 16d = 134.4$$

$$d = 8.4 \text{ in} \quad 20-d = 11.6 \text{ in.}$$



Areas under shear diagram

$$A \text{ to } D \quad \int V dx = \left(\frac{1}{2}\right)(8.4)(6.72 \times 10^3) = 28.224 \times 10^3 \text{ lb}\cdot\text{in}$$

$$D \text{ to } B \quad \int V dx = \left(\frac{1}{2}\right)(11.6)(-9.28 \times 10^3) = -53.824 \times 10^3 \text{ lb}\cdot\text{in}$$

$$B \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(8)(6.4 \times 10^3) = 25.6 \times 10^3 \text{ lb}\cdot\text{in}$$

Bending moments:  $M_A = 0$

$$M_D = 0 + 28.224 \times 10^3 = 28.224 \times 10^3 \text{ lb}\cdot\text{in}$$

$$M_B = 28.224 \times 10^3 - 53.824 \times 10^3 = -25.6 \times 10^3 \text{ lb}\cdot\text{in}$$

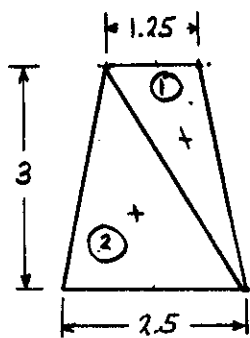
$$M_C = -25.6 \times 10^3 + 25.6 \times 10^3 = 0$$

Maximum  $|M| = 28.224 \times 10^3 \text{ lb}\cdot\text{in}$

locate centroid of cross section

$$\bar{y} = \frac{7.5}{5.625} = 1.3333 \text{ in. from bottom}$$

For each triangle  $\bar{I} = \frac{1}{36} b h^3$



Part	$A, \text{in}^2$	$\bar{y}, \text{in}$	$A\bar{y}, \text{in}^3$	$d, \text{in}$	$Ad^2, \text{in}^4$	$\bar{I}, \text{in}^4$
①	1.875	2	3.75	0.6667	0.8333	0.9375
②	3.75	1	3.75	0.3333	0.4167	1.875
$\Sigma$	5.625		7.5		1.25	2.8125

Moment of inertia

$$I = \Sigma \bar{I} + \Sigma Ad^2$$

$$= 1.25 + 2.8125 = 4.0625 \text{ in}^4$$

Normal stress

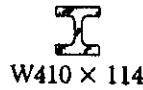
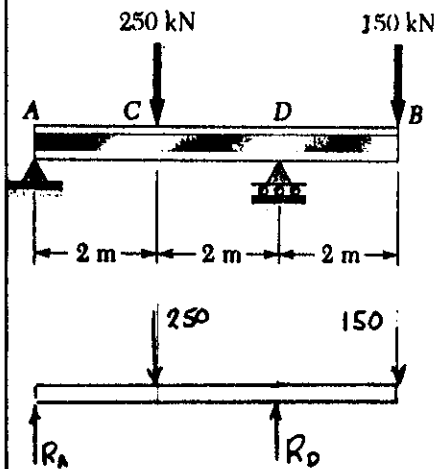
$$\sigma = \frac{Mc}{I} = \frac{(28.224 \times 10^3)(1.6667)}{4.0625}$$

$$= 11.58 \times 10^3 \text{ psi}$$

$$= 11.58 \text{ ksi}$$

PROBLEM 5.67

5.67 and 5.68 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$w = 0$

$\sum M_D = 0$

$-4R_A + (2)(250) - (2)(150) = 0$

$R_A = 50 \text{ kN } \uparrow$

$\sum M_A = 0$

$4R_D - (2)(250) - (6)(150) = 0$

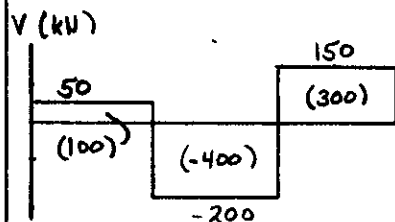
$R_D = 350 \text{ kN } \uparrow$

Shear:  $V_A = 50 \text{ kN}$

A to C  $V = 50 \text{ kN}$

C to D  $V = 50 - 250 = -200 \text{ kN}$

D to B  $V = -200 + 350 = 150 \text{ kN}$

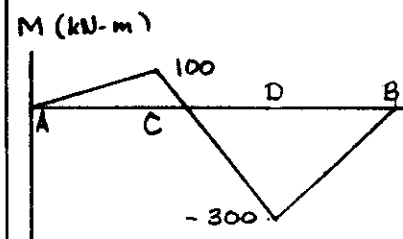


Areas of shear diagram

A to C  $\int V dx = (50)(2) = 100 \text{ kN}\cdot\text{m}$

C to D  $\int V dx = (-200)(2) = -400 \text{ kN}\cdot\text{m}$

D to B  $\int V dx = (150)(2) = 300 \text{ kN}\cdot\text{m}$



Bending moments:  $M_A = 0$

$M_C = M_A + \int V dx = 0 + 100 = 100 \text{ kN}\cdot\text{m}$

$M_D = M_C + \int V dx = 100 - 400 = -300 \text{ kN}\cdot\text{m}$

$M_B = M_D + \int V dx = -300 + 300 = 0$

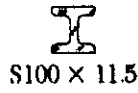
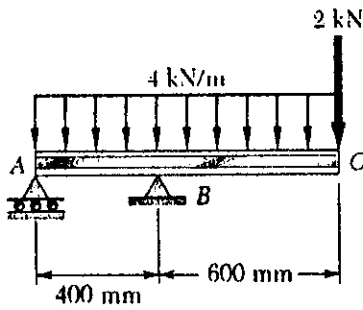
maximum  $|M| = 300 \text{ kN}\cdot\text{m} = 300 \times 10^3 \text{ N}\cdot\text{m}$

For W410 x 114 rolled steel section  $S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa} = 136.4 \text{ MPa}$

PROBLEM 5.68

5.67 and 5.68 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



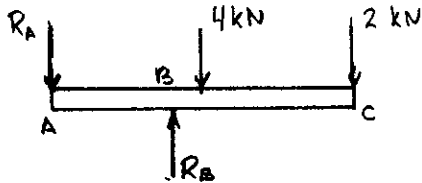
SOLUTION

$$\sum M_B = 0 \quad (0.4)(R_A) - (0.1)(4) - (0.6)(2) = 0$$

$$R_A = 4 \text{ kN} \downarrow$$

$$\sum M_A = 0 \quad (0.4)(R_B) - (0.5)(4) - (1)(2) = 0$$

$$R_B = 10 \text{ kN} \uparrow$$



A to B  $0 < x < 0.4 \text{ m}$

$$\frac{dV}{dx} = -w = -4 \text{ kN/m}$$

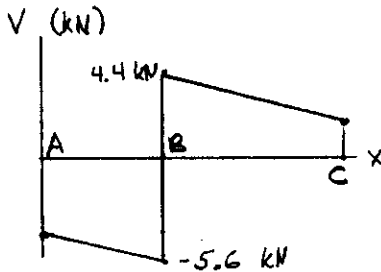
$$V = -4x - 4 \text{ kN}$$

$$\text{At } x = 0.4 \text{ m} \quad V_B = -5.6 \text{ kN}$$

$$\frac{dM}{dx} = -4x - 4$$

$$M = M_A - 2x^2 - 4x = 0 - 2x^2 - 4x$$

$$\text{At } x = 0.4 \text{ m} \quad M_B = 1.92 \text{ kN}\cdot\text{m}$$



B to C  $0.4 \text{ m} < x < 1.0 \text{ m}$

$$\frac{dV}{dx} = -w = -4 \text{ kN/m}$$

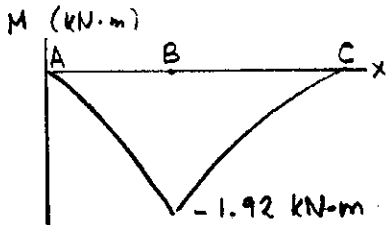
$$V = 4.4 - 4(x - 0.4) = 6 - 4x \text{ kN}$$

$$\frac{dM}{dx} = 6 - 4x$$

$$M = 6x - 2x^2 + C_1 \quad \text{kN}\cdot\text{m}$$

$$M = 0 \text{ at } x = 1 \therefore C_1 = 4 \text{ kN}\cdot\text{m}$$

$$M = 4 + 6x - 2x^2 \quad \text{kN}\cdot\text{m}$$



$$|M|_{\max} = 1.92 \text{ kN}\cdot\text{m} = 1.92 \times 10^3 \text{ N}\cdot\text{m}$$

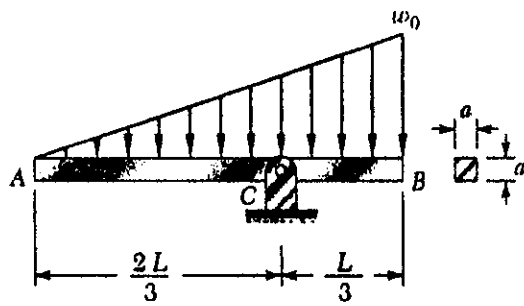
For S 100 x 11.5 rolled steel section  $S_x = 49.6 \times 10^3 \text{ mm}^3 = 49.6 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{\max}}{S_x} = \frac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} = 38.7 \text{ MPa}$$



PROBLEM 5.69

5.69 Beam  $AB$ , of length  $L$  and square cross section of side  $a$ , is supported by a pivot at  $C$  and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum normal stress due to bending occurs at  $C$  and is equal to  $w_0 L^2 / (1.5a)^3$ .



SOLUTION

Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

For the triangular distribution the centroid lies at  $x = \frac{2L}{3}$ .

$$W = \frac{1}{2} w_0 L$$

$$\sum F_y = 0 \quad R_0 - W = 0 \quad R_0 = \frac{1}{2} w_0 L$$

$$\sum M_c = 0 \quad 0 = 0 \quad \text{equilibrium}$$

$$V = 0, \quad M = 0 \quad \text{at } x = 0$$

$$0 < x < \frac{2L}{3}$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$

Just to the left of  $C$

$$V = -\frac{w_0 (2L/3)^2}{2L} = -\frac{2}{9} w_0 L$$

Just to the right of  $C$

$$V = -\frac{2}{9} w_0 L + R_0 = \frac{5}{18} w_0 L$$

Note sign change. Maximum  $|M|$  occurs at  $C$ .

$$M_c = -\frac{w_0 (2L/3)^3}{6L} = -\frac{4}{81} w_0 L^2$$

$$\text{Maximum } |M| = \frac{4}{81} w_0 L^2$$

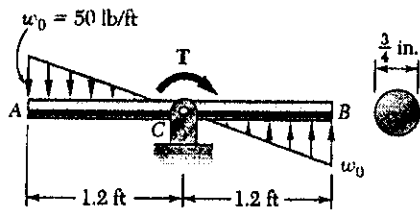
For square cross section

$$I = \frac{1}{12} a^4 \quad c = \frac{1}{2} a$$

$$\sigma_m = \frac{|M|_{\max} c}{I} = \frac{\frac{4}{81} w_0 L^2 \cdot \frac{a}{2}}{\frac{1}{12} a^4} = \frac{8}{27} \frac{w_0 L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_0 L^2}{a^3} = \frac{w_0 L^2}{(1.5a)^3}$$

PROBLEM 5.70

5.70 Knowing that rod  $AB$  is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



SOLUTION

A to C  $0 < x < 1.2 \text{ ft}$

$$w = 50 \left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

$$V = V_A + \int_0^x (41.667x - 50) dx$$

$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x) dx$$

$$= 6.944x^3 - 25x^2$$

At  $x = 1.2 \text{ ft}$ ,  $V = -30 \text{ lb}$ ,  
 $M = -24 \text{ lb}\cdot\text{ft}$

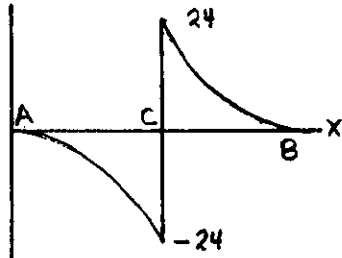
C to B Use symmetry conditions.

Maximum  $|M| = 24 \text{ lb}\cdot\text{ft} = 288 \text{ lb}\cdot\text{in}$

V (lb.)



M (lb-ft)



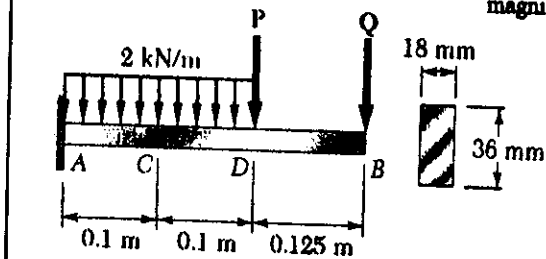
Cross section  $c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375 \text{ in}$

$$I = \frac{\pi}{4} c^4 = \left(\frac{\pi}{4}\right)(0.375)^4 = 15.532 \times 10^{-3} \text{ in}^4$$

Normal stress  $\sigma = \frac{M|c}{I} = \frac{(288)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3 \text{ psi}$   
 $= 6.95 \text{ ksi}$

PROBLEM 5.71

5.71 Beam AB supports a uniformly distributed load of 2 kN/m and two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending on the bottom edge of the beam is -56.9 MPa at A and -29.9 MPa at C. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads P and Q.



SOLUTION

$$I = \frac{1}{12}(18)(36)^3 = 69,984 \times 10^3 \text{ mm}^4$$

$$c = \frac{1}{2}d = 18 \text{ mm}$$

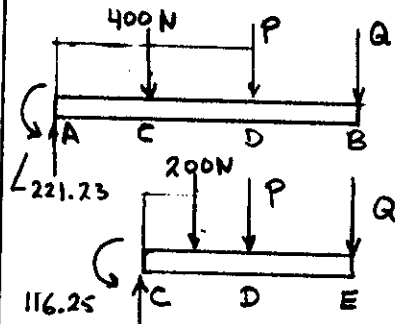
$$S = \frac{I}{c} = 3.888 \times 10^3 \text{ mm}^3 = 3.888 \times 10^{-6} \text{ m}^3$$

At A  $M_A = S \sigma_A$

$$M_A = (3.888 \times 10^{-6})(-56.9) = -221.25 \text{ N}\cdot\text{m}$$

At C  $M_C = S \sigma_C$

$$M_C = (3.888 \times 10^{-6})(-29.9) = -116.25 \text{ N}\cdot\text{m}$$



$$\sum M_A = 0$$

$$221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \quad (1)$$

$$+\sum M_C = 0$$

$$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

$$0.1P + 0.225Q = 106.25 \quad (2)$$

Solving (1) and (2) simultaneously

$$P = 500 \text{ N} \quad \blacktriangleleft$$

$$Q = 250 \text{ N} \quad \blacktriangleleft$$

Reaction force at A

$$R_A - 400 - 500 - 250 = 0$$

$$R_A = 1150 \text{ N}\cdot\text{m}$$

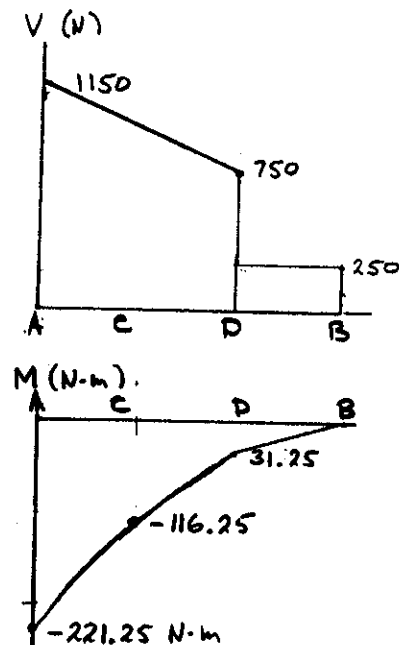
$$V_A = 1150 \text{ N}$$

$$V_D = 250$$

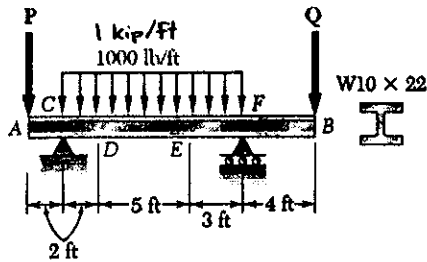
$$M_A = -221.25 \text{ N}\cdot\text{m}$$

$$M_C = -116.25 \text{ N}\cdot\text{m}$$

$$M_D = -31.25 \text{ N}\cdot\text{m}$$



PROBLEM 5.72



5.72 Beam AB supports a uniformly distributed load of 1000 lb/ft and two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending on the bottom edge of the lower flange of the W 10 × 22 rolled-steel beam is +2.07 ksi at D and +0.776 ksi at E. (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending which occurs in the beam.

SOLUTION

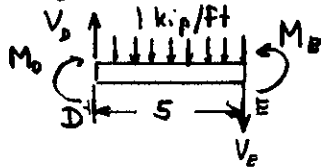
For W10 × 22 rolled steel section  $S = 23.2 \text{ in}^3$

Bending moments at D and E  $M = S\sigma$

$$M_D = (23.2)(2.07) = 48.0 \text{ kip}\cdot\text{in} = 4.00 \text{ kip}\cdot\text{ft}$$

$$M_E = (23.2)(0.776) = 18.0 \text{ kip}\cdot\text{in} = 1.50 \text{ kip}\cdot\text{ft}$$

Using portion DE as a free body



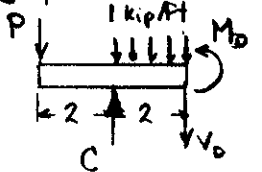
$$\sum M_E = 0 \quad -M_D + M_E - 5V_D + (2.5)(5)(1) = 0$$

$$V_D = 2 \text{ kips}$$

$$+\sum M_D = 0 \quad -M_D + M_E - 5V_E - (2.5)(5)(1) = 0$$

$$V_E = -3 \text{ kips}$$

Using portion ACD as a free body



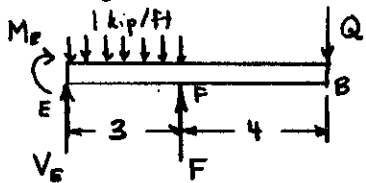
$$\sum M_C = 0 \quad 2P + (1)(2)(1) + M_D - 2V_D = 0$$

$$P = 1 \text{ kip}$$

$$+\uparrow \sum F_y = 0 \quad -P + C - (2)(1) - V_D = 0$$

$$C = 5 \text{ kips}$$

Using portion EFB as a free body



$$+\sum M_F = 0 \quad -4Q + (1.5)(3)(1) - 3V_E - M_E = 0$$

$$Q = 3 \text{ kips}$$

$$+\uparrow \sum F_y = 0 \quad F + V_E - (3)(1) - Q = 0$$

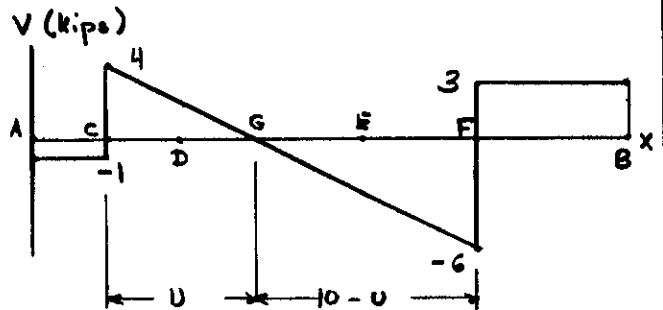
$$F = 9 \text{ kips}$$

Shear: A to C  $V = -1 \text{ kips}$   
 C<sup>+</sup>  $V = -1 + 5 = 4 \text{ kips}$   
 F<sup>-</sup>  $V = 4 - (10)(1) = -6 \text{ kips}$   
 F<sup>+</sup>  $V = -6 + 9 = 3 \text{ kips}$   
 F to B  $V = 3 \text{ kips}$

Locate point G where  $V = 0$

$$\frac{U}{4} = \frac{10-U}{6} \quad 10U = 40$$

$$U = 4 \text{ ft} \quad 10-U = 6 \text{ ft}$$



continued

Problem 5.72 continued

Areas under shear diagram

A to C  $\int V dx = (2)(-1) = -2 \text{ kip}\cdot\text{ft}$   
 C to G  $\int V dx = (\frac{1}{2})(4)(4) = 8 \text{ kip}\cdot\text{ft}$   
 G to F  $\int V dx = (\frac{1}{2})(6)(-6) = -18 \text{ kip}\cdot\text{ft}$   
 F to B  $\int V dx = (4)(3) = 12 \text{ kip}\cdot\text{ft}$

Bending moments  $M_A = 0$

$M_C = 0 - 2 = -2 \text{ kip}\cdot\text{ft}$

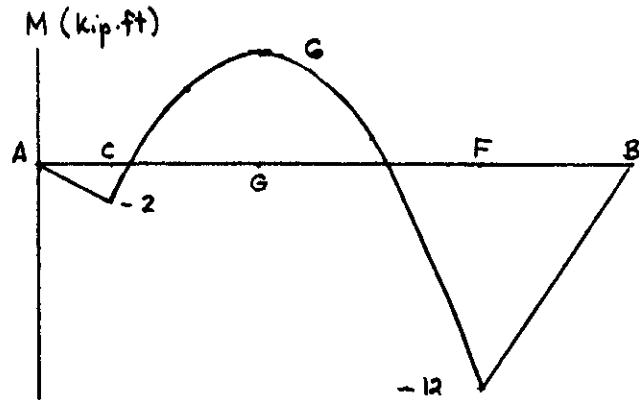
$M_G = -2 + 8 = 6 \text{ kip}\cdot\text{ft}$

$M_F = 6 - 18 = -12 \text{ kip}\cdot\text{ft}$

$M_B = -12 + 12 = 0$

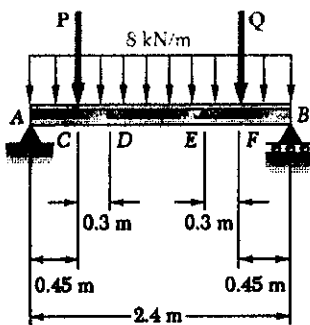
Maximum  $|M| = 12 \text{ kip}\cdot\text{ft}$   
 $= 144 \text{ kip}\cdot\text{in.}$

Normal stress  $\sigma = \frac{M}{S} = \frac{144}{23.2} = 6.21 \text{ ksi}$



PROBLEM 5.73

\* 5.73 Beam AB supports a uniformly distributed load of 8 kN/m and two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending on the bottom edge of the lower flange of the W 200 x 52 rolled-steel beam is 100 MPa at D and 70 MPa at E. (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending which occurs in the beam.



W200 x 52

SOLUTION

For W 200 x 52 rolled steel section

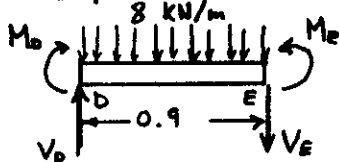
$S = 512 \times 10^3 \text{ mm}^3 = 512 \times 10^{-6} \text{ m}^3$

Bending moments at D and E  $M = S\sigma$

$M_D = (512 \times 10^{-6})(100 \times 10^6) = 51.2 \times 10^3 \text{ N}\cdot\text{m}$

$M_E = (512 \times 10^{-6})(70 \times 10^6) = 35.84 \times 10^3 \text{ N}\cdot\text{m}$

Using portion DE as a free body



$\sum M_E = 0 \quad -0.9 V_D - M_D + M_E + (0.45)(0.9)(8) = 0$

$V_D = -13.467 \text{ kN}$

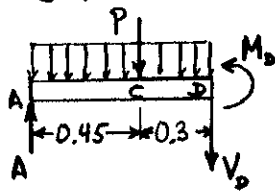
$\sum F_y = 0 \quad V_D - V_E - (0.9)(8) = 0$

$V_E = -20.667 \text{ kN}$

continued

Problem 5.73 continued

Using portion ACD as a free body



$$+\circlearrowleft \sum M_A = 0$$

$$-0.45 P - (0.375)(0.75)(8) - 0.75 V_D + M_D = 0$$

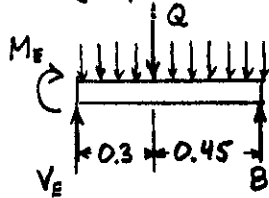
$$P = 131.222 \text{ kN}$$

$$+\uparrow \sum F_y = 0$$

$$A - P - V_D - (0.75)(8) = 0$$

$$A = 123.756 \text{ kN}$$

Using portion EFB as a free body



$$+\circlearrowleft \sum M_B = 0$$

$$0.45 Q + (0.375)(0.75)(8) - 0.75 V_E - M_E = 0$$

$$Q = 40.2 \text{ kN}$$

$$+\uparrow \sum F_y = 0$$

$$V_E - Q - (0.75)(8) + B = 0$$

$$B = 66.867 \text{ kN}$$

Shear:  $V_A = 123.756 \text{ kN}$

$$V_{C^-} = 123.756 - (0.45)(8) = 120.155 \text{ kN}$$

$$V_{C^+} = 120.155 - 131.222 = -11.067 \text{ kN}$$

$$V_{F^-} = -11.067 - (1.5)(8) = -23.067 \text{ kN}$$

$$V_{F^+} = -23.067 - 40.2 = -63.267 \text{ kN}$$

$$V_B = -63.267 - (0.45)(8) = -66.867 \text{ kN}$$

Areas under shear diagram

$$A \text{ to } C: \frac{1}{2}(0.45)(123.756 + 120.155) = 54.88 \text{ kN}\cdot\text{m}$$

$$C \text{ to } F: \frac{1}{2}(1.5)(-11.067 - 23.067) = -25.6 \text{ kN}\cdot\text{m}$$

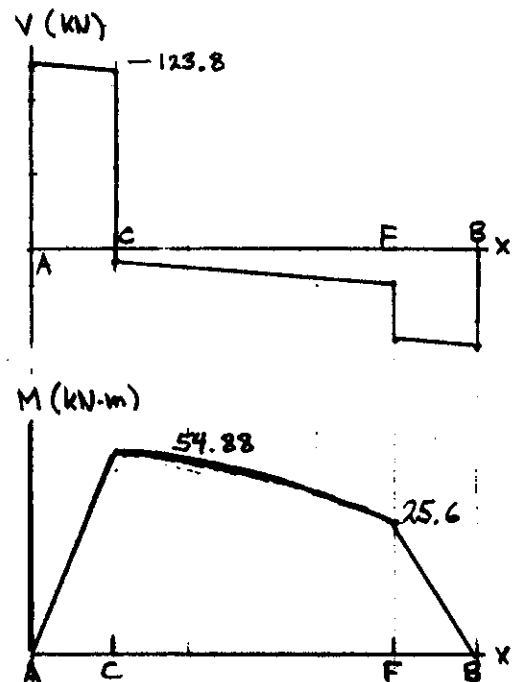
$$F \text{ to } B: \frac{1}{2}(0.45)(-63.267 - 66.867) = -29.28 \text{ kN}\cdot\text{m}$$

Bending moments:  $M_A = 0$

$$M_C = 0 + 54.88 = 54.88 \text{ kN}\cdot\text{m}$$

$$M_F = 54.88 - 25.6 = 29.28 \text{ kN}\cdot\text{m}$$

$$M_B = 29.28 - 29.28 = 0$$

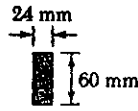
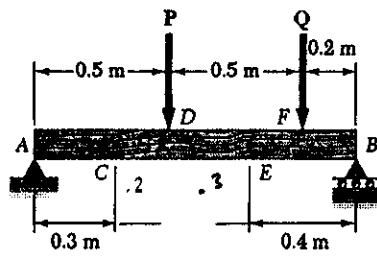


$$\text{Maximum } |M| = 54.88 \text{ kN}\cdot\text{m} = 54.88 \times 10^3 \text{ N}\cdot\text{m}$$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{54.88 \times 10^3}{512 \times 10^{-6}} = 107.2 \times 10^6 \text{ Pa} = 107.2 \text{ MPa}$$

PROBLEM 5.74

\* 5.74 Beam AB supports two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending on the bottom edge of the beam is +15 MPa at C and +22 MPa at E. (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending which occurs in the beam.



SOLUTION

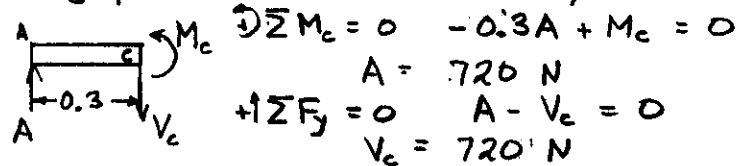
For rectangular cross section  $S = \frac{1}{6}bh^2$   
 $S = (\frac{1}{6})(24)(60)^2 = 14.4 \times 10^3 \text{ mm}^3 = 14.4 \times 10^{-6} \text{ m}^3$

Bending moments at C and E  $M = S\sigma$

$M_C = (14.4 \times 10^{-6})(15 \times 10^6) = 216 \text{ N}\cdot\text{m}$

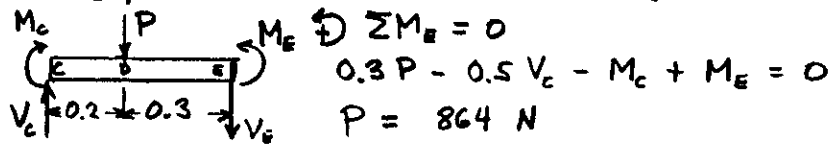
$M_E = (14.4 \times 10^{-6})(22 \times 10^6) = 316.8 \text{ N}\cdot\text{m}$

Using portion AC as a free body



$\sum M_C = 0 \quad -0.3A + M_C = 0$   
 $A = 720 \text{ N}$   
 $\sum F_y = 0 \quad A - V_c = 0$   
 $V_c = 720 \text{ N}$

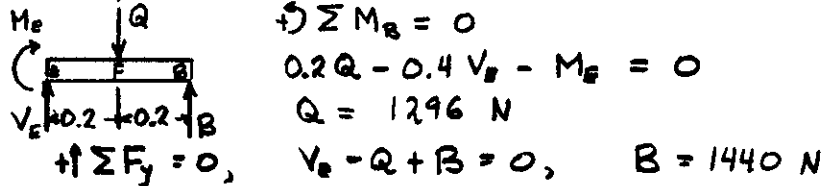
Using portion CDE as a free body



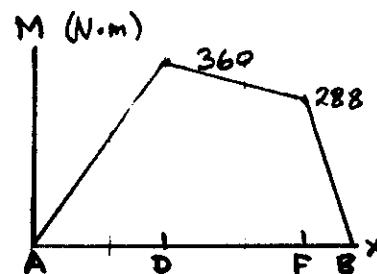
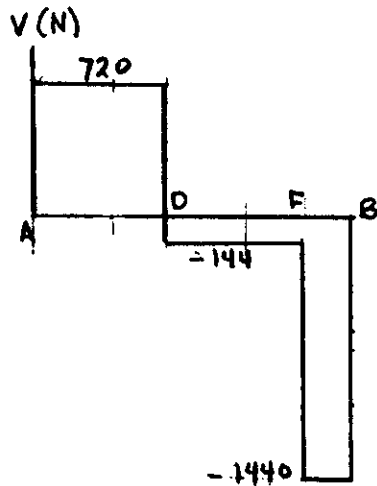
$\sum M_E = 0$   
 $0.3P - 0.5V_c - M_C + M_E = 0$   
 $P = 864 \text{ N}$

$\sum F_y = 0, \quad V_c - P - V_E = 0, \quad V_E = -144 \text{ N}$

Using portion EFB as a free body



$\sum M_B = 0$   
 $0.2Q - 0.4V_E - M_E = 0$   
 $Q = 1296 \text{ N}$   
 $\sum F_y = 0, \quad V_E - Q + B = 0, \quad B = 1440 \text{ N}$



Areas under shear diagram

A to D  $(0.5)(720) = 360 \text{ N}\cdot\text{m}$

D to F  $(0.5)(-144) = -72 \text{ N}\cdot\text{m}$

F to B  $(0.2)(-1440) = -288 \text{ N}\cdot\text{m}$

Bending moments:  $M_A = 0$

$M_D = 0 + 360 = 360 \text{ N}\cdot\text{m}$

$M_F = 360 - 72 = 288 \text{ N}\cdot\text{m}$

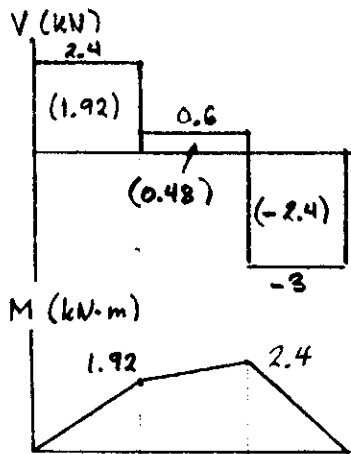
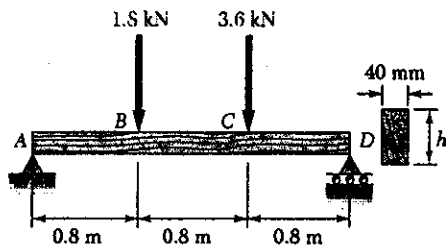
$M_B = 288 - 288 = 0$

Maximum  $|M| = 360 \text{ N}\cdot\text{m}$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{360}{14.4 \times 10^{-6}} = 25 \times 10^6 \text{ Pa} = 25 \text{ MPa}$

PROBLEM 5.75

5.75 and 5.76 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



SOLUTION

$$+\circlearrowleft \sum M_D = 0$$

$$-2.4A + (1.6)(1.8) + (0.8)(3.6) = 0$$

$$A = 2.4 \text{ kN}$$

$$+\circlearrowright \sum M_A = 0$$

$$-(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0$$

$$D = 3 \text{ kN}$$

Construct shear and bending moment diagrams

$$|M|_{\max} = 2.4 \text{ kN}\cdot\text{m} = 2.4 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3$$

$$= 200 \times 10^3 \text{ mm}^3$$

$$S = \frac{1}{2}bh^2 = \frac{1}{2}(40)h^2 = 200 \times 10^3$$

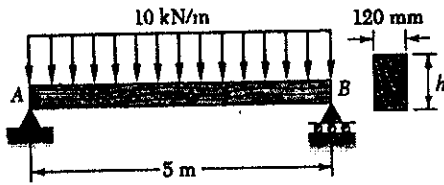
$$h^2 = \frac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$$

$$h = 173.2 \text{ mm}$$



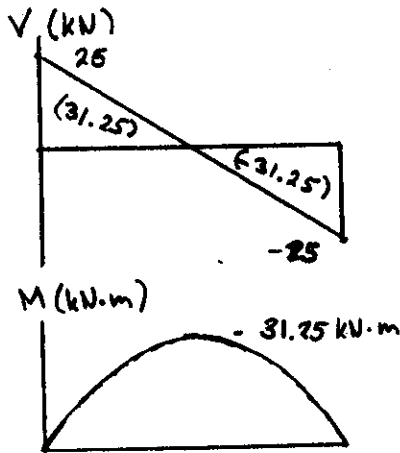
PROBLEM 5.76

5.75 and 5.76 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



SOLUTION

Reactions:  $A = B$  by symmetry  
 $\uparrow \Sigma F_y = 0 \quad A + B - (5)(10) = 0$   
 $A = B = 25 \text{ kN}$



From bending moment diagram

$$|M|_{\max} = 31.25 \text{ kN}\cdot\text{m} = 31.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3} \text{ m}^3$$

$$= 2.604 \times 10^6 \text{ mm}^3$$

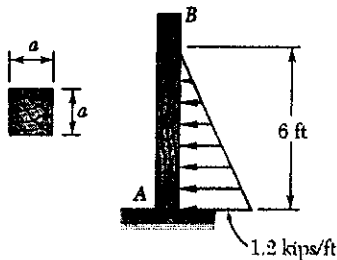
$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(120) h^2 = 2.604 \times 10^6$$

$$h^2 = \frac{(6)(2.604 \times 10^6)}{120} = 130.21 \times 10^3 \text{ mm}^2$$

$$h = 361 \text{ mm}$$

**PROBLEM 5.77**

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.



**SOLUTION**

Equivalent concentrated load

$$P = \left(\frac{1}{2}\right)(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A

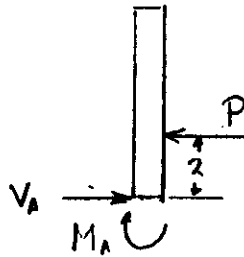
$$M_A = (2)(3.6) = 7.2 \text{ kip}\cdot\text{ft} = 86.4 \text{ kip}\cdot\text{in}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

For a square section  $S = \frac{1}{6} a^3$

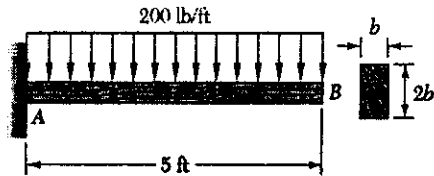
$$a = \sqrt[3]{6S}$$

$$a_{\min} = \sqrt[3]{(6)(49.37)} = 6.67 \text{ in.}$$



**PROBLEM 5.78**

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.



**SOLUTION**

Construct shear and bending moment curves.

$$|M|_{\max} = 2500 \text{ lb}\cdot\text{ft} = 2.5 \text{ kip}\cdot\text{ft} \\ = 30 \text{ kip}\cdot\text{in.}$$

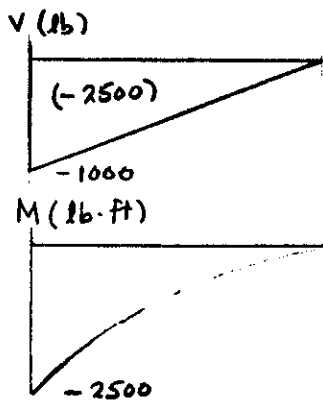
$$\sigma_{\text{all}} = 1.75 \text{ ksi}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{30}{1.75} = 17.143 \text{ in}^3$$

$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (2b)^2 = \frac{2}{3} b^3 = 17.143$$

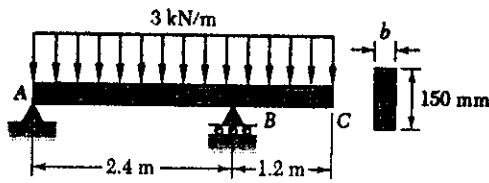
$$b^3 = \frac{(3)(17.143)}{2} = 25.7 \text{ in}^3,$$

$$b = 2.95 \text{ in.}$$



PROBLEM 5.79

5.79 and 5.80 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 Mpa.



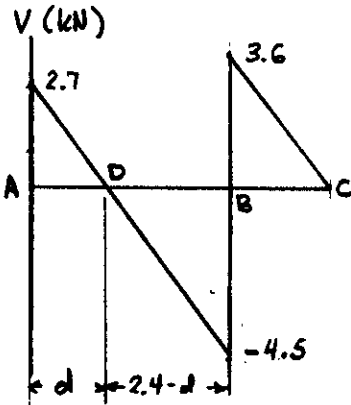
SOLUTION

$$+\circlearrowleft M_B = 0$$

$$-2.4 A + (0.6)(3.6)(3) = 0 \quad A = 2.7 \text{ kN}$$

$$+\circlearrowleft M_A = 0$$

$$-(1.8)(3.6)(3) + 2.4 B = 0 \quad B = 8.1 \text{ kN}$$



Shear:  $V_A = 2.7 \text{ kN}$   
 $V_B = 2.7 - (2.4)(3) = -4.5 \text{ kN}$   
 $V_B = -4.5 + 8.1 = 3.6 \text{ kN}$   
 $V_C = 3.6 - (1.2)(3) = 0$

locate point D where  $V = 0$

$$\frac{d}{2.7} = \frac{2.4-d}{4.5} \quad 7.2 d = 6.48$$

$$d = 0.9 \text{ m} \quad 2.4 - d = 1.5 \text{ m}$$

Areas under shear curve

A to D  $\int V dx = (\frac{1}{2})(0.9)(2.7) = 1.215 \text{ kN}\cdot\text{m}$   
 D to B  $\int V dx = (\frac{1}{2})(1.5)(-4.5) = -3.375 \text{ kN}\cdot\text{m}$   
 B to C  $\int V dx = (\frac{1}{2})(1.2)(3.6) = 2.16 \text{ kN}\cdot\text{m}$

Bending moments:  $M_A = 0$

$$M_B = 0 + 1.215 = 1.215 \text{ kN}\cdot\text{m}$$

$$M_B = 1.215 - 3.375 = -2.16 \text{ kN}\cdot\text{m}$$

$$M_C = -2.16 + 2.16 = 0$$

Maximum  $|M| = 2.16 \text{ kN}\cdot\text{m} = 2.16 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$

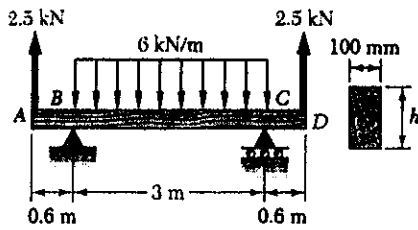
For rectangular section

$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (150)^2 = 180 \times 10^3$$

$$b = \frac{(6)(180 \times 10^3)}{150^2} = 48 \text{ mm}$$

PROBLEM 5.80

5.79 and 5.80 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



SOLUTION

By symmetry  $B = C$

$$+\uparrow \sum F_y = 0 \quad B + C + 2.5 + 2.5 - (3)(6) = 0$$

$$B = C = 6.5 \text{ kN}$$

Shear: A to B  $V = 2.5 \text{ kN}$

$$V_B = 2.5 + 6.5 = 9 \text{ kN}$$

$$V_C = 9 - (3)(6) = -9 \text{ kN}$$

C to D  $V = -9 + 6.5 = -2.5 \text{ kN}$

Areas under shear diagram

A to B  $\int V dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$

B to E  $\int V dx = (\frac{1}{2})(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$

E to C  $\int V dx = -6.75 \text{ kN}\cdot\text{m}$

C to D  $\int V dx = -1.5 \text{ kN}\cdot\text{m}$

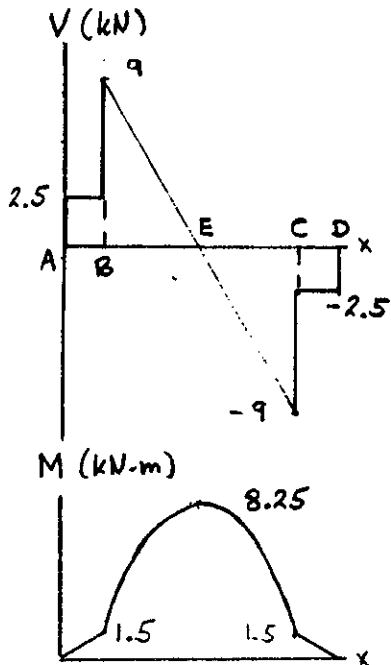
Bending moments  $M_A = 0$

$$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$$

$$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$$

$$M_D = 1.5 - 1.5 = 0$$



$$\text{Maximum } |M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|_{\text{max}}}{\sigma_{\text{all}}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

For a rectangular section  $S = \frac{1}{6} b h^2$

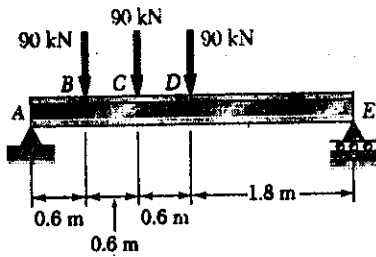
$$687.5 \times 10^3 = (\frac{1}{6})(100) h^2$$

$$h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$$

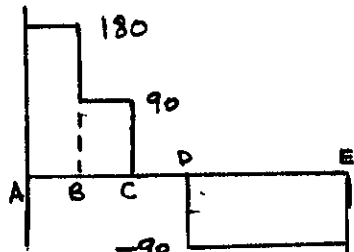
$$h = 203 \text{ mm}$$

PROBLEM 5.81

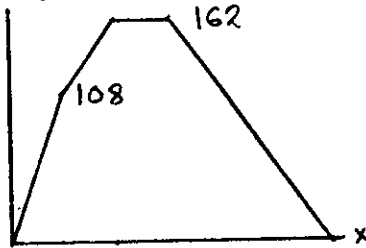
5.81 and 5.82 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.



V (kN)



M (kN·m)



SOLUTION

$$\sum M_E = 0 \quad -3.6A + (3)(90) + (2.4)(90) + (1.8)(90) = 0$$

$$A = 180 \text{ kN}$$

$$\sum M_A = 0 \quad 3.6E - (1.8)(90) - (1.2)(90) - (0.6)(90) = 0$$

$$E = 90 \text{ kN}$$

Shear: A to B  $V = 180 \text{ kN}$

B to C  $V = 180 - 90 = 90 \text{ kN}$

C to D  $V = 90 - 90 = 0$

D to E  $V = 0 - 90 = -90 \text{ kN}$

Areas under shear diagram

A to B  $\int V dx = (0.6)(180) = 108 \text{ kN}\cdot\text{m}$

B to C  $\int V dx = (0.6)(90) = 54 \text{ kN}\cdot\text{m}$

C to D  $\int V dx = 0$

D to E  $\int V dx = (1.8)(-90) = -162 \text{ kN}\cdot\text{m}$

Bending moments  $M_A = 0$

$M_B = 0 + 108 = 108 \text{ kN}\cdot\text{m}$

$M_C = 108 + 54 = 162 \text{ kN}\cdot\text{m}$

$M_D = 162 + 0 = 162 \text{ kN}\cdot\text{m}$

$M_E = 162 - 162 = 0$

Maximum  $|M| = 162 \text{ kN}\cdot\text{m} = 162 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{162 \times 10^3}{160 \times 10^6} = 1.0125 \times 10^{-3} \text{ m}^3 = 1012.5 \times 10^3 \text{ mm}^3$

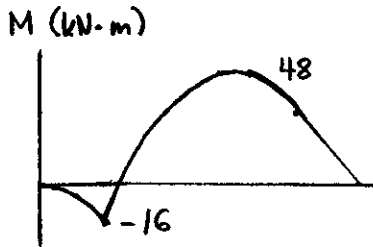
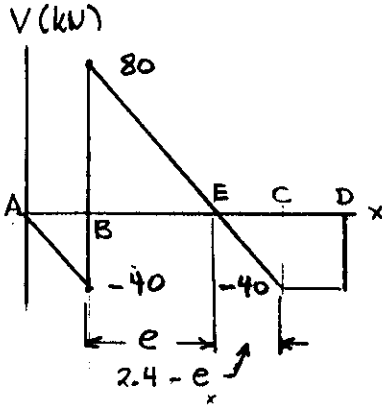
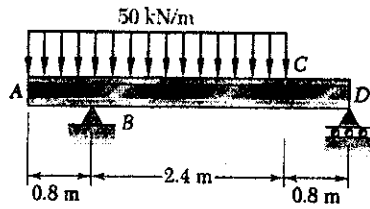
Shape	S ( $10^3 \text{ mm}^3$ )
W 460 x 74	1460
W 410 x 60	1060
W 360 x 64	1030
W 310 x 74	1060

lightest wide flange beam

W 410 x 60 @ 60 kg/m

PROBLEM 5.82

5.81 and 5.82 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.



SOLUTION

$$\begin{aligned} \sum M_D = 0 & \quad -3.2B + (2.4)(3.2)(50) = 0 \\ & \quad B = 120 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_B = 0 & \quad 3.2D - (0.8)(3.2)(50) = 0 \\ & \quad D = 40 \text{ kN} \end{aligned}$$

Shear:  $V_A = 0$   
 $V_B^- = 0 - (0.8)(50) = -40 \text{ kN}$   
 $V_B^+ = -40 + 120 = 80 \text{ kN}$   
 $V_C = 80 - (2.4)(50) = -40 \text{ kN}$   
 $V_D = -40 + 0 = -40 \text{ kN}$

Locate point E where  $V = 0$

$$\frac{e}{80} = \frac{2.4 - e}{40} \quad 120e = 192$$

$$e = 1.6 \text{ m} \quad 2.4 - e = 0.8 \text{ m}$$

Areas: A to B,  $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN}\cdot\text{m}$   
 B to E  $\int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN}\cdot\text{m}$   
 E to C  $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN}\cdot\text{m}$   
 C to D  $\int V dx = (0.8)(-40) = -32 \text{ kN}\cdot\text{m}$

Bending moments:  $M_A = 0$   
 $M_B = 0 - 16 = -16 \text{ kN}\cdot\text{m}$   
 $M_E = -16 + 64 = 48 \text{ kN}\cdot\text{m}$   
 $M_C = 48 - 16 = 32 \text{ kN}\cdot\text{m}$   
 $M_D = 32 - 32 = 0$

Maximum  $|M| = 48 \text{ kN}\cdot\text{m} = 48 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

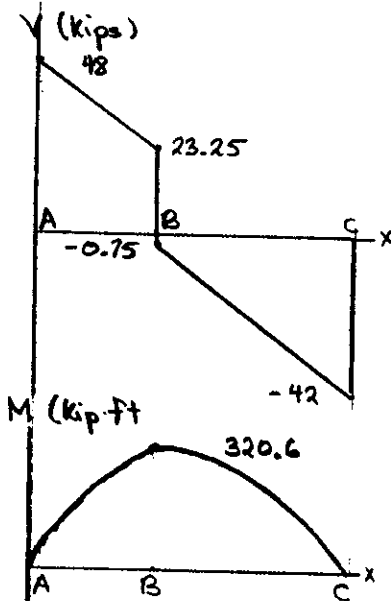
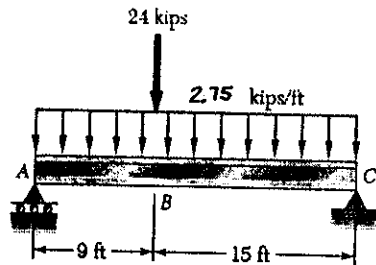
$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$

Shape	$S$ ( $10^3 \text{ mm}^3$ )
W 310 x 32.7	415
W 250 x 28.4	308
W 200 x 35.9	342

Lightest wide flange beam  
 W 250 x 28.4 @ 28.4 kg/m

PROBLEM 5.83

5.83 and 5.84 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



SOLUTION

$$+\circlearrowleft \sum M_C = 0 \quad -24A + (112)(24)(2.75) + (15)(24) = 0$$

$$A = 48 \text{ kips}$$

$$+\circlearrowleft \sum M_A = 0 \quad 24C - (12)(24)(2.75) - (9)(24) = 0$$

$$C = 42 \text{ kips}$$

Shear:  $V_A = 48$

$$V_B^- = 48 - (9)(2.75) = 23.25 \text{ kips}$$

$$V_B^+ = 23.25 - 24 = -0.75 \text{ kips}$$

$$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$$

Areas under shear diagram

$$A \text{ to } B \quad \int V dx = \left(\frac{1}{2}\right)(9)(48 + 23.25) = 320.6 \text{ kip-ft}$$

$$B \text{ to } C \quad \int V dx = \left(\frac{1}{2}\right)(15)(-0.75 - 42) = -320.6 \text{ kip-ft}$$

Bending moments:  $M_A = 0$

$$M_B = 0 + 320.6 = 320.6 \text{ kip-ft}$$

$$M_C = 320.6 - 320.6 = 0$$

$$\text{Maximum } |M| = 320.6 \text{ kip-ft} = 3848 \text{ kip-in}$$

$$\sigma_{all} = 24 \text{ ksi}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

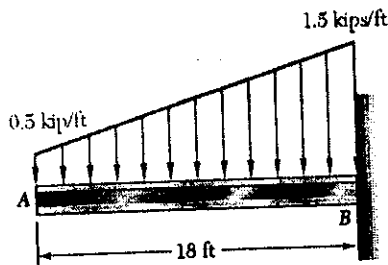
Shape	S (in <sup>3</sup> )
W 30 x 99	269
W 27 x 84	213
W 24 x 104	258
W 21 x 101	227
W 18 x 106	204

Lightest wide flange beam

W 27 x 84 @ 84 lb/ft

PROBLEM 5.84

5.83 and 5.84 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



SOLUTION

$$w = 0.5 + \frac{(1.5 - 0.5)x}{18} = 0.5 + 0.0555x$$

$$\frac{dV}{dx} = -w = -0.5 + 0.05556x$$

$$V = 0 - 0.5x - 0.02778x^2 = \frac{dM}{dx}$$

$$M = 0 - 0.25x^2 - 0.009259x^3$$

Maximum  $|M|$  occurs at  $x = 18$  ft.

$$|M|_{\max} = (0.25)(18)^2 + (0.009259)(18)^3 = 135 \text{ kip}\cdot\text{ft} = 1620 \text{ kip}\cdot\text{in}$$

$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\min} = \frac{|M|_{\max}}{\sigma_{\text{all}}} = \frac{1620}{24} = 67.5 \text{ in}^3$$

Shape	$S$ (in <sup>3</sup> )
W21 x 44	81.6
W18 x 50	88.9
W16 x 57	92.2
W14 x 53	77.8
W12 x 72	97.4
W10 x 68	75.7

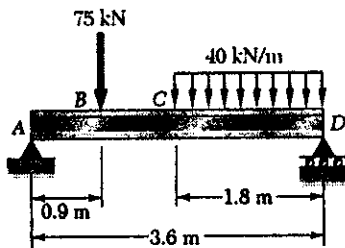
Lightest wide flange beam

W18 x 50 @ 50 lb/ft



PROBLEM 5.85

5.85 and 5.86 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric S-shape beam to support the loading shown.



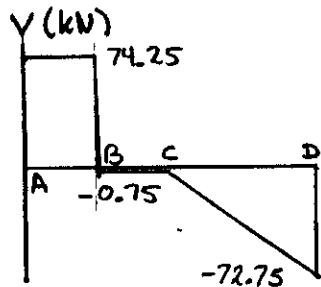
SOLUTION

$$+\circlearrowleft \sum M_D = 0 \quad -3.6 A + (2.7)(75) + (0.9)(1.8)(40) = 0$$

$$A = 74.25 \text{ kN}$$

$$+\circlearrowright \sum M_A = 0 \quad 3.6 D - (0.9)(75) - (2.7)(1.8)(40) = 0$$

$$D = 72.75 \text{ kN}$$



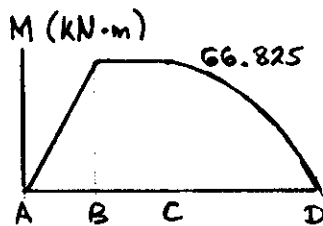
Shear: A to B  $V = 74.25 \text{ kN}$   
 B to C  $V = 74.25 - 75 = -0.75 \text{ kN}$   
 $V_D = -0.75 - (1.8)(40) = -72.75 \text{ kN}$

Areas under shear diagram

A to B  $\int V dx = (0.9)(74.25) = 66.825 \text{ kN}\cdot\text{m}$

B to C  $\int V dx = (0.9)(-0.75) = -0.675 \text{ kN}\cdot\text{m}$

C to D  $(\frac{1}{2})(1.8)(-0.75 - 72.75) = -66.15 \text{ kN}\cdot\text{m}$



Bending moments:  $M_A = 0$

$M_B = 0 + 66.825 = 66.825 \text{ kN}\cdot\text{m}$

$M_C = 66.825 - 0.675 = 66.15 \text{ kN}\cdot\text{m}$

$M_D = 66.15 - 66.15 = 0$

Maximum  $|M| = 66.825 \text{ kN}\cdot\text{m} = 66.825 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.825 \times 10^3}{160 \times 10^6} = 417.7 \times 10^{-6} \text{ m}^3 = 417.7 \times 10^3 \text{ mm}^3$

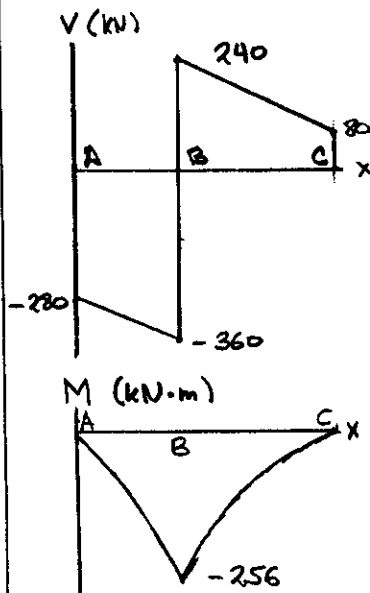
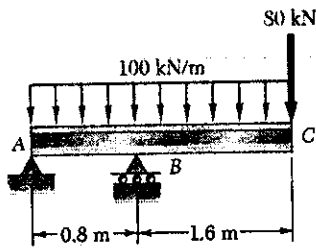
Shape	$S (10^3 \text{ mm}^3)$
S 380 x 64	971
S 310 x 47.3	593
S 250 x 52	482

Lightest S-section

S 310 x 47.3 @ 47.3 kg/m

PROBLEM 5.86

5.85 and 5.86 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric S-shape beam to support the loading shown.



SOLUTION

$$+\circlearrowleft \sum M_B = 0 \quad 0.8 A - (0.4)(2.4)(100) - (1.6)(80) = 0$$

$$A = 280 \text{ kN } \downarrow$$

$$+\circlearrowleft \sum M_A = 0 \quad 0.8 B - (1.2)(2.4)(100) - (2.4)(80) = 0$$

$$B = 600 \text{ kN } \uparrow$$

Shear:  $V_A = -280 \text{ kN}$

$$V_{B^-} = -280 - (0.8)(100) = -360 \text{ kN}$$

$$V_{B^+} = -360 + 600 = 240 \text{ kN}$$

$$V_C = 240 - (1.6)(100) = 80 \text{ kN}$$

Areas under shear diagram

$$A \text{ to } B \quad \left(\frac{1}{2}\right)(0.8)(-280 - 360) = -256 \text{ kN}\cdot\text{m}$$

$$B \text{ to } C \quad \left(\frac{1}{2}\right)(1.6)(240 + 80) = 256 \text{ kN}\cdot\text{m}$$

Bending moments:  $M_A = 0$

$$M_B = 0 - 256 = -256 \text{ kN}\cdot\text{m}$$

$$M_C = -256 + 256 = 0$$

$$\text{Maximum } |M| = 256 \text{ kN}\cdot\text{m} = 256 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

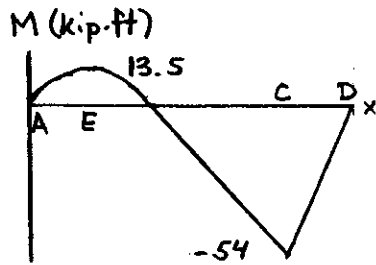
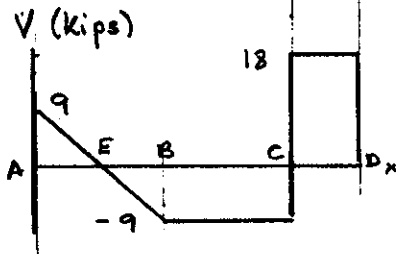
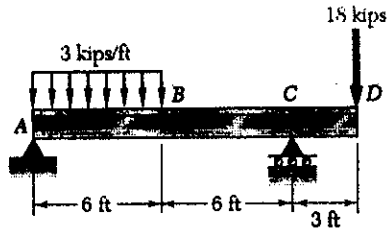
Shape	$S (10^3 \text{ mm}^3)$
S 510 × 98.3	1950
S 460 × 104	1685

Lightest S-section

S 510 × 98.3

PROBLEM 5.87

5.87 and 5.88 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.



SOLUTION

$$+\circlearrowleft \sum M_C = 0 \quad -12A + (9)(6)(3) - (3)(18) = 0$$

$$A = 9 \text{ kips}$$

$$+\circlearrowleft \sum M_A = 0 \quad 12C - (3)(6)(3) - (15)(18) = 0$$

$$C = 27 \text{ kips}$$

Shear:  $V_A = 9 \text{ kips}$

$$B \text{ to } C \quad V = 9 - (6)(3) = -9 \text{ kips}$$

$$C \text{ to } D \quad V = -9 + 27 = 18 \text{ kips}$$

$$\text{Areas: } A \text{ to } E \quad \left(\frac{1}{2}\right)(3)(9) = 13.5 \text{ kip}\cdot\text{ft}$$

$$E \text{ to } B \quad \left(\frac{1}{2}\right)(3)(-9) = -13.5 \text{ kip}\cdot\text{ft}$$

$$B \text{ to } C \quad (6)(-9) = -54 \text{ kip}\cdot\text{ft}$$

$$C \text{ to } D \quad (3)(18) = 54 \text{ kip}\cdot\text{ft}$$

Bending moments:  $M_A = 0$

$$M_E = 0 + 13.5 = 13.5 \text{ kip}\cdot\text{ft}$$

$$M_B = 13.5 - 13.5 = 0$$

$$M_C = 0 + 54 = 54 \text{ kip}\cdot\text{ft}$$

$$M_D = 54 - 54 = 0$$

$$\text{Maximum } |M| = 54 \text{ kip}\cdot\text{ft} = 648 \text{ kip}\cdot\text{in}$$

$$\sigma_{\text{all}} = 24 \text{ ksi}$$

$$S_{\text{min}} = \frac{648}{24} = 27 \text{ in}^3$$

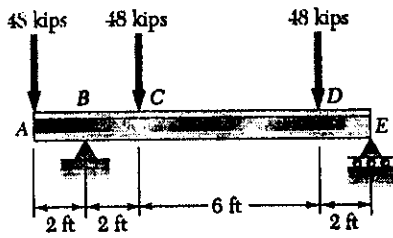
Shape	$S(\text{in}^3)$
S 12 x 31.8	36.4
S 10 x 35	29.4

Lightest S-shaped beam

S 12 x 31.8

PROBLEM 5.88

5.87 and 5.88 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.



SOLUTION

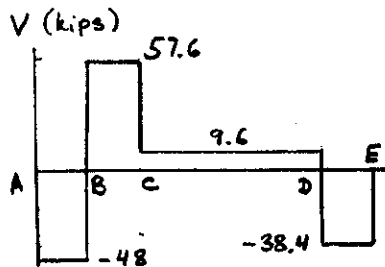
$$+\sum M_E = 0 \quad (12)(48) - 10B + (8)(48) + (2)(48) = 0$$

$$B = 105.6 \text{ kips}$$

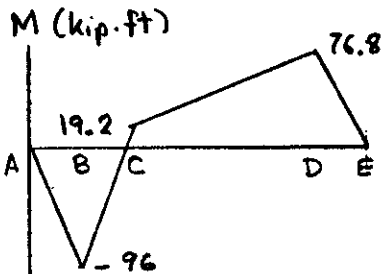
$$+\sum M_B = 0 \quad (2)(48) - (2)(48) - (8)(48) + 10E = 0$$

$$E = 38.4 \text{ kips}$$

Shear: A to B  $V = -48$  kips  
 B to C  $V = -48 + 105.6 = 57.6$  kips  
 C to D  $V = 57.6 - 48 = 9.6$  kips  
 D to E  $V = 9.6 - 48 = -38.4$  kips



Areas: A to B  $(2)(-48) = -96$  kip-ft  
 B to C  $(2)(57.6) = 115.2$  kip-ft  
 C to D  $(6)(9.6) = 57.6$  kip-ft  
 D to E  $(2)(-38.4) = -76.8$  kip-ft



Bending moments:  $M_A = 0$   
 $M_B = 0 - 96 = -96$  kip-ft  
 $M_C = -96 + 115.2 = 19.2$  kip-ft  
 $M_D = 19.2 + 57.6 = 76.8$  kip-ft  
 $M_E = 76.8 - 76.8 = 0$

Maximum  $|M| = 96$  kip-ft = 1152 kip-in

$\sigma_{all} = 24$  ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$$

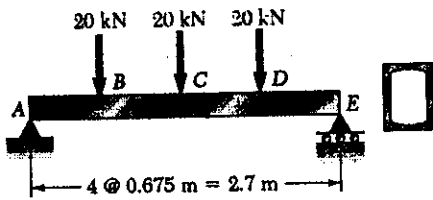
Shape	$S$ (in <sup>3</sup> )
S 15 x 42.9	59.6
S 12 x 50	50.8

Lightest S-shaped beam

S 15 x 42.9

PROBLEM 5.89

5.89 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.



SOLUTION

By symmetry  $A = E$

$$+\uparrow \sum F_y = 0 \quad A + E - 20 - 20 - 20 = 0$$

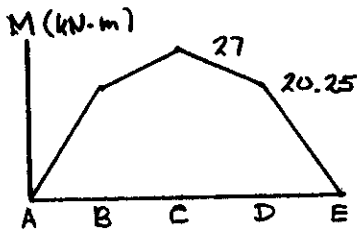
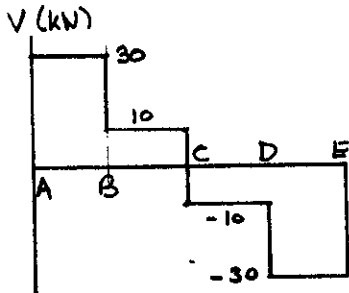
$$A = E = 30 \text{ kN}$$

Shear:

A to B	$V = 30 \text{ kN}$
B to C	$V = 30 - 20 = 10 \text{ kN}$
C to D	$V = 10 - 20 = -10 \text{ kN}$
D to E	$V = -10 - 20 = -30 \text{ kN}$

Areas:

A to B	$(0.675)(30) = 20.25 \text{ kN}\cdot\text{m}$
B to C	$(0.675)(10) = 6.75 \text{ kN}\cdot\text{m}$
C to D	$(0.675)(-10) = -6.75 \text{ kN}\cdot\text{m}$
D to E	$(0.675)(-30) = -20.25 \text{ kN}\cdot\text{m}$



Bending moments:

$$M_A = 0$$

$$M_B = 0 + 20.25 = 20.25 \text{ kN}\cdot\text{m}$$

$$M_C = 20.25 + 6.75 = 27 \text{ kN}\cdot\text{m}$$

$$M_D = 27 - 6.75 = 20.25 \text{ kN}\cdot\text{m}$$

$$M_E = 20.25 - 20.25 = 0$$

$$\text{Maximum } |M| = 27 \text{ kN}\cdot\text{m} = 27 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{\text{all}} = 150 \text{ MPa} = 150 \times 10^6 \text{ Pa}$$

For a section consisting of two channels

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

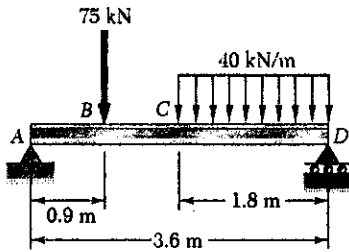
For each channel  $S_{\text{min}} = (\frac{1}{2})(180 \times 10^3) = 90 \times 10^3 \text{ mm}^3$

Shape	$S (10^3 \text{ mm}^3)$
C 180 × 14.6	99.2
C 150 × 19.3	93.6

Lightest channel section  
C 180 × 14.6

**PROBLEM 5.90**

5.90 Two metric rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 190 MPa, determine the most economical channels that can be used.



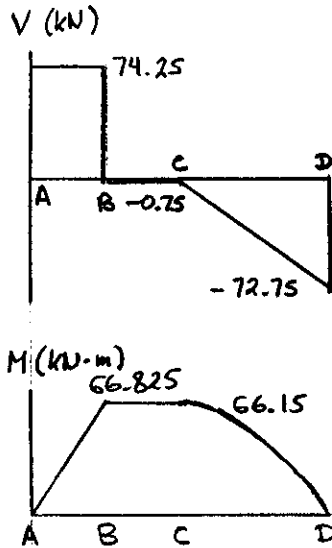
**SOLUTION**

$$+\circlearrowleft \sum M_D = 0 \quad -3.6A + (2.7)(75) + (0.9)(1.8)(40) = 0$$

$$A = 74.25 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0 \quad 3.6D - (0.9)(75) - (2.7)(1.8)(40) = 0$$

$$D = 72.75 \text{ kN}$$



Shear: A to B  $V = 74.25 \text{ kN}$   
 B to C  $V = 74.25 - 75 = -0.75 \text{ kN}$   
 $V_D = -0.75 - (1.8)(40) = -72.75 \text{ kN}$

Areas: A to B  $(0.9)(74.25) = 66.825 \text{ kN}\cdot\text{m}$   
 B to C  $(0.9)(-0.75) = -0.675 \text{ kN}\cdot\text{m}$   
 C to D  $(\frac{1}{2})(1.8)(-0.75 - 72.75) = -66.15$

Bending moments:  $M_A = 0$   
 $M_B = 0 + 66.825 = 66.825 \text{ kN}\cdot\text{m}$   
 $M_C = 66.825 - 0.675 = 66.15 \text{ kN}\cdot\text{m}$   
 $M_D = 66.15 - 66.15 = 0$

Maximum  $|M| = 66.825 \text{ kN}\cdot\text{m} = 66.825 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 190 \text{ MPa} = 190 \times 10^6 \text{ Pa}$

For double channel

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.825 \times 10^3}{190 \times 10^6} = 351.7 \times 10^{-6} \text{ m}^3$$

$$= 351.7 \times 10^3 \text{ mm}^3$$

For each channel

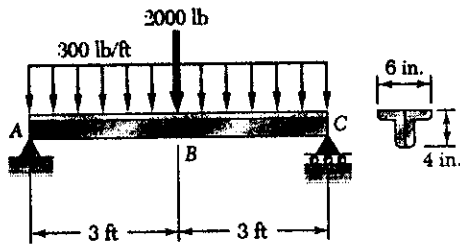
$$S_{min} = (\frac{1}{2})(351.7 \times 10^3) = 175.9 \times 10^3 \text{ mm}^3$$

Shape	$S (10^3 \text{ mm}^3)$
C 230 x 22	185
C 200 x 27.9	179

lightest channel section  
 C 230 x 22

PROBLEM 5.91

5.91 Two L 4 × 3 rolled-steel angles are bolted together to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



SOLUTION

By symmetry  $A = C$

$$+\sum F_y = 0 \quad A + C - 2000 - (6)(300) = 0$$

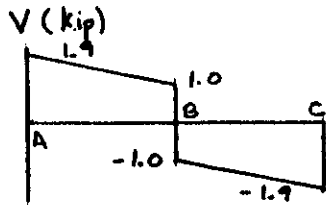
$$A = C = 1900 \text{ lb.}$$

Shear:  $V_A = 1900 \text{ lb.} = 1.9 \text{ kips}$

$$V_B^- = 1900 - (3)(300) = 1000 \text{ lb} = 1 \text{ kip}$$

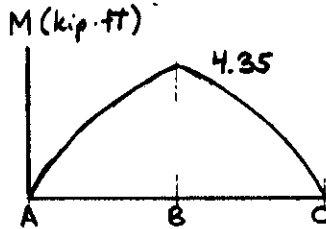
$$V_B^+ = 1000 - 2000 = -1000 \text{ lb} = -1 \text{ kip}$$

$$V_C = -1000 - (3)(300) = -1900 \text{ lb} = -1.9 \text{ kip}$$



Areas: A to B  $(\frac{1}{2})(3)(1.9 + 1) = 4.35 \text{ kip}\cdot\text{ft}$

B to C  $(\frac{1}{2})(3)(-1 - 1.9) = -4.35 \text{ kip}\cdot\text{ft}$



Bending moments:  $M_A = 0$

$$M_B = 0 + 4.35 = 4.35 \text{ kip}\cdot\text{ft}$$

$$M_C = 4.35 - 4.35 = 0$$

Maximum  $|M| = 4.35 \text{ kip}\cdot\text{ft} = 52.2 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$

For section consisting of two angles  $S_{min} = \frac{|M|}{\sigma_{all}} = \frac{52.2}{24} = 2.175 \text{ in}^3$

For each angle  $S_{min} = (\frac{1}{2})(2.175) = 1.0875 \text{ in}^3$

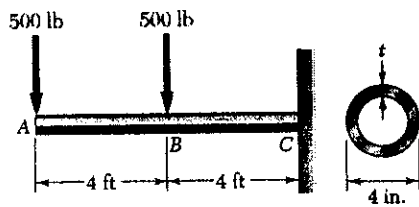
Angle section	S (in <sup>3</sup> )
L 4 × 3 × $\frac{1}{4}$	1.89
L 4 × 3 × $\frac{3}{8}$	1.46
L 4 × 3 × $\frac{1}{2}$	1.00

Smallest allowable thickness

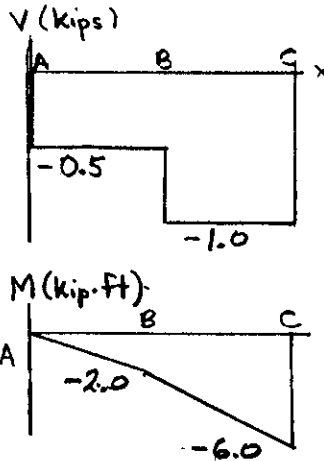
$$t = \frac{3}{8} \text{ in.}$$

PROBLEM 5.92

5.92 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available have thicknesses varying from  $\frac{1}{4}$  in. to 1 in. in  $\frac{1}{8}$ -in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness  $t$  that can be used.



SOLUTION



Shear: A to B  $V = -500 \text{ lb} = -0.5 \text{ kip}$   
 B to C  $V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$

Areas: A to B  $(4)(-0.5) = -2.0 \text{ kip}\cdot\text{ft}$   
 B to C  $(4)(-1.0) = -4.0 \text{ kip}\cdot\text{ft}$

Bending moments:  $M_A = 0$   
 $M_B = 0 - 2.0 = -2.0 \text{ kip}\cdot\text{ft}$   
 $M_C = -2.0 - 4.0 = -6.0 \text{ kip}\cdot\text{ft}$

Maximum  $|M| = 6.0 \text{ kip}\cdot\text{ft} = 72 \text{ kip}\cdot\text{in.}$

$\sigma_{all} = 24 \text{ ksi}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$

$I = \frac{\pi}{4} (C_2^4 - C_1^4) \quad C = C_2 \quad C_2 = \frac{1}{2} d = 2.0 \text{ in.}$

$S = \frac{I}{C} = \frac{\pi}{4} \frac{C_2^4 - C_1^4}{C_2} = \frac{\pi}{4} \frac{2^4 - C_1^4}{2} = 3 \text{ in}^3$

$C_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606 \text{ in}^4 \quad C_1 = 1.7004 \text{ in.}$

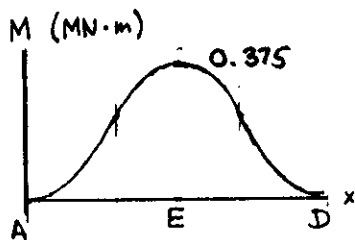
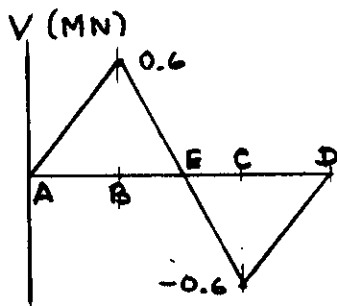
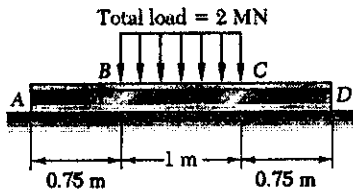
$t_{min} = C_2 - C_1 = 2.0 - 1.7004 = 0.2996 \text{ in}$

Using  $\frac{1}{8}$  in. increments for design  $t = \frac{3}{8} \text{ in.}$



PROBLEM 5.93

5.93 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical metric wide-flange beam to support the loading shown.



SOLUTION

Downward distributed load  $w = \frac{2}{1.0} = 2 \text{ MN/m}$

Upward distributed reaction  $q = \frac{2}{2.5} = 0.8 \text{ MN/m}$

Net distributed load over BC  $1.2 \text{ MN/m}$

Shear:  $V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6 \text{ MN}$

$V_C = 0.6 - (1.0)(1.2) = -0.6 \text{ MN}$

$V_D = -0.6 + (0.75)(0.8) = 0$

Areas: A to B  $(\frac{1}{2})(0.75)(0.6) = 0.225 \text{ MN}\cdot\text{m}$

B to E  $(\frac{1}{2})(0.5)(0.6) = 0.150 \text{ MN}\cdot\text{m}$

E to C  $(\frac{1}{2})(0.5)(-0.6) = -0.150 \text{ MN}\cdot\text{m}$

C to D  $(\frac{1}{2})(0.75)(-0.6) = -0.225 \text{ MN}\cdot\text{m}$

Bending moments:  $M_A = 0$

$M_B = 0 + 0.225 = 0.225 \text{ MN}\cdot\text{m}$

$M_E = 0.225 + 0.150 = 0.375 \text{ MN}\cdot\text{m}$

$M_C = 0.375 - 0.150 = 0.225 \text{ MN}\cdot\text{m}$

$M_D = 0.225 - 0.225 = 0$

Maximum  $|M| = 0.375 \text{ MN}\cdot\text{m} = 375 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 170 \text{ MPa} = 170 \times 10^6 \text{ Pa}$

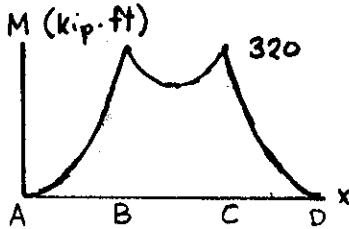
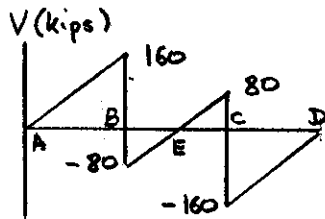
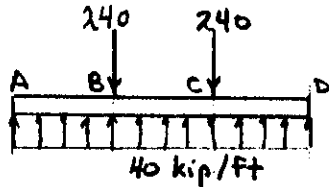
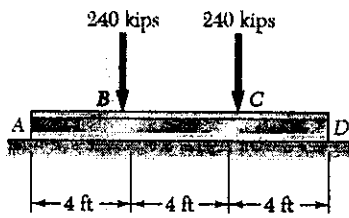
$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$

Shape	$S(10^3 \text{ mm}^3)$
W 690 × 125	3510
W 610 × 101	2530
W 530 × 150	3720
W 460 × 113	2400

Lightest wide flange section

W 610 × 101

PROBLEM 5.94



5.94 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

Distributed reaction  $q = \frac{480}{12} = 40 \text{ kip/ft}$

Shear:  $V_A = 0$

$V_B^- = 0 + (4)(40) = 160 \text{ kips}$

$V_B^+ = 160 - 240 = -80 \text{ kips}$

$V_C^- = -80 + (4)(40) = 80 \text{ kips}$

$V_C^+ = 80 - 240 = -160 \text{ kips}$

$V_D = -160 + (4)(40) = 0$

Areas: A to B  $(\frac{1}{2})(4)(160) = 320 \text{ kip}\cdot\text{ft}$

B to E  $(\frac{1}{2})(2)(-80) = -80 \text{ kip}\cdot\text{ft}$

E to C  $(\frac{1}{2})(2)(80) = 80 \text{ kip}\cdot\text{ft}$

C to D  $(\frac{1}{2})(4)(-160) = -320 \text{ kip}\cdot\text{ft}$

Bending moments:  $M_A = 0$

$M_B = 0 + 320 = 320 \text{ kip}\cdot\text{ft}$

$M_E = 320 - 80 = 240 \text{ kip}\cdot\text{ft}$

$M_C = 240 + 80 = 320 \text{ kip}\cdot\text{ft}$

$M_D = 320 - 320 = 0$

Maximum  $|M| = 320 \text{ kip}\cdot\text{ft} = 3840 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160 \text{ in}^3$

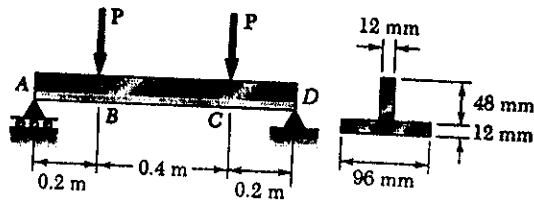
Shape	$S \text{ (in}^3\text{)}$
S 24 x 80	175
S 20 x 96	165

Lightest S-shaped section

S 24 x 80

PROBLEM 5.95

5.95 and 5.96 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and -140 MPa in compression.



SOLUTION

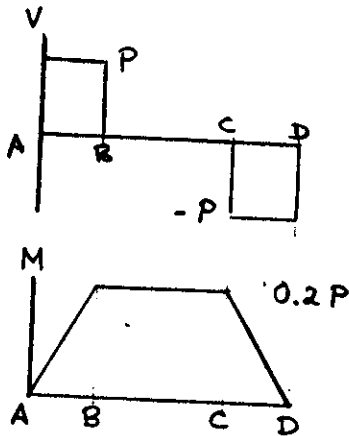
Reactions:  $A = D = P$

Shear: A to B  $V = P$   
 B to C  $V = P - P = 0$   
 C to D  $V = 0 - P = -P$

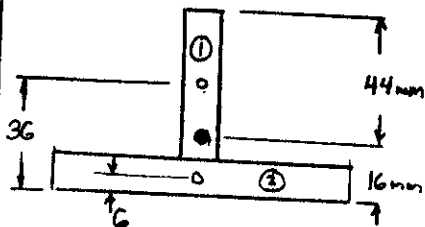
Areas: A to B  $0.2P$   
 B to C  $0$   
 C to D  $-0.2P$

Bending moments:  $M_A = 0$   
 $M_B = 0 + 0.2P = 0.2P$   
 $M_C = 0.2P + 0 = 0.2P$   
 $M_D = 0.2P - 0.2P = 0$

Largest positive bending moment:  $0.2P$   
 Largest negative bending moment:  $0$



Centroid and moment of inertia



Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (mm <sup>3</sup> )	d (mm)	$Ad^2$ (mm <sup>4</sup> )	$\bar{I}$ mm <sup>4</sup>
①	576	36	20736	20	$230.4 \times 10^3$	$110.6 \times 10^3$
②	1152	6	6912	10	$115.2 \times 10^3$	$13.8 \times 10^3$
$\Sigma$	1728		27648		$345.6 \times 10^3$	$124.4 \times 10^3$

$\bar{Y} = \frac{27648}{1728} = 16 \text{ mm}$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 470.6 \times 10^3 \text{ mm}^4$

Top  $c = 44 \text{ mm}$   $S = \frac{I}{c} = \frac{470.6 \times 10^3}{44} = 10.68 \times 10^3 \text{ mm}^3 = 10.68 \times 10^{-6} \text{ m}^3$

Allowable pos. M.  $M = |c_t| S = (140 \times 10^6)(10.68 \times 10^{-6}) = 1495 \text{ N}\cdot\text{m}$

Bot.  $c = 16 \text{ mm}$   $S = \frac{I}{c} = \frac{470.6 \times 10^3}{16} = 29.38 \times 10^3 \text{ mm}^3 = 29.38 \times 10^{-6} \text{ m}^3$

Allowable pos. M  $M = |c_c| S = (80 \times 10^6)(29.38 \times 10^{-6}) = 2350 \text{ N}\cdot\text{m}$

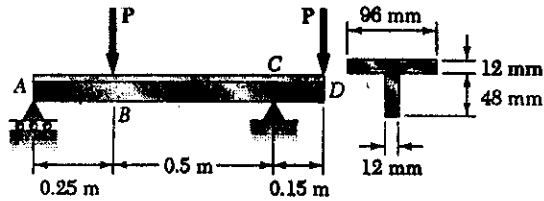
Smaller value  $M = 1495 \text{ N}\cdot\text{m}$

Allowable value of P  $0.2P = 1495$

$P = 7475 \text{ N} = 7.48 \text{ kN}$

PROBLEM 5.96

5.95 and 5.96 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and -140 MPa in compression



SOLUTION

$$\sum M_C = 0 \quad -0.75A + 0.5P - 0.15P = 0$$

$$A = 0.46667 P$$

$$\sum M_A = 0 \quad 0.75C - 0.25P - 0.9P = 0$$

$$C = 1.53333 P$$

Shear: A to B  $V = 0.46667 P$

B to C  $V = 0.46667 P - P = -0.53333 P$

C to D  $V = -0.53333 P + 1.53333 P = P$

Areas: A to B  $(.25)(0.46667 P) = 0.11667 P$

B to C  $(0.5)(-0.53333 P) = -0.26667 P$

C to D  $(0.15)P = 0.15 P$

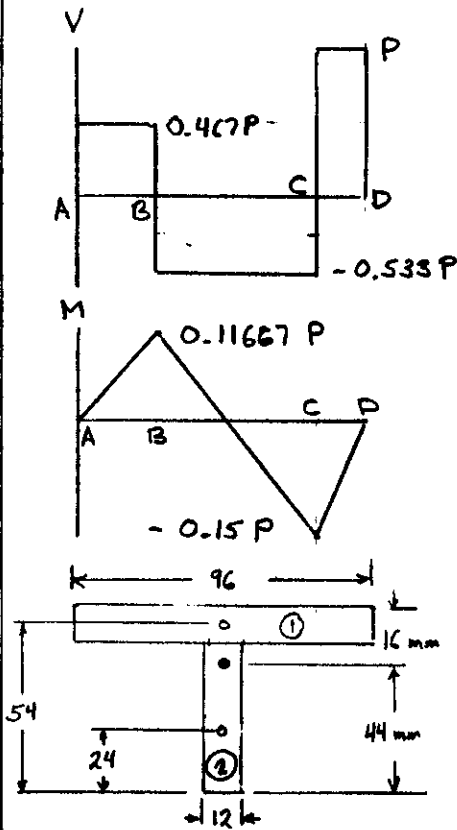
Bending moments:  $M_A = 0$

$M_B = 0 + 0.11667 P = 0.11667 P$

$M_C = 0.11667 P - 0.26667 P = -0.15 P$

$M_D = -0.15 P + 0.15 P = 0$

Centroid and moment of inertia.



Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (mm <sup>3</sup> )	d (mm)	$Ad^2$ (mm <sup>4</sup> )	$\bar{I}$ (mm <sup>4</sup> )
①	1152	54	62208	10	115200	13824
②	576	24	13824	20	230400	110592
$\Sigma$	1728		76032		345600	124416

$$\bar{Y} = \frac{76032}{1728} = 44 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 470016 \text{ mm}^4$$

Top:  $y = 16 \text{ mm} \quad \frac{I}{y} = \frac{470016}{16} = 29.376 \times 10^3 \text{ mm}^3 = 29.376 \times 10^{-6} \text{ m}^3$

Bottom:  $y = -44 \text{ mm} \quad \frac{I}{y} = \frac{470016}{44} = 10.682 \times 10^3 \text{ mm}^3 = -10.682 \times 10^{-6} \text{ m}^3$

Bending moment limits:  $M = -\frac{IS}{y}$

Tension at B  $= (-10.682 \times 10^{-6})(80 \times 10^6) = 854.56 \text{ N}\cdot\text{m} \quad \triangleleft B$

Comp. at B  $= -(29.376 \times 10^{-6})(-140 \times 10^6) = 4.1126 \times 10^3 \text{ N}\cdot\text{m}$

Tension at C  $= -(29.376 \times 10^{-6})(80 \times 10^6) = -2.35 \times 10^3 \text{ N}\cdot\text{m}$

Comp. at C  $= (-10.682 \times 10^{-6})(-140 \times 10^6) = 1.4955 \times 10^3 \text{ N}\cdot\text{m} \quad \triangleleft C$

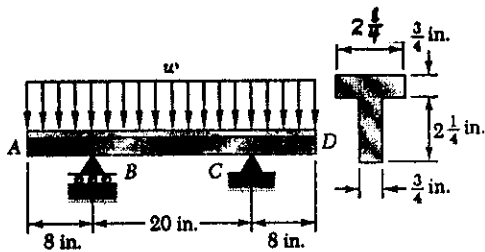
Allowable load:  $0.11667 P = 854.56 \quad P = 7.32 \times 10^3 \text{ N}$

$-0.15 P = -1.4955 \times 10^3 \quad P = 9.97 \times 10^3 \text{ N}$

The smaller value is  $P = 7.32 \text{ kN}$

PROBLEM 5.97

5.97 Determine the largest permissible uniformly distributed load  $w$  for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and -19.5 ksi in compression.



SOLUTION

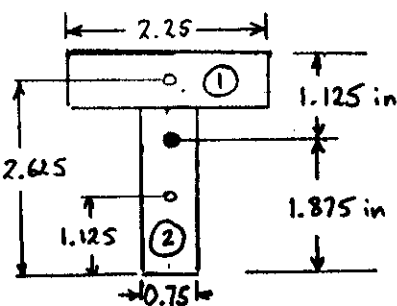
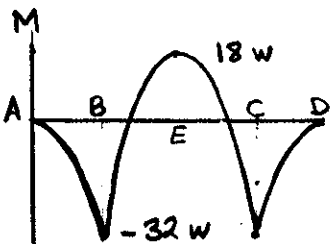
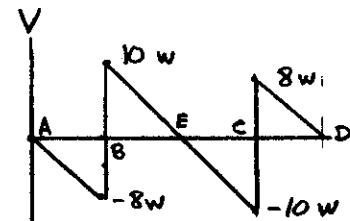
Reactions:  $B + C - 36w = 0$        $B = C = 18w$

Shear:  $V_A = 0$   
 $V_{B^-} = 0 - 8w = -8w$   
 $V_{B^+} = -8w + 18w = 10w$   
 $V_{C^-} = 10w - 20w = -10w$   
 $V_{C^+} = -10w + 18w = 8w$   
 $V_D = 8w - 8w = 0$

Areas: A to B  $(\frac{1}{2})(8)(-8w) = -32w$   
 B to E  $(\frac{1}{2})(10)(10w) = 50w$

Bending moments:  $M_A = 0$   
 $M_B = 0 - 32w = -32w$   
 $M_E = -32w + 50w = 18w$

Centroid and moment of inertia



Part	A (in <sup>2</sup> )	$\bar{y}$ (in)	A $\bar{y}$ (in <sup>3</sup> )	d (in)	Ad <sup>2</sup> (in <sup>4</sup> )	$\bar{I}$ (in <sup>4</sup> )
①	1.6875	2.625	4.4297	0.75	0.9492	0.0791
②	1.6875	1.125	1.8984	0.75	0.9492	0.7119
$\Sigma$	3.375		6.3281		1.8984	0.7910

$\bar{Y} = \frac{6.3281}{3.375} = 1.875$  in

$I = \Sigma Ad^2 + \Sigma \bar{I} = 2.6894$  in<sup>4</sup>

Top:  $y = 1.125$   
 Bottom:  $y = -1.875$

$I/y = 2.3906$  in<sup>3</sup>  
 $I/y = -1.4343$  in<sup>3</sup>

Bending moment limits

$M = -\sigma I/y$

Tension at B and C  
 Comp. at B and C  
 Tension at E  
 Compression at E

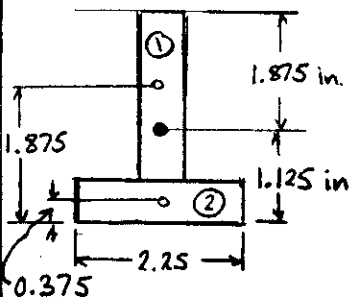
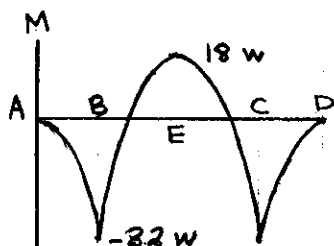
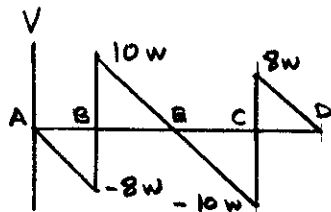
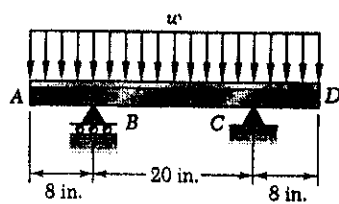
$-(12)(2.3906) = -28.687$  kip·in  
 $-(-19.5)(-1.4343) = -27.969$  kip·in  
 $-(12)(-1.4343) = 17.212$  kip·in  
 $-(-19.5)(2.3906) = 46.6$  kip·in

Allowable load  $w$

B & C       $-32w = -27.969$        $w = 0.874$  kip/in  
 E             $18w = 17.212$              $w = 0.956$  kip/in

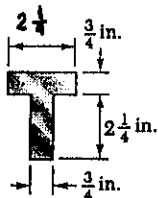
Smallest  $w = 0.874$  kip/in = 10.49 kip/ft

PROBLEM 5.98



5.97 Determine the largest permissible uniformly distributed load  $w$  for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and -19.5 ksi in compression.

5.98 Solve Prob. 5.97, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.



SOLUTION

Reactions  $B + C - 36w = 0$   $B = C = 18w$

Shear:  $V_A = 0$   
 $V_B^- = 0 - 8w = -8w$   
 $V_B^+ = -8w + 18w = 10w$   
 $V_C^- = 10w - 20w = -10w$   
 $V_C^+ = -10w + 18w = 8w$   
 $V_D = 8w - 8w = 0$

Areas: A to B  $(\frac{1}{2})(8)(-8w) = -32w$   
 B to E  $(\frac{1}{2})(10)(10w) = 50w$

Bending moments:  $M_A = 0$   
 $M_B = 0 - 32w = -32w$   
 $M_E = -32w + 50w = 18w$

Centroid and moment of inertia

Part	A (in <sup>2</sup> )	$\bar{y}$ (in)	$A\bar{y}$ (in <sup>3</sup> )	d (in)	$Ad^2$ (in <sup>4</sup> )	$\bar{I}$ (in <sup>4</sup> )
①	1.6875	1.875	3.1641	0.75	0.9492	0.7119
②	1.6875	0.375	0.6328	0.75	0.9492	0.0791
$\Sigma$	3.375		3.7969		1.8984	0.7910

$\bar{Y} = \frac{3.7969}{3.375} = 1.125$  in.

$I = \Sigma Ad^2 + \Sigma \bar{I} = 2.6894$  in<sup>4</sup>

Top:  $y = 1.875$  in  $I/y = 1.4343$  in<sup>3</sup>  
 Bottom  $y = -1.125$   $-I/y = -2.3906$  in<sup>3</sup>

Bending moment limits  $M = -\sigma I/y$

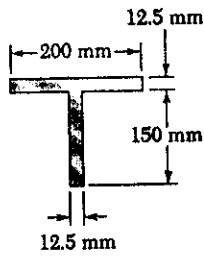
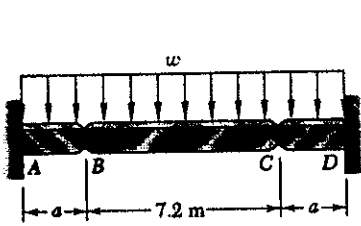
Tension at B and C  $-(12)(1.4343) = -17.212$  kip-in ←  
 Comp. at B and C  $-(-19.5)(-2.3906) = -46.6$  kip-in  
 Tension at E  $-(12)(-2.3906) = 28.687$  kip-in  
 Compression at E  $-(-19.5)(1.4343) = 27.969$  kip-in ←

Allowable load  $w$ :  
 B & C  $-32w = -17.212$   $w = 0.539$  kip/in  
 E  $18w = 27.969$   $w = 1.554$  kip/in

Smallest  $w = 0.539$  kip/in = 6.45 kip/ft ←

PROBLEM 5.99

5.99 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of w if beam BC is not to be overstressed, (b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.

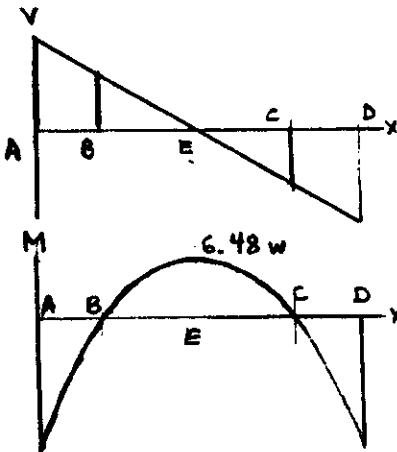


SOLUTION

(a)  $M_B = M_C = 0$   
 $V_B = -V_C = (\frac{1}{2})(7.2)w = 3.6w$

Area B to E of shear diagram  
 $(\frac{1}{2})(3.6)(3.6w) = 6.48w$

$M_E = 0 + 6.48w = 6.48w$



Centroid and moment of inertia

Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (mm <sup>3</sup> )	d (mm)	$Ad^2$ (mm <sup>4</sup> )	$\bar{I}$ (mm <sup>4</sup> )
①	2500	156.25	390625	34.82	$3.031 \times 10^6$	$0.0326 \times 10^6$
②	1875	75	140625	46.43	$4.042 \times 10^6$	$3.516 \times 10^6$
$\Sigma$	4375		531250		$7.073 \times 10^6$	$3.548 \times 10^6$

$\bar{Y} = \frac{531250}{4375} = 121.43 \text{ mm}$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4$

Location	y (mm)	$I/y$ ( $10^3 \text{ mm}^3$ )	← also ( $10^{-6} \text{ m}^3$ )
top	41.07	258.6	
bottom	-121.43	-87.47	

Bending moment limits  $M = -\sigma I / y$

Tension at E:  $-(110 \times 10^6)(-87.47 \times 10^{-6}) = 9.622 \times 10^3 \text{ N}\cdot\text{m}$   
 Comp. at E:  $-(-150 \times 10^6)(258.6 \times 10^{-6}) = 38.8 \times 10^3 \text{ N}\cdot\text{m}$   
 Tension at A & D:  $-(110 \times 10^6)(258.6 \times 10^{-6}) = -28.45 \times 10^3 \text{ N}\cdot\text{m}$   
 Comp. at A & D:  $-(-150 \times 10^6)(-87.47 \times 10^{-6}) = -13.121 \times 10^3 \text{ N}\cdot\text{m}$

Allowable load w

$6.48w = 9.622 \times 10^3$

$w = 1.485 \times 10^3 \text{ N/m}$   
 $= 1.485 \text{ kN/m}$

Shear at A  $V_A = (a + 3.6)w$

Area A to B of shear diagram  $\frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a + 7.2)w$

Bending moment at A (also D)  $M_A = \frac{1}{2} \times \frac{1}{2}a(a + 7.2)w$

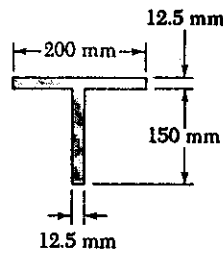
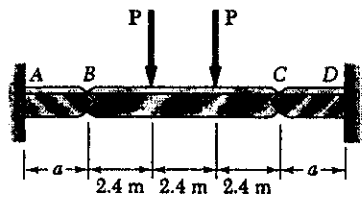
$-\frac{1}{2}a(a + 7.2)(1.485 \times 10^3) = -13.121 \times 10^3$

$\frac{1}{2}a^2 + 3.6a - 8.837 = 0$

$a = 1.935 \text{ m}$

PROBLEM 5.100

5.100 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of P if beam BC is not to be overstressed, (b) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed



SOLUTION

(a)  $M_B = M_C = 0$   
 $V_B = -V_C = P$

Area B to E of shear diagram.  
 $2.4 P$

$M_E = 0 + 2.4 P = 2.4 P = M_F$

Centroid and moment of inertia

Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (mm <sup>3</sup> )	d (mm)	$Ad^2$ (mm <sup>4</sup> )	$\bar{I}$ (mm <sup>4</sup> )
①	2500	156.25	390625	34.82	$3.031 \times 10^6$	$0.0326 \times 10^6$
②	1875	75	140625	46.43	$4.042 \times 10^6$	$3.516 \times 10^6$
$\Sigma$	4375		531250		$7.073 \times 10^6$	$3.548 \times 10^6$

$\bar{Y} = \frac{531250}{4375} = 121.43 \text{ mm}$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4$

Location	y (mm)	$I/y$ ( $10^3 \text{ mm}^3$ ) ← also ( $10^{-6} \text{ m}^3$ )
top	41.07	258.6
bottom	-121.43	-87.47

Bending moment limits  $M = -\sigma I/y$

Tension at E & F:  $-(110 \times 10^6)(-87.47 \times 10^{-6}) = 9.622 \times 10^3 \text{ N}\cdot\text{m}$   
 Comp. at E & F:  $-(-150 \times 10^6)(258.6 \times 10^{-6}) = 38.8 \times 10^3 \text{ N}\cdot\text{m}$   
 Tension at A & D:  $-(110 \times 10^6)(258.6 \times 10^{-6}) = -28.45 \times 10^3 \text{ N}\cdot\text{m}$   
 Comp. at A & D:  $-(-150 \times 10^6)(-87.47 \times 10^{-6}) = -13.121 \times 10^3 \text{ N}\cdot\text{m}$

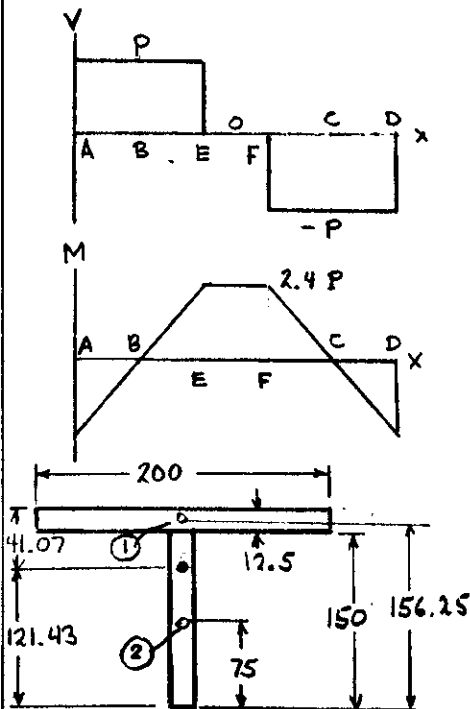
Allowable load P  $2.4 P = 9.622 \times 10^3$   $P = 4.01 \times 10^3 \text{ N} = 4.01 \text{ kN}$

Shear at A  $V_A = P$

Area A to B of shear diagram  $aV_A = aP$

Bending moment at A  $M_A = -aP = -4.01 \times 10^3 a$

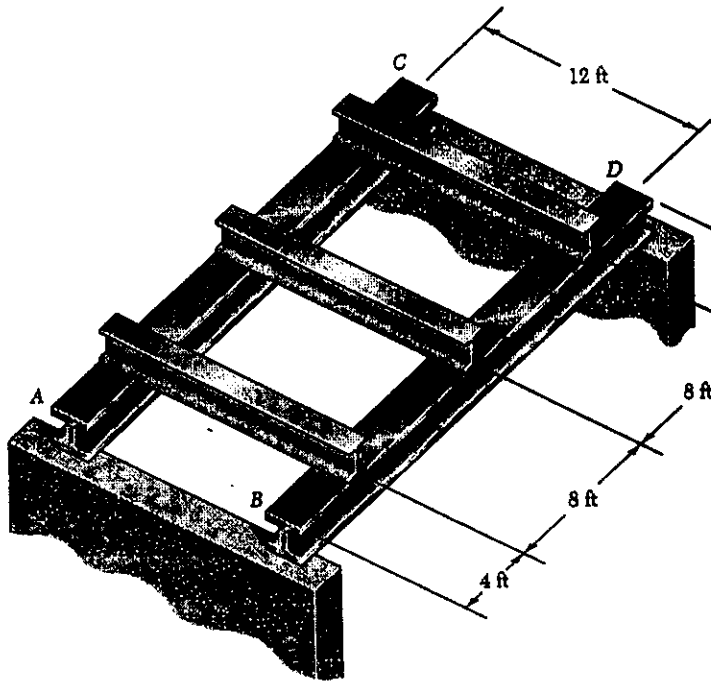
Distance a  $-4.01 \times 10^3 a = -13.121 \times 10^3$   $a = 3.27 \text{ m}$





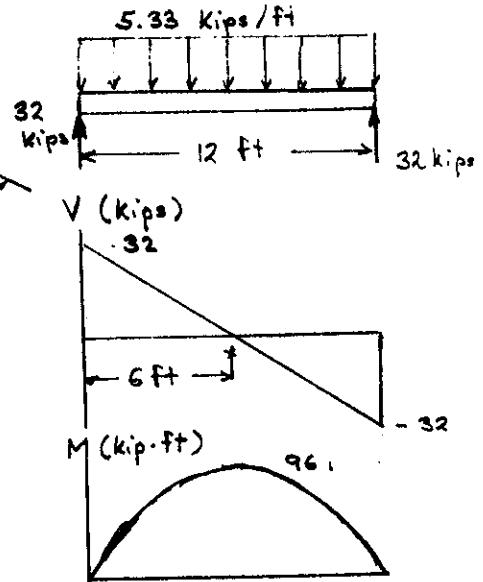
**PROBLEM 5.101**

5.101 Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders AC and BD. Knowing that the allowable normal stress for the steel used is 24 ksi, select (a) the most economical S-shape for the three beams, (b) the most economical W-shape for the two girders.



**SOLUTION**

Beams 1, 2, and 3



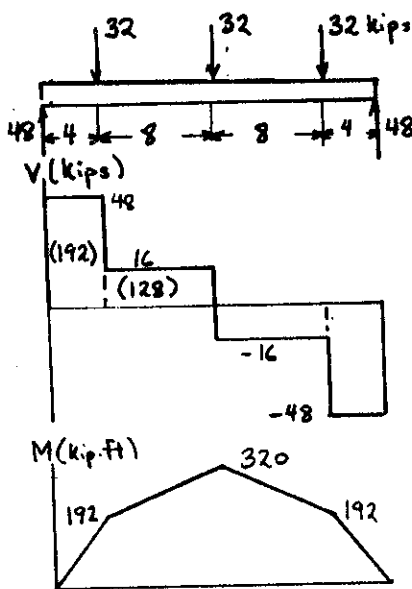
Maximum  $M = (\frac{1}{2})(c)(32) = 96 \text{ kip}\cdot\text{ft} = 1152 \text{ kip}\cdot\text{in}$

$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{1152}{24} = 48 \text{ in}^3$

(a) Use S 15 x 42.9

Shape	$S \text{ (in}^3\text{)}$
S 15 x 42.9	59.6
S 12 x 50	50.8

Beams AC and BC



Areas under shear diagram

$(4)(48) = 192 \text{ kip}\cdot\text{ft}$

$(8)(16) = 128 \text{ kip}\cdot\text{ft}$

Maximum  $M = 192 + 128 = 320 \text{ kip}\cdot\text{ft} = 3840 \text{ kip}\cdot\text{in}$

$S_{\min} = \frac{|M|}{\sigma_{\text{all}}} = \frac{3840}{24} = 160 \text{ in}^3$

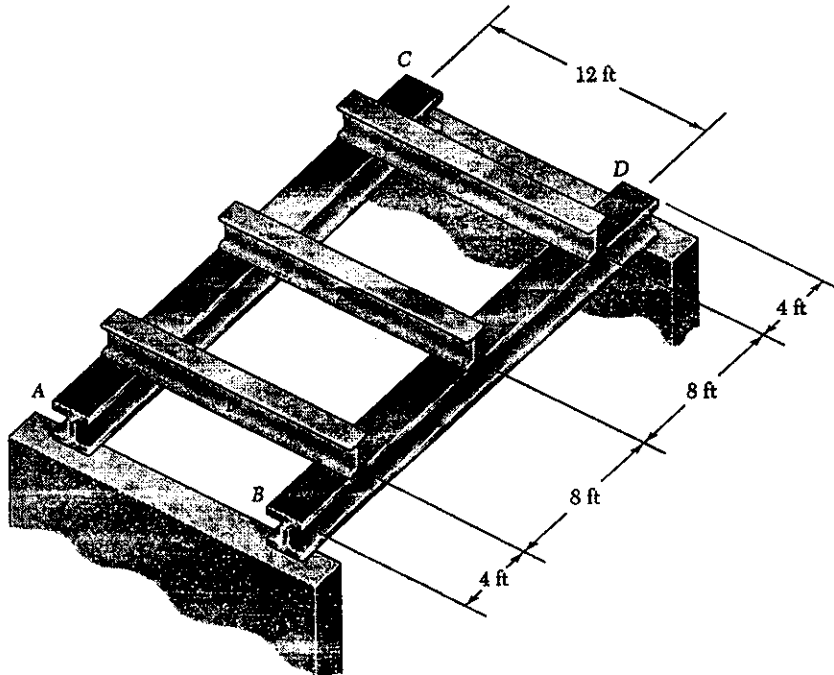
Shape	$S \text{ (in}^3\text{)}$
W 30 x 99	269
W 27 x 84	213
W 24 x 104	258
W 21 x 101	227
W 18 x 106	204

(b) Use W 27 x 84

PROBLEM 5.102

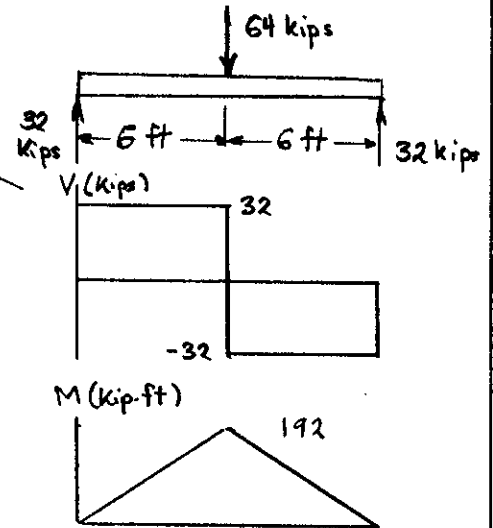
5.101 Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders AC and BD. Knowing that the allowable normal stress for the steel used is 24 ksi, select (a) the most economical S-shape for the three beams, (b) the most economical W-shape for the two girders.

5.102 Solve Prob. 5.101, assuming that the 64-kip distributed loads are replaced by 64-kip concentrated loads applied at the midpoints of the three beams.



SOLUTION

Beams 1, 2, and 3



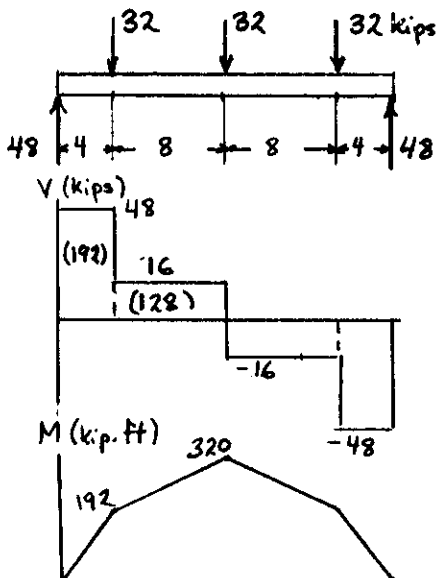
Maximum  $M = (6)(32) = 192 \text{ kip}\cdot\text{ft} = 2304 \text{ kip}\cdot\text{in}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2304}{24} = 96 \text{ in}^3$

(a) Use S 20 x 66

Shape	$S (\text{in}^3)$
S 20 x 66	119
S 18 x 70	103

Beams AC and BD



Areas under shear diagram

$(4)(48) = 192 \text{ kip}\cdot\text{ft}$

$(8)(16) = 128 \text{ kip}\cdot\text{ft}$

Maximum  $M = 192 + 128 = 320 \text{ kip}\cdot\text{ft} = 3840 \text{ kip}\cdot\text{in}$

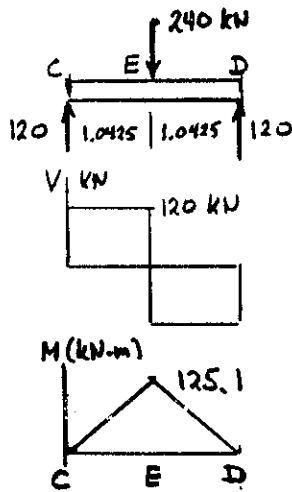
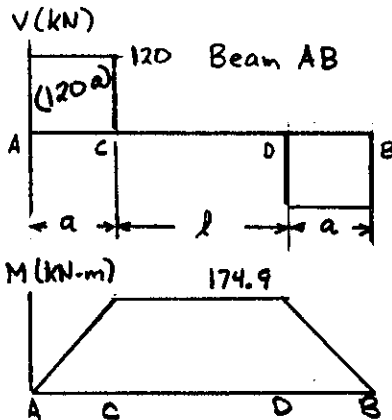
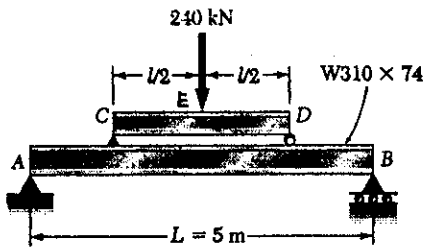
$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160 \text{ in}^3$

Shape	$S (\text{in}^3)$
W 30 x 99	269
W 27 x 84	213
W 24 x 104	258
W 21 x 101	227
W 18 x 106	204

(b) Use W 27 x 84

**PROBLEM 5.103**

5.103 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (a) the smallest allowable length  $l$  of beam  $CD$  if the W 310 × 74 beam  $AB$  is not to be overstressed, (b) the W shape which should be used for beam  $CD$ . Neglect the weight of both beams.



**SOLUTION**

For rolled steel section W 310 × 74 of beam AB

$$S = 1060 \times 10^3 \text{ mm}^3 = 1060 \times 10^{-6} \text{ m}^3$$

$$\sigma_{all} = 165 \text{ MPa} = 165 \times 10^6 \text{ Pa}$$

Allowable bending moment

$$M_{all} = S \sigma_{all} = (1060 \times 10^{-6}) (165 \times 10^6) = 174.9 \times 10^3 \text{ N}\cdot\text{m} = 174.9 \text{ kN}\cdot\text{m}$$

(a) Beam AB

Area A to C of shear diagram = 120 a

Bending moment at C = 120 a

$$120 a = 174.9 = 1.4575 \text{ m}$$

$$\text{Geometry: } 2a + l = 5 \quad l = 5 - 2a = 2.085 \text{ m}$$

(b) Beam CD (midpoint E)

Area C to E of shear diagram =  $(1.0425)(120) = 125.1 \text{ kN}\cdot\text{m}$

Bending moment at E

$$M = 125.1 \text{ kN}\cdot\text{m} = 125.1 \times 10^3 \text{ N}\cdot\text{m}$$

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6} \text{ m}^3 = 758.2 \times 10^3 \text{ mm}^3$$

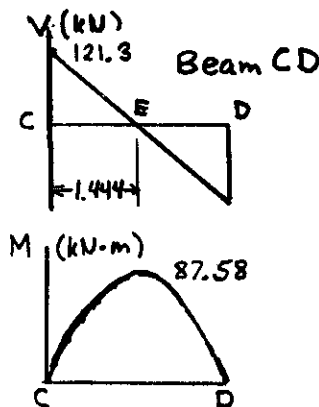
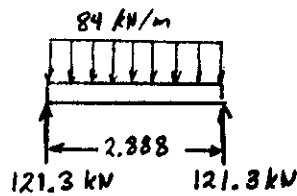
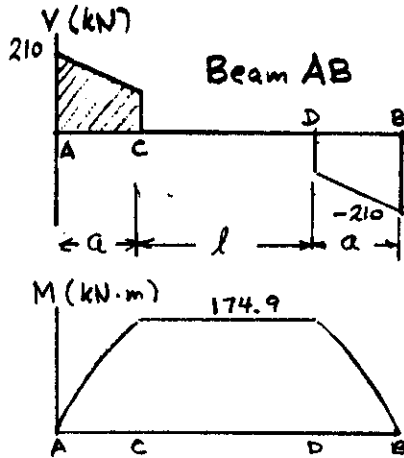
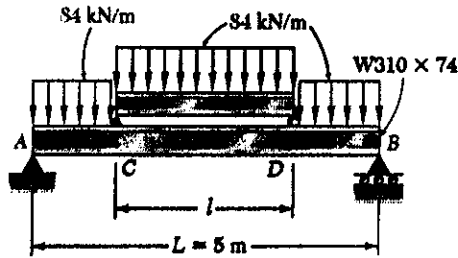
Shape	$S$ ( $10^3 \text{ mm}^3$ )
W 460 × 52	942
W 410 × 46.1	774 ←
W 360 × 57.8	899
W 310 × 60	851
W 250 × 67	809
W 200 × 86	853

Answer

W 410 × 46.1 ←

**PROBLEM 5.104**

5.104 A uniformly distributed load of 84 kN/m is to be supported over the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (a) the smallest allowable length  $l$  of beam CD if the W 310 × 74 beam AB is not to be overstressed, (b) the W shape which should be used for beam CD. Neglect the weight of both beams.



Shape	$S$ ( $10^3 \text{ mm}^3$ )
W 460 × 52	942
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 59	582

**SOLUTION**

For rolled steel section W310×74 of beam AB

$$S = 1060 \times 10^3 \text{ mm}^3 = 1060 \times 10^{-6} \text{ m}^3$$

$$\sigma_{all} = 165 \text{ MPa} = 165 \times 10^6 \text{ Pa}$$

Allowable bending moment

$$M_{all} = S \sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3 \text{ N}\cdot\text{m} = 174.9 \text{ kN}\cdot\text{m}$$

By symmetry reactions A and B are equal.

$$+\uparrow \Sigma F_y = 0 \quad A + B - (5)(84) = 0$$

$$A = B = 210 \text{ kN}$$

By symmetry, reaction at C and D are equal.

$$+\uparrow \Sigma F_y = 0 \quad C + D - 84l = 0$$

$$C = D = 42l$$

$$\text{Geometry} \quad a = \frac{1}{2}(5-l)$$

Beam AB: Area A to C of shear diagram

$$\frac{1}{2}(a)(A+C) = \frac{1}{2} \cdot \frac{1}{2}(5-l)(210 + 42l) = \frac{1}{4}(1050 - 42l^2)$$

Bending moment at C  $\frac{1}{4}(1050 - 42l^2)$

$$\frac{1}{4}(1050 - 42l^2) = 174.9 \quad l^2 = 8.3429 \text{ m}^2$$

$$(a) \quad l = 2.888 \text{ m}$$

$$C = D = 42l = 121.3 \text{ kN}$$

Beam CD (midpoint E)

$$\text{Area C to E of shear diagram} \quad \frac{1}{2}(1.444)(121.3) = 87.58 \text{ kN}\cdot\text{m}$$

Bending moment at E

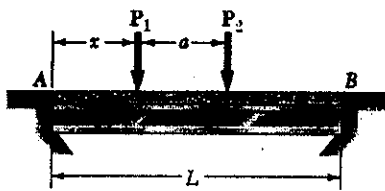
$$M = 87.58 \text{ kN}\cdot\text{m} = 87.58 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{all} = 165 \times 10^6 \text{ Pa}$$

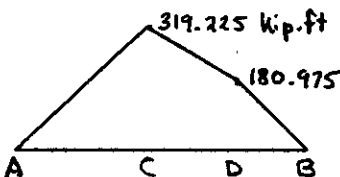
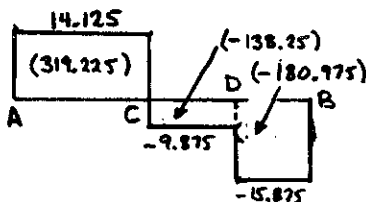
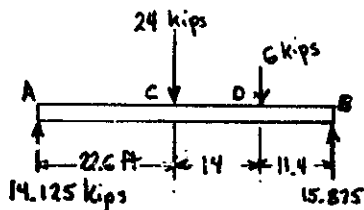
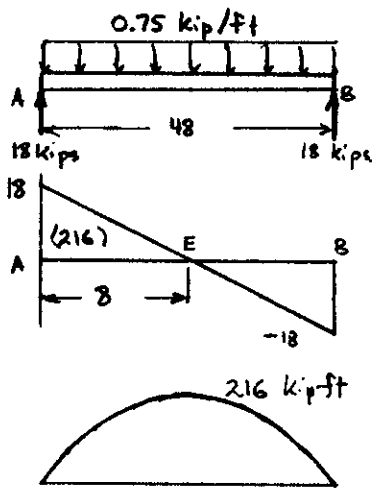
$$S_{min} = \frac{M}{\sigma_{all}} = \frac{87.58 \times 10^3}{165 \times 10^6} = 531 \times 10^{-6} \text{ m}^3 = 531 \times 10^3 \text{ mm}^3$$

(b) Use W 310 × 38.7

PROBLEM 5.105



$L = 48 \text{ ft}$   
 $a = 14 \text{ ft}$   
 $P_1 = 24 \text{ kips}$   
 $P_2 = 6 \text{ kips}$   
 $w = 0.75 \text{ kip/ft}$



\*5.105 A bridge of length  $L = 48 \text{ ft}$  is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength  $\sigma_U = 60 \text{ ksi}$ . The combined weight of the slab and beams can be approximated by a uniformly distributed load  $w = 0.75 \text{ kip/ft}$  on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance  $a = 14 \text{ ft}$  from each other will be driven across the bridge and that the resulting concentrated loads  $P_1$  and  $P_2$  exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using *LRFD* with the load factors  $\gamma_D = 1.25$ ,  $\gamma_L = 1.75$  and the resistance factor  $\phi = 0.9$ . [Hint. It can be shown that the maximum value of  $|M_L|$  occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to  $aP_2/2(P_1 + P_2)$ .]

SOLUTION

Dead load:  $R_A = R_B = (\frac{1}{2})(48)(0.75) = 18 \text{ kips}$

Area A to E of shear diagram  $(\frac{1}{2})(8)(18) = 216 \text{ kip}\cdot\text{ft}$

$M_{max} = 216 \text{ kip}\cdot\text{ft} = 2592 \text{ kip}\cdot\text{in.}$  at point E

live load:  $U = \frac{aP_2}{2(P_1 + P_2)} = \frac{(14)(6)}{2(24 + 6)} = 1.4 \text{ ft}$

$x = \frac{L}{2} - U = 24 - 1.4 = 22.6 \text{ ft}$

$x + a = 22.6 + 14 = 36.6 \text{ ft}$

$L - x - a = 48 - 36.6 = 11.4 \text{ ft.}$

$\sum M_B = 0 \quad -48 R_A + (25.4)(24) + (11.4)(6) = 0$   
 $R_A = 14.125 \text{ kips}$

Shear: A to C  $V = 14.125 \text{ kips}$   
 C to D  $V = 14.125 - 24 = -9.875 \text{ kips}$   
 D to B  $V = -15.875 \text{ kips}$

Area A to C  $(22.6)(14.125) = 319.225 \text{ kip}\cdot\text{ft}$

Bending moment:  $M_C = 319.225 \text{ kip}\cdot\text{ft} = 3831 \text{ kip}\cdot\text{in.}$

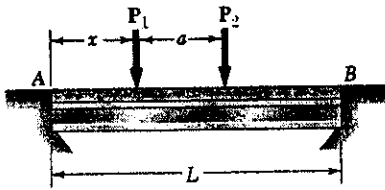
Design:  $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_{ult} S_{min}$

$S_{min} = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_{ult}} = \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$   
 $= 184.2 \text{ in}^3$

Shape	$S \text{ (in}^3\text{)}$
W 30 x 99	269
W 27 x 84	213 ←
W 24 x 104	258
W 21 x 101	227
W 18 x 106	204

W 27 x 84

## PROBLEM 5.106



$$\begin{aligned} L &= 48 \text{ ft} \\ a &= 14 \text{ ft} \\ P_1 &= 24 \text{ kips} \\ P_2 &= 6 \text{ kips} \\ w &= 0.75 \text{ kip/ft} \end{aligned}$$

\*5.105 A bridge of length  $L = 48$  ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength  $\sigma_u = 60$  ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load  $w = 0.75$  kip/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance  $a = 14$  ft from each other will be driven across the bridge and that the resulting concentrated loads  $P_1$  and  $P_2$  exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using *LRFD* with the load factors  $\gamma_D = 1.25$ ,  $\gamma_L = 1.75$  and the resistance factor  $\phi = 0.9$ . [Hint. It can be shown that the maximum value of  $|M_L|$  occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to  $aP_2/2(P_1 + P_2)$ .]

\*5.106 Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.105, determine how much heavier a truck could safely cross the bridge designed in that problem.

## SOLUTION

See solution to PROBLEM 5.105 for calculation of the following:

$$M_D = 2592 \text{ kip}\cdot\text{in} \quad M_L = 3831 \text{ kip}\cdot\text{in}$$

$$\text{For rolled steel section } W27 \times 84 \quad S = 213 \text{ in}^3$$

Allowable live load moment  $M_L^*$

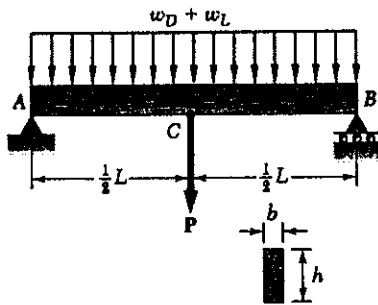
$$\gamma_D M_D + \gamma_L M_L^* = \phi M_u = \phi \sigma_u S$$

$$M_L^* = \frac{\phi \sigma_u S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75} = 4721 \text{ kip}\cdot\text{in}$$

$$\text{Ratio } \frac{M_L^*}{M_L} = \frac{4721}{3831} = 1.232 = 1 + 0.232$$

$$\text{Increase } 23.2\%$$

PROBLEM 5.107



\*5.107 A roof structure consisting of plywood and roofing material is supported by several timber beams of length  $L = 16$  m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load  $w_D = 350$  N/m. The live loads consist of the snow load, represented by a uniformly distributed load  $w_L = 600$  N/m, and a 6-kN concentrated load  $P$  applied at the midpoint  $C$  of each beam. Knowing that the ultimate strength for the timber used is  $\sigma_u = 50$  MPa and that the width of the beams is  $b = 75$  mm, determine the minimum allowable depth  $h$  of the beams, using *LRFD* with the load factors  $\gamma_D = 1.2$ ,  $\gamma_L = 1.6$  and the resistance factor  $\phi = 0.9$ .

SOLUTION

$L = 16$  m,  $w_D = 350$  N/m = 0.35 kN/m  
 $w_L = 600$  N/m = 0.6 kN/m,  $P = 6$  kN

Dead load:  $R_A = (\frac{1}{2})(16)(0.35) = 2.8$  kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(2.8) = 11.2$  kN·m

Bending moment at C:  $11.2$  kN·m =  $11.2 \times 10^3$  N·m

Live load:  $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8$  kN

Shear at C<sup>-</sup>  $V = 7.8 - (8)(0.6) = 3$  kV

Area A to C of shear diagram

$(\frac{1}{2})(8)(7.8 + 3) = 43.2$  kN·m

Bending moment at C:  $43.2$  kN·m =  $43.2 \times 10^3$  N·m

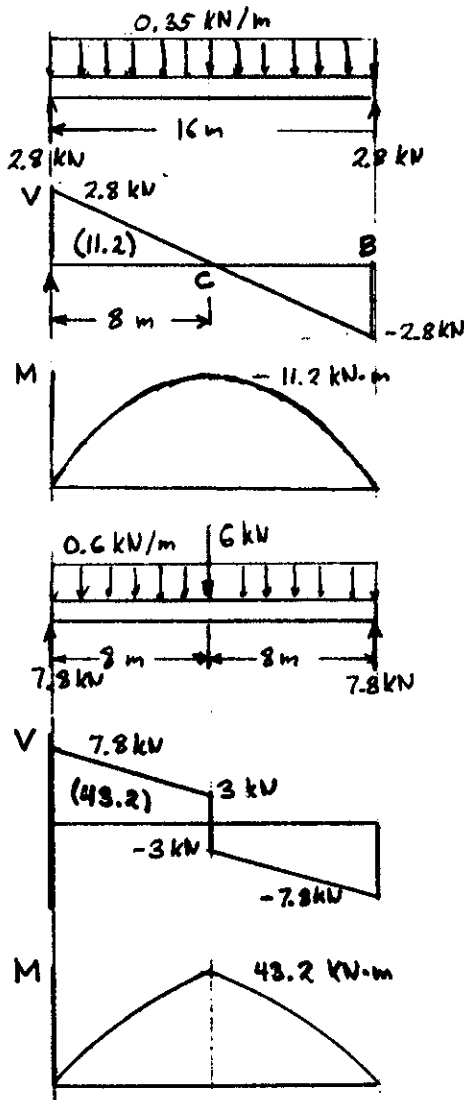
Design  $\gamma_D M_D + \gamma_L M_L = \phi M_u = \phi \sigma_u S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_u} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

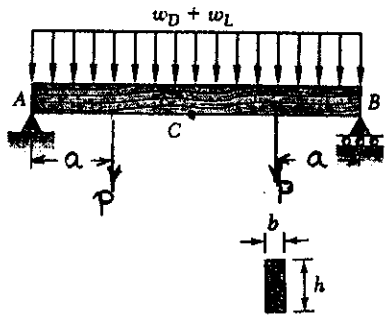
$$= 1.8347 \times 10^{-3} \text{ m}^3 = 1.8347 \times 10^6 \text{ mm}^3$$

For a rectangular section  $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} = 383 \text{ mm}$$



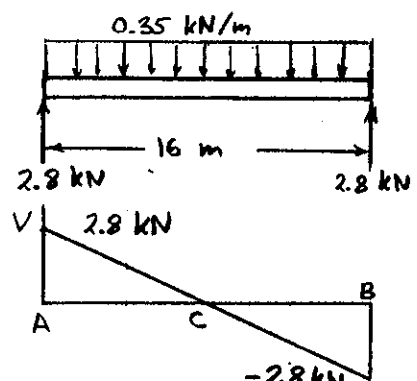
**PROBLEM 5.108**



\*5.107 A roof structure consisting of plywood and roofing material is supported by several timber beams of length  $L = 16$  m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load  $w_D = 350$  N/m. The live loads consist of the snow load, represented by a uniformly distributed load  $w_L = 600$  N/m, and a 6-kN concentrated load  $P$  applied at the midpoint  $C$  of each beam. Knowing that the ultimate strength for the timber used is  $\sigma_u = 50$  MPa and that the width of the beams is  $b = 75$  mm, determine the minimum allowable depth  $h$  of the beams, using *LRFD* with the load factors  $\gamma_D = 1.2$ ,  $\gamma_L = 1.6$  and the resistance factor  $\phi = 0.9$ .

\*5.108 Solve Prob. 5.107, assuming that the 6-kN concentrated loads are replaced by 3-kN concentrated loads  $P_1$  and  $P_2$  applied at a distance of 4 m from each end of the beams.

**SOLUTION**

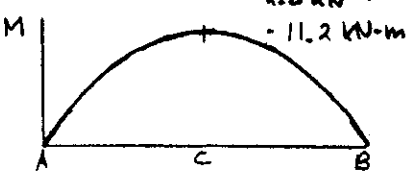


$L = 16$  m,  $a = 4$  m,  $w_D = 350$  N/m =  $0.35$  kN/m  
 $w_L = 600$  N/m =  $0.6$  kN/m  $P = 3$  kN

Dead load:  $R_A = (\frac{1}{2})(16)(0.35) = 2.8$  kN

Area A to C of shear diagram  
 $(\frac{1}{2})(8)(2.8) = 11.2$  kN-m

Bending moment at C:  $11.2$  kN-m =  $11.2 \times 10^3$  N-m



Live load:  $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8$  kN

Shear at  $D^-$   $7.8 - (4)(0.6) = 5.4$  kN  
 Shear at  $D^+$   $5.4 - 3 = 2.4$  kN

Area A to D  $(\frac{1}{2})(4)(7.8 + 5.4) = 26.4$  kN-m

Area D to C  $(\frac{1}{2})(4)(2.4) = 4.8$  kN-m

Bending moment at C =  $26.4 + 4.8 = 31.2$  kN-m  
 =  $31.2 \times 10^3$  N-m

Design  $\gamma_D M_D + \gamma_L M_L = \phi M_u = \phi \sigma_u S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_u} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(31.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

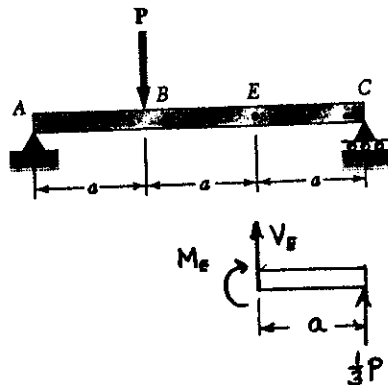
$$= 1.408 \times 10^{-3} \text{ m}^3 = 1.408 \times 10^6 \text{ mm}^3$$

For a rectangular section  $S = \frac{1}{6} b h^3$

$$h = \sqrt[3]{\frac{6S}{b}} = \sqrt[3]{\frac{(6)(1.408 \times 10^6)}{75}} = 336 \text{ mm}$$



PROBLEM 5.109



5.109 through 5.111 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment at point  $E$  and check your answer by drawing the free-body diagram of the portion of beam to the right of  $E$ .

SOLUTION

$$\uparrow \sum M_C = 0 \quad -3aA + 2aP = 0 \quad A = \frac{2}{3}P$$

$$V = \frac{2}{3}P - P\langle x-a \rangle^0$$

$$M = \frac{2}{3}Px - P\langle x-a \rangle^1$$

At point  $E \quad x = 2a$

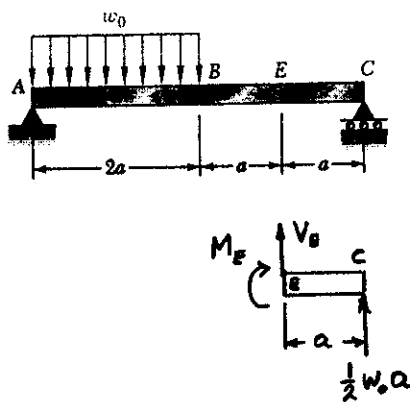
$$M_E = \frac{2}{3}P(2a) - Pa = \frac{1}{3}Pa$$

$$\sum M_A = 0 \quad 3aC - aP = 0 \quad C = \frac{1}{3}P$$

$$\uparrow \sum M_E = 0 \quad -M_E + (a)(\frac{1}{3}P) = 0$$

$$M_E = \frac{1}{3}Pa$$

PROBLEM 5.110



5.109 through 5.111 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment at point  $E$  and check your answer by drawing the free-body diagram of the portion of beam to the right of  $E$ .

SOLUTION

$$\uparrow \sum M_C = 0 \quad -4aA + (3a)(2a(w_0)) = 0 \quad A = \frac{3}{2}w_0a$$

$$w = w_0 - w_0\langle x-2a \rangle^0 = -\frac{dV}{dx}$$

$$V = -w_0x + w_0\langle x-2a \rangle^1 + \frac{3}{2}w_0a = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0\langle x-2a \rangle^2 + \frac{3}{2}w_0ax + 0$$

At point  $E \quad x = 3a$

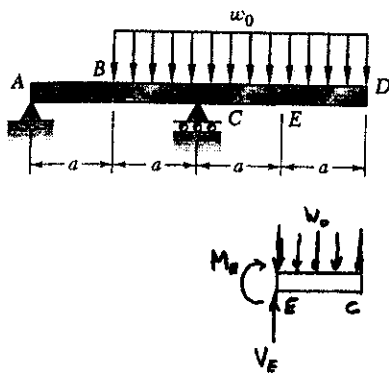
$$M_E = -\frac{1}{2}w_0(3a)^2 + \frac{1}{2}w_0a^2 + \frac{3}{2}w_0a(3a) = \frac{1}{2}w_0a^2$$

$$\uparrow \sum M_A = 0 \quad 4aC - (a)(2aw_0) = 0 \quad C = \frac{1}{2}w_0a$$

$$\uparrow \sum M_E = 0 \quad -M_E + (a)(\frac{1}{2}w_0a) = 0$$

$$M_E = \frac{1}{2}w_0a^2$$

PROBLEM 5.111



5.109 through 5.111 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment at point  $E$  and check your answer by drawing the free-body diagram of the portion of beam to the right of  $E$ .

SOLUTION

$$+\circlearrowleft \sum M_C = 0 \quad -2aA - \left(\frac{3}{2}\right)(3aw_0) = 0 \quad A = -\frac{3}{4}w_0a$$

$$+\circlearrowleft \sum M_A = 0 \quad 2aC + \left(\frac{5a}{2}\right)(3aw_0) = 0 \quad C = \frac{15}{4}w_0a$$

$$w = w_0 \langle x-a \rangle^0 = -\frac{dV}{dx}$$

$$V = -w_0 \langle x-a \rangle^1 - \frac{3}{4}w_0a + \frac{15}{4}w_0a \langle x-2a \rangle^0 = \frac{dM}{dx}$$

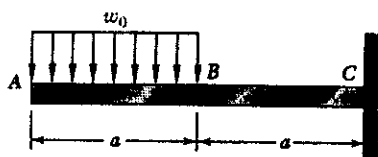
$$M = -\frac{1}{2}w_0 \langle x-a \rangle^2 - \frac{3}{4}w_0ax + \frac{15}{4}w_0a \langle x-2a \rangle^1 + 0$$

At point  $E \quad x = 3a$

$$M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0a(3a) + \frac{15}{4}w_0a(a) \\ = -\frac{1}{2}w_0a^2$$

Check:  $+\circlearrowleft \sum M_E = 0 \quad -M_E - \frac{a}{2}(w_0a) = 0$   
 $M_E = -\frac{1}{2}w_0a^2$

PROBLEM 5.112



5.112 through 5.114 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment at point  $C$  and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

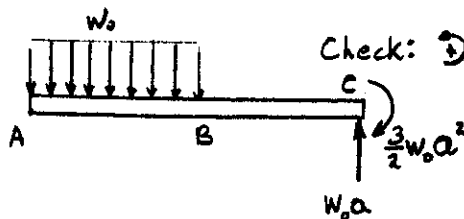
$$w = w_0 - w_0 \langle x-a \rangle^0 = -\frac{dV}{dx}$$

$$V = -w_0x + w_0 \langle x-a \rangle^1 = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0 \langle x-a \rangle^2$$

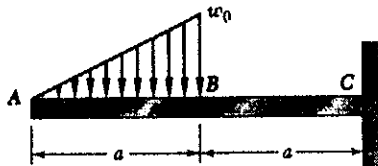
At point  $C \quad x = 2a$

$$M_C = -\frac{1}{2}w_0(2a)^2 + \frac{1}{2}w_0a^2 = -\frac{3}{2}w_0a^2$$



Check:  $+\circlearrowleft \sum M_C = 0 \quad \left(\frac{3a}{2}\right)(w_0a) + M_C = 0$   
 $M_C = -\frac{3}{2}w_0a^2$

**PROBLEM 5.113**



5.112 through 5.114 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

**SOLUTION**

$$W = \frac{w_0 x}{a} - w_0 \langle x-a \rangle^0 - \frac{w_0}{a} \langle x-a \rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{w_0 x^2}{2a} + w_0 \langle x-a \rangle^1 + \frac{w_0}{2a} \langle x-a \rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2} \langle x-a \rangle^2 + \frac{w_0}{6a} \langle x-a \rangle^3$$

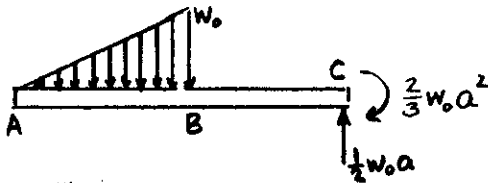
At point C  $x = 2a$

$$M_c = -\frac{w_0 (2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} = -\frac{2}{3} w_0 a^2$$

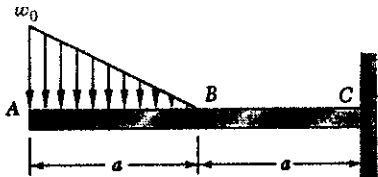
Check:  $\sum M_c = 0$

$$\left(\frac{4a}{3}\right) \left(\frac{1}{2} w_0 a\right) + M_c = 0$$

$$M_c = -\frac{2}{3} w_0 a^2$$



**PROBLEM 5.114**



5.112 through 5.114 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

**SOLUTION**

$$W = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a} \langle x-a \rangle^1 = -\frac{dV}{dx}$$

$$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a} \langle x-a \rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a} \langle x-a \rangle^3$$

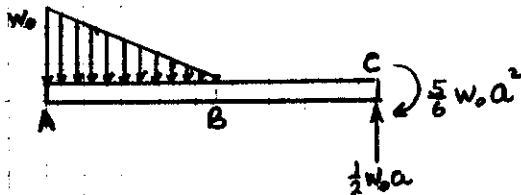
At point C  $x = 2a$

$$M_c = -\frac{w_0 (2a)^2}{2} + \frac{w_0 (2a)^3}{6a} - \frac{w_0 a^3}{6a} = -\frac{5}{6} w_0 a^2$$

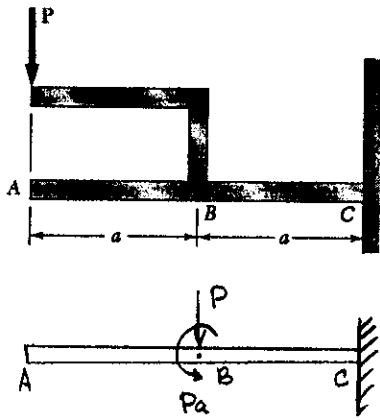
Check:

$$\sum M_c = 0 \quad \left(\frac{5}{3} a\right) \left(\frac{1}{2} w_0 a\right) + M_c = 0$$

$$M_c = -\frac{5}{6} w_0 a^2$$



PROBLEM 5.115



5.115 and 5.116 (a) Using singularity functions, write the equations defining the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment just to the right of point B.

SOLUTION

$$V = -P\langle x-a \rangle^0$$

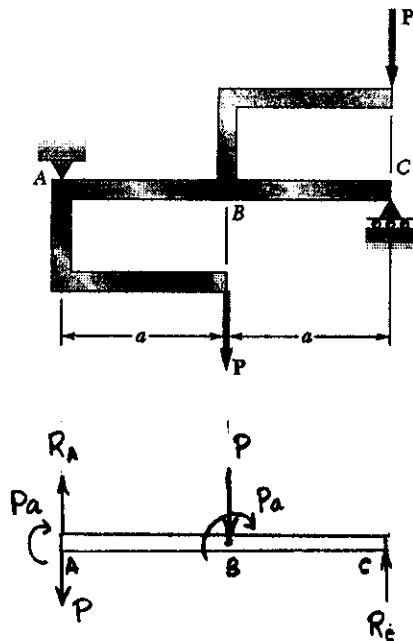
$$\frac{dM}{dx} = -P\langle x-a \rangle^0$$

$$M = -P\langle x-a \rangle^1 - Pa\langle x-a \rangle^0$$

Just to the right of B  $x = a^+$

$$M = -0 - Pa = -Pa$$

PROBLEM 5.116



5.115 and 5.116 (a) Using singularity functions, write the equations defining the shear and bending moment for beam ABC under the loading shown. (b) Use the equation obtained for  $M$  to determine the bending moment just to the right of point B.

SOLUTION

$$\sum M_C = 0 \quad (2a)P + aP - 2(Pa) - 2aR_A = 0$$

$$R_A = \frac{1}{2}P$$

$$V = (R_A - P) - P\langle x-a \rangle^0$$

$$= -\frac{1}{2}P - P\langle x-a \rangle^0$$

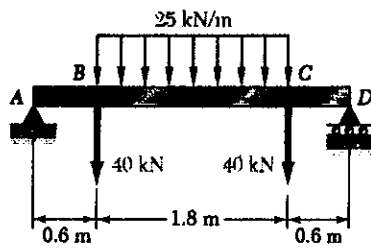
$$\frac{dM}{dx} = -\frac{1}{2}P - P\langle x-a \rangle^0$$

$$M = -\frac{1}{2}Px - P\langle x-a \rangle^1 + Pa + Pa\langle x-a \rangle^0$$

Just to the right of point B  $x = a^+$

$$M = -\frac{1}{2}Pa - 0 + Pa + Pa = \frac{3}{2}Pa$$

**PROBLEM 5.117**



5.117 through 5.120 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

**SOLUTION**

By symmetry  $R_A = R_D$

$$+\uparrow \sum F_y = 0 \quad R_A + R_D - 40 - (1.8)(25) - 40 = 0$$

$$R_A = R_D = 62.5 \text{ kN}$$

$$w = 25 \langle x - 0.6 \rangle^0 - 25 \langle x - 2.4 \rangle^0 = -\frac{dV}{dx}$$

$$V = 62.5 - 25 \langle x - 0.6 \rangle^1 + 25 \langle x - 2.4 \rangle^1 - 40 \langle x - 0.6 \rangle^0 - 40 \langle x - 2.4 \rangle^0 \text{ kN} \quad \blacktriangleleft$$

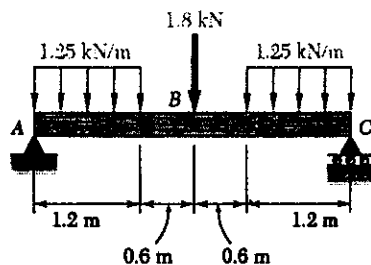
$$M = 62.5x - 12.5 \langle x - 0.6 \rangle^2 + 12.5 \langle x - 2.4 \rangle^2 - 40 \langle x - 0.6 \rangle^1 - 40 \langle x - 2.4 \rangle^1 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

Locate point where  $V = 0$ . Assume  $0.6 < x^* < 1.8$

$$0 = 62.5 - 25(x^* - 0.6) + 0 - 40 - 0 \quad x^* = 1.5 \text{ m}$$

$$M = (62.5)(1.5) - (25)(0.9)^2 + 0 - (40)(0.9) - 0 = 47.625 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 5.118**



5.117 through 5.120 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.

**SOLUTION**

By symmetry  $R_A = R_C$

$$+\uparrow \sum F_y = 0 \quad R_A + R_C - (1.2)(1.25) - 1.8 - (1.2)(1.25) = 0$$

$$R_A = R_C = 2.4 \text{ kN}$$

$$w = 1.25 - 1.25 \langle x - 1.2 \rangle^0 + 1.25 \langle x - 2.4 \rangle^0 = -\frac{dV}{dx}$$

$$V = -1.25x + 1.25 \langle x - 1.2 \rangle^1 - 1.25 \langle x - 2.4 \rangle^1 + 2.4 - 1.8 \langle x - 1.8 \rangle^0 \text{ kN} \quad \blacktriangleleft$$

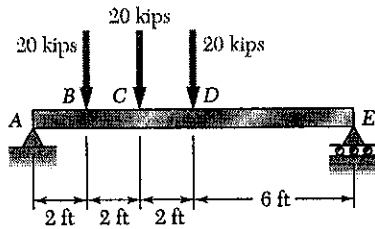
$$M = -0.625x^2 + 0.625 \langle x - 1.2 \rangle^2 - 0.625 \langle x - 2.4 \rangle^2 + 2.4x - 1.8 \langle x - 1.8 \rangle^0 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$M_{\max}$  occurs at  $x = 1.8 \text{ m}$

$$M_{\max} = -(0.625)(1.8)^2 + (0.625)(0.6)^2 + 0 + (2.4)(1.8) - 0 = 2.52 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

**PROBLEM 5.119**

5.117 through 5.120 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



**SOLUTION**

$$+\sum M_E = 0 \quad -12A + (10)(20) + (8)(20) + (6)(20) = 0$$

$$A = 40 \text{ kips.}$$

$$V = 40 - 20\langle x-2 \rangle^0 - 20\langle x-4 \rangle^0 - 20\langle x-6 \rangle^0 \quad \text{kips}$$

$$M = 40x - 20\langle x-2 \rangle^1 - 20\langle x-4 \rangle^1 - 20\langle x-6 \rangle^1 \quad \text{kip}\cdot\text{ft}$$

Values of V

- A to B  $V = 40 \text{ kip}$   
 B to C  $V = 40 - 20 = 20 \text{ kips}$   
 C to D  $V = 40 - 20 - 20 = 0$   
 D to E  $V = 40 - 20 - 20 - 20 = -20 \text{ kip}$

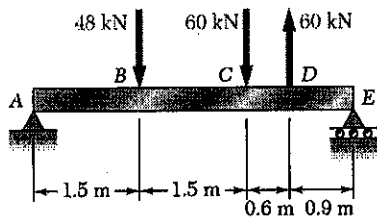
[www.konkur.in](http://www.konkur.in)

Bending moment is constant and maximum over C to D.

At C  $x = 4 \text{ ft} \quad M = (40)(4) - (20)(2) - 0 - 0 = 120 \text{ kip}\cdot\text{ft}$

**PROBLEM 5.120**

5.117 through 5.120 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



**SOLUTION**

$$+\sum M_E = 0 \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$$

$$R_A = 40 \text{ kN}$$

$$V = 40 - 48\langle x-1.5 \rangle^0 - 60\langle x-3.0 \rangle^0 + 60\langle x-3.6 \rangle^0 \quad \text{kN}$$

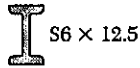
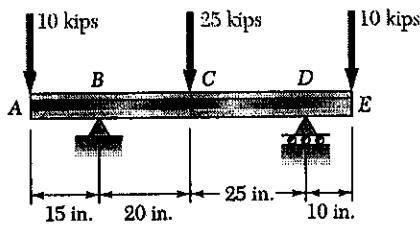
$$M = 40x - 48\langle x-1.5 \rangle^1 - 60\langle x-3.0 \rangle^1 + 60\langle x-3.6 \rangle^1 \quad \text{kN}\cdot\text{m}$$

Pt.	x (m)	M (kN·m)
A	0	0
B	1.5	$(40)(1.5) = 60 \text{ kN}\cdot\text{m}$
C	3.0	$(40)(3.0) - (48)(1.5) = 48 \text{ kN}\cdot\text{m}$
D	3.6	$(40)(3.6) - (48)(2.1) - (60)(0.6) = 7.2 \text{ kN}\cdot\text{m}$
E	4.5	$(40)(4.5) - (48)(3.0) - (60)(1.5) + (60)(0.9) = 0$

$M_{\max} = 60 \text{ kN}\cdot\text{m}$

**PROBLEM 5.121**

5.121 and 5.122 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



**SOLUTION**

$$+\circlearrowleft \sum M_D = 0$$

$$(60)(10) + 45 R_B + (25)(25) - (10)(10) = 0$$

$$R_B = 25 \text{ kips}$$

$$+\circlearrowleft \sum M_B = 0$$

$$(15)(10) - (20)(25) + 45 R_D - (55)(10) = 0$$

$$R_D = 20 \text{ kips}$$

$$V = -10 + 25 \langle x-15 \rangle^0 - 25 \langle x-35 \rangle^0 + 20 \langle x-60 \rangle^0 \quad \text{kips}$$

$$M = -10x + 25 \langle x-15 \rangle^1 - 25 \langle x-35 \rangle^1 + 20 \langle x-60 \rangle^1 \quad \text{kip} \cdot \text{in}$$

Pt	x (ft)	M (kip·in)
B	15	$-(10)(15) = -150 \text{ kip} \cdot \text{in}$
C	35	$-(10)(35) + (25)(20) = 150 \text{ kip} \cdot \text{in}$
D	60	$-(10)(60) + (25)(45) - (25)(25) = -100 \text{ kip} \cdot \text{in}$
E	70	$-(10)(70) + (25)(55) - (25)(35) + (20)(10) = 0 \quad \text{checks}$

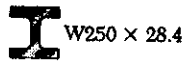
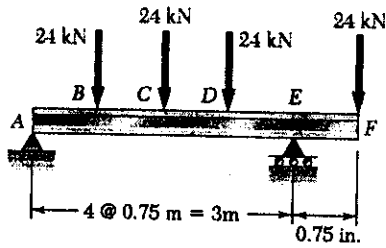
Maximum  $|M| = 150 \text{ kip} \cdot \text{in}$

For  $S6 \times 12.5$  rolled steel section  $S = 7.37 \text{ in}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{150}{7.37} = 20.35 \text{ ksi}$

PROBLEM 5.122

5.121 and 5.122 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



SOLUTION

$$\begin{aligned} \sum M_E = 0 \\ -3R_A + (2.25)(24) - (1.5)(24) \\ - (0.75)(24) + (0.75)(24) = 0 \end{aligned}$$

$$R_A = 30 \text{ kips}$$

$$\sum M_A = 0 \quad - (0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$$

$$R_E = 66 \text{ kips}$$

$$V = 30 - 24\langle x - 0.75 \rangle^0 - 24\langle x - 1.5 \rangle^0 - 24\langle x - 2.25 \rangle^0 + 66\langle x - 3 \rangle^0 \quad \text{kN}$$

$$M = 30x - 24\langle x - 0.75 \rangle^1 - 24\langle x - 1.5 \rangle^1 - 24\langle x - 2.25 \rangle^1 + 66\langle x - 3 \rangle^1 \quad \text{kN}\cdot\text{m}$$

Pt	x (m)	M (kN·m)
B	0.75	$(30)(0.75) = 22.5 \text{ kN}\cdot\text{m}$
C	1.5	$(30)(1.5) - (24)(0.75) = 27 \text{ kN}\cdot\text{m}$
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5 \text{ kN}\cdot\text{m}$
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18 \text{ kN}\cdot\text{m}$
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$

$$\text{Maximum } |M| = 27 \text{ kN}\cdot\text{m} = 27 \times 10^3 \text{ N}\cdot\text{m}$$

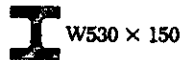
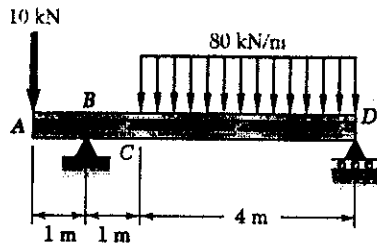
For rolled steel section W250 x 28.4  $S = 308 \times 10^3 \text{ mm}^3 = 308 \times 10^{-6} \text{ m}^3$

$$\text{Normal stress } \sigma = \frac{|M|}{S} = \frac{27 \times 10^3}{308 \times 10^{-6}} = 87.7 \times 10^6 \text{ Pa} = 87.7 \text{ MPa}$$



PROBLEM 5.123

5.123 and 5.124 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



SOLUTION

$$+\circlearrowleft M_D = 0$$

$$(6)(10) - 5R_B + (2)(4)(80) = 0$$

$$R_B = 140 \text{ kN}$$

$$w = 80 \langle x-2 \rangle^0 \text{ kN/m} = -dv/dx$$

$$V = -10 + 140 \langle x-1 \rangle^0 - 80 \langle x-2 \rangle^1 \text{ kN}$$

A to B  $V = -10 \text{ kN}$

B to C  $V = -10 + 140 = 130 \text{ kN}$

D ( $x=6$ )  $V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at B and at point E ( $x=x_E$ ) between C and D.

$$V = 0 = -10 + 140 \langle x_E-1 \rangle^0 - 80 \langle x_E-2 \rangle^1$$

$$= -10 + 140 - 80(x_E-2) \quad x_E = 3.625 \text{ m}$$

$$M = -10x + 140 \langle x-1 \rangle^1 - 40 \langle x-2 \rangle^2 \text{ kN}\cdot\text{m}$$

At pt. B  $x=1 \quad M_B = -(10)(1) = -10 \text{ kN}\cdot\text{m}$

At pt. E  $x=3.625$

$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN}\cdot\text{m}$$

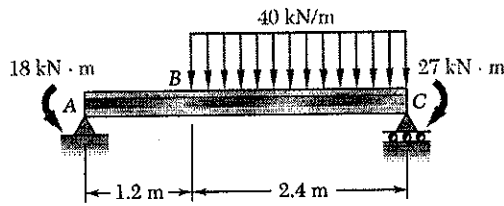
$$|M|_{\max} = 225.6 \text{ kN}\cdot\text{m} \quad \text{at } x = 3.625 \text{ m}$$

For W 530 x 150  $S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$   
 $= 60.6 \text{ MPa}$

PROBLEM 5.124

5.123 and 5.124 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.



SOLUTION

$$\sum M_c = 0$$

$$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$$

$$R_A = 29.5 \text{ kN}$$

$$V = 29.5 - 40 \langle x - 1.2 \rangle' \text{ kN}$$

Point D  $V = 0$   $29.5 - 40(x_D - 1.2) = 0$   
 $x_D = 1.9375 \text{ m}$

$$M = -18 + 29.5x - 20 \langle x - 1.2 \rangle^2 \text{ kN}\cdot\text{m}$$

$$M_A = -18 \text{ kN}\cdot\text{m}$$

$$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN}\cdot\text{m}$$

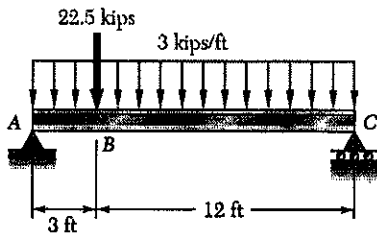
$$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN}\cdot\text{m}$$

Maximum  $|M| = 28.278 \text{ kN}\cdot\text{m}$  at  $x = 1.9375 \text{ m}$  ◀ ▶

For S 310 x 52 rolled steel section  $S = 625 \times 10^3 \text{ mm}^3$   
 $= 625 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{|M|}{S} = \frac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$  ◀

**PROBLEM 5.125**



5.125 and 5.126 A beam is being designed to be supported and loaded as shown. (a) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam (b) Knowing that the allowable stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that should be selected.

**SOLUTION**

$$\sum M_c = 0 \quad -15 R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$R_A = 40.5 \text{ kips.}$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3 \rangle^0 \text{ kips}$$

Location of point D where  $V = 0$ . Assume  $3 < x_D < 12$

$$0 = 40.5 - 3x_D - 22.5 \quad x_D = 6 \text{ ft}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3 \rangle^1 \text{ kip}\cdot\text{ft}$$

$$\begin{aligned} \text{At point D } (x = 6 \text{ ft}) \quad M &= (40.5)(6) - (1.5)(6)^2 - (22.5)(3) \\ &= 121.5 \text{ kip}\cdot\text{ft} = 1458 \text{ kip}\cdot\text{in.} \end{aligned}$$

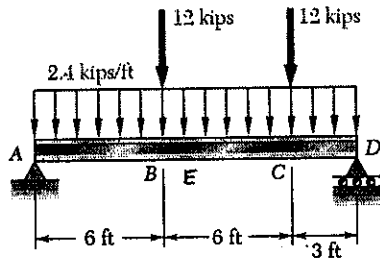
Maximum  $|M| = 121.5 \text{ kip}\cdot\text{ft}$  at  $x = 6 \text{ ft}$ . ▶

$$S_{\min} = \frac{M}{\sigma_{\text{all}}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

Shape	$S$ (in <sup>3</sup> )
W 21 x 44	81.6
W 18 x 50	88.9
W 16 x 40	64.7 <span style="margin-left: 20px;">←</span>
W 14 x 43	62.7
W 12 x 50	64.7
W 10 x 68	75.7

Answer W 16 x 40 ▶

**PROBLEM 5.126**



5.125 and 5.126 A beam is being designed to be supported and loaded as shown. (a) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam (b) Knowing that the allowable stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that should be selected.

**SOLUTION**

$$\sum M_C = 0$$

$$-15 R_A + (7.5)(15)(2.4) - (9)(12) - (3)(12) = 0$$

$$R_A = 27.6 \text{ kips}$$

$$w = 2.4 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 27.6 - 2.4x - 12\langle x-6 \rangle^0 - 12\langle x-12 \rangle^0 \text{ kips}$$

$$V_{B^-} = 27.6 - (2.4)(6) = 13.2 \text{ kips}$$

$$V_{B^+} = 27.6 - (2.4)(6) - 12 = 1.2 \text{ kips}$$

$$V_C^- = 27.6 - (2.4)(12) - 12 = -13.2 \text{ kips}$$

} Point where  $V=0$  lies between B and C.

locate point E where  $V=0$

$$0 = 27.6 - 2.4x_E - 12 - 0 \quad x_E = 6.5 \text{ ft.}$$

$$M = 27.6x - 1.2x^2 - 12\langle x-6 \rangle^1 - 12\langle x-12 \rangle^1 \text{ kip-ft}$$

$$\text{At point E } (x=6.5 \text{ ft}) \quad M = (27.6)(6.5) - (1.2)(6.5)^2 - (12)(0.5) - 0$$

$$= 122.7 \text{ kip-ft} = 1472.4 \text{ kip-in.}$$

Maximum  $|M|$  122.7 kip-ft at  $x = 6.5$  ft. ▶

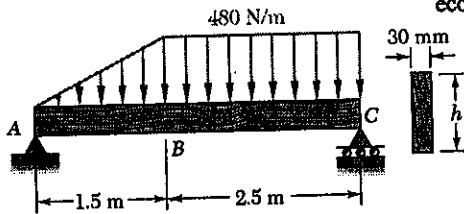
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1472.4}{24} = 61.35 \text{ in}^3$$

Shape	$S$ (in <sup>3</sup> )
W21 x 44	81.6
W18 x 50	88.9
W16 x 40	64.7
W14 x 43	62.7
W12 x 50	64.7
W10 x 68	75.7

Answer: W16 x 40 ▶

PROBLEM 5.127

5.127 and 5.128. A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with a 12-MPa allowable stress and a rectangular cross section of 30-mm width and depth  $h$  varying from 80 to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



SOLUTION

$$480 \text{ N/m} = 0.48 \text{ kN/m}$$

$$+\circlearrowleft \sum M_C = 0$$

$$-4 R_A + (3)(\frac{1}{2})(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$$

$$R_A = 0.645 \text{ kN}$$

$$W = \frac{0.48}{1.5} x - \frac{0.48}{1.5} \langle x - 1.5 \rangle^1 = 0.32 x - 0.32 \langle x - 1.5 \rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16 x^2 + 0.16 \langle x - 1.5 \rangle^2 \text{ kN}$$

Locate point D where  $V = 0$ . Assume  $1.5 \text{ m} < x_D < 4 \text{ m}$

$$0 = 0.645 - 0.16 x_D^2 + 0.16 (x_D - 1.5)^2$$

$$= 0.645 - \cancel{0.16 x_D^2} + \cancel{0.16 x_D^2} - 0.48 x_D + 0.36$$

$$x_D = 2.09375 \text{ m}$$

$$M = 0.645 x - 0.05333 x^3 + 0.05333 \langle x - 1.5 \rangle^3 \text{ kN}\cdot\text{m}$$

At point D  $M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$

$$= 0.87211 \text{ kN}\cdot\text{m}$$

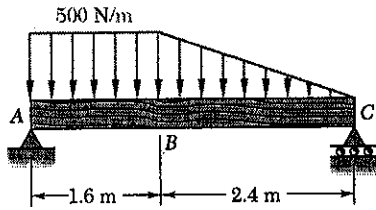
$$S_{min} = \frac{M_D}{\sigma_{all}} = \frac{0.87211 \times 10^3}{12 \times 10^6} = 72.6758 \times 10^{-6} \text{ m}^3 = 72.6758 \times 10^3 \text{ mm}^3$$

For a rectangular cross section  $S = \frac{1}{6} b h^2$   $h = \sqrt{\frac{6S}{b}}$

$$h_{min} = \sqrt{\frac{(6)(72.6758 \times 10^3)}{30}} = 120.56 \text{ mm}$$

At next larger 10-mm increment  $h = 130 \text{ mm}$

PROBLEM 5.128



5.127 and 5.128. A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with a 12-MPa allowable stress and a rectangular cross section of 30-mm width and depth  $h$  varying from 80 to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$500 \text{ N/m} = 0.5 \text{ kN/m}$

$\sum M_C = 0$

$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$

$R_A = 0.880 \text{ kN}$

$w = 0.5 - \frac{0.5}{2.4} \langle x - 1.6 \rangle^1 = 0.5 - 0.20833 \langle x - 1.6 \rangle^1 \text{ kN/m} = -\frac{dv}{dx}$

$V = 0.880 - 0.5x + 0.104167 \langle x - 1.6 \rangle^2 \text{ kN}$

$V_A = 0.880 \text{ kN}$

$V_B = 0.880 - (0.5)(1.6) = 0.080 \text{ kN}$

$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN}$  } Sign change

locate point D where  $V = 0$

$0 = 0.880 - 0.5x_D + 0.104167 (x_D - 1.6)^2$

$0.104167 x_D^2 - 0.83333 x_D + 1.14667 = 0$

$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)}$

$= 4.0 \pm 2.2342 = \cancel{6.2342}, 1.7658 \text{ m}$

$M = 0.880x - 0.25x^2 + 0.347222 \langle x - 1.6 \rangle^3 \text{ kN}\cdot\text{m}$

$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.347222)(0.1658)^3 = 0.776 \text{ kN}\cdot\text{m}$

$M_{max} = 0.776 \text{ kN}\cdot\text{m}$  at  $x = 1.7658 \text{ m}$

$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{0.776 \times 10^3}{12 \times 10^6} = 64.66 \times 10^{-6} \text{ m}^3 = 64.66 \times 10^3 \text{ mm}^3$

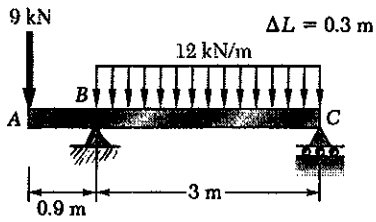
For a rectangular cross section  $S = \frac{1}{6}bh^2$   $h = \frac{6S}{b}$

$h_{min} = \sqrt{\frac{(6)(64.66 \times 10^3)}{30}} = 113.7 \text{ mm}$

At next higher 10-mm increment  $h = 120 \text{ mm}$

**PROBLEM 5.129**

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments  $\Delta L$ , starting at point  $A$  and ending at the right-hand support.



**SOLUTION**

$$+\circlearrowleft M_C = 0 \quad (3.9)(9) - 3R_B + (1.5)(3.0)(12) = 0$$

$$R_B = 29.7 \text{ kN}$$

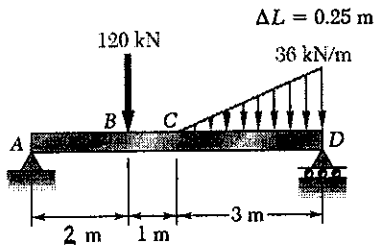
$$W = 12 \langle x - 0.9 \rangle^0$$

$$V = -9 + 29.7 \langle x - 0.9 \rangle^0 + 12 \langle x - 0.9 \rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = -9x + 29.7 \langle x - 0.9 \rangle^1 - 6 \langle x - 0.9 \rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

x m	V kN	M kN·m
0.0	-9.0	0.00
0.3	-9.0	-2.70
0.6	-9.0	-5.40
0.9	20.7	-8.10
1.2	17.1	-2.43
1.5	13.5	2.16
1.8	9.9	5.67
2.1	6.3	8.10
2.4	2.7	9.45
2.7	-0.9	9.72
3.0	-4.5	8.91
3.3	-8.1	7.02
3.6	-11.7	4.05
3.9	-15.3	0.00

**PROBLEM 5.130**



5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments  $\Delta L$ , starting at point  $A$  and ending at the right-hand support.

**SOLUTION**

$$\begin{aligned} \uparrow \sum M_D = 0 \\ -6 R_A + (4)(120) + (1)\left(\frac{1}{2}\right)(3)(36) = 0 \end{aligned}$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3} \langle x-3 \rangle' = 12 \langle x-3 \rangle'$$

$$V = 89 - 120 \langle x-2 \rangle^0 - 6 \langle x-3 \rangle^2 \text{ kN}$$

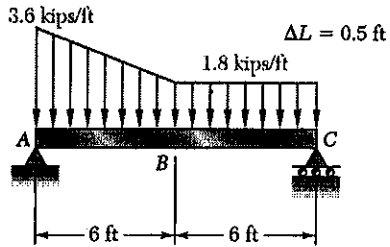
$$M = 89x - 120 \langle x-2 \rangle' - 2 \langle x-3 \rangle^3 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0



**PROBLEM 5.131**

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments  $\Delta L$ , starting at point  $A$  and ending at the right-hand support.



**SOLUTION**

$$\rightarrow \sum M_C = 0$$

$$-12 R_A + (6)(12)(1.8) + (10)(\frac{1}{2})(6)(1.8) = 0$$

$$R_A = 15.3 \text{ kips.}$$

$$W = 3.6 - \frac{1.8}{6}x + \frac{1.8}{6}\langle x-6 \rangle'$$

$$= 3.6 - 0.3x + 0.3\langle x-6 \rangle'$$

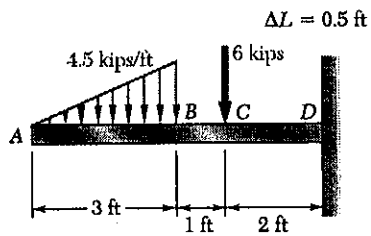
$$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6 \rangle^2 \text{ kips}$$

$$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6 \rangle^3 \text{ kip}\cdot\text{ft}$$

x ft	V kips	M kip·ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	-0.90	37.8
6.5	-1.80	37.1
7.0	-2.70	36.0
7.5	-3.60	34.4
8.0	-4.50	32.4
8.5	-5.40	29.9
9.0	-6.30	27.0
9.5	-7.20	23.6
10.0	-8.10	19.8
10.5	-9.00	15.5
11.0	-9.90	10.8
11.5	-10.80	5.6
12.0	-11.70	0.0

**PROBLEM 5.132**

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments  $\Delta L$ , starting at point  $A$  and ending at the right-hand support.



**SOLUTION**

$$W = \frac{4.5}{3}x - \frac{4.5}{3}\langle x-3 \rangle^1 - 4.5\langle x-3 \rangle^0$$

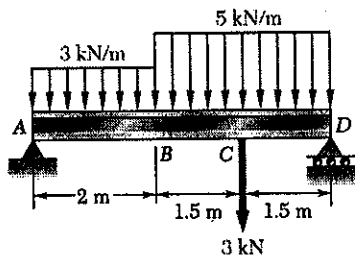
$$= 1.5x - 1.5\langle x-3 \rangle^1 - 4.5\langle x-3 \rangle^0$$

$$V = -0.75x^2 + 0.75\langle x-3 \rangle^2 + 4.5\langle x-3 \rangle^1 - 6\langle x-4 \rangle^0 \quad \text{kips}$$

$$M = -0.25x^3 + 0.25\langle x-3 \rangle^3 + 2.25\langle x-3 \rangle^2 - 6\langle x-4 \rangle^1 \quad \text{kip}\cdot\text{ft}$$

x ft	V kips	M kip·ft
0.0	0.00	0.00
0.5	-0.19	-0.03
1.0	-0.75	-0.25
1.5	-1.69	-0.84
2.0	-3.00	-2.00
2.5	-4.69	-3.91
3.0	-6.75	-6.75
3.5	-6.75	-10.13
4.0	-12.75	-13.50
4.5	-12.75	-19.88
5.0	-12.75	-26.25
5.5	-12.75	-32.63
6.0	-12.75	-39.00

PROBLEM 5.133



W200 x 22.5  
L = 5 m  
 $\Delta L = 0.25$  m

5.133 and 5.134 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from  $x = 0$  to  $x = L$ , using the increments  $\Delta L$  indicated, (b) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.

SOLUTION

$$\uparrow \Sigma M_D = 0$$

$$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$$

$$R_A = 10.2 \text{ kN}$$

$$w = 3 + 2 \langle x-2 \rangle^0 \text{ kN/m} = -\frac{dw}{dx}$$

$$V = 10.2 - 3x - 2 \langle x-2 \rangle^1 - 3 \langle x-3.5 \rangle^0 \text{ kN}$$

$$M = 10.2x - 1.5x^2 - \langle x-2 \rangle^2 - 3 \langle x-3.5 \rangle^1 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3
2.85	-0.05	16.164	83.3

For rolled steel section  
W 200 x 22.5

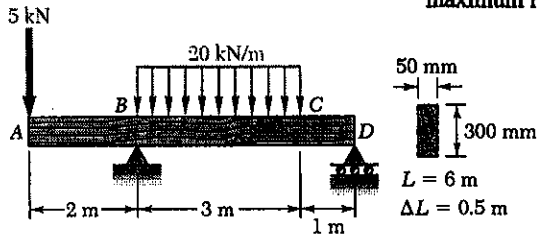
$$S = 194 \times 10^3 \text{ mm}^3$$

$$S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$$

$$\begin{aligned} \sigma_{max} &= \frac{M_{max}}{S} = \frac{16.164 \times 10^3}{194 \times 10^{-6}} \\ &= 83.3 \times 10^6 \text{ Pa} \\ &= 83.3 \text{ MPa} \end{aligned}$$

**PROBLEM 5.134**

5.133 and 5.134 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from  $x = 0$  to  $x = L$ , using the increments  $\Delta L$  indicated, (b) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



**SOLUTION**

$$+\circlearrowleft \sum M_D = 0$$

$$-4R_B + (6)(5) + (2.5)(3)(20) = 0$$

$$R_B = 45 \text{ kN}$$

$$w = 20 \langle x-2 \rangle^0 - 20 \langle x-5 \rangle^0 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = -5 + 45 \langle x-2 \rangle^0 + 20 \langle x-2 \rangle^1 + 20 \langle x-5 \rangle^1 \text{ kN} \quad \blacktriangleleft$$

$$M = -5x + 45 \langle x-2 \rangle^1 - 10 \langle x-2 \rangle^2 + 10 \langle x-5 \rangle^2 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0 ←
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

Maximum  $|M| = 30 \text{ kN}\cdot\text{m}$   
at  $x = 4.0 \text{ m}$

For rectangular cross section

$$S = \frac{1}{6} b h^2 = \left(\frac{1}{6}\right)(50)(300)^2$$

$$= 750 \times 10^3 \text{ mm}^3$$

$$= 750 \times 10^{-6} \text{ m}^3$$

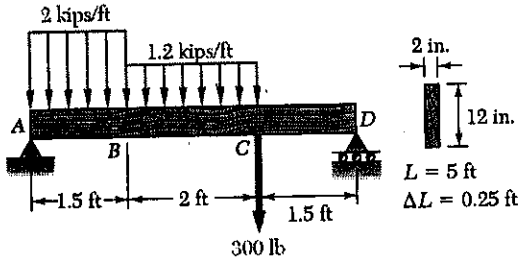
$$\sigma_{\max} = \frac{M_{\max}}{S} = \frac{30 \times 10^3}{750 \times 10^{-6}}$$

$$= 40 \times 10^6 \text{ Pa} = 40 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma = \frac{|M|}{S}$$

**PROBLEM 5.135**

5.135 and 5.136 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from  $x = 0$  to  $x = L$ , using the increments  $\Delta L$  indicated, (b) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



**SOLUTION**

$300 \text{ lb} = 0.3 \text{ kips}$

$+\sum M_D = 0$

$-5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$

$R_A = 3.84 \text{ kips.}$

$w = 2 - 0.8\langle x - 1.5 \rangle' - 1.2\langle x - 3.5 \rangle' \text{ kip/ft}$

$V = 3.84 - 2x + 0.8\langle x - 1.5 \rangle' + 1.2\langle x - 3.5 \rangle' - 0.3\langle x - 3.5 \rangle' \text{ kips}$

$M = 3.84x - x^2 + 0.4\langle x - 1.5 \rangle'^2 + 0.6\langle x - 3.5 \rangle'^2 - 0.3\langle x - 3.5 \rangle' \text{ kip}\cdot\text{ft}$

x ft	V kips	M kip·ft	sigma ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	-0.06	3.80	0.951
2.50	-0.36	3.75	0.937
2.75	-0.66	3.62	0.906
3.00	-0.96	3.42	0.855
3.25	-1.26	3.14	0.786
3.50	-1.86	2.79	0.697
3.75	-1.86	2.32	0.581
4.00	-1.86	1.86	0.465
4.25	-1.86	1.39	0.349
4.50	-1.86	0.93	0.232
4.75	-1.86	0.46	0.116
5.00	-1.86	-0.00	-0.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951 ←
2.30	-0.12	3.80	0.949

Maximum  $|M| = 3.804 \text{ kip}\cdot\text{ft}$   
 $= 45.648 \text{ kip}\cdot\text{in}$   
 at  $x = 2.20 \text{ ft}$

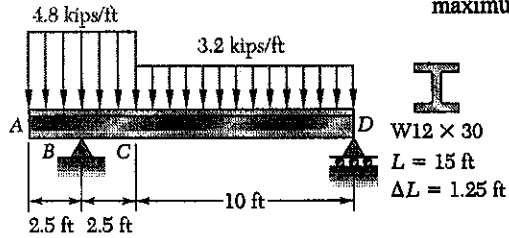
Rectangular section  
 2 in × 12 in.

$S = \frac{1}{6}bh^2 = \frac{1}{6}(2)(12)^2$   
 $= 48 \text{ in}^3$

$\sigma = \frac{M}{S} = \frac{45.648}{48}$   
 $= 0.951 \text{ ksi}$

**PROBLEM 5.136**

5.135 and 5.136 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from  $x = 0$  to  $x = L$ , using the increments  $\Delta L$  indicated, (b) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



**SOLUTION**

$$\rightarrow \sum M_b = 0$$

$$-12.5 R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$$

$$R_B = 36.8 \text{ kips.}$$

$$w = 4.8 - 1.6 \langle x - 5 \rangle^0 \text{ kips/ft}$$

$$V = -4.8x + 36.8 \langle x - 2.5 \rangle^0 + 1.6 \langle x - 5 \rangle^1 \text{ kips}$$

$$M = -2.4x^2 + 36.8 \langle x - 2.5 \rangle^1 + 0.8 \langle x - 5 \rangle^2 \text{ kip}\cdot\text{ft}$$

x ft	V kips	M kip·ft	sigma ksi
0.00	0.0	0.00	0.00
1.25	-6.0	-3.75	-1.17
2.50	24.8	-15.00	-4.66
3.75	18.8	12.25	3.81
5.00	12.8	32.00	9.95
6.25	8.8	45.50	14.15
7.50	4.8	54.00	16.79
8.75	0.8	57.50	17.88
10.00	-3.2	56.00	17.41
11.25	-7.2	49.50	15.39
12.50	-11.2	38.00	11.81
13.75	-15.2	21.50	6.68
15.00	-19.2	0.00	0.00
8.90	0.32	57.58	17.90
9.00	-0.00	57.60	17.91 ←
9.10	-0.32	57.58	17.90

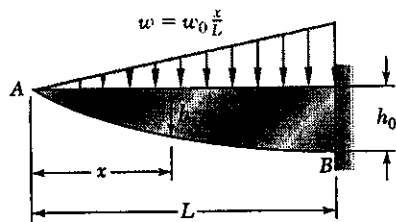
Maximum  $M = 57.6 \text{ kip}\cdot\text{ft}$   
 $= 691.2 \text{ kip}\cdot\text{in}$   
 at  $x = 9.0 \text{ ft.}$

For rolled steel section W12x30  
 $S = 38.6 \text{ in}^3$

Maximum normal stress

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6} = 17.91 \text{ ksi} \leftarrow$$

PROBLEM 5.137



5.137 and 5.138 The cantilever beam  $AB$ , consisting of a cast-iron plate of uniform thickness  $b$  and length  $L$ , is to support the distributed load  $w(x)$  shown. (a) Knowing that the beam is to be of constant strength, express  $h$  in terms of  $x$ ,  $L$ , and  $h_0$ . (b) Determine the smallest allowable value of  $h_0$  if  $L = 750$  mm,  $b = 30$  mm,  $w_0 = 300$  kN/m, and  $\sigma_{all} = 250$  MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section  $S = \frac{1}{6}bh^2$

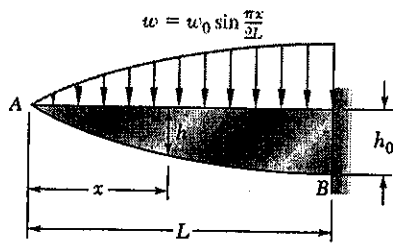
Equating  $\frac{1}{6}bh^2 = \frac{w_0 x^3}{6L\sigma_{all}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} bL}}$

At  $x = L \qquad h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}} \qquad \therefore h = h_0 \left(\frac{x}{L}\right)^{3/2}$

Data:  $L = 750$  mm =  $0.75$  m,  $b = 30$  mm =  $0.030$  m  
 $w_0 = 300$  kN/m =  $300 \times 10^3$  N/m,  $\sigma_{all} = 250$  MPa =  $250 \times 10^6$  Pa

$$h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(250 \times 10^6)(0.030)}} = 150 \times 10^{-3} \text{ m} = 150 \text{ mm}$$

**PROBLEM 5.138**



5.137 and 5.138 The cantilever beam  $AB$ , consisting of a cast-iron plate of uniform thickness  $b$  and length  $L$ , is to support the distributed load  $w(x)$  shown. (a) Knowing that the beam is to be of constant strength, express  $h$  in terms of  $x$ ,  $L$ , and  $h_0$ . (b) Determine the smallest allowable value of  $h_0$  if  $L = 750$  mm,  $b = 30$  mm,  $w_0 = 300$  kN/m, and  $\sigma_{all} = 250$  Mpa.

**SOLUTION**

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{2L}$$

$$V = -\frac{2w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \text{ at } x = 0 \rightarrow C_1 = -\frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi} (1 - \cos \frac{\pi x}{2L})$$

$$M = -\frac{2w_0 L}{\pi} (x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}) \quad |M| = \frac{2w_0 L}{\pi} (x - \frac{2L}{\pi} \sin \frac{\pi x}{2L})$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0 L}{\pi \sigma_{all}} (x - \frac{2L}{\pi} \sin \frac{\pi x}{2L})$$

For a rectangular cross section  $S = \frac{1}{6} b h^2$

Equating  $\frac{1}{6} b h^2 = \frac{2w_0 L}{\pi \sigma_{all}} (x - \frac{2L}{\pi} \sin \frac{\pi x}{2L})$

$$h = \left\{ \frac{12 w_0 L}{\pi \sigma_{all} b} (x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}) \right\}^{1/2}$$

At  $x = L$   $h = h_0 = \left\{ \frac{12 w_0 L^2}{\pi \sigma_{all} b} (1 - \frac{2}{\pi}) \right\}^{1/2} = 1.178 \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

(a)  $h = h_0 \left[ \left( \frac{x}{L} - \frac{2}{\pi} \sin \frac{\pi x}{2L} \right) / \left( 1 - \frac{2}{\pi} \right) \right]^{1/2}$

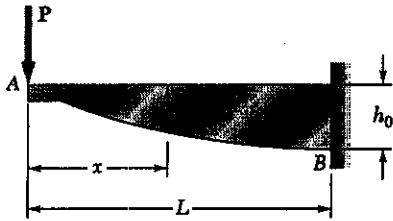
Data:  $L = 750$  mm =  $0.75$  m,  $b = 30$  mm =  $0.030$  m  
 $w_0 = 300$  kN/m =  $300 \times 10^3$  N/m,  $\sigma_{all} = 250$  MPa =  $250 \times 10^6$  Pa

(b)  $h_0 = 1.178 \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(250 \times 10^6)(0.030)}} = 176.7 \times 10^{-3}$  m =  $176.7$  mm

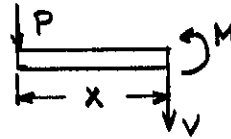


**PROBLEM 5.139**

5.139 and 5.140 The beam  $AB$ , consisting of a cast-iron plate of uniform thickness  $b$  and length  $L$ , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express  $h$  in terms of  $x$ ,  $L$ , and  $h_0$ . (b) Determine the maximum allowable load if  $L = 36$  in.,  $h_0 = 12$  in.,  $b = 1.25$  in., and  $\sigma_{all} = 36$  ksi.



**SOLUTION**



$$V = -P$$

$$M = -Px \quad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P}{\sigma_{all}} x$$

For a rectangular cross section  $S = \frac{1}{6}bh^2$

Equating  $\frac{1}{6}bh^2 = \frac{Px}{\sigma_{all}} \quad h = \left(\frac{6Px}{\sigma_{all}b}\right)^{1/2}$

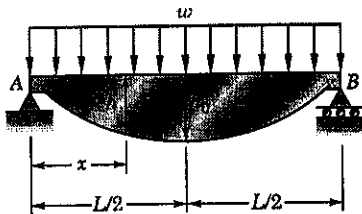
At  $x = L \quad h = h_0 = \left\{\frac{6PL}{\sigma_{all}b}\right\}^{1/2}$

$h = h_0 \frac{x}{L}$

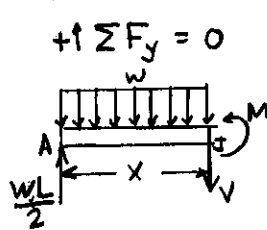
Solving for  $P \quad P = \frac{\sigma_{all}bh_0^2}{6L} = \frac{(36)(1.25)(12)^2}{(6)(36)} = 30 \text{ kips}$

**PROBLEM 5.140**

5.139 and 5.140 The beam  $AB$ , consisting of a cast-iron plate of uniform thickness  $b$  and length  $L$ , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express  $h$  in terms of  $x$ ,  $L$ , and  $h_0$ . (b) Determine the maximum allowable load if  $L = 36$  in.,  $h_0 = 12$  in.,  $b = 1.25$  in., and  $\sigma_{all} = 36$  ksi.



**SOLUTION**



$\uparrow \sum F_y = 0 \quad R_A + R_B - wL = 0 \quad R_A = R_B = \frac{wL}{2}$

$\curvearrowright \sum M_J = 0$   
 $\frac{wL}{2}x - wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}x(L-x)$

$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$

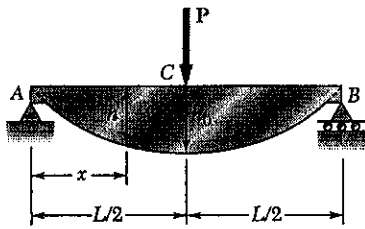
For a rectangular cross section  $S = \frac{1}{6}bh^2$

Equating  $\frac{1}{6}bh^2 = \frac{wx(L-x)}{2\sigma_{all}} \quad h = \left\{\frac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$

At  $x = \frac{L}{2} \quad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all}b}\right\}^{1/2} \quad h = h_0 \left[\frac{x}{L} \left(1 - \frac{x}{L}\right)\right]^{1/2}$

Solving for  $w \quad w = \frac{4\sigma_{all}bh_0^2}{3L^2} = \frac{(4)(36)(1.25)(12)^2}{(3)(36)} = 6.67 \text{ kip/in}$

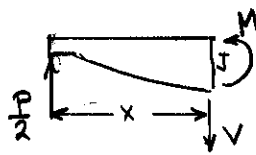
PROBLEM 5.141



5.141 and 5.142 The beam  $AB$ , consisting of a cast-aluminum plate of uniform thickness  $b$  and length  $L$ , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express  $h$  in terms of  $x$ ,  $L$ , and  $h_0$  for portion  $AC$  of the beam. (b) Determine the maximum allowable load if  $L = 800$  mm,  $h_0 = 200$  mm,  $b = 25$  mm, and  $\sigma_{all} = 72$  MPa.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$\begin{aligned} \sum M_J = 0 \\ -\frac{P}{2}x + M = 0 \\ M = \frac{Px}{2} \quad (0 < x < \frac{L}{2}) \end{aligned}$$

$$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$$

For a rectangular cross section  $S = \frac{1}{6}bh^2$

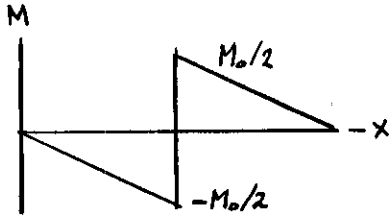
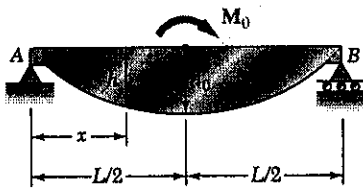
Equating  $\frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \quad h = \sqrt{\frac{3Px}{\sigma_{all}b}}$

(a) At  $x = \frac{L}{2} \quad h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all}b}} \quad h = h_0 \sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$

For  $x > \frac{L}{2}$  replace  $x$  by  $L-x$

(b) Solving for  $P \quad P = \frac{2\sigma_{all}bh_0^2}{3L} = \frac{(2)(72 \times 10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60 \times 10^3 \text{ N} = 60 \text{ kN}$

PROBLEM 5.142

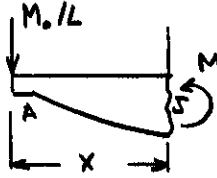


5.141 and 5.142 The beam  $AB$ , consisting of a cast-aluminum plate of uniform thickness  $b$  and length  $L$ , is to support the load shown. (a) Knowing that the beam is to be of constant strength, express  $h$  in terms of  $x$ ,  $L$ , and  $h_0$  for portion  $AC$  of the beam. (b) Determine the maximum allowable load if  $L = 800$  mm,  $h_0 = 200$  mm,  $b = 25$  mm, and  $\sigma_{all} = 72$  Mpa.

SOLUTION

$$R_A = M_0/L \downarrow$$

$$R_B = M_0/L \uparrow$$



$$\sum M_J = 0$$

$$\frac{M_0}{L}x + M = 0$$

$$M = -\frac{M_0 x}{L} \quad (0 < x < \frac{L}{2})$$

$$M = \frac{M_0(L-x)}{L} \quad (\frac{L}{2} < x < L)$$

For  $x > \frac{L}{2}$

For  $x > \frac{L}{2}$  replace  $x$  by  $L-x$ .

$$S = \frac{|M|}{\sigma_{all}} = \frac{M_0 x}{\sigma_{all} L} \quad \text{for } (0 < x < \frac{L}{2})$$

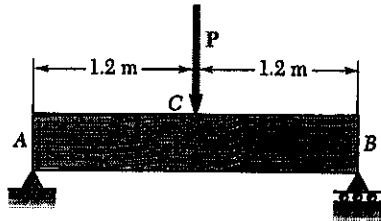
For a rectangular cross section  $S = \frac{1}{6}bh^2$

$$\text{Equating } \frac{1}{6}bh^2 = \frac{M_0 x}{\sigma_{all} L} \quad h = \sqrt{\frac{6M_0 x}{\sigma_{all} bL}}$$

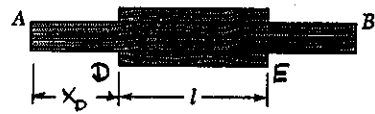
$$\text{At } x = \frac{L}{2} \quad h = h_0 = \sqrt{\frac{3M_0}{\sigma_{all} b}} \quad h = h_0 \sqrt{2x/L}$$

$$\text{Solving for } M_0 \quad M_0 = \frac{\sigma_{all} b h_0^2}{3} = \frac{(72 \times 10^6)(0.025)(0.200)^2}{3} = 24 \times 10^3 \text{ N}\cdot\text{m} = 24 \text{ kN}\cdot\text{m}$$

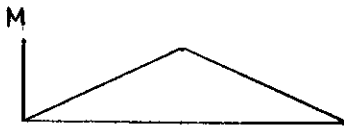
PROBLEM 5.143



(a)



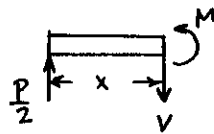
(b)



5.143 and 5.144 A preliminary design based on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part a of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part b of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length  $l$  of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$0 < x < \frac{l}{2}$$

$$\sum \Sigma M_J = 0 \quad -\frac{P}{2}x + M = 0$$

$$M = \frac{Px}{2} \quad \text{or} \quad M = \frac{M_{max}x}{1.2}$$

Bending moment diagram is two straight lines.

At C  $S_c = \frac{1}{6}bh_c^2$

$M_c = M_{max}$

At D  $S_D = \frac{1}{6}bh_D^2$

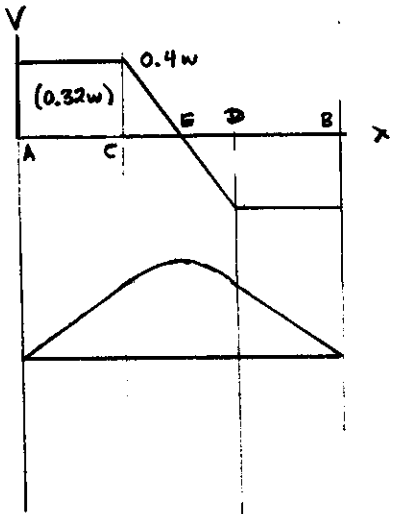
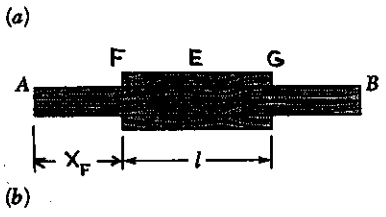
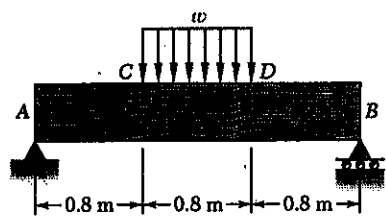
$M_D = \frac{M_{max}x_D}{1.2}$

$$\frac{S_D}{S_c} = \frac{h_D^2}{h_c^2} = \left(\frac{100 \text{ mm}}{200 \text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_c} = \frac{x_D}{1.2}$$

$x_D = 0.3 \text{ m}$

$$\frac{l}{2} = 1.2 - x_D = 0.9 \quad l = 1.800 \text{ m}$$

**PROBLEM 5.144**



5.143 and 5.144 A preliminary design based on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part a of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part b of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length  $l$  of the two outer pieces of timber that will yield the same factor of safety as the original design.

**SOLUTION**

$$R_A = R_B = \frac{0.8w}{2} = 0.4w$$

Shear: A to C  $V = 0.4w$   
 D to B  $V = -0.4w$

Areas: A to C  $(0.8)(0.4)w = 0.32w$   
 C to E  $(\frac{1}{2})(0.4)(0.4)w = 0.08w$

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Bending moments.  
 At C  $M_c = 0.40w$

A to C  $M = 0.40wx$

At C  $S_c = \frac{1}{6}bh_c^2$   $M_c = M_{max} = 0.40w$

At F  $S_F = \frac{1}{6}bh_F^2$   $M_F = 0.40wX_F$

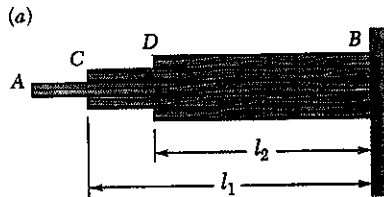
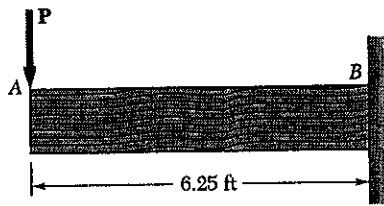
$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_c} = \frac{0.40wX_F}{0.40w}$$

$$X_F = 0.25\text{ m} \quad \frac{l}{2} = 1.2 - X_F = 0.95\text{ m}$$

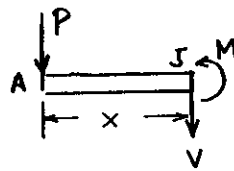
$$l = 1.900\text{ m}$$

**PROBLEM 5.145**

5.145 and 5.146 A preliminary design based on the use of a cantilever prismatic beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 2-in. cross section. Determine the respective lengths  $l_1$  and  $l_2$  of the two inner and two outer pieces of timber that will yield the same factor of safety as the original design.



**SOLUTION**



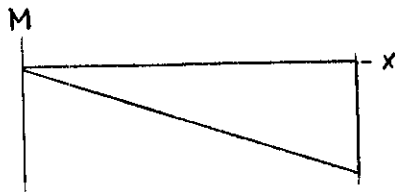
$$\begin{aligned} +\circlearrowleft \sum M_J &= 0 \\ P_x + M &= 0 \quad M = -Px \\ |M| &= Px \end{aligned}$$

At B  $|M|_B = M_{max}$   
 At C  $|M|_C = M_{max} x_c / 6.25$   
 At D  $|M|_D = M_{max} x_D / 6.25$

$$S_B = \frac{1}{6} b h^2 = \frac{1}{6} \cdot b (5b)^2 = \frac{25}{6} b^3$$

$$\text{A to C} \quad S_C = \frac{1}{6} \cdot b (b)^2 = \frac{1}{6} b^3$$

$$\text{C to D} \quad S_D = \frac{1}{6} b (3b)^2 = \frac{9}{6} b^3$$



$$\frac{|M|_C}{|M|_B} = \frac{x_c}{6.25} = \frac{S_C}{S_B} = \frac{1}{25}$$

$$x_c = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

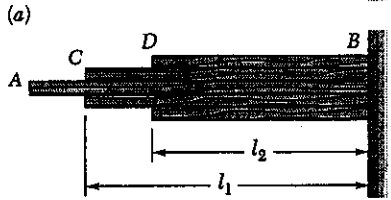
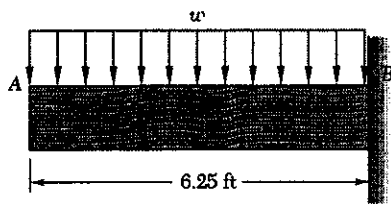
$$l_1 = 6.25 - 0.25 = 6.00 \text{ ft} \quad \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25}$$

$$x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

$$l_2 = 6.25 - 2.25 = 4.00 \text{ ft} \quad \blacktriangleleft$$

**PROBLEM 5.146**

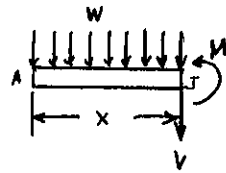


$$\frac{|M|_C}{|M|_B} = \left(\frac{x_c}{6.25}\right)^2 = \frac{S_c}{S_B} = \frac{1}{25}$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25}$$

5.145 and 5.146 A preliminary design based on the use of a cantilever prismatic beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part a of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part b of the figure, five pieces of the same timber as the original beam and of 2 × 2-in. cross section. Determine the respective lengths  $l_1$  and  $l_2$  of the two inner and two outer pieces of timber that will yield the same factor of safety as the original design.

**SOLUTION**



$$\sum M_J = 0 \quad wx \frac{x}{2} + M = 0$$

$$M = -\frac{wx^2}{2} \quad |M| = \frac{wx^2}{2}$$

At B  $|M|_B = |M|_{\min}$

At C  $|M|_C = |M|_{\max} (x_c/6.25)^2$

At D  $|M|_D = |M|_{\max} (x_D/6.25)^2$

At B  $S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C  $S_c = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to D  $S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

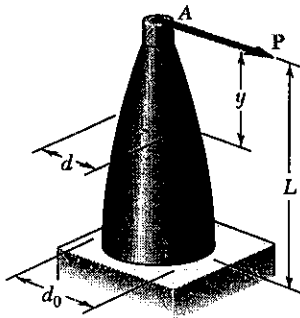
$$x_c = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

$$l_1 = 6.25 - 1.25 \text{ ft} = 5.00 \text{ ft} \quad \blacktriangleleft$$

$$x_D = \frac{6.25 \sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

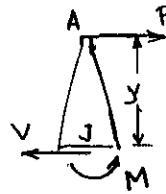
$$l_2 = 6.25 - 3.75 \text{ ft} = 2.50 \text{ ft} \quad \blacktriangleleft$$

**PROBLEM 5.147**



5.147 A cantilevered machine element of cast aluminum and in the shape of a solid of revolution of variable diameter  $d$  is being designed to support a horizontal concentrated load  $P$  as shown. (a) Knowing that the machine element is to be of constant strength, express  $d$  in terms of  $y$ ,  $L$ , and  $d_0$ . (b) Determine the maximum allowable value of  $P$  if  $L = 300$  mm,  $d_0 = 60$  mm, and  $\sigma_{all} = 72$  Mpa.

**SOLUTION**



$$\sum M_J = 0 \quad M - Py = 0$$

$$M = Py$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{Py}{\sigma_{all}}$$

For a solid circular cross section  $c = \frac{d}{2} \quad I = \frac{\pi}{4} c^4$

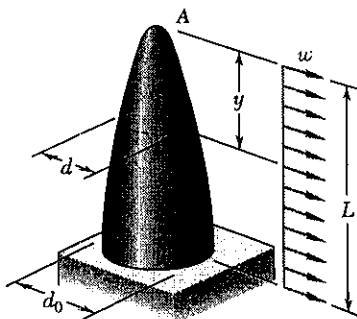
$$S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi d^3}{32}$$

Equating  $\frac{\pi d^3}{32} = \frac{Py}{\sigma_{all}} \quad d = \left( \frac{32 Py}{\pi \sigma_{all}} \right)^{1/3}$

At  $y = L \quad d = d_0 = \left( \frac{32 PL}{\pi \sigma_{all}} \right)^{1/3} \quad \frac{d}{d_0} = \left( \frac{y}{L} \right)^{1/3}$

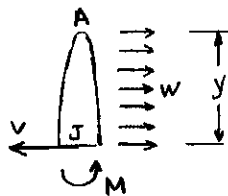
Solving for  $P \quad P = \frac{\pi d_0^3 \sigma_{all}}{32 L} = \frac{\pi (0.060)^3 (72 \times 10^6)}{(32)(0.300)} = 5.09 \times 10^3 \text{ N} = 5.09 \text{ kN}$

**PROBLEM 5.148**



5.148 A cantilevered machine element of cast aluminum and in the shape of a solid of revolution of variable diameter  $d$  is being designed to support a horizontal distributed load  $w$  as shown. (a) Knowing that the machine element is to be of constant strength, express  $d$  in terms of  $y$ ,  $L$ , and  $d_0$ . (b) Determine the smallest allowable value of  $d_0$  if  $L = 300$  mm,  $w = 20$  kN/m, and  $\sigma_{all} = 72$  MPa.

**SOLUTION**



$$\sum M_J = 0$$

$$M - \frac{y}{2} wy = 0 \quad M = \frac{wy^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wy^2}{2\sigma_{all}}$$

For a solid circular cross section  $c = \frac{d}{2}$

$$I = \frac{\pi}{4} c^4 \quad S = \frac{I}{c} = \frac{\pi c^3}{4} = \frac{\pi d^3}{32}$$

Equating  $\frac{\pi d^3}{32} = \frac{wy^2}{2\sigma_{all}} \quad d = \left( \frac{16 wy^2}{\pi \sigma_{all}} \right)^{1/3}$

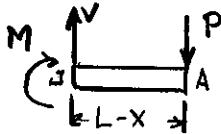
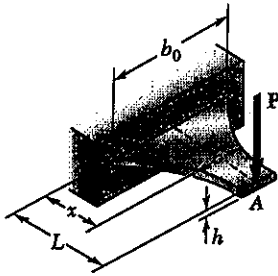
At  $x = L \quad d = d_0 = \left( \frac{16 w L^2}{\pi \sigma_{all}} \right)^{1/3} \quad d = d_0 \left( \frac{y}{L} \right)^{2/3}$

Using the data  $d_0 = \left\{ \frac{(16)(20 \times 10^3)(0.300)^2}{\pi (72 \times 10^6)} \right\}^{1/3} = 50.3 \times 10^{-3} \text{ m} = 50.3 \text{ mm}$



**PROBLEM 5.149**

5.149 A cantilever beam  $AB$  consisting of a steel plate of uniform depth  $h$  and variable width  $b$  is to support a concentrated load  $P$  at point  $A$ . (a) Knowing that the beam is to be of constant strength, express  $b$  in terms of  $x$ ,  $L$ , and  $b_0$ . (b) Determine the smallest allowable value of  $h$  if  $L = 12$  in.,  $b_0 = 15$  in.,  $P = 3.2$  kips, and  $\sigma_{all} = 24$  ksi.



**SOLUTION**

$$\circlearrowleft \sum M_B = 0 \quad -M - P(L-x) = 0 \quad M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P(L-x)}{\sigma_{all}}$$

For a rectangular cross section  $S = \frac{1}{6}bh^2$

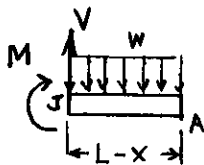
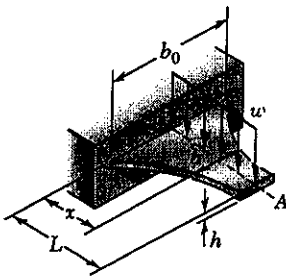
Equating  $\frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{all}} \quad b = \frac{6P(L-x)}{\sigma_{all}h^2}$

At  $x = 0 \quad b = b_0 = \frac{6PL}{\sigma_{all}h^2} \quad b = b_0 \left(1 - \frac{x}{L}\right)$

Solving for  $h \quad h = \sqrt{\frac{6PL}{\sigma_{all}b_0}} = \sqrt{\frac{(6)(3.2)(12)}{(24)(15)}} = 0.800$  in

**PROBLEM 5.150**

5.150 A cantilever beam  $AB$  consisting of a steel plate of uniform depth  $h$  and variable width  $b$  is to support a distributed load  $w$  along its center line  $AB$ . (a) Knowing that the beam is to be of constant strength, express  $b$  in terms of  $x$ ,  $L$ , and  $b_0$ . (b) Determine the maximum allowable value of  $w$  if  $L = 15$  in.,  $b_0 = 18$  in.,  $h = 0.75$  in., and  $\sigma_{all} = 24$  ksi



**SOLUTION**

$$\circlearrowleft \sum M_B = 0 \quad -M - w(L-x) \frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \quad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w(L-x)^2}{2\sigma_{all}}$$

For a rectangular cross section  $S = \frac{1}{6}bh^2$

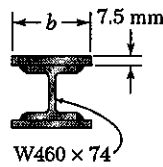
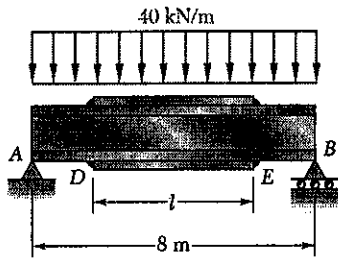
$$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{all}} \quad b = \frac{3w(L-x)^2}{\sigma_{all}h^2}$$

At  $x = 0 \quad b = b_0 = \frac{3wL^2}{\sigma_{all}h^2} \quad b = b_0 \left(1 - \frac{x}{L}\right)^2$

Solving for  $w \quad w = \frac{\sigma_{all}b_0h^2}{3L^2} = \frac{(24)(18)(0.75)^2}{(3)(15)^2} = 0.360$  kip/in  
 $= 360$  lb/in

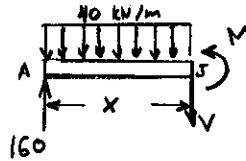
**PROBLEM 5.151**

5.151 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that  $l = 5$  m and  $b = 200$  mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



**SOLUTION**

$$R_A = R_B = 160 \text{ kN}$$



$$+\circlearrowleft \sum M_f = 0$$

$$-160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$

At center of beam  $x = 4\text{m}$   $M_c = 320 \text{ kN}\cdot\text{m}$

At D  $x = \frac{1}{2}(8-l) = 1.5\text{m}$   $M_D = 195 \text{ kN}\cdot\text{m}$

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At center of beam

$$I = I_{\text{beam}} + 2I_{\text{plate}}$$

$$= 333 \times 10^6 + 2 \left\{ (200)(7.5) \left( \frac{457}{2} + \frac{7.5}{2} \right)^2 + \frac{1}{12} (200)(7.5)^3 \right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

$$S = \frac{I}{c} = \frac{2097 \times 10^3 \text{ mm}^3}{236} = 2097 \times 10^3 \text{ mm}^3$$

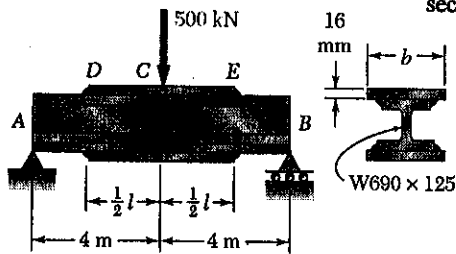
Normal stress  $\sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^3} = 152.6 \times 10^6 \text{ Pa} = 152.6 \text{ MPa}$   $\blacktriangleleft$

At D  $S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^3 \text{ mm}^3$

Normal stress  $\sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^3} = 133.6 \times 10^6 \text{ Pa} = 133.6 \text{ MPa}$   $\blacktriangleleft$

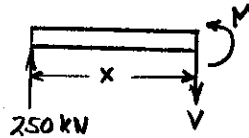
**PROBLEM 5.152**

5.152 Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively,  $l = 4$  m and  $b = 285$  mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



**SOLUTION**

$$R_A = R_B = 250 \text{ kN}$$



$$\sum M_J = 0$$

$$-250x + M = 0$$

$$M = 250x \text{ kN}\cdot\text{m}$$

At center of beam  $x = 4$  m  $M_c = (250)(4) = 1000 \text{ kN}\cdot\text{m}$

At D  $x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2$  m  $M_D = 500 \text{ kN}\cdot\text{m}$

At center of beam  $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$= 1190 \times 10^6 + 2 \left\{ (285)(16) \left( \frac{678}{2} + \frac{16}{2} \right)^2 + \frac{1}{12} (285)(16)^3 \right\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

$$c = \frac{678}{2} + 16 = 355 \text{ mm} \quad S = \frac{I}{c} = \frac{2288 \times 10^6}{355} = 6445 \times 10^3 \text{ mm}^3$$

$$= 6445 \times 10^{-6} \text{ m}^3$$

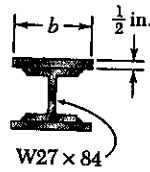
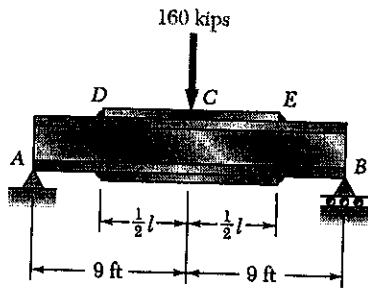
Normal stress  $\sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6 \text{ Pa} = 155.2 \text{ MPa}$   $\blacktriangleleft$

At D  $S = 3510 \times 10^3 \text{ mm}^3 = 3510 \times 10^{-6} \text{ m}^3$

Normal stress  $\sigma = \frac{M}{S} = \frac{500 \times 10^3}{3510 \times 10^{-6}} = 142.4 \times 10^6 \text{ Pa} = 142.4 \text{ MPa}$   $\blacktriangleleft$

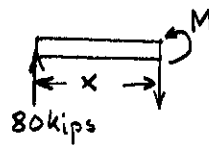
**PROBLEM 5.153**

5.153 Two cover plates, each  $\frac{1}{2}$ -in. thick, are welded to a W27  $\times$  84 beam as shown. Knowing that  $l = 10$  ft and  $b = 10.5$  in., determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



**SOLUTION**

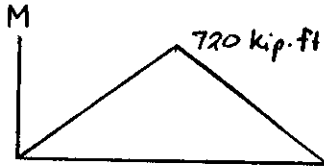
$$R_A = R_B = 80 \text{ kips}$$



$$+\circlearrowleft \sum M_y = 0$$

$$-80x + M = 0$$

$$M = 80x \text{ kip-ft}$$



At C  $x = 9$  ft  $M_C = 720 \text{ kip-ft} = 8640 \text{ kip-in}$

At D  $x = 9 - 5 = 4$  ft

$$M_D = (80)(4) = 320 \text{ kip-ft} = 3840 \text{ kip-in}$$

At center of beam

$$I = I_{beam} + 2 I_{plate}$$

$$I = 2850 + 2 \left\{ (10.5)(0.500) \left( \frac{26.71}{2} + \frac{0.500}{2} \right)^2 + \frac{1}{12} (10.5)(0.500)^3 \right\}$$

$$= 4794 \text{ in}^4$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

Normal stress  $\sigma = \frac{Mc}{I} = \frac{(8640)(13.855)}{4794} = 25.0 \text{ ksi}$

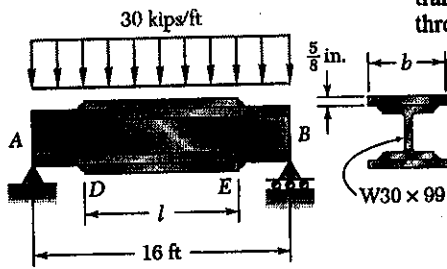
At point D

$$S = 213 \text{ in}^3$$

Normal stress  $\sigma = \frac{M}{S} = \frac{3840}{213} = 18.03 \text{ ksi}$

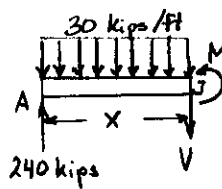
PROBLEM 5.154

5.154 Two cover plates, each  $\frac{5}{8}$ -in. thick, are welded to a W30 × 99 beam as shown. Knowing that  $l = 9$  ft and  $b = 12$  in., determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



SOLUTION

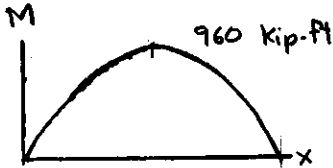
$$R_A = R_B = 240 \text{ kips}$$



$$\sum M_J = 0$$

$$-240x + 30x \frac{x}{2} + M = 0$$

$$M = 240x - 15x^2 \text{ kip}\cdot\text{ft}$$



At center of beam  $x = 8$  ft

$$M_c = 960 \text{ kip}\cdot\text{ft} = 11520 \text{ kip}\cdot\text{in}$$

At point D,  $x = \frac{1}{2}(16-9) = 3.5$  ft

$$M_D = 656.25 \text{ kip}\cdot\text{ft} = 7875 \text{ kip}\cdot\text{in}$$

At center of beam  $I = I_{\text{beam}} + 2I_{\text{plate}}$

$$I = 3990 + 2 \left\{ (12)(0.625) \left( \frac{29.65}{2} + \frac{0.625}{2} \right)^2 + \frac{1}{12} (12)(0.625)^3 \right\} = 7428 \text{ in}^4$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

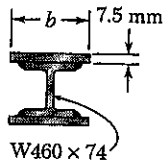
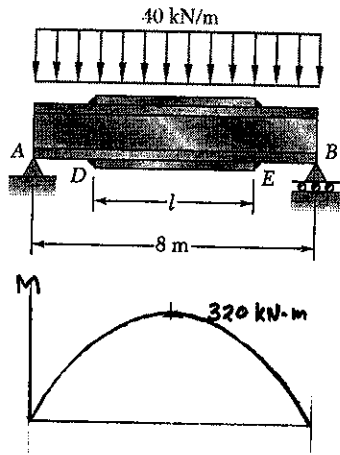
Normal stress  $\sigma = \frac{M_c}{I} = \frac{(11520)(15.45)}{7428} = 24.0 \text{ ksi}$

At point D  $S = 269 \text{ in}^3$

Normal stress  $\sigma = \frac{M}{S} = \frac{7875}{269} = 29.3 \text{ ksi}$

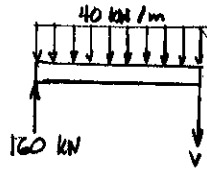
PROBLEM 5.155

5.155 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that  $\sigma_{all} = 150$  MPa for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.



SOLUTION

$$R_A = R_B = 160 \text{ kN}$$



$$+\circlearrowleft \sum M_J = 0$$

$$-160x + (40x)\left(\frac{x}{2}\right) + M = 0$$

$$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$$

For W 460 × 74 rolled steel beam

$$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$$

Allowable bending moment

$$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$$

$$= 219 \times 10^3 \text{ N}\cdot\text{m} = 219 \text{ kN}\cdot\text{m}$$

To locate points D and E, set  $M = M_{all}$

$$160x - 20x^2 = 219$$

$$20x^2 - 160x + 219 = 0$$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} = \begin{matrix} 1.753 \text{ m} \\ 6.247 \text{ m} \end{matrix}$$

$$x_D = 1.753 \text{ m} \quad x_E = 6.247 \text{ m} \quad l = x_E - x_D = 4.49 \text{ m}$$

At center of beam  $M = 320 \text{ kN}\cdot\text{m} = 320 \times 10^3 \text{ N}\cdot\text{m}$

$$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{ m}^3 = 2133 \times 10^3 \text{ mm}^3$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm}$$

Required moment of inertia  $I = Sc = 503.4 \times 10^6 \text{ mm}^4$

But  $I = I_{beam} + 2I_{plate}$

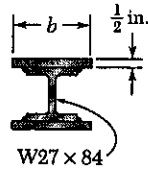
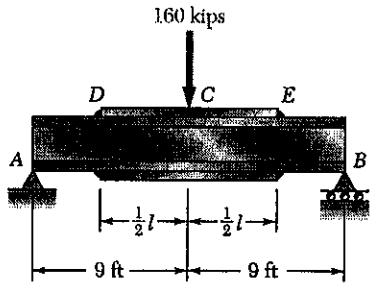
$$503.4 \times 10^6 = 333 \times 10^6 + 2 \left\{ (b)(7.5) \left( \frac{457}{2} + \frac{7.5}{2} \right)^2 + \frac{1}{12} (b)(7.5)^3 \right\}$$

$$= 333 \times 10^6 + 809.2 \times 10^3 b$$

$$b = 211 \text{ mm}$$

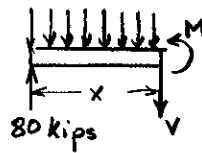
**PROBLEM 5.156**

5.156 Two cover plates, each  $\frac{1}{2}$ -in. thick, are welded to a W27 x 84 beam as shown. Knowing that  $\sigma_{all} = 24$  ksi for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.

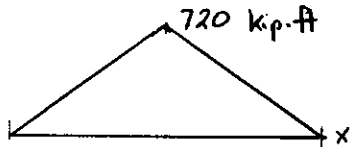


**SOLUTION**

$$R_A = R_B = 80 \text{ kips}$$



$$\begin{aligned} \sum M_J = 0 \\ -80x + M = 0 \\ M = 80x \text{ kip}\cdot\text{ft} \end{aligned}$$



At D  $S = 213 \text{ in}^3$

Allowable bending moment

$$\begin{aligned} M_{all} = \sigma_{all} S &= (24)(213) = 5112 \text{ kip}\cdot\text{in} \\ &= 426 \text{ kip} \end{aligned}$$

Set  $M_D = M_{all}$

$$80x_0 = 426 \quad x_0 = 5.325 \text{ ft.}$$

$$l = 18 - 2x_0 = 7.35 \text{ ft}$$

At center of beam

$$M = (80)(9) = 720 \text{ kip}\cdot\text{ft} = 8640 \text{ kip}\cdot\text{in.}$$

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

Required moment of inertia

$$I = Sc = 4987.8 \text{ in}^4$$

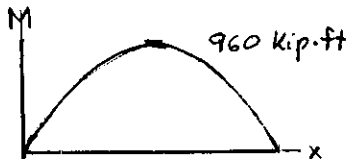
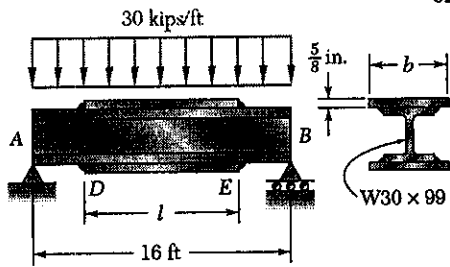
But  $I = I_{beam} + 2I_{plate}$

$$\begin{aligned} 4987.8 &= 2850 + 2 \left\{ (b)(0.500) \left( \frac{26.71}{2} + \frac{0.500}{2} \right)^2 + \frac{1}{12} (b)(0.500)^3 \right\} \\ &= 2850 + 185.12 b \end{aligned}$$

$$b = 11.55 \text{ in.}$$

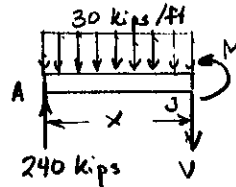
PROBLEM 5.157

5.157 Two cover plates, each  $\frac{5}{8}$ -in. thick, are welded to a W30 x 99 beam as shown. Knowing that  $\sigma_{all} = 22$  ksi for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.



SOLUTION

$$R_A = R_B = 240 \text{ kips}$$



$$\begin{aligned} \sum M_J = 0 \\ -240x + 30 \times \frac{x}{2} + M = 0 \\ M = 240x - 15x^2 \text{ kip-ft} \end{aligned}$$

For W 30 x 99 rolled steel section

$$S = 269 \text{ in}^3$$

Allowable bending moment

$$\begin{aligned} M_{all} &= \sigma_{all} S = (22)(269) = 5918 \text{ kip-in} \\ &= 493.167 \text{ kip-ft} \end{aligned}$$

To locate points D and E, set  $M = M_{all}$

$$240x - 15x^2 = 493.167 \quad 15x^2 - 240x + 493.167 = 0$$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, 13.58 \text{ ft}$$

$$l = x_E - x_D = 13.58 - 2.42 = 11.16 \text{ ft}$$

Center of beam  $M = 960 \text{ kip-ft} = 11520 \text{ kip-in}$

$$S = \frac{M}{\sigma_{all}} = \frac{11520}{22} = 523.64 \text{ in}^3$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

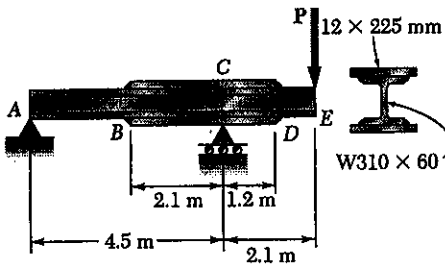
Required moment of inertia  $I = Sc = 8090 \text{ in}^4$

But  $I = I_{beam} + 2 I_{plate}$

$$\begin{aligned} 8090 &= 3990 + 2 \left\{ (b)(0.625) \left( \frac{29.65}{2} + \frac{0.625}{2} \right)^2 + \frac{1}{12} (b)(0.625)^3 \right\} \\ &= 3990 + 286.47 b \end{aligned}$$

$$b = 14.31 \text{ in}$$





SOLUTION

$$\rightarrow \sum M_E = 0 \quad -4.5 R_A - 2.1 P = 0$$

$$R_A = -0.46667 P \quad \text{ie } 0.46667 P \downarrow$$

$$+\circlearrowleft \sum M_A = 0 \quad 4.5 R_D - 6.6 P = 0$$

$$R_D = 1.46667 P$$

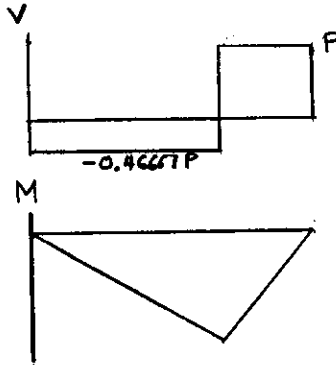
Shear: A to C  $V = -0.46667 P$   
 C to E  $V = P$

Bending moments:

$$M_C = -(4.5)(0.46667 P) = -2.10 P \text{ KN}\cdot\text{m}$$

$$M_B = \frac{2.4}{4.5} M_C = -1.12 P \text{ KN}\cdot\text{m}$$

$$M_D = \frac{0.9}{2.1} M_C = -0.9 P \text{ KN}\cdot\text{m}$$



At B and D  $S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

$$\sigma_{all} = \frac{|M|}{S} = \frac{1.120 P_{all}}{851 \times 10^{-6}} = 165 \times 10^6 \quad \text{at B}$$

$$P_{all} = 125.4 \text{ KN}$$

At C  $I = I_{beam} + 2 I_{plate}$

$$= 129 \times 10^6 + 2 \left\{ (225)(12) \left( \frac{310}{2} + \frac{12}{2} \right)^2 + \frac{1}{12} (225)(12)^3 \right\}$$

$$= 269 \times 10^6 \text{ mm}^4$$

$$c = \frac{310}{2} + 12 = 167 \text{ mm}$$

$$S = \frac{I}{c} = \frac{1611 \times 10^3 \text{ mm}^4}{167 \text{ mm}}$$

$$= 1611 \times 10^{-6} \text{ m}^3$$

$$\sigma_{all} = \frac{|M|}{S} = \frac{2.10 P}{1611 \times 10^{-6}} = 165 \times 10^6$$

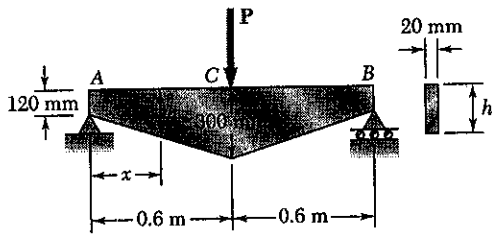
$$P_{all} = 126.6 \text{ KN}$$

Allowable load is the smaller value

$$P = 125.4 \text{ KN}$$

PROBLEM 5.159

5.159 For the tapered beam shown, and knowing that  $P = 150 \text{ kN}$ , determine (a) the transverse section in which the maximum normal stress occurs, (b) the corresponding value of the normal stress.



SOLUTION

$$R_A = R_B = \frac{P}{2}$$

$$\sum M_j = 0 \quad -\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \quad (0 < x < \frac{1}{2})$$

For a tapered rectangular beam  $h = a + kx \quad (0 < x < \frac{1}{2})$

$$S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$$

Bending stress  $\sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set  $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b} \frac{d}{dx} \left\{ \frac{x}{(a+kx)^2} \right\} = \frac{3P}{b} \frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b} \cdot \frac{a - kx}{(a+kx)^3} = 0 \quad x_m = \frac{a}{k}$$

Data:  $a = 120 \text{ mm}$ ,  $k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$

$$x_m = \frac{120}{300} = 0.400 \text{ m}$$

$$M_m = \frac{Px_m}{2} = \frac{(150)(0.400)}{2} = 30 \text{ kN}\cdot\text{m}$$

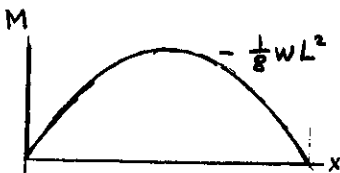
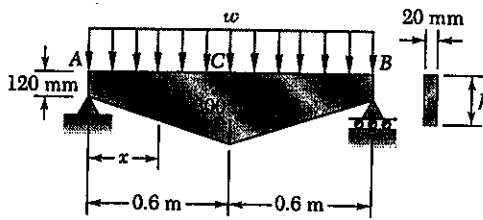
$$h_m = a + kx_m = 120 + (300)(0.400) = 240 \text{ mm}$$

$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)(20)(240)^2 = 192 \times 10^3 \text{ mm}^3 = 192 \times 10^{-6} \text{ m}^3$$

$$\sigma_m = \frac{M_m}{S_m} = \frac{30 \times 10^3}{192 \times 10^{-6}} = 156.3 \times 10^6 \text{ Pa} = 156.3 \text{ MPa}$$

PROBLEM 5.160

5.160 For the tapered beam shown, and knowing that  $w = 160 \text{ kN/m}$ , determine (a) the transverse section in which the maximum normal stress occurs, (b) the corresponding value of the normal stress.



SOLUTION

$$R_A = R_B = \frac{1}{2} wL$$

$$\begin{aligned} \sum M_J = 0 \\ -\frac{1}{2} wLx + wx \frac{x}{2} + M = 0 \\ M = \frac{w}{2} (Lx - x^2) \\ = \frac{w}{2} x (L - x) \end{aligned}$$

where  $w = 160 \text{ kN/m}$  and  $L = 1.2 \text{ m}$ .

For the tapered beam  $h = a + kx$

$$a = 120 \text{ mm} \quad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For a rectangular cross section  $S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

$$\text{Bending stress } \sigma = \frac{M}{S} = \frac{3w}{b} \frac{Lx - x^2}{(a + kx)^2}$$

To find location of maximum bending stress set  $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a + kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a + kx)^2 (L - 2x) - (Lx - x^2) 2(a + kx)k}{(a + kx)^4} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL + 2ax - kLx}{(a + kx)^3} \right\} = 0$$

$$x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

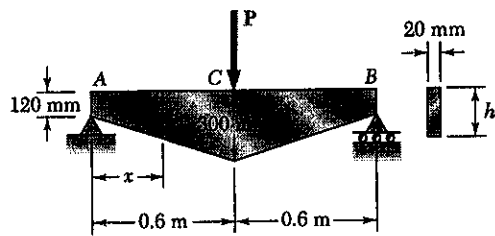
$$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6} (20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

$$M_m = \frac{w}{2} x_m (L - x_m) = \frac{160 \times 10^3}{2} (0.24)(0.96) = 18.432 \times 10^3 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \text{Maximum bending stress } \sigma_m &= \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6 \text{ Pa} \\ &= 150 \text{ MPa} \end{aligned}$$

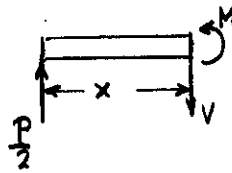
PROBLEM 5.161

5.161 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest concentrated load  $P$  that can be applied, knowing that  $\sigma_{all} = 140 \text{ MPa}$ .



SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$\sum M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \quad (0 < x < \frac{L}{2})$$

For a tapered beam  $h = a + kx$

For rectangular cross section  $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress  $\sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set  $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b} \frac{d}{dx} \left\{ \frac{x}{(a+kx)^2} \right\} = \frac{3P}{b} \cdot \frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b} \frac{a-kx}{(a+kx)^3} = 0 \quad x_m = \frac{a}{k}$$

Then  $M_m = \frac{Px_m}{2} = \frac{Pa}{2k}$

$$h_m = a + kx_m = 2a$$

$$S_m = \frac{1}{6}bh_m^2 = \frac{2}{3}ba^2$$

Data:  $a = 120 \text{ mm}$   $k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$   $b = 20 \text{ mm}$

$$x_m = \frac{120 \text{ mm}}{300 \text{ mm/m}} = 0.400 \text{ m}$$

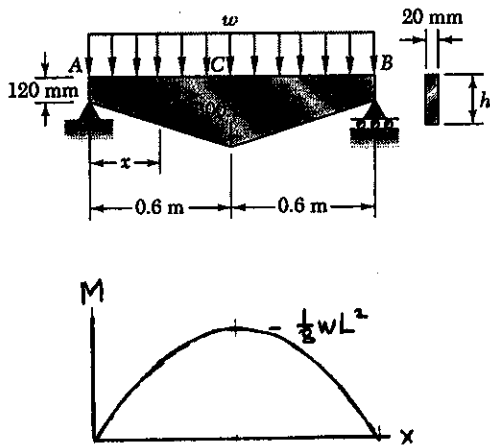
$$S_m = \frac{2}{3}(20)(120)^2 = 192 \times 10^3 \text{ mm}^3 = 192 \times 10^{-6} \text{ m}^3$$

$$M_{mx} = \sigma_{all} S_m = (140 \times 10^6)(192 \times 10^{-6}) = 26.88 \times 10^3 \text{ N}\cdot\text{m}$$

$$P = \frac{2M_m}{x_m} = \frac{(2)(26.88 \times 10^3)}{0.400} = 134.4 \times 10^3 \text{ N} = 134.4 \text{ kN}$$

**PROBLEM 5.162**

5.162 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest distributed load  $w$  that can be applied, knowing that  $\sigma_{all} = 140 \text{ MPa}$ .



**SOLUTION**

$$R_A = R_B = \frac{1}{2} wL \quad L = 1.2 \text{ m}$$

$$\begin{aligned} \sum M_J = 0 \\ -\frac{1}{2} wL + wx \frac{x}{2} + M = 0 \\ M = \frac{w}{2} (Lx - x^2) \\ = \frac{w}{2} x(L-x) \end{aligned}$$

For the tapered beam  $h = a + kx$

$$a = 120 \text{ mm} \quad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$$

For rectangular cross section  $S = \frac{1}{6} bh^2 = \frac{1}{6} b(a + kx)^2$

Bending stress  $\sigma = \frac{M}{S} = \frac{3w}{b} \frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress set  $\frac{d\sigma}{dx} = 0$

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a + kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a + kx)^2 (L - 2x) - (Lx - x^2) 2(a + kx)k}{(a + kx)^3} \right\} \\ &= \frac{3w}{b} \left\{ \frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^2} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^2} \right\} \\ &= \frac{3w}{b} \left\{ \frac{aL - (2a + kL)x}{(a + kx)^2} \right\} = 0 \end{aligned}$$

$$x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

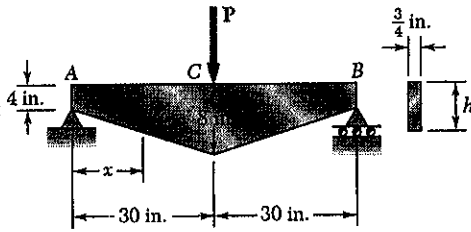
$$S_m = \frac{1}{6} b h_m^2 = \frac{1}{6} (20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

Allowable value of  $M_m$   $M_m = S_m \sigma_{all} = (122.88 \times 10^{-6})(140 \times 10^6)$   
 $= 17.2032 \times 10^3 \text{ N}\cdot\text{m}$

Allowable value of  $w$   $w = \frac{2M_m}{x_m(L - x_m)} = \frac{(2)(17.2032 \times 10^3)}{(0.24)(0.96)}$   
 $= 149.3 \times 10^3 \text{ N/m} = 149.3 \text{ kN/m}$

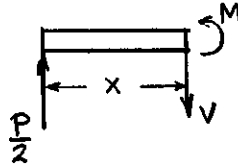
PROBLEM 5.163

5.163 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest concentrated load  $P$  that can be applied, knowing that  $\sigma_{all} = 24$  ksi.



SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$\sum M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \quad (0 < x < \frac{L}{2})$$

For a tapered beam  $h = a + kx$

For a rectangular cross section  $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress  $\sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set  $\frac{d\sigma}{dx} = 0$

$$\begin{aligned} \frac{d\sigma}{dx} &= \frac{3P}{b} \frac{d}{dx} \left\{ \frac{x}{(a+kx)^2} \right\} = \frac{3P}{b} \frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4} \\ &= \frac{3P}{b} \frac{a - kx}{(a+kx)^3} = 0 \quad x_m = \frac{a}{k} \end{aligned}$$

Data:  $a = 4$  in.,  $k = \frac{8-4}{30} = 0.13333$  in/in

$$x_m = \frac{4}{0.13333} = 30 \text{ in.}$$

$$h_m = a + kx_m = 8 \text{ in}$$

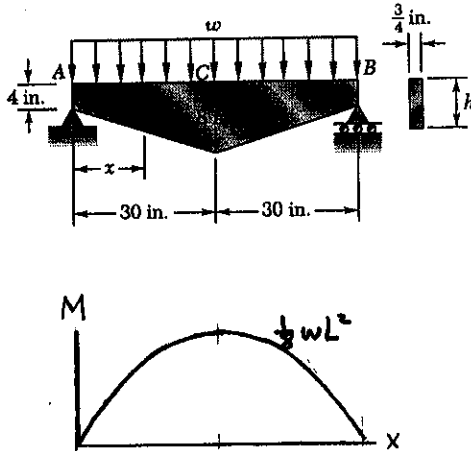
$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = \sigma_{all} S_m = (24)(8) = 192 \text{ kip}\cdot\text{in}$$

$$P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8 \text{ kips}$$

PROBLEM 5.164

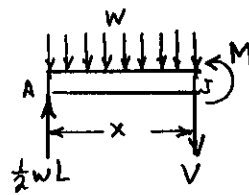
5.164 For the tapered beam shown, determine (a) the transverse section in which the maximum normal stress occurs, (b) the largest distributed load  $w$  that can be applied, knowing that  $\sigma_{all} = 24 \text{ ksi}$



SOLUTION

$$R_A = R_B = \frac{1}{2} wL$$

$$L = 60 \text{ in.}$$



$$\sum M_f = 0$$

$$-\frac{1}{2} wLx + wx \frac{x}{2} + M = 0$$

$$M = \frac{w}{2} (Lx - x^2)$$

$$= \frac{w}{2} x(L-x)$$

For the tapered beam  $h = a + kx$

$$a = 4 \text{ in.} \quad k = \frac{8-4}{30} = \frac{2}{15} \text{ in./in.}$$

For a rectangular cross section  $S = \frac{1}{6} bh^2 = \frac{1}{6} b(a+kx)^2$

$$\text{Bending stress} \quad \sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a+kx)^2}$$

To find location of maximum bending stress set  $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b} \frac{d}{dx} \left\{ \frac{Lx - x^2}{(a+kx)^2} \right\} = \frac{3w}{b} \left\{ \frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3} \right\}$$

$$= \frac{3w}{b} \left\{ \frac{aL - (2a+kL)x}{(a+kx)^3} \right\} = 0$$

$$x_m = \frac{aL}{2a+kL} = \frac{(4)(60)}{(2)(4) + (\frac{2}{15})(60)} = 15 \text{ in.}$$

$$h_m = a + kx_m = 4 + (\frac{2}{15})(15) = 6.00 \text{ in.}$$

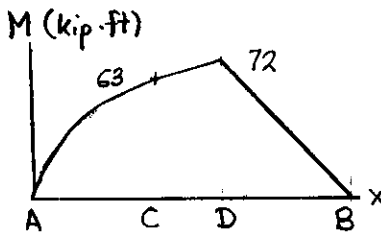
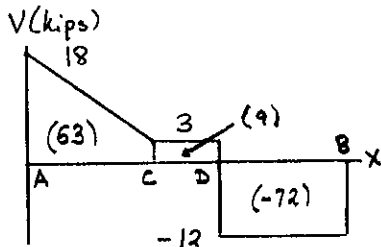
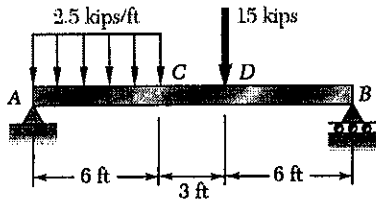
$$S_m = \frac{1}{6} bh_m^3 = (\frac{1}{6})(\frac{3}{4})(6.00)^2 = 4.50 \text{ in}^3$$

$$\text{Allowable value of } M_m = S_m \sigma_{all} = (4.50)(24) = 108.0 \text{ kip}\cdot\text{in}$$

$$\text{Allowable value of } w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320 \text{ kip/in.}$$

$$= 320 \text{ lb/in.}$$

PROBLEM 5.165



5.165 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

SOLUTION

$$\begin{aligned} \sum M_B = 0 \\ -15 R_A + (12)(6)(2.5) + (6)(15) = 0 \\ R_A = 18 \text{ kips} \end{aligned}$$

$$\begin{aligned} \sum M_A = 0 \\ 15 R_B - (3)(6)(2.5) - (9)(15) = 0 \\ R_B = 12 \text{ kips} \end{aligned}$$

Shear:  $V_A = 18 \text{ kips}$   
 $V_C = 18 - (6)(2.5) = 3 \text{ kips}$   
 C to D  $V = 3 \text{ kips}$   
 D to B  $V = 3 - 15 = -12 \text{ kips}$

Areas under shear diagram  
 A to C  $\int V dx = (\frac{1}{2})(6)(18+3) = 63 \text{ kip}\cdot\text{ft}$   
 C to D  $\int V dx = (3)(3) = 9 \text{ kip}\cdot\text{ft}$   
 D to B  $\int V dx = (6)(-12) = -72 \text{ kip}\cdot\text{ft}$

Bending moments:  $M_A = 0$   
 $M_C = 0 + 63 = 63 \text{ kip}\cdot\text{ft}$   
 $M_D = 63 + 9 = 72 \text{ kip}\cdot\text{ft}$   
 $M_B = 72 - 72 = 0$

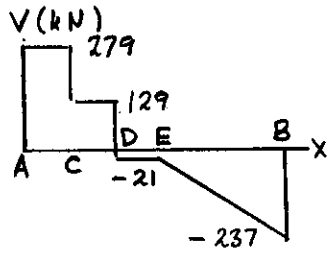
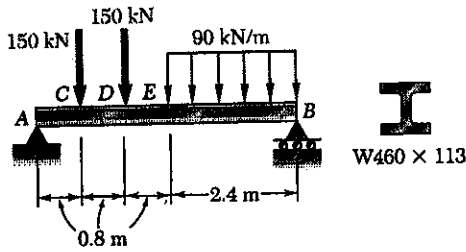
$|V|_{\max} = 18 \text{ kips}$

$|M|_{\max} = 72 \text{ kip}\cdot\text{ft}$



PROBLEM 5.166

5.166 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at D.



SOLUTION

$$\begin{aligned} \sum M_B = 0 \\ -4.8 R_A + (4.0)(150) + (3.2)(150) + (1.2)(2.4)(90) = 0 \\ R_A = 279 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum M_A = 0 \\ 4.8 R_B - (0.8)(150) - (1.6)(150) - (3.6)(2.4)(90) = 0 \\ R_B = 237 \text{ kN} \end{aligned}$$

Shear:

A to C	$V = 279$
C to D	$V = 279 - 150 = 129 \text{ kN}$
D to E	$V = 129 - 150 = -21 \text{ kN}$
$V_E$	$-21 \text{ kN}$
$V_B$	$-21 - (2.4)(90) = -237 \text{ kN}$

Areas under shear diagram

A to C  $\int V dx = (0.8)(279) = 223.2 \text{ kN}\cdot\text{m}$

C to D  $\int V dx = (0.8)(129) = 103.2 \text{ kN}\cdot\text{m}$

Maximum bending moment occurs at point D where V changes sign.

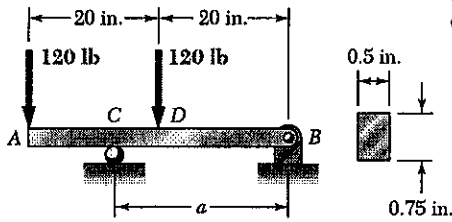
$$\begin{aligned} M_D &= M_A + \int_A^D V dx \\ &= 0 + 223.2 + 103.2 = 326.4 \text{ kN}\cdot\text{m} \end{aligned}$$

For rolled steel section W 460 x 113

$$S = 2400 \times 10^3 \text{ mm}^3 = 2400 \times 10^{-6} \text{ m}^3$$

Normal stress  $\sigma = \frac{M}{S} = \frac{326.4 \times 10^3}{2400 \times 10^{-6}} = 136.0 \times 10^6 \text{ Pa} = 136.0 \text{ MPa}$

PROBLEM 5.167

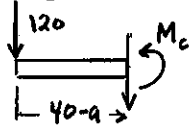


5.167 Determine (a) the distance  $a$  for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (Hint: Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

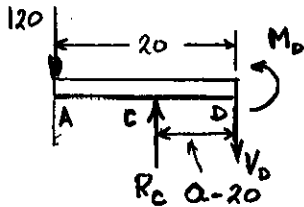
$$\begin{aligned} \sum M_B = 0 \\ -R_c a + (40)(120) + (20)(120) = 0 \\ R_c = \frac{7200}{a} \end{aligned}$$

Bending moment at C



$$\begin{aligned} \sum M_c = 0 \\ M_c + 120(40 - a) = 0 \\ M_c = -4800 + 120a \end{aligned}$$

Bending moment at D



$$\begin{aligned} \sum M_D = 0 \\ M_D + (20)(120) - R_c(a - 20) = 0 \\ M_D = R_c(a - 20) - 2400 \\ = R_c a - 20R_c - 2400 \\ = 7200 - \frac{(20)(7200)}{a} - 2400 \\ = 4800 - \frac{144000}{a} \end{aligned}$$

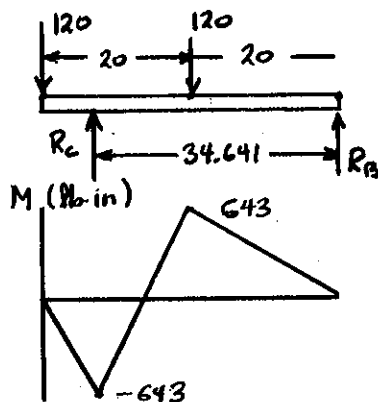
Equate  $-M_c = M_D$

$$4800 - 120a = 4800 - \frac{144000}{a}$$

$$120a^2 = 144000 \quad a = 34.641 \text{ in.}$$

$$M_c = -4800 + (120)(34.641) = -643.08 \text{ lb}\cdot\text{in}$$

$$M_D = 4800 - \frac{144000}{34.641} = 643.08 \text{ lb}\cdot\text{in}$$



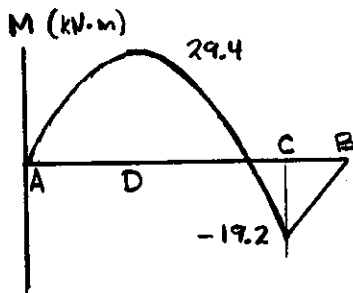
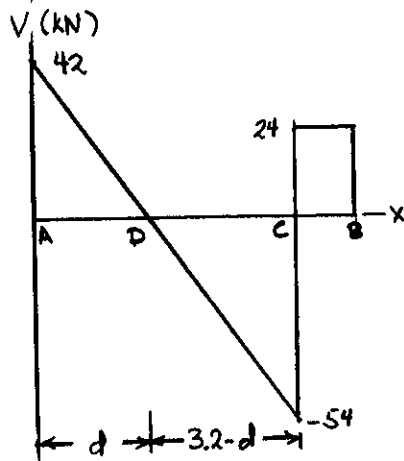
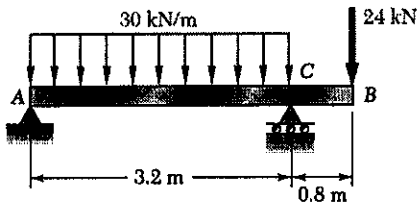
For rectangular section  $S = \frac{1}{6}bh^2$

$$S = \left(\frac{1}{6}\right)(0.5)(0.75)^2 = 0.046875 \text{ in}^3$$

Maximum normal stress

$$\begin{aligned} \sigma_{\max} = \frac{M_{\max}}{S} = \frac{643.08}{0.046875} = 13.72 \times 10^3 \text{ psi} \\ = 13.72 \text{ ksi} \end{aligned}$$

PROBLEM 5.168



5.168 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

SOLUTION

$$+\circlearrowleft \sum M_C = 0$$

$$- 3.2 R_A + (1.6)(3.2)(30) - (0.8)(24) = 0$$

$$R_A = 42 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0$$

$$3.2 R_C - (1.6)(3.2)(30) - (4.0)(24) = 0$$

$$R_C = 78 \text{ kN}$$

Shear:  $V_A = 42 \text{ kN}$   
 $V_C^- = 42 - (3.2)(30) = -54 \text{ kN}$   
 $V_C^+ = -54 + 78 = 24 \text{ kN}$   
 C to B  $V = 24 \text{ kN}$

locate point D where  $V = 0$

$$\frac{d}{42} = \frac{3.2-d}{54} \quad 96d = 134.4$$

$$d = 1.4 \text{ m} \quad 3.2-d = 1.8 \text{ m}$$

Areas under shear diagram

A to D  $\int V dx = (\frac{1}{2})(1.4)(42) = 29.4 \text{ kN}\cdot\text{m}$   
 D to C  $\int V dx = (\frac{1}{2})(1.8)(54) = -48.6 \text{ kN}\cdot\text{m}$   
 C to B  $\int V dx = (0.8)(24) = 19.2 \text{ kN}\cdot\text{m}$

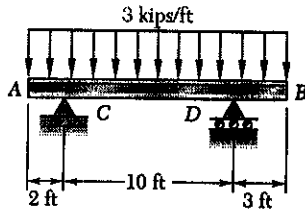
Bending moments:  $M_A = 0$   
 $M_D = 0 + 29.4 = 29.4 \text{ kN}\cdot\text{m}$   
 $M_C = 29.4 - 48.6 = -19.2 \text{ kN}\cdot\text{m}$   
 $M_B = 19.2 - 19.2 = 0 \text{ checks}$

Maximum  $|V| = 54 \text{ kN}$  ▶

Maximum  $|M| = 29.4 \text{ kN}\cdot\text{m}$  ▶

PROBLEM 5.169

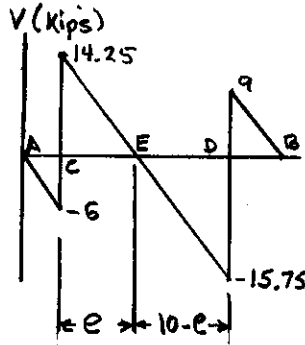
5.169 For the beam and loading shown, determine (a) the maximum value of the bending moment, (b) the maximum normal stress due to bending.



SOLUTION

$$\begin{aligned} \rightarrow \sum M_D = 0 \quad & -10 R_C + (4.5)(15)(3) = 0 \\ R_C = & 20.25 \text{ kips} \end{aligned}$$

$$\begin{aligned} \rightarrow \sum M_C = 0 \quad & 10 R_D - (5.5)(15)(3) = 0 \\ R_D = & 24.75 \text{ kips} \end{aligned}$$



Shear:

$$\begin{aligned} V_A &= 0 \\ V_C^- &= 0 - (2)(3) = -6 \text{ kips} \\ V_C^+ &= -6 + 20.25 = 14.25 \text{ kips} \\ V_D^- &= 14.25 - (10)(3) = -15.75 \text{ kips} \\ V_D^+ &= -15.75 + 24.75 = 9 \text{ kips} \\ V_B &= 9 - (3)(3) = 0 \quad \text{checks} \end{aligned}$$

locate point E where  $V = 0$

$$\begin{aligned} \frac{e}{14.25} &= \frac{10-e}{15.75} \quad 30e = 142.5 \\ e &= 4.75 \text{ ft.} \quad 10-e = 5.25 \text{ ft} \end{aligned}$$

Areas under shear diagram

$$\begin{aligned} \text{A to C} \quad \int V dx &= \left(\frac{1}{2}\right)(2)(-6) = -6 \text{ kip}\cdot\text{ft} \\ \text{C to E} \quad \int V dx &= \left(\frac{1}{2}\right)(4.75)(14.25) = 33.84 \text{ kip}\cdot\text{ft} \\ \text{E to D} \quad \int V dx &= \left(\frac{1}{2}\right)(5.25)(15.75) = -41.34 \text{ kip}\cdot\text{ft} \\ \text{D to B} \quad \int V dx &= \left(\frac{1}{2}\right)(3)(9) = 13.5 \text{ kip}\cdot\text{ft} \end{aligned}$$

Bending moments:

$$\begin{aligned} M_A &= 0 \\ M_C &= 0 - 6 = -6 \text{ kip}\cdot\text{ft} \\ M_E &= -6 + 33.84 = 27.84 \text{ kip}\cdot\text{ft} \\ M_D &= 27.84 - 41.34 = -13.5 \text{ kip}\cdot\text{ft} \\ M_B &= -13.5 + 13.5 = 0 \quad \text{checks} \end{aligned}$$

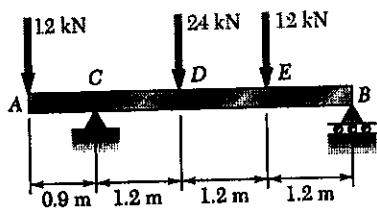
$$\begin{aligned} \text{Maximum } |M| &= 27.84 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft \\ &= 334.1 \text{ kip}\cdot\text{in} \end{aligned}$$

For rolled steel section S10 x 25.4  $S = 24.7 \text{ in}^3$

Maximum normal stress  $\sigma_{\max} = \frac{M_{\max}}{S} = \frac{334.1}{24.7} = 13.53 \text{ ksi} \quad \blacktriangleleft$

**PROBLEM 5.170**

5.170.(a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.



**SOLUTION**

$$+\circlearrowleft \sum M_B = 0$$

$$(4.5)(12) - 3.6 R_c + (2.4)(24) + (1.2)(12) = 0$$

$$R_c = 35 \text{ kN}$$

$$V = -12 + 35 \langle x - 0.9 \rangle^0 - 24 \langle x - 2.1 \rangle^0 - 12 \langle x - 3.3 \rangle^0 \quad \blacktriangleleft$$

$$M = -12x + 35 \langle x - 0.9 \rangle^1 - 24 \langle x - 2.1 \rangle^1 - 12 \langle x - 3.3 \rangle^1 \quad \blacktriangleleft$$

Pt C ( $x = 0.9 \text{ m}$ )  $M_c = -(12)(0.9) = -10.8 \text{ kN}\cdot\text{m}$

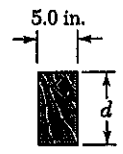
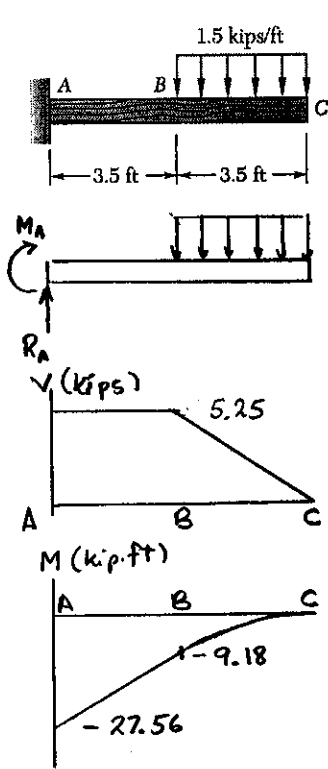
Pt D ( $x = 2.1 \text{ m}$ )  $M_D = -(12)(2.1) + 35(1.2) = 16.8 \text{ kN}\cdot\text{m}$

Pt E ( $x = 3.3 \text{ m}$ )  $M_E = -(12)(3.3) + (35)(2.4) - (24)(1.2) = 15.6 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$

Maximum  $|M| = 16.8 \text{ kN}\cdot\text{m} \quad \blacktriangleleft$

**PROBLEM 5.171**

5.171 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.



**SOLUTION**

$$+\uparrow \sum F_y = 0 \quad R_A - (3.5)(1.5) = 0$$

$$R_A = 5.25 \text{ kN}$$

$$+\curvearrowright \sum M_A = 0 \quad -M_A - (5.25)(3.5)(1.5) = 0$$

$$M_A = -27.56 \text{ kN}$$

Shear: A to B  $V = 5.25$  kips  
 B to C  $V = 5.25 - (3.5)(1.5) = 0$  checks

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Area under shear diagram  
 A to B  $\int V dx = (3.5)(5.25) = 18.38 \text{ kip}\cdot\text{ft}$

B to C  $\int V dx = (\frac{1}{2})(3.5)(5.25) = 9.19 \text{ kip}\cdot\text{ft}$

Bending moments  $M_A = -27.56 \text{ kip}\cdot\text{ft}$

$M_B = -27.56 + 18.38 = -9.18$

$M_C = -9.18 + 9.19 \approx 0$  checks.

Maximum  $|M| = 27.56 \text{ kip}\cdot\text{ft} = 330.7 \text{ kip}\cdot\text{in}$

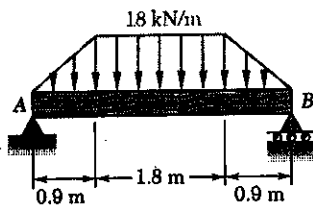
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{330.7}{1.75} = 189.0 \text{ in}^3$$

For a rectangular cross section  $S = \frac{1}{6} b d^2$   $d = \sqrt{\frac{6S}{b}}$

$$d = \sqrt{\frac{(6)(189.0)}{5}} = 15.06 \text{ in.}$$

**PROBLEM 5.172**

5.172 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



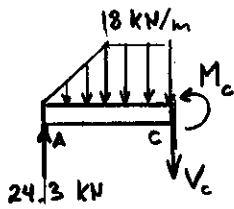
**SOLUTION**

By symmetry  $R_A = R_B$        $+\Sigma F_y = 0$

$$R_A - \left(\frac{1}{2}\right)(0.9)(18) - (1.8)(18) - \left(\frac{1}{2}\right)(0.9)(18) + R_B = 0$$

$$R_A = R_B = 24.3 \text{ kN}$$

Maximum bending moment occurs at the center



$$\curvearrowleft \Sigma M_c = 0$$

$$-(1.8)(24.3) + (1.2)\left(\frac{1}{2}\right)(0.9)(18) + (0.45)(0.9)(18) + M_c = 0$$

$$M_c = 26.73 \text{ kN}\cdot\text{m} = 26.73 \times 10^3 \text{ N}\cdot\text{m}$$

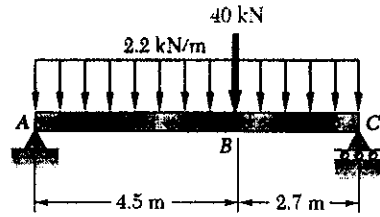
$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{26.73 \times 10^3}{12 \times 10^6} = 2.2275 \times 10^{-3} \text{ m}^3 = 2227.5 \times 10^3 \text{ mm}^3$$

For a rectangular section  $S = \frac{1}{6} b d^2 = \frac{1}{6} \left(\frac{1}{3} d\right) d^2 = \frac{1}{18} d^3$

$$\frac{1}{18} d^3 = 2227.5 \times 10^3 \quad d^3 = 40.09 \times 10^6 \quad d = 342 \text{ mm} \quad \blacktriangleleft$$

PROBLEM 5.173

5.173 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.



SOLUTION

$$\sum M_c = 0$$

$$-7.2 R_A + (3.6)(7.2)(2.2) + (2.7)(40) = 0$$

$$R_A = 22.92 \text{ kN}$$

Shear:  $V_A = 22.92 \text{ kN}$

$$V_B^- = 22.92 - (4.5)(2.2) = 13.02 \text{ kN}$$

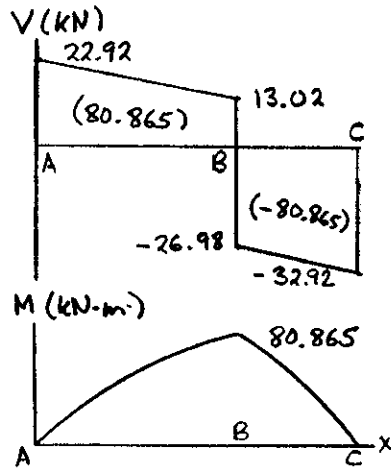
$$V_B^+ = 13.02 - 40 = -26.98 \text{ kN}$$

$$V_C = -26.98 - (2.7)(2.2) = -32.92 \text{ kN}$$

Areas under shear diagram

$$\text{A to B } \int V dx = \left(\frac{1}{2}\right)(4.5)(22.92 + 13.02) = 80.865 \text{ kN}\cdot\text{m}$$

$$\text{B to C } \int V dx = \left(\frac{1}{2}\right)(2.7)(-26.98 - 32.92) = -80.865 \text{ kN}\cdot\text{m}$$



Bending moments:  $M_A = 0$

$$M_B = 0 + 80.865 = 80.865 \text{ kN}\cdot\text{m}$$

$$M_C = 80.865 - 80.865 = 0 \text{ checks}$$

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{160 \times 10^6} = 505 \times 10^{-6} \text{ m}^3 = 505 \times 10^3 \text{ mm}^3$$

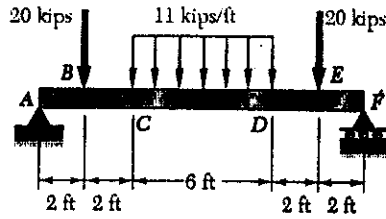
Shape	$S (10^3 \text{ mm}^3)$
W 410 x 38.8	637
W 360 x 39	578
W 310 x 38.7	549
W 250 x 44.8	535
W 200 x 52	512

Use W 310 x 38.7



**PROBLEM 5.174**

5.174 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



**SOLUTION**

By symmetry  $R_A = R_F$

$$+\uparrow \sum F_y = 0 \quad R_A - 20 - (6)(11) - 20 + R_F = 0$$

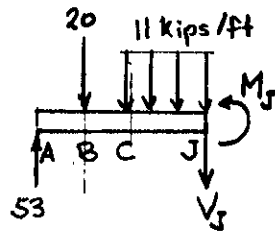
$$R_A = R_F = 50 \text{ kips.}$$

Maximum bending moment occurs at center of beam.

$$\rightarrow \sum M_J = 0 \quad -(7)(53) + (5)(20) + (1.5)(3)(11) + M_J = 0$$

$$M_J = 221.5 \text{ kip}\cdot\text{ft} = 2658 \text{ kip}\cdot\text{in.}$$

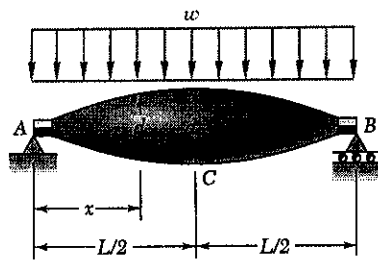
$$S_{\min} = \frac{M_{\max}}{\sigma_{\text{all}}} = \frac{2658}{24} = 110.75 \text{ in}^3$$



Shape	$S (\text{in}^3)$
W 24 x 68	154
W 21 x 62	127
W 18 x 76	146
W 16 x 77	134
W 14 x 82	123
W 12 x 96	131

Use W 21 x 62

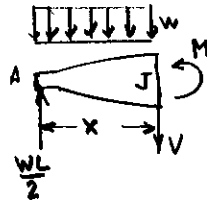
PROBLEM 5.175



5.175 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter  $d$  is being designed to support a distributed load  $w$  as shown. (a) Knowing that the machine element is to be of constant strength, express  $d$  in terms of  $x$ ,  $L$ , and  $d_0$ . (b) Determine the smallest allowable value of  $d_0$  if  $L = 250$  mm,  $w = 30$  kN/m, and  $\sigma_{all} = 72$  MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$



$$\rightarrow \sum M_J = 0$$

$$-\frac{wL}{2}x + wx \frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

For a solid circular cross section  $c = \frac{d}{2}$   $I = \frac{\pi}{4}c^3$   $S = \frac{I}{c} = \frac{\pi d^3}{32}$

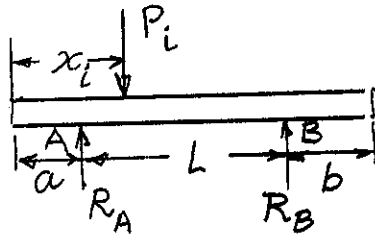
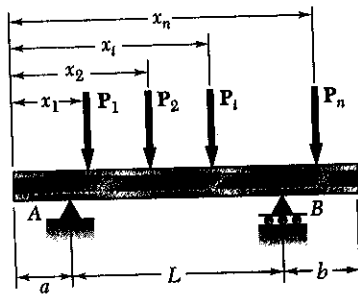
Equating  $\frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{all}}$   $d = \left\{ \frac{16wx(L-x)}{\pi\sigma_{all}} \right\}^{1/3}$

At  $x = \frac{L}{2}$   $d = d_0 = \left\{ \frac{4wL^2}{\pi\sigma_{all}} \right\}^{1/3}$

$$d = d_0 \left\{ 4 \frac{x}{L} \left( 1 - \frac{x}{L} \right) \right\}^{1/3}$$

Using the data  $d_0 = \frac{(4)(30 \times 10^3)(0.250)^2}{\pi(72 \times 10^6)} = 32.1 \times 10^{-3} \text{ m} = 32.1 \text{ mm}$

**PROBLEM 5.C1**



**5.C1** Several concentrated loads  $P_i (i = 1, 2, \dots, n)$  can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.23, 5.27, and 5.29. (Hint: Maximum values will occur at a support or under a load.)

**SOLUTION**

REACTIONS AT A AND B

$$\uparrow \sum M_A = 0: R_B L - \sum_i P_i (x_i - a)$$

$$R_B = (1/L) \sum_i P_i (x_i - a)$$

$$R_A = \sum_i P_i - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text.)

WE DEFINE: IF  $x \geq a$  THEN  $STPA = 1$  ELSE  $STPA = 0$

IF  $x \geq a + L$  THEN  $STPB = 1$  ELSE  $STPB = 0$

IF  $x \geq x_i$  THEN  $STP(I) = 1$  ELSE  $STP(I) = 0$

$$V = R_A STPA + R_B STPB - \sum_i P_i STP(I)$$

$$M = R_A (x - a) STPA + R_B (x - a - L) STPB - \sum_i P_i (x - x_i) STP(I)$$

$\sigma = M/S$ , where  $S$  is obtained from Appendix C.

PROGRAM OUTPUTS

Prob. 5.23

RA=80.0 kN RB=80.0 kN

X m	V kN	M kN.m	Sigma MPa
2.00	0.00	104.00	128.55

Prob. 5.27

R1 = 44.0 kN R2 = 16.0 kN

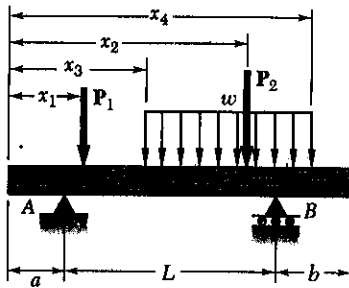
X m	V kN	M kN.m	Sigma MPa
0.00	-20.00	0.00	0.00
1.60	24.00	-32.00	-31.07
4.00	-16.00	25.60	24.85
5.60	-16.00	0.00	0.00

Prob. 5.29

R1 = 52.5 kips R2 = 22.5 kips

X ft	V kips	M kip.ft	Sigma ksi
0.00	-25.00	0.00	0.00
1.00	27.50	-25.00	-7.85
3.00	2.50	30.00	9.42
9.00	-22.50	45.00	14.14
11.00	0.00	0.00	0.00

**PROBLEM 5.C2**



**5.C2** A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value  $\sigma_{all}$ . Write a computer program that can be used to calculate at given intervals  $\Delta L$  the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals  $\Delta L$  indicated: (a) Prob. 5.75 ( $\Delta L = 0.1$  m), (b) Prob. 5.79 ( $\Delta L = 0.2$  m), (c) Prob. 5.80 ( $\Delta L = 0.3$  m).

**SOLUTION**

REACTIONS AT A AND B

$$\rightarrow \sum M_A = 0: R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3) \left( \frac{x_4 + x_3}{2} - a \right) = 0$$

$$R_B = \frac{1}{L} [P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2} w(x_4 - x_3)(x_4 + x_3 - 2a)]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET  $n = (a + b + L) / \Delta L$

FOR  $i = 0$  TO  $n$ :  $x = (\Delta L) i$

WE DEFINE: IF  $x \geq a$  THEN  $STPA = 1$  ELSE  $STPA = 0$

IF  $x \geq a + L$ , THEN  $STPB = 1$  ELSE,  $STPB = 0$

IF  $x \geq x_1$ , THEN  $STP1 = 1$  ELSE  $STP1 = 0$

IF  $x \geq x_2$  THEN  $STP2 = 1$  ELSE  $STP2 = 0$

IF  $x \geq x_3$  THEN  $STP3 = 1$  ELSE  $STP3 = 0$

IF  $x \geq x_4$  THEN  $STP4 = 1$  ELSE  $STP4 = 0$

$$V = R_A STPA + R_B STPB - P_1 STP1 - P_2 STP2 - w(x - x_3) STP3 + w(x - x_4) STP4$$

$$M = R_A(x - a) STPA + R_B(x - a - L) STPB - P_1(x - x_1) STP1 - P_2(x - x_2) STP2 - \frac{1}{2} w(x - x_3)^2 STP3 + \frac{1}{2} w(x - x_4)^2 STP4$$

$$S_{min} = |M| / \sigma_{all}$$

IF UNKNOWN DIMENSION IS h:

From  $S = \frac{1}{6} t h^2$ , we have  $h = \sqrt{6S/t}$

IF UNKNOWN DIMENSION IS t:

From  $S = \frac{1}{6} t h^2$ , we have  $t = 6S/h^2$

(CONTINUED)

PROBLEM 5.C2 CONTINUED

PROGRAM OUTPUTS

Prob. 5.75

RA =	2.40 kN	RB =	3.00 kN
X	V	M	H
m	kN	kN.m	mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	-3.00	2.400	173.21
1.70	-3.00	2.100	162.02
1.80	-3.00	1.800	150.00
1.90	-3.00	1.500	136.93
2.00	-3.00	1.200	122.47
2.10	-3.00	0.900	106.07
2.20	-3.00	0.600	86.60
2.30	-3.00	0.300	61.24
2.40	0.00	0.000	0.05

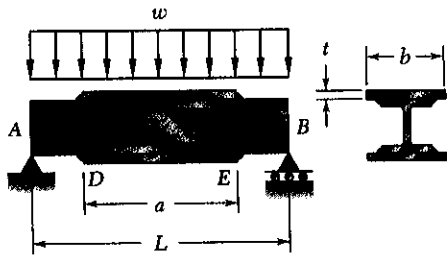
Prob. 5.79

RA =	2.70 kN	RB =	8.10 kN
X	V	M	T
m	kN	kN.m	mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	-0.30	1.200	26.67
1.20	-0.90	1.080	24.00
1.40	-1.50	0.840	18.67
1.60	-2.10	0.480	10.67
1.80	-2.70	0.000	0.00
2.00	-3.30	-0.600	13.33
2.20	-3.90	-1.320	29.33
2.40	3.60	-2.160	48.00
2.60	3.00	-1.500	33.33
2.80	2.40	-0.960	21.33
3.00	1.80	-0.540	12.00
3.20	1.20	-0.240	5.33
3.40	0.60	-0.060	1.33
3.60	0.00	-0.000	0.00

Prob. 5.80

RA =	6.50 kN	RB =	6.50 kN
X	V	M	H
m	kN	kN.m	mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	-0.00	8.250	203.10
2.40	-1.80	7.980	199.75
2.70	-3.60	7.170	189.34
3.00	-5.40	5.820	170.59
3.30	-7.20	3.930	140.18
3.60	-2.50	1.500	86.60
3.90	-2.50	0.750	61.24
4.20	0.00	0.000	0.06

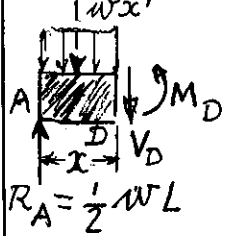
**PROBLEM 5.C3**



**5.C3** Two cover plates, each of thickness  $t$ , are to be welded to a wide-flange beam of length  $L$ , which is to support a uniformly distributed load  $w$ . Denoting by  $\sigma_{all}$  the allowable normal stress in the beam and in the plates, by  $d$  the depth of the beam, and by  $I_b$  and  $S_b$ , respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (a) the length  $a$  of the plates, (b) the width  $b$  of the plates. Use this program to solve Probs. 5.155 and 5.157.

**SOLUTION**

(a) Required Length of Plates



FB = AD:  $\sum M_D = 0: M_D + wx \left(\frac{x}{2}\right) - R_A x = 0$

But:  $R_A = \frac{1}{2} wL$  and  $M_D = S \sigma_{all}$ . Divide by  $\frac{1}{2} w$ :

$x^2 - Lx + \left(\frac{2S \sigma_{all}}{w}\right) = 0$ . Set  $k = \frac{2S \sigma_{all}}{w}$

$x^2 - Lx + k = 0$

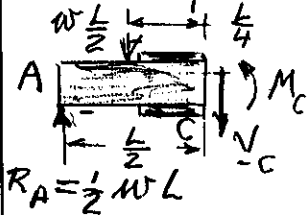
Solving the quadratic:  $x = \frac{L - \sqrt{L^2 - 4k}}{2}$

Compute  $x$  and

$a = L - 2x$

(b) Required Width of Plates

At midpoint C of beam:

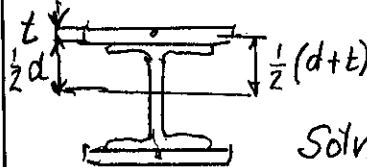


FB = AC:  $\sum M_C = 0: M_C + \frac{wL}{2} \frac{L}{4} - \frac{wL}{2} \frac{L}{2} = 0$

Compute  $M_C = \frac{1}{8} wL^2$

Compute:  $C = t + \frac{1}{2}d$

From  $\sigma_{all} = \frac{M_C C}{I}$  compute  $I = \frac{M_C C}{\sigma_{all}}$



But  $I = I_{beam} + I_{plates} = I_b + 2 \left[ \frac{1}{12} b t^3 + b t \left(\frac{d+t}{2}\right)^2 \right]$

Solving for  $b$ :  $b = \frac{6(I - I_b)}{t [t^2 + 3(d+t)^2]}$

PROGRAM OUTPUTS

PROB. 5.155:  $W 460 \times 74$ ,  $\sigma_{all} = 150 \text{ MPa}$   
 $w = 40 \text{ kN/m}$ ,  $L = 8 \text{ m}$ ,  $t = 7.5 \text{ mm}$   
 $d = 457 \text{ mm}$ ,  $I_b = 333 \times 10^6 \text{ mm}^4$ ,  $S = 1460 \times 10^3 \text{ mm}^3$

PROB. 5.157:  $W 30 \times 99$ ,  $\sigma_{all} = 22 \text{ ksi}$   
 $w = 30 \text{ kips/ft}$ ,  $L = 16 \text{ ft}$ ,  $t = 5/8 \text{ in.}$   
 $d = 29.65 \text{ in.}$ ,  $I_b = 3990 \text{ in}^4$ ,  $S = 269 \text{ in}^3$

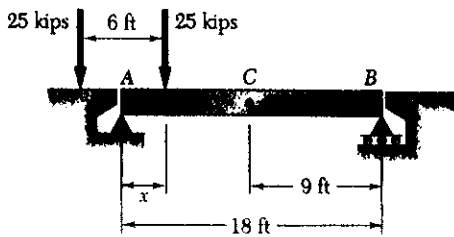
Prob. 5.155

$a = 4.49 \text{ m}$   
 $b = 211 \text{ mm}$

Prob. 5.157

$a = 11.16 \text{ ft}$   
 $b = 14.31 \text{ in.}$

**PROBLEM 5.C4**



**5.C4** Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of  $x$  from 0 to 24 ft at intervals  $\Delta x = 1.5$  ft.

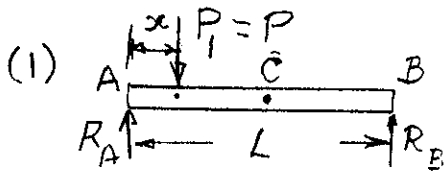
**SOLUTION**

NOTATION: Length of beam =  $L = 18$  ft

Loads:  $P_1 = P_2 = P = 25$  kips

Distance between loads =  $d = 6$  ft

We note that  $d < L/2$



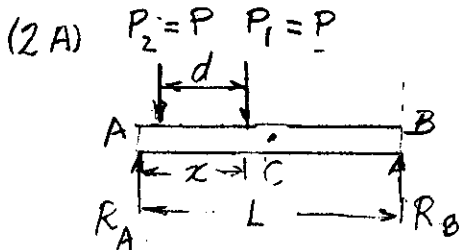
(1) FROM  $x = 0$  TO  $x = d$ :

$$\rightarrow \sum M_B = 0: P(L-x) - R_A L = 0$$

$$R_A = P(L-x)/L$$

Under  $P_1$ :  $M_1 = R_A x$

At C:  $M_C = R_A(L/2) - P(L/2 - x)$



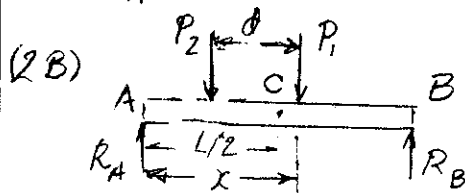
(2) FROM  $x = d$  TO  $x = L$ :

$$\rightarrow \sum M_B = 0: P(L-x) + P(L-x+d) - R_A L = 0$$

$$R_A = P(2L - 2x + d)/L$$

Under  $P_1$ :  $M_1 = R_A x - Pd$

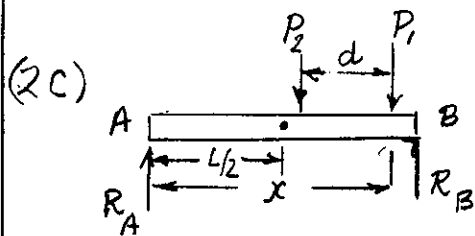
Under  $P_2$ :  $M_2 = R_A(x-d)$



(2A) FROM  $x = d$  TO  $x = L/2$ :

$$M_C = R_A(L/2) - P(L/2 - x) - P(L/2 - x + d)$$

$$= R_A(L/2) - P(L - 2x + d)$$

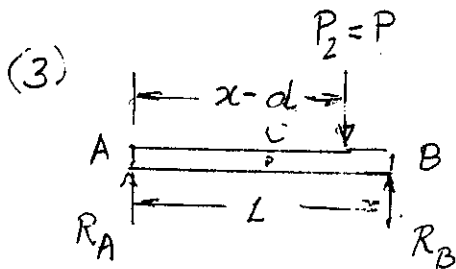


(2B) FROM  $x = L/2$  TO  $x = L/2 + d$ :

$$M_C = R_A(L/2) - P(L/2 - x + d)$$

(2C) FROM  $x = L/2 + d$  TO  $x = L$ :

$$M_C = R_A L/2$$



(3) FROM  $x = L$  TO  $x = L + d$ :

$$\rightarrow \sum M_B = 0: P(L-x+d) - R_A L = 0$$

$$R_A = P(L-x+d)/L$$

Under  $P_2$ :  $M_2 = R_A(x-d)$

At C:  $M_C = R_A(L/2)$

(CONTINUED)

PROBLEM 5.C4 CONTINUED

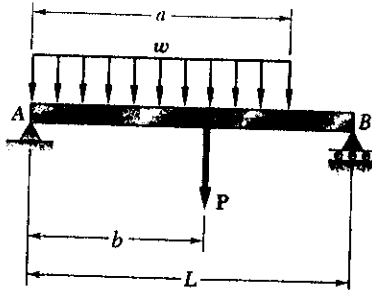
PROGRAM OUTPUT

P = 25 kips, L = 18 ft, D = 6 ft

x ft	MC kip.ft	M1 kip.ft	M2 kip.ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.00	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00

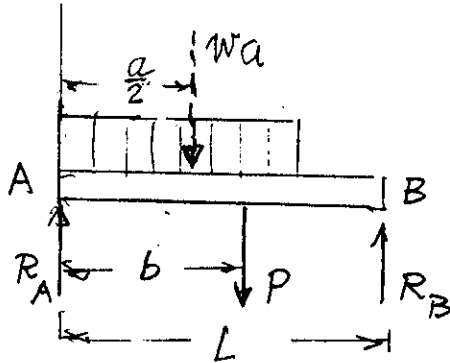


PROBLEM 5.C5



5.C5 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval  $\Delta L = 0.2$  ft to the beam and loading of (a) Prob. 5.83, (b) Prob. 5.125.

SOLUTION



REACTIONS AT A AND B

USING FB DIAGRAM OF BEAM:

$$+\circlearrowleft \sum M_A = 0: R_B L - Pb - wa(a/2) = 0$$

$$R_B = (1/L)(Pb + \frac{1}{2}wa^2)$$

$$R_A = P + wa - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text).

SET  $n = L/\Delta L$ . FOR  $i = 0$  TO  $n$ :  $x = (\Delta L)i$

WE DEFINE: IF  $x \geq a$  THEN  $STPA = 1$  ELSE  $STPA = 0$   
IF  $x \geq b$  THEN  $STPB = 1$  ELSE  $STPB = 0$

$$V = R_A - wx + w(x-a)STPA - PSTPB$$

$$M = R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w(x-a)^2STPA - P(x-b)STPB$$

LOCATE AND PRINT  $(x, V)$  AND  $(x, M)$

SEE NEXT PAGES FOR PROGRAM OUTPUTS

(CONTINUED)

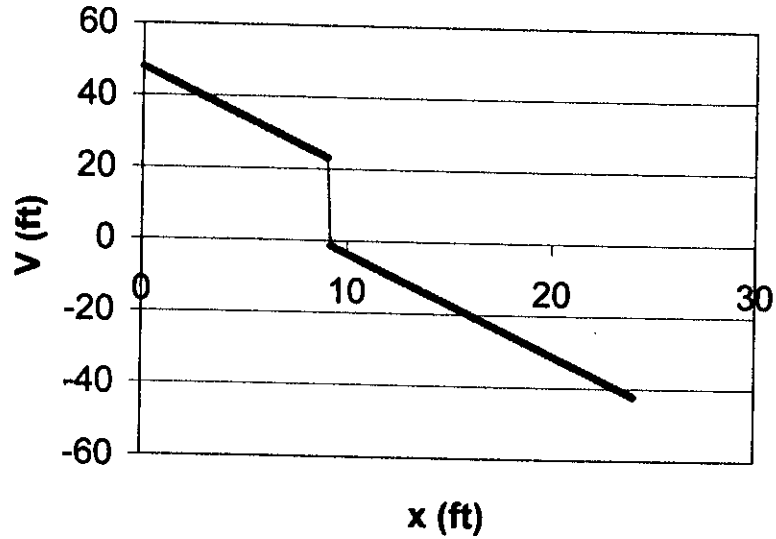
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT FOR P5.83

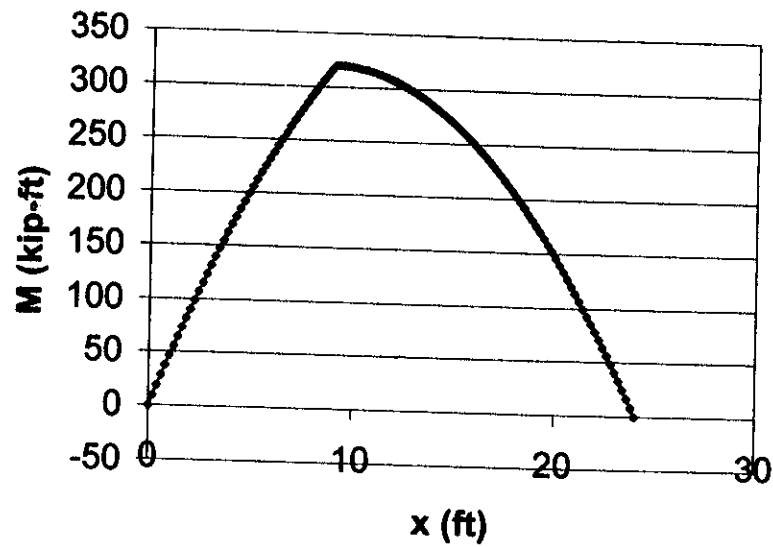
PROBLEM 5.83

RA = 48.00 kips RB = 42.00 kips

### Shear Diagram



### Moment Diagram



(CONTINUED)

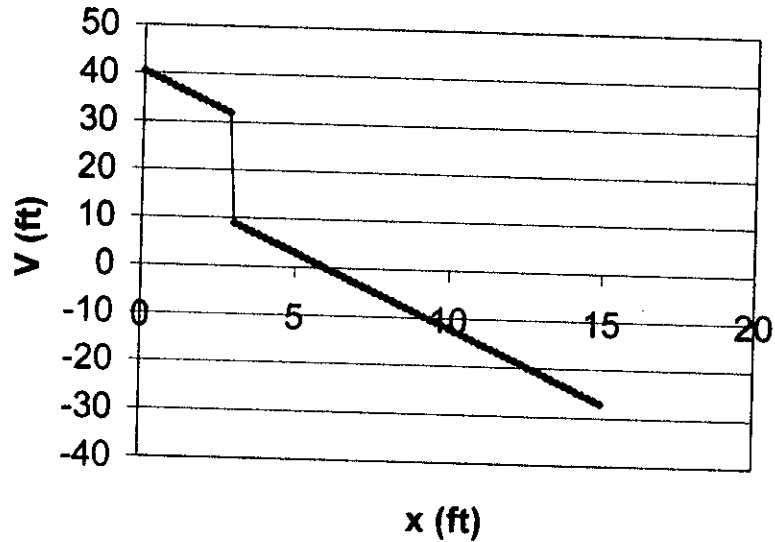
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT FOR P5,125

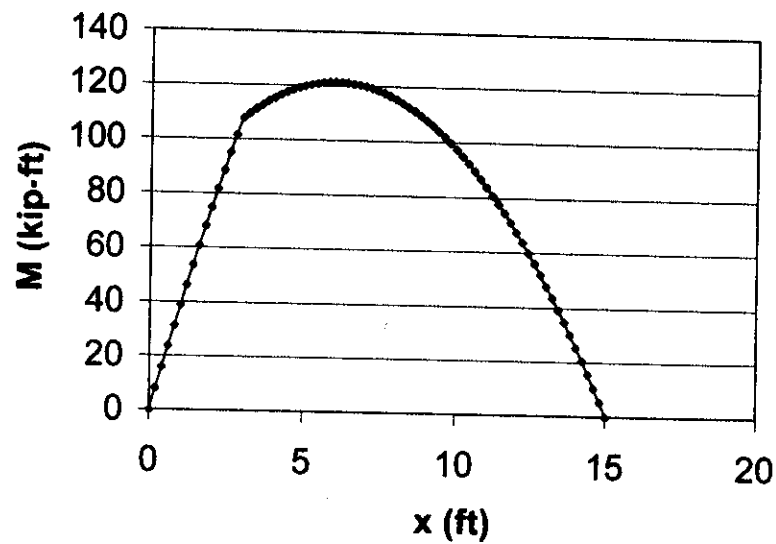
PROBLEM 5.125

RA = 40.50 kips RB = 27.00 kips

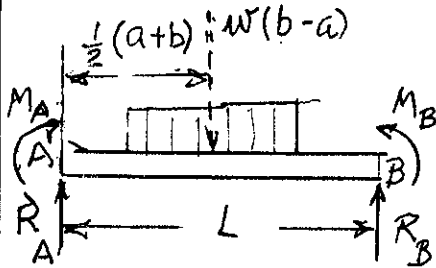
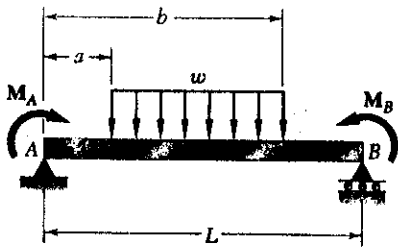
### Shear Diagram



### Moment Diagram



## PROBLEM 5.C6



**5.C6** Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval  $\Delta L = 0.025$  m to the beam and loading of Prob. 5.124.

## SOLUTION

REACTIONS AT A AND B

$$\uparrow \sum M_A = 0:$$

$$R_B L + M_B - M_A - w(b-a) \frac{1}{2}(a+b) = 0$$

$$R_B = (1/L) [M_A - M_B + \frac{1}{2} w(b^2 - a^2)]$$

$$R_A = w(b-a) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET  $n = L/\Delta L$ , FOR  $i = 0$  TO  $n$ :  $x = (\Delta L)i$

WE DEFINE: IF  $x \geq a$  THEN  $STPA = 1$  ELSE  $STPA = 0$

IF  $x \geq b$  THEN  $STPB = 1$  ELSE  $STPB = 0$

$$V = R_A - w(x-a)STPA + w(x-b)STPB$$

$$M = M_A + R_A x - \frac{1}{2} w(x-a)^2 STPA + \frac{1}{2} w(x-b)^2 STPB$$

LOCATE AND PRINT  $(x, V)$  AND  $(x, M)$

PROGRAM OUTPUT ON NEXT PAGE

(CONTINUED)

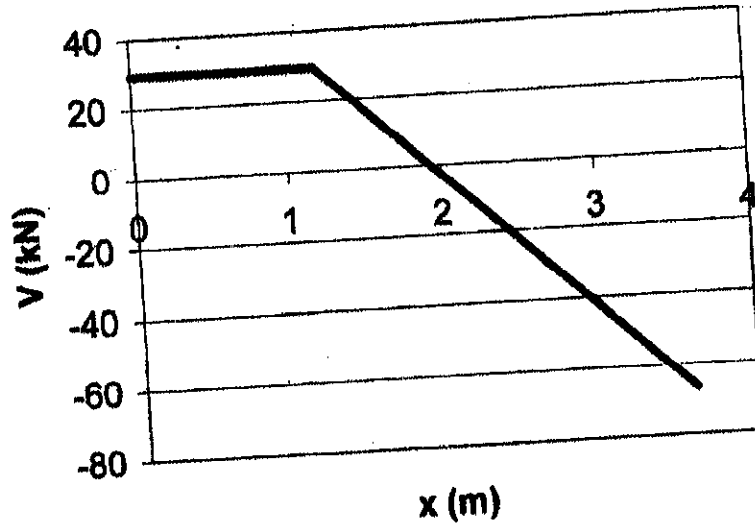
PROBLEM 5.C6 CONTINUED

PROGRAM OUTPUT

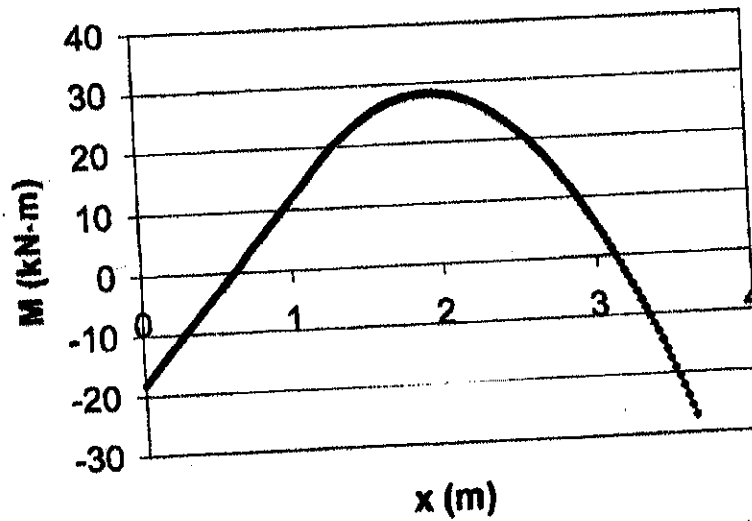
PROBLEM 5.124

RA = 29.50 kips RB = 66.50 kips

**Shear Diagram**



**Moment Diagram**

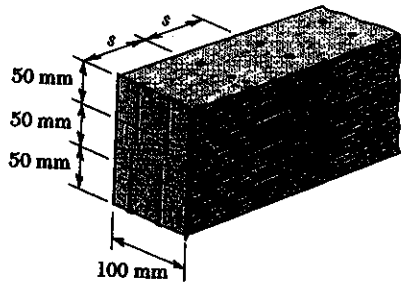


# CHAPTER 6

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**PROBLEM 6.1**

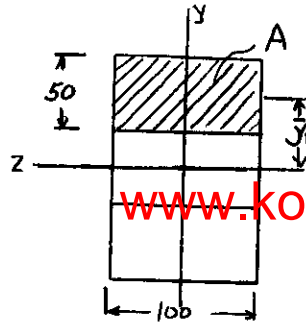
6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing  $s$  that can be used between each pair of nails.



**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

$$= 28.125 \times 10^{-6} \text{ m}^4$$



$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

$$Q = A \bar{y}_1 = 250 \times 10^3 \text{ mm}^3$$

$$= 250 \times 10^{-6} \text{ m}^3$$

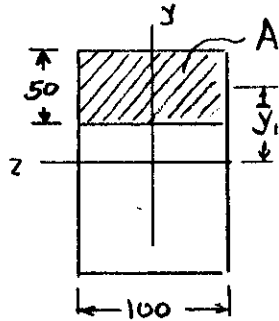
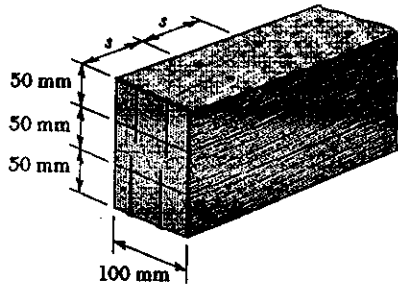
$$q = \frac{VQ}{I} = \frac{(1500)(250 \times 10^{-6})}{28.125 \times 10^{-6}} = 13.333 \times 10^3 \text{ N/m}$$

$$qs = 2 F_{\text{nail}}$$

$$s = \frac{2 F_{\text{nail}}}{q} = \frac{(2)(400)}{13.333 \times 10^3} = 60 \times 10^{-3} \text{ m}$$

$$= 60 \text{ mm}$$

PROBLEM 6.2



6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing  $s$  that can be used between each pair of nails.

6.2 For the built-up beam of Prob. 6.1, determine the allowable shear if the spacing between each pair of nails is  $s = 45$  mm.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$

$$= 28.125 \times 10^{-6} \text{ m}^4$$

$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

$$Q = A \bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} \qquad q s = 2 F_{\text{nail}}$$

Eliminating  $q$   $\frac{VQ}{I} = \frac{2 F_{\text{nail}}}{s}$

Solving for  $V$

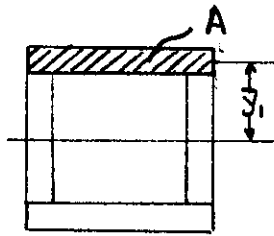
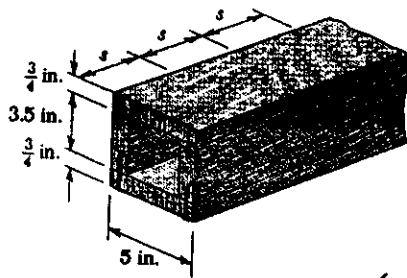
$$V = \frac{2 I F_{\text{nail}}}{Q s} = \frac{(2)(28.125 \times 10^{-6})(400)}{(250 \times 10^{-6})(45 \times 10^{-3})}$$

$$= 2 \times 10^3 \text{ N} = 2 \text{ kN}$$

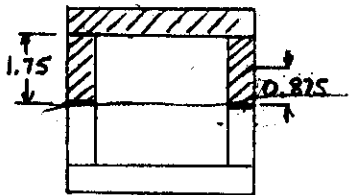


**PROBLEM 6.3**

6.3 A square box beam is made of two  $\frac{3}{4} \times 3.5$ -in. planks and two  $\frac{3}{4} \times 5$ -in. planks nailed together as shown. Knowing that the spacing between nails is  $s = 1.25$  in. and that the vertical shear in the beam is  $V = 250$  lb, determine (a) the shearing force in each nail, (b) the maximum shearing stress in the beam.



$$q s = 2 F_{\text{nail}}$$



**SOLUTION**

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12} (5)(5)^3 - \frac{1}{12} (3.5)(3.5)^3 = 39.578 \text{ in}^4$$

(a)  $A = (5)\left(\frac{3}{4}\right) = 3.75 \text{ in}^2$

$$\bar{y}_1 = 2.5 - \frac{s}{2} = 2.125 \text{ in}$$

$$Q_1 = A \bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$$

$$q = \frac{V Q_1}{I} = \frac{(250)(7.969)}{39.578} = 50.34 \text{ lb/in}$$

$$F_{\text{nail}} = \frac{q s}{2} = \frac{(50.34)(1.25)}{2} = 31.5 \text{ lb.}$$

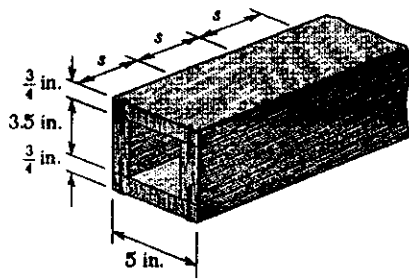
(b)  $Q_2 = Q_1 + (2)(1.75)\left(\frac{3}{4}\right)(0.875)$

$$= 7.969 + 2.297 = 10.266 \text{ in}^3$$

$$t = (2)\left(\frac{3}{4}\right) = 1.5 \text{ in.}$$

$$\tau_{\text{max}} = \frac{V Q}{I t} = \frac{(250)(10.266)}{(39.578)(1.5)} = 43.2 \text{ psi}$$

PROBLEM 6.4

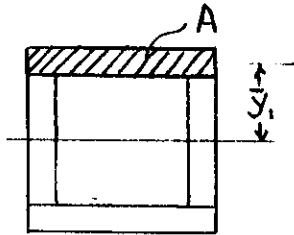


6.4 A square box beam is made of two  $\frac{3}{4} \times 3.5$ -in. planks and two  $\frac{3}{4} \times 5$ -in. planks nailed together as shown. Knowing that the spacing between nails is  $s = 2$  in. and that the allowable shearing force in each nail is 75 lb, determine (a) the largest allowable vertical shear in the beam, (b) the corresponding maximum shearing stress in the beam.

SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12} (5)(5)^3 - \frac{1}{12} (3.5)(3.5)^3 = 39.578 \text{ in}^4$$



(a)  $A = (5)\left(\frac{3}{4}\right) = 3.75 \text{ in}^2$

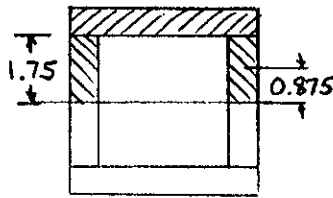
$$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in.}$$

$$Q_1 = A \bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$$

$$q_{\text{allow}} = \frac{2F_{\text{nail}}}{s} = \frac{(2)(75)}{2} = 75 \text{ lb/in}$$

$$V_{\text{allow}} = \frac{I q_{\text{allow}}}{Q_1} = \frac{(39.578)(75)}{7.969} = 372 \text{ lb}$$

$$q = \frac{VQ}{I}$$



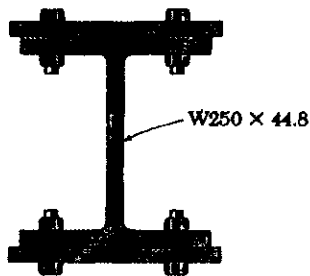
(b)  $Q = Q_1 + (2)(1.75)\left(\frac{3}{4}\right)(0.875)$

$$= 7.969 + 2.297 = 10.266 \text{ in}^3$$

$$t = (2)\left(\frac{3}{4}\right) = 1.5 \text{ in}$$

$$\tau_{\text{max}} = \frac{VQ}{It} = \frac{(372)(10.266)}{(39.578)(1.5)} = 64.4 \text{ psi}$$

PROBLEM 6.5\*



6.5 The beam shown has been reinforced by attaching to it two 12 × 175-mm plates, using bolts of 18-mm diameter spaced longitudinally every 125 mm. Knowing that the average allowable shearing stress in the bolts is 85 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia

Part	A (mm <sup>2</sup> )	d (mm)	Ad <sup>2</sup> (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
Top plate	2100	* 139	40.574	0.025
W250 × 44.8	5720	0	0	71.1
Bot. plate	2100	* 139	40.574	0.025
Σ			81.148	71.15

$$* d = \frac{266}{2} + \frac{12}{2} = 139 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 152.30 \times 10^6 \text{ mm}^4 = 152.30 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate} d_{plate} = (2100)(139) = 291.9 \times 10^3 \text{ mm}^3 = 291.9 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

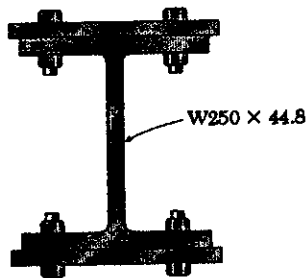
$$F_{bolt} = \tau_{all} A_{bolt} = (85 \times 10^6)(254.47 \times 10^{-6}) = 21.63 \times 10^3 \text{ N}$$

$$q = \frac{2F_{bolt}}{s} = \frac{(2)(21.63 \times 10^3)}{125 \times 10^{-3}} = 346.1 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{Iq}{Q} = \frac{(152.30 \times 10^{-6})(346.1 \times 10^3)}{291.9 \times 10^{-6}} = 180.6 \times 10^3 \text{ N}$$

$$= 180.6 \text{ kN}$$

PROBLEM 6.6



6.5 The beam shown has been reinforced by attaching to it two  $12 \times 175$ -mm plates, using bolts of 18-mm diameter spaced longitudinally every 125 mm. Knowing that the average allowable shearing stress in the bolts is 85 MPa, determine the largest permissible vertical shearing force.

6.6 Solve Prob. 6.5, assuming that the reinforcing plates are only 9 mm thick.

SOLUTION

Calculate moment of inertia

Part	A (mm <sup>2</sup> )	d (mm)	Ad <sup>2</sup> (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
Top plate	1575	137.5	29.777	0.011
W250 x 44.8	5720	0	0	71.1
Bot. plate	1575	137.5	29.777	0.011
$\Sigma$			59.555	71.121

$$* d = \frac{266}{2} + \frac{9}{2} = 137.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 130.68 \times 10^6 \text{ mm}^4 = 130.68 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}} d_{\text{plate}} = (1575)(137.5) = 216.56 \times 10^3 \text{ mm}^3 = 216.56 \times 10^{-6} \text{ m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

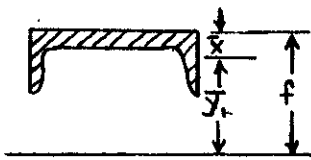
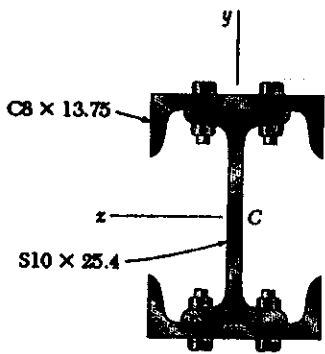
$$F_{\text{bolt}} = \tau_{\text{all}} A_{\text{bolt}} = (85 \times 10^6)(254.47 \times 10^{-6}) = 21.63 \times 10^3 \text{ N}$$

$$q = \frac{2F_{\text{bolt}}}{s} = \frac{(2)(21.63 \times 10^3)}{125 \times 10^{-3}} = 346.1 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{Iq}{Q} = \frac{(130.68 \times 10^{-6})(346.1 \times 10^3)}{216.56 \times 10^{-6}} = 209 \times 10^3 \text{ N}$$

$$= 209 \text{ kN}$$

PROBLEM 6.7



6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of  $\frac{3}{4}$ -in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.

SOLUTION

Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{10}{2} + 0.308 = 5.308 \text{ in}$$

$$\bar{x} = 0.533 \text{ in}$$

$$\bar{y}_i = f - \bar{x} = 5.308 - 0.533 = 4.775 \text{ in}$$

Determine moment of inertia.

Part	A (in <sup>2</sup> )	d (in)	Ad <sup>2</sup> (in <sup>4</sup> )	$\bar{I}$ (in <sup>4</sup> )
C8 x 13.75	4.04	4.770	91.92	1.53
S10 x 25.4	7.46	0	0	124
C8 x 13.75	4.04	4.770	91.92	1.53
$\Sigma$			183.84	127.06

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 183.84 + 127.06 = 310.9 \text{ in}^4$$

$$Q = A \bar{y}_i = (4.04)(4.770) = 19.271 \text{ in}^3$$

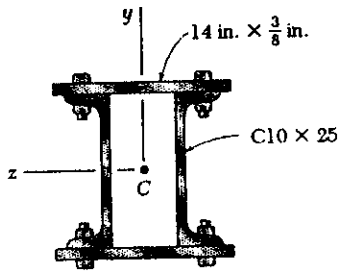
$$q = \frac{VQ}{I} = \frac{(30)(19.271)}{310.9} = 1.8595 \text{ kip/in}$$

$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.8595)(5) = 4.649 \text{ kip}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} \left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.649}{0.4418} = 10.52 \text{ ksi}$$

PROBLEM 6.8



6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of  $\frac{3}{4}$ -in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis

SOLUTION

Calculate moment of inertia

Part	A (in <sup>2</sup> )	d (in)	Ad <sup>2</sup> (in <sup>4</sup> )	$\bar{I}$ (in <sup>4</sup> )
Top plate	5.25	*5.1875	141.28	0.06
C10 x 25	7.35	0		91.2
C10 x 25	7.35	0		91.2
Bot. plate	5.25	*5.1875	141.28	0.06
$\Sigma$			282.56	182.52

$$* d = \frac{10}{2} + \frac{1}{2}\left(\frac{3}{8}\right) = 5.1875 \text{ in} = \bar{y}_1$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 282.56 + 182.52 = 465.08 \text{ in}^4$$

$$Q = A_{plate} \bar{y}_1 = (5.25)(5.1875) = 27.234 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(27.234)}{465.08} = 1.7567 \text{ kips/in}$$

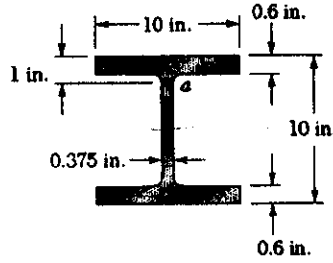
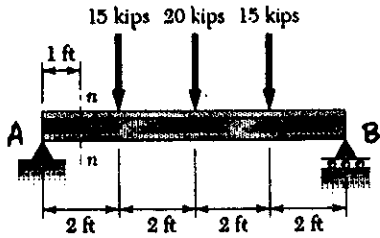
$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.7567)(5) = 4.392 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} \left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.392}{0.4418} = 9.94 \text{ ksi}$$

**PROBLEM 6.9**

6.9 through 6.12 For the beam and loading shown, consider section  $n-n$  and determine (a) the largest shearing stress in that section, (b) the shearing stress at point  $a$ .



**SOLUTION**

By symmetry  $R_A = R_B$

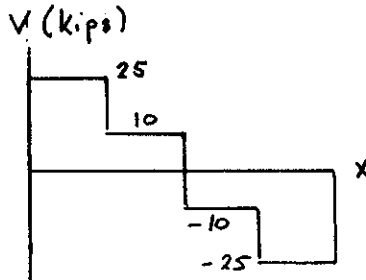
$$\uparrow \sum F_y = 0$$

$$R_A + R_B - 15 - 20 - 15 = 0$$

$$R_A = R_B = 25 \text{ kips}$$

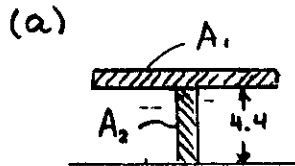
From shear diagram  $V = 30 \text{ kips}$  at  $n-n$ .

Determine moment of inertia.



Part	$A \text{ (in}^2\text{)}$	$d \text{ (in)}$	$Ad^2 \text{ (in}^4\text{)}$	$\bar{I} \text{ (in}^4\text{)}$
Top Flng	6	4.7	132.54	0.18
Web	3.30	0	0	21.30
Bot. Flng	6	4.7	132.54	0.18
$\Sigma$			265.08	21.66

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 286.74 \text{ in}^4$$



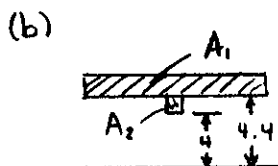
Part	$A \text{ (in}^2\text{)}$	$\bar{y} \text{ (in)}$	$A\bar{y} \text{ (in}^3\text{)}$
①	6	4.7	28.2
②	1.65	2.2	3.63
$\Sigma$			31.83

$$Q = \Sigma A\bar{y}$$

$$= 31.83 \text{ in}^3$$

$$t = 0.375 \text{ in}$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{(25)(31.83)}{(286.74)(0.375)} = 7.40 \text{ ksi}$$



Part	$A \text{ (in}^2\text{)}$	$\bar{y} \text{ (in)}$	$A\bar{y} \text{ (in}^3\text{)}$
①	6	4.7	28.2
②	0.15	4.2	0.63
$\Sigma$			28.83

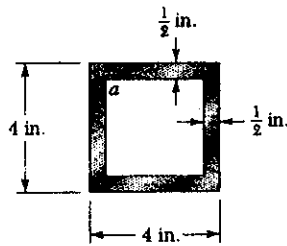
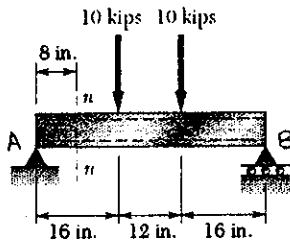
$$Q = \Sigma A\bar{y} = 28.83 \text{ in}^3$$

$$t = 0.375 \text{ in}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(28.83)}{(286.74)(0.375)} = 6.70 \text{ ksi}$$

PROBLEM 6.10

6.9 through 6.12 For the beam and loading shown, consider section  $n-n$  and determine (a) the largest shearing stress in that section, (b) the shearing stress at point  $a$ .

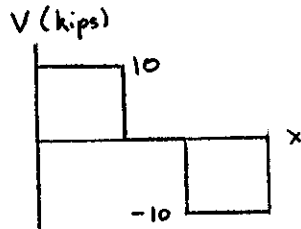


SOLUTION

By symmetry  $R_A = R_B$

$$+\uparrow \sum F_y = 0 \quad R_A + R_B - 10 - 10 = 0$$

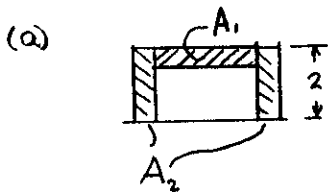
$$R_A = R_B = 10 \text{ kips}$$



From the shear diagram  $V = 10$  kips at  $n-n$ .

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12} (4)(4)^3 - \frac{1}{12} (3)(3)^3 = 14.583 \text{ in}^4$$

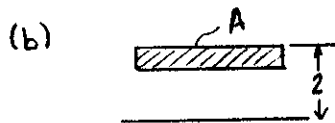


$$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1)$$

$$= 4.625 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17 \text{ ksi}$$



$$Q = A \bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3$$

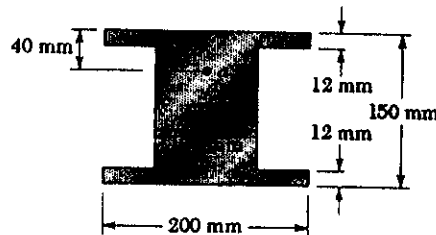
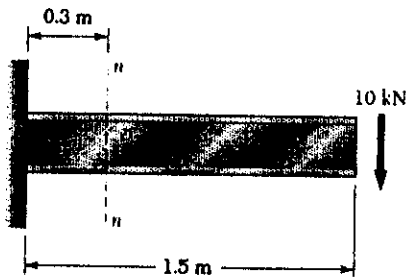
$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40 \text{ ksi}$$



PROBLEM 6.11

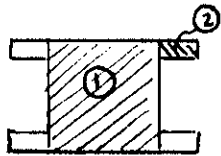
6.9 through 6.12 For the beam and loading shown, consider section  $n-n$  and determine (a) the largest shearing stress in that section, (b) the shearing stress at point  $a$ .



SOLUTION

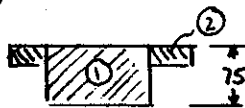
At section  $n-n$

$$V = 10 \text{ kN}$$



$$\begin{aligned} I &= I_1 + 4 I_2 \\ &= \frac{1}{12} b_1 h_1^3 + 4 \left( \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 \right) \\ &= \frac{1}{12} (100)(150)^3 + 4 \left[ \frac{1}{12} (50)(12)^3 + (50)(12)(69)^2 \right] \\ &= 28.125 \times 10^6 + 4 \left[ 0.0072 \times 10^6 + 2.8566 \times 10^6 \right] \\ &= 39.58 \times 10^6 \text{ mm}^4 = 39.58 \times 10^{-6} \text{ m}^4 \end{aligned}$$

(a)



$$\begin{aligned} Q &= A_1 \bar{y}_1 + 2 A_2 \bar{y}_2 \\ &= (100)(75)(37.5) + (2)(50)(12)(69) \\ &= 364.05 \times 10^3 \text{ mm}^3 = 364.05 \times 10^{-6} \text{ m}^3 \\ t &= 100 \text{ mm} = 0.100 \text{ m} \end{aligned}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10 \times 10^3)(364.05 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 920 \times 10^3 \text{ Pa} = 920 \text{ kPa}$$

(b)

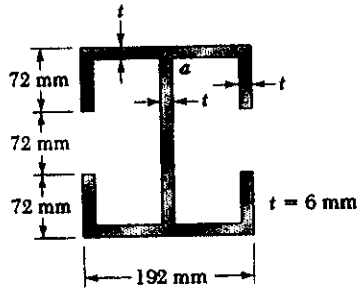
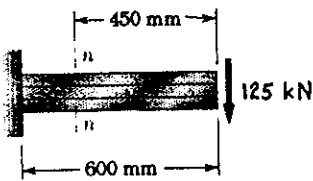


$$\begin{aligned} Q &= A_1 \bar{y}_1 + 2 A_2 \bar{y}_2 \\ &= (100)(40)(55) + (2)(50)(12)(69) \\ &= 302.8 \times 10^3 \text{ mm}^3 = 302.8 \times 10^{-6} \text{ m}^3 \\ t &= 100 \text{ mm} = 0.100 \text{ m} \end{aligned}$$

$$\tau = \frac{VQ}{It} = \frac{(10 \times 10^3)(302.8 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 765 \times 10^3 \text{ Pa} = 765 \text{ kPa}$$

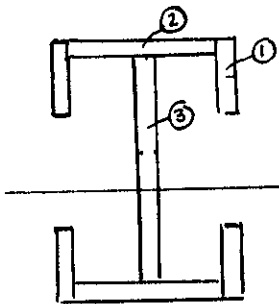
PROBLEM 6.12

6.9 through 6.12 For the beam and loading shown, consider section  $n-n$  and determine (a) the largest shearing stress in that section, (b) the shearing stress at point  $a$ .



SOLUTION

At section  $n-n$   $V = 125 \text{ kN}$



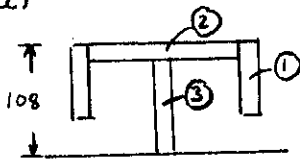
$$I_1 = \frac{1}{12} (6)(72)^3 + (6)(72)(72)^2 = 2.4261 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} (180)(6)^3 + (180)(6)(105)^2 = 11.910 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12} (6)(204)^3 = 4.2448 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + I_3 = 37.77 \times 10^6 \text{ mm}^4 = 37.77 \times 10^{-6} \text{ m}^4$$

(a)



$$Q = 2A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3$$

$$= (2)(6)(72)(72) + (180)(6)(105) + (6)(102)(51)$$

$$= 206.82 \times 10^3 \text{ mm}^3 = 206.82 \times 10^{-6} \text{ m}^3$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{(125 \times 10^3)(206.82 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 114.1 \times 10^6 \text{ Pa} = 114.1 \text{ MPa} \blacktriangleleft$$

(b)

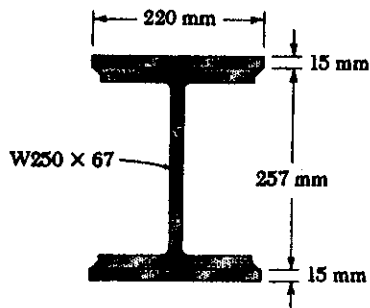
$$Q = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= (2)(6)(72)(72) + (180)(6)(105) = 175.61 \times 10^3 \text{ mm}^3 = 175.61 \times 10^{-6} \text{ m}^3$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(125 \times 10^3)(175.61 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 96.9 \times 10^6 \text{ Pa} = 96.9 \text{ MPa} \blacktriangleleft$$

**PROBLEM 6.13**



$$* d = \frac{257}{2} + \frac{15}{2} = 136 \text{ mm}$$

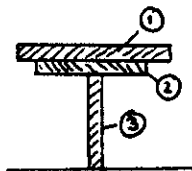
6.13 Two steel plates of 15 × 220-mm rectangular cross section are welded to the W250 × 67 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 100 MPa.

**SOLUTION**

Calculate moment of inertia

Part	A (mm <sup>2</sup> )	d (mm)	Ad <sup>2</sup> (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
Top plate	3300	* 136	61.036	0.062
W250 × 67		0	0	104
Bot. plate	3300	136	61.036	0.062
$\Sigma$			122.072	104.124

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 226.2 \times 10^6 \text{ mm}^4 = 226.2 \times 10^{-6} \text{ m}^4$$



Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	A $\bar{y}$ (10 <sup>3</sup> mm <sup>3</sup> )
① Top plate	3300	136	448.8
② Top flange	3203	120.65	386.4
③ Half web	1004	56.40	56.6
$\Sigma$			891.8

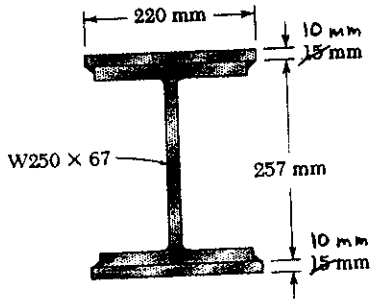
$$Q = \Sigma A\bar{y} = 891.8 \times 10^3 \text{ mm}^3 = 891.8 \times 10^{-6} \text{ m}^3$$

$$t = t_w = 8.9 \text{ mm} = 8.9 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{max}}{Q} = \frac{(226.2 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{891.8 \times 10^{-6}} = 226 \times 10^3 \text{ N} = 226 \text{ kN}$$

PROBLEM 6.14



$$* d = \frac{257}{2} + \frac{10}{2} = 133.5 \text{ mm}$$

6.13 Two steel plates of  $15 \times 220$ -mm rectangular cross section are welded to the W250  $\times$  67 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 100 MPa.

6.14 Solve Prob. 6.13, assuming that the two steel plates are (a) replaced by steel plates of  $10 \times 220$ -mm rectangular cross section, (b) removed.

SOLUTION

Calculate moment of inertia for part (a)

Part	A (mm <sup>2</sup> )	d (mm)	Ad <sup>2</sup> (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
Top plate	2200	*133.5	39.209	0.018
W 250 $\times$ 67		0	0	104
Bot. plate	2200	*133.5	39.209	0.018
$\Sigma$			78.42	104.04

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 182.46 \times 10^6 \text{ mm}^4 = 182.46 \times 10^{-6} \text{ m}^4$$

Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (10 <sup>3</sup> mm <sup>3</sup> )
① Top plate	2200	133.5	293.7
② Top flange	3203	120.65	386.4
③ Half web	1004	56.40	56.6
$\Sigma$			736.7

$$Q = \Sigma A\bar{y} = 736.7 \times 10^3 \text{ mm}^3 = 736.7 \times 10^{-6} \text{ m}^3$$

$$t = t_w = 8.9 \text{ mm} = 8.9 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{max}}{Q} = \frac{(182.46 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{736.7 \times 10^{-6}} = 220 \times 10^3 \text{ N} = 220 \text{ kN}$$

(b)  $I = 104 \times 10^6 \text{ mm}^4 = 104 \times 10^{-6} \text{ m}^4$

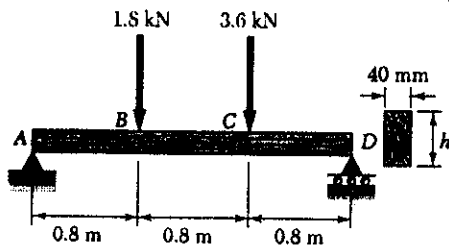
Consider Q for top flange and half web

$$Q = A_2\bar{y}_2 + A_3\bar{y}_3 = 386.4 \times 10^3 + 56.6 \times 10^3 = 443 \times 10^3 \text{ mm}^3 = 443 \times 10^{-6} \text{ m}^3$$

$$V = \frac{It\tau_{max}}{Q} = \frac{(104 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{443 \times 10^{-6}} = 209 \times 10^3 \text{ N} = 209 \text{ kN}$$

**PROBLEM 6.15**

6.15 Knowing that the allowable shearing stress for the timber used is 825 kPa, check whether the design obtained for the beam indicated is acceptable and, if not, redesign the cross section of the beam. Consider the beam of (a) Prob. 5.75, (b) Prob. 5.76.



(a) SOLUTION

From solution to PROBLEM 5.75

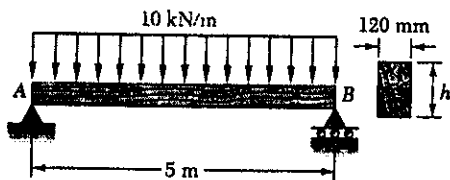
$$|V|_{\max} = 2.4 \text{ kN} \quad h = 173.2 \text{ mm}$$

$$A = bh = (40)(173.2) = 6928 \text{ mm}^2 \\ = 6928 \times 10^{-6} \text{ m}^2$$

For a rectangular cross section  $\tau_{\max} = \frac{3}{2} \frac{|V|_{\max}}{A}$

$$\tau_{\max} = \frac{3}{2} \frac{2.4 \times 10^3}{6928 \times 10^{-6}} = 520 \times 10^3 \text{ Pa} = 520 \text{ kPa} < 825 \text{ kPa}$$

Design is acceptable. ▶



(b) SOLUTION

From solution to PROBLEM 5.76

$$|V|_{\max} = 25 \text{ kN} \quad h = 361 \text{ mm}$$

$$A = bh = (120)(361) = 43.32 \times 10^3 \text{ mm}^2 \\ = 43.32 \times 10^{-3} \text{ m}^2$$

For a rectangular cross section  $\tau_{\max} = \frac{3}{2} \frac{|V|_{\max}}{A}$

$$\tau_{\max} = \frac{3}{2} \frac{25 \times 10^3}{43.32 \times 10^{-3}} = 865 \times 10^3 \text{ Pa} = 865 \text{ kPa} > 825 \text{ kPa}$$

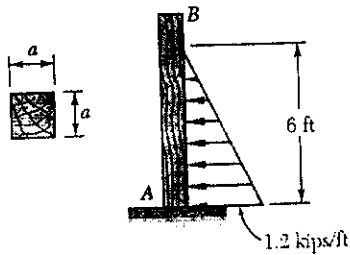
Design is not acceptable. ▶

Resign 
$$A = \frac{3}{2} \frac{|V|_{\max}}{\tau_{\text{all}}} = \frac{3}{2} \frac{25 \times 10^3}{825 \times 10^3} = 45.45 \times 10^{-3} \text{ m}^2 \\ = 45.45 \times 10^3 \text{ mm}^2$$

$$h = \frac{A}{b} = \frac{45.45 \times 10^3}{120} = 379 \text{ mm} \quad h = 379 \text{ mm} \quad \blacktriangleleft$$

PROBLEM 6.16

6.16 Knowing that the allowable shearing stress for the timber used is 130 psi, check whether the design obtained for the beam indicated is acceptable and, if not, redesign the cross section of the beam. Consider the beam of (a) Prob. 5.77, (b) Prob. 5.78.



(a) SOLUTION

$$V_{\max} = \frac{1}{2}(6)(1.2) = 3.6 \text{ kips}$$

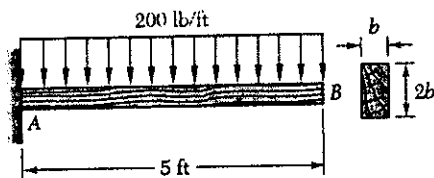
From solution to PROBLEM 5.77

$$a = 6.67 \text{ in.} \quad A = a^2 = 44.45 \text{ in}^2$$

For a rectangular section  $\tau_{\max} = \frac{3}{2} \frac{V_{\max}}{A}$

$$\tau_{\max} = \frac{3}{2} \frac{3.6}{44.45} = 0.1215 \text{ ksi} = 121.5 \text{ psi} < 130 \text{ psi}$$

Design is acceptable.



(b) SOLUTION

From solution to PROBLEM 5.78

$$|V|_{\max} = 1000 \text{ lb} \quad b = 2.95 \text{ in.}$$

$$A = (b)(2b) = 2b^2 = 17.40 \text{ in}^2$$

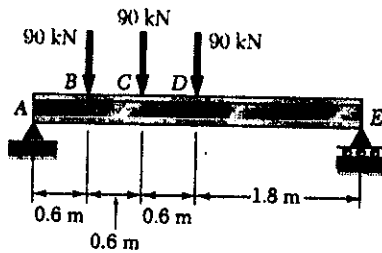
For a rectangular cross section  $\tau_{\max} = \frac{3}{2} \frac{|V|_{\max}}{A}$

$$\tau_{\max} = \frac{3}{2} \frac{1000}{17.40} = 86.2 \text{ psi} < 130 \text{ psi}$$

Design is acceptable.

**PROBLEM 6.17**

6.17 Determine the average shearing stress in the web of the beam indicated and check whether the design obtained earlier for that beam is acceptable, knowing that the allowable shearing stress for the steel used is 100 MPa. Consider the beam of (a) Prob. 5.81, (b) Prob. 5.82.



(a) SOLUTION

From the solution to PROBLEM 5.81

$$|V|_{\max} = 180 \text{ kN}$$

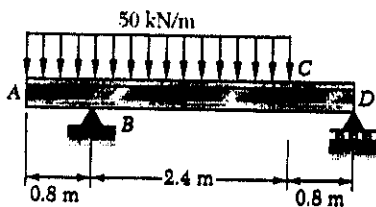
The selected section is W 410 x 60

For that section  $t_w = 7.7 \text{ mm}$   $d = 407 \text{ mm}$

$$A_{\text{web}} = t_w d = 3.13 \times 10^3 \text{ mm}^2 = 3.13 \times 10^{-3} \text{ m}^2$$

$$\tau_{\text{ave}} = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{180 \times 10^3}{3.13 \times 10^{-3}} = 57.4 \times 10^6 \text{ Pa} = 57.4 \text{ MPa} < 100 \text{ MPa}$$

Design is acceptable.



(b) SOLUTION

From the solution to PROBLEM 5.82

$$|V|_{\max} = 80 \text{ kN}$$

The selected section is W 250 x 28.4

For that section  $t_w = 6.4 \text{ mm}$   $d = 260 \text{ mm}$

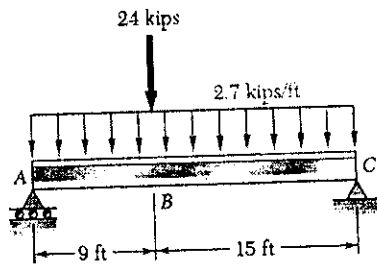
$$A_{\text{web}} = t_w d = (6.4)(260) = 1664 \text{ mm}^2 = 1664 \times 10^{-6} \text{ m}^2$$

$$\tau_{\text{ave}} = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{80 \times 10^3}{1664 \times 10^{-6}} = 48.1 \times 10^6 \text{ Pa} = 48.1 \text{ MPa} < 100 \text{ MPa}$$

Design is acceptable.

PROBLEM 6.18

6.18 Determine the average shearing stress in the web of the beam indicated and check whether the design obtained earlier for that beam is acceptable, knowing that the allowable shearing stress for the steel used is 14.5 ksi. Consider the beam of (a) Prob. 5.83, (b) Prob. 5.84



(a) SOLUTION

From the solution to PROBLEM 5.83

$$|V|_{\max} = 48 \text{ kips}$$

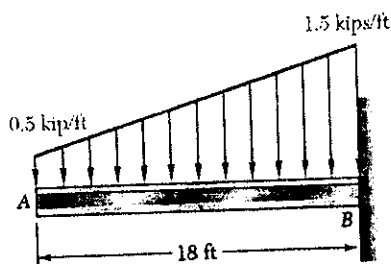
The selected section is W 27 x 84

For that section  $t_w = 0.460 \text{ in.}$   $d = 26.71 \text{ in.}$

$$A_{\text{web}} = t_w d = (0.460)(26.71) = 12.29 \text{ in}^2$$

$$\tau_{\text{ave}} = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{48}{12.29} = 3.91 \text{ ksi} < 14.5 \text{ ksi}$$

Design is acceptable.



(b) SOLUTION

From the solution to PROBLEM 5.84

$$|V|_{\max} = \frac{1}{2}(18)(0.5 + 1.5) = 18 \text{ kips}$$

The selected section is W 18 x 50

For that section  $t_w = 0.355 \text{ in.}$   $d = 17.99 \text{ in.}$

$$A_{\text{web}} = t_w d = (0.355)(17.99) = 6.39 \text{ in}^2$$

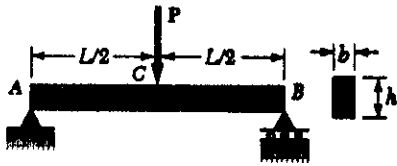
$$\tau_{\text{ave}} = \frac{|V|_{\max}}{A_{\text{web}}} = \frac{18}{6.39} = 2.82 \text{ ksi} < 14.5 \text{ ksi}$$

Design is acceptable.



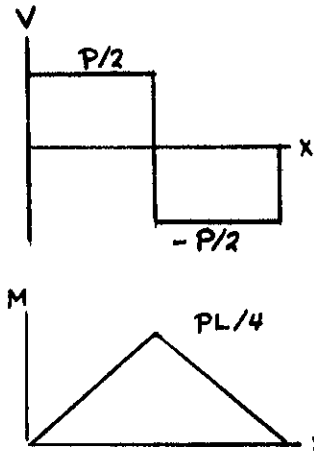
PROBLEM 6.19

6.19 A simply supported timber beam  $AB$  of rectangular cross section carries a single concentrated load  $P$  at its midpoint  $C$ . (a) Show that the ratio  $\tau_m / \sigma_m$  of the maximum values of the shearing and normal stresses in the beam is equal to  $h/2L$ , where  $h$  and  $L$  are, respectively, the depth and the length of the beam. (b) Determine the depth  $h$  and width  $b$  of the beam, knowing that  $L = 2$  m,  $P = 40$  kN,  $\tau_m = 960$  kPa, and  $\sigma_m = 12$  MPa.



SOLUTION

Reactions  $R_A = R_B = P/2$



(1)  $V_{max} = R_A = \frac{P}{2}$

(2)  $A = bh$  for rectangular section

(3)  $\tau_m = \frac{3}{2} \frac{V_{max}}{A} = \frac{3P}{4bh}$  for rectangular section

(4)  $M_{max} = \frac{PL}{4}$

(5)  $S = \frac{1}{6} bh^2$  for rectangular section

(6)  $\sigma_m = \frac{M_{max}}{S} = \frac{3PL}{2bh^2}$

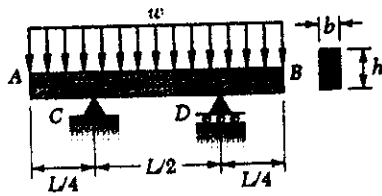
(a)  $\frac{\tau_m}{\sigma_m} = \frac{h}{2L}$

(b) Solving for  $h$   $h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960 \times 10^3)}{12 \times 10^6} = 320 \times 10^{-3}$  m  
 $= 320$  mm

Solving equation (3) for  $b$

$b = \frac{3P}{4h\tau_m} = \frac{(3)(40 \times 10^3)}{(4)(320 \times 10^{-3})(960 \times 10^3)} = 97.7 \times 10^{-3}$  m  
 $= 97.7$  mm

PROBLEM 6.20



6.20 A timber beam  $AB$  of length  $L$  and rectangular cross section carries a uniformly distributed load  $w$  and is supported as shown. (a) Show that the ratio  $\tau_m/\sigma_m$  of the maximum values of the shearing and normal stresses in the beam is equal to  $2h/L$ , where  $h$  and  $L$  are, respectively, the depth and the length of the beam. (b) Determine the depth  $h$  and width  $b$  of the beam, knowing that  $L = 5$  m,  $w = 8$  kN/m,  $\tau_m = 1.08$  MPa, and  $\sigma_m = 12$  MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$

From shear diagram  $|V|_m = \frac{wL}{4}$  (1)

For rectangular section  $A = bh$  (2)

$$I_m = \frac{3}{2} \frac{V_m}{A} = \frac{3wL}{8bh}$$
 (3)

From bending moment diagram

$$|M|_m = \frac{wL^2}{32}$$
 (4)

For a rectangular cross section

$$S = \frac{1}{6}bh^2$$
 (5)

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2}$$
 (6)

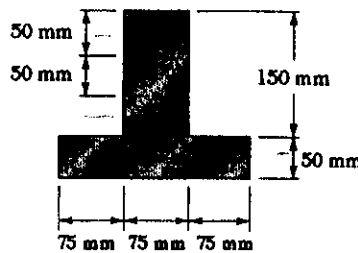
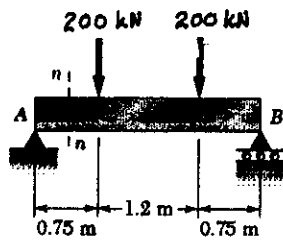
(a) Dividing eq. (3) by eq. (6)  $\frac{\tau_m}{\sigma_m} = \frac{2h}{L}$

(b) Solving for  $h$   $h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08 \times 10^6)}{2(12 \times 10^6)} = 225 \times 10^{-3} \text{ m} = 225 \text{ mm}$

Solving eq. (3) for  $b$   $b = \frac{3wL}{8hI_m} = \frac{(3)(8 \times 10^3)(5)}{(8)(225 \times 10^{-3})(1.08 \times 10^6)} = 61.7 \times 10^{-3} \text{ m} = 61.7 \text{ mm}$

PROBLEM 6.21

6.21 and 6.22 For the beam and loading shown, consider section  $n-n$  and determine the shearing stress at (a) point  $a$ , (b) point  $b$ .

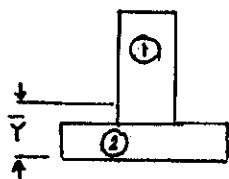


SOLUTION

$$R_A = R_B = 200 \text{ kN}$$

$$\text{At section } n-n \quad V = 200 \text{ kN}$$

locate centroid and calculate moment of inertia.

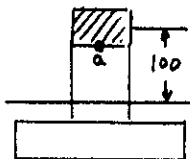


Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (10 <sup>3</sup> mm <sup>3</sup> )	d (mm)	$Ad^2$ (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
①	11250	125	1406.25	50	28.125	21.094
②	11250	25	281.25	50	28.125	2.344
$\Sigma$	22500		1687.5		56.25	23.438

$$\bar{Y} = \frac{1687.5 \times 10^3}{22500} = 75 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 79.688 \times 10^6 \text{ mm}^4 = 79.688 \times 10^{-6} \text{ m}^4$$

(a)

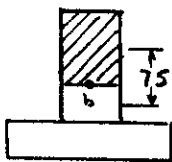


$$Q_a = A\bar{y} = (75)(50)(100) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(375 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 12.55 \times 10^6 \text{ Pa} = 12.55 \text{ MPa}$$

(b)



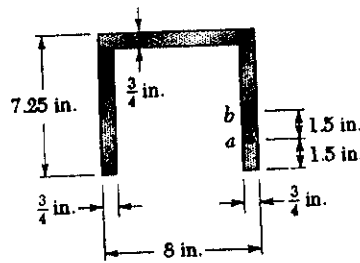
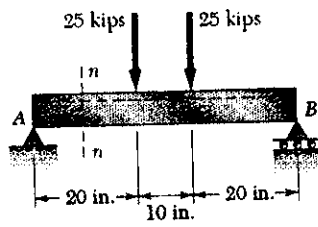
$$Q_b = A\bar{y} = (75)(100)(75) = 562.5 \times 10^3 \text{ mm}^3 = 562.5 \times 10^{-6} \text{ m}^3$$

$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(200 \times 10^3)(562.5 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 18.82 \times 10^6 \text{ Pa} = 18.82 \text{ MPa}$$

PROBLEM 6.22

6.21 and 6.22 For the beam and loading shown, consider section  $n-n$  and determine the shearing stress at (a) point a, (b) point b.

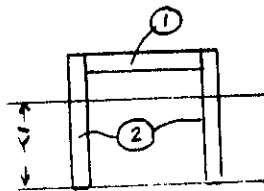


SOLUTION

$R_A = R_B = 25 \text{ kips}$

At section  $n-n$   $V = 25 \text{ kips}$ .

Locate centroid and calculate moment of inertia.

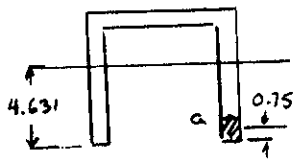


Part	$A \text{ (in}^2\text{)}$	$\bar{y} \text{ (in)}$	$A\bar{y} \text{ (in}^3\text{)}$	$d \text{ (in)}$	$Ad^2 \text{ (in}^4\text{)}$	$\bar{I} \text{ (in}^4\text{)}$
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
$\Sigma$	15.75		72.94		35.56	47.86

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in.}$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$

(a)

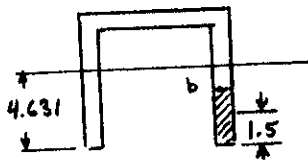


$Q_a = A\bar{y} = \left(\frac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$

$t = \frac{3}{4} = 0.75 \text{ in}$

$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} = 1.745 \text{ ksi}$

(b)



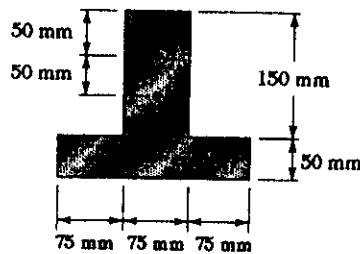
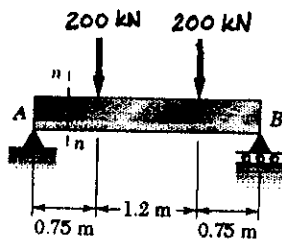
$Q_b = A\bar{y} = \left(\frac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$

$t = 0.75 \text{ in.}$

$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} = 2.82 \text{ ksi}$

PROBLEM 6.23

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section n-n.

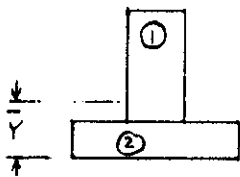


SOLUTION

$$R_A = R_B = 200 \text{ kN}$$

$$\text{At section n-n } V = 200 \text{ kN}$$

Locate centroid and calculate moment of inertia.

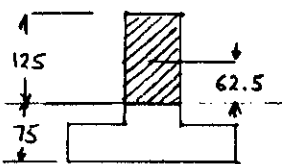


Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ (10 <sup>3</sup> mm <sup>3</sup> )	d (mm)	$A d^2$ (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
①	11250	125	1406.25	50	28.125	21.094
②	11250	25	281.25	50	28.125	2.344
$\Sigma$	22500		1687.5		56.25	23.438

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1687.5 \times 10^3}{22500} = 75 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 79.688 \times 10^6 \text{ mm}^4 = 79.688 \times 10^{-6} \text{ m}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



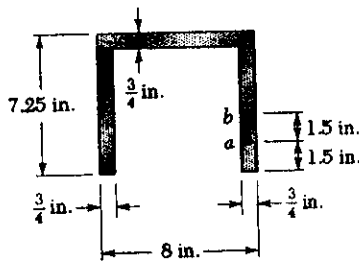
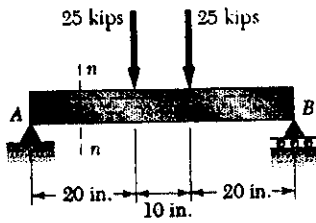
$$Q = A\bar{y} = (75)(125)(62.5) = 585.94 \times 10^3 \text{ mm}^3 = 585.94 \times 10^{-6} \text{ m}^3$$

$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(200 \times 10^3)(585.94 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 19.61 \times 10^6 \text{ Pa} = 19.61 \text{ MPa}$$

PROBLEM 6.24

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section n-n.

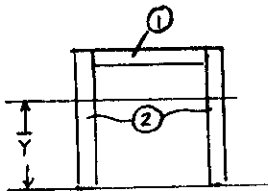


SOLUTION

$$R_A = R_B = 25 \text{ kips}$$

$$\text{At section } n-n \quad V = 25 \text{ kips}$$

Locate centroid and calculate moment of inertia

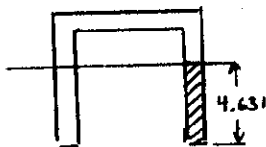


Part	A (in <sup>2</sup> )	$\bar{y}$ (in)	$A\bar{y}$ (in <sup>3</sup> )	d (in)	$Ad^2$ (in <sup>4</sup> )	$\bar{I}$ (in <sup>4</sup> )
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
$\Sigma$	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



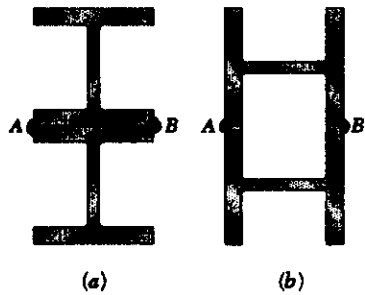
$$Q = A\bar{y} = \left(\frac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \frac{3}{4} = 0.75 \text{ in}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$

**PROBLEM 6.25**

6.25 Two W200 × 46.1 rolled steel sections are to be welded at A and B in either of the two ways shown to form a composite beam. Knowing that for each weld the allowable horizontal shearing force is 500 kN per meter of weld, determine the maximum allowable shear in the composite beam for each of the two arrangements shown.



**SOLUTION**

For rolled steel section W 200 × 46.1

$$A = 5860 \text{ mm}^2 \quad d = 203 \text{ mm} \quad b_f = 203 \text{ mm}$$

$$I_x = 45.5 \times 10^6 \text{ mm}^4 \quad I_y = 15.3 \times 10^6 \text{ mm}^4$$

$$(a) \quad I = 2 \left[ I_x + A \left( \frac{d}{2} \right)^2 \right] = 2 \left[ 45.5 \times 10^6 + (5860) \left( \frac{203}{2} \right)^2 \right] = 211.7 \times 10^6 \text{ mm}^4 \\ = 211.7 \times 10^{-6} \text{ m}^4$$

$$Q = A \frac{d}{2} = (5860) \left( \frac{203}{2} \right) = 594.8 \times 10^3 \text{ mm}^3 = 594.8 \times 10^{-6} \text{ m}^3$$

$$q = 500 \text{ kN/m for one weld. For 2 welds } q_{all} = 1000 \text{ kN/m}$$

$$q_{all} = \frac{VQ}{I} \quad V_{all} = \frac{I q_{all}}{Q} = \frac{(211.7 \times 10^{-6})(1000 \times 10^3)}{594.8 \times 10^{-6}} = 356 \times 10^3 \text{ N} \\ = 356 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad I = 2 \left[ I_y + A \left( \frac{b_f}{2} \right)^2 \right] = 2 \left[ 15.3 \times 10^6 + 5860 \left( \frac{203}{2} \right)^2 \right] = 151.34 \times 10^6 \text{ mm}^4 \\ = 151.34 \times 10^{-6} \text{ m}^4$$

$$Q = A \frac{b_f}{2} = (5860) \left( \frac{203}{2} \right) = 594.8 \times 10^3 \text{ mm}^3 = 594.8 \times 10^{-6} \text{ m}^3$$

$$V_{all} = \frac{I q_{all}}{Q} = \frac{(151.34 \times 10^{-6})(1000 \times 10^3)}{594.8 \times 10^{-6}} = 254 \times 10^3 \text{ N} = 254 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 6.26

6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear  $V$ . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant  $k$  in the following expression for the maximum shearing stress



$$\tau_{\max} = k \frac{V}{A}$$

where  $A$  is the cross-sectional area of the beam.

SOLUTION

$$I = \frac{\pi}{4} c^4 \quad \text{and} \quad A = \pi c^2$$



For semicircle  $A_s = \frac{\pi}{2} c^2 \quad \bar{y} = \frac{4c}{3\pi}$

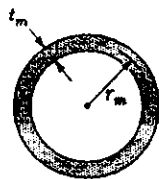
$$Q = A_s \bar{y} = \frac{\pi}{2} c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3} c^3$$

$\tau_{\max}$  occurs at center where  $t = 2c$

$$\tau_{\max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3} c^3}{\frac{\pi}{4} c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3} \frac{V}{A} \quad k = \frac{4}{3} = 1.333 \quad \blacktriangleleft$$

PROBLEM 6.27

6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear  $V$ . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant  $k$  in the following expression for the maximum shearing stress



$$\tau_{\max} = k \frac{V}{A}$$

where  $A$  is the cross-sectional area of the beam.

SOLUTION

For a thin walled circular section  $A = 2\pi r_m t_m$

$$J = A r_m^2 = 2\pi r_m^3 t_m \quad I = \frac{1}{2} J = \pi r_m^3 t_m$$



For a semicircular arc  $\bar{y} = \frac{2r_m}{\pi}$

$$A_s = \pi r_m t_m \quad Q = A_s \bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$$

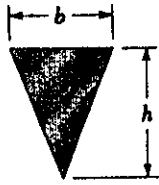
$$t = 2t_m$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A} \quad k = 2.00 \quad \blacktriangleleft$$



PROBLEM 6.28

6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear  $V$ . Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant  $k$  in the following expression for the maximum shearing stress

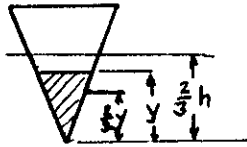


$$\tau_{\max} = k \frac{V}{A}$$

where  $A$  is the cross-sectional area of the beam.

SOLUTION

$$A = \frac{1}{2} b h \quad I = \frac{1}{36} b h^3$$



For a cut at location  $y$

$$A(y) = \frac{1}{2} \left( \frac{b}{h} y \right) y = \frac{b y^2}{2h}$$

$$\bar{y}(y) = \frac{2}{3} h - \frac{2}{3} y$$

$$Q(y) = A \bar{y} = \frac{b y^2}{3} (h - y)$$

$$t(y) = \frac{b y}{h}$$

$$\tau(y) = \frac{VQ}{It} = \frac{V \frac{b y^2}{3} (h - y)}{\left( \frac{1}{36} b h^3 \right) \frac{b y}{h}} = \frac{12 V y (h - y)}{b h^3} = \frac{12 V}{b h^3} (h y - y^2)$$

To find location of maximum of  $\tau$ , set  $\frac{d\tau}{dy} = 0$

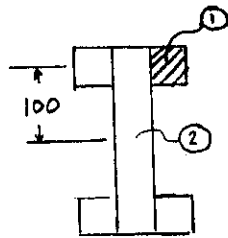
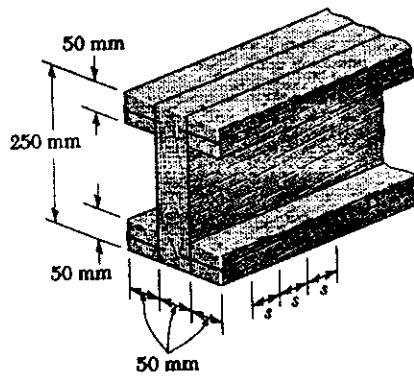
$$\frac{d\tau}{dy} = \frac{12 V}{b h^3} (h - 2y_m) = 0 \quad y_m = \frac{1}{2} h$$

$$\tau_m = \frac{12 V}{b h^3} (h y_m - y_m^2) = \frac{12 V}{b h^3} \left[ \frac{1}{2} h^2 - \left( \frac{1}{2} h \right)^2 \right] = \frac{3 V}{b h^2} = \frac{3}{2} \frac{V}{A}$$

$$k = \frac{3}{2} = 1.500 \quad \blacktriangleleft$$

PROBLEM 6.29

6.29 The built-up wooden beam shown is subjected to a vertical shear of 5 kN. Knowing that the longitudinal spacing of the nails is  $s = 45$  mm and that each nail is 90 mm long, determine the shearing force in each nail.



SOLUTION

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12} (50)(50)^3 + (50)(50)(100)^2 = 25.52 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12} (50)(250)^3 = 65.10 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + I_2 = 167.18 \times 10^6 \text{ mm}^4 = 167.18 \times 10^{-6} \text{ m}^4$$

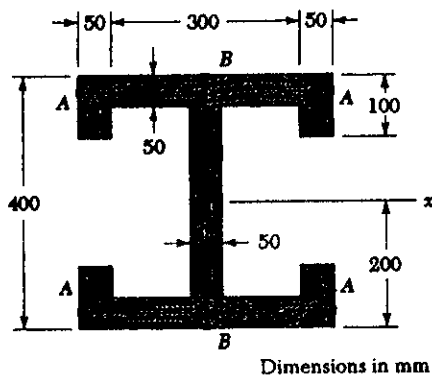
$$Q = Q_1 = A_1 \bar{y}_1 = (50)(50)(100) = 250 \times 10^3 \text{ mm}^3$$

$$= 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(5 \times 10^3)(250 \times 10^{-6})}{167.18 \times 10^{-6}} = 7.477 \times 10^3 \text{ N/m}$$

$$F_{\text{nail}} = qs = (7.477 \times 10^3)(45 \times 10^{-3}) = 336 \text{ N}$$

**PROBLEM 6.30**



6.30 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (a) at A, (b) at B. (Given:  $I_x = 1.504 \times 10^9 \text{ mm}^4$ .)

**SOLUTION**

$$I_x = 1.504 \times 10^9 \text{ mm}^4 = 1504 \times 10^{-6} \text{ m}^4$$

$$s_A = 60 \text{ mm} = 0.060 \text{ m}$$

$$s_B = 25 \text{ mm} = 0.025 \text{ m}$$

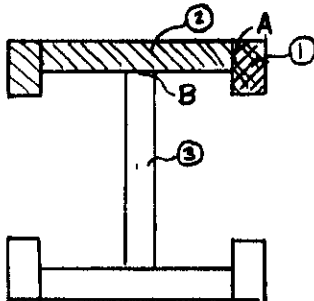
$$(a) \quad Q_A = Q_1 = A_1 \bar{y}_1 = (50)(100)(150) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$$

$$\begin{aligned} F_A &= q_A s_A \\ &= \frac{V Q_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}} \\ &= 239 \text{ N} \end{aligned}$$

$$(b) \quad Q_2 = A_2 \bar{y}_2 = (300)(50)(175) = 2625 \times 10^3 \text{ mm}^3$$

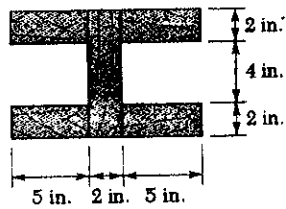
$$\begin{aligned} Q_B &= 2Q_1 + Q_2 = 4125 \times 10^3 \text{ mm}^3 \\ &= 4125 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$F_B = q_B s_B = \frac{V Q_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}} = 549 \text{ N}$$



PROBLEM 6.31

6.31 The built-up beam shown is made up by gluing together five planks. Knowing that the allowable average shearing stress in the glued joints is 60 psi, determine the largest permissible vertical shear in the beam.



SOLUTION

$$I_1 = \frac{1}{12}(5)(2)^3 + (5)(2)(3)^2 = 93.33 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(8)^3 = 85.33 \text{ in}^4$$

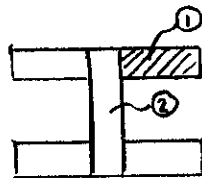
$$I = 4I_1 + I_2 = 458.66 \text{ in}^4$$

$$Q = A_1 \bar{y}_1 = (5)(2)(3) = 30 \text{ in}^3$$

For each glued joint  $t = 2 \text{ in.}$

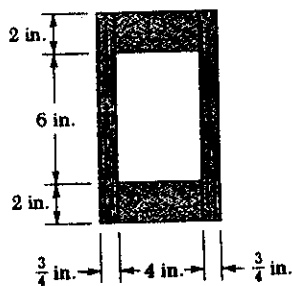
$$V = \frac{I t \tau}{Q} = \frac{(458.66)(2)(60)}{30} = 1835 \text{ lb.}$$

$$\tau = \frac{VQ}{It}$$



PROBLEM 6.32

6.32 The built-up beam shown is made up by gluing together two  $\frac{3}{4} \times 10$ -in. plywood strips and two  $2 \times 4$ -in. planks. Knowing that the allowable average shearing stress in the glued joints is 50 psi, determine the largest permissible vertical shear in the beam.



SOLUTION

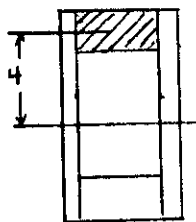
$$I = \frac{1}{12}(5.5)(10)^3 - \frac{1}{12}(4)(6)^3 = 386.33 \text{ in}^4$$

$$Q = A \bar{y} = (4)(2)(4) = 32 \text{ in}^3$$

$$t = 2 + 2 = 4 \text{ in}$$

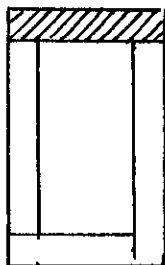
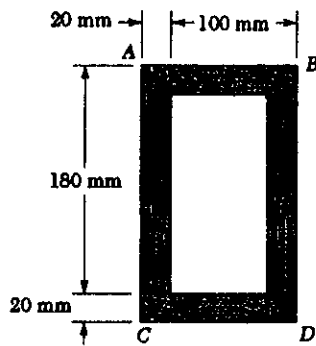
$$\tau = \frac{VQ}{It}$$

$$V = \frac{I t \tau}{Q} = \frac{(386.33)(4)(50)}{32} = 2410 \text{ lb.}$$



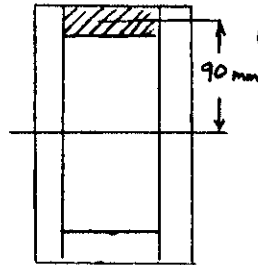
**PROBLEM 6.33**

6.33 Two  $20 \times 100$ -mm and two  $20 \times 180$ -mm boards are glued together as shown to form a  $120 \times 200$ -mm box beam. Knowing that the beam is subjected to a vertical shear of  $3.5$  kN, determine the average shearing stress in the glued joint (a) at A, (b) at B.



**SOLUTION**

$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4 = 52.693 \times 10^{-6} \text{ m}^4$$



(a)  $Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3 = 144 \times 10^{-6} \text{ m}^3$

$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$\tau_A = \frac{VQ_A}{It_A} = \frac{(3.5 \times 10^3)(144 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} = 239 \times 10^3 \text{ Pa} = 239 \text{ kPa}$

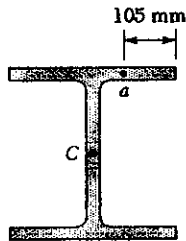
(b)  $Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$

$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$\tau_B = \frac{VQ_B}{It_B} = \frac{(3.5 \times 10^3)(216 \times 10^{-6})}{(52.693 \times 10^{-6})(0.040)} = 359 \times 10^3 \text{ Pa} = 359 \text{ kPa}$

PROBLEM 6.34

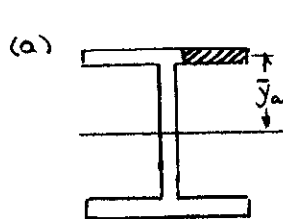
6.34 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (a) at point A, (b) at the centroid C of the section.



SOLUTION

For W360 × 122,  $d = 363 \text{ mm}$ ,  $b_f = 257 \text{ mm}$ ,  $t_f = 21.70 \text{ mm}$ ,  $t_w = 13.0 \text{ mm}$

$$I = 365 \times 10^6 \text{ mm}^4 = 365 \times 10^{-6} \text{ m}^4$$



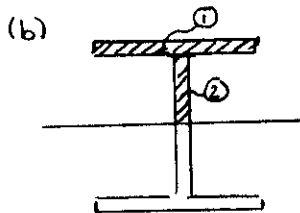
$$A_a = (105)(21.70) = 2278.5 \text{ mm}^2$$

$$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$Q_a = A_a \bar{y}_a = 388.8 \times 10^3 \text{ mm}^3 = 388.8 \times 10^{-6} \text{ m}^3$$

$$t_a = t_f = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6 \text{ Pa} = 12.27 \text{ MPa}$$



$$A_1 = b_f t_f = (257)(21.70) = 5577 \text{ mm}^2$$

$$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$$

$$A_2 = t_w \left( \frac{d}{2} - t_f \right) = (13.0)(159.8) = 2077 \text{ mm}^2$$

$$\bar{y}_2 = \frac{1}{2} \left( \frac{d}{2} - t_f \right) = 79.9 \text{ mm}$$

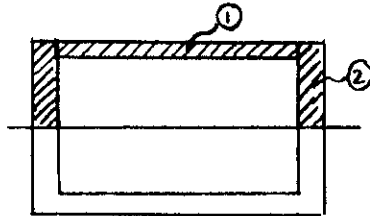
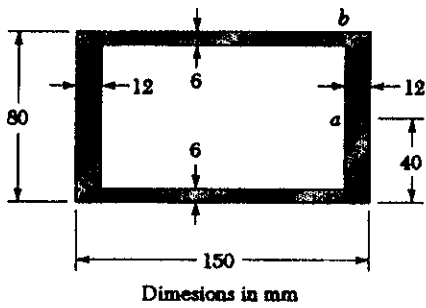
$$Q_c = \sum A \bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3 = 1117.7 \times 10^{-6} \text{ m}^3$$

$$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa}$$

PROBLEM 6.35

6.35 and 6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (a) point a, (b) point b.



SOLUTION

$$I = \frac{1}{12} (150)(80)^3 - \frac{1}{12} (126)(68)^3$$

$$= 3.098 \times 10^6 \text{ mm}^4 = 3.098 \times 10^{-6} \text{ m}^4$$

$$(a) \quad Q_a = A_1 \bar{y}_1 + 2A_2 \bar{y}_2$$

$$= (126)(6)(37) + (2)(12)(40)(20)$$

$$= 47.172 \times 10^3 \text{ mm}^3 = 47.172 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(150 \times 10^3)(47.172 \times 10^{-6})}{(3.098 \times 10^{-6})(0.024)}$$

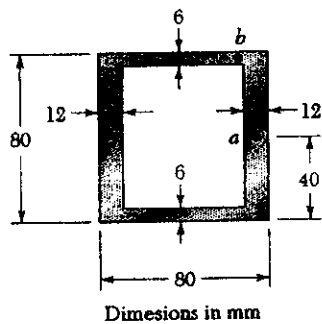
$$= 95.2 \times 10^6 \text{ Pa} = 95.2 \text{ MPa}$$

$$(b) \quad Q_b = A_1 \bar{y}_1 = (126)(6)(37) = 27.97 \times 10^3 \text{ mm}^3 = 27.97 \times 10^{-6} \text{ m}^3$$

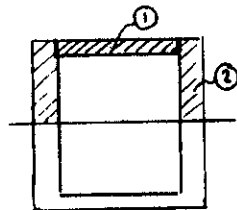
$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(150 \times 10^3)(27.97 \times 10^{-6})}{(3.098 \times 10^{-6})(0.012)} = 112.9 \times 10^6 \text{ Pa} = 112.9 \text{ MPa}$$

PROBLEM 6.36



Dimensions in mm



6.35 and 6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (a) point a, (b) point b.

SOLUTION

$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(56)^3 = 1.9460 \times 10^6 \text{ mm}^4$$

$$= 1.946 \times 10^{-6} \text{ m}^4$$

$$(a) Q_a = A_1 \bar{y}_1 + 2A_2 \bar{y}_2$$

$$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3$$

$$= 31.632 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(150 \times 10^3)(31.632 \times 10^{-6})}{(1.946 \times 10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa}$$

$$= 101.6 \text{ MPa}$$

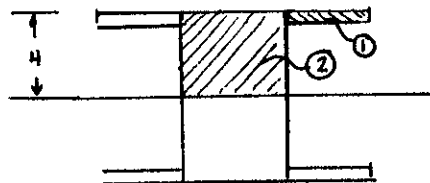
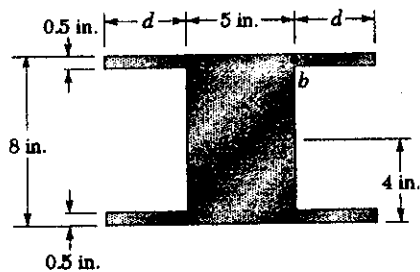
$$(b) Q_b = A_1 \bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3$$

$$= 12.432 \times 10^{-6} \text{ m}^3$$

$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(150 \times 10^3)(12.432 \times 10^{-6})}{(1.946 \times 10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa} = 79.9 \text{ MPa}$$

PROBLEM 6.37



6.37 The vertical shear is 1200 lb in a beam having the cross section shown. Knowing that  $d = 4$  in., determine the shearing stress (a) at point a, (b) at point b.

SOLUTION

$$I_1 = \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(3.75)^2 = 28.167 \text{ in}^4$$

$$I_2 = \frac{1}{3}(5)(4)^3 = 106.67 \text{ in}^4$$

$$I = 4I_1 + 2I_2 = 326 \text{ in}^4$$

$$(a) Q_a = 2A_1 \bar{y}_1 + A_2 \bar{y}_2$$

$$= (2)(4)(0.5)(3.75) + (5)(4)(2) = 55 \text{ in}^3$$

$$t_a = 5 \text{ in.}$$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(1200)(55)}{(326)(5)} = 40.5 \text{ psi}$$

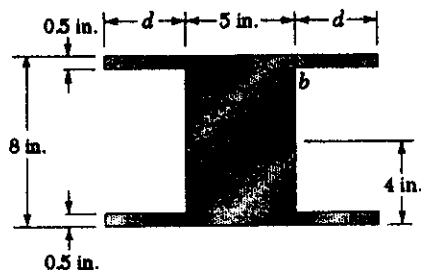
$$(b) Q_b = A_1 \bar{y}_1 = (4)(0.5)(3.75) = 7.5 \text{ in}^3 \quad t_b = 0.5 \text{ in.}$$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(1200)(7.5)}{(326)(0.5)} = 55.2 \text{ psi}$$



PROBLEM 6.38

6.38 The vertical shear is 1200 lb in a beam having the cross section shown. Determine (a) the distance  $d$  for which  $\tau_a = \tau_b$ . (b) the corresponding shearing stress at points  $a$  and  $b$ .



SOLUTION

$$A_1 = 0.5 d \text{ in}^2, \bar{y}_1 = 3.75 \text{ in} \quad t_b = 0.5 \text{ in}$$

$$A_2 = (5)(4) = 20 \text{ in}^2, \bar{y}_2 = 2 \text{ in} \quad t_a = 5 \text{ in}$$

$$Q_b = A_1 \bar{y}_1 = 1.875 d \text{ in}^3$$

$$\tau_b = \frac{V Q_b}{I t_b} = \frac{V}{I} \frac{1.875 d}{0.5} = 3.75 \frac{V d}{I}$$

$$Q_a = A_2 \bar{y}_2 + 2 Q_b = (20)(2) + (2)(1.875 d) \\ = 40 + 3.75 d$$

$$t_a = 5 \text{ in.}$$

$$(a) \quad \tau_a = \frac{V Q_a}{I t_a} = \frac{V(40 + 3.75 d)}{I (5)} = 8 \frac{V}{I} + 0.75 \frac{V d}{I} = \tau_b = 3.75 \frac{V d}{I}$$

$$8 + 0.75 d = 3.75 d \quad d = \frac{8}{3} = 2.667 \text{ in.}$$

$$(b) \quad I_1 = \frac{1}{12} (2.667)(0.5)^3 + (2.667)(0.5)(3.75)^2 = 18.78 \text{ in}^4$$

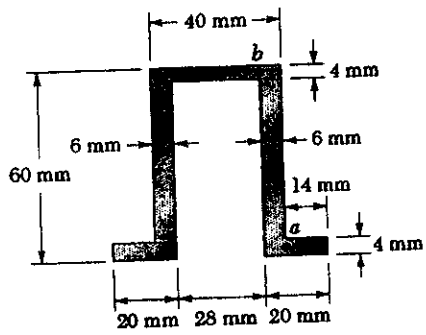
$$I_2 = \frac{1}{3} (5)(4)^3 = 106.67 \text{ in}^4$$

$$I = 4 I_1 + 2 I_2 = 288.45 \text{ in}^4$$

$$\tau_a = \tau_b = 3.75 \frac{V d}{I} = \frac{(3.75)(1200)(2.667)}{288.45} = 41.6 \text{ psi}$$

PROBLEM 6.39

6.39 Knowing that a given vertical shear  $V$  causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress (a) at point  $a$ , (b) at point  $b$ .

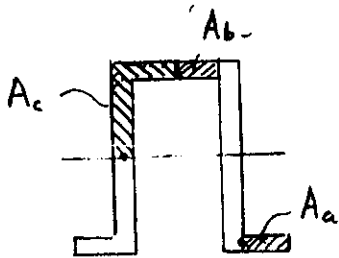


SOLUTION

Neutral axis lies 30 mm above bottom

$$\tau_c = \frac{VQ_c}{It} \quad \tau_a = \frac{VQ_a}{It_a} \quad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a} \quad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$



$$Q_c = (6)(30)(15) + (14)(4)(28) = 4260 \text{ mm}^3$$

$$t_c = 6 \text{ mm}$$

$$Q_a = (14)(4)(28) = 1568 \text{ mm}^3$$

$$t_a = 4 \text{ mm}$$

$$Q_b = (14)(4)(28) = 1568 \text{ mm}^3$$

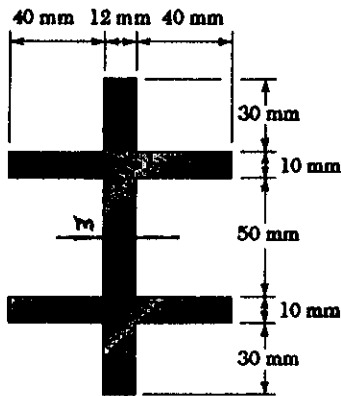
$$t_b = 4 \text{ mm}$$

$$\tau_c = 75 \text{ MPa}$$

$$\tau_a = \frac{Q_a}{Q_c} \cdot \frac{t_c}{t_a} \cdot \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

$$\tau_b = \frac{Q_b}{Q_c} \cdot \frac{t_c}{t_b} \cdot \tau_c = \frac{1568}{4260} \cdot \frac{6}{4} \cdot 75 = 41.4 \text{ MPa}$$

PROBLEM 6.40



6.40 Knowing that a given vertical shear  $V$  causes a maximum shearing stress of 50 MPa in a thin-walled member having the cross section shown, determine the corresponding shearing stress (a) at point  $a$ , (b) at point  $b$ , (c) at point  $c$ .

SOLUTION

$$Q_a = (12)(30)(25 + 10 + 15) = 18 \times 10^3 \text{ mm}^3$$

$$Q_b = (40)(10)(25 + 5) = 12 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_a + 2Q_b + (12)(10)(25 + 5) = 45.6 \times 10^3 \text{ mm}^3$$

$$Q_m = Q_c + (12)(25)\left(\frac{25}{2}\right) = 49.35 \times 10^3 \text{ mm}^3$$

$$t_a = t_c = t_m = 12 \text{ mm}$$

$$t_b = 10 \text{ mm}$$

$$\tau_m = 50 \text{ MPa}$$

$$(a) \quad \frac{\tau_a}{\tau_m} = \frac{Q_a}{Q_m} \cdot \frac{t_m}{t_a} = \frac{18}{49.35} \cdot \frac{12}{12} = 0.3647$$

$$\tau_a = 18.23 \text{ MPa} \quad \blacktriangleleft$$

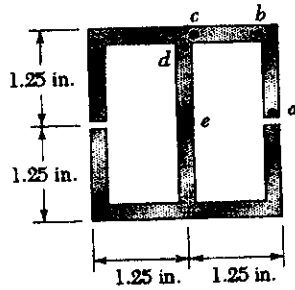
$$(b) \quad \frac{\tau_b}{\tau_m} = \frac{Q_b}{Q_m} \cdot \frac{t_m}{t_b} = \frac{12}{49.35} \cdot \frac{12}{10} = 0.2918$$

$$\tau_b = 14.59 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \frac{\tau_c}{\tau_m} = \frac{Q_c}{Q_m} \cdot \frac{t_m}{t_c} = \frac{45.6}{49.35} \cdot \frac{12}{12} = 0.9240$$

$$\tau_c = 46.2 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 6.41



6.41 and 6.42 The extruded beam shown has a uniform wall thickness of  $\frac{1}{8}$  in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.

SOLUTION

$$I = \frac{1}{12} (2.50)(2.50)^3 - \frac{1}{12} (2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

$$t = 0.125 \text{ in at all sections.}$$

$$V = 2 \text{ kips}$$

$$Q_a = 0 \quad \tau_a = \frac{VQ_a}{It} = 0$$

$$Q_b = (0.125)(1.25) \left( \frac{1.25}{2} \right) = 0.09766 \text{ in}^3$$

$$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} = 1.26 \text{ ksi}$$

$$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in}^3$$

$$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} = 3.30 \text{ ksi}$$

$$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$$

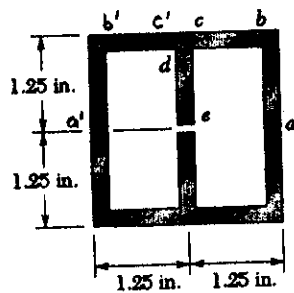
$$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} = 6.84 \text{ ksi}$$

$$Q_e = Q_d + (0.125)(1.125) \left( \frac{1.125}{2} \right) = 0.60839$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} = 7.86 \text{ ksi}$$

**PROBLEM 6.42**

6.41 and 6.42 The extruded beam shown has a uniform wall thickness of  $\frac{1}{8}$  in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



**SOLUTION**

$$I = \frac{1}{12} (2.50)(2.50)^3 - \frac{1}{12} (2.125)(2.125)^3 = 1.2382 \text{ in}^4$$

Add symmetric points  $c'$ ,  $b'$ , and  $a'$ .

$$Q_e = 0$$

$$Q_d = (0.125)(1.125)(\frac{1.125}{2}) = 0.07910 \text{ in}^3 \quad t_d = 0.125 \text{ in}$$

$$Q_c = Q_e = (0.125)^2(1.1875) = 0.09765 \text{ in}^3 \quad t_c = 0.25 \text{ in.}$$

$$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3 \quad t_b = 0.25 \text{ in.}$$

$$Q_a = Q_b + (2)(0.125)(1.25)(\frac{1.25}{2}) = 0.60839 \text{ in}^3 \quad t_a = 0.25 \text{ in}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60839)}{(1.2382)(0.25)} = 3.93 \text{ ksi} \quad \blacktriangleleft$$

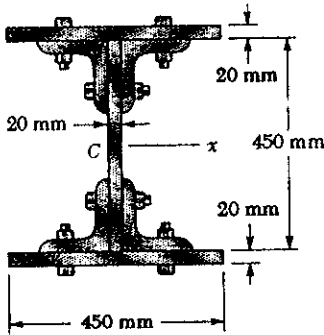
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.125)} = 1.02 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_e = \frac{VQ_e}{It_e} = 0 \quad \blacktriangleleft$$

PROBLEM 6.43



6.43 Three  $20 \times 450$ -mm steel plates are bolted to four  $L152 \times 152 \times 19.0$  angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (Given:  $I_x = 1896 \times 10^6 \text{ mm}^4$ .)

SOLUTION

Flange:  $I_f = \frac{1}{12} (450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6 \text{ mm}^4$

Web:  $I_w = \frac{1}{12} (20)(450)^3 = 151.9 \times 10^6 \text{ mm}^4$

Angle:  $\bar{I} = 11.6 \times 10^6 \text{ m}^4$ ,  $A = 5420 \text{ mm}^2$   
 $y = 44.9 \text{ mm}$   $d = 225 - 44.9 = 180.1 \text{ mm}$

$I_a = \bar{I} + Ad^2 = 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6 \text{ mm}^4$

$I = 2I_f + I_w + 4I_a = 1896 \times 10^6 \text{ mm}^4 = 1896 \times 10^{-6} \text{ m}^4$



$Q_f = (450)(20)(235) = 2115 \times 10^3 \text{ mm}^3$

$Q_a = (5420)(180.1) = 976 \times 10^3 \text{ mm}^3$

$Q = Q_f + 2Q_a = 4067 \times 10^3 \text{ mm}^3 = 4067 \times 10^{-6} \text{ m}^3$

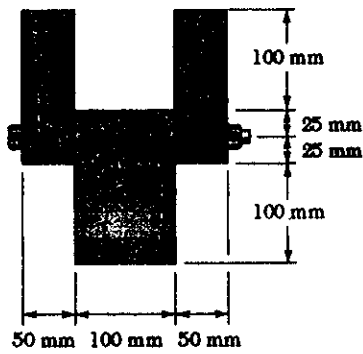
$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} (22)^2 = 380.1 \text{ mm}^2 = 380.1 \times 10^{-6} \text{ m}^2$

$F_{bolt} = 2 \sum_{bolt} A_b = (2)(90 \times 10^6)(380.1 \times 10^{-6}) = 68.42 \times 10^3 \text{ N}$

$q_{bolt} = \frac{F_{bolt}}{s} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3 \text{ N/m}$

$q = \frac{VQ}{I}$   $V_{all} = \frac{I q_{bolt}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3 \text{ N}$   
 $= 255 \text{ kN}$

PROBLEM 6.44



6.44 A beam consists of three planks connected by steel bolts with a longitudinal spacing of 225 mm. Knowing that the shear in the beam is vertical and equal to 6 kN and that the allowable average shearing stress in each bolt is 60 MPa, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A(\text{mm}^2)$	$\bar{y}(\text{mm})$	$A\bar{y}^2 (10^6 \text{mm}^4)$	$\bar{I} (10^6 \text{mm}^4)$
①	7500	50	18.75	14.06
②	7500	50	18.75	14.06
③	15000	-50	37.50	28.12
$\Sigma$			75.00	56.25

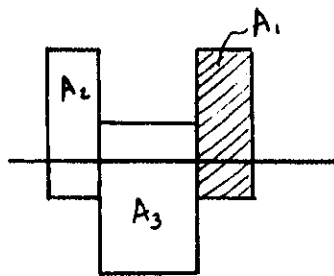
$$I = \Sigma A\bar{y}^2 + \Sigma \bar{I} = 131.25 \times 10^6 \text{ mm}^4 = 131.25 \times 10^{-6} \text{ m}^4$$

$$Q = A_1 \bar{y}_1 = (7500)(50) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

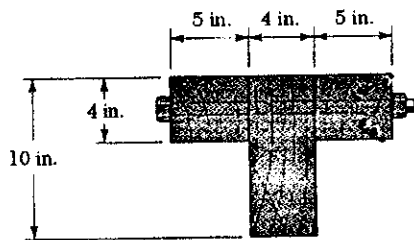
$$F_{\text{bolt}} = \tau_{\text{bolt}} A_{\text{bolt}} = q_s = \frac{VQS}{I}$$

$$A_{\text{bolt}} = \frac{VQS}{\tau_{\text{bolt}} I} = \frac{(6 \times 10^3)(375 \times 10^{-6})(0.225)}{(60 \times 10^6)(131.25 \times 10^{-6})} = 64.286 \times 10^{-6} \text{ m}^2 = 64.286 \text{ mm}^2$$

$$d_{\text{bolt}} = \sqrt{\frac{4A_{\text{bolt}}}{\pi}} = \sqrt{\frac{(4)(64.286)}{\pi}} = 9.05 \text{ mm}$$



PROBLEM 6.45



6.45 and 6.46 Three planks are connected as shown by bolts of  $\frac{3}{8}$ -in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

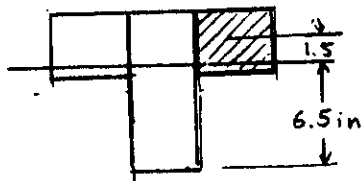
SOLUTION

Locate neutral axis.

$$\Sigma A = (2)(5)(4) + (4)(10) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(5)(4)(8) + (4)(10)(5) = 520 \text{ in}^3$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 6.5 \text{ in}$$



$$Q = (5)(4)(1.5) = 30 \text{ in}^3$$

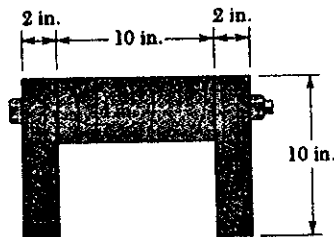
$$I = 2 \left[ \frac{1}{12} (5)(4)^3 + (5)(4)(1.5)^2 \right] + \frac{1}{12} (4)(10)^3 + (4)(10)(1.5)^2 = 566.7 \text{ in}^4$$

$$F = q_s = \frac{VQS}{I} = \frac{(2.5)(30)(6)}{566.7} = 0.7941 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{bolt} = \frac{F}{A_{bolt}} = \frac{0.7941}{0.1104} = 7.19 \text{ ksi}$$

PROBLEM 6.46



6.45 and 6.46 Three planks are connected as shown by bolts of  $\frac{3}{8}$ -in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

SOLUTION

Locate neutral axis

$$\Sigma A = (2)(2)(10) + (10)(4) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(2)(10)(5) + (10)(4)(8) = 520 \text{ in}^3$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{520}{80} = 6.5 \text{ in}$$

$$I = 2 \left[ \frac{1}{12} (2)(10)^3 + (2)(10)(1.5)^2 \right] + \frac{1}{12} (10)(4)^3 + (10)(4)(1.5)^2 = 566.7 \text{ in}^4$$

$$Q = (2)(10)(1.5) = 30 \text{ in}^3$$

$$F = q_s = \frac{VQS}{I} = \frac{(2.5)(30)(6)}{566.7} = 0.7941 \text{ kips}$$

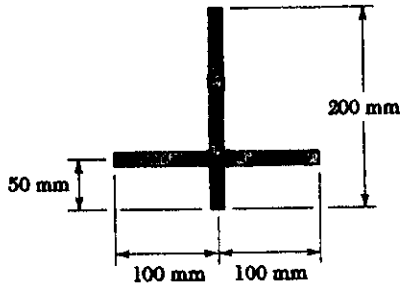
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{bolt} = \frac{F}{A_{bolt}} = \frac{0.7941}{0.1104} = 7.19 \text{ ksi}$$



PROBLEM 6.47

6.47 Three plates, each 12-mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.



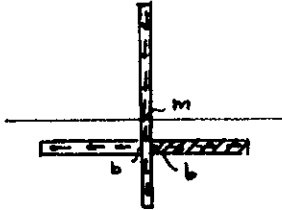
SOLUTION

Locate neutral axis

$$\Sigma A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^2$$

$$\Sigma A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 75.77 \text{ mm}$$



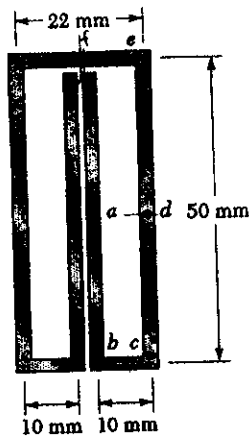
$$I = \frac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2 \left[ \frac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2 \right] = 10.934 \times 10^6 \text{ mm}^4 = 10.934 \times 10^{-6} \text{ m}^4$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

PROBLEM 6.48

6.48 A plate of 2-mm thickness is bent as shown and then used as a beam. For a vertical shear of 5 kN, determine the shearing stress at the five points indicated and sketch the shear flow in the cross section.



SOLUTION

$$I = 2 \left[ \frac{1}{12} (2)(48)^3 + \frac{1}{12} (2)(52)^3 + \frac{1}{12} (20)(2)^3 + (20)(2)(25)^2 \right]$$

$$= 133.76 \times 10^3 \text{ mm}^4 = 133.75 \times 10^{-9} \text{ m}^4$$

$$Q_a = (2)(24)(12) = 576 \text{ mm}^3 = 576 \times 10^{-9} \text{ m}^3$$

$$Q_b = 0$$

$$Q_c = Q_b - (12)(2)(25) = -600 \text{ mm}^3 = -600 \times 10^{-9} \text{ m}^3$$

$$Q_d = Q_c - (2)(24)(12) = -1.176 \times 10^3 \text{ mm}^3 = -1.176 \times 10^{-6} \text{ m}^3$$

$$Q_e = Q_d + (2)(26)(13) = -600 \text{ mm}^3 = -500 \times 10^{-9} \text{ m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(5 \times 10^3)(576 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 10.77 \times 10^6 \text{ Pa} = 10.76 \text{ MPa}$$

$$\tau_b = \frac{VQ_b}{It} = 0$$

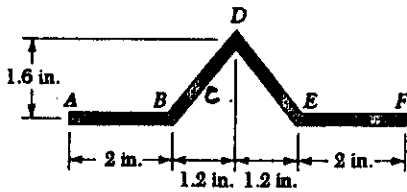
$$\tau_c = \frac{VQ_c}{It} = \frac{(5 \times 10^3)(600 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 11.21 \times 10^6 \text{ Pa} = 11.21 \text{ MPa}$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(5 \times 10^3)(1.176 \times 10^{-6})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 22.0 \times 10^6 \text{ Pa} = 22.0 \text{ MPa}$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(5 \times 10^3)(500 \times 10^{-9})}{(133.75 \times 10^{-9})(2 \times 10^{-3})} = 9.35 \times 10^6 \text{ Pa} = 9.35 \text{ MPa}$$

PROBLEM 6.49

6.49 A plate of  $\frac{1}{4}$ -in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (a) the maximum shearing stress in the section, (b) the shearing stress at point B. Also sketch the shear flow in the cross section.



SOLUTION

[www.konkur.in](http://www.konkur.in)

$$L_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in}$$

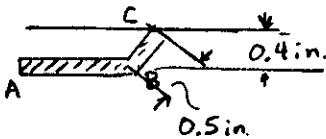
$$A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

Part	$A \text{ (in}^2\text{)}$	$\bar{y} \text{ (in)}$	$A\bar{y} \text{ (in}^3\text{)}$	$d \text{ (in)}$	$Ad^2 \text{ (in}^4\text{)}$	$\bar{I} \text{ (in}^4\text{)}$	$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{0.8}{2.0} = 0.4 \text{ in}$  $* \frac{1}{12} A_{BD} h^2 = \frac{1}{12} (0.5)(1.6)^2 = 0.1067 \text{ in}^4$  $I = \sum Ad^2 + \sum \bar{I} = 0.5333 \text{ in}^4$
AB	0.5	0	0	0.4	0.080	neglect	
BD	0.5	0.8	0.4	0.4	0.080	*0.1067	
DE	0.5	0.8	0.4	0.4	0.080	*0.1067	
EF	0.5	0	0	0.4	0.080	neglect	
$\Sigma$	2.0		0.8		0.320	0.2133	

(a)

$$Q_m = Q_{AB} + Q_{BC}$$



$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$

$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$

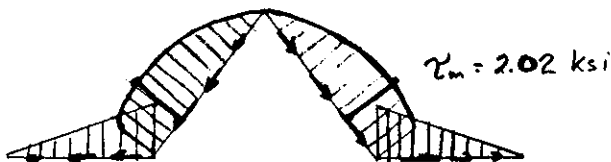
$$Q_m = 0.225 \text{ in}^3$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5333)(0.25)} = 2.025 \text{ ksi}$$

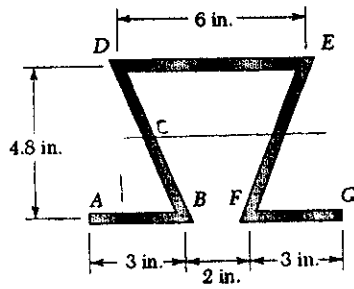
(b)  $Q_B = Q_{AB} = 0.2 \text{ in}^3$

$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5333)(0.25)} = 1.80 \text{ ksi}$$

$$\tau_D = 0$$



PROBLEM 6.50



6.50 A plate of thickness  $t$  is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (a) the thickness  $t$  for which the maximum shearing stress is 300 psi, (b) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.

SOLUTION

$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above AB

Calculate  $I$

$$I_{AB} = (3t)(2.4)^2 = 17.28 t$$

$$I_{BD} = \frac{1}{12}(5.2t)(4.8)^2 = 9.984 t$$

$$I_{DE} = (6t)(2.4)^2 = 34.56 t$$

$$I_{EF} = I_{DB} = 9.984 t$$

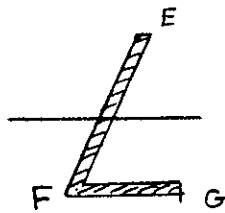
$$I_{FG} = I_{AB} = 17.28 t$$

$$I = \Sigma I = 89.09 t$$

(a) At point C  $Q_c = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6t)(1.2) = 10.32 t$

$$\tau = \frac{VQ_c}{It} \quad \therefore t = \frac{VQ}{\tau I} = \frac{(600)(10.32 t)}{(300)(89.09 t)} = 0.23168 \text{ in}$$

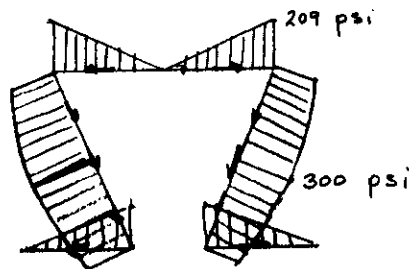
(b)  $I = (89.09)(0.23168) = 20.64 \text{ in}^3$



$$Q_E = Q_{EF} + Q_{FG}$$

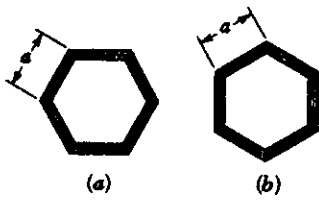
$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^2$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)} = 209 \text{ psi}$$

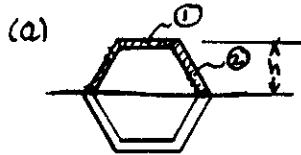


PROBLEM 6.51

6.51 and 6.52 An extruded beam has a uniform wall thickness  $t$ . Denoting by  $V$  the vertical shear and by  $A$  the cross-sectional area of the beam, express the maximum shearing stress as  $\tau_{max} = k(V/A)$  and determine the constant  $k$  for each of the two orientations shown.



SOLUTION



$$h = \frac{\sqrt{3}}{2} a$$

$$A_1 = A_2 = at$$

$$I_1 = A_1 h^2 = at h^2 = \frac{3}{4} a^3 t$$

$$I_2 = \frac{1}{3} A_2 h^2 = \frac{1}{3} at \frac{3}{4} a^2 = \frac{1}{4} a^3 t$$

$$I = 2I_1 + 4I_2 = \frac{5}{2} a^3 t$$

$$Q_1 = A_1 h = \frac{\sqrt{3}}{2} a^2 t$$

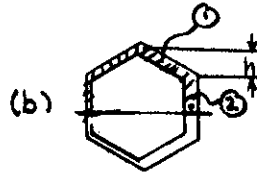
$$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4} a^2 t$$

$$Q_m = Q_1 + 2Q_2 = \sqrt{3} a^2 t$$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \sqrt{3} a^2 t}{(\frac{5}{2} a^3 t)(2t)} = \frac{\sqrt{3}}{5} \frac{V}{at}$$

$$= \frac{\sqrt{3}}{5} \frac{V}{6at} = \frac{6\sqrt{3}}{5} \frac{V}{A} = k \frac{V}{A}$$

$$k = \frac{6\sqrt{3}}{5} = 2.08 \quad \blacktriangleleft$$



$$h = \frac{a}{2}$$

$$A_1 = at \quad A_2 = \frac{1}{2} at$$

$$I_1 = \bar{I}_1 + A_1 d^2$$

$$= \frac{1}{12} at h^2 + at \left(\frac{a}{2} + \frac{h}{2}\right)^2$$

$$= \frac{1}{12} a^3 t + \frac{9}{16} a^3 t = \frac{7}{12} a^3 t$$

$$I_2 = \frac{1}{3} t \left(\frac{a}{2}\right)^3 = \frac{1}{24} a^3 t$$

$$I = 4I_1 + 4I_2 = \frac{5}{2} a^3 t$$

$$Q_1 = at \left(\frac{a}{2} + \frac{h}{2}\right) = \frac{3}{4} a^2 t$$

$$Q_2 = \left(\frac{1}{2} at\right) \left(\frac{a}{4}\right) = \frac{1}{8} a^2 t$$

$$Q = 2Q_1 + 2Q_2 = \frac{7}{4} a^2 t$$

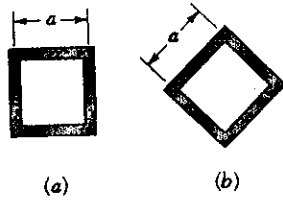
$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4} a^2 t}{(\frac{5}{2} a^3 t)(2t)}$$

$$= \frac{7}{20} \frac{V}{at} = \frac{42}{20} \frac{V}{6at} = \frac{21}{10} \frac{V}{A}$$

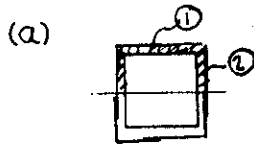
$$= k \frac{V}{A} \quad k = \frac{21}{10} = 2.10 \quad \blacktriangleleft$$

PROBLEM 6.52

6.51 and 6.52 An extruded beam has a uniform wall thickness  $t$ . Denoting by  $V$  the vertical shear and by  $A$  the cross-sectional area of the beam, express the maximum shearing stress as  $\tau_{max} = k(V/A)$  and determine the constant  $k$  for each of the two orientations shown.



SOLUTION



$$I_1 = (at) \left(\frac{a}{2}\right)^2 = \frac{1}{4} a^3 t$$

$$I_2 = \frac{1}{3} t \left(\frac{a}{2}\right)^3 = \frac{1}{24} a^3 t$$

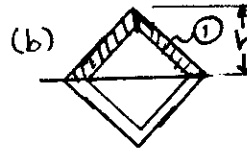
$$I = 2I_1 + 4I_2 = \frac{2}{3} a^3 t$$

$$Q_1 = (at) \left(\frac{a}{2}\right) = \frac{1}{2} a^2 t$$

$$Q_2 = \left(\frac{1}{2} at\right) \left(\frac{a}{4}\right) = \frac{1}{8} a^2 t$$

$$Q = Q_1 + 2Q_2 = \frac{3}{4} a^2 t$$

$$\begin{aligned} \tau_{max} &= \frac{VQ}{I(2t)} = \frac{V\left(\frac{3}{4} a^2 t\right)}{\left(\frac{2}{3} a^3 t\right)(2t)} = \\ &= \frac{9}{16} \frac{V}{at} = \frac{9}{4} \frac{V}{4at} = \frac{9}{4} \frac{V}{A} \\ &= k \frac{V}{A} \therefore k = \frac{9}{4} = 2.25 \end{aligned}$$



$$h = \frac{1}{2} \sqrt{2} a$$

$$I_1 = \frac{1}{3} A_1 h^2 = \left(\frac{1}{3} at\right) \left(\frac{\sqrt{2}}{2} a\right)^2 = \frac{1}{6} a^3 t$$

$$I = 4I_1 = \frac{2}{3} a^3 t$$

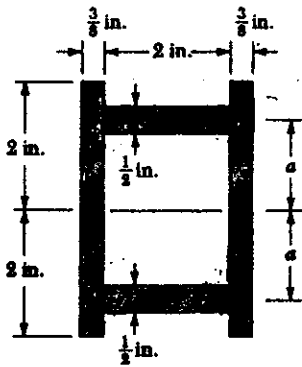
$$Q_1 = at \left(\frac{1}{2}\right) = \frac{1}{2} at$$

$$Q = 2Q_1 = \frac{1}{2} \sqrt{2} a^2 t$$

$$\begin{aligned} \tau_{max} &= \frac{VQ}{I(2t)} = \frac{V\left(\frac{1}{2} \sqrt{2} a^2 t\right)}{\left(\frac{2}{3} a^3 t\right)(2t)} \\ &= \frac{3\sqrt{2}}{8} \frac{V}{at} = \frac{3\sqrt{2}}{2} \frac{V}{4at} \\ &= \frac{3\sqrt{2}}{2} \frac{V}{A} = k \frac{V}{A} \end{aligned}$$

$$k = \frac{3\sqrt{2}}{2} = 2.12$$

**PROBLEM 6.53**



6.53 The design of a beam calls for connecting two vertical rectangular  $\frac{3}{8} \times 4$ -in. plates by welding them to two horizontal  $\frac{1}{2} \times 2$ -in. plates as shown. For a vertical shear  $V$ , determine the dimension  $a$  for which the shear flow through the welded surfaces is maximum.

**SOLUTION**

$$I = (2)(\frac{1}{2})(\frac{3}{8})(4)^3 + (2)(\frac{1}{2})(2)(\frac{1}{2})^3 + (2)(2)(\frac{1}{2})a^2$$

$$= 4.041667 + 2a^2 \quad \text{in}^4$$

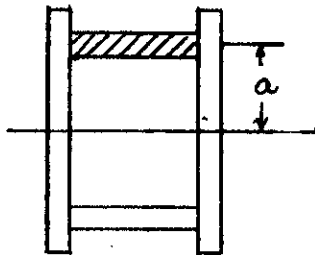
$$Q = (2)(\frac{1}{2})a = a \quad \text{in}^2$$

$$q = \frac{VQ}{I} = \frac{Va}{4.041667 + 2a^2} \quad \text{Set } \frac{dq}{da} = 0$$

$$\frac{dq}{da} = \left[ \frac{(4.041667 + 2a^2) - (a)(4a)}{(4.041667 + 2a^2)^2} \right] V = 0$$

$$2a^2 = 4.041667$$

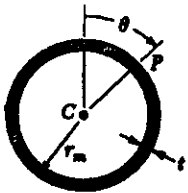
$$a = 1.422 \text{ in.}$$



**PROBLEM 6.54**

6.54 (a) Determine the shearing stress at point  $P$  of a thin-walled pipe of the cross section shown caused by a vertical shear  $V$ . (b) Show that the maximum shearing stress occurs for  $\theta = 90^\circ$  and is equal to  $2V/A$ , where  $A$  is the cross-sectional area of the pipe.

**SOLUTION**



$$A = 2\pi r_m t \quad J = Ar_m^2 = 2\pi r_m^3 t \quad I = \frac{1}{2}J = \pi r_m^3 t$$

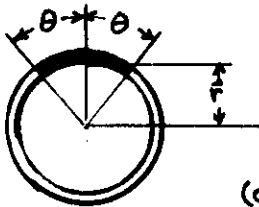
$$\bar{r} = \frac{\sin \theta}{\theta} \quad \text{for a circular arc}$$

$$A_p = 2r\theta t$$

$$Q_p = A_p \bar{r} = 2rt \sin \theta$$

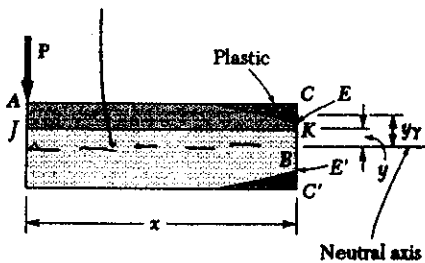
$$(a) \quad \tau_p = \frac{VQ_p}{I(2t)} = \frac{(V)(2rt \sin \theta)}{(\pi r_m^3 t)(2t)} = \frac{V \sin \theta}{\pi r_m^2 t}$$

$$(b) \quad \tau_m = \frac{2V \sin \frac{\pi}{2}}{2\pi r_m t} = \frac{2V}{A}$$



**PROBLEM 6.55**

6.55 Consider the cantilever beam  $AB$  discussed in Sec. 6.8 and the portion  $ACKJ$  of the beam that is located to the left of the transverse section  $CC'$  and above the horizontal plane  $JK$ , where  $K$  is a point at a distance  $y < y_r$  above the neutral axis (Fig. P6.55). (a) Recalling that  $\sigma_x = \sigma_y$  between  $C$  and  $E$  and  $\sigma_x = (\sigma_r/y_r)y$  between  $E$  and  $K$ , show that the magnitude of the horizontal shearing force  $H$  exerted on the lower face of the portion of beam  $ACKJ$  is



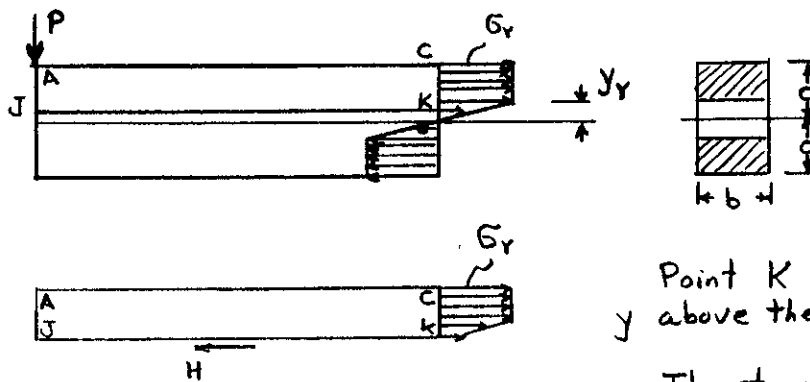
$$H = \frac{1}{2} b \sigma_r \left( 2c - y_r - \frac{y^2}{y_r} \right)$$

(b) Observing that the shearing stress at  $K$  is

$$\tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta H}{\Delta A} = \lim_{\Delta x \rightarrow 0} \frac{1}{b} \frac{\Delta H}{\Delta x} = \frac{1}{b} \frac{\partial H}{\partial x}$$

and recalling that  $y_r$  is a function of  $x$  defined by Eq. (6.14), derive Eq. (6.15).

**SOLUTION**



Point  $K$  is located a distance  $y$  above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_r \frac{y}{y_r} \text{ for } 0 \leq y < y_r \text{ and } \sigma = \sigma_r \text{ for } y_r \leq y \leq c.$$

For equilibrium of horizontal forces acting on  $ACKJ$

$$\begin{aligned} H &= \int \sigma dA = \int_y^{y_r} \frac{\sigma_r y b}{y_r} dy + \int_{y_r}^c \sigma_r b dy = \frac{\sigma_r b}{y_r} \left( \frac{y_r^2 - y^2}{2} \right) + \sigma_r b (c - y_r) \\ &= \frac{1}{2} b \sigma_r \left( 2c - y_r - \frac{y^2}{y_r} \right) \end{aligned} \quad \leftarrow (a)$$

Note that  $y_r$  is a function of  $x$

$$\tau_{xy} = \frac{1}{b} \frac{\partial H}{\partial x} = \frac{1}{2} \sigma_r \left( -\frac{\partial y_r}{\partial x} + \frac{y^2}{y_r^2} \frac{dy_r}{dx} \right) = -\frac{1}{2} \sigma_r \left( 1 - \frac{y^2}{y_r^2} \right) \frac{dy_r}{dx}$$

But  $M = Px = \frac{3}{2} M_y \left( 1 - \frac{1}{3} \frac{y_r^2}{c^2} \right)$

Differentiating  $\frac{dM}{dx} = P = \frac{3}{2} M_y \left( -\frac{2}{3} \frac{y_r}{c^2} \frac{dy_r}{dx} \right)$

$$\frac{dy_r}{dx} = -\frac{Pc^2}{y_r M_y} = -\frac{Pc^2}{y_r \frac{3}{2} \sigma_r b c^2} = -\frac{3P}{2 \sigma_r b y_r}$$

Then  $\tau_{xy} = \frac{1}{2} \sigma_r \left( 1 - \frac{y^2}{y_r^2} \right) \frac{3P}{2 \sigma_r b y_r} = \frac{3P}{4b y_r} \left( 1 - \frac{y^2}{y_r^2} \right) \quad \leftarrow (b)$



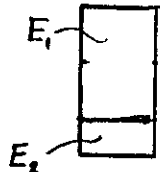
PROBLEM 6.56

6.56 For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

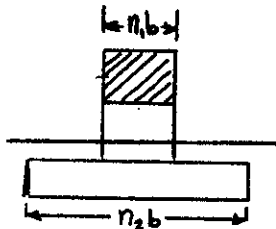
$$\tau = \frac{VQ}{It}$$

remains valid provided that both  $Q$  and  $I$  are computed using the transformed section of the beam (see Sec. 4.6) and provided further that  $t$  is the actual width of the beam at the point where  $\tau$  is computed.

SOLUTION



Actual Section



Transformed Section

Let  $E_{ref}$  be a reference modulus of elasticity

$$n_1 = \frac{E_1}{E_{ref}}, \quad n_2 = \frac{E_2}{E_{ref}}, \quad \text{etc.}$$

Widths  $b$  of actual section are multiplied by  $n$ 's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where  $I$  is the moment of inertia of the transformed cross section and  $y$  is measured from the centroid of the transformed section

The horizontal shearing force over length  $\Delta x$  is

$$\Delta H = -\int (\Delta \sigma_x) dA = -\int \frac{n(\Delta M)y}{I} dA = -\frac{(\Delta M)}{I} \int ny dA = -\frac{Q(\Delta M)}{I}$$

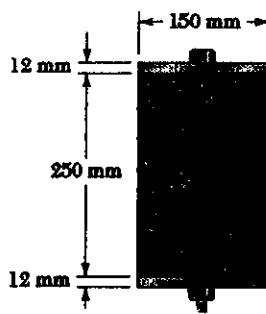
$$Q = \int ny dA = \text{first moment of transformed section.}$$

Shear flow  $q = \frac{\Delta H}{\Delta x} = \frac{\Delta M}{\Delta x} \frac{Q}{I} = \frac{VQ}{I}$

$q$  is distributed over actual width  $t$ , thus  $\tau = \frac{q}{t}$

$$\tau = \frac{VQ}{It}$$

PROBLEM 6.57



6.57 and 6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. Determine the average shearing stress in the bolts caused by a vertical shearing force of 4 kN. (Hint. Use the method indicated in Prob. 6.56.)

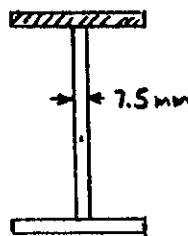
SOLUTION

$$\text{Let } E_{\text{ref}} = E_s = 200 \text{ GPa}$$

$$n_s = 1 \quad n_w = \frac{E_w}{E_s} = \frac{10 \text{ GPa}}{200 \text{ GPa}} = \frac{1}{20}$$

Widths of transformed section

$$b_s = 150 \text{ mm} \quad b_w = \left(\frac{1}{20}\right)(150) = 7.5 \text{ mm}$$



$$I = 2 \left[ \frac{1}{12} (150)(12)^3 + (150)(12)(125+6)^2 \right] + \frac{1}{12} (7.5)(250)^3$$

$$= 2 \left[ 0.0216 \times 10^6 + 30.890 \times 10^6 \right] + 9.766 \times 10^6$$

$$71.589 \times 10^6 \text{ mm}^4 = 71.589 \times 10^{-6} \text{ m}^4$$

$$Q = (150)(12)(125+6) = 235.8 \times 10^3 \text{ mm}^3$$

$$= 235.8 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(4 \times 10^3)(235.8 \times 10^{-6})}{71.589 \times 10^{-6}} = 13.175 \times 10^3 \text{ N/m}$$

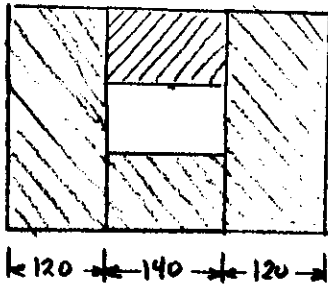
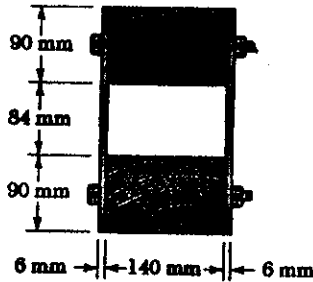
$$F_{\text{bolt}} = qS = (23.175 \times 10^3)(200 \times 10^{-3}) = 2.635 \times 10^3 \text{ N}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \left(\frac{\pi}{4}\right)(12)^2 = 113.1 \text{ mm}^2 = 113.1 \times 10^{-6} \text{ m}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{2.635 \times 10^3}{113.1 \times 10^{-6}} = 23.3 \times 10^6 \text{ Pa} = 23.3 \text{ MPa}$$

**PROBLEM 6.58**

6.57 and 6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. Determine the average shearing stress in the bolts caused by a vertical shearing force of 4 kN. (Hint. Use the method indicated in Prob. 6.56.)



**SOLUTION**

Let wood be the reference material

$$n_w = 1.0 \quad n_s = \frac{E_s}{E_w} = \frac{200 \text{ GPa}}{10 \text{ GPa}} = 20$$

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12} (380)(264)^3 - \frac{1}{12} (140)(84)^3 = 575.7 \times 10^6 \text{ mm}^4$$

$$= 575.7 \times 10^{-6} \text{ m}^4$$

$$Q = (140)(90)(42 + 45) = 1.0962 \times 10^6 \text{ mm}^3$$

$$= 1.096 \times 10^{-3} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(4 \times 10^3)(1.096 \times 10^{-3})}{575.7 \times 10^{-6}} = 7.615 \times 10^3 \text{ N/m}$$

$$F_{bolt} = qS = (7.615 \times 10^3)(200 \times 10^{-3}) = 1.523 \times 10^3 \text{ N}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4} (12)^2 = 113.1 \text{ mm}^2 = 113.1 \times 10^{-6} \text{ m}^2$$

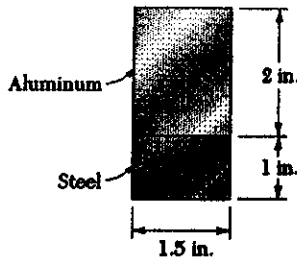
Double shear

$$\tau_{bolt} = \frac{F_{bolt}}{2A_{bolt}} = \frac{1.523 \times 10^3}{2(113.1 \times 10^{-6})} = 6.73 \times 10^6 \text{ Pa}$$

$$= 6.73 \text{ MPa}$$

PROBLEM 6.59

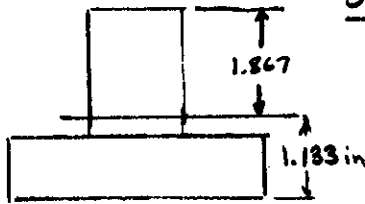
6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is  $29 \times 10^6$  psi for the steel and  $10.6 \times 10^6$  psi for the aluminum, determine (a) the average stress at the bonded surface, (b) the maximum stress in the beam. (Hint. Use the method indicated in Prob. 6.56.)



SOLUTION

$n = 1$  in aluminum  $n = \frac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$  in steel

Part	$nA$ (in <sup>2</sup> )	$\bar{y}$ (in)	$nA\bar{y}$ (in <sup>3</sup> )	$d$ (in)	$nAd^2$ (in <sup>4</sup> )	$n\bar{I}$ (in <sup>4</sup> )
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
$\Sigma$	7.1038		8.0519		3.8994	1.3420



$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335$

$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$

(a) At the bonded surface  $Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

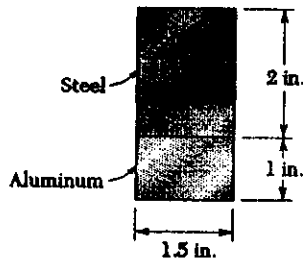
$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} = 1.323 \text{ ksi}$

(b) At the neutral axis  $Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^3$

$\tau_{max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2414)(1.5)} = 1.329 \text{ ksi}$

PROBLEM 6.60

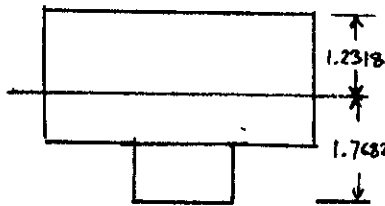
6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is  $29 \times 10^6$  psi for the steel and  $10.6 \times 10^6$  psi for the aluminum, determine (a) the average stress at the bonded surface, (b) the maximum stress in the beam. (Hint. Use the method indicated in Prob. 6.56.)



SOLUTION

$n = 1$  in aluminum  $n = \frac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$  in steel

Part	$nA$ (in <sup>2</sup> )	$\bar{y}$ (in)	$nA\bar{y}$ (in <sup>3</sup> )	$d$ (in)	$nAd^2$ (in <sup>4</sup> )	$n\bar{I}$ (in <sup>4</sup> )
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.2682	2.4125	0.1250
$\Sigma$	9.7074		17.1648		2.8535	2.8608



$\bar{y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682$  in

$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143$  in<sup>4</sup>

(a) At the bonded surface  $Q = (1.5)(1.2682) = 1.9023$  in<sup>3</sup>

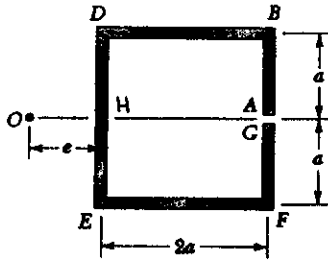
$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888$  ksi

(b) At the neutral axis  $Q = (2.7358)(1.5)(1.2318) \times \frac{1.2318}{2} = 3.1133$  in<sup>3</sup>

$\tau_{max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453$  ksi

PROBLEM 6.61

6.61 through 6.64 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

$$I_{AB} = I_{FE} = \frac{1}{3} t a^3 \quad I_{DE} = I_{BF} = 2ata^2 + \frac{1}{12} 2att^3 \approx 2ta^3$$

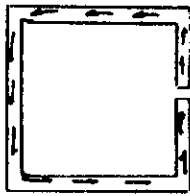
$$I_{DE} = \frac{1}{12} t (2a)^3 = \frac{2}{3} ta^3 \quad I = \sum I = \frac{16}{3} ta^3$$

Part AB  $A = ty \quad \bar{y} = \frac{y}{2} \quad Q = \frac{1}{2} ty^2$

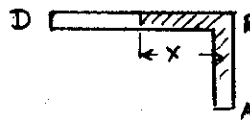
$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2} ty^2}{\frac{16}{3} ta^3 t} = \frac{3Vy^2}{32a^3 t}$$



$$F_1 = \int \tau dA = \int_0^a \tau t dy = \frac{3V}{32a^3} \int_0^a y^2 dt = \frac{1}{32} V$$



Part BD



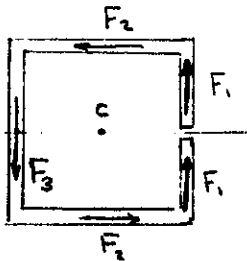
$$Q = Q_B + txa = \frac{1}{2} ta^2 + tax$$

$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3} a^3 t} \left( \frac{1}{2} a^2 + ax \right)$$

$$= \frac{3V}{32a^2} (a + 2x)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{3V}{32a^2} (a + 2x) dx$$

$$= \frac{3V}{32a^2} (ax + x^2) \Big|_0^{2a} = \frac{3V}{32a^2} (2a^2 + 4a^2) = \frac{9}{16} V$$

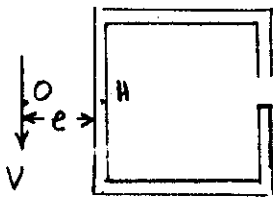


$$\sum M_H = \sum M_H$$

$$Ve = (2a)(2F_1) + (2a)(F_2)$$

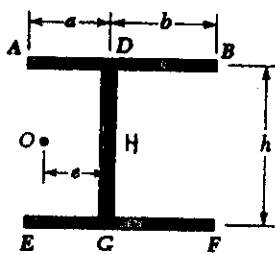
$$= \frac{1}{8} Va + \frac{9}{8} Va = \frac{5}{4} Va$$

$$e = \frac{5}{4} a$$



PROBLEM 6.62

6.61 through 6.64 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



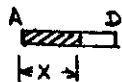
SOLUTION

$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^3$$

$$I_{OG} = \frac{1}{12}th^3 \quad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^3$$

Part AD

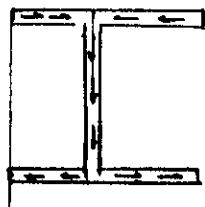
$$Q = tx \frac{h}{2} = \frac{1}{2}thx$$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

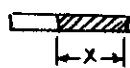
$$F_1 = \int \tau dA = \int_0^a \frac{Vhx}{2I} t dx = \frac{Vht}{2I} \int_0^a x dx$$

$$= \frac{Vht}{2I} \left. \frac{x^2}{2} \right|_0^a = \frac{Vhta^2}{4I}$$



Part BD

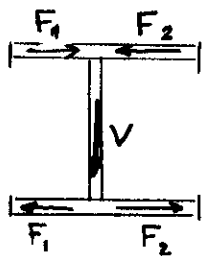
$$Q = tx \frac{h}{2} = \frac{1}{2}thx$$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau dA = \int_0^b \frac{Vhx}{2I} t dx = \frac{Vht}{2I} \int_0^b x^2 dx$$

$$= \frac{Vht}{2I} \left. \frac{x^2}{2} \right|_0^b = \frac{Vhtb^2}{4I}$$

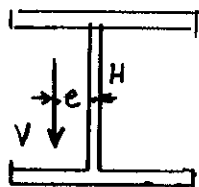


$$\Sigma M_H = \Sigma M_H$$

$$Ve = F_2 h - F_1 h = \frac{Vht^2(b^2 - a^2)}{4I}$$

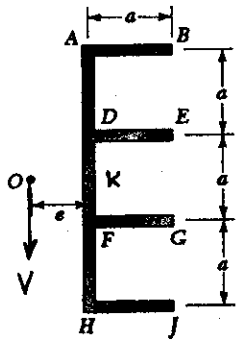
$$= \frac{Vht^2(b^2 - a^2)}{4 \cdot \frac{1}{12}t(6a+6b+h)h^3} = \frac{3V(b^2 - a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2 - a^2)}{6(a+b) + h}$$



PROBLEM 6.63

6.61 through 6.64 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

$$I_{AB} = I_{HJ} = at \left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at \left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^3$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3 \quad I = \sum I = \frac{29}{4}ta^3$$

Part AB  $A = tx \quad \bar{y} = \frac{3a}{2} \quad Q = \frac{3}{2}atx$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{3}{2}atx}{\frac{29}{4}ta^3t} = \frac{6Vx}{29a^2t}$$

$$F_1 = \int \tau dA = \int_0^a \frac{6Vx}{29a^2t} t dx = \frac{6V}{29a^2} \int_0^a x dx = \frac{3}{29}V$$

Part DE  $A = tx \quad \bar{y} = \frac{a}{2} \quad Q = \frac{1}{2}atx$

$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2}atx}{\frac{29}{4}ta^3t} = \frac{2Vx}{29a^2t}$$

$$F_2 = \int \tau dA = \int_0^a \frac{2Vx}{29a^2t} t dx = \frac{2V}{29a^2} \int_0^a x dx = \frac{1}{29}V$$

$$\sum M_K = \sum M_K$$

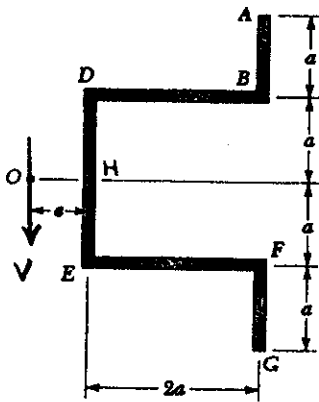
$$Ve = F_1(3a) + F_2(a) = \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a$$



PROBLEM 6.64

6.61 through 6.64 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



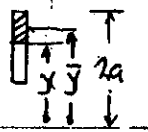
SOLUTION

$$I_{AB} = I_{FG} = \frac{1}{12} t a^3 + (t a) \left(\frac{3a}{2}\right)^2 = \frac{7}{3} t a^3$$

$$I_{DE} = I_{EF} = (2at)a^2 + \frac{1}{12}(2a)t^3 \approx 2a^3t$$

$$I_{DE} = \frac{1}{12} t (2a)^3 = \frac{2}{3} t a^3 \quad I = \Sigma I = \frac{28}{3} t a^3$$

Part AB  $A = t(2a - y), \quad \bar{y} = \frac{2a + y}{2}$



$$Q = A\bar{y} = \frac{1}{2} t (2a - y)(2a + y) = \frac{1}{2} t (4a^2 - y^2)$$

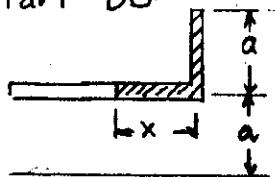
$$\tau = \frac{VQ}{It} = \frac{V}{2I} (4a^2 - y^2)$$

$$F_1 = \int \tau dA = \int_a^{2a} \frac{V}{2I} (4a^2 - y^2) t dy$$

$$= \frac{Vt}{2I} \left( 4a^2 y - \frac{y^3}{3} \right) \Big|_a^{2a} = \frac{Vt a^3}{2I} \left[ (4)(2) - \frac{(2)^3}{3} - (4)(1) + \frac{(1)^3}{3} \right]$$

$$= \frac{5}{6} \frac{Vt a^3}{I} = \frac{5}{96} V$$

Part DB



$$Q = (ta) \frac{3a}{2} + t x a = ta \left( \frac{3a}{2} + x \right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I} \left( \frac{3a}{2} + x \right)$$

$$F_2 = \int \tau dA = \int_0^{2a} \frac{Va}{I} \left( \frac{3a}{2} + x \right) t dx = \frac{Vta}{I} \int_0^{2a} \left( \frac{3a}{2} + x \right) dx$$

$$= \frac{Vta}{I} \left( \frac{3ax}{2} + \frac{x^2}{2} \right) \Big|_0^{2a} = \frac{Vta^3}{I} \left[ \frac{(3)(2)}{2} + \frac{(2)^2}{2} \right]$$

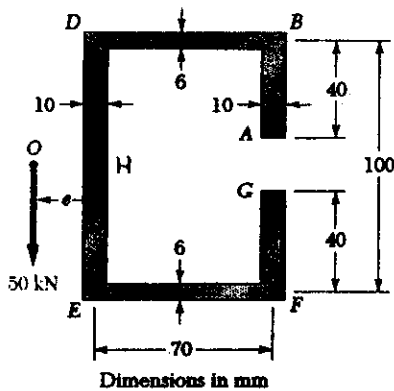
$$= 5 \frac{Vta^3}{I} = \frac{15}{28} V$$

$$\rightarrow \Sigma M_H = \ominus \Sigma M_H$$

$$Ve = F_2(2a) - 2F_1(2a) = \frac{30}{28} Va - \frac{20}{96} Va = \frac{5}{7} Va$$

$$e = \frac{5}{7} a$$

PROBLEM 6.65



6.65 and 6.66 An extruded beam has the cross section shown. Determine (a) the location of the shear center O, (b) the distribution of the shearing stresses caused by a 50-kN vertical shearing force applied at O.

SOLUTION

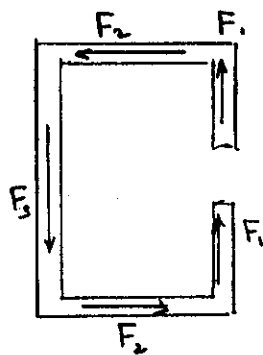
All quantities in mm, mm<sup>2</sup>, etc.

$$I_{AB} = \frac{1}{12}(10)(40)^3 + (10)(40)(30)^2 = 0.41333 \times 10^6 \text{ mm}^4$$

$$I_{OB} \approx (70)(6)(50)^2 + \frac{1}{12}(70)(6)^3 = 1.05126 \times 10^6 \text{ mm}^4$$

$$I_{DE} = \frac{1}{12}(10)(100)^3 = 0.83333 \times 10^6 \text{ mm}^4$$

$$I = \Sigma I = 3.7625 \times 10^6 \text{ mm}^4$$



Part AB:

$$A = 10(y-10)$$

$$\bar{y} = \frac{1}{2}(y+10)$$

$$Q = A\bar{y} = 5(y-10)(y+10) = 5(y^2-100)$$

$$Q_B = 5(50^2-100) = 12 \times 10^3 \text{ mm}^3$$

$$\tau = \frac{VQ}{Iz}$$

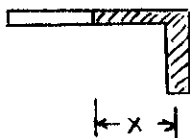
$$F_1 = \int \tau dA = \int_{10}^{50} \frac{V 5(y^2-100)}{I z} z dy$$

$$\frac{F_1}{V} = \frac{5}{I} \int_{10}^{50} (100-y^2) dy = \frac{5}{I} \left( \frac{y^3}{3} - 100y \right) \Big|_{10}^{50} = \frac{5}{I} \left[ \frac{50^3}{3} - (100)(50) - \frac{10^3}{3} + (100)(10) \right]$$

$$= \frac{(5)(36.667 \times 10^3)}{3.7625 \times 10^6} = 0.048726$$

Part DB

$$Q = Q_B + (6x)(50) = 12 \times 10^3 + 300x$$



$$\tau = \frac{VQ}{Iz}$$

$$Q_D = 12 \times 10^3 + (300)(70) = 33 \times 10^3 \text{ mm}^3$$

$$F_2 = \int \tau dA = \int_0^{70} \frac{V(12 \times 10^3 + 300x)}{I z} z dx = \frac{V}{I} \int_0^{70} (12 \times 10^3 + 300x) dx$$

$$\frac{F_2}{V} = \frac{1}{I} \left[ (12 \times 10^3)x + 300 \frac{x^2}{2} \right]_0^{70} = \frac{(12 \times 10^3)(70) + (300)(70^2)/2}{3.7625 \times 10^6}$$

$$= 0.41860$$

$$\Sigma M_H = \Sigma M_H$$

$$Ve = 2F_1(70) + F_2(100) = (2)(0.048726V)(70) + (0.41860V)(100)$$

$$= 48.7V \quad e = 48.7 \text{ mm}$$

At point H

$$Q_H = Q_D + (10)(50)(25) = 33 \times 10^3 + 12.5 \times 10^3 = 45.5 \times 10^3 \text{ mm}^3$$

continued

PROBLEM 6.65 (continued)

$$V = 50 \times 10^3 \text{ N} \quad I = 3.7625 \times 10^6 \text{ mm}^4 = 3.7625 \times 10^{-6} \text{ m}^4$$

Part AB, Point A  $Q = 0 \quad \tau = 0$  ▶

Part AB, Point B  $Q = Q_B = 12 \times 10^3 \text{ mm}^3 = 12 \times 10^{-6} \text{ m}^3, \quad t = 10 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(12 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 15.95 \times 10^6 \text{ Pa} = 15.95 \text{ MPa} \quad \blacktriangleleft$$

Part BD, Point B  $Q = 12 \times 10^{-6} \text{ m}^3 \quad t = 6 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(12 \times 10^{-6})}{(3.7625 \times 10^{-6})(6 \times 10^{-3})} = 26.6 \times 10^6 \text{ Pa} = 26.6 \text{ MPa} \quad \blacktriangleleft$$

Part BD, Point D  $Q = 33 \times 10^{-6} \text{ m}^3 \quad t = 6 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(33 \times 10^{-6})}{(3.7625 \times 10^{-6})(6 \times 10^{-3})} = 73.1 \times 10^6 \text{ Pa} = 73.1 \text{ MPa} \quad \blacktriangleleft$$

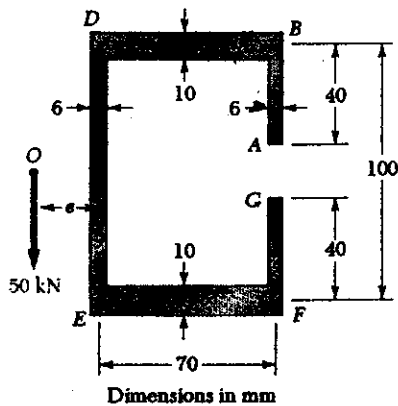
Part DE, Point D  $Q = 33 \times 10^{-6} \text{ m}^3 \quad t = 10 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(33 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 43.9 \times 10^6 \text{ Pa} = 43.9 \text{ MPa} \quad \blacktriangleleft$$

Point H  $Q = 45.5 \times 10^{-6} \text{ m}^3 \quad t = 10 \times 10^{-3}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(45.5 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 60.5 \times 10^6 \text{ Pa} = 60.5 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 6.66**



6.65 and 6.66 An extruded beam has the cross section shown. Determine (a) the location of the shear center O, (b) the distribution of the shearing stresses caused by a 50-kN vertical shearing force applied at O.

**SOLUTION**

All quantities in mm, mm<sup>2</sup>, etc

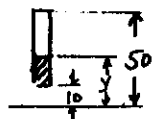
$$I_{AB} = \frac{1}{12}(6)(40)^3 + (6)(40)(30)^2 = 0.248 \times 10^6 \text{ mm}^4$$

$$I_{BD} = (70)(10)(50)^2 + \frac{1}{12}(70)(10)^3 = 1.75583 \times 10^6 \text{ mm}^4$$

$$I_{DE} = \frac{1}{12}(6)(100)^3 = 0.500 \times 10^6 \text{ mm}^4$$

$$I = \Sigma I = 4.5076 \times 10^6 \text{ mm}^4$$

Part AB:  $A = 6(y-10)$      $\bar{y} = \frac{1}{2}(y+10)$



$$Q = A\bar{y} = 3(y-10)(y+10) = 3(y^2 - 100)$$

$$Q_B = 3(50^2 - 100) = 7.2 \times 10^3 \text{ mm}^3$$

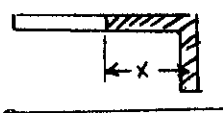
$$\tau = \frac{VQ}{It}$$

$$F_1 = \int \tau dA = \int_{10}^{50} \frac{V}{I} \frac{3(y^2 - 100)}{6} \cdot 6 dy = \frac{3V}{I} \int_{10}^{50} (y^2 - 100) dy$$

$$\frac{F_1}{V} = \frac{3}{I} \int_{10}^{50} (y^2 - 100) dy = \frac{3}{I} \left( \frac{y^3}{3} - 100y \right) \Big|_{10}^{50} = \frac{3}{I} \left[ \frac{50^3}{3} - (100)(50) - \frac{10^3}{3} + (100)(10) \right]$$

$$= \frac{(3)(36.667 \times 10^3)}{4.5076 \times 10^6} = 0.02440$$

Part DB:  $Q = Q_B + (10x)(50) = 7.2 \times 10^3 + 500x$



$$\tau = \frac{VQ}{It}$$

$$Q_D = 7.2 \times 10^3 + (500)(70) = 42.2 \times 10^3 \text{ mm}^3$$

$$F_2 = \int \tau dA = \int_0^{70} \frac{V}{I} (7.2 \times 10^3 + 500x) \cdot 10 dx$$

$$\frac{F_2}{V} = \frac{1}{I} \int_0^{70} [(7.2 \times 10^3) + 500x] dx = \frac{1}{I} \left[ (7.2 \times 10^3)x + 500 \frac{x^2}{2} \right] = \frac{1}{I} \left[ (7.2 \times 10^3)(70) + (500) \frac{(70)^2}{2} \right]$$

$$= \frac{1.729 \times 10^6}{4.5076 \times 10^6} = 0.38357$$

$$\odot \Sigma M_H = \odot M_H \quad V e = (2F_1)(70) + (F_2)(100) = (2)(0.02440V)(70) + (0.38357V)(100)$$

$$= 41.8 V \quad e = 41.8 \text{ mm} \quad \blacktriangleleft$$

At point H  $Q_H = Q_D + (6)(50)(25) = 42.2 \times 10^3 + 7.5 \times 10^3 = 49.7 \times 10^3 \text{ mm}^3$

continued

PROBLEM 6.66 (continued)

$$V = 50 \times 10^3 \text{ N} \quad I = 4.5076 \times 10^6 \text{ mm}^4 = 4.5076 \times 10^{-6} \text{ m}^4$$

Point A  $Q = 0 \quad z = 0$

Part AB, Point B  $Q_B = 7.2 \times 10^{-6} \text{ m}^3 \quad z = 6 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{Iz} = \frac{(50 \times 10^3)(7.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(6 \times 10^{-3})} = 13.31 \times 10^6 \text{ Pa} = 13.31 \text{ MPa}$$

Part BD, Point B  $Q = 7.2 \times 10^{-6} \text{ m}^3 \quad z = 10 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{Iz} = \frac{(50 \times 10^3)(7.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(10 \times 10^{-3})} = 7.99 \times 10^6 \text{ Pa} = 7.99 \text{ MPa}$$

Part BD, Point D  $Q = 42.2 \times 10^{-6} \text{ m}^3 \quad z = 10 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{Iz} = \frac{(50 \times 10^3)(42.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(10 \times 10^{-3})} = 46.8 \times 10^6 \text{ Pa} = 46.8 \text{ MPa}$$

Part DE, Point D  $Q = 42.2 \times 10^{-6} \text{ m}^3 \quad z = 6 \times 10^{-3} \text{ m}$

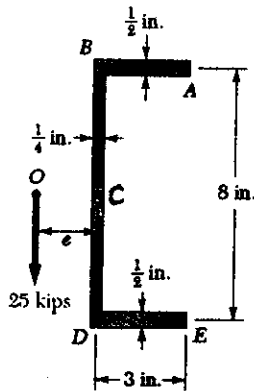
$$\tau = \frac{VQ}{Iz} = \frac{(50 \times 10^3)(42.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(6 \times 10^{-3})} = 78.0 \times 10^6 \text{ Pa} = 78.0 \text{ MPa}$$

Point H  $Q = 49.7 \times 10^{-6} \text{ m}^3 \quad z = 6 \times 10^{-3} \text{ m}$

$$\tau = \frac{VQ}{Iz} = \frac{(50 \times 10^3)(49.7 \times 10^{-6})}{(4.5076 \times 10^{-6})(6 \times 10^{-3})} = 91.9 \times 10^6 \text{ Pa} = 91.9 \text{ MPa}$$

PROBLEM 6.67

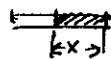
6.67 and 6.68 An extruded beam has the cross section shown. Determine (a) the location of the shear center  $O$ , (b) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at  $O$ .



SOLUTION

$$I = 2 \left[ \frac{1}{12} \cdot (3) \left( \frac{1}{2} \right)^3 + (3) \left( \frac{1}{2} \right) (4)^2 \right] + \frac{1}{12} \cdot \left( \frac{1}{4} \right) (8)^3 = 58.729 \text{ in}^4$$

Part AB  $A = \frac{1}{2}x$ ,  $\bar{y} = 4$ ,  $Q = A\bar{y} = 2x$



$$\tau = \frac{VQ}{It} = \frac{(25)(2x)}{(58.729)(\frac{1}{2})} = 1.7027x$$

Point A  $x = 0$   $\tau = 0$

Point B  $x = 3$   $\tau = 5.11 \text{ ksi}$

$$F_1 = \int \tau dA = \int_0^3 1.7027x \cdot \frac{1}{2} dx = \left. \frac{1.7027}{4} x^2 \right|_0^3$$

$$= \frac{(1.7027)(3)^2}{4} = 3.8311 \text{ kips}$$

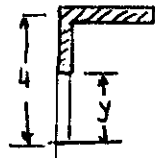
$$\sum M_H = \sum M_H \quad 25e = (F_1)(8)$$

$$e = \frac{(3.8311)(8)}{25} = 1.226 \text{ in}$$



Part BD  $Q = (2)(3) + \left( \frac{1}{4} \right) (4-y) \left( \frac{4+y}{2} \right)$

$$= 6 + \frac{1}{8} (16 - y^2) = 8 - \frac{1}{8} y^2$$



$$\tau = \frac{VQ}{It} = \frac{25 \left( 8 - \frac{1}{8} y^2 \right)}{(58.729) \left( \frac{1}{4} \right)}$$

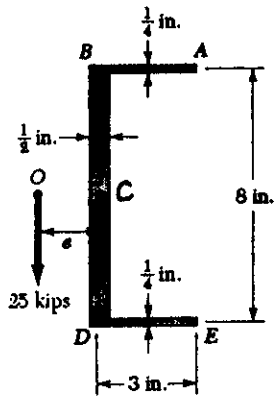
$$= 13.622 - 0.2128 y^2$$

Point B  $y = 4 \text{ in}$   $\tau = 10.22 \text{ ksi}$

Point C  $y = 0$   $\tau = 13.62 \text{ ksi}$

PROBLEM 6.68

6.67 and 6.68 An extruded beam has the cross section shown. Determine (a) the location of the shear center  $O$ , (b) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at  $O$ .



SOLUTION

$$I = 2 \left[ \frac{1}{12} (3) \left(\frac{1}{4}\right)^3 + (3) \left(\frac{1}{4}\right) (4)^2 \right] + \frac{1}{12} \left(\frac{1}{2}\right) (8)^3 = 45.341 \text{ in}^4$$

Part AB  $A = \frac{1}{4}x, \bar{y} = 4 \quad Q = A\bar{y} = x$

$$\tau = \frac{VQ}{It} = \frac{(25)(x)}{(45.341)\left(\frac{1}{4}\right)} = 2.2055x$$

Point A  $x = 0 \quad \tau = 0$

Point B  $x = 3 \quad \tau = 6.62 \text{ ksi}$

$$F_1 = \int \tau dA = \int_0^3 (2.2055x) \frac{1}{4} dx = \frac{2.2055}{4} \frac{x^2}{2} \Big|_0^3$$

$$= \frac{(2.2055)(3)^2}{(4)(2)} = 2.4812 \text{ kips}$$

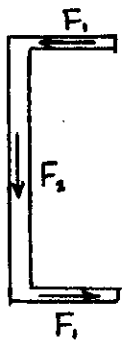
$\rightarrow M_H = \rightarrow M_H \quad 25e = F_1(8)$   
 $e = 0.794 \text{ in.}$

Part BD  $Q = 3 + \frac{1}{2}(4-y) \frac{4+y}{2} =$   
 $= 3 + \frac{1}{4}(16-y^2) = 7 - \frac{1}{4}y^2$

$$\tau = \frac{VQ}{It} = \frac{(25)(7 - \frac{1}{4}y^2)}{(45.341)\left(\frac{1}{2}\right)} = 7.7193 - 0.2757y^2$$

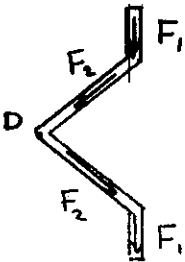
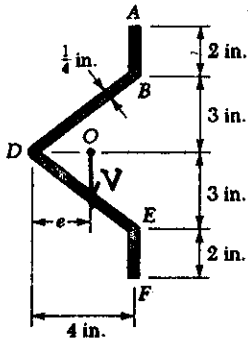
Point B  $y = 4 \quad \tau = 3.31 \text{ ksi}$

Point C  $y = 0 \quad \tau = 7.72 \text{ ksi}$



PROBLEM 6.69

6.69 through 6.74 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

$$L_{DB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \quad A_{DB} = L_{DB}t = (5)\left(\frac{1}{4}\right) = 1.25 \text{ in}^2$$

$$I_{DB} = \frac{1}{3} A_{DB} h^2 = \left(\frac{1}{3}\right)(1.25)(3)^2 = 3.75 \text{ in}^4$$

$$I_{AB} = \frac{1}{12} \left(\frac{1}{4}\right)(2)^3 + \left(\frac{1}{4}\right)(2)(4)^2 = 8.1667 \text{ in}^4$$

$$I = (2)(3.75) + (2)(8.1667) = 23.833 \text{ in}^4$$

Part AB:  $A = \frac{1}{4}(5-y) \text{ in}^2$ ,  $\bar{y} = \frac{1}{2}(5+y) \text{ in}$

$$Q = A\bar{y} = \frac{1}{8}(5-y)(5+y) = \frac{1}{8}(25-y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(25-y^2)}{(8)(23.833)(0.25)} = \frac{V(25-y^2)}{47.667}$$

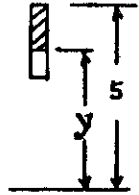
$$F_1 = \int \tau dA = \int_3^5 \frac{V(25-y^2)}{47.667} \cdot \frac{1}{4} dy$$

$$= \frac{V}{190.667} \left[ 25y - \frac{1}{3}y^3 \right]_3^5 =$$

$$= \frac{V}{190.667} \left[ (25)(5) - \frac{1}{3}(5)^3 - (25)(3) + \frac{1}{3}(3)^3 \right] = 0.09091V$$

$$\odot M_D = \odot M_D \quad -Ve = -2F_1(4) = -0.7273V$$

$$e = 0.727 \text{ in.}$$

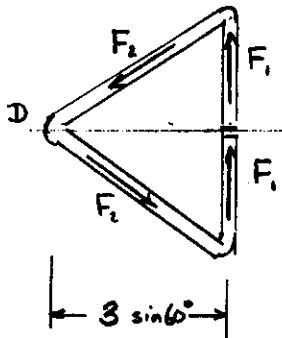
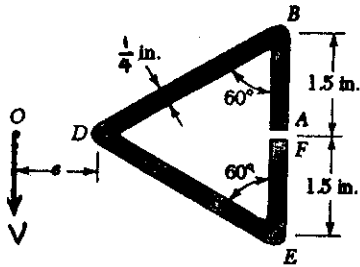




PROBLEM 6.70

6.69 through 6.74 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



$$I_{AB} = \frac{1}{3} \left(\frac{1}{4}\right) (1.5)^3 = 0.28125 \text{ in}^4$$

$$L_{BD} = 3 \text{ in} \quad A_{BD} = (3) \left(\frac{1}{4}\right) = 0.75 \text{ in}^2$$

$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3} (0.75) (1.5)^2 = 0.5625 \text{ in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875 \text{ in}^4$$

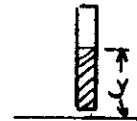
Part AB:  $A = \frac{1}{4} y \quad \bar{y} = \frac{1}{2} y \quad Q = A\bar{y} = \frac{1}{8} y^2$

$$\tau = \frac{VQ}{It} = \frac{V y^2}{(8)(1.6875)(0.25)} = \frac{V y^2}{3.375}$$

$$F_1 = \int \tau dA = \int_0^{1.5} \frac{V y^2}{3.375} \cdot (0.25 dy)$$

$$= \frac{(0.25)V}{3.375} \frac{y^3}{3} \Big|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.08333 V$$



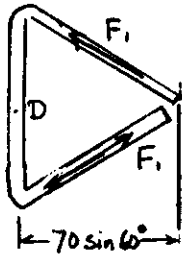
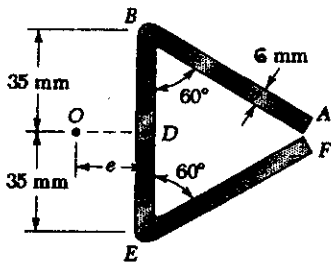
$$\sum M_D = \sum M_D \quad V e = 2 F_1 (3 \sin 60^\circ)$$

$$V e = (2)(0.08333)V(3 \sin 60^\circ)$$

$$e = (2)(0.08333)(3 \sin 60^\circ) = 0.433 \text{ in.}$$

PROBLEM 6.71

6.69 through 6.74 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

$$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$$

$$L_{AB} = 70 \text{ mm} \quad A_{AB} = (70)(6) = 420 \text{ mm}^2$$

$$I_{AB} = \frac{1}{3} A_{AB} h^2 = \left(\frac{1}{3}\right)(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$$

$$I = (2 \times 85.75 \times 10^3) + (2 \times 171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$$

Part AB  $A = ts = 6s$

$$\bar{y} = \frac{1}{2} s \sin 30^\circ = \frac{1}{4} s$$

$$Q = A\bar{y} = \frac{3}{2} s^2$$

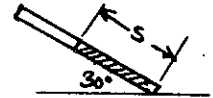
$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{It}$$

$$F_1 = \int \tau dA = \int_0^{70} \frac{3Vs^2}{2It} t ds = \frac{3V}{I} \int_0^{70} s^2 ds$$

$$= \frac{(3)(70)^3}{(2)(3)I} V = \frac{1}{3} V$$

$$\begin{aligned} +\sum M_o = +\sum M_o \quad Ve &= 2(F_1 \cos 60^\circ)(70 \sin 60^\circ) \\ &= 20.2 V \end{aligned}$$

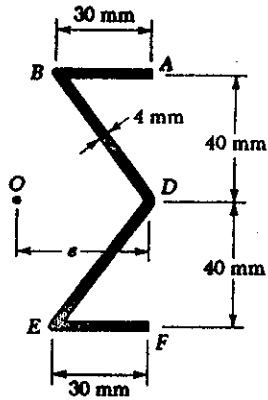
$$e = 20.2 \text{ mm}$$



PROBLEM 6.72

6.69 through 6.74 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.

SOLUTION



$$I_{AB} = (30)(4)(40)^2 = 192 \times 10^3 \text{ mm}^4$$

$$L_{BD} = \sqrt{30^2 + 40^2} = 50 \text{ mm} \quad A_{BD} = (50)(4) = 200 \text{ mm}^2$$

$$I_{BD} = \frac{1}{3} A_{BD} h^2 = \frac{1}{3} (200)(40)^2 = 106.67 \times 10^3 \text{ mm}^4$$

$$I = 2 I_{BD} + 2 I_{AB} = 597.33 \times 10^3 \text{ mm}^4$$

Part AB

$$A = 4x \quad \bar{y} = 40 \quad Q = A\bar{y} = 160x$$

$$\tau = \frac{VQ}{Iz} = \frac{V(160x)}{I(4)} = \frac{40V}{I} x$$

$$F_1 = \int \tau dA = \int_0^{30} \frac{40V}{I} 4 dx = \frac{160V}{I} \int_0^{30} x dx$$

$$= \frac{160V}{2I} x^2 \Big|_0^{30} = \frac{(160)(30)^2}{2(597.33 \times 10^3)} V$$

$$= 0.12054 V$$

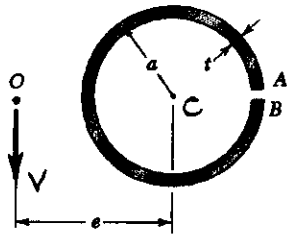
$$+\circlearrowleft M_o = +\circlearrowleft M_o \quad V e = (2) F_1 (40) = 9.64 V$$

$$e = 9.64 \text{ mm}$$

PROBLEM 6.73

6.69 through 6.74 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.

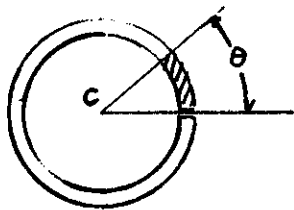
SOLUTION



For whole cross section  $A = 2\pi at$

$J = Aa^2 = 2\pi a^3 t$        $I = \frac{1}{2}J = \pi a^3 t$

Use polar coordinate  $\theta$  for partial cross section.



$A = st = a\theta t$        $s = \text{arc length}$

$\bar{r} = a \frac{\sin \alpha}{\alpha}$  where  $\alpha = \frac{1}{2}\theta$

$\bar{y} = \bar{r} \sin \alpha = a \frac{\sin^2 \alpha}{\alpha}$

$Q = A\bar{y} = a\theta t a \frac{\sin^2 \alpha}{\alpha} = a^2 t 2 \sin^2 \alpha$   
 $= a^2 t 2 \sin^2 \frac{\theta}{2} = a^2 t (1 - \cos \theta)$

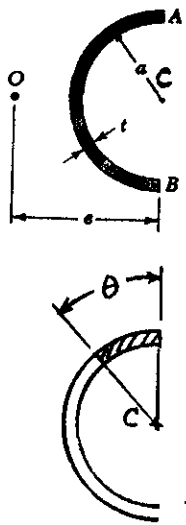
$\tau = \frac{VQ}{It} = \frac{Va^2}{I} (1 - \cos \theta)$

$M_c = \int \bar{a} \tau dA = \int_0^{2\pi} \frac{Va^3}{I} (1 - \cos \theta) t a d\theta = \frac{Va^4 t}{I} (\theta - \sin \theta) \Big|_0^{2\pi}$   
 $= \frac{2\pi Va^4 t}{\pi a^3 t} = 2aV$

But  $M_c = Ve$ , hence  $e = 2a$

PROBLEM 6.74

6.69 through 6.74 Determine the location of the shear center  $O$  of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

For a thin-walled hollow circular cross section  $A = 2\pi at$

$$J = a^2 A = 2\pi a^3 t \quad I = \frac{1}{2} J = \pi a^3 t$$

For the half-pipe section  $I = \frac{\pi}{2} a^3 t$

Use polar coordinate  $\theta$  for partial cross section

$$A = st = a\theta t \quad s = \text{arc length}$$

$$\bar{r} = a \frac{\sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\theta}{2}$$

$$\bar{y} = \bar{r} \cos \alpha = a \frac{\sin \alpha \cos \alpha}{\alpha}$$

$$Q = A\bar{y} = a\theta t a \frac{\sin \alpha \cos \alpha}{\alpha} = a^2 t (2 \sin \alpha \cos \alpha) \\ = a^2 t \sin 2\alpha = a^2 t \sin \theta$$

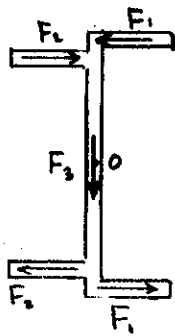
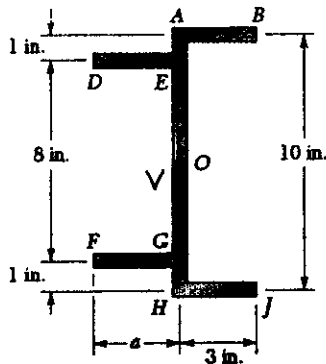
$$\tau = \frac{VQ}{It} = \frac{Va^2}{I} \sin \theta$$

$$M_c = \int a\tau dA = \int_0^\pi a \frac{Va^2}{I} \sin \theta t a d\theta = \frac{Va^4 t}{I} [-\cos \theta]_0^\pi \\ = 2 \frac{Va^4 t}{I} = \frac{4}{\pi} Va$$

But  $M_c = Ve$ , hence  $e = \frac{4}{\pi} a = 1.273 a$

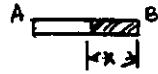
PROBLEM 6.75

6.75 and 6.76 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension  $a$  for which the shear center  $O$  of the cross section is located at the point indicated.



SOLUTION

Part AB  $A = tx \quad \bar{y} = 5 \text{ in} \quad Q = A\bar{y} = 5tx$

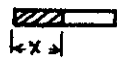


$$\tau = \frac{VQ}{It} = \frac{V \cdot 5tx}{It} = \frac{5Vx}{I}$$

$$F_1 = \int \tau dA = \int_0^3 \frac{5Vx}{I} t dx = \frac{5Vt}{I} \int_0^3 x dx$$

$$= \frac{(5)(3)^2}{2} \frac{Vt}{I} = 22.5 \frac{Vt}{I}$$

Part DE  $A = tx \quad \bar{y} = 4 \text{ in} \quad Q = A\bar{y} = 4tx$



$$\tau = \frac{VQ}{It} = \frac{V \cdot 4tx}{It} = \frac{4Vx}{I}$$

$$F_2 = \int \tau dA = \int_0^a \frac{4Vx}{I} \cdot t dx = \frac{4Vt}{I} \int_0^a x dx$$

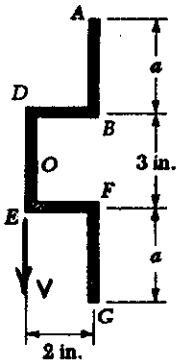
$$= \frac{2Vta^2}{I}$$

$$\sum M_O = 0 \quad 0 = (10)(22.5 \frac{Vt}{I}) - (8) \frac{2Vta^2}{I}$$

$$a^2 = \frac{(10)(22.5)}{(8)(2)} = 14.0625 \text{ in}^2 \quad a = 3.75 \text{ in.}$$

PROBLEM 6.76

6.75 and 6.76 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension  $a$  for which the shear center  $O$  of the cross section is located at the point indicated.



SOLUTION

Part AB Let  $c = 1.5 + a$  as shown.

$$A = t(c - y) \quad \bar{y} = \frac{1}{2}(c + y)$$

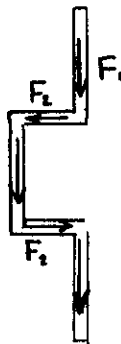
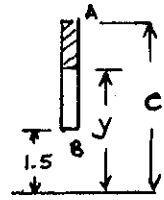
$$Q = A\bar{y} = \frac{1}{2}t(c - y)(c + y) = \frac{1}{2}t(c^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(c^2 - y^2)}{2I}$$

$$F_1 = \int \tau dA = \int_{1.5}^c \frac{V(c^2 - y^2)}{2I} t dy = \frac{Vt}{2I} \int_{1.5}^c (c^2 - y^2) dy$$

$$= \frac{Vt}{2I} \left( c^2 y - \frac{y^3}{3} \right) \Big|_{1.5}^c = \frac{Vt}{2I} \left[ c^3 - \frac{c^3}{3} - 1.5c^2 + \frac{(1.5)^3}{3} \right]$$

$$= \frac{Vt}{2I} \left[ \frac{2}{3}c^3 - 1.5c^2 + 1.125 \right]$$



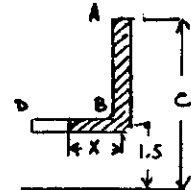
Part BD  $Q = Q_{AB} + t x \bar{y}_{60}$   
 $= \frac{1}{2}t(c^2 - 1.5^2) + t x (1.5)$

$$\tau = \frac{VQ}{It} = \frac{V}{2I} (c^2 - 1.5^2 + 3x)$$

$$F_2 = \int \tau dA = \int_0^2 \frac{V}{2I} (c^2 - 1.5^2 + 3x) t dx$$

$$= \frac{Vt}{2I} \left[ (c^2 - 1.5^2)x + 1.5x^2 \right]_0^2 = \frac{Vt}{2I} \left[ 2c^2 - (2)(1.5)^2 + (1.5)(2)^2 \right]$$

$$= \frac{Vt}{2I} \left[ 2c^2 + 1.5 \right]$$



$$+\circlearrowleft \Sigma M_o = +\circlearrowleft \Sigma M_o \quad 0 = 3F_2 - (2)(2)F_1$$

$$\frac{Vt}{2I} \left\{ 3(2c^2 + 1.5) - 4 \left( \frac{2}{3}c^3 - 1.5c^2 + 1.125 \right) \right\} = 0$$

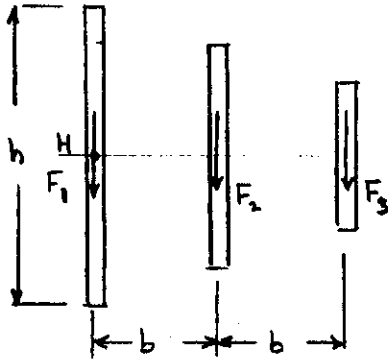
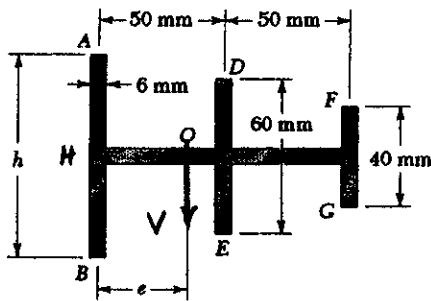
$$-\frac{8}{3}c^3 + 12c^2 = 0$$

$$c = \frac{(12)(3)}{8} = 4.5 \text{ in}$$

$$a = 4.5 - 1.5 = 3.00 \text{ in.}$$

PROBLEM 6.77

6.77 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center  $O$  of the cross section, knowing that  $h = 80$  mm.



SOLUTION

Let  $h_1 = \overline{AB} = h$ ,  $h_2 = \overline{DE}$ ,  $h_3 = \overline{FG}$

$$I = \frac{1}{12} t (h_1^3 + h_2^3 + h_3^3)$$

Part AB:  $A = (\frac{1}{2}h_1 - y)t$

$$\bar{y} = \frac{1}{2}(\frac{1}{2}h_1 + y)$$

$$Q = A\bar{y} = \frac{1}{2}t(\frac{1}{2}h_1 - y)(\frac{1}{2}h_1 + y)$$

$$= \frac{1}{2}t(\frac{1}{4}h_1^2 - y^2)$$

$$\tau = \frac{VQ}{I t} = \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)$$

$$F_1 = \int \tau dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)t dy$$

$$= \frac{Vt}{2I} \left( \frac{1}{4}h_1^2 y - \frac{y^3}{3} \right) \Big|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I} \left( \frac{1}{4}h_1^2 \frac{1}{2}h_1 - \frac{1}{3}(\frac{h_1}{2})^3 \right) = \frac{Vt h_1^3}{12 I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE

$$F_2 = \frac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$$

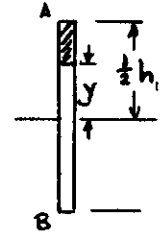
and for Part FG

$$F_3 = \frac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$$

$$+\circlearrowleft \Sigma M_H = +\circlearrowleft \Sigma M_H$$

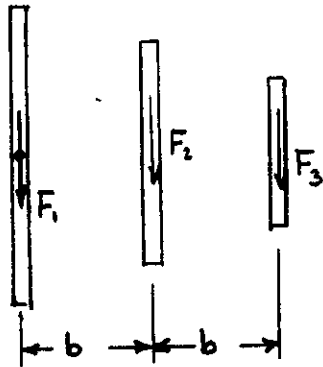
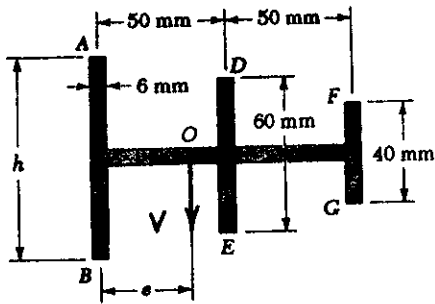
$$Ve = bF_2 + 2bF_3 = \frac{bh_2^3 + 2bh_3^3}{h_1^3 + h_2^3 + h_3^3} V$$

$$e = \frac{h_2^3 + 2h_3^3}{h_1^3 + h_2^3 + h_3^3} b = \frac{(60)^3 + (2)(40)^3}{(80)^3 + (60)^3 + (40)^3} (50) = 21.7 \text{ mm}$$





PROBLEM 6.78



6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension  $h$  for which the shear center  $O$  of the cross section is located at a distance  $e = 25$  mm from the center of the flange  $AB$ .

SOLUTION

Let  $h_1 = \overline{AB} = h$ ,  $h_2 = \overline{DE}$ ,  $h_3 = \overline{FG}$

$$I = \frac{1}{12} t (h_1^3 + h_2^3 + h_3^3)$$

Part AB  $A = (\frac{1}{2}h_1 - y)t$

$$\bar{y} = \frac{1}{2}(\frac{1}{2}h_1 + y)$$

$$Q = A\bar{y} = \frac{1}{2}t(\frac{1}{2}h_1 - y)(\frac{1}{2}h_1 + y) = \frac{1}{2}t(\frac{1}{4}h_1^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)$$

$$F_1 = \int \tau dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}(\frac{1}{4}h_1^2 - y^2)t dy$$

$$= \frac{Vt}{2I}(\frac{1}{4}h_1^2 y - \frac{1}{3}y^3) \Big|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I} \left[ (\frac{1}{4}h_1^2)(\frac{1}{2}h_1) - \frac{1}{3}(\frac{1}{2}h_1)^3 \right] = \frac{Vth_1^3}{12I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE  $F_2 = \frac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$

Likewise, for Part DE

and for Part FG

$$F_3 = \frac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$$

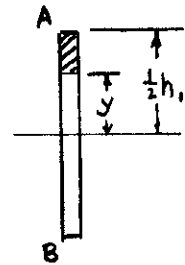
$$+\circlearrowleft \sum M_O = +\circlearrowleft \sum M_O$$

$$0 = eF_1 - (b-e)F_2 - (2b-e)F_3$$

$$\frac{eh_1^3}{h_1^3 + h_2^3 + h_3^3} - \frac{(b-e)h_2^3}{h_1^3 + h_2^3 + h_3^3} - \frac{(2b-e)h_3^3}{h_1^3 + h_2^3 + h_3^3} V = 0$$

$$h_1^3 = \frac{b-e}{e} h_2^3 + \frac{(2b-e)}{e} h_3^3 = \frac{(25)(60)^3}{25} + \frac{(75)(40)^3}{25} = 408 \times 10^3 \text{ mm}^3$$

$$h_1 = 74.2 \text{ mm}$$



## PROBLEM 6.79

6.79 For the angle shape and loading of Sample Prob. 6.5, check that  $\int q dz = 0$  along the horizontal leg of the angle and  $\int q dy = P$  along its vertical leg.

## SOLUTION

Referring to Sample Prob. 6.5

Along horizontal leg  $\tau_t = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3} (a^2 - 4az + 3z^2)$

$$\int q dz = \int_0^a \tau_t t dz = \frac{3P}{4a^3} \int_0^a (a^2 - 4az + 3z^2) dz = \frac{3P}{4a^3} \left( a^2 z - 4a \frac{z^2}{2} + 3 \frac{z^3}{3} \right) \Big|_0^a$$

$$= \frac{3P}{4a^3} (a^3 - 2a^3 + a^3) = 0$$

Along vertical leg  $\tau_c = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3} (a^2 + 4ay - 5y^2)$

$$\int q dy = \int_0^a \tau_c t dy = \frac{3P}{4a^3} \int_0^a (a^2 + 4ay - 5y^2) dy = \frac{3P}{4a^3} \left( a^2 y + 4a \frac{y^2}{2} - 5 \frac{y^3}{3} \right) \Big|_0^a$$

$$= \frac{3P}{4a^3} (a^3 + 2a^3 - \frac{5}{3}a^3) = \frac{3P}{4a^3} \cdot \frac{4}{3}a^3 = P$$

PROBLEM 6.80

6.80 For the angle shape and loading of Sample Prob. 6.5, (a) determine the points where the shearing stress is maximum and the corresponding values of the stress, (b) verify that the points obtained are located on the neutral axis corresponding to the given loading.

SOLUTION

Referring to Sample Prob. 6.5

(a) Along vertical leg  $\tau_c = \frac{3P(a-y)(a+5y)}{4ta^3} = \frac{3P}{4ta^3} (a^2 + 4ay - 5y^2)$

$\frac{d\tau_c}{dy} = \frac{3P}{4ta^3} (4a - 10y) = 0 \quad y = \frac{2}{3}a$

$\tau_m = \frac{3P}{4ta^3} [a^2 + (4a)(\frac{2}{3}a) - (5)(\frac{2}{3}a)^2] = \frac{3P}{4ta^3} (\frac{9}{3}a^2) = \frac{27}{20} \frac{P}{ta}$

Along horizontal leg  $\tau_f = \frac{3P(a-z)(a-3z)}{4ta^3} = \frac{3P}{4ta^3} (a^2 - 4az + 3z^2)$

$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3} (-4a + 6z) = 0 \quad z = \frac{2}{3}a$

$\tau_m = \frac{3P}{4ta^3} [a^2 - (4a)(\frac{2}{3}a) + (3)(\frac{2}{3}a)^2] = \frac{3P}{4ta^3} (-\frac{5}{3}a^2) = -\frac{1}{4} \frac{P}{ta}$

At the corner  $y=0, z=0 \quad \tau = \frac{3}{4} \frac{P}{ta}$

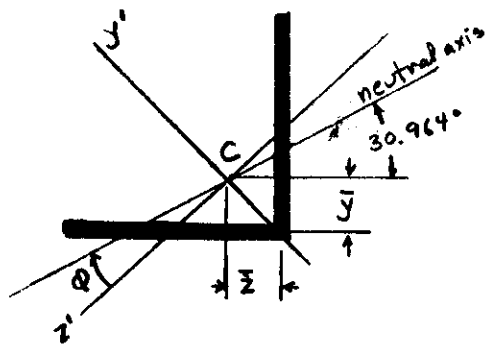
(b)  $I_{y'} = \frac{1}{3} ta^3 \quad I_{z'} = \frac{1}{12} ta^3 \quad \theta = 45^\circ$

$\tan \varphi = \frac{I_{z'}}{I_{y'}} \tan \theta = \frac{1}{4} \quad \varphi = 14.036^\circ$

$\theta - \varphi = 45 - 14.036 = 30.964^\circ$

$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$

$\bar{z} = \frac{\sum A\bar{z}}{\sum A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$



Neutral axis intersects vertical leg at

$y = \bar{y} + \bar{z} \tan 30.964^\circ$

$= (\frac{1}{4} + \frac{1}{4} \tan 30.964^\circ) a = 0.400 a = \frac{2}{5} a$

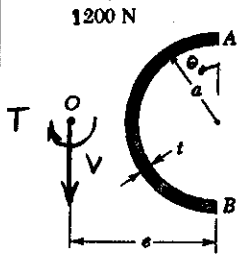
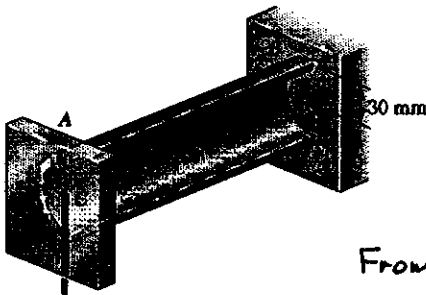
Neutral axis intersects horizontal leg at

$z = \bar{z} + \bar{y} \tan(45^\circ + \varphi)$

$= (\frac{1}{4} + \frac{1}{4} \tan 59.036^\circ) a = 0.6667 a = \frac{2}{3} a$

PROBLEM 6.81

\*6.81 A cantilever beam  $AB$ , consisting of half of a thin-walled pipe of 30-mm mean radius and 6-mm wall thickness, is subjected to a 1200-N vertical load. Knowing that the line of action of the load passes through the centroid  $C$  of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum stress in the beam. (Hint: The location of the shear center of the cross section was determined in Prob. 6.74.)



SOLUTION

From the solution to PROBLEM 6.74

$$e = \frac{4}{\pi} a \quad I = \frac{\pi}{2} a^3 t$$

$$Q = a^2 t \sin \theta \quad \tau = \frac{VQ}{It}$$

$$Q_{max} = a^2 t \text{ at } \theta = 90^\circ$$

Due to shearing force  $\tau_{90^\circ} = \frac{VQ_{max}}{It}$

$$V = 1200 \text{ N} \quad t = 6 \times 10^{-3} \text{ m}$$

$$I = \frac{\pi}{2} (30)^3 (6) = 254.47 \times 10^3 \text{ mm}^4 = 254.47 \times 10^{-9} \text{ m}^4$$

$$Q_{max} = (30)^2 (6) = 5.4 \times 10^3 \text{ mm}^3 = 5.4 \times 10^{-6} \text{ m}^3$$

$$\tau_{90^\circ} = \frac{(1200)(5.4 \times 10^{-6})}{(254.47 \times 10^{-9})(6 \times 10^{-3})} = 4.24 \times 10^6 \text{ Pa} = 4.24 \text{ MPa}$$

$$e = \frac{4}{\pi} a, \quad \bar{x} = \frac{2}{\pi} a \quad e - \bar{x} = \frac{2}{\pi} a$$

$$\text{Torque } T = (e - \bar{x}) V = \frac{2}{\pi} (30 \times 10^{-3})(1200) = 22.92 \text{ N}\cdot\text{m}$$

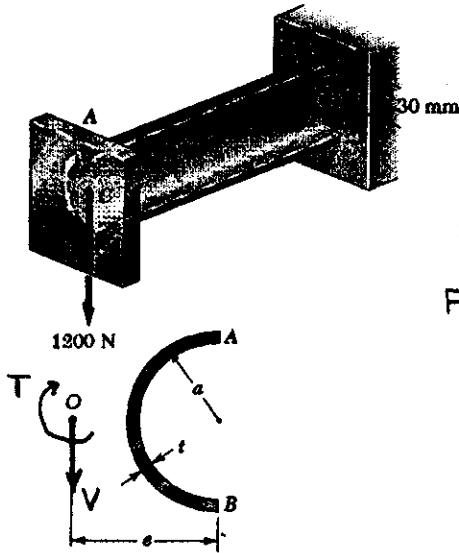
$$l = \pi a = \pi (30) = 94.248 \text{ mm} = 94.248 \times 10^{-3} \text{ m}$$

For torsion of a rectangular bar  $c_1 = c_2 = \frac{1}{3} (1 - 0.630 \frac{t}{l})$   
 $= \frac{1}{3} (1 - \frac{(0.630)(6)}{94.248}) = 0.31996$

$$\tau_{torsion} = \frac{T}{c_1 l t^2} = \frac{22.92}{(0.31996)(94.248 \times 10^{-3})(6 \times 10^{-3})^2} = 21.11 \times 10^6 \text{ Pa} = 21.11 \text{ MPa}$$

By superposition  $\tau_{max} = 4.24 + 21.11 = 25.35 \text{ MPa}$

PROBLEM 6.82



\*6.81 A cantilever beam  $AB$ , consisting of half of a thin-walled pipe of 30-mm mean radius and 6-mm wall thickness, is subjected to a 1200-N vertical load. Knowing that the line of action of the load passes through the centroid  $C$  of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum stress in the beam. (Hint: The location of the shear center of the cross section was determined in Prob. 6.74.)

\*6.82 Solve Prob. 6.81, assuming that the thickness of the beam is reduced to 5 mm.

SOLUTION

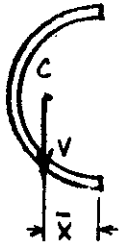
From the solution to PROBLEM 6.74

$$e = \frac{4}{\pi} a \quad I = \frac{\pi}{2} a^3 t$$

$$Q = a^2 t \sin \theta \quad \tau = \frac{VQ}{It} = \frac{Va^2}{I}$$

$$Q_{max} = a^2 t \quad \text{at } \theta = 90^\circ$$

Due to shearing force  $\tau_{90^\circ} = \frac{VQ_{max}}{It}$



$$V = 1200 \text{ N} \quad t = 5 \times 10^{-3} \text{ m}$$

$$I = \frac{\pi}{2} (30)^3 (5) = 212.06 \times 10^3 \text{ mm}^4 = 212.06 \times 10^{-7} \text{ m}^4$$

$$Q_{max} = (30)^2 (5) = 4.5 \times 10^3 \text{ mm}^3 = 4.5 \times 10^{-6} \text{ m}^3$$

$$\tau_{90^\circ} = \frac{(1200)(4.5 \times 10^{-6})}{(212.06 \times 10^{-7})(5 \times 10^{-3})} = 5.09 \times 10^6 \text{ Pa} = 5.09 \text{ MPa}$$

$$e = \frac{4}{\pi} a \quad \bar{x} = \frac{2}{\pi} a \quad e - \bar{x} = \frac{2}{\pi} a$$

Torque  $T = (e - \bar{x})V = \frac{2}{\pi} (30 \times 10^{-3})(1200) = 22.92 \text{ N}\cdot\text{m}$

For torsion of a rectangular bar  $C_1 = C_2 = \frac{1}{3} \left[ 1 - 0.630 \frac{t}{r} \right]$   
 $= \frac{1}{3} \left[ 1 - \frac{(0.630)(5)}{94.248} \right] = 0.32219$

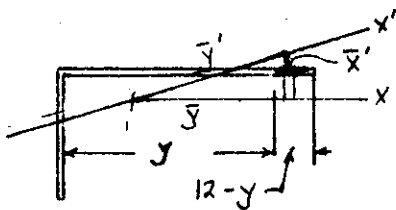
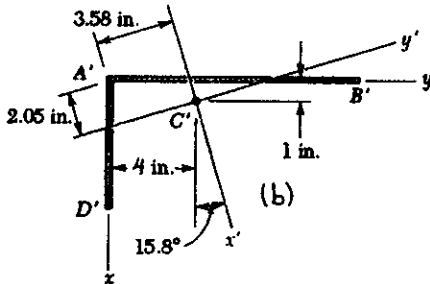
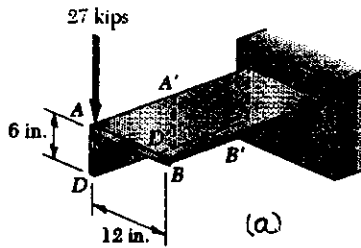
$$\tau_{torsion} = \frac{T}{C_1 t^2} = \frac{22.92}{(0.32219)(94.248 \times 10^{-3})(5 \times 10^{-3})^2} = 30.19 \times 10^6 \text{ Pa}$$

$$= 30.19 \text{ MPa}$$

By superposition  $\tau_{max} = 5.09 + 30.19 = 35.3 \text{ MPa}$

PROBLEM 6.83

\*6.83 The cantilever beam shown consists of an angle shape of  $\frac{3}{8}$ -in. thickness. For the given loading, determine the location and magnitude of the largest shearing stress along line A'B' in the horizontal leg of the angle shape. The  $x'$  and  $y'$  axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are  $I_{x'} = 115.7 \text{ in}^4$  and  $I_{y'} = 12.61 \text{ in}^4$



SOLUTION



$$V = 27 \text{ kips} \quad \beta = 15.8^\circ$$

$$V_{x'} = V \cos \beta \quad V_{y'} = V \sin \beta$$

In the horizontal leg use coordinate  $y$  as shown.

$$A = \frac{3}{8} (12 - y) \text{ in}^2 \quad t = \frac{3}{8} \text{ in}$$

$$\bar{y} = \frac{1}{2} (12 + y) - 4 = 2 + \frac{1}{2} y \text{ in.}$$

$$\bar{x} = 1 \text{ in.}$$

$$\bar{x}' = \bar{x} \cos \beta - \bar{y} \sin \beta$$

$$\bar{y}' = \bar{y} \cos \beta + \bar{x} \sin \beta$$

Due to  $V_{x'}$  
$$\tau_1 = \frac{V_{x'} A \bar{x}'}{I_{y'} t} = \frac{(V \cos \beta) (\frac{3}{8}) (12 - y) [(1 \cos \beta - (2 + \frac{1}{2} y) \sin \beta)]}{(12.61) (\frac{3}{8})}$$

$$= 2.0603 (12 - y) (0.41765 - 0.13614 y) \quad \text{ksi}$$

Due to  $V_{y'}$  
$$\tau_2 = \frac{V_{y'} A \bar{y}'}{I_{x'} t} = \frac{(V \sin \beta) (\frac{3}{8}) (12 - y) [(2 + \frac{1}{2} y) \cos \beta + (1) \sin \beta]}{(115.6) (\frac{3}{8})}$$

$$= 0.063595 (12 - y) (2.19672 + 0.48111 y) \quad \text{ksi}$$

Total 
$$\tau_1 + \tau_2 = (12 - y) (1.000 - 0.250 y) \quad \text{ksi}$$

$y$ (in)	0	2	4	6	8	10	12
$\tau$ (ksi)	12.00	5.00	0	-3.00	-4.00	-3.00	0

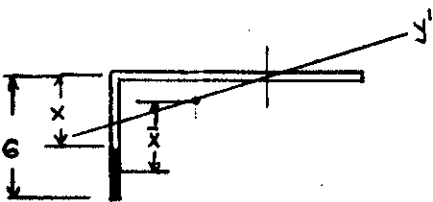
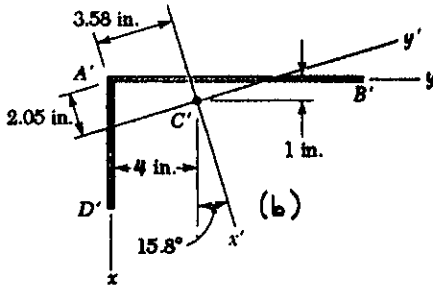
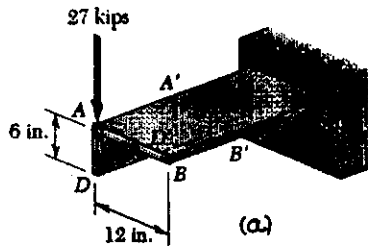
$\tau = 12 \text{ ksi}$  at corner

$\tau = -4 \text{ ksi}$  at  $y = 8 \text{ in}$

$$\frac{d\tau}{dy} = -(0.25)(12 - y_m) + (1 - 0.25 y_m)$$

$$= 0.5 y_m - 4 = 0 \quad y_m = 8 \text{ in}$$

PROBLEM 6.84



\*6.83 The cantilever beam shown consists of an angle shape of  $\frac{3}{8}$  - in. thickness. For the given loading, determine the location and magnitude of the largest shearing stress along line  $A'B'$  in the horizontal leg of the angle shape. The  $x'$  and  $y'$  axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are  $I_{x'} = 115.7 \text{ in}^4$  and  $I_{y'} = 12.61 \text{ in}^4$ .

\*6.84 For the cantilever beam and loading of Prob. 6.83, determine the location and magnitude of the largest shearing stress along line  $A'D'$  in the vertical leg of the angle shape.

SOLUTION

$V = 27 \text{ kips}$        $\beta = 15.8^\circ$

$V_{x'} = V \cos \beta$        $V_{y'} = V \sin \beta$

In vertical leg use coordinate  $x$  as shown.

$A = \frac{3}{8}(6-x) \text{ in}^2$        $t = \frac{3}{8} \text{ in.}$

$\bar{y} = 4 \text{ in.}$

$\bar{x} = \frac{1}{2}(6+x) - 1 = 2 + \frac{1}{2}x$

$\bar{x}' = \bar{x} \cos \beta - \bar{y} \sin \beta$

$\bar{y}' = \bar{y} \cos \beta + \bar{x} \sin \beta$

Due to  $V_{x'}$        $\tau_1 = \frac{V_{x'} A \bar{x}'}{I_{y'} t} = \frac{(V \cos \beta)(\frac{3}{8})(6-x)[(2 + \frac{1}{2}x) \cos \beta - 4 \sin \beta]}{(12.61)(\frac{3}{8})}$

$= 2.0603 (6-x)(0.83531 + 0.48111x)$

Due to  $V_{y'}$        $\tau_2 = \frac{V_{y'} A \bar{y}'}{I_{x'} t} = \frac{(V \sin \beta)(\frac{3}{8})(6-x)[4 \cos \beta + (2 + \frac{1}{2}x) \sin \beta]}{(115.6)(\frac{3}{8})}$

$= 0.06359 (6-x)(4.3934 + 0.13614x)$

Total:  $\tau_1 + \tau_2 = (6-x)(2.000 + 1.000x)$

$x \text{ (in)}$	0	1	2	3	4	5	6
$\tau \text{ (ksi)}$	12.00	15.00	16.00	15.00	12.00	7.00	0

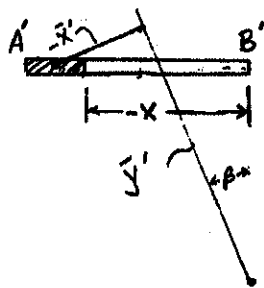
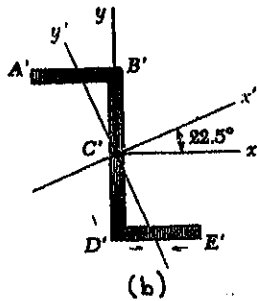
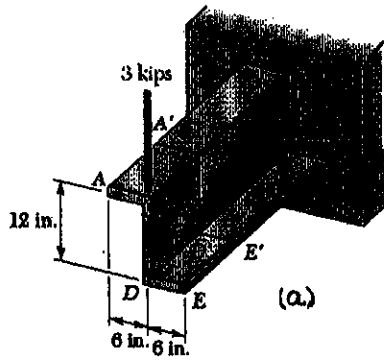
$\tau_{max} = 16 \text{ ksi at } x = 2 \text{ in.}$

$\frac{d\tau}{dx} = (6-x_n)(1) + (2+x_n)(-1)$

$= 4 - 2x_n = 0$        $x_n = 2 \text{ in.}$

**PROBLEM 6.87**

\*6.87 The cantilever beam shown consists of a Z shape of  $\frac{1}{4}$ -in. thickness. For the given loading, determine the distribution of the shearing stresses along line  $A'B'$  in the upper horizontal leg of the Z shape. The  $x'$  and  $y'$  axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are  $I_{x'} = 166.3 \text{ in}^4$  and  $I_{y'} = 13.61 \text{ in}^4$ .



**SOLUTION**

$V = 3 \text{ kips}$        $\beta = 22.5^\circ$

$V_{x'} = V \sin \beta$        $V_{y'} = V \cos \beta$

In upper horizontal leg use coordinate  $x$  ( $-6 \text{ in} \leq x \leq 0$ )

$A = \frac{1}{4} (6+x) \text{ in.}$

$\bar{x} = \frac{1}{2} (-6+x) \text{ in}$

$\bar{y} = 6 \text{ in.}$

$\bar{x}' = \bar{x} \cos \beta + \bar{y} \sin \beta$

$\bar{y}' = \bar{y} \cos \beta - \bar{x} \sin \beta$

Due to  $V_{x'}$        $\tau_1 = \frac{V_{x'} A \bar{x}'}{I_{y'} t}$

$\tau_1 = \frac{(V \sin \beta) (\frac{1}{4}) (-6+x) [\frac{1}{2} (-6+x) \cos \beta + 6 \sin \beta]}{(13.61) (\frac{1}{4})}$

$= 0.084353 (6+x) (-0.47554 + 0.46194 x)$

Due to  $V_{y'}$        $\tau_2 = \frac{V_{y'} A \bar{y}'}{I_{x'} t} = \frac{(V \cos \beta) (\frac{1}{4}) (6+x) [6 \cos \beta + \frac{1}{2} (-6+x) \sin \beta]}{(166.3) (\frac{1}{4})}$

$= 0.0166665 (6+x) [6.69132 - 0.19134 x]$

Total  $\tau_1 + \tau_2 = (6+x) [-0.071411 + 0.085396 x]$

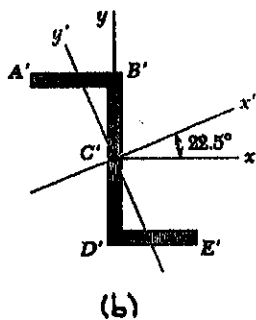
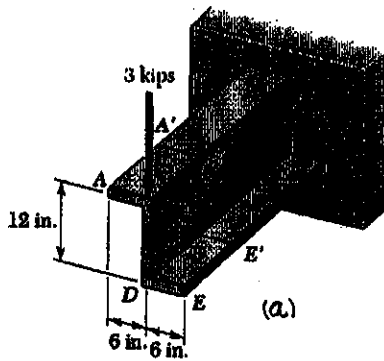
$x$ (in)	-6	-5	-4	-3	-2	-1	0
$\tau$ (ksi)	0	-0.105	-0.140	-0.104	0.003	0.180	0.428



**PROBLEM 6.88**

\*6.87 The cantilever beam shown consists of a Z shape of  $\frac{1}{4}$ -in. thickness. For the given loading, determine the distribution of the shearing stresses along line  $A'B'$  in the upper horizontal leg of the Z shape. The  $x'$  and  $y'$  axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are  $I_{x'} = 166.3 \text{ in}^4$  and  $I_{y'} = 13.61 \text{ in}^4$ .

\*6.88 For the cantilever beam and loading of Prob. 6.87, determine the distribution of the shearing stresses along line  $B'D'$  in the vertical web of the Z shape.



**SOLUTION**

$V = 3 \text{ kips} \quad \beta = 22.5^\circ$

$V_{x'} = V \sin \beta \quad \bar{V}_{y'} = V \cos \beta$

For part  $AB'$   $A = (\frac{1}{4})(6) = 1.5 \text{ in}^2$   
 $\bar{x} = -3 \text{ in}, \bar{y} = 6 \text{ in}.$

For part  $B'Y$

$A = \frac{1}{4}(6-y)$   
 $\bar{x} = 0 \quad \bar{y} = \frac{1}{2}(6+y)$

$x' = x \cos \beta + y \sin \beta$   
 $y' = y \cos \beta - x \sin \beta$

Due to  $V_{x'}$   $\tau_1 = \frac{V_{x'}(A_{As} \bar{x}'_s + A_{Ar} \bar{x}'_r)}{I_{y'} t}$

$\tau_1 = \frac{(V \sin \beta) [(1.5)(-3 \cos \beta + 6 \sin \beta) + \frac{1}{4}(6-y) \frac{1}{2}(6+y) \sin \beta]}{(13.61)(\frac{1}{4})}$   
 $= \frac{(V \sin \beta) [-0.7133 + 1.7221 - 0.047835 y^2]}{3.4025} = 0.3404 - 0.01614 y^2$

Due to  $V_{y'}$   $\tau_2 = \frac{V_{y'}(A_{As} \bar{y}'_s + A_{Ar} \bar{y}'_r)}{I_{x'} t}$

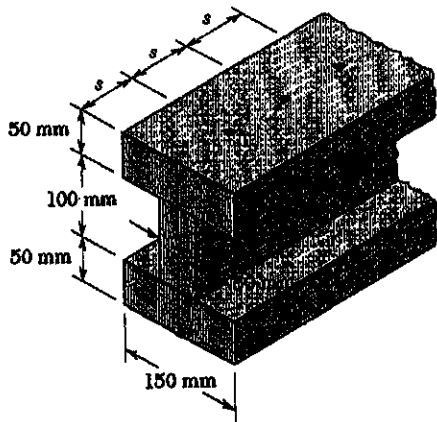
$\tau_2 = \frac{(V \cos \beta) [(1.5)(6 \cos \beta + 3 \sin \beta) + \frac{1}{4}(6-y) \frac{1}{2}(6+y) \cos \beta]}{(166.3)(\frac{1}{4})}$   
 $= \frac{(V \cos \beta) [10.037 + 4.1575 - 0.11548 y^2]}{(166.3)(\frac{1}{4})} = 0.9463 - 0.00770 y^2$

Total  $\tau_1 + \tau_2 = 1.2857 - 0.02384 y^2$

$y \text{ (in)}$	0	$\pm 2$	$\pm 4$	$\pm 6$
$\tau \text{ (ksi)}$	1.287	1.191	0.905	0.428

**PROBLEM 6.89**

6.89 Three boards, each 50 mm thick, are nailed together to form a beam that is subjected to a 1200-N vertical shear. Knowing that the allowable shearing force in each nail is 600 N, determine the largest permissible spacing  $s$  between the nails.



**SOLUTION**

Calculate moment of inertia

Part	$A$ (mm <sup>2</sup> )	$d$ (mm)	$Ad^2$ (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
Top	7500	75	42.19	1.56
Middle	5000	0	0	4.17
Bottom	7500	75	42.19	1.56
$\Sigma$			84.38	7.29

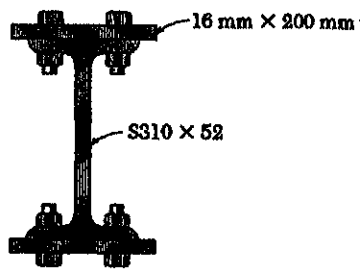
$$I = \Sigma Ad^2 + \Sigma \bar{I} = 91.67 \times 10^6 \text{ mm}^4 = 91.67 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{top}} d_{\text{top}} = (7500)(75) = 562.5 \times 10^3 \text{ mm}^3 = 562.5 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(562.5 \times 10^{-6})}{91.67 \times 10^{-6}} = 7.363 \times 10^3 \text{ N/m}$$

$$F_{\text{nail}} = qs \quad S = \frac{F_{\text{nail}}}{q} = \frac{600}{7.363 \times 10^3} = 81.5 \times 10^{-3} \text{ m} = 81.5 \text{ mm} \blacktriangleleft$$

PROBLEM 6.90



6.90 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using bolts of 18-mm diameter spaced longitudinally every 120 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible shearing force.

SOLUTION

Calculate moment of inertia

Part	A (mm <sup>2</sup> )	d (mm)	Ad <sup>2</sup> (10 <sup>6</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>6</sup> mm <sup>4</sup> )
Top plate	3200	*160.5	82.43	0.07
S310 x 52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
$\Sigma$			164.86	95.44

$$* d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{\text{plate}} d_{\text{plate}} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} (18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

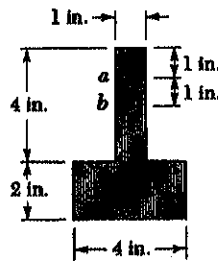
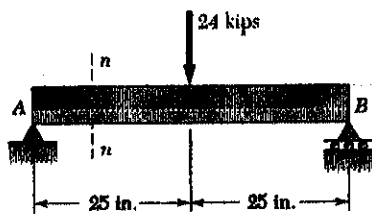
$$F_{\text{bolt}} = \tau_{\text{all}} A_{\text{bolt}} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

$$q_s = 2 F_{\text{bolt}} \quad q = \frac{2 F_{\text{bolt}}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \quad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N} = 193.5 \text{ kN}$$

**PROBLEM 6.91**

6.91 For the beam and loading shown, consider section  $n-n$  and determine the shearing stress at (a) point  $a$ , (b) point  $b$ .

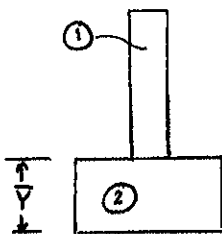


**SOLUTION**

$$R_A = R_B = 12 \text{ kips}$$

$$\text{At section } n-n \quad V = 12 \text{ kips.}$$

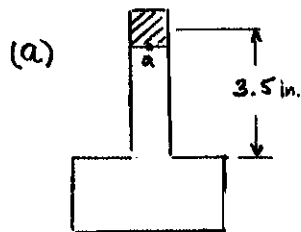
Locate centroid and calculate moment of inertia.



Part	$A \text{ (in}^2\text{)}$	$\bar{y} \text{ (in)}$	$A\bar{y} \text{ (in}^3\text{)}$	$d \text{ (in)}$	$Ad^2 \text{ (in}^4\text{)}$	$\bar{I} \text{ (in}^4\text{)}$
①	4	4	16	2	16	5.33
②	8	1	8	1	8	2.67
$\Sigma$	12		24		24	8

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in.}$$

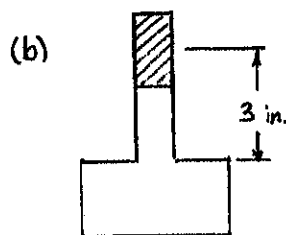
$$I = \Sigma Ad^2 + \Sigma \bar{I} = 24 + 8 = 32 \text{ in}^4$$



$$Q_a = A_a \bar{y}_a = (1)(1)(3.5) = 3.5 \text{ in}^3$$

$$t = 1 \text{ in}$$

$$\tau_a = \frac{VQ_a}{I t} = \frac{(12)(3.5)}{(32)(1)} = 1.313 \text{ ksi}$$



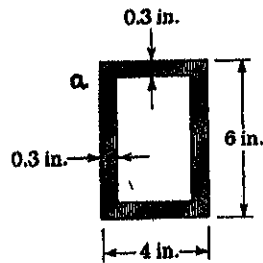
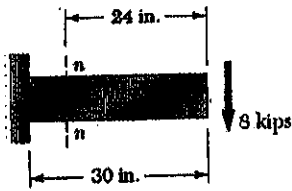
$$Q_b = A_b \bar{y}_b = (1)(2)(3) = 6 \text{ in}^3$$

$$t = 1 \text{ in.}$$

$$\tau_b = \frac{VQ_b}{I t} = \frac{(12)(6)}{(32)(1)} = 2.25 \text{ ksi}$$

PROBLEM 6.92

6.92 For the beam and loading shown, consider section  $n-n$  and determine (a) the largest shearing stress in that section, (b) the shearing stress at point  $a$ .



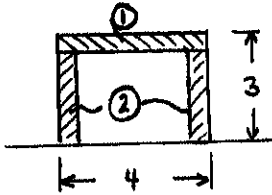
SOLUTION

At section  $n-n$   $V = 8$  kips

Moment of inertia

$$\begin{aligned}
 I &= \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3 \\
 &= \frac{1}{12} (4)(6)^3 - \frac{1}{12} (3.4)(6.4)^3 \\
 &= 27.3852 \text{ in}^4
 \end{aligned}$$

(a) The largest shearing stress occurs on a section through the centroid of the entire cross section.

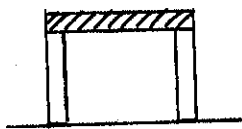


Part	$A$ (in <sup>2</sup> )	$\bar{y}$ (in)	$Q = A\bar{y}$ (in <sup>3</sup> )
①	1.2	2.85	3.42
②	1.62	1.35	2.187
$\Sigma$	2.82		5.607

$$Q_m = 4.5135 \text{ in}^3 \quad t = (2)(0.3) = 0.6 \text{ in.}$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(8)(5.607)}{(27.3852)(0.6)} = 2.73 \text{ ksi}$$

(b)



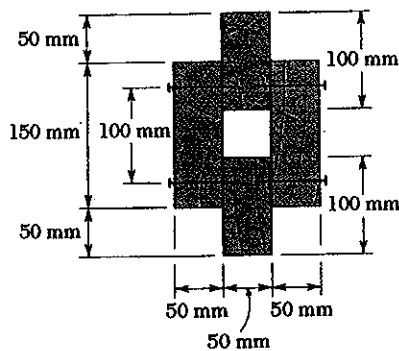
$$Q_a = A_a \bar{y}_a = (1.2)(2.85) = 3.42 \text{ in}^3$$

$$t = (2)(0.3) = 0.6 \text{ in.}$$

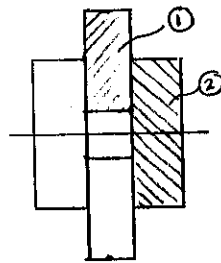
$$\tau_a = \frac{VQ_a}{It} = \frac{(8)(3.42)}{(27.3852)(0.6)} = 1.665 \text{ ksi}$$

**PROBLEM 6.93**

6.93 The built-up timber beam shown is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is  $s = 60$  mm and that each nail is 90 mm long, determine the shearing force in each nail.



**SOLUTION**



$$I_1 = \frac{1}{12} b h^3 + A_1 d^2$$

$$= \frac{1}{12} (50)(100)^3 + (50)(100)(75)^2$$

$$= 32.292 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12} (50)(150)^3$$

$$= 14.0625 \times 10^6 \text{ mm}^4$$

$$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \text{ mm}^4 = 92.71 \times 10^{-6} \text{ m}^4$$

$$Q = Q_1 = A_1 \bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

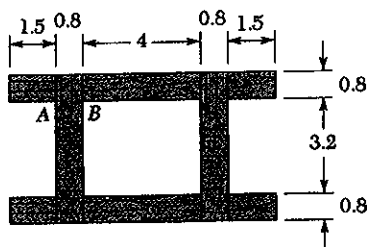
$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \text{ N/m}$$

$$s = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$2F_{\text{nail}} = qs \quad F_{\text{nail}} = \frac{1}{2} qs = \frac{1}{2} (24.27 \times 10^3)(60 \times 10^{-3}) = 728 \text{ N}$$

**PROBLEM 6.94**

6.94 The built-up beam shown was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb vertical shear, determine the average shearing stress in the glued joint (a) at A, (b) at B.



Dimensions in inches

**SOLUTION**

$$I = 2 \left[ \frac{1}{12} (0.8)(4.8)^3 + \frac{1}{12} (7)(0.8)^3 + (7)(0.8)(2.0)^2 \right]$$

$$= 60.143 \text{ in}^4$$

$$(a) \quad A_a = (1.5)(0.8) = 1.2 \text{ in}^2 \quad \bar{y}_a = 2.0 \text{ in.}$$

$$Q_a = A_a \bar{y}_a = 2.4 \text{ in}^3$$

$$t_a = 0.8 \text{ in.}$$

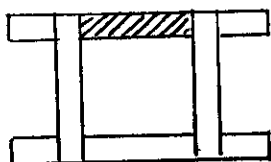
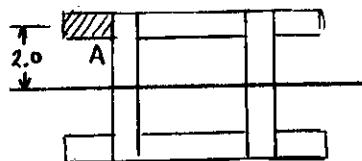
$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(1200)(2.4)}{(60.143)(0.8)} = 59.9 \text{ psi}$$

$$(b) \quad A_b = (4)(0.8) = 3.2 \text{ in}^2 \quad \bar{y}_b = 2.0 \text{ in.}$$

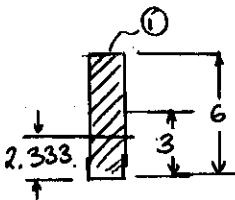
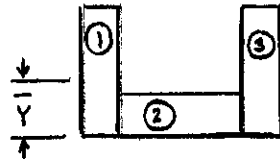
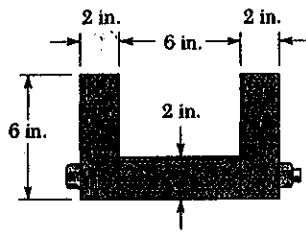
$$Q_b = A_b \bar{y}_b = (3.2)(2.0) = 6.4 \text{ in}^3$$

$$t_b = (2)(0.8) = 1.6 \text{ in.}$$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(1200)(6.4)}{(60.143)(1.6)} = 79.8 \text{ psi}$$



**PROBLEM 6.95**



6.95 A beam consists of three planks connected as shown by  $\frac{3}{8}$ -in.-diameter bolts spaced every 12 in. along the longitudinal axis of the beam. Knowing that the beam is subjected to a 2500-lb vertical shear, determine the maximum shearing stress in the bolts.

**SOLUTION**

Locate neutral axis and compute moment of inertia.

Part	A (in <sup>2</sup> )	$\bar{y}$ (in)	$A\bar{y}$ in <sup>3</sup>	d (in)	$Ad^2$ (in <sup>4</sup> )	$\bar{I}$ (in <sup>4</sup> )
①	12	3	36	0.667	5.333	36
②	12	1	12	1.333	21.333	4
③	12	3	36	0.667	5.333	36
$\Sigma$	36		84		32	76

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{84}{36} = 2.333 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 108 \text{ in}^4$$

$$Q = A_1 \bar{y}_1 = (2)(6)(3 - 2.333) = 8 \text{ in}^3$$

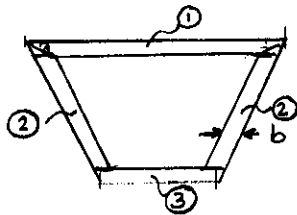
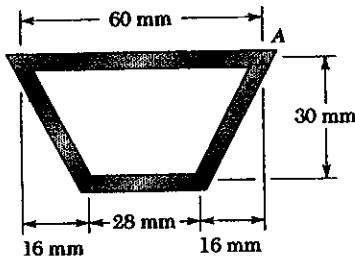
$$q = \frac{VQ}{I} = \frac{(2500)(8)}{108} = 185.2 \text{ lb/in}$$

$$F_{\text{bolt}} = q s = (185.2)(12) = 2.222 \times 10^3 \text{ lb.}$$

$$A_{\text{bolt}} = \frac{\pi}{4} d_{\text{bolt}}^2 = \frac{\pi}{4} \left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{\text{bolt}} = \frac{F_{\text{bolt}}}{A_{\text{bolt}}} = \frac{2.222 \times 10^3}{0.1104} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

PROBLEM 6.96



6.96 An extruded beam with the cross section shown and a 3-mm wall thickness is subjected to a 10-kN vertical shear. Determine (a) the shearing stress at point A, (b) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

SOLUTION

For part (a) height  $h = 30 \text{ mm}$

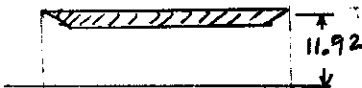
$$b = \frac{\sqrt{16^2 + 30^2}}{30} t = (1.13333)(3) = 3.4 \text{ mm}$$

$$\bar{I}_x = \frac{1}{12} (2b) h^3 = \quad \times 10^3 \text{ mm}^4$$

Part	A (mm <sup>2</sup> )	$\bar{y}$ (mm)	$A\bar{y}$ mm <sup>3</sup>	d (mm)	$Ad^2$ (10 <sup>3</sup> mm <sup>4</sup> )	$\bar{I}$ (10 <sup>3</sup> mm <sup>4</sup> )
①	180	30	5400	11.92	25.58	0.135
②	204	15	3060	3.08	1.94	15.3
③	84	0	0	18.08	27.46	0.063
	468		8460		54.98	15.55

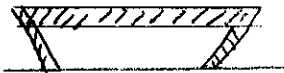
$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{8460}{468} = 18.08 \text{ mm}$$

$$I = \sum Ad^2 + \sum \bar{I} = 70.48 \times 10^3 \text{ mm}^4 = 70.48 \times 10^{-9} \text{ m}^4$$



$$Q_A = (60)(3)(11.92) = 2.146 \times 10^3 \text{ mm}^3 = 2.146 \times 10^{-6} \text{ m}^3$$

$$\tau_A = \frac{VQ_A}{I t} = \frac{(10 \times 10^3)(2.146 \times 10^{-6})}{(70.48 \times 10^{-9})(6 \times 10^{-3})} = 50.7 \times 10^6 \text{ Pa} = 50.7 \text{ MPa}$$

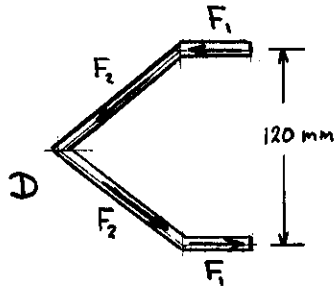
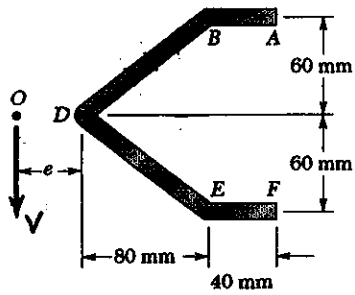


$$Q_m = Q_A + (2)(3.4)(11.92) \frac{11.92}{2} = 2.629 \times 10^3 \text{ mm}^3 = 2.629 \times 10^{-6} \text{ m}^3$$

$$\tau_m = \frac{VQ}{I t} = \frac{(10 \times 10^3)(2.629 \times 10^{-6})}{(70.48 \times 10^{-9})(6 \times 10^{-3})} = 62.6 \times 10^6 \text{ Pa} = 62.6 \text{ MPa}$$



**PROBLEM 6.97**



6.97 and 6.98 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center  $O$  of the cross section.

**SOLUTION**

$$I_{AB} = (40t)(60)^2 = 144 \times 10^3 t$$

$$L_{DE} = \sqrt{80^2 + 60^2} = 100 \text{ mm} \quad A_{DE} = 100t$$

$$I_{DE} = \frac{1}{3} A_{DE} h^2 = \frac{1}{3} (100t)(60)^2 = 120 \times 10^3 t$$

$$I = 2I_{AB} + 2I_{DE} = 528 \times 10^3 t$$

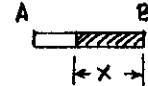
Part AB:  $A = tx \quad \bar{y} = 60 \text{ mm}$

$$Q = A\bar{y} = 60tx \text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$

$$F_1 = \int \tau dA = \int_0^{40} \frac{60Vx}{I} t dx = \frac{60Vt}{I} \int_0^{40} x dx$$

$$= \frac{60Vt}{I} \frac{x^2}{2} \Big|_0^{40} = \frac{(60)(30)^2 Vt}{(2)(528 \times 10^3)t} = 0.051136 V$$



$$\sum M_o = \sum M_o$$

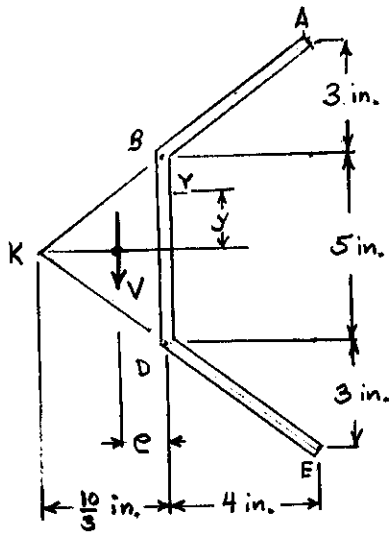
$$Ve = (0.051136 V)(120)$$

$$e = (0.051136)(120) = 6.14 \text{ mm}$$

PROBLEM 6.98

6.97 and 6.98 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center  $O$  of the cross section.

SOLUTION



$$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \quad A_{AB} = 5t$$

$$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12} (5t)(3)^2 + (5t)(4)^2 = 83.75 t \text{ in}^4$$

$$I_{BD} = \frac{1}{12} (t)(5)^3 = 10.417 t \text{ in}^4$$

$$I = 2I_{AB} + I_{BD} = 177.917 t \text{ in.}$$

In part BD  $Q = Q_{AB} + Q_{BY}$

$$Q = (5t)(4) + (2.5 - y)t\left(\frac{1}{2}\right)(2.5 + y) = 20t + 3.125t - \frac{1}{2}ty^2 = (23.125 - \frac{1}{2}y^2)t$$

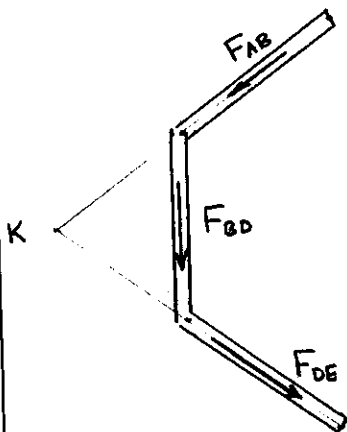
$$\tau = \frac{VQ}{It} \quad F_{BD} = \int \tau dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{I} \cdot t dy = \frac{Vt}{I} \int_{-2.5}^{2.5} (23.125 - \frac{1}{2}y^2) dy = \frac{Vt}{I} [23.125y - \frac{1}{6}y^3]_{-2.5}^{2.5} = \frac{Vt}{I} \cdot 2 \left[ (23.125)(2.5) - \frac{(2.5)^3}{6} \right] = \frac{Vt}{177.917t} (110.417) = 0.62061 V$$

$$\sum M_K = \sum M_K$$

$$-V \left( \frac{10}{3} - e \right) = -\frac{10}{3} (0.62061 V)$$

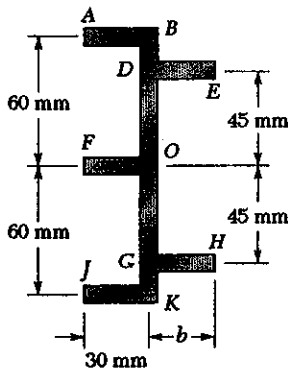
$$e = \frac{10}{3} [1 - 0.62061] = 1.265 \text{ in.}$$

Note that the lines of action of  $F_{AB}$  and  $F_{DE}$  pass through point  $K$ . Thus, these forces have zero moment about point  $K$ .



PROBLEM 6.99

6.99 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension  $b$  for which the shear center  $O$  of the cross section is located at the point indicated.



SOLUTION

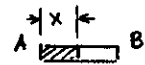
Part AB:  $A = tx$   $\bar{y} = 60 \text{ mm}$

$Q = A\bar{y} = 60tx \text{ mm}^2$

$\tau = \frac{VQ}{It} = \frac{60Vx}{I}$

$F_1 = \int \tau dA = \int_0^{30} \frac{60Vx}{I} t dx = \frac{60Vt}{I} \int_0^{30} x dx$

$= \frac{60Vt}{I} \left[ \frac{x^2}{2} \right]_0^{30} = \frac{(60)(30)^2}{2} \frac{Vt}{I} = 27 \times 10^3 \frac{Vt}{I}$

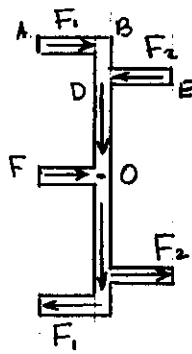
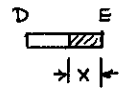


Part DE  $A = tx$   $\bar{y} = 45 \text{ mm}$

$Q = A\bar{y} = 45tx$

$\tau = \frac{VQ}{It} = \frac{45Vx}{I}$

$F_2 = \int \tau dA = \int_0^b \frac{45Vx}{I} t dx = \frac{45Vt}{I} \int_0^b x dx = \frac{45b^2 Vt}{2I}$



$\rightarrow \Sigma M_O = + \Sigma M_O$   $0 = (2)(45)F_2 - (2)(60)F_1$

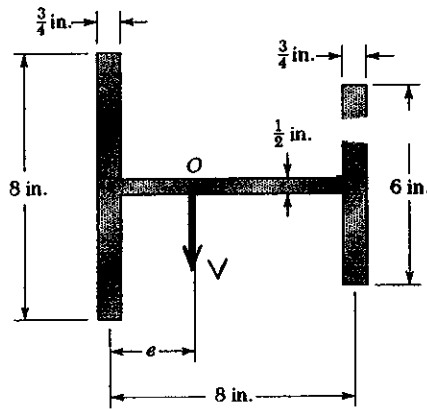
$[(45)^2 b^2 - (2)(60)(27 \times 10^3)] \frac{Vt}{I} = 0$

$b^2 = \frac{(2)(60)(27 \times 10^3)}{45^2} = 1600 \text{ mm}^2$   $b = 40 \text{ mm}$

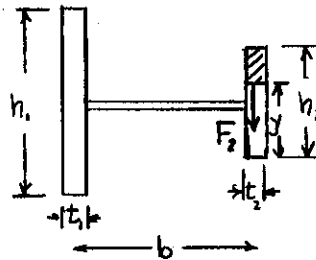
Note that the pair of  $F_1$  forces form a couple. Likewise, the pair of  $F_2$  forces. The lines of action of the forces in BDOGK pass through point  $O$ .

PROBLEM 6.100

6.100 A thin-walled beam has the cross section shown. Determine the location of the shear center  $O$  of the cross section.



SOLUTION



$$I = \frac{1}{2} t_1 h_1^3 + \frac{1}{2} t_2 h_2^3$$

Right flange

$$A = (\frac{1}{2} h_2 - y) t_2$$

$$\bar{y} = \frac{1}{2} (\frac{1}{2} h_2 + y) t_2$$

$$Q = A \bar{y}$$

$$= \frac{1}{2} (\frac{1}{2} h_2 - y) (\frac{1}{2} h_2 + y) t_2$$

$$= \frac{1}{2} (\frac{1}{4} h_2^2 - y^2) t_2$$

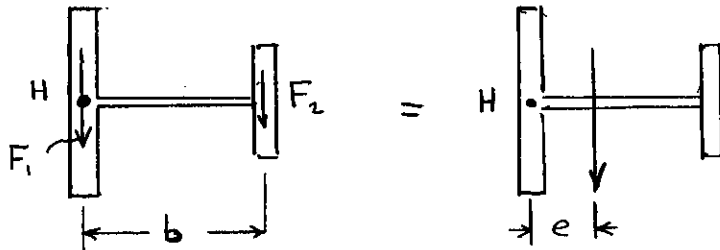
$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2} (\frac{1}{4} h_2^2 - y^2) t_2$$

$$F_2 = \int \tau dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2It_2} (\frac{1}{4} h_2^2 - y^2) t_2 dy = \frac{Vt_2}{2I} (\frac{1}{4} h_2^2 y - \frac{y^3}{3}) \Big|_{-h_2/2}^{h_2/2}$$

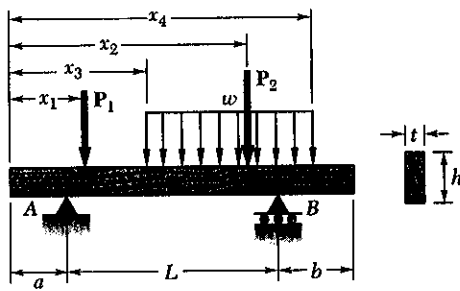
$$= \frac{Vt_2}{2I} \left\{ \frac{1}{4} h_2^2 \frac{h_2}{2} - \frac{1}{3} (\frac{h_2}{2})^3 + \frac{1}{4} h_2^2 \frac{h_2}{2} - \frac{1}{3} (\frac{h_2}{2})^3 \right\} = \frac{Vt_2 h_2^3}{12I} = \frac{Vt_2 h_2^3}{t_1 h_1^3 + t_2 h_2^3}$$

$$\sum M_H = + \sum M_H \quad -Ve = -F_2 b = -V \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3}$$

$$e = \frac{t_2 h_2^3 b}{t_1 h_1^3 + t_2 h_2^3} = \frac{(0.75)(6)^3 (8)}{(0.75)(8)^3 + (0.75)(6)^3} = 2.37 \text{ in.}$$



PROBLEM 6.C1



6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values  $\sigma_{all}$  and  $\tau_{all}$ . Measuring  $x$  from end A and using SI units, write a computer program to calculate for successive cross sections, from  $x = 0$  to  $x = L$  and using given increments  $\Delta x$ , the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming  $\sigma_{all} = 12 \text{ MPa}$  and  $\tau_{all} = 825 \text{ kPa}$ , and using the increments indicated: (a) Prob. 5.75 ( $\Delta x = 0.1 \text{ m}$ ), (b) Prob. 5.76 ( $\Delta x = 0.2 \text{ m}$ ).

SOLUTION

See solution of P 5.C2 for the determination of  $R_A$ ,  $R_B$ ,  $V(x)$ , and  $M(x)$   
 We recall that

$$V(x) = R_A \text{STPA} + R_B \text{STPB} - P_1 \text{STP1} - P_2 \text{STP2} - w(x-x_3) \text{STP3} + w(x-x_4) \text{STP4}$$

$$M(x) = R_A(x-a) \text{STPA} + R_B(x-a-L) \text{STPB} - P_1(x-x_1) \text{STP1} - P_2(x-x_2) \text{STP2} - \frac{1}{2}w(x-x_3)^2 \text{STP3} + \frac{1}{2}w(x-x_4)^2 \text{STP4}$$

where  $\text{STPA}$ ,  $\text{STPB}$ ,  $\text{STP1}$ ,  $\text{STP2}$ ,  $\text{STP3}$ , and  $\text{STP4}$  are step functions defined in P 5.C2

(1) TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

If unknown dimension is  $h$ :

$$S_{min} = |M|/\sigma_{all} \cdot \text{From } S = \frac{1}{6}th^2, \text{ we have } h_0 = h = \sqrt{6S/t}$$

If unknown dimension is  $t$ :

$$S_{min} = |M|/\sigma_{all} \cdot \text{From } S = \frac{1}{6}th^2, \text{ we have } t_0 = t = 6S/h^2$$

(2) TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:  $\tau_{max} = \frac{3V}{2A} = \frac{3|V|}{2th}$

If unknown dimension is  $h$ :  $h_0 = h = \frac{3M}{2t\tau_{all}}$

If unknown dimension is  $t$ :  $t_0 = t = \frac{3M}{2h\tau_{all}}$

(CONTINUED)

PROBLEM 6.C1 CONTINUED

PROGRAM OUTPUTS

Prob. 5.75

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	-3.00	2.400	173.21	136.36
1.70	-3.00	2.100	162.02	136.36
1.80	-3.00	1.800	150.00	136.36
1.90	-3.00	1.500	136.93	136.36
2.00	-3.00	1.200	122.47	136.36
2.10	-3.00	0.900	106.07	136.36
2.20	-3.00	0.600	86.60	136.36
2.30	-3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.05	0.00

Prob. 5.76

RA = 25.00 kN RB = 25.00 kN

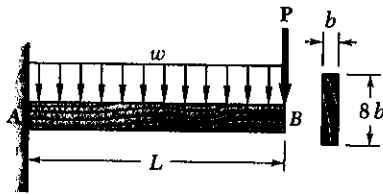
X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	25.00	0.000	0.00	378.79
0.20	23.00	4.800	141.42	348.48
0.40	21.00	9.200	195.79	318.18
0.60	19.00	13.200	234.52	287.88
0.80	17.00	16.800	264.58	257.58
1.00	15.00	20.000	288.68	227.27
1.20	13.00	22.800	308.22	196.97
1.40	11.00	25.200	324.04	166.67
1.60	9.00	27.200	336.65	136.36
1.80	7.00	28.800	346.41	106.06
2.00	5.00	30.000	353.55	75.76
2.20	3.00	30.800	358.24	45.45
2.40	1.00	31.200	360.56	15.15
2.60	-1.00	31.200	360.56	15.15
2.80	-3.00	30.800	358.24	45.45
3.00	-5.00	30.000	353.55	75.76
3.20	-7.00	28.800	346.41	106.06
3.40	-9.00	27.200	336.65	136.36
3.60	-11.00	25.200	324.04	166.67
3.80	-13.00	22.800	308.22	196.97
4.00	-15.00	20.000	288.68	227.27
4.20	-17.00	16.800	264.58	257.58
4.40	-19.00	13.200	234.52	287.88
4.60	-21.00	9.200	195.79	318.18
4.80	-23.00	4.800	141.42	348.48
5.00	0.00	0.000	0.00	0.00

The smallest allowable value of  $h$  is the largest of the values shown in the last two columns.

For Prob. 5.75,  $h = h_G = 173.2$  mm.

For Prob. 5.76,  $h = h_E = 379$  mm.

**PROBLEM 6.C2**



**6.C2** A cantilever timber beam  $AB$  of length  $L$  and of the uniform rectangular section shown supports a concentrated load  $P$  at its free end and a uniformly distributed load  $w$  along its entire length. Write a computer program to determine the length  $L$  and the width  $b$  of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming  $\sigma_{all} = 1.8$  ksi and  $\tau_{all} = 120$  psi, use this program to determine the dimensions  $L$  and  $b$  when (a)  $P = 1000$  lb and  $w = 0$ , (b)  $P = 0$  and  $w = 12.5$  lb/in., (c)  $P = 500$  lb and  $w = 12.5$  lb/in.

**SOLUTION**

Both the maximum shear and the maximum bending moment occur at  $A$ . We have

$$V_A = P + wL$$

$$M_A = PL + \frac{1}{2} wL^2$$

TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

$$\sigma_{all} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{8} b(8b)^2} = \frac{3M_A}{32b^3}$$

$$b_G = b = \left[ \frac{3}{32} \frac{M_A}{\sigma_{all}} \right]^{1/3}$$

TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:

$$\tau_{all} = \frac{3V}{2A} = \frac{3}{2} \frac{V_A}{b(8b)} = \frac{3V_A}{16b^2}$$

$$b_G = b = \left[ \frac{3}{16} \frac{V_A}{\tau_{all}} \right]^{1/2}$$

PROGRAM

For  $L=0$ ,  $V_A = P$  and  $b_G > 0$ , while  $M_A = 0$  and  $b_G = 0$ . Starting with  $L=0$  and using increments  $\Delta L = 0.001$  in., we increase  $L$  until  $b_G$  and  $b_G$  become equal. We then print  $L$  and  $b$ .

PROGRAM OUTPUTS

For  $P = 1000$  lb,  $w = 0.0$  lb/in.

Increment = 0.0010 in.

$L = 37.5$  in.,  $b = 1.250$  in.

For  $P = 0$  lb,  $w = 12.5$  lb/in.

Increment = 0.0010 in.

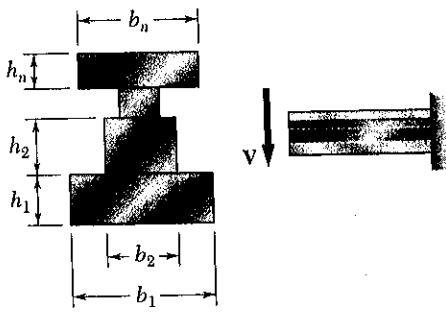
$L = 70.3$  in.,  $b = 1.172$  in.

For  $P = 500$  lb,  $w = 12.5$  lb/in.

Increment = 0.0010 in.

$L = 59.8$  in.,  $b = 1.396$  in.

**PROBLEM 6.C3**



**6.C3** A beam having the cross section shown is subjected to a vertical shear  $V$ . Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (a) Prob. 6.10, (b) Prob. 6.11, (c) Prob. 6.21, (d) Prob. 6.23.

**SOLUTION**

1. Enter  $V$  and the number  $n$  of rectangles.
2. For  $i = 1$  to  $n$ , enter the dimensions  $b_i$  and  $h_i$ .
3. Determine the area  $A_i = b_i h_i$  of each rectangle.
4. Determine the elevation of the centroid of each rectangle

$$\bar{y}_i = \sum_{k=1}^i h_k - 0.5 h_i$$

and the elevation  $\bar{y}$  of the centroid of the entire section

$$\bar{y} = \left( \sum_i A_i \bar{y}_i \right) / \left( \sum_i A_i \right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[ \frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2 \right]$$

6. For each surface separating two rectangles  $i$  and  $i+1$ , determine  $Q_i$  of the area below that surface

$$Q_i = \sum_{k=1}^i A_k (\bar{y}_k - \bar{y})$$

7. Select for  $t_i$  the smaller of  $b_i$  and  $b_{i+1}$ .

The shearing stress on the surface between the rectangles  $i$  and  $i+1$  is

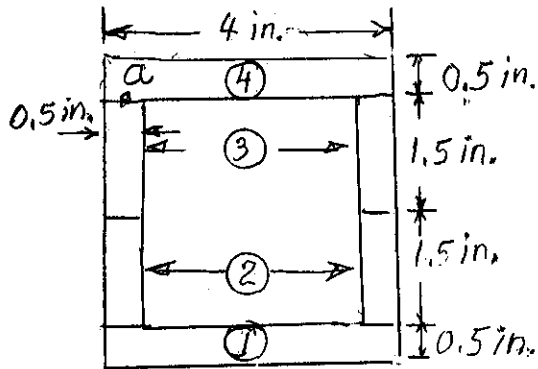
$$\tau_i = \frac{V Q_i}{I t_i} \quad \blacktriangleleft$$

(CONTINUED)



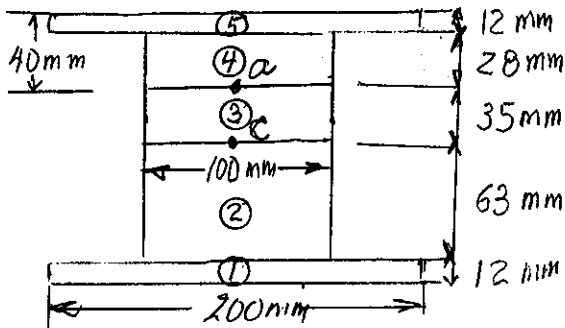
PROBLEM 6.C3 CONTINUED

PROGRAM OUTPUTS



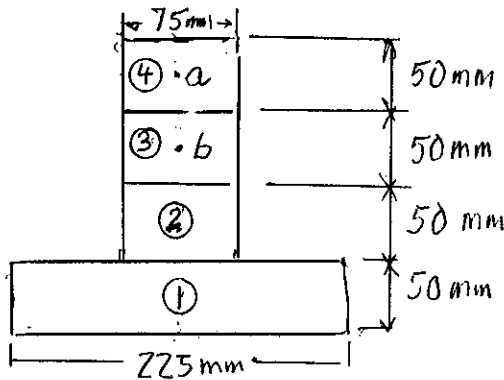
Problem 6.10

$V = 10.00$  kips  
 $Y_{BAR}$  of Section = 2.000 in.  
 $I = 14.583 \text{ in}^4$   
 Between elements 1 and 2:  
 $\tau = 2.400$  ksi  
 Between elements 2 and 3:  $\tau = 3.171$  ksi (a)  
 Between elements 3 and 4:  
 $\tau = 2.400$  ksi (b)



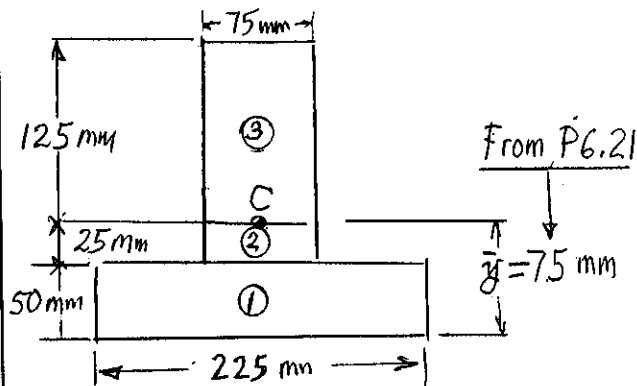
Problem 6.11

$V = 10.00$  kN  
 $Y_{BAR}$  of Section = 75.00 mm  
 $I = 39.580 \times 10^{-6} \text{ m}^4$   
 Between elements 1 and 2:  
 $\tau = 418.39$  kPa  
 Between elements 2 and 3:  $\tau = 919.78$  kPa (a)  
 Between elements 3 and 4:  
 $\tau = 765.03$  kPa (b)  
 Between elements 4 and 5:  
 $\tau = 418.39$  kPa



Problem 6.21

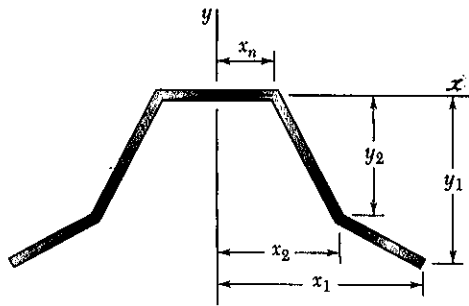
$V = 200.00$  kN  
 $Y_{BAR}$  of Section = 75.00 mm  
 $I = 79.687 \times 10^{-6} \text{ m}^4$   
 Between elements 1 and 2:  
 $\tau = 18.82$  MPa  
 Between elements 2 and 3:  $\tau = 18.82$  MPa (b)  
 Between elements 3 and 4:  
 $\tau = 12.55$  MPa (a)



Problem 6.23

$V = 200.00$  kN  
 $Y_{BAR}$  of Section = 75.00 mm  
 $I = 79.688 \times 10^{-6} \text{ m}^4$   
 Between elements 1 and 2:  
 $\tau = 18.82$  MPa  
 Between elements 2 and 3:  
 $\tau = 19.61$  MPa

**PROBLEM 6.C4**



**6.C4** A plate of uniform thickness  $t$  is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear  $V$ . Use this program (a) to solve Prob. 6.49, (b) to find the shearing stress at point  $E$  for the shape and load of Prob. 6.50, assuming a thickness  $t = \frac{1}{4}$  in.

**SOLUTION**

For each element on the right-hand side, we compute (for  $i=1$  to  $n$ ):

$$\text{Length of element} = L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

$$\text{Area of element} = A_i = t L_i \quad \text{where } t = \frac{1}{4} \text{ in.}$$

$$\text{Distance from } x \text{ axis to centroid of element} = \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$$

Distance from  $x$  axis to centroid of section:

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$

Note that  $y_n = 0$  and that  $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$I = 2 \sum A_i \left[ \frac{1}{12} (y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2 \right]$$

Computation of  $Q$  at point  $P$  where stress is desired

$Q = \sum A_i (\bar{y}_i - \bar{y})$  where sum extends to the areas located between one end of section and point  $P$ .

Shearing stress at  $P$ :

$$\tau = \frac{VQ}{It}$$

NOTE:  $\tau_{\max}$  occurs on neutral axis, i.e., for  $y_p = \bar{y}$ .

PROGRAM OUTPUTS

Part (a):

$I = 0.5333 \text{ in}^4$   
 $\tau_{\max} = 2.02 \text{ ksi}$   
 $\tau_B = 1.800 \text{ ksi}$

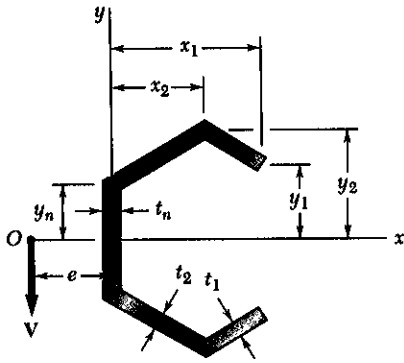


Part (b):

$I = 22.27 \text{ in}^4$   
 $\tau_E = 194.0 \text{ psi}$



**PROBLEM 6.C5**



**6.C5** The cross section of an extruded beam is symmetric with respect to the  $x$  axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (a) the location of the shear center  $O$ , (b) the distribution of shearing stresses caused by a vertical force applied at  $O$ . Use this program to solve Probs. 6.65, 6.68, 6.69, and 6.70.

**SOLUTION**

SINCE SECTION IS SYMMETRIC WITH  $x$  AXIS,  
COMPUTATIONS WILL BE DONE FOR TOP  
HALF.

FOR  $L=1$  TO  $n+1$  (NOTE:  $n+1$  IS THE ORIGIN)

ENTER  $x_L, y_L, y_L$

COMPUTE LENGTH OF EACH SEGMENT

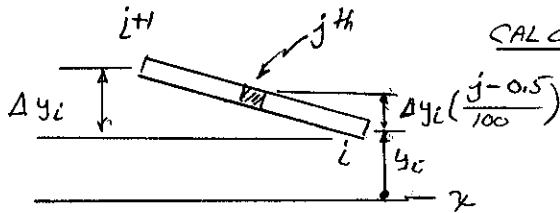
FOR  $L=1$  TO  $n$

$$\Delta x_L = x_{L+1} - x_L$$

$$\Delta y_L = y_{L+1} - y_L$$

$$L = (\Delta x_L^2 + \Delta y_L^2)^{1/2}$$

CALCULATE MOMENT OF INERTIA  $I_x$



CONSIDER EACH SEGMENT AS MADE  
OF 100 EQUAL PARTS

FOR  $L=1$  TO  $n$

$$\Delta \text{AREA} = L_L t_L / 100$$

FOR  $j=1$  TO 100

$$y = y_L + \Delta y_L (j-0.5) / 100$$

$$\Delta I = (\Delta \text{AREA}) y^2$$

$$I_x = I_x + \Delta I$$

SINCE ONLY TOP HALF WAS USED

$$I_x = 2 I_x$$

CALCULATE SHEARING STRESS AT ENDS OF  
SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR  $L=1$  TO  $n$

$$\Delta \text{AREA} = L_L t_L / 100, \tau_{\text{new}} = \tau_{\text{next}}$$

FOR  $j=1$  TO 100

$$y = y_L + \Delta y_L (j-0.5) / 100$$

$$\Delta Q = (\Delta \text{AREA}) y$$

$$\tau_{\text{old}} = \tau_{\text{new}}, Q = Q + \Delta Q$$

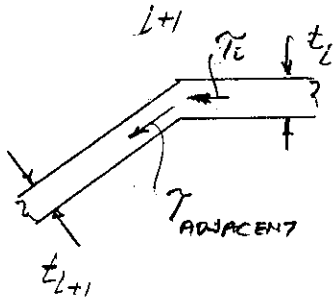
$$\tau_{\text{new}} = VQ / I_x t_L$$

$$\tau_{\text{ave}} = 0.5 (\tau_{\text{old}} + \tau_{\text{new}})$$

$$\tau = \tau + \tau_{\text{ave}}$$

CONTINUED

PROBLEM 6.C5 - CONTINUED



$$\begin{aligned} \text{FORCE}_i &= \tau_i (\text{AREA}) \\ \tau_i &= VQ / I_x t_i \\ (\tau_{ADJACENT})_L &= VQ / I_x t_{L+1} \\ Q_L &= Q \\ \tau_{\text{MAX}} &= (\tau_{ADJACENT})_L \end{aligned}$$

LOCATION OF SHEAR CENTER  
CALCULATE MOMENT OF SHEAR FORCES ABOUT ORIGIN

FOR \$L = 1\$ TO \$N\$

$$\begin{aligned} (F_x)_L &= \text{FORCE}_L (\Delta x_L) / L_L \\ (F_y)_L &= \text{FORCE}_L (\Delta y_L) / L_L \\ \text{MOMENT}_L &= -(F_x)_L y_L + (F_y)_L x_L \\ \text{MOMENT} &= \text{MOMENT} + \text{MOMENT}_L \end{aligned}$$

FOR WHOLE SECTION MOMENT = 2 (MOMENT)  
SHEAR CENTER IS AT  
\$e = \text{MOMENT} / V\$

PROGRAM OUTPUT

Prob.	6.65			
	T(i)	X(i)	Y(i)	L(i)
	mm	mm	mm	mm
1	10.00	70.00	10.00	40.000
2	6.00	70.00	50.00	70.000
3	10.00	0.00	50.00	50.000
4	10.00	0.00	0.00	

Moment of inertia: \$I\_x = 3759956 \text{ mm}^4\$ Shear = 50000 N

Junction of segments	Q	Tau Before	Tau After	Force in segment
	mm <sup>3</sup>	MPa	MPa	kN
1 and 2	12000.000	15.96	26.60	2482.37
2 and 3	33000.000	73.14	43.88	20888.54
3 and 4	45500.000	60.51	60.51	27372.75

Moment of shear forces about origin: \$M = 2436.386 \text{ N}\cdot\text{m}\$  
+ counterclockwise

Distance from origin to shear center: \$e = 48.728 \text{ mm}\$

CONTINUED

**PROBLEM 6.C5 - PROGRAM PRINTOUTS CONTINUED**

Prob. 6.68

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	3.00	4.00	3.000
2	0.50	0.00	4.00	4.000
3	0.50	0.00	0.00	

Moment of inertia:  $I_x = 45.3328 \text{ in}^4$  Shear = 25.000 kips

Junction of segments	Q $\text{in}^3$	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	3.000	6.62	3.31	2.48
2 and 3	7.000	7.72	7.72	12.47

Moment of shear forces about origin:  
+ counterclockwise  $M = 19.853 \text{ kip}\cdot\text{in.}$

Distance from origin to shear center:  $e = 0.7941 \text{ in.}$

Prob. 6.69

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	4.00	5.00	2.000
2	0.25	4.00	3.00	5.000
3	0.25	0.00	0.00	

Moment of inertia:  $I_x = 23.8331 \text{ in}^4$  Shear = 10.000 kips

Junction of segments	Q $\text{in}^3$	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	2.000	3.36	3.36	0.91
2 and 3	3.875	6.50	6.50	6.80

Moment of shear forces about origin:  
+ counterclockwise  $M = -7.273 \text{ kip}\cdot\text{in.}$

Distance from origin to shear center:  $e = -0.7273 \text{ in.}$

Prob. 6.70

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	2.60	0.00	1.500
2	0.25	2.60	1.50	3.002
3	0.25	0.00	0.00	

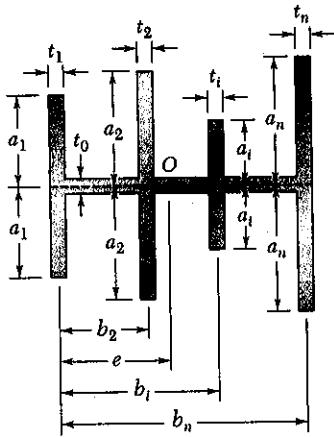
Moment of inertia:  $I_x = 1.6881 \text{ in}^4$  Shear = 10.000 kips

Junction of segments	Q $\text{in}^3$	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	0.281	6.66	6.66	0.83
2 and 3	0.844	20.00	20.00	11.65

Moment of shear forces about origin:  
+ counterclockwise  $M = 4.332 \text{ kip}\cdot\text{in.}$

Distance from origin to shear center:  $e = 0.4332 \text{ in.}$

**PROBLEM 6.C6**



**6.C6** A thin-walled beam has the cross section shown. Write a computer program that, for dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center  $O$  of the cross section. Use this program to solve Prob. 6.100.

**SOLUTION**

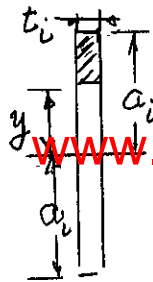
Distribution of shearing stresses in element  $i$

Let  $V$  = shear in cross section

$\bar{I}$  = Centroidal moment of inertia of section

We have for shaded area

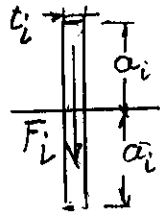
$$Q = A \bar{y} = t_i (a_i - y) \frac{a_i + y}{2}$$



$$\frac{VQ}{I t_i} (a_i^2 - y^2)$$

$$\tau = \frac{QV}{I t_i} = \frac{V}{2I} (a_i^2 - y^2)$$

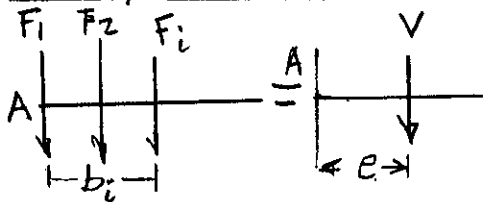
Force exerted on element  $i$



$$F_i = \int_{-a_i}^{a_i} \tau (t_i dy) = \frac{V t_i}{2I} \int_{-a_i}^{a_i} (a_i^2 - y^2) dy$$

$$= \frac{V t_i}{I} \int_0^{a_i} (a_i^2 - y^2) dy = \frac{V t_i}{I} (a_i^3 - \frac{1}{3} a_i^3) = \frac{2}{3} \frac{V}{I} t_i a_i^3$$

The system of the forces  $F_i$  must be equivalent to  $V$  at shear center.



$$\sum F_i = \sum F: \frac{2}{3} \frac{V}{I} \sum t_i a_i^3 = V \quad (1)$$

$$\sum M_A = \sum M_A: \frac{2}{3} \frac{V}{I} \sum t_i a_i^3 b_i = e V \quad (2)$$

Divide (2) by (1): 
$$e = \frac{\sum t_i a_i^3 b_i}{\sum t_i a_i^3}$$

PROGRAM OUTPUT:

Prob. 6.100

For element 1:

$$t = 0.75 \text{ in.}, a = 4 \text{ in.}, b = 0$$

For element 2:

$$t = 0.75 \text{ in.}, a = 3 \text{ in.}, b = 8 \text{ in.}$$

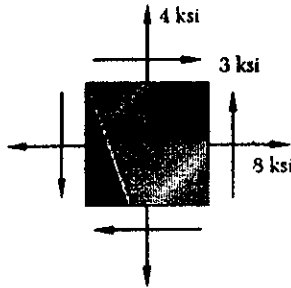
Answer:  $e = 2.37 \text{ in.}$

# CHAPTER 7

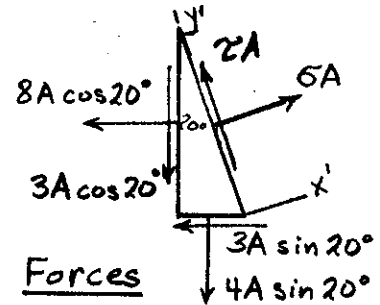
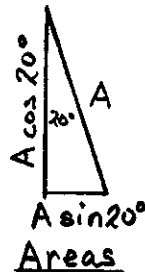
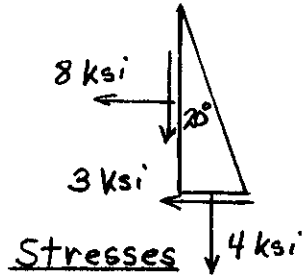
[www.konkur.in](http://www.konkur.in)

**PROBLEM 7.1**

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



**SOLUTION**



$$\rightarrow \Sigma F = 0$$

$$\sigma A - 8A \cos 20^\circ \cos 20^\circ - 3A \cos 20^\circ \sin 20^\circ - 3A \sin 20^\circ \cos 20^\circ - 4A \sin 20^\circ \sin 20^\circ = 0$$

$$\sigma = 8 \cos^2 20^\circ + 3 \cos 20^\circ \sin 20^\circ + 3 \sin 20^\circ \cos 20^\circ + 4 \sin^2 20^\circ = 9.48 \text{ ksi} \quad \blacktriangleleft$$

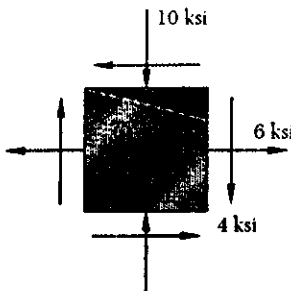
$$\uparrow \Sigma F = 0$$

$$\tau A + 8A \cos 20^\circ \sin 20^\circ - 3A \cos 20^\circ \cos 20^\circ + 3A \sin 20^\circ \sin 20^\circ - 4A \sin 20^\circ \cos 20^\circ = 0$$

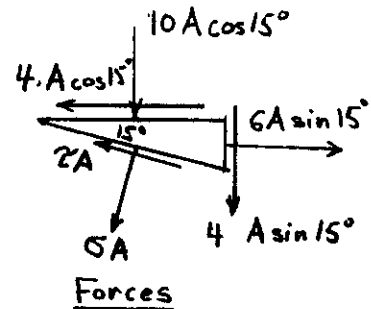
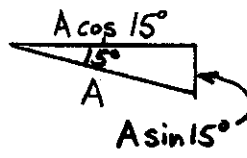
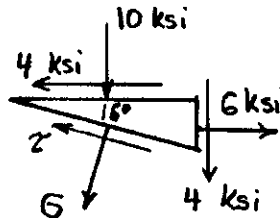
$$\tau = -8 \cos 20^\circ \sin 20^\circ + 3(\cos^2 20^\circ - \sin^2 20^\circ) + 4 \sin 20^\circ \cos 20^\circ = 1.013 \text{ ksi} \quad \blacktriangleright$$

**PROBLEM 7.2**

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



**SOLUTION**



$$\uparrow \Sigma F = 0$$

$$\sigma A + 4A \cos 15^\circ \sin 15^\circ + 10A \cos 15^\circ \cos 15^\circ - 6A \sin 15^\circ \sin 15^\circ + 4A \sin 15^\circ \cos 15^\circ = 0$$

$$\sigma = -4 \cos 15^\circ \sin 15^\circ - 10 \cos^2 15^\circ + 6 \sin^2 15^\circ - 4 \sin 15^\circ \cos 15^\circ = 10.93 \text{ ksi} \quad \blacktriangleleft$$

$$\rightarrow \Sigma F = 0$$

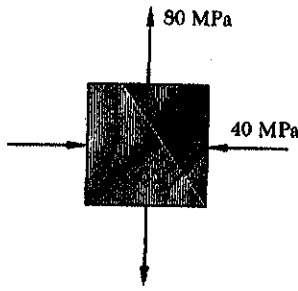
$$\tau A + 4A \cos 15^\circ \cos 15^\circ - 10A \cos 15^\circ \sin 15^\circ - 6A \sin 15^\circ \cos 15^\circ - 4A \sin 15^\circ \sin 15^\circ = 0$$

$$\tau = -4(\cos^2 15^\circ - \sin^2 15^\circ) + (10 + 6) \cos 15^\circ \sin 15^\circ = 0.536 \text{ ksi} \quad \blacktriangleright$$

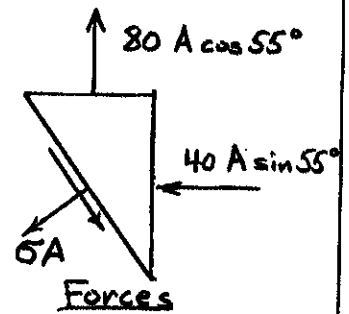
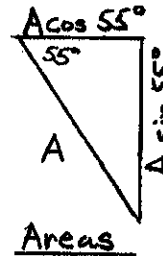
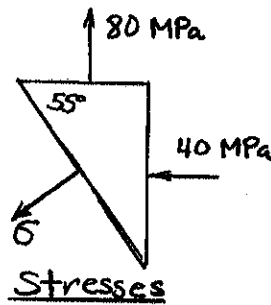


PROBLEM 7.3

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$$+\nearrow \Sigma F = 0$$

$$\sigma A - 80 A \cos 55^\circ \cos 55^\circ + 40 A \sin 55^\circ \sin 55^\circ = 0$$

$$\sigma = 80 \cos^2 55^\circ - 40 \sin^2 55^\circ = -0.521 \text{ MPa}$$

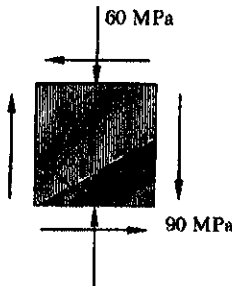
$$+\searrow \Sigma F = 0$$

$$\tau A - 80 A \cos 55^\circ \sin 55^\circ - 40 A \sin 55^\circ \cos 55^\circ$$

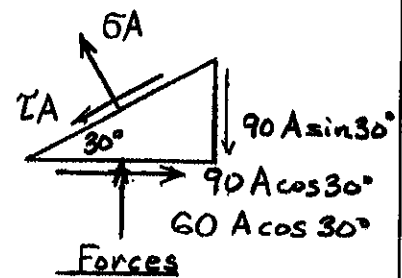
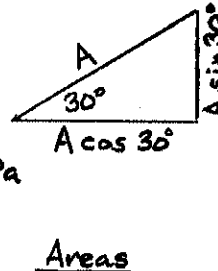
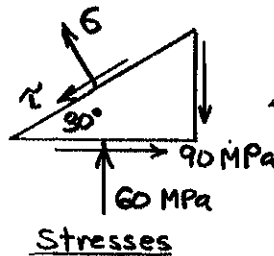
$$\tau = 120 \cos 55^\circ \sin 55^\circ = 56.4 \text{ MPa}$$

PROBLEM 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$$+\nearrow \Sigma F = 0$$

$$\sigma A - 90 A \sin 30^\circ \cos 30^\circ - 60 A \cos 30^\circ \sin 30^\circ + 60 A \cos 30^\circ \cos 30^\circ = 0$$

$$\sigma = 180 \sin 30^\circ \cos 30^\circ - 60 \cos^2 30^\circ = 32.9 \text{ MPa}$$

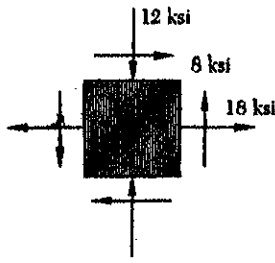
$$+\searrow \Sigma F = 0$$

$$\tau A + 90 A \sin 30^\circ \sin 30^\circ - 90 A \cos 30^\circ \cos 30^\circ - 60 A \cos 30^\circ \sin 30^\circ = 0$$

$$\tau = 90 (\cos^2 30^\circ - \sin^2 30^\circ) + 60 \cos 30^\circ \sin 30^\circ = 71.0 \text{ MPa}$$

**PROBLEM 7.5**

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



**SOLUTION**

$$\sigma_x = 18 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = 8 \text{ ksi}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(8)}{18 - (-12)} = 0.5333$$

$$2\theta_p = 28.07^\circ \quad \theta_p = 14.04^\circ, 104.04^\circ$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{18 - 12}{2} \pm \sqrt{\left(\frac{18 + 12}{2}\right)^2 + (8)^2}$$

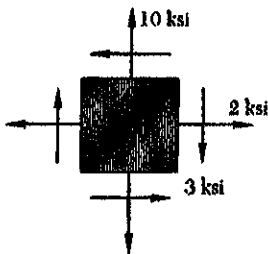
$$= 3 \pm 17 \text{ ksi}$$

$$\sigma_{max} = 20 \text{ ksi}$$

$$\sigma_{min} = -14 \text{ ksi}$$

**PROBLEM 7.6**

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



**SOLUTION**

$$\sigma_x = 2 \text{ ksi} \quad \sigma_y = 10 \text{ ksi} \quad \tau_{xy} = -3 \text{ ksi}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-3)}{2 - 10} = 0.750$$

$$2\theta_p = 36.87^\circ \quad \theta_p = 18.43^\circ, 108.43^\circ$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2 + 10}{2} \pm \sqrt{\left(\frac{2 - 10}{2}\right)^2 + (-3)^2}$$

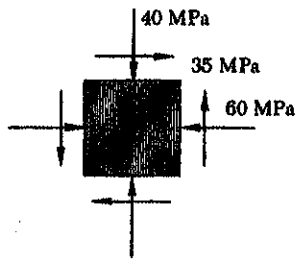
$$= 6 \pm 5 \text{ ksi}$$

$$\sigma_{max} = 11 \text{ ksi}$$

$$\sigma_{min} = 1 \text{ ksi}$$

**PROBLEM 7.7**

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



**SOLUTION**

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(35)}{-60 + 40} = -3.50$$

$$2\theta_p = -74.05^\circ \quad \theta_p = -37.03^\circ, 52.97^\circ \quad \blacktriangleleft$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-60 - 40}{2} \pm \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2}$$

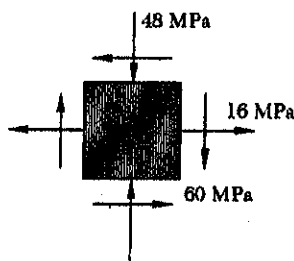
$$= -50 \pm 36.4 \text{ MPa}$$

$$\sigma_{max} = -13.60 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = -86.4 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 7.8**

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.



**SOLUTION**

$$\sigma_x = 16 \text{ MPa} \quad \sigma_y = -48 \text{ MPa} \quad \tau_{xy} = -60 \text{ MPa}$$

$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{16 + 48} = -1.875$$

$$2\theta_p = -61.93^\circ \quad \theta_p = -30.96^\circ, 59.04^\circ \quad \blacktriangleleft$$

$$(b) \sigma_{max, min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

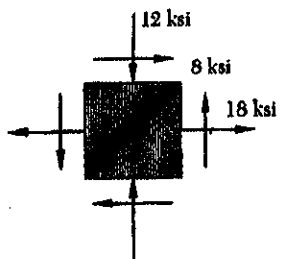
$$= \frac{16 - 48}{2} \pm \sqrt{\left(\frac{16 + 48}{2}\right)^2 + (-60)^2}$$

$$= -16 \pm 68$$

$$\sigma_{max} = 52 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = -84 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 7.9**



7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

**SOLUTION**

$$\sigma_x = 18 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = 8 \text{ ksi}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{18 - (-12)}{2(8)} = -1.875$$

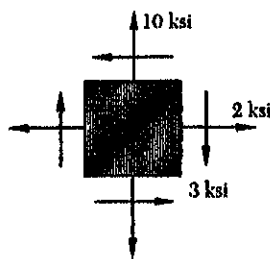
$$2\theta_s = -61.93^\circ \quad \theta_s = -30.96^\circ, 59.04^\circ \quad \blacktriangleleft$$

$$(b) \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{18 - (-12)}{2}\right)^2 + (8)^2} = 17 \text{ ksi} \quad \blacktriangleleft$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{18 - 12}{2} = 3 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 7.10**



7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

**SOLUTION**

$$\sigma_x = 2 \text{ ksi} \quad \sigma_y = 10 \text{ ksi} \quad \tau_{xy} = -3 \text{ ksi}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{2 - 10}{2(-3)} = -1.3333$$

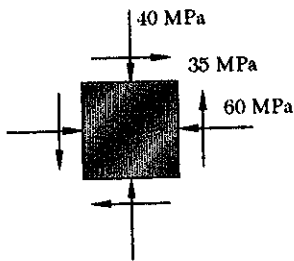
$$2\theta_s = -53.13^\circ \quad \theta_s = -26.57^\circ, 63.43^\circ \quad \blacktriangleleft$$

$$(b) \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{2 - 10}{2}\right)^2 + (-3)^2} = 5 \text{ ksi} \quad \blacktriangleleft$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{2 + 10}{2} = 6 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 7.11



7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{-60 + 40}{2(35)} = 0.2857$$

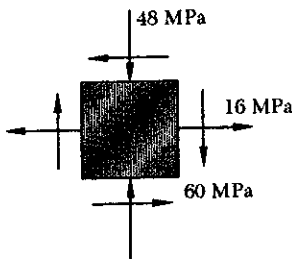
$$2\theta_s = 15.95^\circ \quad \theta_s = 7.97^\circ, 97.97^\circ \quad \blacktriangleleft$$

$$(b) \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-60 + 40}{2}\right)^2 + (35)^2} = 36.4 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60 - 40}{2} = -50 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.12



7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

SOLUTION

$$\sigma_x = 16 \text{ MPa} \quad \sigma_y = -48 \text{ MPa} \quad \tau_{xy} = -60 \text{ MPa}$$

$$(a) \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{16 + 48}{2(-60)} = 0.5333$$

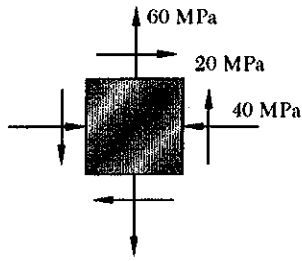
$$2\theta_s = 28.07^\circ \quad \theta_s = 14.04^\circ, 104.04^\circ \quad \blacktriangleleft$$

$$(b) \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{16 + 48}{2}\right)^2 + (-60)^2} = 68 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{16 - 48}{2} = -16 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 7.13**



**7.13 through 7.16** For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a)  $25^\circ$  clockwise, (b)  $10^\circ$  counterclockwise.

**SOLUTION**

$$\sigma_x = -40 \text{ MPa} \quad \sigma_y = 60 \text{ MPa} \quad \tau_{xy} = 20 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = 10 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = -50 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a)  $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = 10 - 50 \cos(-50^\circ) + 20 \sin(-50^\circ) = -37.5 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = +50 \sin(-50^\circ) + 20 \cos(-50^\circ) = -25.4 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = 10 + 50 \cos(-50^\circ) - 20 \sin(-50^\circ) = 57.5 \text{ MPa} \quad \blacktriangleright$$

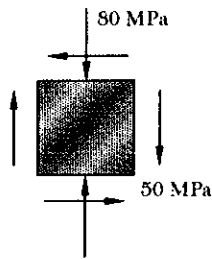
(b)  $\theta = 10^\circ \quad 2\theta = 20^\circ$

$$\sigma_{x'} = 10 - 50 \cos(20^\circ) + 20 \sin(20^\circ) = -30.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = +50 \sin(20^\circ) + 20 \cos(20^\circ) = 35.9 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = 10 + 50 \cos(20^\circ) - 20 \sin(20^\circ) = 50.1 \text{ MPa} \quad \blacktriangleright$$

PROBLEM 7.14



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a)  $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = -40 + 40 \cos(-50^\circ) - 50 \sin(-50^\circ) = 24.0 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -40 \sin(-50^\circ) + 50 \cos(-50^\circ) = -1.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40 \cos(-50^\circ) + 50 \sin(-50^\circ) = -104.0 \text{ MPa} \quad \blacktriangleleft$$

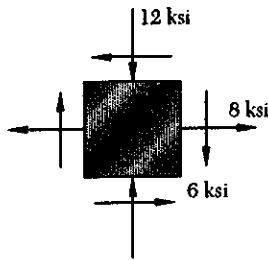
(b)  $\theta = 10^\circ \quad 2\theta = 20^\circ$

$$\sigma_{x'} = -40 + 40 \cos(20^\circ) - 50 \sin(20^\circ) = -19.5 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -40 \sin(20^\circ) - 50 \cos(20^\circ) = -60.7 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = -40 - 40 \cos(20^\circ) + 50 \sin(20^\circ) = -60.5 \text{ MPa} \quad \blacktriangleleft$$

## PROBLEM 7.15



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a)  $25^\circ$  clockwise, (b)  $10^\circ$  counterclockwise.

## SOLUTION

$$\sigma_x = 8 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = 10 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a)  $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = -2 + 10 \cos(-50^\circ) - 6 \sin(-50^\circ) = 9.02 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -10 \sin(-50^\circ) - 6 \cos(-50^\circ) = 3.80 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{y'} = -2 - 10 \cos(-50^\circ) + 6 \sin(-50^\circ) = -13.02 \text{ ksi} \quad \blacktriangleleft$$

(b)  $\theta = 10^\circ \quad 2\theta = 20^\circ$

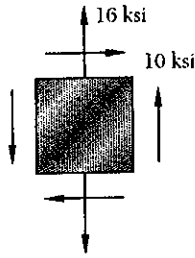
$$\sigma_{x'} = -2 + 10 \cos(20^\circ) - 6 \sin(20^\circ) = 5.34 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -10 \sin(20^\circ) - 6 \cos(20^\circ) = -9.06 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{y'} = -2 - 10 \cos(20^\circ) + 6 \sin(20^\circ) = -9.34 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 7.16



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = 16 \text{ ksi} \quad \tau_{xy} = 10 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = 8 \text{ ksi} \quad \frac{\sigma_x - \sigma_y}{2} = -8 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(a)  $\theta = -25^\circ \quad 2\theta = -50^\circ$

$$\sigma_{x'} = 8 - 8 \cos(-50^\circ) + 10 \sin(-50^\circ) = -4.80 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{x'y'} = 8 \sin(-50^\circ) + 10 \cos(-50^\circ) = 0.30 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{y'} = 8 + 8 \cos(-50^\circ) - 10 \sin(-50^\circ) = 20.80 \text{ ksi} \quad \blacktriangleleft$$

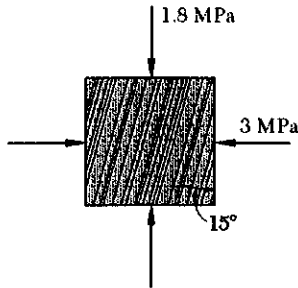
(b)  $\theta = 10^\circ \quad 2\theta = 20^\circ$

$$\sigma_{x'} = 8 - 8 \cos(20^\circ) + 10 \sin(20^\circ) = 3.90 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{x'y'} = 8 \sin(20^\circ) + 10 \cos(20^\circ) = 12.13 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{y'} = 8 + 8 \cos(20^\circ) - 10 \sin(20^\circ) = 12.10 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 7.17



7.17 and 7.18 The grain of a wooden member forms an angle of  $15^\circ$  with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

SOLUTION

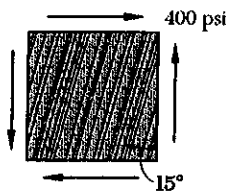
$$\sigma_x = -3 \text{ MPa} \quad \sigma_y = -1.8 \text{ MPa} \quad \tau_{xy} = 0$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$\begin{aligned} \text{(a) } \tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{-3 + 1.8}{2} \sin(-30^\circ) + 0 \\ &= -0.300 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{(b) } \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-3 - 1.8}{2} + \frac{-3 + 1.8}{2} \cos(-30^\circ) + 0 \\ &= -2.92 \text{ MPa} \end{aligned}$$

PROBLEM 7.18



7.17 and 7.18 The grain of a wooden member forms an angle of  $15^\circ$  with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = 400 \text{ psi}$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

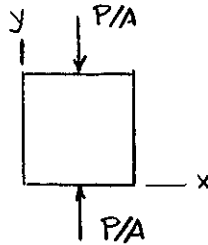
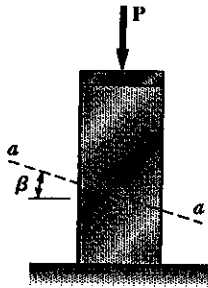
$$\begin{aligned} \text{(a) } \tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -0 + 400 \cos(-30^\circ) \\ &= 346 \text{ psi} \end{aligned}$$

$$\begin{aligned} \text{(b) } \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= 0 + 0 + 400 \sin(-30^\circ) \\ &= -200 \text{ psi} \end{aligned}$$

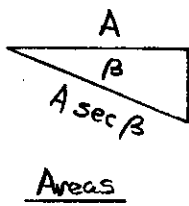
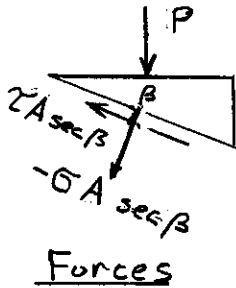
PROBLEM 7.19

7.19 The centric force  $P$  is applied to a short post as shown. Knowing that the stresses on plane  $a-a$  are  $\sigma = -15$  ksi and  $\tau = 5$  ksi, determine (a) the angle  $\beta$  that plane  $a-a$  forms with the horizontal, (b) the maximum compressive stress in the post.

SOLUTION



$$\begin{aligned} \sigma_x &= 0 \\ \tau_{xy} &= 0 \\ \sigma_y &= \sigma_{\text{max comp.}} = -\frac{P}{A} \end{aligned}$$



Force Triangle

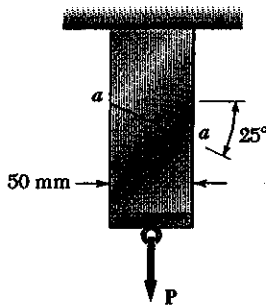
$$\tan \beta = \frac{\tau A \sec \beta}{-\sigma A \sec \beta} = \frac{5}{-15} = -\frac{1}{3}$$

(a)  $\beta = \arctan \frac{1}{3} = 18.4^\circ$

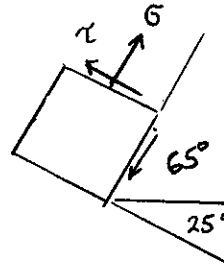
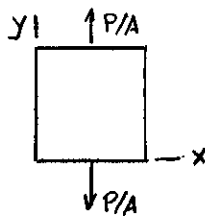
(b)  $P = (-\sigma A \sec \beta)(\sec \beta) \quad \frac{P}{A} = \frac{-\sigma}{\cos^2 \beta} = \frac{15}{\cos^2 18.4^\circ} = 16.67 \text{ ksi}$

PROBLEM 7.20

7.20 Two members of uniform cross section  $50 \times 80$  mm are glued together along plane  $a-a$ , which forms an angle of  $25^\circ$  with the horizontal. Knowing that the allowable stresses for the glued joint are  $\sigma = 800$  kPa and  $\tau = 600$  kPa, determine the largest axial load  $P$  that can be applied.



SOLUTION



For plane  $a-a \quad \theta = 65^\circ$

$$\sigma_x = 0 \quad \tau_{xy} = 0 \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta = 0 + \frac{P}{A} \sin^2 65^\circ + 0$$

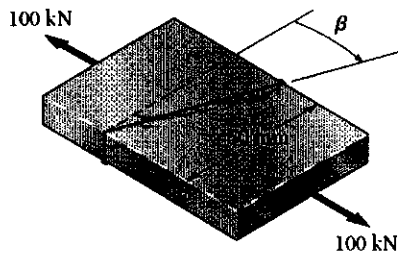
$$P = \frac{A\sigma}{\sin^2 65^\circ} = \frac{(50 \times 10^{-3})(80 \times 10^{-3})(800 \times 10^3)}{\sin^2 65^\circ} = 3.90 \times 10^3 \text{ N}$$

$$\tau = -(\sigma_x - \sigma_y) \sin \theta \cos \theta + \tau_{xy} (\cos^2 \theta - \sin^2 \theta) = \frac{P}{A} \sin 65^\circ \cos 65^\circ + 0$$

$$P = \frac{A\tau}{\sin 65^\circ \cos 65^\circ} = \frac{(50 \times 10^{-3})(80 \times 10^{-3})(600 \times 10^3)}{\sin 65^\circ \cos 65^\circ} = 6.27 \times 10^3 \text{ N}$$

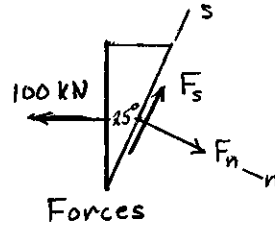
Allowable value of  $P$  is the smaller.  $P = 3.90 \times 10^3 \text{ N} = 3.90 \text{ kN}$

**PROBLEM 7.21**



7.21 Two steel plates of uniform cross section  $10 \times 80$  mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that  $\beta = 25^\circ$ , determine (a) the in-plane shearing stress parallel to the weld, (b) the normal stress perpendicular to the weld.

**SOLUTION**



Area of weld

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos 25^\circ} = 882.7 \times 10^{-6} \text{ m}^2$$

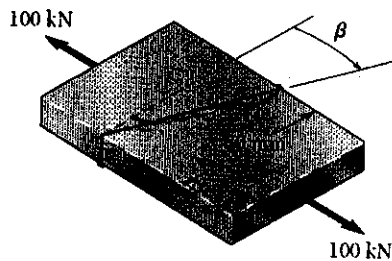
(a)  $\Sigma F_s = 0 \quad F_s - 100 \sin 25^\circ = 0 \quad F_s = 42.26 \text{ kN}$

$$\tau_w = \frac{F_s}{A_w} = \frac{42.26 \times 10^3}{882.7 \times 10^{-6}} = 47.9 \times 10^6 \text{ Pa} = 47.9 \text{ MPa}$$

(b)  $\Sigma F_n = 0 \quad F_n - 100 \cos 25^\circ = 0 \quad F_n = 90.63 \text{ kN}$

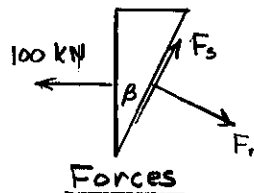
$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63 \times 10^3}{882.7 \times 10^{-6}} = 102.7 \times 10^6 \text{ Pa} = 102.7 \text{ MPa}$$

**PROBLEM 7.22**



7.22 Two steel plates of uniform cross section  $10 \times 80$  mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (a) the angle  $\beta$ , (b) the corresponding normal stress perpendicular to the weld.

**SOLUTION**



Area of weld

$$A_w = \frac{(10 \times 10^{-3})(80 \times 10^{-3})}{\cos \beta} = \frac{800 \times 10^{-6}}{\cos \beta} \text{ m}^2$$

(a)  $\Sigma F_s = 0 \quad F_s - 100 \sin \beta = 0 \quad F_s = 100 \sin \beta \text{ kN} = 100 \times 10^3 \sin \beta \text{ N}$

$$\tau_w = \frac{F_s}{A_w} \quad 30 \times 10^6 = \frac{100 \times 10^3 \sin \beta}{800 \times 10^{-6} / \cos \beta} = 125 \times 10^6 \sin \beta \cos \beta$$

$$\sin \beta \cos \beta = \frac{1}{2} \sin 2\beta = \frac{30 \times 10^6}{125 \times 10^6} = 0.240 \quad \beta = 14.34^\circ$$

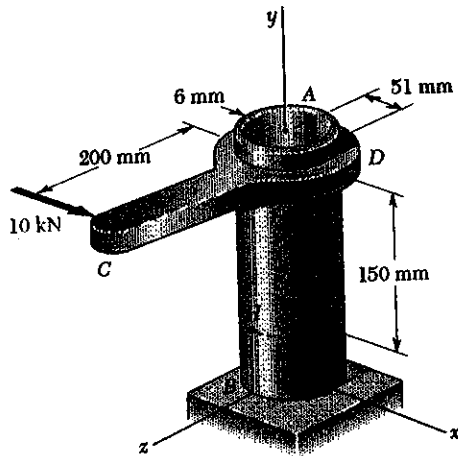
(b)  $\Sigma F_n = 0 \quad F_n - 100 \cos \beta = 0 \quad F_n = 100 \cos 14.34^\circ = 96.88 \text{ kN}$

$$A_w = \frac{800 \times 10^{-6}}{\cos 14.34} = 825.74 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88 \times 10^3}{825.74 \times 10^{-6}} = 117.3 \times 10^6 \text{ Pa} = 117.3 \text{ MPa}$$

**PROBLEM 7.23**

7.23 The steel pipe  $AB$  has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm  $CD$  is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point  $H$ .



**SOLUTION**

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

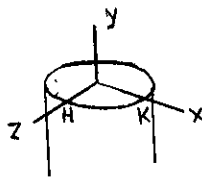
$$I = \frac{1}{2} J = 2.0927 \text{ m}^4$$

Force-couple system at center of tube in the plane containing points  $H$  and  $K$ .

$$F_x = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$

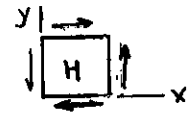


Torsion

$$T = M_y = 2000 \text{ N}\cdot\text{m}$$

$$c = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa}$$



Transverse Shear

For semicircle

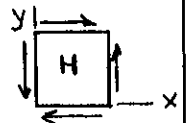
$$A = \frac{\pi}{2} r^2 \quad \bar{y} = \frac{4}{3\pi} r$$

$$Q = A\bar{y} = \frac{2}{3} r^3$$

$$Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$$

$$V = F_x = 10 \times 10^3 \text{ N} \quad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \times 10^6 \text{ Pa}$$



Bending: Point  $H$  lies on neutral axis.  $\sigma_y = 0$

Total stresses at point  $H$ :  $\sigma_x = 0, \sigma_y = 0$   
 $\tau_{xy} = 24.37 \times 10^6 + 11.02 \times 10^6 = 35.39 \times 10^6 \text{ Pa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0 \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.39 \times 10^6 \text{ Pa}$$

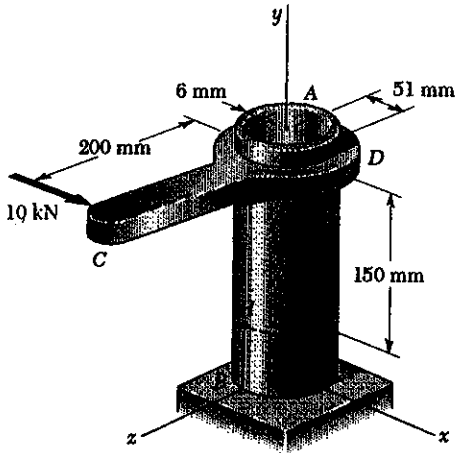
$$\sigma_{max} = \sigma_{ave} + R = 35.39 \times 10^6 \text{ Pa} = 35.4 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -35.39 \times 10^6 \text{ Pa} = -35.4 \text{ MPa}$$

$$\tau_{max} = R = 35.4 \text{ MPa}$$

PROBLEM 7.24

7.24 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

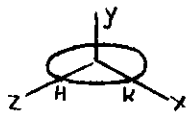
$$F_x = 10 \text{ kN} = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

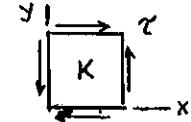
$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$

At point K, place local x-axis in negative global z-direction

$$T = M_y = 2000 \text{ N}\cdot\text{m} \quad C = r_o = 51 \times 10^{-3} \text{ m}$$



Torsion:



$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6} = 24.37 \times 10^6 \text{ Pa} = 24.37 \text{ MPa}$$

Transverse Shear: Stress due to transverse shear  $V = F_x$  is zero at pt. K.

$$\text{Bending: } |\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \times 10^6 \text{ Pa} = 36.56 \text{ MPa}$$

Point K lies on compression side of neutral axis:  $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K  $\sigma_x = 0 \quad \sigma_y = -36.56 \text{ MPa}, \tau_{xy} = 24.37 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

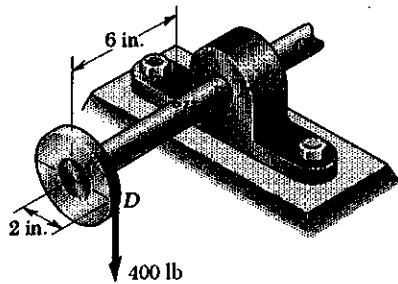
$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = +12.18 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa} \quad \blacktriangleright$$

$$\tau_{max} = R = 30.46 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.25

7.25 A 400-lb vertical force is applied at  $D$  to a gear attached to the solid one-inch diameter shaft  $AB$ . Determine the principal stresses and the maximum shearing stress at point  $H$  located as shown on top of the shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point  $H$ .

$$V = 400 \text{ lb.} \quad M = (400)(6) = 2400 \text{ lb}\cdot\text{in}$$

$$T = (400)(2) = 800 \text{ lb}\cdot\text{in.}$$

Shaft cross section.

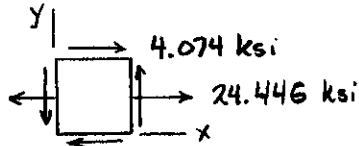
$$d = 1 \text{ in} \quad c = \frac{1}{2}d = 0.5 \text{ in}$$

$$J = \frac{\pi}{2}c^3 = 0.098175 \text{ in}^4 \quad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion:  $\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$

Bending:  $\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$

Transverse shear: Stress at point  $H$  is zero.



$$\sigma_x = 24.446 \text{ ksi} \quad \sigma_y = 0 \quad \tau_{xy} = 4.074 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2}$$

$$= 12.884 \text{ ksi}$$

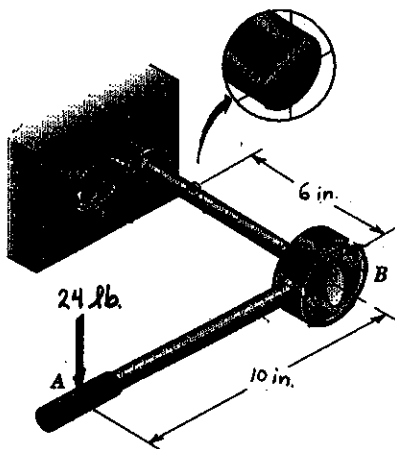
$$\sigma_a = \sigma_{ave} + R = 25.107 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -0.661 \text{ ksi}$$

$$\tau_{max} = R = 12.884 \text{ ksi}$$

**PROBLEM 7.26**

7.26 A mechanic uses a crowfoot wrench to loosen a bolt at  $E$ . Knowing that the mechanic applies a vertical 24-lb force at  $A$ , determine the principal stresses and the maximum shearing stress at point  $H$  located as shown on top of the  $\frac{3}{4}$ -in. diameter shaft.



**SOLUTION**

Equivalent force-couple system at center of shaft in section at point  $H$ .

$$V = 24 \text{ lb.} \quad M = (24)(6) = 144 \text{ lb}\cdot\text{in}$$

$$T = (24)(10) = 240 \text{ lb}\cdot\text{in}$$

Shaft cross section:  $d = 0.75 \text{ in.}$ ,  $c = \frac{1}{2}d = 0.375 \text{ in.}$

$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \quad I = \frac{1}{2}J = 0.015532 \text{ in}^4$$

Torsion:  $\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$

Bending:  $\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$

Transverse Shear: At point  $H$  stress due to transverse shear is zero.

Resultant stresses:  $\sigma_x = 3.477 \text{ ksi}$ ,  $\sigma_y = 0$ ,  $\tau_{xy} = 2.897 \text{ ksi}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 5.116 \text{ ksi}$$

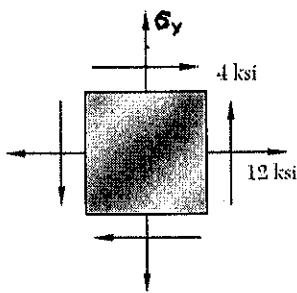
$$\sigma_b = \sigma_{ave} - R = -1.640 \text{ ksi}$$

$$\tau_{max} = R = 3.378 \text{ ksi}$$



**PROBLEM 7.27**

7.27 For the state of plane stress shown, determine the largest value of  $\sigma_y$  for which the maximum in-plane shearing stress is equal to or less than 15 ksi.



**SOLUTION**

$$\sigma_x = 12 \text{ ksi}, \quad \sigma_y = ?, \quad \tau_{xy} = 4 \text{ ksi}$$

$$\text{Let } U = \frac{\sigma_x - \sigma_y}{2} \quad \sigma_y = \sigma_x - 2U$$

$$R = \sqrt{U^2 + \tau_{xy}^2} = 15 \text{ ksi}$$

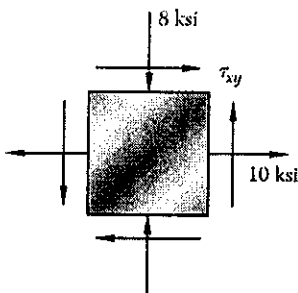
$$U = \pm \sqrt{R^2 - \tau_{xy}^2} = \pm \sqrt{15^2 - 4^2} = \pm 14.457 \text{ ksi}$$

$$\sigma_y = \sigma_x - 2U = 12 \mp (2)(14.457) = 40.9 \text{ ksi}, \quad -16.91 \text{ ksi}$$

Largest value for  $\sigma_y$  is required.  $\sigma_y = 40.9 \text{ ksi}$  ▶

**PROBLEM 7.28**

7.28 For the state of plane stress shown, determine (a) the largest value of  $\tau_{xy}$  for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (b) the corresponding principal stresses.



**SOLUTION**

$$\sigma_x = 10 \text{ ksi}, \quad \sigma_y = -8 \text{ ksi}, \quad \tau_{xy} = ?$$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10 - (-8)}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{9^2 + \tau_{xy}^2} = 12 \text{ ksi}$$

$$(a) \quad \tau_{xy} = \sqrt{12^2 - 9^2} = 7.94 \text{ ksi} \quad \blacktriangleleft$$

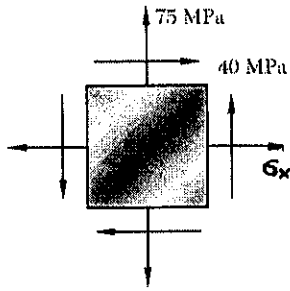
$$(b) \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 1 + 12 = 13 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_{ave} - R = 1 - 12 = -11 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 7.29**

7.29 Determine the range of values of  $\sigma_x$  for which the maximum in-plane shearing stress is equal to or less than 50 MPa.



**SOLUTION**

$$\sigma_x = ? , \sigma_y = 75 \text{ MPa} , \tau_{xy} = 40 \text{ MPa}$$

$$\text{Let } u = \frac{\sigma_x - \sigma_y}{2} \quad \sigma_x = \sigma_y + 2u$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{\max} = 50 \text{ MPa}$$

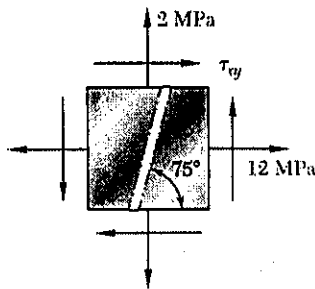
$$u = \pm \sqrt{R^2 - \tau_{xy}^2} = \pm \sqrt{50^2 - 40^2} = \pm 30 \text{ MPa}$$

$$\sigma_x = \sigma_y + 2u = 75 \pm (2 \times 30) = 135 \text{ MPa} , 15 \text{ MPa}$$

$$\text{Allowable range } 15 \text{ MPa} \leq \sigma_x \leq 135 \text{ MPa}$$

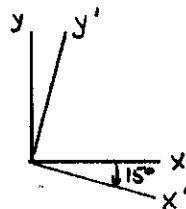
**PROBLEM 7.30**

7.30 For the state of plane stress shown, determine (a) the value of  $\tau_{xy}$  for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.



**SOLUTION**

$$\sigma_x = 12 \text{ MPa} , \sigma_y = 2 \text{ MPa} , \tau_{xy} = ?$$



Since  $\tau_{x'y'} = 0$ ,  $x'$ -direction is a principal direction.

$$\theta_p = -15^\circ$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$(a) \tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y) \tan 2\theta_p = \frac{1}{2}(12 - 2) \tan(-30^\circ) = -2.89 \text{ MPa}$$

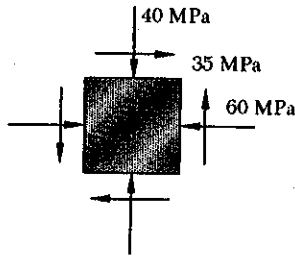
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7 \text{ MPa}$$

$$(b) \sigma_a = \sigma_{ave} + R = 7 + 5.7735 = 12.77 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 7 - 5.7735 = 1.226 \text{ MPa}$$

PROBLEM 7.31



7.31 Solve Probs. 7.7 and 7.11, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-50 \text{ MPa}, 0)$$

$$\tan \beta = \frac{GX}{CG} = \frac{35}{10} = 3.500$$

$$\beta = 74.05^\circ$$

$$\theta_B = -\frac{1}{2}\beta = -37.03^\circ$$

$$\alpha = 180^\circ - \beta = 105.95^\circ$$

$$\theta_A = \frac{1}{2}\alpha = 52.97^\circ$$

$$R = \sqrt{CG^2 + GX^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -50 - 36.4 = -86.4 \text{ MPa}$$

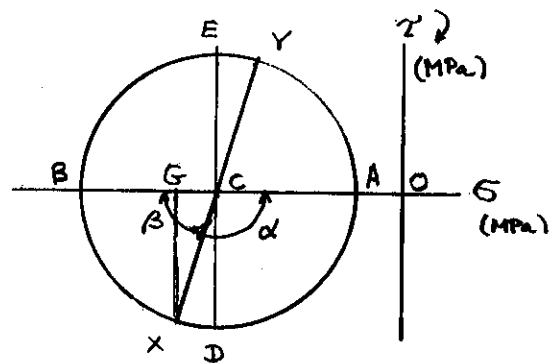
$$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4 = -13.6 \text{ MPa}$$

$$\theta_D = \theta_B + 45^\circ = 7.97^\circ$$

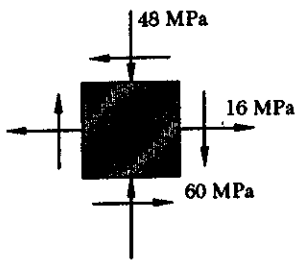
$$\theta_E = \theta_A + 45^\circ = 97.97^\circ$$

$$\tau_{max} = R = 36.4 \text{ MPa}$$

$$\sigma' = \sigma_{ave} = -50 \text{ MPa}$$



PROBLEM 7.32



7.32 Solve Probs. 7.8 and 7.12, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

SOLUTION

$$\sigma_x = 16 \text{ MPa} \quad \sigma_y = -48 \text{ MPa} \quad \tau_{xy} = -60 \text{ MPa}$$

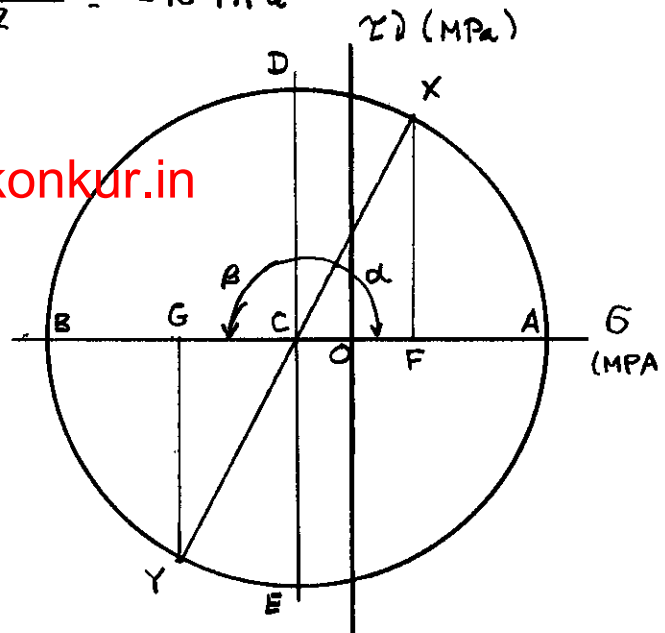
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -16 \text{ MPa}$$

Points:

$$X: (\sigma_x, -\tau_{xy}) = (16 \text{ MPa}, 60 \text{ MPa})$$

$$Y: (\sigma_y, \tau_{xy}) = (-48 \text{ MPa}, -60 \text{ MPa})$$

$$C: (\sigma_{ave}, 0) = (-16 \text{ MPa}, 0)$$



$$\tan \alpha = \frac{FX}{CF} = \frac{60}{32} = 1.875$$

$$\alpha = 61.93^\circ$$

$$\theta_A = -\frac{1}{2}\alpha = -30.96^\circ \quad \blacktriangleleft$$

$$\beta = 180^\circ - \alpha = 118.07^\circ$$

$$\theta_B = \frac{1}{2}\beta = 59.04^\circ \quad \blacktriangleleft$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{32^2 + 60^2} = 68 \text{ MPa}$$

$$\sigma_A = \sigma_{max} = \sigma_{ave} + R = -16 + 68 = 52 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_B = \sigma_{min} = \sigma_{ave} - R = -16 - 68 = -84 \text{ MPa} \quad \blacktriangleleft$$

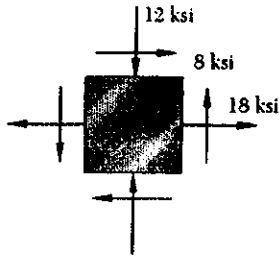
$$\theta_D = \theta_A + 45^\circ = 14.04^\circ \quad \blacktriangleleft$$

$$\theta_E = \theta_B + 45^\circ = 104.04^\circ \quad \blacktriangleleft$$

$$\tau_{max} = R = 68 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma' = \sigma_{ave} = -16 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.33



7.33 Solve Prob. 7.9, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.

SOLUTION

$$\sigma_x = 18 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = 8 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 3 \text{ ksi}$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (18 \text{ ksi}, -8 \text{ ksi})$$

$$Y: (\sigma_y, \tau_{xy}) = (-12 \text{ ksi}, 8 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (3 \text{ ksi}, 0)$$

$$\tan \alpha = \frac{FX}{CF} = \frac{8}{15} = 0.5333$$

$$\alpha = 28.07^\circ$$

$$\theta_A = \frac{1}{2} \alpha = 14.04^\circ$$

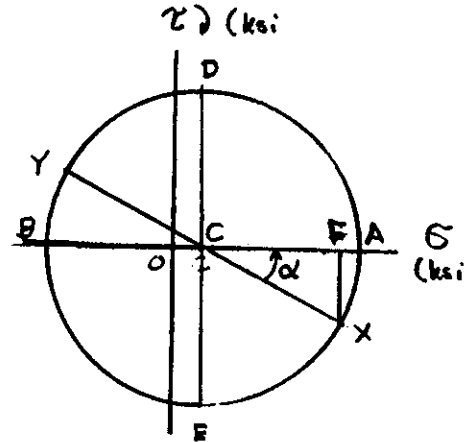
$$\theta_D = \theta_A + 45^\circ = 59.04^\circ \quad \blacktriangleleft$$

$$\theta_E = \theta_A - 45^\circ = -30.96^\circ \quad \blacktriangleleft$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{15^2 + 8^2} = 17 \text{ ksi}$$

$$\tau_{max} = R = 17 \text{ ksi} \quad \blacktriangleleft$$

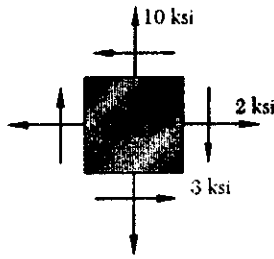
$$\sigma' = \sigma_{ave} = 3 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 7.34

7.34 Solve Prob. 7.10, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



SOLUTION

$$\sigma_x = 2 \text{ ksi} \quad \sigma_y = 10 \text{ ksi} \quad \tau_{xy} = -3 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{2 + 10}{2} = 6 \text{ ksi}$$

Points

$$X: (\sigma_x, \tau_{xy}) = (2 \text{ ksi}, 3 \text{ ksi})$$

$$Y: (\sigma_y, \tau_{xy}) = (10 \text{ ksi}, -3 \text{ ksi})$$

$$C: (\sigma_{ave}, 0) = (6 \text{ ksi}, 0)$$

$$\tan \alpha = \frac{FX}{FC} = \frac{3}{4} = 0.75$$

$$\alpha = 36.87^\circ$$

$$\theta_B = \frac{1}{2} \alpha = 18.43^\circ$$

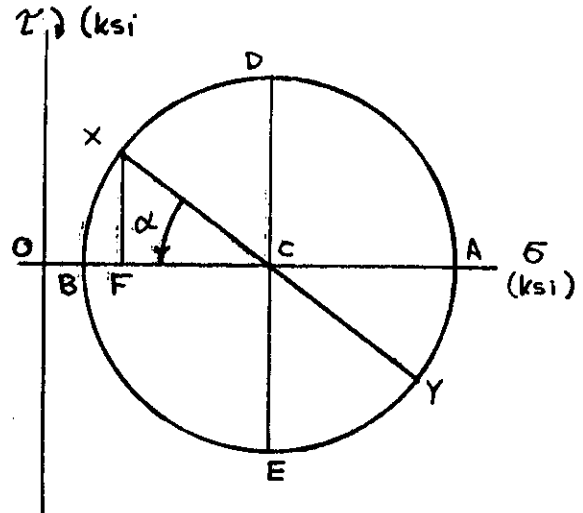
$$\theta_D = \theta_B - 45^\circ = -26.57^\circ$$

$$\theta_E = \theta_B + 45^\circ = 63.43^\circ$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{4^2 + 3^2} = 5 \text{ ksi}$$

$$\tau_{max} = R = 5 \text{ ksi}$$

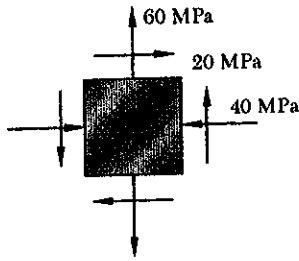
$$\sigma' = \sigma_{ave} = 6 \text{ ksi}$$



PROBLEM 7.35

7.35 Solve Prob. 7.13, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



SOLUTION

$$\sigma_x = -40 \text{ MPa} \quad \sigma_y = 60 \text{ MPa} \quad \tau_{xy} = 20 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 10 \text{ MPa}$$

Points

$$X: (-40 \text{ MPa}, -20 \text{ MPa})$$

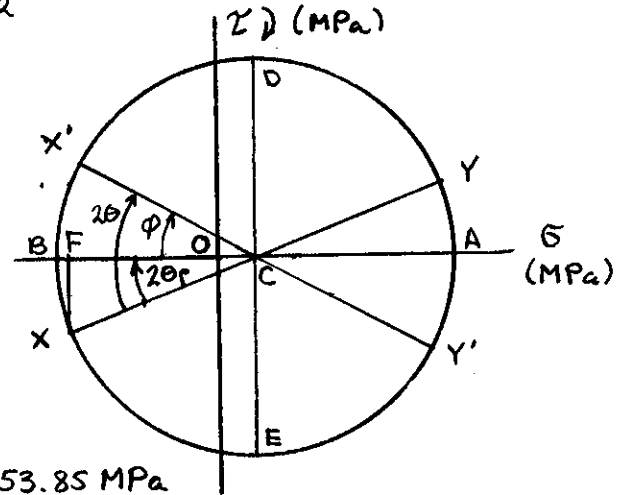
$$Y: (60 \text{ MPa}, 20 \text{ MPa})$$

$$C: (10 \text{ MPa}, 0)$$

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{20}{50} = 0.4$$

$$2\theta_p = 21.80^\circ \quad \theta_p = 10.90^\circ$$

$$R = \sqrt{FC^2 + FX^2} = \sqrt{50^2 + 20^2} = 53.85 \text{ MPa}$$



(a)  $\theta = 25^\circ \Rightarrow 2\theta = 50^\circ$

$$\phi = 2\theta - 2\theta_p = 50^\circ - 21.80^\circ = 28.20^\circ$$

$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -37.5 \text{ MPa}$$

$$\tau_{x'y'} = -R \sin \phi = -25.4 \text{ MPa}$$

$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 57.5 \text{ MPa}$$

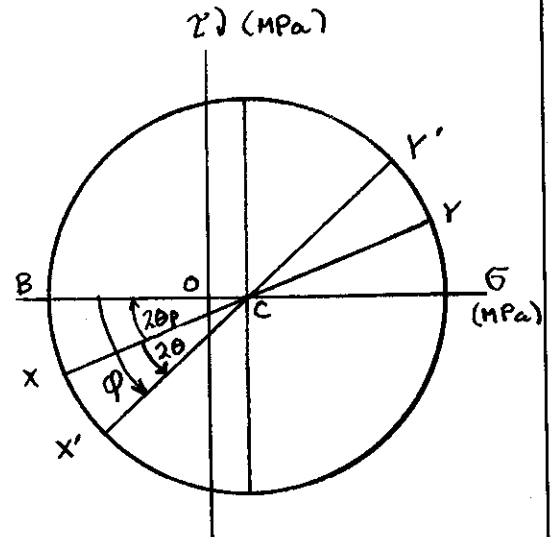
(b)  $\theta = 10^\circ \Rightarrow 2\theta = 20^\circ$

$$\phi = 2\theta_p + 2\theta = 21.80^\circ + 20^\circ = 41.80^\circ$$

$$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -30.1 \text{ MPa}$$

$$\tau_{x'y'} = R \sin \phi = 35.9 \text{ MPa}$$

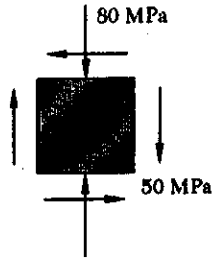
$$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 50.1 \text{ MPa}$$



PROBLEM 7.36

7.36 Solve Prob. 7.14, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



SOLUTION

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$$

Points

- X: (0, 50 MPa)
- Y: (-80 MPa, -50 MPa)
- C: (-40 MPa, 0)

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{50}{40} = 1.25$$

$$2\theta_p = 51.34^\circ$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{40^2 + 50^2} = 64.03 \text{ MPa}$$

(a)  $\theta = 25^\circ \curvearrowright \quad 2\theta = 50^\circ \curvearrowright$

$$\phi = 51.34^\circ - 50^\circ = 1.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 24.0 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -R \sin \phi = -1.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -104.0 \text{ MPa} \quad \blacktriangleleft$$

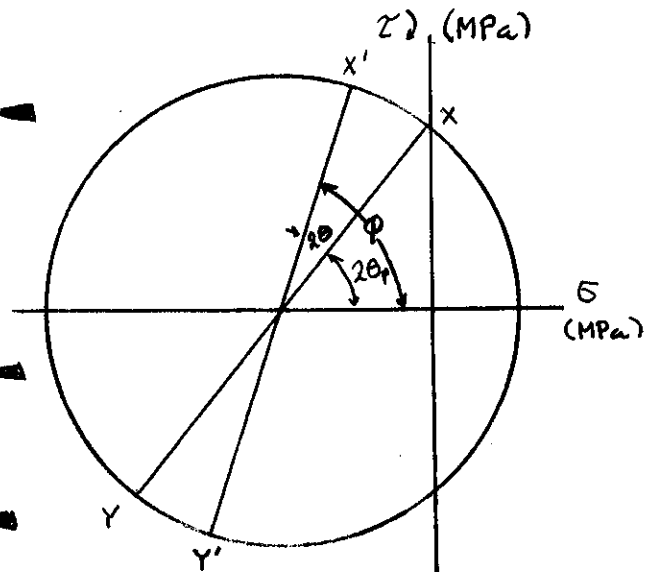
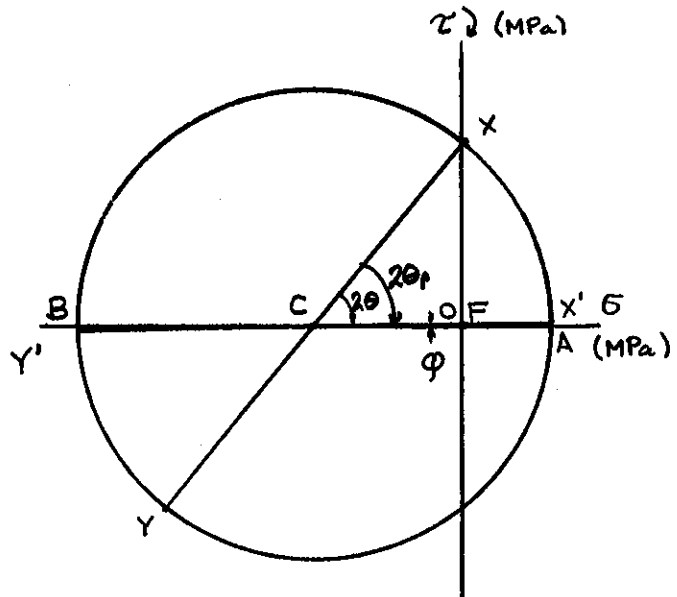
(b)  $\theta = 10^\circ \curvearrowleft \quad 2\theta = 20^\circ \curvearrowleft$

$$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = -19.5 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = +R \sin \phi = -60.7 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -60.5 \text{ MPa} \quad \blacktriangleleft$$

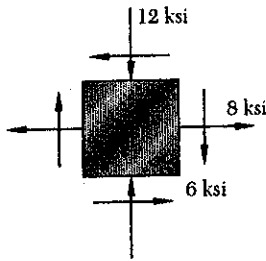




PROBLEM 7.37

7.37 Solve Prob. 7.15, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



SOLUTION

$$\sigma_x = 8 \text{ ksi} \quad \sigma_y = -12 \text{ ksi} \quad \tau_{xy} = -6 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi}$$

Points

$$X: (8 \text{ ksi}, 6 \text{ ksi})$$

$$Y: (-12 \text{ ksi}, -6 \text{ ksi})$$

$$C: (-2 \text{ ksi}, 0)$$

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{6}{10} = 0.6$$

$$2\theta_p = 30.96^\circ$$

$$R = \sqrt{CF^2 + FX^2} = \sqrt{10^2 + 6^2} = 11.66 \text{ ksi}$$

(a)  $\theta = 25^\circ \rightarrow 2\theta = 50^\circ$

$$\phi = 50^\circ - 30.96^\circ = 19.04^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 9.02 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{xy'} = R \sin \phi = 3.80 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -13.02 \text{ ksi} \quad \blacktriangleleft$$

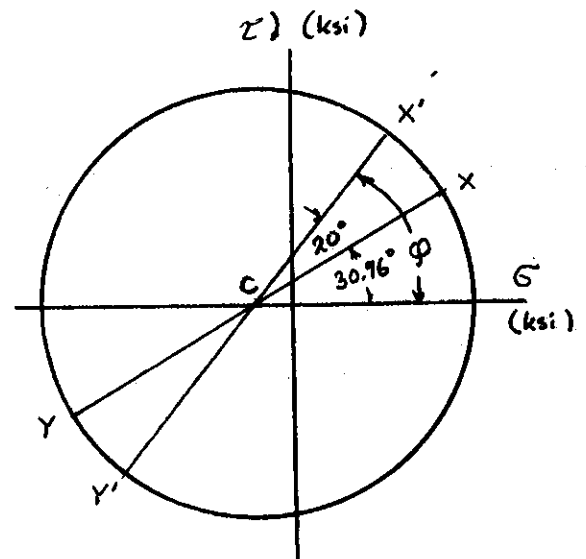
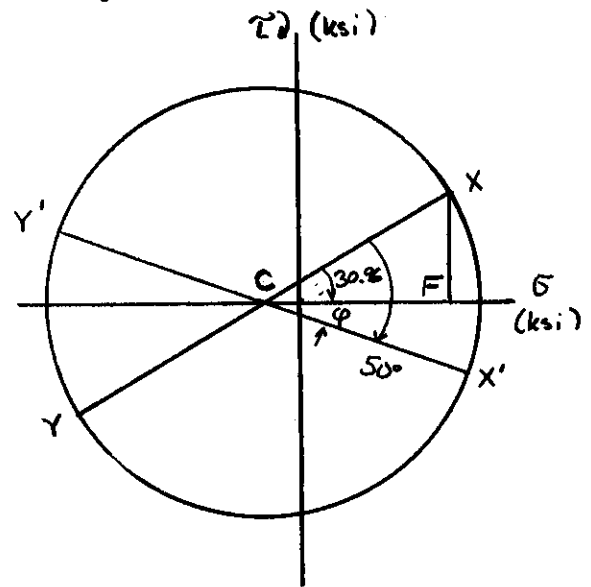
(b)  $\theta = 10^\circ \rightarrow 2\theta = 20^\circ$

$$\phi = 30.96^\circ + 20^\circ = 50.96^\circ$$

$$\sigma_{x'} = \sigma_{ave} + R \cos \phi = 5.34 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{xy'} = -R \sin \phi = -9.06 \text{ ksi} \quad \blacktriangleleft$$

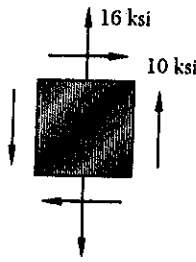
$$\sigma_{y'} = \sigma_{ave} - R \cos \phi = -9.34 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 7.38

7.38 Solve Prob. 7.16, using Mohr's circle:

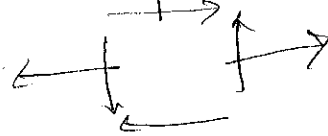
7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



SOLUTION

$\sigma_x = 0$        $\sigma_y = 16 \text{ ksi}$        $\tau_{xy} = 10 \text{ ksi}$

$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 8 \text{ ksi}$



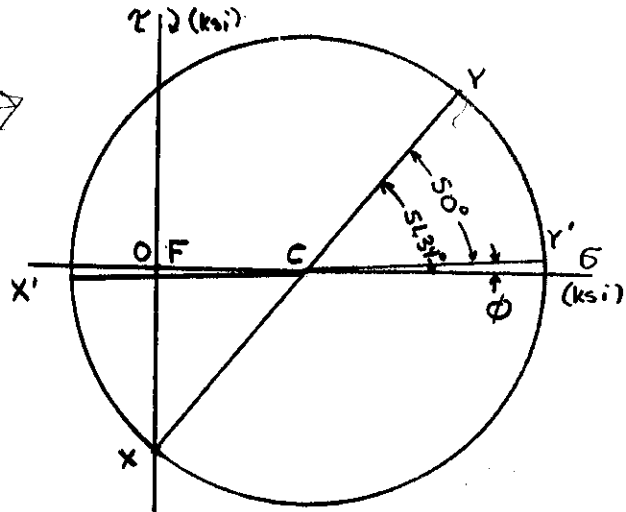
Points:

- X: (0, -10 ksi)
- Y: (16 ksi, 10 ksi)
- C: (8 ksi, 0)

$\tan 2\theta_p = \frac{FX}{FC} = \frac{10}{8} = 1.25$

$2\theta_p = 51.34^\circ$

$R = \sqrt{FC^2 + FX^2} = \sqrt{8^2 + 10^2} = 12.81 \text{ ksi}$



(a)  $\theta = 25^\circ \rightarrow 2\theta = 50^\circ \rightarrow$

$\phi = 51.34^\circ - 50^\circ = 1.34^\circ$

$\sigma_{x'} = \sigma_{ave} - R \cos \phi = -4.81 \text{ ksi} \leftarrow$

$\tau_{x'y'} = R \sin \phi = 0.30 \text{ ksi} \leftarrow$

$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 20.81 \text{ ksi} \leftarrow$

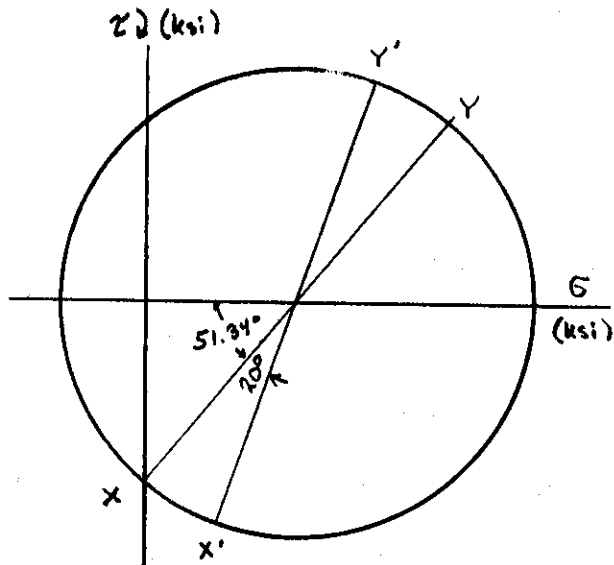
(b)  $\theta = 10^\circ \rightarrow 2\theta = 20^\circ \rightarrow$

$\phi = 51.34^\circ + 20^\circ = 71.34^\circ$

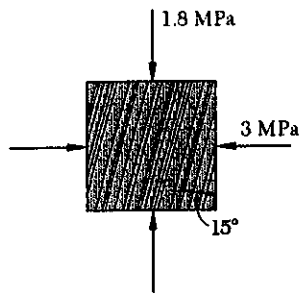
$\sigma_{x'} = \sigma_{ave} - R \cos \phi = 3.90 \text{ ksi} \leftarrow$

$\tau_{x'y'} = R \sin \phi = 12.14 \text{ ksi} \leftarrow$

$\sigma_{y'} = \sigma_{ave} + R \cos \phi = 12.10 \text{ ksi} \leftarrow$



**PROBLEM 7.39**



7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

**SOLUTION**

$$\sigma_x = -3 \text{ MPa} \quad \sigma_y = -1.8 \text{ MPa} \quad \tau_{xy} = 0$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2.4 \text{ MPa}$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (-3 \text{ MPa}, 0)$$

$$Y: (\sigma_y, \tau_{xy}) = (-1.8 \text{ MPa}, 0)$$

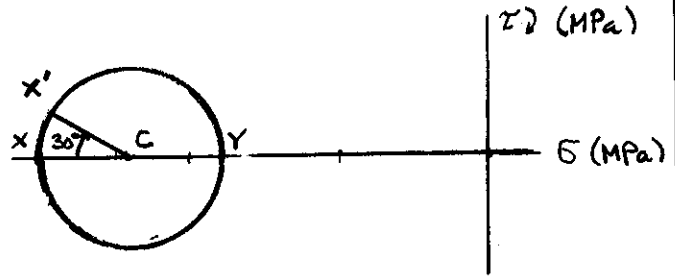
$$C: (\sigma_{ave}, 0) = (-2.4 \text{ MPa}, 0)$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

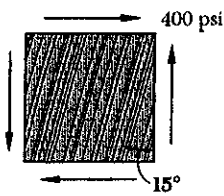
$$\bar{CX} = 0.6 \text{ MPa} \quad R = 0.6 \text{ MPa}$$

$$(a) \tau_{x'y'} = -\bar{CX}' \sin 30^\circ = -R \sin 30^\circ = -0.6 \sin 30^\circ = -0.3 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \sigma_{x'} = \sigma_{ave} - \bar{CX}' \cos 30^\circ = -2.4 - 0.6 \cos 30^\circ = -2.92 \text{ MPa} \quad \blacktriangleleft$$



**PROBLEM 7.40**



7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.

**SOLUTION**

$$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = 400 \text{ psi}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Points

$$X: (\sigma_x, -\tau_{xy}) = (0, -400 \text{ psi})$$

$$Y: (\sigma_y, \tau_{xy}) = (0, 400 \text{ psi})$$

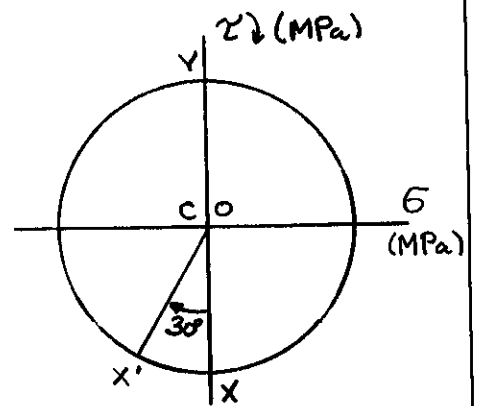
$$C: (\sigma_{ave}, 0) = (0, 0)$$

$$\theta = -15^\circ \quad 2\theta = -30^\circ$$

$$\bar{CX} = R = 400 \text{ psi}$$

$$(a) \tau_{x'y'} = R \cos 30^\circ = 400 \cos 30^\circ = 346 \text{ psi} \quad \blacktriangleleft$$

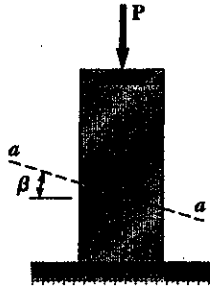
$$(b) \sigma_{x'} = \sigma_{ave} - R \sin 30^\circ = -400 \sin 30^\circ = -200 \text{ psi} \quad \blacktriangleleft$$



PROBLEM 7.41

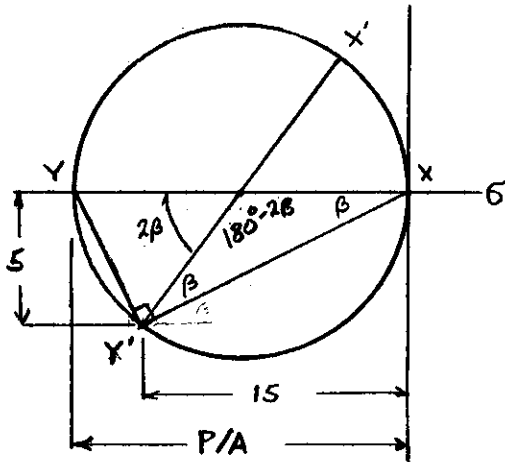
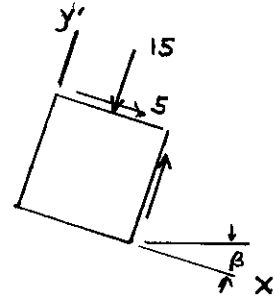
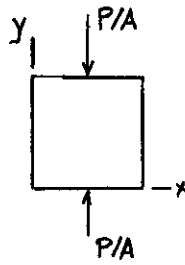
7.41 Solve Prob. 7.19, using Mohr's circle.

7.19 The centric force  $P$  is applied to a short post as shown. Knowing that the stresses on plane  $a-a$  are  $\sigma = -15$  ksi and  $\tau = 5$  ksi, determine (a) the angle  $\beta$  that plane  $a-a$  forms with the horizontal, (b) the maximum compressive stress in the post.



SOLUTION

$$\begin{aligned} \sigma_x &= 0 \\ \tau_{xy} &= 0 \\ \sigma_y &= -P/A \end{aligned}$$



From the Mohr's circle

$$\tan \beta = \frac{5}{15} = 0.3333 \quad \beta = 18.4^\circ$$

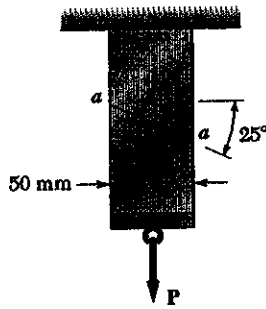
$$-5 = \frac{P}{2A} + \frac{P}{2A} \cos 2\beta$$

$$\begin{aligned} \frac{P}{A} &= \frac{2(-5)}{1 + \cos 2\beta} = \frac{(2)(15)}{1 + \cos 2\beta} \\ &= 16.67 \text{ ksi} \end{aligned}$$

PROBLEM 7.42

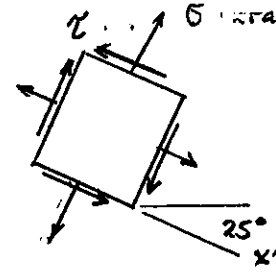
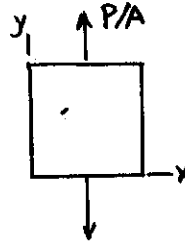
7.42 Solve Prob. 7.20, using Mohr's circle.

7.20 Two members of uniform cross section  $50 \times 80$  mm are glued together along plane  $a-a$ , which forms an angle of  $25^\circ$  with the horizontal. Knowing that the allowable stresses for the glued joint are  $\sigma = 800$  kPa and  $\tau = 600$  kPa, determine the largest axial load  $P$  that can be applied.

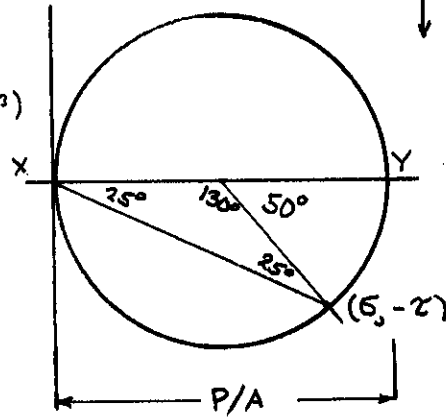


SOLUTION

$$\begin{aligned} \sigma_x &= 0 \\ \sigma_y &= P/A \\ \tau_{xy} &= 0 \end{aligned}$$



$$A = (50 \times 10^{-3})(80 \times 10^{-3}) = 4 \times 10^{-3} \text{ m}^2$$



$$\sigma = \frac{P}{2A} (1 + \cos 50^\circ)$$

$$P = \frac{2A\sigma}{1 + \cos 50^\circ}$$

$$P \leq \frac{(2)(4 \times 10^{-3})(800 \times 10^3)}{1 + \cos 50^\circ}$$

$$P \leq 3.90 \times 10^3 \text{ N}$$

$$\tau = \frac{P}{2A} \sin 50^\circ$$

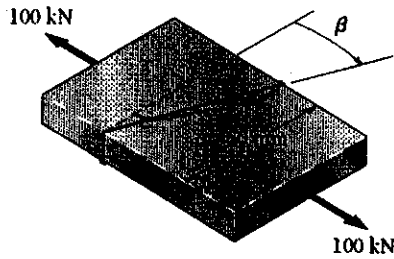
$$P = \frac{2A\tau}{\sin 50^\circ} \leq \frac{(2)(4 \times 10^{-3})(600 \times 10^3)}{\sin 50^\circ} = 6.27 \times 10^3 \text{ N}$$

Choosing the smaller value  $P \leq 3.90 \times 10^3 \text{ N} = 3.90 \text{ kN}$

PROBLEM 7.43

7.43 Solve Prob. 7.21, using Mohr's circle.

7.21 Two steel plates of uniform cross section  $10 \times 80$  mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that  $\beta = 25^\circ$ , determine (a) the in-plane shearing stress parallel to the weld, (b) the normal stress perpendicular to the weld.



SOLUTION

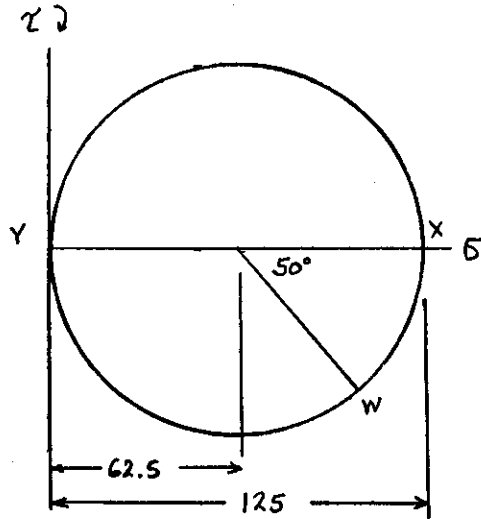
$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \quad \tau_{xy} = 0$$

From Mohr's circle

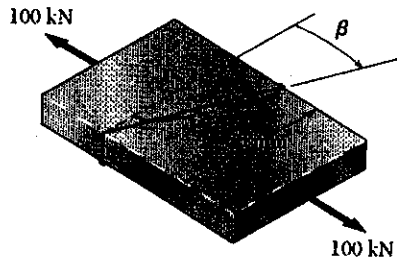
$$(a) \tau_w = 62.5 \sin 50^\circ = 47.9 \text{ MPa}$$

$$(b) \sigma_w = 62.5 + 62.5 \cos 50^\circ = 102.7 \text{ MPa}$$



PROBLEM 7.44

7.44 Solve Prob. 7.22, using Mohr's circle.



7.22 Two steel plates of uniform cross section  $10 \times 80$  mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (a) the angle  $\beta$ , (b) the corresponding normal stress perpendicular to the weld.

SOLUTION

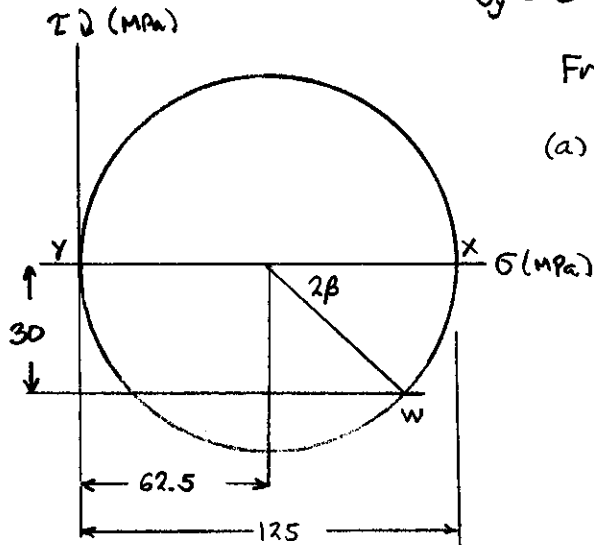
$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \quad \tau_{xy} = 0$$

From Mohr's circle

$$(a) \sin 2\beta = \frac{30}{62.5} = 0.48 \quad \beta = 14.3^\circ \quad \blacktriangleleft$$

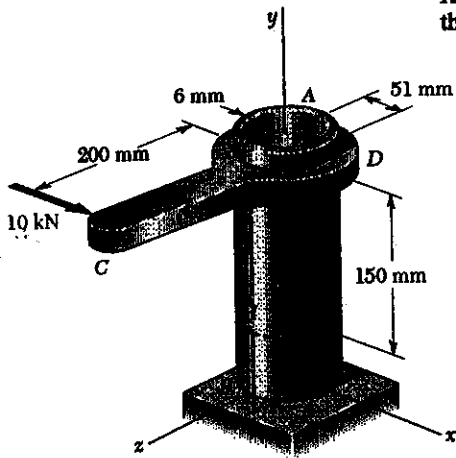
$$(b) \sigma = 62.5 + 62.5 \cos 2\beta = 117.3 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H.



SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

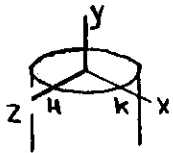
$$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

$$F_x = 10 \times 10^3 \text{ N}$$

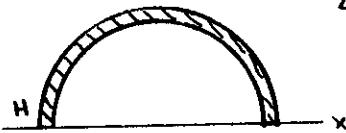
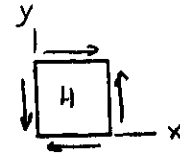
$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$



Torsion:  $T = M_y = 2000 \text{ N}\cdot\text{m}$   
 $C = r_o = 51 \times 10^{-3} \text{ m}$

$$\tau_{xy} = \frac{TC}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \text{ MPa}$$



Transverse Shear:  
For semicircle

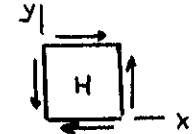
$$A = \frac{\pi}{2} r^2 \quad \bar{y} = \frac{4}{3\pi} r$$

$$Q = A\bar{y} = \frac{2}{3} r^3$$

For pipe  $Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$

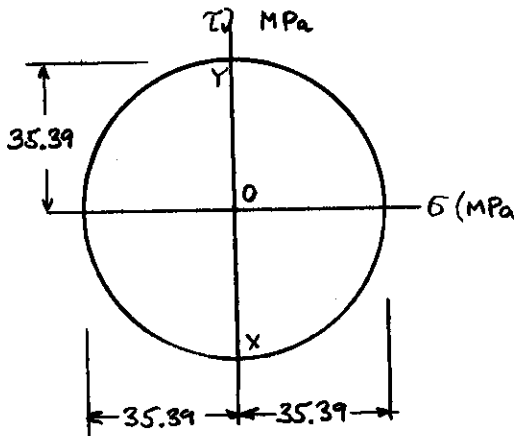
$$V = F_x = 10 \times 10^3 \text{ N} \quad t = (2)(6) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \text{ MPa}$$



Bending: Point H lies on neutral axis  $\sigma_y = 0$

Total stresses at point H  $\sigma_x = 0, \sigma_y = 0 \quad \tau_{xy} = 24.37 + 11.02 = 35.39 \text{ MPa}$



$$\sigma_{ave} = 0$$

$$R = 35.39 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 35.39 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -35.39 \text{ MPa}$$

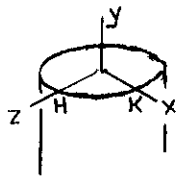
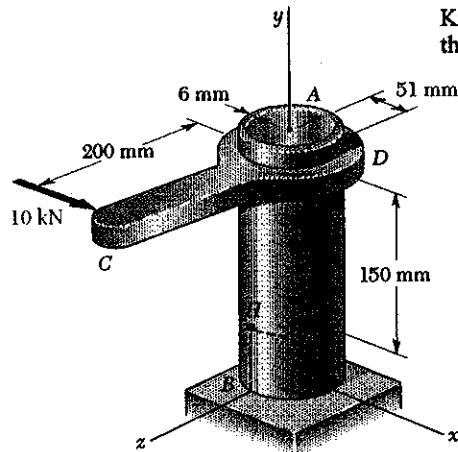
$$\tau_{max} = R = 35.39 \text{ MPa}$$



PROBLEM 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

7.24 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \quad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2} (r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K

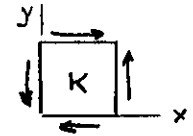
$$F_x = 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$$

Torsion:  $T = M_y = 2000 \text{ N}\cdot\text{m}$   
 $C = r_o = 51 \times 10^{-3} \text{ m}$

$$\tau_{xy} = \frac{TC}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \text{ MPa}$$



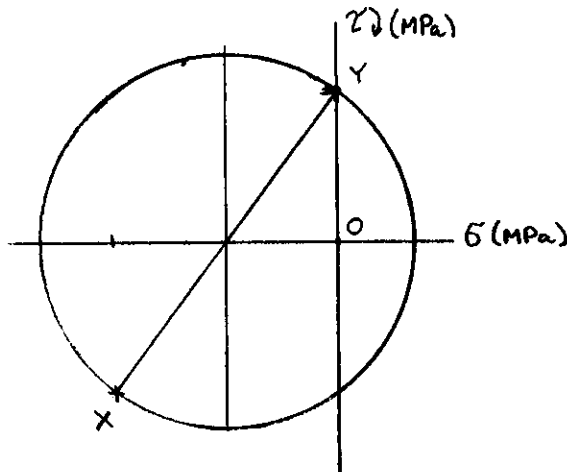
Note that local x-axis is taken along negative global z-direction.

Transverse Shear: Stress due to  $V = F_x$  is zero at point K.

Bending:  $|\sigma_y| = \frac{M_z C}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \text{ MPa}$

Point K lies on compression side of neutral axis.  $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K  $\sigma_x = 0$ ,  $\sigma_y = -36.56 \text{ MPa}$ ,  $\tau_{xy} = 24.37 \text{ MPa}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = 12.18 \text{ MPa}$$

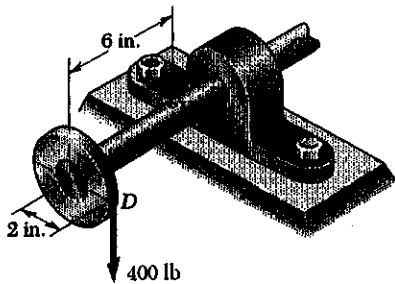
$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa}$$

$$\tau_{max} = R = 30.46 \text{ MPa}$$

PROBLEM 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 A 400-lb vertical force is applied at  $D$  to a gear attached to the solid one-inch diameter shaft  $AB$ . Determine the principal stresses and the maximum shearing stress at point  $H$  located as shown on top of the shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point  $H$ .

$$V = 400 \text{ lb} \quad M = (400)(6) = 2400 \text{ lb}\cdot\text{in}$$

$$T = (400)(2) = 800 \text{ lb}\cdot\text{in}$$

Shaft cross section

$$d = 1 \text{ in} \quad c = \frac{1}{2}d = 0.5 \text{ in}$$

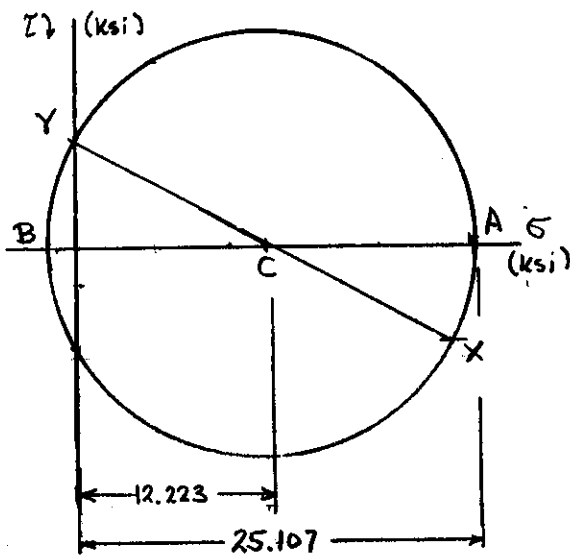
$$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \quad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion:  $\tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$

Bending:  $\sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$

Transverse Shear: Stress at point  $H$  is zero.

Resultant stresses:  $\sigma_x = 24.446 \text{ ksi}$ ,  $\sigma_y = 0$ ,  $\tau_{xy} = 4.074 \text{ ksi}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

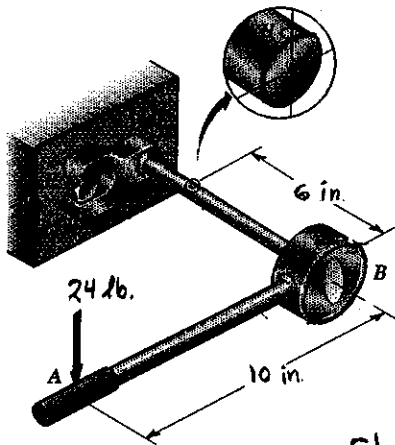
$$= \sqrt{(12.223)^2 + (4.074)^2} = 12.884 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 25.107 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -0.661 \text{ ksi}$$

$$\tau_{max} = R = 12.88 \text{ ksi}$$

PROBLEM 7.48



7.48 Solve Prob. 7.26, using Mohr's circle.

7.26 A mechanic uses a crowfoot wrench to loosen a bolt at  $E$ . Knowing that the mechanic applies a vertical 24-lb force at  $A$ , determine the principal stresses and the maximum shearing stress at point  $H$  located as shown on top of the  $\frac{3}{4}$ -in. diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point  $H$ .

$$V = 24 \text{ lb.} \quad M = (24)(6) = 144 \text{ lb}\cdot\text{in}$$

$$T = (24)(10) = 240 \text{ lb}\cdot\text{in}$$

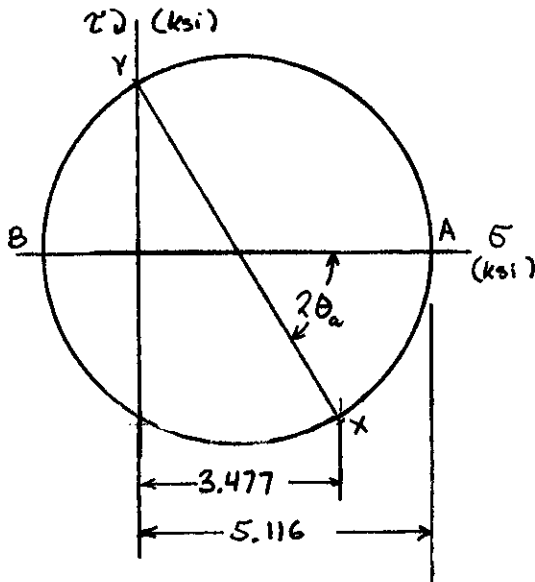
Shaft cross section:  $d = 0.75 \text{ in.} \quad c = \frac{1}{2}d = 0.375 \text{ in}$   
 $J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \quad I = \frac{1}{2}J = 0.015532 \text{ in}^4$

Torsion:  $\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$

Bending:  $\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$

Transverse Shear: At point  $H$  stress due to transverse shear is zero.

Resultant stresses:  $\sigma_x = 3.477 \text{ ksi}, \sigma_y = 0, \tau_{xy} = 2.897 \text{ ksi}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 5.116 \text{ ksi} \quad \blacktriangle$$

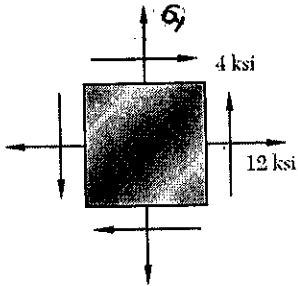
$$\sigma_b = \sigma_{ave} - R = -1.640 \text{ ksi} \quad \blacktriangle$$

$$\tau_{max} = R = 3.378 \text{ ksi} \quad \blacktriangle$$

PROBLEM 7.49

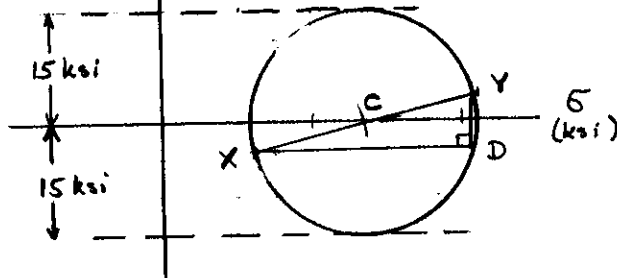
7.49 Solve Prob. 7.27, using Mohr's circle.

7.27 For the state of plane stress shown, determine the largest value of  $\sigma_y$  for which the maximum in-plane shearing stress is equal to or less than 15 ksi.



SOLUTION

$$\sigma_x = 12 \text{ ksi}, \quad \sigma_y = ?, \quad \tau_{xy} = 4 \text{ ksi}$$



Given:  $\tau_{max} = R = 15 \text{ ksi}$

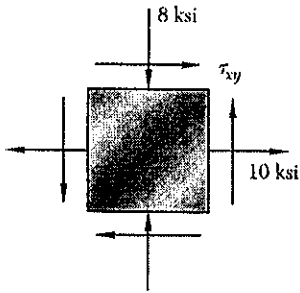
$$\overline{XY} = 2R = 30 \text{ ksi}$$

$$\overline{DY} = (2)(\tau_{xy}) = 8 \text{ ksi}$$

$$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{30^2 - 8^2} = 28.9 \text{ ksi}$$

$$\sigma_y = \sigma_x + \overline{XD} = 12 + 28.9 = 40.9 \text{ ksi}$$

PROBLEM 7.50

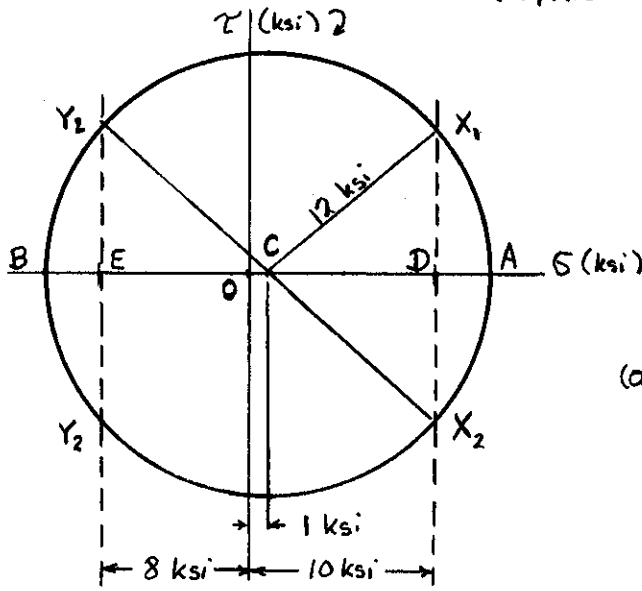


\*7.50 Solve Prob. 7.28, using Mohr's circle.

7.28 For the state of plane stress shown, determine (a) the largest value of  $\tau_{xy}$  for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (b) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point C with coordinates  $(\frac{\sigma_x + \sigma_y}{2}, 0) = (\frac{10 + 8}{2}, 0) = (9, 0 \text{ ksi})$ . The radius of the circle is  $\tau_{max(in-plane)} = 12 \text{ ksi}$ .



The stress point  $(\sigma_x, -\tau_{xy})$  lie along the line  $X_1 X_2$  of the Mohr circle diagram. The extreme points with  $R \leq 12 \text{ ksi}$  are  $X_1$  and  $X_2$ .

(a) The largest allowable value of  $\tau_{xy}$  is obtained from triangle  $CDX_1$ ,

$$\overline{DX_1}^2 = \overline{DX_2}^2 = \sqrt{\overline{CX_1}^2 - \overline{CD}^2}$$

$$\tau_{xy}^2 = \sqrt{12^2 - 9^2} = 7.94 \text{ ksi}$$

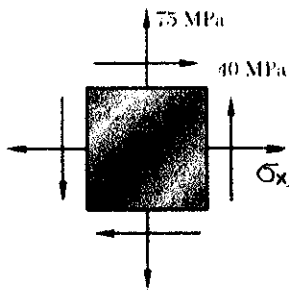
(b) The principal stresses are  $\sigma_a = 1 + 12 = 13 \text{ ksi}$

$$\sigma_b = 1 - 12 = -11 \text{ ksi}$$

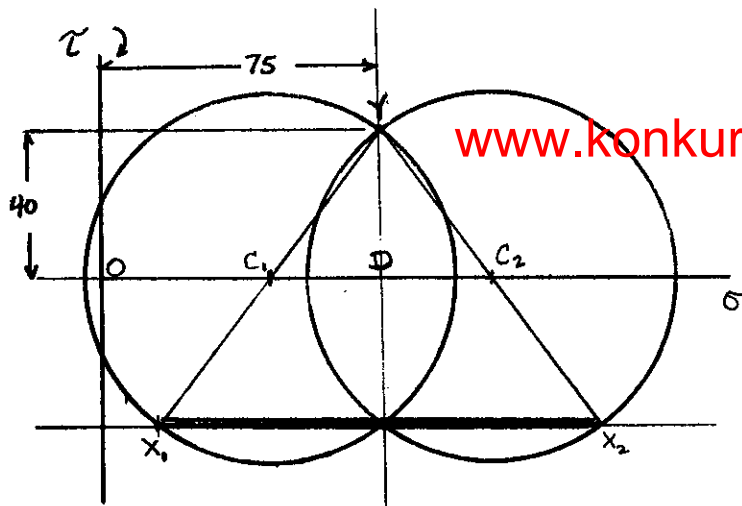
**PROBLEM 7.51**

7.51 Solve Prob. 7.29, using Mohr's circle.

7.29 Determine the range of values of  $\sigma_x$  for which the maximum in-plane shearing stress is equal to or less than 50 MPa.



**SOLUTION**



For the Mohr's circle, point Y lies at (75 MPa, 40 MPa).

The radius of limiting circles is  $R = 50$  MPa

Let  $C_1$  be the location of the left most limiting circle and  $C_2$  be that of the right most one.

$$\overline{C_1 Y} = 50 \text{ MPa}$$

$$\overline{C_2 Y} = 50 \text{ MPa}$$

Noting right triangles  $C_1 D Y$  and  $C_2 D Y$

$$\overline{C_1 D}^2 + \overline{D Y}^2 = \overline{C_1 Y}^2 \quad \overline{C_2 D}^2 = 40^2 = 50^2 \quad \overline{C_1 D} = 30$$

Coordinates of point  $C_1$  are  $(0, 75 - 30) = (0, 45 \text{ MPa})$

Likewise, coordinates of point  $C_2$  are  $= (0, 75 + 30) = (0, 105 \text{ MPa})$

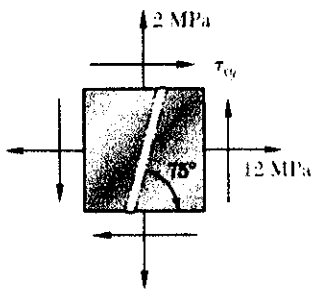
Coordinates of point  $X_1$   $(45 - 30, -40) = (15 \text{ MPa}, -40 \text{ MPa})$

Coordinates of point  $X_2$   $(105 + 30, -40) = (135 \text{ MPa}, -40 \text{ MPa})$

The point  $(\sigma_x, -\tau_{xy})$  must lie on the line  $X_1 X_2$

Thus  $15 \text{ MPa} \leq \sigma_x \leq 135 \text{ MPa}$

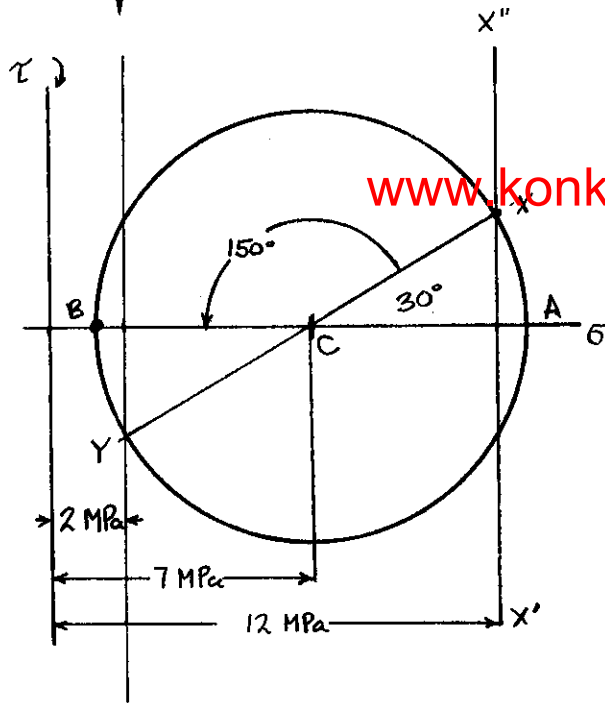
PROBLEM 7.52



7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 For the state of plane stress shown, determine (a) the value of  $\tau_{xy}$  for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.

SOLUTION



Point X of Mohr's circle must lie on  $X'X''$  so that  $\sigma_x = 12 \text{ MPa}$ . Likewise, point Y lies on line  $Y'Y''$  so that  $\sigma_y = 2 \text{ MPa}$ . The coordinates of C are  $\frac{2+12}{2}, 0 = (7 \text{ MPa}, 0)$ .

Counterclockwise rotation through  $150^\circ$  brings line CX to CB, where  $\tau = 0$ .

$$R = \frac{\sigma_x - \sigma_y}{2} \sec 30^\circ$$

$$= \frac{12 - 2}{2} \sec 30^\circ$$

$$= 5.77 \text{ MPa}$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2} \tan 30^\circ$$

$$= -\frac{12 - 2}{2} \tan 30^\circ$$

$$= -2.89 \text{ MPa}$$

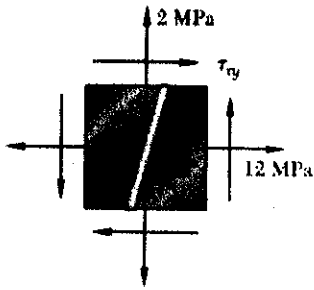
$$\sigma_A = \sigma_{ave} + R = 7 + 5.77 = 12.77 \text{ MPa}$$

$$\sigma_B = \sigma_{ave} - R = 7 - 5.77 = 1.23 \text{ MPa}$$

**PROBLEM 7.53**

7.53 Solve Prob. 7.30, using Mohr's circle and assuming that the weld forms an angle of  $60^\circ$  with the horizontal

7.30 For the state of plane stress shown, determine (a) the value of  $\tau_{xy}$  for which the in-plane shearing stress parallel to the weld is zero, (b) the corresponding principal stresses.

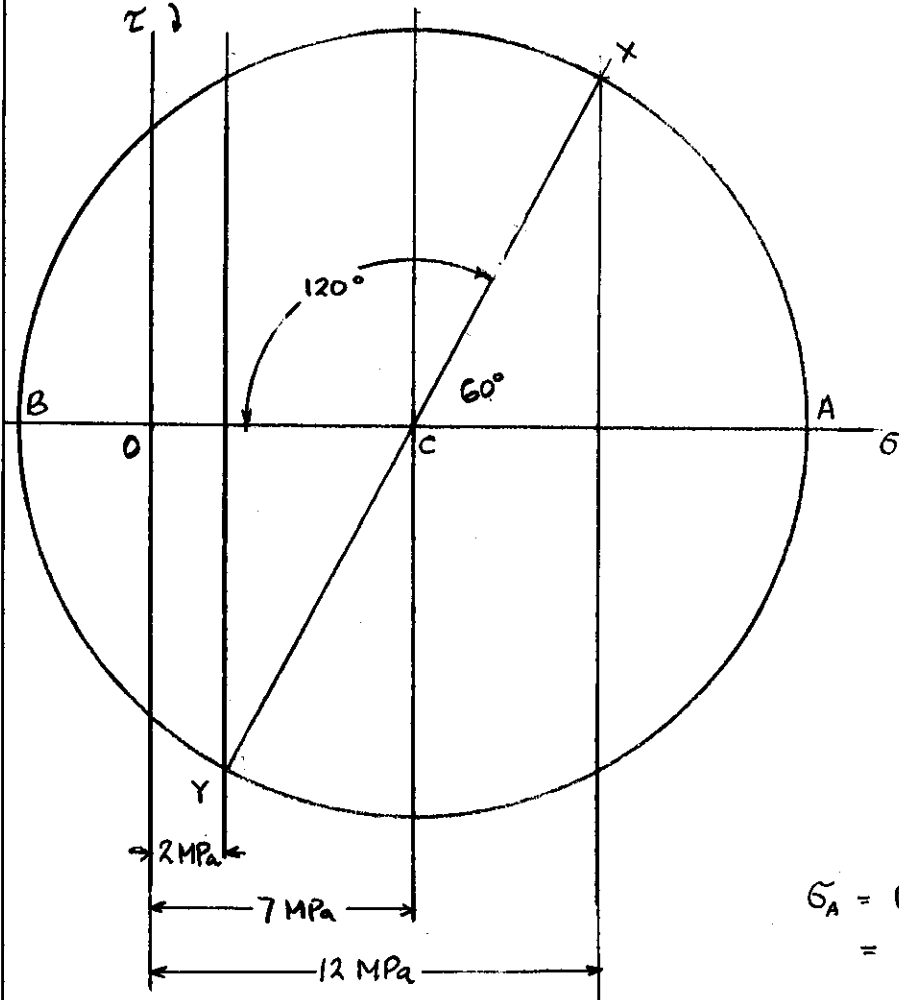


**SOLUTION**

Locate point C  
at  $\sigma = \frac{12+2}{2} = 7 \text{ MPa}$   
with  $\tau = 0$ .

Angle  $XCB = 120^\circ$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12 - 2}{2} = 5 \text{ MPa}$$



$$R = 5 \sec 60^\circ = 10 \text{ MPa}$$

$$\tau_{xy} = -5 \tan 60^\circ = -8.66 \text{ MPa}$$

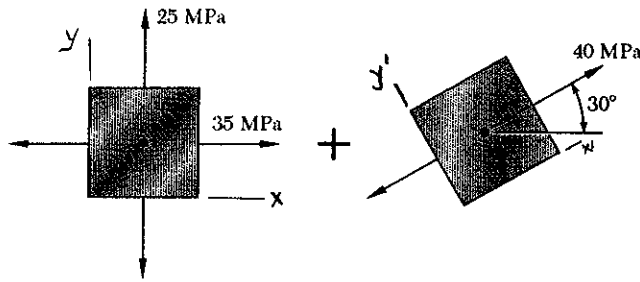
$$\sigma_A = \sigma_{ave} + R = 7 + 10 = 17 \text{ MPa}$$

$$\sigma_B = \sigma_{ave} - R = 7 - 10 = -3 \text{ MPa}$$



**PROBLEM 7.54**

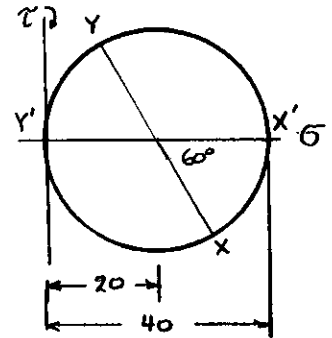
7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



**SOLUTION**

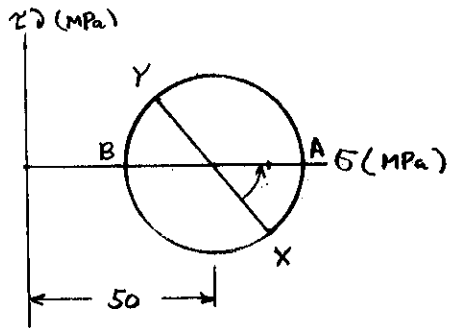
Mohr's circle for 2nd stress state

$$\begin{aligned} \sigma_x &= 20 + 20 \cos 60^\circ \\ &= 30 \text{ MPa} \\ \sigma_y &= 20 - 20 \cos 60^\circ \\ &= 10 \text{ MPa} \\ \tau_{xy} &= 20 \sin 60^\circ \\ &= 17.32 \text{ MPa} \end{aligned}$$



Resultant stresses

$$\begin{aligned} \sigma_x &= 35 + 30 = 65 \text{ MPa} \\ \sigma_y &= 25 + 10 = 35 \text{ MPa} \\ \tau_{xy} &= 0 + 17.32 = 17.32 \text{ MPa} \end{aligned}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(65 + 35) = 50 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(17.32)}{65 - 35} = 1.1547$$

$$2\theta_p = 49.11^\circ \quad \theta_a = 24.6^\circ \quad \theta_b = 114.6^\circ$$

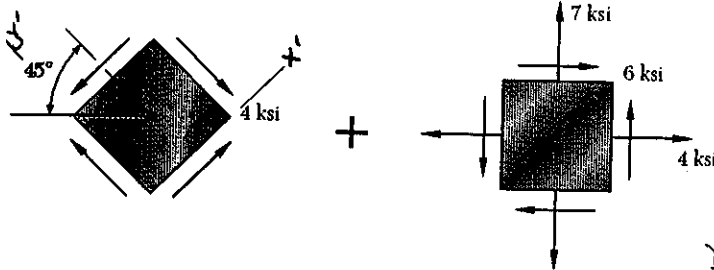
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22.91 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 72.91 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 27.09 \text{ MPa}$$

PROBLEM 7.55

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

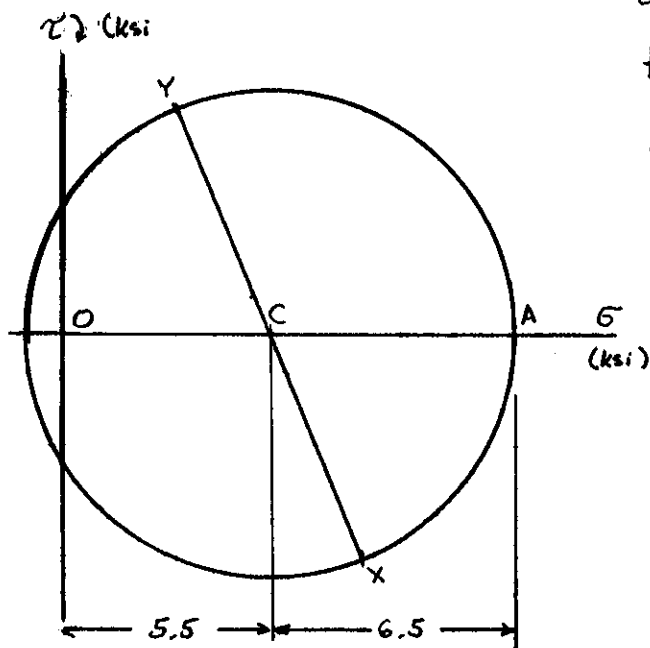


Resultant stresses

$$\sigma_x = 4 + 4 = 8 \text{ ksi}$$

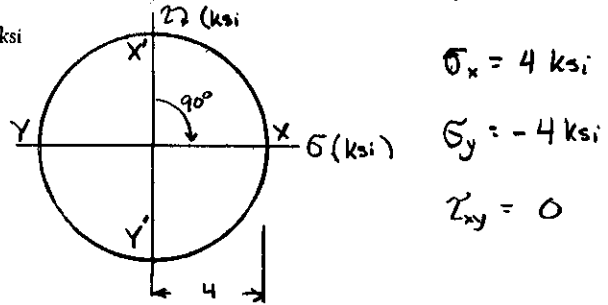
$$\sigma_y = -4 + 7 = 3 \text{ ksi}$$

$$\tau_{xy} = 6 + 0 = 6 \text{ ksi}$$



SOLUTION

Mohr's circle for 1st stress state.



$$\sigma_x = 4 \text{ ksi}$$

$$\sigma_y = -4 \text{ ksi}$$

$$\tau_{xy} = 0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{5} = 2.4$$

$$2\theta_p = 67.38^\circ \quad \theta_a = 33.69^\circ$$

$$\theta_b = 123.69^\circ$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

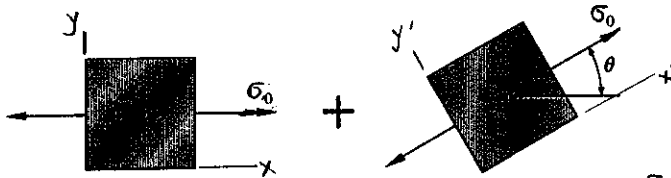
$$= \sqrt{2.5^2 + 6^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 12 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -1 \text{ ksi}$$

PROBLEM 7.56

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



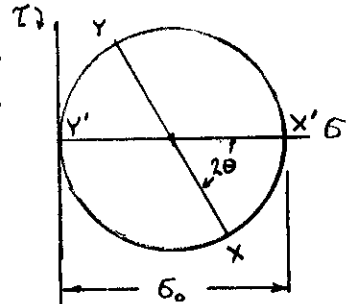
SOLUTION

Mohr's circle for 2nd stress state

$$\sigma_x = \frac{1}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\sigma_y = \frac{1}{2} \sigma_0 - \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\tau_{xy} = \frac{1}{2} \sigma_0 \sin 2\theta$$



Resultant stresses

$$\sigma_x = \sigma_0 + \frac{1}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta = \frac{3}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\sigma_y = 0 + \frac{1}{2} \sigma_0 - \frac{1}{2} \sigma_0 \cos 2\theta = \frac{1}{2} \sigma_0 - \frac{1}{2} \sigma_0 \cos 2\theta$$

$$\tau_{xy} = 0 + \frac{1}{2} \sigma_0 \sin 2\theta = \frac{1}{2} \sigma_0 \sin 2\theta$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = \sigma_0$$

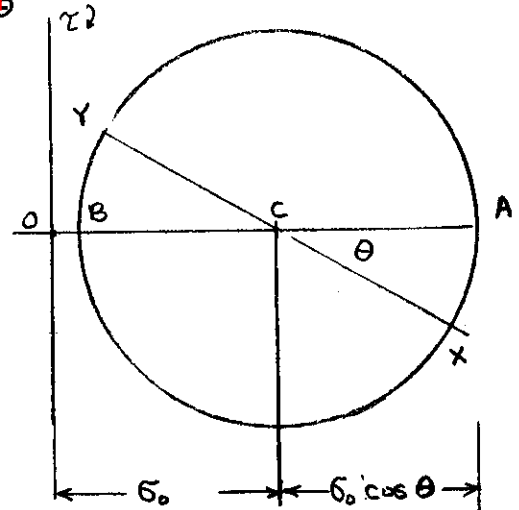
$$\begin{aligned} \tan 2\theta_p &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta} \\ &= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta \end{aligned}$$

$$\theta_p = \frac{1}{2} \theta$$

$$\begin{aligned} R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \\ &= \sqrt{\left(\frac{1}{2} \sigma_0 + \frac{1}{2} \sigma_0 \cos 2\theta\right)^2 + \left(\frac{1}{2} \sigma_0 \sin 2\theta\right)^2} \\ &= \frac{1}{2} \sigma_0 \sqrt{1 + 2 \cos 2\theta + \cos^2 2\theta + \sin^2 2\theta} \\ &= \frac{\sqrt{2}}{2} \sigma_0 \sqrt{1 + \cos 2\theta} = \sigma_0 |\cos \theta| \end{aligned}$$

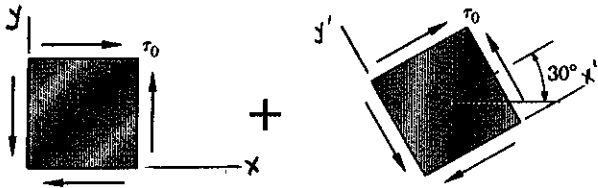
$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0 \cos \theta \quad \blacktriangleleft$$

$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0 \cos \theta \quad \blacktriangleleft$$



PROBLEM 7.57

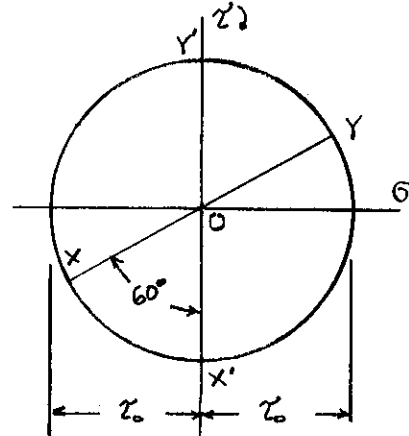
7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



SOLUTION

Mohr's circle for 2nd state of stress

$$\begin{aligned}\sigma_{x'} &= 0 \\ \sigma_{y'} &= 0 \\ \tau_{x'y'} &= \tau_0\end{aligned}$$



$$\begin{aligned}\sigma_x &= -\tau_0 \sin 60^\circ = -\frac{\sqrt{3}}{2}\tau_0 \\ \sigma_y &= \tau_0 \sin 60^\circ = \frac{\sqrt{3}}{2}\tau_0 \\ \tau_{xy} &= \tau_0 \cos 60^\circ = \frac{1}{2}\tau_0\end{aligned}$$

Resultant stresses

$$\begin{aligned}\sigma_x &= 0 - \frac{\sqrt{3}}{2}\tau_0 = -\frac{\sqrt{3}}{2}\tau_0 \\ \sigma_y &= 0 + \frac{\sqrt{3}}{2}\tau_0 = \frac{\sqrt{3}}{2}\tau_0 \\ \tau_{xy} &= \tau_0 + \frac{1}{2}\tau_0 = \frac{3}{2}\tau_0\end{aligned}$$

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$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

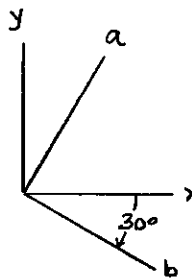
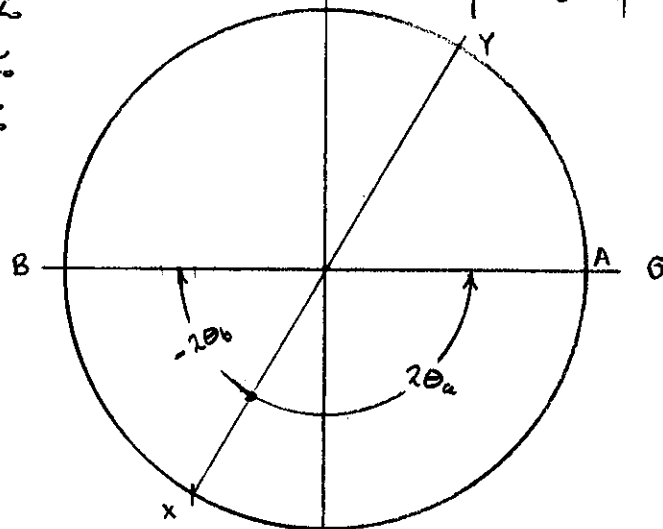
$$\begin{aligned}R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{\left(\frac{\sqrt{3}\tau_0}{2}\right)^2 + \left(\frac{3}{2}\tau_0\right)^2} \\ &= \sqrt{3}\tau_0\end{aligned}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \cdot \frac{3}{2}\tau_0}{-\sqrt{3}\tau_0} = -\sqrt{3}$$

$$2\theta_p = -60^\circ \quad \theta_b = -30^\circ \quad \theta_a = 60^\circ$$

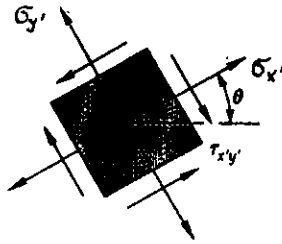
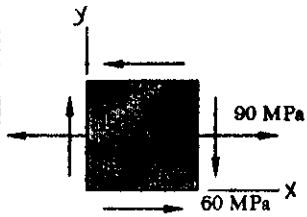
$$\sigma_a = \sigma_{ave} + R = \sqrt{3}\tau_0$$

$$\sigma_b = \sigma_{ave} - R = -\sqrt{3}\tau_0$$



**PROBLEM 7.58**

7.58 For the state of stress shown, determine the range of values of  $\theta$  for which the normal stress  $\sigma_x'$  is equal to or less than 100 MPa.



**SOLUTION**

$$\sigma_x = 90 \text{ MPa}, \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

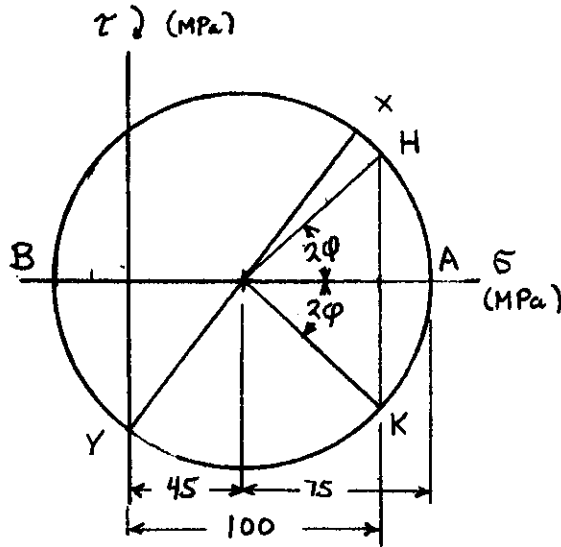
$$2\theta_p = -53.13^\circ$$

$$\theta_a = -26.565^\circ$$

$\sigma_x' \leq 100 \text{ MPa}$  for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$$R \cos 2\phi = 100 - 45 = 55 \text{ MPa}$$

$$\cos 2\phi = \frac{55}{75} = 0.73333$$



$$2\phi = 42.833^\circ \quad \phi = 21.417^\circ$$

$$\theta_H = \theta_a + \phi = -26.565^\circ + 21.417^\circ = -5.15^\circ$$

$$2\theta_K = 2\theta_H + 360^\circ - 4\phi = -10.297^\circ + 360^\circ - 85.666^\circ = 264.037^\circ$$

$$\theta_K = 132.02^\circ$$

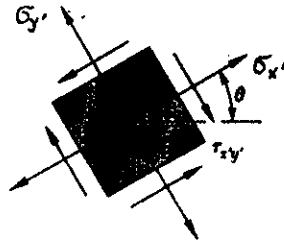
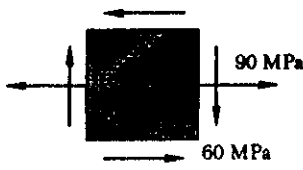
Permissible range of  $\theta$  is  $\theta_H \leq \theta \leq \theta_K$

$$-5.15^\circ \leq \theta \leq 132.02^\circ$$

$$\text{Also } 174.85^\circ \leq \theta \leq 312.02^\circ$$

PROBLEM 7.59

7.59 For the state of stress shown, determine the range of values of  $\theta$  for which the normal stress  $\sigma_x'$  is equal to or less than 50 MPa.



SOLUTION

$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = 0$$

$$\tau_{xy} = -60 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(-60)}{90} = -\frac{4}{3}$$

$$2\theta_p = -53.13^\circ$$

$$\theta_a = -26.565^\circ$$

$\sigma_{x'} \leq 50 \text{ MPa}$  for states of stress corresponding to the arc HBK of Mohr's circle. From the circle

$$R \cos 2\phi = 50 - 45 = 5 \text{ MPa}$$

$$\cos 2\phi = \frac{5}{75} = 0.066667$$

$$2\phi = 86.177^\circ \quad \phi = 43.089^\circ$$

$$\theta_H = \theta_a + \phi = -26.565^\circ + 43.089^\circ = 16.524^\circ$$

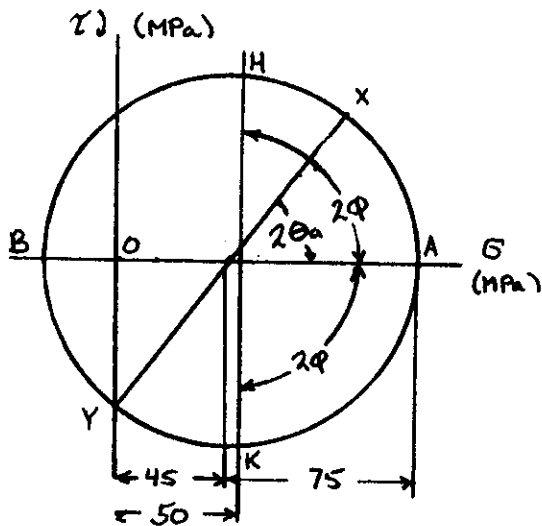
$$2\theta_K = 2\theta_H + 360^\circ - 4\phi = 32.524^\circ + 360^\circ - 172.355^\circ = 220.169^\circ$$

$$\theta_K = 110.085^\circ$$

Permissible range of  $\theta$  is  $\theta_H \leq \theta \leq \theta_K$

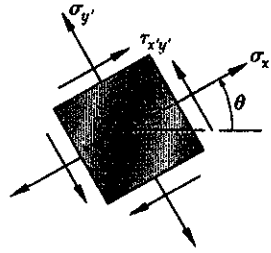
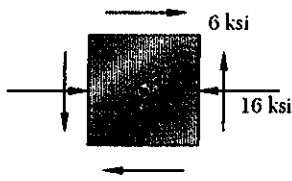
$$16.524^\circ \leq \theta \leq 110.085^\circ$$

Also  $196.524^\circ \leq \theta \leq 290.085^\circ$



PROBLEM 7.60

7.60 For the state of stress shown, determine the range of values of  $\theta$  for which the magnitude of the shearing stress  $\tau_{xy}$  is equal to or less than 8 ksi.



SOLUTION

$$\sigma_x = -16 \text{ ksi}, \quad \sigma_y = 0$$

$$\tau_{xy} = 6 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -8 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-8)^2 + (6)^2} = 10 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{-16} = -0.75$$

$$2\theta_p = -36.870^\circ$$

$$\theta_b = -18.435^\circ$$

$|\tau_{xy}| \leq 8 \text{ ksi}$  for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle  $\phi$  is calculated from

$$R \sin 2\phi = 8$$

$$\sin 2\phi = \frac{8}{10} = 0.8$$

$$2\phi = 53.130^\circ \quad \phi = 26.565^\circ$$

$$\theta_H = \theta_b - \phi = -18.435^\circ - 26.565^\circ = -45^\circ$$

$$\theta_K = \theta_b + \phi = -18.435^\circ + 26.565^\circ = 8.13^\circ$$

$$\theta_U = \theta_H + 90^\circ = 45^\circ$$

$$\theta_V = \theta_K + 90^\circ = 98.13^\circ$$

Permissible ranges of  $\theta$

$$\theta_H \leq \theta \leq \theta_K$$

$$-45^\circ \leq \theta \leq 8.13^\circ$$

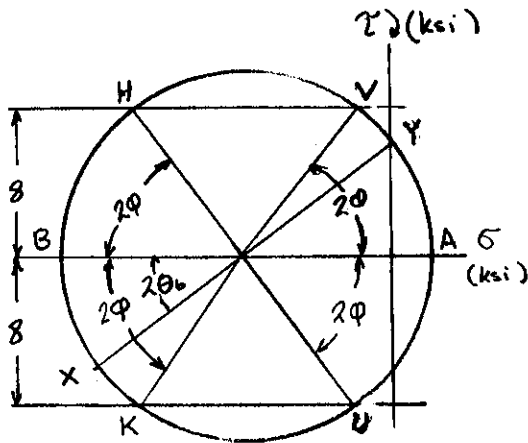
$$\theta_U \leq \theta \leq \theta_V$$

$$45^\circ \leq \theta \leq 98.13^\circ$$

Also

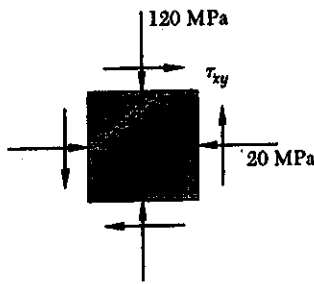
$$135^\circ \leq \theta \leq 188.13^\circ$$

$$225^\circ \leq \theta \leq 278.13^\circ$$



PROBLEM 7.61

7.61 For the element shown, determine the range of values of  $\tau_{xy}$  for which the maximum tensile stress is equal to or less than 60 MPa.



SOLUTION

$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -70 \text{ MPa}$$

$$\text{Set } \sigma_{max} = 60 \text{ MPa} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 130 \text{ MPa}$$

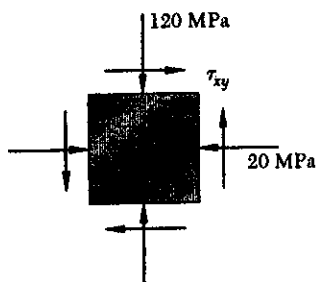
$$\text{But } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{130^2 - 50^2} = 120 \text{ MPa}$$

$$\text{Range of } \tau_{xy} \quad -120 \text{ MPa} \leq \tau_{xy} \leq 120 \text{ MPa}$$

PROBLEM 7.62

7.62 For the element shown, determine the range of values of  $\tau_{xy}$  for which the maximum in plane shearing stress is equal to or less than 150 MPa.



SOLUTION

$$\sigma_x = -20 \text{ MPa} \quad \sigma_y = -120 \text{ MPa}$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = 50 \text{ MPa}$$

$$\text{Set } \tau_{max(in-plane)} = R = 150 \text{ MPa}$$

$$\text{But } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

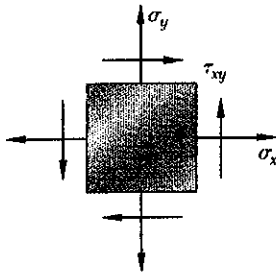
$$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{150^2 - 50^2} = 141.4 \text{ MPa}$$

$$\text{Range of } \tau_{xy} \quad -141.4 \text{ MPa} \leq \tau_{xy} \leq 141.4 \text{ MPa}$$



PROBLEM 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that  $\sigma_x = 14$  ksi,  $\sigma_y = 9$  ksi, and  $\sigma_{min} = 5$  ksi. Determine (a) the orientation of the principal planes, (b) the principal stress  $\sigma_{max}$ , (c) the maximum in-plane shearing stress.



SOLUTION

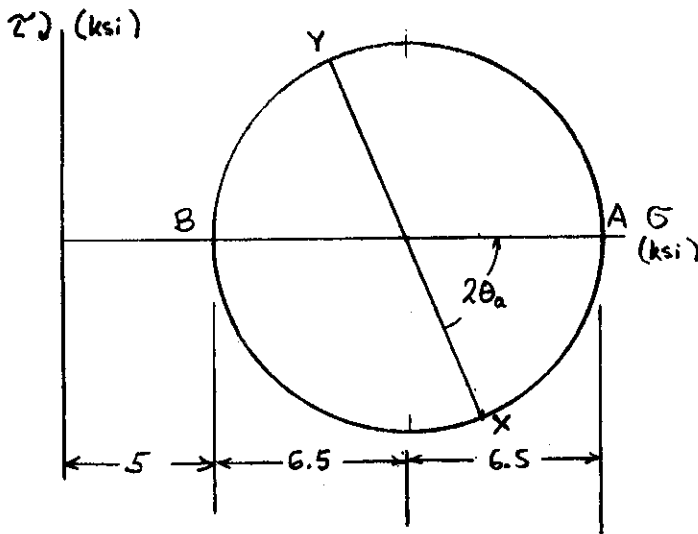
$$\sigma_x = 14 \text{ ksi}, \sigma_y = 9 \text{ ksi}, \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.5 \text{ ksi}$$

$$\sigma_{min} = \sigma_{ave} - R \therefore R = \sigma_{ave} - \sigma_{min} = 11.5 - 5 = 6.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm \sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$

But it is given that  $\tau_{xy}$  is positive, thus  $\tau_{xy} = +6$  ksi



$$(a) \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{5} = 2.4$$

$$2\theta_p = 67.38^\circ$$

$$\theta_a = 33.69^\circ$$

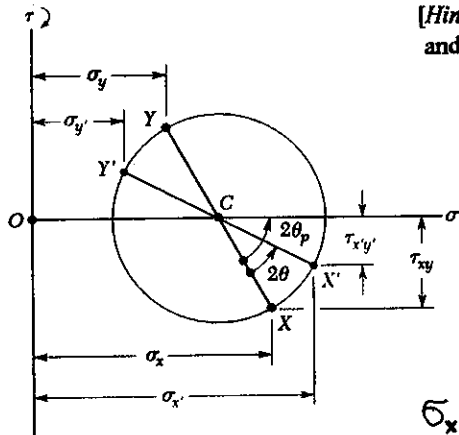
$$\theta_b = 123.69^\circ$$

$$(b) \sigma_{max} = \sigma_{ave} + R = 18 \text{ ksi}$$

$$(c) \tau_{max(in-plane)} = R = 6.5 \text{ ksi}$$

PROBLEM 7.64

7.64 The Mohr circle shown corresponds to the state of stress given in Fig. *xxa* and *b*, page *yyy*. Noting that  $\sigma_x = OC + (CX') \cos(2\theta_p - 2\theta)$  and that  $\tau_{xy} = (CX') \sin(2\theta_p - 2\theta)$ , derive the expressions for  $\sigma_x$  and  $\tau_{xy}$  given in Eqs. (7.5) and (7.6), respectively. [Hint: Use  $\sin(A+B) = \sin A \cos B + \cos A \sin B$  and  $\cos(A+B) = \cos A \cos B - \sin A \sin B$ .]



SOLUTION

$$\begin{aligned} \overline{OC} &= \frac{1}{2}(\sigma_x + \sigma_y) & \overline{CX'} &= \overline{CX} \\ \overline{CX'} \cos 2\theta_p &= \overline{CX} \cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2} \\ \overline{CX'} \sin 2\theta_p &= \overline{CX} \sin 2\theta_p = \tau_{xy} \end{aligned}$$

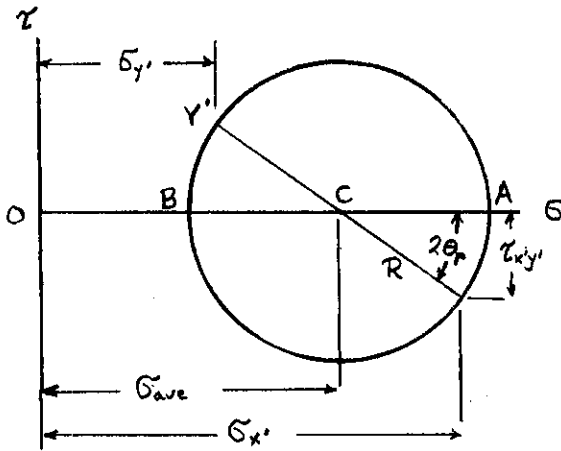
$$\begin{aligned} \sigma_x &= \overline{OC} + \overline{CX'} \cos(2\theta_p - 2\theta) \\ &= \overline{OC} + \overline{CX'} (\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta) \\ &= \overline{OC} + \overline{CX'} \cos 2\theta_p \cos 2\theta + \overline{CX'} \sin 2\theta_p \sin 2\theta \\ &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \end{aligned}$$

$$\begin{aligned} \tau_{x'y'} &= \overline{CX'} \sin(2\theta_p - 2\theta) = \overline{CX'} (\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta) \\ &= \overline{CX'} \sin 2\theta_p \cos 2\theta - \overline{CX'} \cos 2\theta_p \sin 2\theta \\ &= \tau_{xy} \cos 2\theta - \frac{\sigma_x - \sigma_y}{2} \sin 2\theta \end{aligned}$$

PROBLEM 7.65

7.65 (a) Prove that the expression  $\sigma_x \sigma_y - \tau_{xy}^2$ , where  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  are components of stress along the rectangular axes  $x'$  and  $y'$ , is independent of the orientation of these axes. Also, show that the given expression Mohr's circle represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (b) Using the invariance property established in part a, express the shearing stress  $\tau_{xy}$  in terms of  $\sigma_x$ ,  $\sigma_y$ , and the principal stresses  $\sigma_{max}$  and  $\sigma_{min}$ .

SOLUTION



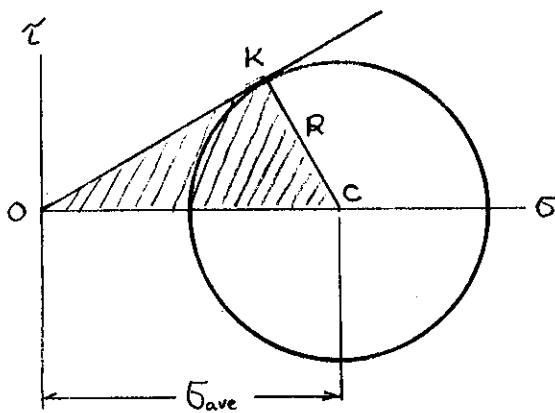
(a) From Mohr's circle

$$\tau_{xy'} = R \sin 2\theta_p$$

$$\sigma_{x'} = \sigma_{ave} + R \cos 2\theta_p$$

$$\sigma_{y'} = \sigma_{ave} - R \cos 2\theta_p$$

$$\begin{aligned} \sigma_{x'} \sigma_{y'} - \tau_{xy'}^2 &= \sigma_{ave}^2 - R^2 \cos^2 2\theta_p - R^2 \sin^2 2\theta_p \\ &= \sigma_{ave}^2 - R^2; \text{ independent of } \theta_p. \end{aligned}$$



Draw line  $\overline{OK}$  from origin tangent to the circle at  $K$ . Triangle  $OCK$  is a right triangle

$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$

$$\begin{aligned} \overline{OK}^2 &= \overline{OC}^2 - \overline{CK}^2 \\ &= \sigma_{ave}^2 - R^2 \\ &= \sigma_x \sigma_y - \tau_{xy}^2 \end{aligned}$$

(b) Applying above to  $\sigma_x, \sigma_y$ , and  $\tau_{xy}$  and to  $\sigma_a, \sigma_b$ ,

$$\sigma_x \sigma_y - \tau_{xy}^2 = \sigma_a \sigma_b - \tau_{ab}^2 = \sigma_{ave}^2 - R^2$$

But  $\tau_{ab} = 0$ ,  $\sigma_a = \sigma_{max}$ ,  $\sigma_b = \sigma_{min}$

$$\sigma_x \sigma_y - \tau_{xy}^2 = \sigma_{max} \sigma_{min}$$

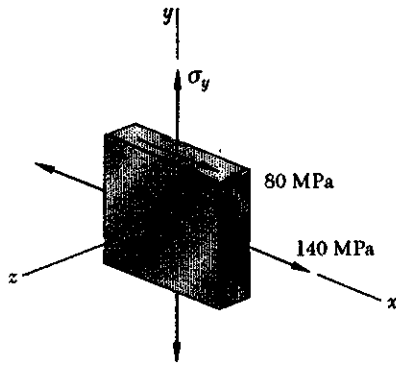
$$\tau_{xy}^2 = \sigma_x \sigma_y - \sigma_{max} \sigma_{min}$$

$$\tau_{xy} = \pm \sqrt{\sigma_x \sigma_y - \sigma_{max} \sigma_{min}}$$

The sign cannot be determined from above equations.

PROBLEM 7.66

7.66 For the state of plane stress shown, determine the maximum shearing stress when (a)  $\sigma_x = 20$  MPa, (b)  $\sigma_x = 140$  MPa. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



SOLUTION

(a)  $\sigma_x = 140$  MPa,  $\sigma_y = 20$  MPa  
 $\tau_{xy} = 80$  MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

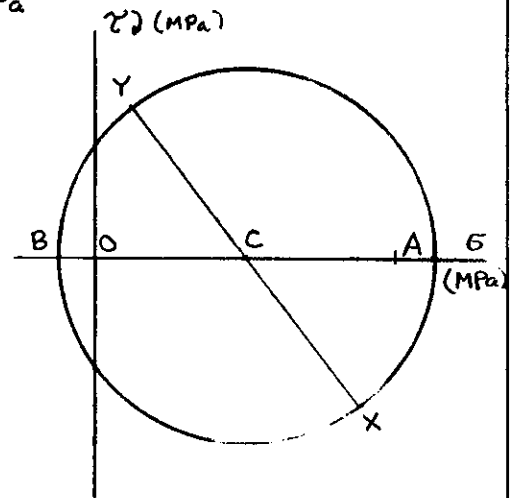
$$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180 \text{ MPa (max)}$$

$$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20 \text{ MPa (min)}$$

$$\sigma_c = 0$$

$$\tau_{max(in-plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 100 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 100 \text{ MPa} \quad \blacktriangleleft$$



(b)  $\sigma_x = 140$  MPa,  $\sigma_y = 140$  MPa

$$\tau_{xy} = 80 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 140 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$

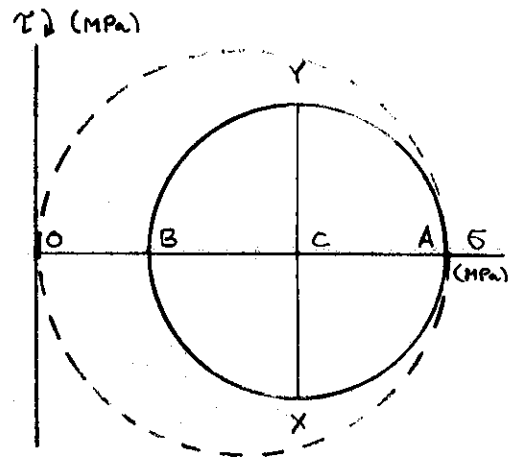
$$\sigma_a = \sigma_{ave} + R = 220 \text{ MPa (max)}$$

$$\sigma_b = \sigma_{ave} - R = 60 \text{ MPa}$$

$$\sigma_c = 0 \quad (\text{min})$$

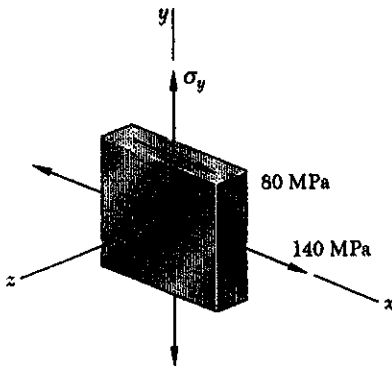
$$\tau_{max(in-plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = 80 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.67

7.67 For the state of plane stress shown, determine the maximum shearing stress when (a)  $\sigma_y = 40$  MPa, (b)  $\sigma_y = 120$  MPa. (Hint: Consider both in-plane and out-of-plane shearing stresses.)



SOLUTION

(a)  $\sigma_x = 140$  MPa     $\sigma_y = 40$  MPa     $\tau_{xy} = 80$  MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$

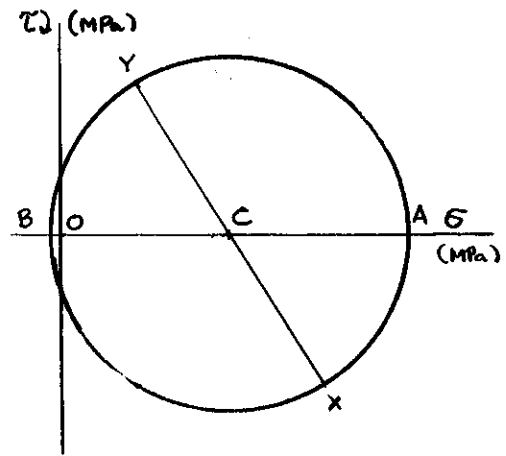
$$\sigma_a = \sigma_{ave} + R = 184.34 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = -4.34 \text{ MPa} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\tau_{max(\text{in-plane})} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.34 \text{ MPa} \quad \blacktriangleleft$$



(b)  $\sigma_x = 140$  MPa,  $\sigma_y = 120$  MPa,  $\tau_{xy} = 80$  MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 210.62 \text{ MPa} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = 49.38 \text{ MPa}$$

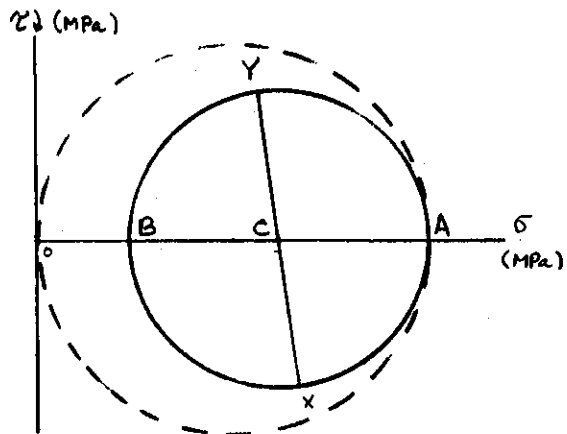
$$\sigma_c = 0 \quad (\text{min})$$

$$\sigma_{max} = \sigma_a = 210.62 \text{ MPa}$$

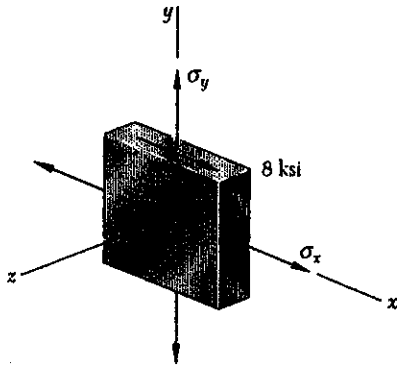
$$\sigma_{min} = \sigma_c = 0$$

$$\tau_{max(\text{in-plane})} = R = 80.62 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 105.31 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.68



7.68 For the state of plane stress shown, determine the maximum shearing stress when (a)  $\sigma_x = 6$  ksi and  $\sigma_y = 18$  ksi, (b)  $\sigma_x = 14$  ksi and  $\sigma_y = 2$  ksi. (Hint: Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(a)  $\sigma_x = 6$  ksi     $\sigma_y = 18$  ksi     $\tau_{xy} = 8$  ksi

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + 8^2} = 10 \text{ ksi}$$

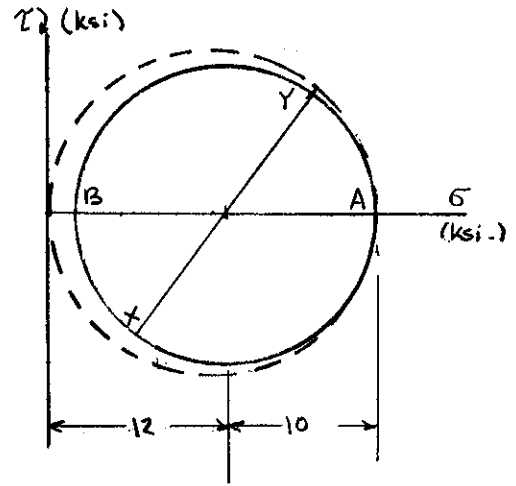
$$\sigma_a = \sigma_{ave} + R = 12 + 10 = 22 \text{ ksi (max)}$$

$$\sigma_b = \sigma_{ave} - R = 12 - 10 = 2 \text{ ksi}$$

$$\sigma_c = 0 \text{ (min)}$$

$$\tau_{max(in-plane)} = R = 10 \text{ ksi}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11 \text{ ksi} \quad \blacktriangleleft$$



(b)  $\sigma_x = 14$  ksi     $\sigma_y = 2$  ksi     $\tau_{xy} = 8$  ksi

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + 8^2} = 10 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 18 \text{ ksi (max)}$$

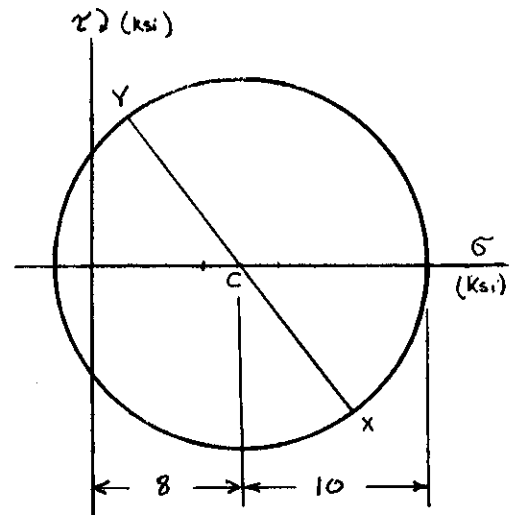
$$\sigma_b = \sigma_{ave} - R = -2 \text{ ksi (min)}$$

$$\sigma_c = 0$$

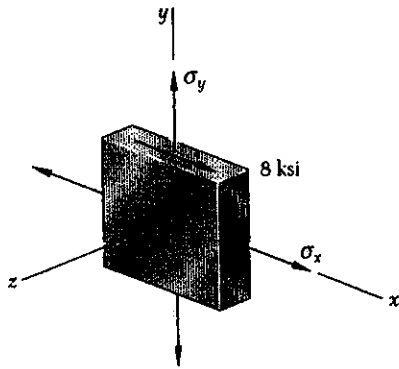
$$\sigma_{max} = 18 \text{ ksi}$$

$$\sigma_{min} = -2 \text{ ksi}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 7.69



7.69 For the state of plane stress shown, determine the maximum shearing stress when (a)  $\sigma_x = 0$  and  $\sigma_y = 12$  ksi, (b)  $\sigma_x = 21$  ksi and  $\sigma_y = 9$  ksi. (Hint: Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(a)  $\sigma_x = 0$ ,  $\sigma_y = 12$  ksi,  $\tau_{xy} = 8$  ksi

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 6 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-6)^2 + 8^2} = 10 \text{ ksi}$$

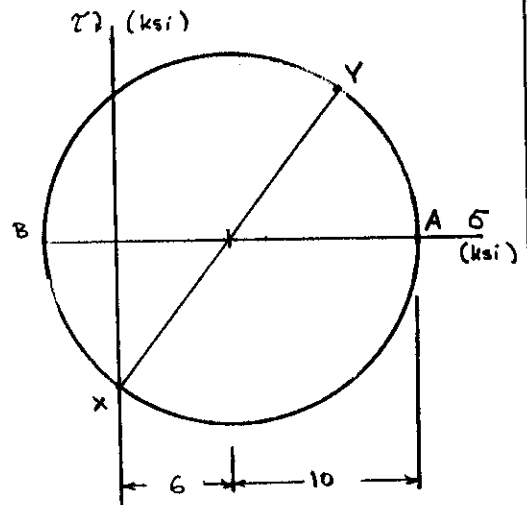
$$\sigma_a = \sigma_{ave} + R = 16 \text{ ksi} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = -4 \text{ ksi} \quad (\text{min})$$

$$\sigma_c = 0$$

$$\sigma_{max} = 16 \text{ ksi} \quad \sigma_{min} = -4 \text{ ksi}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10 \text{ ksi} \quad \blacktriangleleft$$



(b)  $\sigma_x = 21$  ksi,  $\sigma_y = 9$  ksi,  $\tau_{xy} = 8$  ksi

$$\sigma_{ave} = 15 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(-6)^2 + 8^2} = 10 \text{ ksi}$$

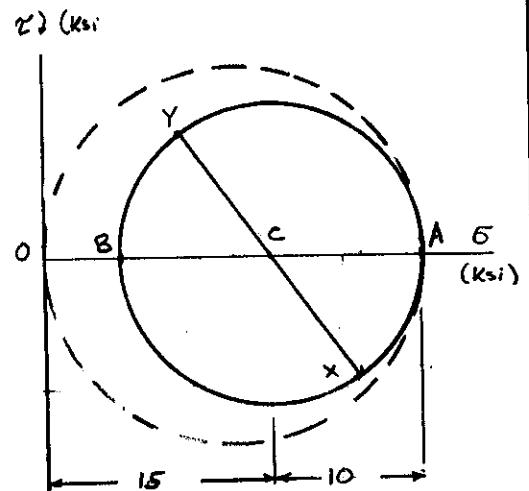
$$\sigma_a = \sigma_{ave} + R = 25 \text{ ksi} \quad (\text{max})$$

$$\sigma_b = \sigma_{ave} - R = 5 \text{ ksi}$$

$$\sigma_c = 0 \quad (\text{min})$$

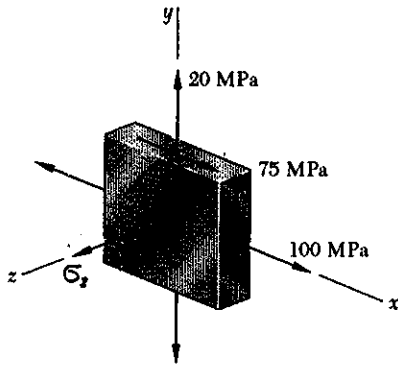
$$\sigma_{max} = 25 \text{ ksi}, \quad \sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 12.5 \text{ ksi} \quad \blacktriangleleft$$



PROBLEM 7.70

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a)  $\sigma_z = 0$ , (b)  $\sigma_z = +45$  MPa, (c)  $\sigma_z = -45$  MPa.



SOLUTION

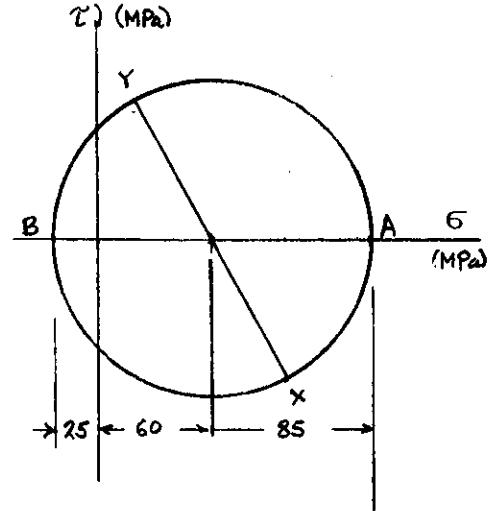
$$\sigma_x = 100 \text{ MPa}, \quad \sigma_y = 20 \text{ MPa}, \quad \tau_{xy} = 75 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 60 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$



(a)  $\sigma_z = 0, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{max} = 145 \text{ MPa}, \quad \sigma_{min} = -25 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85 \text{ MPa} \quad \blacktriangleleft$$

(b)  $\sigma_z = +45 \text{ MPa}, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{max} = 145 \text{ MPa}, \quad \sigma_{min} = -25 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85 \text{ MPa} \quad \blacktriangleleft$$

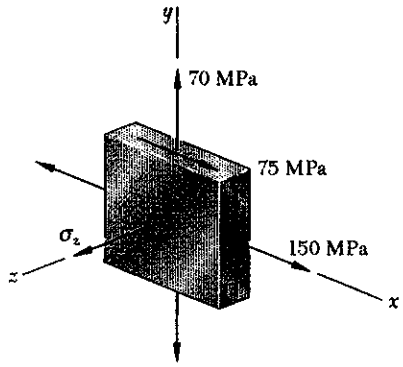
(c)  $\sigma_z = -45 \text{ MPa}, \quad \sigma_a = 145 \text{ MPa}, \quad \sigma_b = -25 \text{ MPa}$

$$\sigma_{max} = 145 \text{ MPa}, \quad \sigma_{min} = -45 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 95 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a)  $\sigma_z = 0$ , (b)  $\sigma_z = +45$  MPa, (c)  $\sigma_z = -45$  MPa.



SOLUTION

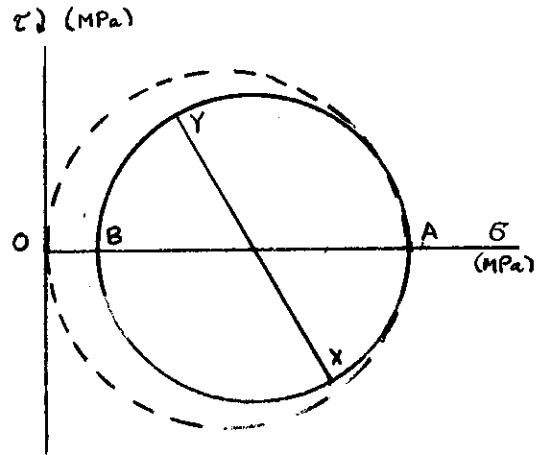
$$\sigma_x = 150 \text{ MPa}, \quad \sigma_y = 70 \text{ MPa}, \quad \tau_{xy} = 75 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 110 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 195 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 25 \text{ MPa}$$



(a)  $\sigma_z = 0$ ,  $\sigma_a = 195$  MPa,  $\sigma_b = 25$  MPa

$$\sigma_{max} = 195 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 97.5 \text{ MPa} \quad \blacktriangleleft$$

(b)  $\sigma_z = +45$  MPa,  $\sigma_a = 195$  MPa,  $\sigma_b = 25$  MPa

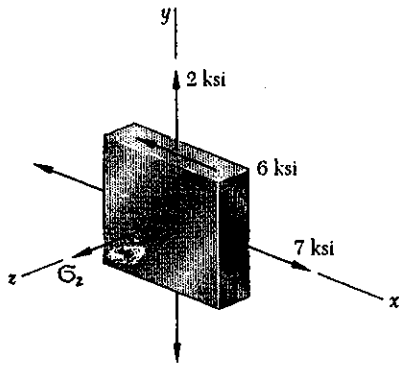
$$\sigma_{max} = 195 \text{ MPa}, \quad \sigma_{min} = 25 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85 \text{ MPa} \quad \blacktriangleleft$$

(c)  $\sigma_z = -45$  MPa,  $\sigma_a = 195$  MPa,  $\sigma_b = 25$  MPa

$$\sigma_{max} = 195 \text{ MPa}, \quad \sigma_{min} = -45 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 120 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.72

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a)  $\sigma_z = +4$  ksi, (b)  $\sigma_z = -4$  ksi, (c)  $\sigma_z = 0$ .



SOLUTION

$$\sigma_x = 7 \text{ ksi}, \quad \sigma_y = 2 \text{ ksi}, \quad \tau_{xy} = -6 \text{ ksi}$$

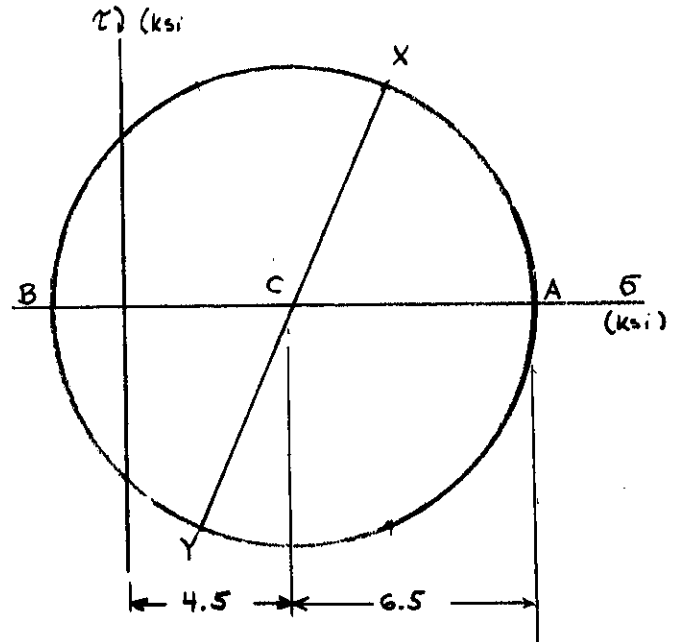
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 11 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$$



(a)  $\sigma_z = 4$  ksi,  $\sigma_a = 11$  ksi,  $\sigma_b = -2$  ksi

$$\sigma_{max} = 11 \text{ ksi}, \quad \sigma_{min} = -2 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5 \text{ ksi}$$

(b)  $\sigma_z = -4$  ksi,  $\sigma_a = 11$  ksi,  $\sigma_b = -2$  ksi

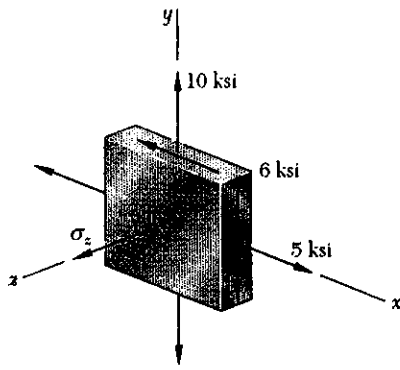
$$\sigma_{max} = 11 \text{ ksi}, \quad \sigma_{min} = -4 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5 \text{ ksi}$$

(c)  $\sigma_z = 0$ ,  $\sigma_a = 11$  ksi,  $\sigma_b = -2$  ksi

$$\sigma_{max} = 11 \text{ ksi}, \quad \sigma_{min} = -2 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5 \text{ ksi}$$

PROBLEM 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a)  $\sigma_z = +4$  ksi, (b)  $\sigma_z = -4$  ksi, (c)  $\sigma_z = 0$ .



SOLUTION

$$\sigma_x = 5 \text{ ksi}, \quad \sigma_y = 10 \text{ ksi}, \quad \tau_{xy} = -6 \text{ ksi}$$

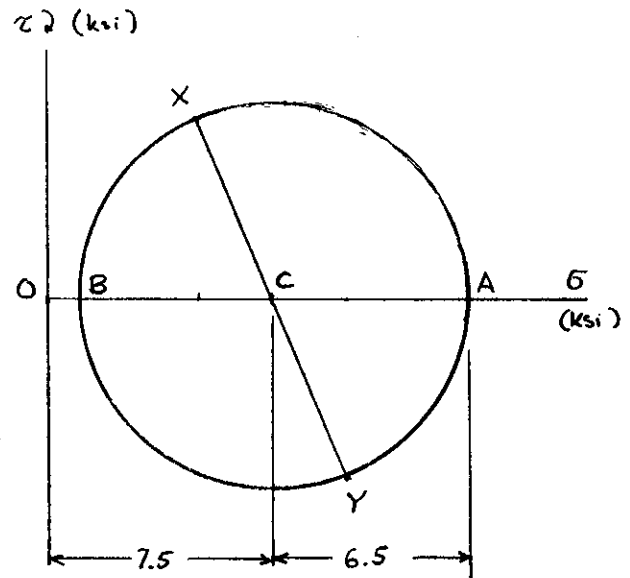
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-2.5)^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 14 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = 1 \text{ ksi}$$



(a)  $\sigma_z = +4$  ksi,  $\sigma_a = 14$  ksi,  $\sigma_b = 1$  ksi

$$\sigma_{max} = 14 \text{ ksi}, \quad \sigma_{min} = 1 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5 \text{ ksi} \quad \blacktriangleleft$$

(b)  $\sigma_z = -4$  ksi,  $\sigma_a = 14$  ksi,  $\sigma_b = 1$  ksi

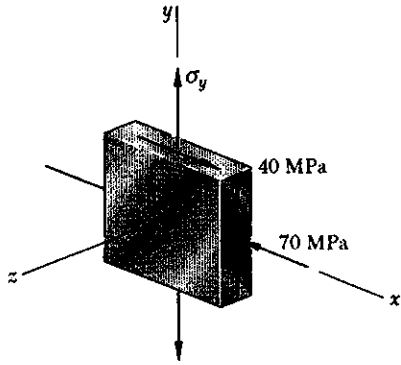
$$\sigma_{max} = 14 \text{ ksi}, \quad \sigma_{min} = -4 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 9 \text{ ksi} \quad \blacktriangleleft$$

(c)  $\sigma_z = 0$ ,  $\sigma_a = 14$  ksi,  $\sigma_b = 1$  ksi

$$\sigma_{max} = 14 \text{ ksi}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 7.74

7.74 For the state of stress shown, determine two values of  $\sigma_x$  for which the maximum shearing stress is 75 MPa.



SOLUTION

$$\sigma_x = -70 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa}$$

$$\text{Let } U = \frac{\sigma_y - \sigma_x}{2} \quad \sigma_y = 2U + \sigma_x$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + U$$

$$R = \sqrt{U^2 + \tau_{xy}^2} \quad U = \pm \sqrt{R^2 - \tau_{xy}^2}$$

Case 1  $\tau_{max} = R = 75 \text{ MPa}, \quad U = \pm \sqrt{75^2 - 40^2} = \pm 63.44 \text{ MPa}$

(1a)  $U = +63.44 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = 56.88 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 68.44 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -81.56 \text{ MPa}$$

$$\sigma_c = 0 \quad \sigma_{max} = 68.44 \text{ MPa}, \quad \sigma_{min} = -81.56 \text{ MPa} \quad \tau_{max} = 75 \text{ MPa}$$

(1b)  $U = -63.44 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = -196.88 \text{ MPa}$  (reject)

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44 \text{ MPa} \quad \sigma_a = \sigma_{ave} + R = -58.44 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -208.44 \text{ MPa}, \quad \sigma_c = 0, \quad \sigma_{max} = 0$$

$$\sigma_{min} = -208.44 \text{ MPa}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22 \text{ MPa} \neq 75 \text{ MPa}$$

Case (2) Assume  $\sigma_{max} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75 \text{ MPa}$

$$\sigma_{min} = -150 \text{ MPa} = \sigma_b$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x + U - \sqrt{U^2 + \tau_{xy}^2}$$

$$\sqrt{U^2 + \tau_{xy}^2} = -\sigma_x + U - \sigma_b$$

$$U^2 + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)U + U^2$$

$$2U = \frac{\tau_{xy}^2 + (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} = \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -160 \text{ MPa}$$

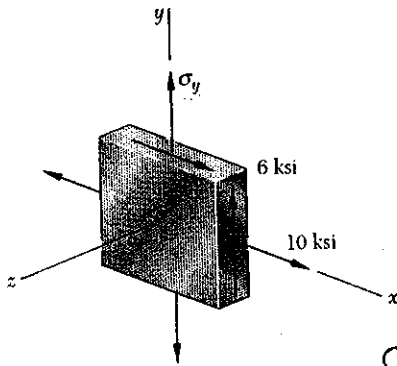
$$U = -80 \text{ MPa} \quad \sigma_y = 2U + \sigma_x = -130 \text{ MPa}$$

$$R = \sqrt{U^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\sigma_a = \sigma_b + 2R = -150 + 100 = -50 \text{ MPa} \quad \text{O.K.}$$

PROBLEM 7.75

7.75 For the state of stress shown, determine two values of  $\sigma_y$  for which the maximum shearing stress is 7.5 ksi.



SOLUTION

$$\sigma_x = 10 \text{ ksi}, \quad \tau_{xy} = 6 \text{ ksi}, \quad \tau_{max} = 7.5 \text{ ksi}$$

$$\text{Let } U = \frac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2U + \sigma_x$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + U$$

$$R = \sqrt{U^2 + \tau_{xy}^2} \qquad U = \pm \sqrt{R^2 - \tau_{xy}^2}$$

Case 1  $\tau_{max} = R = 7.5 \text{ ksi}, \quad U = \pm 4.5 \text{ ksi}$

(1a)  $U = +4.5 \text{ ksi} \qquad \sigma_y = 2U + \sigma_x = 19 \text{ ksi} \qquad \text{reject}$   
 $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 14.5 \text{ ksi}, \quad \sigma_a = \sigma_{ave} + R = 22 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 7 \text{ ksi}$   
 $\sigma_{max} = 22 \text{ ksi}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11 \text{ ksi} \neq 7.5 \text{ ksi}$

(1b)  $U = -4.5 \text{ ksi} \qquad \sigma_y = 2U + \sigma_x = 1 \text{ ksi}$   
 $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5 \text{ ksi}, \quad \sigma_a = \sigma_{ave} + R = 13 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$   
 $\sigma_{max} = 13 \text{ ksi}, \quad \sigma_{min} = -2 \text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5 \text{ ksi} \quad \text{OK.}$

Case 2 Assume  $\sigma_{min} = 0 \qquad \sigma_{max} = 2\tau_{max} = 15 \text{ ksi} = \sigma_a$

$$\sigma_a = \sigma_{ave} + R = \sigma_x + U + \sqrt{U^2 + \tau_{xy}^2}$$

$$\sigma_a - \sigma_x - U = \sqrt{U^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - U)^2 = U^2 + \tau_{xy}^2$$

$$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)U + U^2 = U^2 + \tau_{xy}^2$$

$$2U = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(15 - 10)^2 - 6^2}{15 - 10} = -2.2 \text{ ksi}$$

$$U = -1.1 \text{ ksi} \qquad \sigma_y = 2U + \sigma_x = 7.8 \text{ ksi}$$

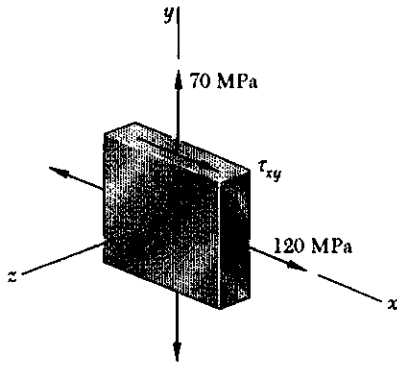
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8.9 \text{ ksi} \qquad R = \sqrt{U^2 + \tau_{xy}^2} = 6.1 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 15 \text{ ksi} \quad \checkmark \qquad \sigma_b = \sigma_{ave} - R = 2.8 \text{ ksi}$$

$$\sigma_{max} = 15 \text{ ksi}, \quad \sigma_{min} = 0 \qquad \tau_{max} = 7.5 \text{ ksi} \quad \checkmark$$

PROBLEM 7.76

7.76 For the state of stress shown, determine the value of  $\tau_{xy}$  for which the maximum shearing stress is 80 MPa.



SOLUTION

$$\sigma_x = 120 \text{ MPa} \quad \sigma_y = 70 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 95 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{120 - 70}{2} = 25 \text{ MPa}$$

Assume  $\sigma_{min} = 0 \quad \sigma_{max} = 2\tau_{max} = 160 \text{ MPa}$

$$\sigma_a = \sigma_{max} = \sigma_{ave} + R$$

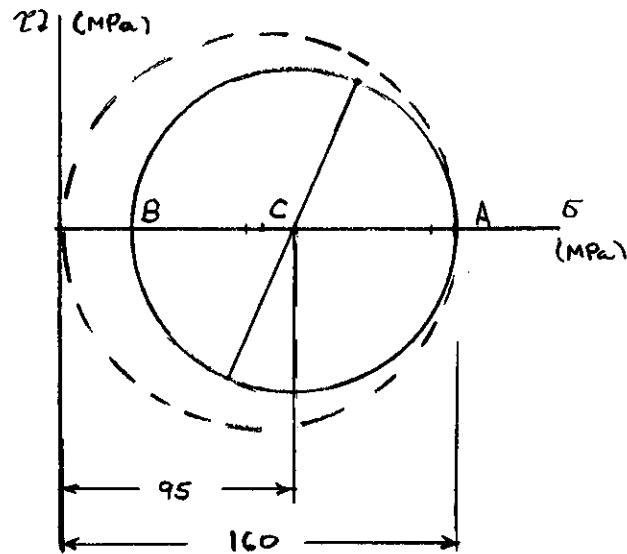
$$R = \sigma_{max} - \sigma_{ave} = 160 - 95 = 65 \text{ MPa}$$

$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$$

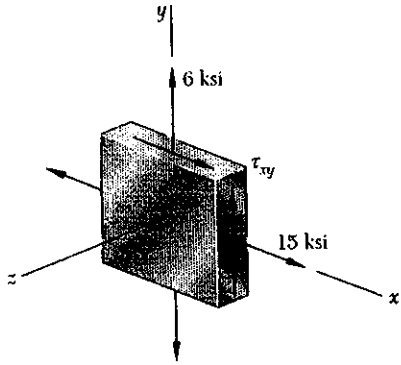
$$\tau_{xy} = \pm 60 \text{ ksi}$$

$$\sigma_b = \sigma_a - 2R = 160 - 130 = 30 \text{ MPa} \geq 0 \quad \text{O.K.}$$



PROBLEM 7.77

7.77 For the state of stress shown, determine the value of  $\tau_{xy}$  for which the maximum shearing stress is (a) 9 ksi, (b) 12 ksi.



SOLUTION

$$\sigma_x = 15 \text{ ksi} \quad \sigma_y = 6 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = 10.5 \text{ ksi}$$

$$U = \frac{\sigma_x - \sigma_y}{2} = 4.5 \text{ ksi}$$

$$\tau_{xy} \text{ (ksi)}$$

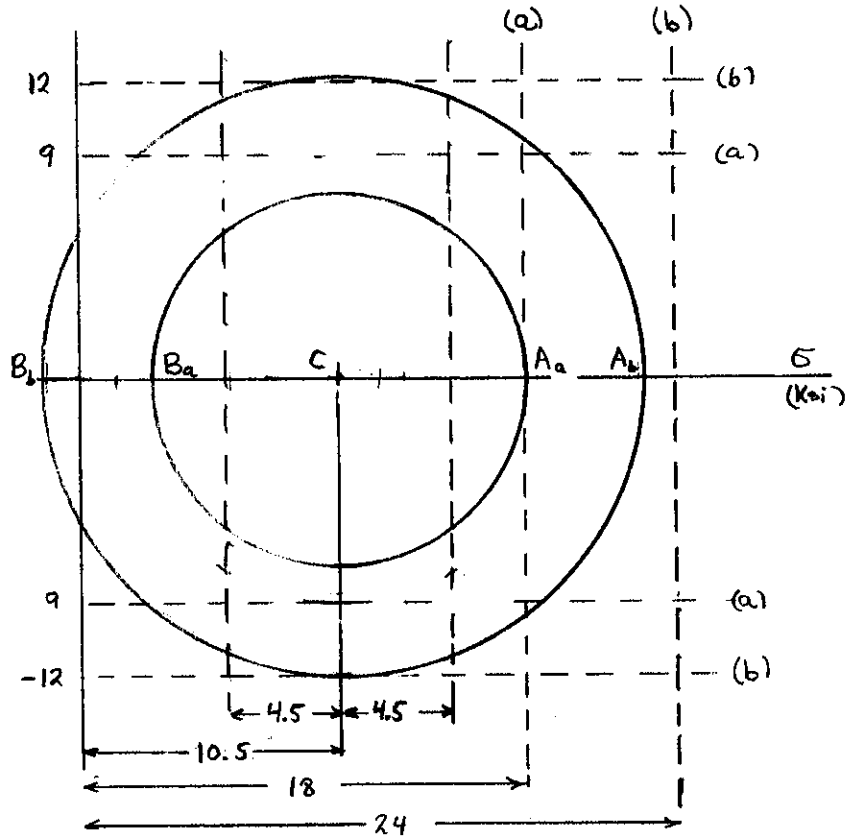
(a) For  $\tau_{max} = 9 \text{ ksi}$

Center of Mohr's circle lies at point C. Lines marked (a) show the limits on  $\tau_{max}$ . Limit on  $\sigma_{max}$  is  $\sigma_a = \sigma_{max} = 2\tau_{max} = 18 \text{ ksi}$ . For the Mohr's circle  $\sigma_a = \sigma_{max}$  corresponds to point  $A_a$ .

$$R = \sigma_a - \sigma_{ave} = 18 - 10.5 = 7.5 \text{ ksi}$$

$$R = \sqrt{U^2 + \tau_{xy}^2}$$

$$\begin{aligned} \tau_{xy} &= \pm \sqrt{R^2 - U^2} \\ &= \pm \sqrt{7.5^2 - 4.5^2} \\ &= \pm 6 \text{ ksi} \end{aligned}$$



(b) For  $\tau_{max} = 12 \text{ ksi}$ .

Center of Mohr's circle lies at point C.  $R = 12 \text{ ksi}$

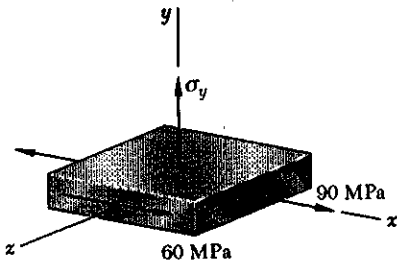
$$\tau_{xy} = \pm \sqrt{R^2 - U^2} = \pm 11.24 \text{ ksi}$$

Checking  $\sigma_a = 10.5 + 12 = 22.5 \text{ ksi}$   $\sigma_b = 10.5 - 12 = -1.5 \text{ ksi}$   
 $\sigma_c = 0$

$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min}) = 12 \text{ ksi} \quad \text{O.K.}$$

PROBLEM 7.78

7.78 For the state of stress shown, determine two values of  $\sigma_y$  for which the maximum shearing stress is 80 MPa.



SOLUTION

$$\sigma_x = 90 \text{ MPa} \quad \sigma_z = 0 \quad \tau_{xz} = 60 \text{ MPa}$$

Mohr's circle for stresses in  $xz$ -plane

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

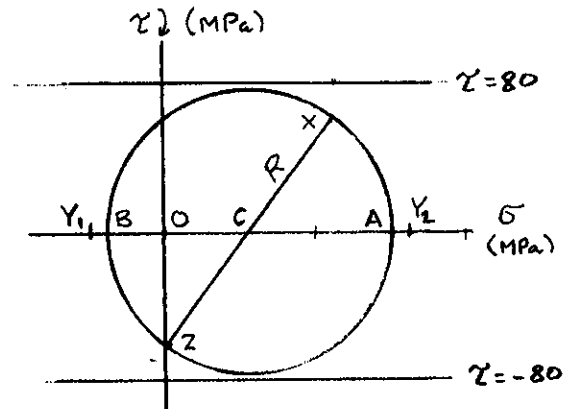
$$\sigma_a = \sigma_{ave} + R = 120 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -30 \text{ MPa}$$

Assume  $\sigma_{max} = \sigma_a = 120 \text{ MPa}$

$$\begin{aligned} \sigma_y = \sigma_{min} &= \sigma_{max} - 2\tau_{max} \\ &= 120 - (2)(80) = -40 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

Assume  $\sigma_{min} = \sigma_b = -30 \text{ MPa}$

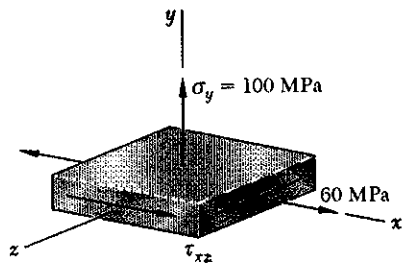
$$\begin{aligned} \sigma_y = \sigma_{max} &= \sigma_{min} + 2\tau_{max} \\ &= -30 + (2)(8) = 130 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$





PROBLEM 7.79

7.79 For the state of stress shown, determine the range of values of  $\tau_{xy}$  for which the maximum shearing stress is equal to or less than 60 MPa.



SOLUTION

$$\sigma_x = 60 \text{ MPa}, \quad \sigma_z = 0, \quad \sigma_y = 100 \text{ MPa}$$

For Mohr's circle of stresses in ZX-plane

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 30 \text{ MPa}$$

$$U = \frac{\sigma_x - \sigma_z}{2} = 30$$

Assume  $\sigma_{max} = \sigma_y = 100 \text{ MPa}$

$$\begin{aligned} \sigma_{min} = \sigma_b &= \sigma_{max} - 2\tau_{max} \\ &= 100 - (2)(60) = -20 \text{ MPa} \end{aligned}$$

$$\begin{aligned} R &= \sigma_{ave} - \sigma_b \\ &= 30 - (-20) = 50 \text{ MPa} \end{aligned}$$

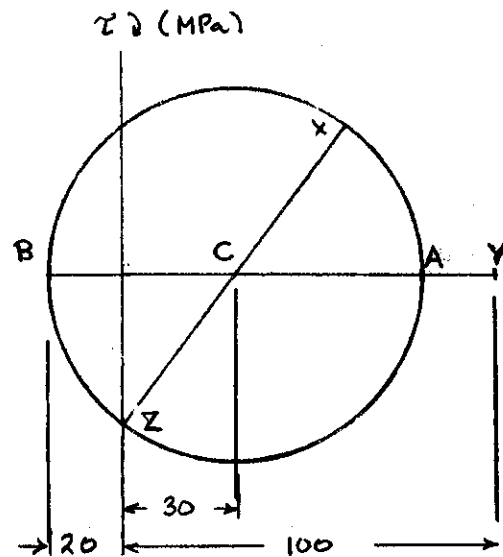
$$\begin{aligned} \sigma_a &= \sigma_{ave} + R \\ &= 30 + 50 = 80 \text{ MPa} < \sigma_y \end{aligned}$$

O.K.

$$R = \sqrt{U^2 + \tau_{xz}^2}$$

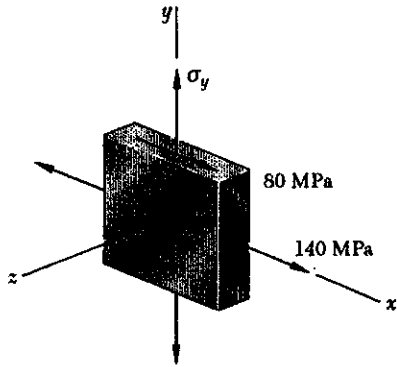
$$\begin{aligned} \tau_{xz} &= \pm \sqrt{R^2 - U^2} \\ &= \pm \sqrt{50^2 - 30^2} = \pm 40 \text{ MPa} \end{aligned}$$

$$-40 \text{ MPa} \leq \tau_{xz} \leq 40 \text{ MPa}$$



PROBLEM 7.80

\*7.80 For the state of stress of Prob. 6.66, determine (a) the value of  $\alpha$ , for which the maximum shearing stress is as small as possible, (b) the corresponding value of the shearing stress.



SOLUTION

Let  $U = \frac{\sigma_x - \sigma_y}{2}$        $\sigma_y = \sigma_x - 2U$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - U$

$R = \sqrt{U^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R = \sigma_x - U + \sqrt{U^2 + \tau_{xy}^2}$

$\sigma_b = \sigma_{ave} - R = \sigma_x - U - \sqrt{U^2 + \tau_{xy}^2}$

Assume  $\tau_{max}$  is the in-plane shearing stress       $\tau_{max} = R$

Then  $\tau_{max}(\text{in-plane})$  is minimum if  $U = 0$

$\sigma_y = \sigma_x - 2U = \sigma_x = 140 \text{ MPa}$  ,       $\sigma_{ave} = \sigma_x - U = 140 \text{ MPa}$

$R = |\tau_{xy}| = 80 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = 140 + 80 = 220 \text{ MPa}$

$\sigma_b = \sigma_{ave} - R = 140 - 80 = 60 \text{ MPa}$

$\sigma_{max} = 220 \text{ MPa}$  ,  $\sigma_{min} = 0$  ,  $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$

Assumption is incorrect.

Assume  $\sigma_{max} = \sigma_a = \sigma_{ave} + R = \sigma_x - U + \sqrt{U^2 + \tau_{xy}^2}$

$\sigma_{min} = 0$        $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}\sigma_a$

$\frac{d\sigma_a}{dU} = -1 + \frac{U}{\sqrt{U^2 + \tau_{xy}^2}} \neq 0$       (no minimum)

Optimum value for  $U$  occurs when  $\tau_{max}(\text{out-of-plane}) = \tau_{max}(\text{in-plane})$

$\frac{1}{2}(\sigma_a + R) = R$  or  $\sigma_a = R$  or  $\sigma_x - U = \sqrt{U^2 + \tau_{xy}^2}$

$(\sigma_x - U)^2 = \sigma_x^2 - 2U\sigma_x + U^2 = U^2 + \tau_{xy}^2$

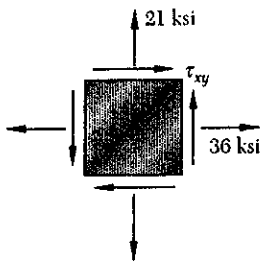
$2U = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa}$        $U = 47.14 \text{ MPa}$

$\sigma_y = \sigma_x - 2U = 140 - 94.3 = 45.7 \text{ MPa}$       ◀

$R = \sqrt{U^2 + \tau_{xy}^2} = \tau_{max} = 92.9 \text{ MPa}$       ◀

PROBLEM 7.81

7.81 The state of plane stress shown occurs in a machine component made of a steel with  $\sigma_y = 45$  ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a)  $\tau_{xy} = 9$  ksi, (b)  $\tau_{xy} = 18$  ksi, (c)  $\tau_{xy} = 20$  ksi. If yield does not occur, determine the corresponding factor of safety.



SOLUTION

$$\sigma_x = 36 \text{ ksi} \quad \sigma_y = 21 \text{ ksi} \quad \sigma_z = 0$$

For stresses in xy-plane  $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5 \text{ ksi}$

$$\frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$$

(a)  $\tau_{xy} = 9 \text{ ksi}$   $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (9)^2} = 11.715 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 40.215 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 16.875 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 34.977 \text{ ksi} < 45 \text{ ksi} \quad (\text{No yielding})$$

$$F.S. = \frac{45}{34.977} = 1.287$$

(b)  $\tau_{xy} = 18 \text{ ksi}$   $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (18)^2} = 19.5 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 48 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 9 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 44.193 \text{ ksi} < 45 \text{ ksi} \quad (\text{No yielding})$$

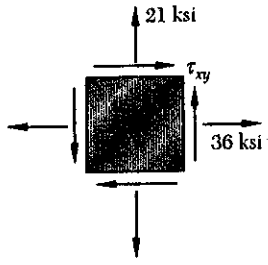
$$F.S. = \frac{45}{44.193} = 1.018$$

(c)  $\tau_{xy} = 20 \text{ ksi}$   $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (20)^2} = 21.36 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 49.86, \quad \sigma_b = \sigma_{ave} - R = 7.14 \text{ ksi}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 46.732 \text{ ksi} > 45 \text{ ksi} \quad (\text{Yielding occurs})$$

PROBLEM 7.82



7.81 The state of plane stress shown occurs in a machine component made of a steel with  $\sigma_y = 45$  ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a)  $\tau_{xy} = 9$  ksi, (b)  $\tau_{xy} = 18$  ksi, (c)  $\tau_{xy} = 20$  ksi. If yield does not occur, determine the corresponding factor of safety.

7.82 Solve Prob. 7.81, using the maximum-shearing-stress criterion.

SOLUTION

$$\sigma_x = 36 \text{ ksi} \quad \sigma_y = 21 \text{ ksi} \quad \sigma_z = 0$$

For stresses in  $xy$ -plane  $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5 \text{ ksi}$

$$\frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$$

(a)  $\tau_{xy} = 9 \text{ ksi} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 11.715 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 40.215 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 16.875 \text{ ksi}$$

$$\sigma_{max} = 40.215 \text{ ksi}, \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 40.215 \text{ ksi} < 45 \text{ ksi} \quad (\text{No yielding})$$

$$F.S. = \frac{45}{40.215} = 1.119$$

(b)  $\tau_{xy} = 18 \text{ ksi} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 19.5 \text{ ksi}$

$$\sigma_a = \sigma_{ave} + R = 48 \text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 9 \text{ ksi}$$

$$\sigma_{max} = 48 \text{ ksi} \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 48 \text{ ksi} > 45 \text{ ksi} \quad (\text{Yielding occurs})$$

(c)  $\tau_{xy} = 20 \text{ ksi} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.36 \text{ ksi}$

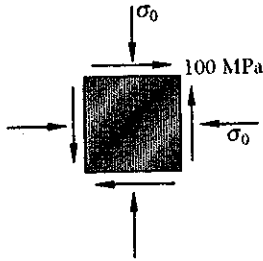
$$\sigma_a = \sigma_{ave} + R = 49.86 \text{ ksi} \quad \sigma_b = \sigma_{ave} - R = 7.14 \text{ ksi}$$

$$\sigma_{max} = 49.86 \text{ ksi} \quad \sigma_{min} = 0$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 49.86 \text{ ksi} > 45 \text{ ksi} \quad (\text{Yielding occurs})$$

PROBLEM 7.83

7.83 The state of plane stress shown occurs in a machine component made of a steel with  $\sigma_y = 325$  MPa. Using the maximum-shearing-stress criterion, determine whether yield occurs when (a)  $\sigma_0 = 200$  MPa, (b)  $\sigma_0 = 240$  MPa, (c)  $\sigma_0 = 280$  MPa. If yield does not occur, determine the corresponding factor of safety.



SOLUTION

$$\sigma_{ave} = -\sigma_0 \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a)  $\sigma_0 = 200$  MPa,  $\sigma_{ave} = -200$  MPa

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa} \quad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -300 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300 \text{ MPa} < 325 \text{ MPa} \quad (\text{No yielding})$$

$$F.S. = \frac{325}{300} = 1.083$$

(b)  $\sigma_0 = 240$  MPa,  $\sigma_{ave} = -240$  MPa

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -340 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

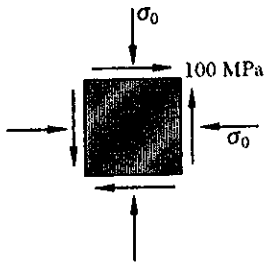
(c)  $\sigma_0 = 280$  MPa,  $\sigma_{ave} = -280$  MPa

$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sigma_{max} = 0, \quad \sigma_{min} = -380 \text{ MPa}$$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380 \text{ MPa} > 325 \text{ MPa} \quad (\text{Yielding occurs})$$

PROBLEM 7.84



7.83 The state of plane stress shown occurs in a machine component made of a steel with  $\sigma_y = 325$  MPa. Using the maximum-shearing-stress criterion, determine whether yield occurs when (a)  $\sigma_0 = 200$  MPa, (b)  $\sigma_0 = 240$  MPa, (c)  $\sigma_0 = 280$  MPa. If yield does not occur, determine the corresponding factor of safety.

7.84 Solve Prob. 7.83, using the maximum-distortion-energy criterion.

SOLUTION

$$\sigma_{ave} = -\sigma_0 \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a)  $\sigma_0 = 200$  MPa  $\sigma_{ave} = -200$  MPa

$$\sigma_a = \sigma_{ave} + R = -100 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -300 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa} \text{ (No yielding)}$$

$$F.S. = \frac{325}{264.56} = 1.228 \quad \blacktriangleleft$$

(b)  $\sigma_0 = 240$  MPa  $\sigma_{ave} = -240$  MPa

$$\sigma_a = \sigma_{ave} + R = -140 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -340 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa} \text{ (No yielding)}$$

$$F.S. = \frac{325}{295.97} = 1.098 \quad \blacktriangleleft$$

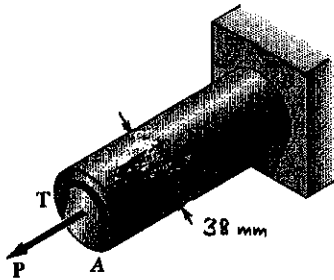
(c)  $\sigma_0 = 280$  MPa  $\sigma_{ave} = -280$  MPa

$$\sigma_a = \sigma_{ave} + R = -180 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -380 \text{ MPa}$$

$$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa} \text{ (Yielding occurs)}$$

PROBLEM 7.85

7.85 The 38-mm-diameter shaft  $AB$  is made of a grade of steel for which the yield strength is  $\sigma_y = 250$  MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque  $T$  for which yield occurs when  $P = 240$  kN.



SOLUTION

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = \frac{1}{2} \sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4} \sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_y$$

$$4\tau_{xy}^2 = \sigma_y^2 - \sigma_x^2 \quad \tau_{xy} = \frac{1}{2} \sqrt{\sigma_y^2 - \sigma_x^2} = \frac{1}{2} \sqrt{250^2 - 211.6^2}$$

$$= 66.568 \text{ MPa} = 66.568 \times 10^6 \text{ Pa}$$

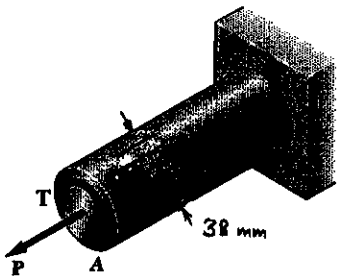
From torsion  $\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J\tau_{xy}}{c}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(66.568 \times 10^6)}{19 \times 10^{-3}} = 717 \text{ N}\cdot\text{m}$$

**PROBLEM 7.86**



**7.85** The 38-mm-diameter shaft *AB* is made of a grade of steel for which the yield strength is  $\sigma_Y = 250$  MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque *T* for which yield occurs when  $P = 240$  kN.

**7.86** Solve Prob. 7.85, using the maximum-distortion-energy criterion.

**SOLUTION**

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\begin{aligned} \sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b &= \frac{1}{4}\sigma_x^2 + \sigma_x \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &\quad + \frac{1}{4}\sigma_x^2 - \sigma_x \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &\quad - \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ &= \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2 \end{aligned}$$

$$\tau_{xy}^2 = \frac{1}{3}(\sigma_Y^2 - \sigma_x^2)$$

$$\tau_{xy} = \frac{1}{\sqrt{3}} \sqrt{250^2 - 211.6^2} = 76.867 \text{ MPa} = 76.867 \times 10^6 \text{ Pa}$$

From torsion  $\tau_{xy} = \frac{Tc}{J}$   $T = \frac{J\tau_{xy}}{c}$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

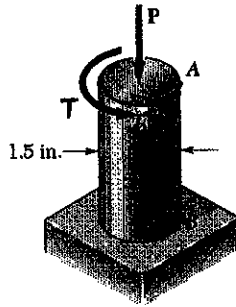
$$c = \frac{1}{2}d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(76.876 \times 10^6)}{19 \times 10^{-3}} = 828 \text{ N}\cdot\text{m}$$



PROBLEM 7.87

7.87 The 1.5-in-diameter shaft  $AB$  is made of a grade of steel for which the yield strength is  $\sigma_Y = 42$  ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque  $T$  for which yield occurs when  $P = 60$  kips.



SOLUTION

$$P = 60 \text{ kips} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$$

$$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \quad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2}$$

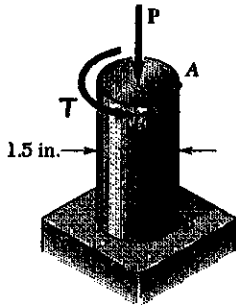
$$= 12.361 \text{ ksi}$$

From torsion  $\tau_{xy} = \frac{Tc}{J} \quad T = \frac{J\tau_{xy}}{c}$

$$c = \frac{1}{2}d = 0.75 \text{ in} \quad J = \frac{\pi}{2} c^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(12.361)}{0.75} = 8.19 \text{ kip}\cdot\text{in}$$

PROBLEM 7.88



7.87 The 1.5-in-diameter shaft  $AB$  is made of a grade of steel for which the yield strength is  $\sigma_y = 42$  ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque  $T$  for which yield occurs when  $P = 60$  kips.

7.88 Solve Prob. 7.87, using the maximum-distortion-energy criterion.

SOLUTION

$$P = 60 \text{ kips} \quad A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$$

$$\sigma_y = 0 \quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

$$\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b = (\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R)$$

$$= \sigma_{ave}^2 + 2\sigma_{ave}R + R^2 + \sigma_{ave}^2 - 2\sigma_{ave}R + R^2 - \sigma_{ave}^2 + R^2$$

$$= \sigma_{ave}^2 + 3R^2$$

$$= \frac{1}{4}\sigma_x^2 + 3\left(\frac{1}{4}\sigma_x^2 + \tau_{xy}^2\right) = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_y^2$$

$$3\tau_{xy}^2 = \sigma_y^2 - \sigma_x^2$$

$$\tau_{xy} = \frac{1}{\sqrt{3}}(\sigma_y^2 - \sigma_x^2) = \frac{1}{\sqrt{3}}\sqrt{42^2 - 33.953^2}$$

$$= 14.273 \text{ ksi}$$

From torsion

$$\tau_{xy} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{xy}}{c}$$

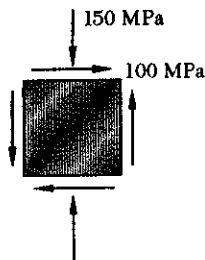
$$c = \frac{1}{2}d = 0.75 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(14.273)}{0.75} = 9.46 \text{ kip}\cdot\text{in.}$$

PROBLEM 7.89

7.89 and 7.90 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used  $\sigma_{UT} = 160$  MPa and  $\sigma_{UC} = 320$  MPa and using Mohr's criterion, determine whether rupture of the component will occur.



SOLUTION

$$\sigma_x = 0 \quad \sigma_y = -150 \text{ MPa} \quad \tau_{xy} = 100 \text{ MPa}$$

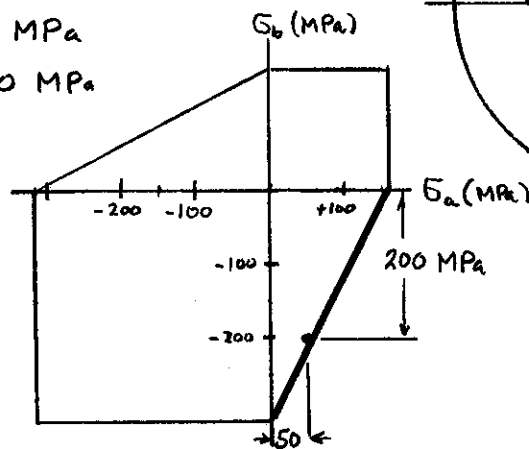
$$\sigma_{ave} = \frac{1}{2} (\sigma_x + \sigma_y) = -75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{75^2 + 100^2} = 125 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 50 \text{ MPa}$$

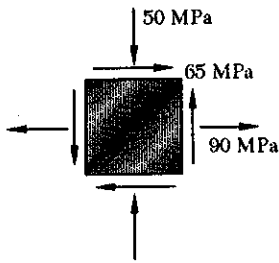
$$\sigma_b = \sigma_{ave} - R = -200 \text{ MPa}$$



Equation of the 4th quadrant boundary is  $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$$\frac{50}{160} - \frac{(-200)}{320} = 0.9375 < 1 \quad , \quad \text{No rupture.}$$

**PROBLEM 7.90**



7.89 and 7.90 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used  $\sigma_{UT} = 160$  MPa and  $\sigma_{UC} = 320$  MPa and using Mohr's criterion, determine whether rupture of the component will occur.

**SOLUTION**

$$\sigma_x = 90 \text{ MPa}, \quad \sigma_y = -50 \text{ MPa}, \quad \tau_{xy} = 65 \text{ MPa}$$

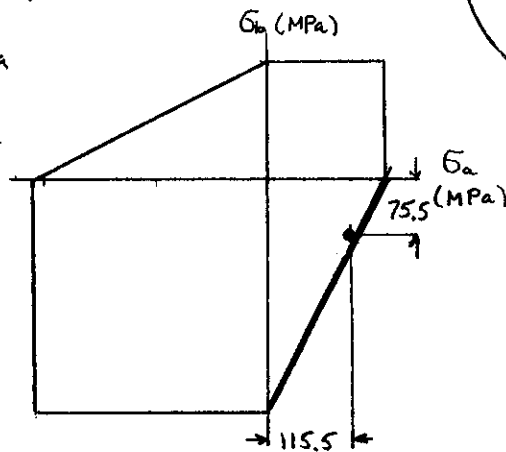
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 20 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{70^2 + 65^2} = 95.5 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 115.5 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -75.5 \text{ MPa}$$



Equation of 4th quadrant boundary

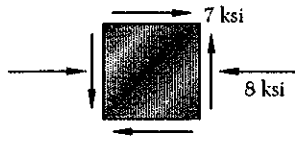
$$\frac{115.5}{160} - \frac{(-75.5)}{320} = 0.958 < 1$$

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

No rupture

PROBLEM 7.91

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used  $\sigma_{UT} = 10$  ksi and  $\sigma_{UC} = 30$  ksi and using Mohr's criterion, determine whether rupture of the component will occur.



SOLUTION

$$\sigma_x = -8 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 7 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -4 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = -4 + 8.062 = 4.062 \text{ ksi}$$

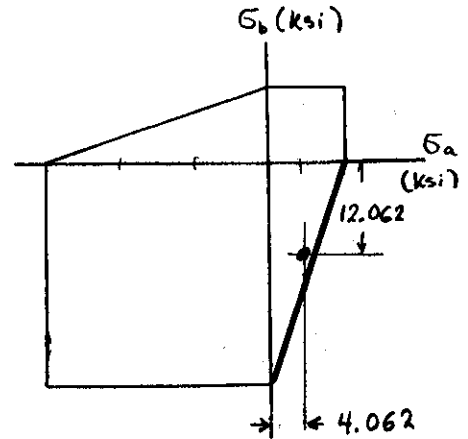
$$\sigma_b = \sigma_{ave} - R = -4 - 8.062 = -12.062 \text{ ksi}$$

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

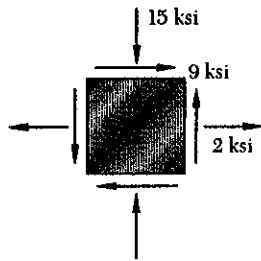
$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

(No rupture)



PROBLEM 7.92

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used  $\sigma_{UT} = 10$  ksi and  $\sigma_{UC} = 30$  ksi and using Mohr's criterion, determine whether rupture of the component will occur.



SOLUTION

$$\sigma_x = 2 \text{ ksi} \quad \sigma_y = -15 \text{ ksi} \quad \tau_{xy} = 9 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 5.879 \text{ ksi}$$

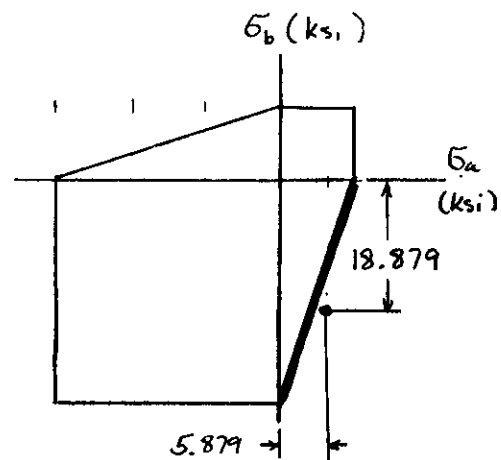
$$\sigma_b = \sigma_{ave} - R = -18.879 \text{ ksi}$$

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

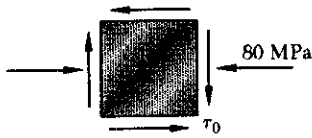
$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur.



PROBLEM 7.93

7.93 The state of plane stress shown will occur at a critical point in a cast pipe made of an aluminum alloy for which  $\sigma_{UT} = 75$  MPa and  $\sigma_{UC} = 150$  MPa. Using Mohr's criterion, determine the shearing stress  $\tau_0$  for which failure should be expected.



SOLUTION

$$\sigma_x = -80 \text{ MPa} \quad \sigma_y = 0 \quad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R, \quad \sigma_b = \sigma_{ave} - R, \quad \tau_0 = \pm \sqrt{R^2 - 40^2}$$

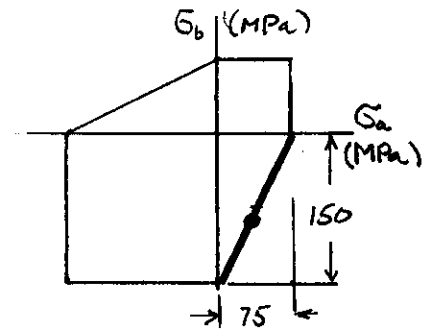
Since  $|\sigma_{ave}| < R$ , stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$

$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$$R = 63.33 \text{ MPa}, \quad \tau_0 = \pm \sqrt{63.33^2 - 40^2} = \pm 49.1 \text{ MPa}$$



PROBLEM 7.94

7.94 The state of plane stress shown will occur in an aluminum casting that is made of an alloy for which  $\sigma_{UT} = 10$  ksi and  $\sigma_{UC} = 25$  ksi. Using Mohr's criterion, determine the shearing stress  $\tau_0$  for which failure should be expected.



SOLUTION

$$\sigma_x = 8 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = \tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_0^2}, \quad \tau_0 = \pm \sqrt{R^2 - 4^2}$$

$$\sigma_a = \sigma_{ave} + R = (4 + R) \text{ ksi} \quad \sigma_b = \sigma_{ave} - R = (4 - R) \text{ ksi}$$

Since  $|\sigma_{ave}| < R$ , stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

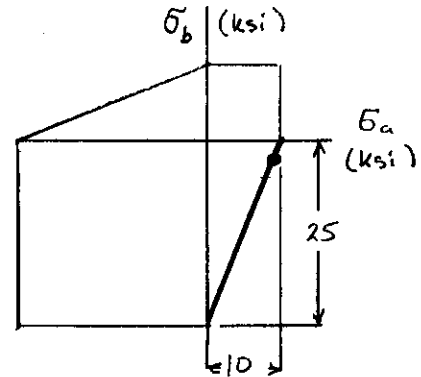
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{4+R}{10} - \frac{4-R}{25} = 1$$

$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$

$$R = 5.429 \text{ ksi}$$

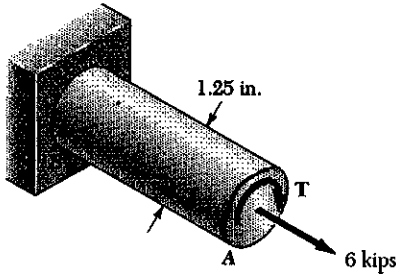
$$\tau_0 = \pm \sqrt{5.429^2 - 4^2} = \pm 3.67 \text{ ksi}$$





PROBLEM 7.95

7.95 The cast-aluminum rod shown is made of an alloy for which  $\sigma_{UT} = 8$  ksi and  $\sigma_{UC} = 16$  ksi. Using Mohr's criterion, determine the magnitude of the torque  $T$  for which rupture should be expected.



SOLUTION

$$P = 6 \text{ kips}, \quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$\sigma_x = \frac{P}{A} = 4.889 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{TC}{J}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.4446 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5.976 + \tau_{xy}^2} \text{ ksi}, \quad \tau_{xy} = \pm \sqrt{R^2 - 5.976} \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 2.4446 + R \text{ ksi}, \quad \sigma_b = 2.4446 - R \text{ ksi}$$

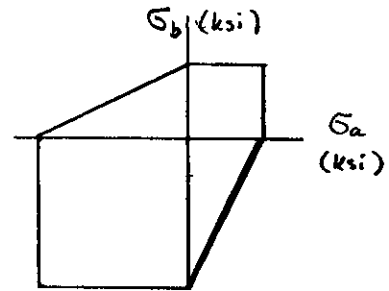
Since  $|\sigma_{ave}| < R$ , stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{2.4446 + R}{8} - \frac{2.4446 - R}{16} = 1$$

$$\left(\frac{1}{8} + \frac{1}{16}\right)R = 1 - \frac{2.4446}{8} + \frac{2.4446}{16}$$

$$R = 4.5185 \text{ ksi}, \quad \tau_{xy} = \pm \sqrt{4.5185^2 - 5.976} = 3.80 \text{ ksi}$$



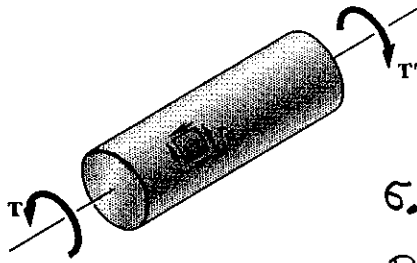
For torsion:  $c = \frac{1}{2}d = 0.625 \text{ in}$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968 \text{ in}^4$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(0.23968)(3.80)}{0.625} = 1.457 \text{ kip}\cdot\text{in}$$

PROBLEM 7.96

7.96 The cast-aluminum rod shown is made of an alloy for which  $\sigma_{UT} = 70$  MPa and  $\sigma_{UC} = 175$  MPa. Knowing that the magnitude  $T$  of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress  $\tau_0$  which should be expected at rupture.



SOLUTION

$$\sigma_x = 0, \quad \sigma_y = 0, \quad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_0^2} = |\tau_{xy}|$$

$$\sigma_a = \sigma_{ave} + R = R$$

$$\sigma_b = \sigma_{ave} - R = -R$$

Since  $|\sigma_{ave}| < R$ , stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

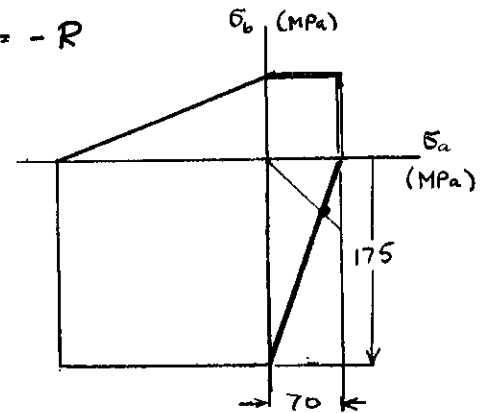
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{R}{70} - \frac{-R}{175} = 1$$

$$\left(\frac{1}{70} + \frac{1}{175}\right)R = 1$$

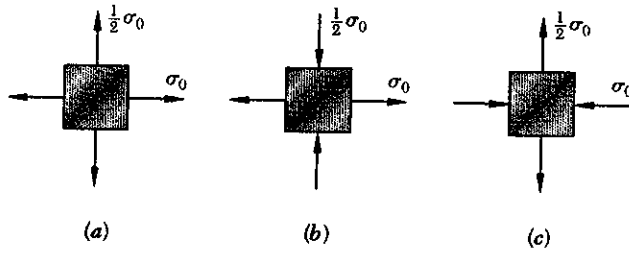
$$R = 50 \text{ MPa}$$

$$|\tau_{xy}| = 50 \text{ MPa}$$



PROBLEM 7.97

7.97 A machine component is made of a grade of cast iron for which  $\sigma_{UT} = 8$  ksi and  $\sigma_{UC} = 20$  ksi. For each of the states of plane stress shown, and using Mohr's criterion, determine the normal stress  $\sigma_0$  at which rupture of the component should be expected.



SOLUTION

(a)  $\sigma_a = \sigma_0, \sigma_b = \frac{1}{2}\sigma_0$

Stress point lies in 1st quadrant.

$\sigma_a = \sigma_0 = \sigma_{UT} = 8$  ksi

(b)  $\sigma_a = \sigma_0, \sigma_b = -\frac{1}{2}\sigma_0$

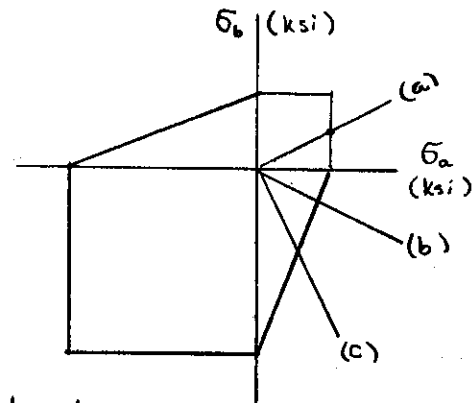
Stress point lies in 4th quadrant.  
Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{8} - \frac{-\frac{1}{2}\sigma_0}{20} = 1 \quad \sigma_0 = 6.67 \text{ ksi}$$

(c)  $\sigma_a = \frac{1}{2}\sigma_0, \sigma_b = -\sigma_0$ , 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{8} - \frac{-\sigma_0}{20} = 1 \quad \sigma_0 = 8.89 \text{ ksi}$$



PROBLEM 7.98

7.98 Determine the normal stress in a basketball of 9.5-in. diameter and 0.125-in. wall thickness that is inflated to a gage pressure of 9 psi.

SOLUTION

$$r = \frac{1}{2}d - t = (\frac{1}{2})(9.5) - 0.125 = 4.625 \text{ in}$$

$$\sigma_1 = \sigma_2 = \frac{Pr}{2t} = \frac{(9)(4.625)}{(2)(0.125)} = 166.5 \text{ psi}$$

**PROBLEM 7.99**

7.99 A spherical gas container made of steel has an 18-ft diameter and a wall thickness of  $\frac{3}{8}$  in. Knowing that the internal pressure is 60 psi, determine the maximum normal stress and the maximum shearing stress in the container.

**SOLUTION**

$$d = 18 \text{ ft} = 216 \text{ in} \quad r = \frac{1}{2}d - t = 107.625 \text{ in.}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(60)(107.625)}{(2)(0.375)} = 8610 \text{ psi} = 8.61 \text{ ksi}$$

$$\tau_{\text{max (out-of-plane)}} = \frac{1}{2}\sigma_1 = 4.31 \text{ ksi}$$

**PROBLEM 7.100**

7.100 The maximum gage pressure is known to be 8 MPa in a spherical steel pressure vessel having a 250-mm diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is  $\sigma_u = 400$  MPa, determine the factor of safety with respect to tensile failure.

**SOLUTION**

$$p = 8 \text{ MPa} = 8 \times 10^6 \text{ Pa} \quad t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(250) - 6 = 119 \text{ mm} = 0.119 \text{ m}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(8 \times 10^6)(0.119)}{(2)(6 \times 10^{-3})} = 79.33 \times 10^6 \text{ Pa} = 79.33 \text{ MPa}$$

$$\text{F.S.} = \frac{\sigma_u}{\sigma_1} = \frac{400}{79.33} = 5.04$$

**PROBLEM 7.101**

7.101 A spherical pressure vessel of 900-mm outside diameter is to be fabricated from a steel having an ultimate stress  $\sigma_u = 400$  MPa. Knowing that a factor of safety of 4 is desired and that the gage pressure can reach 3.5 MPa, determine the smallest wall thickness that should be used.

**SOLUTION**

$$p = 3.5 \text{ MPa}, \quad r = \frac{1}{2}d - t = \left(\frac{1}{2}\right)(900) - t = 450 - t \text{ mm}$$

$$\sigma_1 = \sigma_2 = \frac{\sigma_u}{\text{F.S.}} = \frac{400}{4} = 100 \text{ MPa}$$

$$\sigma_1 = \frac{pr}{2t} \quad \therefore t = \frac{pr}{2\sigma_1} = \frac{(3.5)(450 - t)}{(2)(100)} = 7.875 - 0.0175t$$

$$1.0175t = 7.875 \quad t = 7.74 \text{ mm}$$

PROBLEM 7.102

7.102 A spherical gas container having a diameter of 5 m and a wall thickness of 24 mm is made of a steel for which  $E = 200$  GPa and  $\nu = 0.29$ . Knowing that the gage pressure in the container is increased from zero to 1.8 MPa, determine (a) the maximum normal stress in the container, (b) the increase in the diameter of the container.

SOLUTION

$$p = 1.8 \text{ MPa} \quad r = \frac{1}{2}d - t = \frac{1}{2}(5) - 24 \times 10^{-3} = 2.476 \text{ m}$$

$$(a) \sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.8)(2.476)}{(2)(24 \times 10^{-3})} = 92.85 \text{ MPa}$$

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - 2\sigma_2) = \frac{1-\nu}{E} \sigma_1 = \frac{1-0.29}{200 \times 10^9} (92.85 \times 10^6) = 329.6 \mu$$

$$\Delta d = d \epsilon_1 = (5)(329.6 \times 10^{-6}) = 1.648 \times 10^{-3} \text{ m} = 1.648 \text{ mm}$$

PROBLEM 7.103

7.103 A spherical pressure vessel is 3 m in diameter and has a wall thickness of 12 mm. Knowing that for the steel used  $\sigma_{all} = 80$  MPa,  $E = 200$  GPa and  $\nu = 0.29$ , determine (a) the allowable gage pressure, (b) the corresponding increase in the diameter of the vessel.

SOLUTION

$$r = \frac{1}{2}d - t = \frac{1}{2}(3000) - 12 = 1488 \text{ mm}$$

$$\sigma_1 = \sigma_2 = \sigma_{all} = 80 \text{ MPa}$$

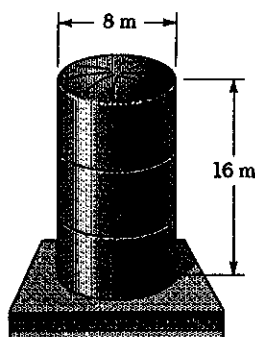
$$(a) \sigma_1 = \sigma_2 = \frac{pr}{2t} \quad p = \frac{2t\sigma_1}{r} = \frac{(2)(12)(80)}{1488} = 1.290 \text{ MPa}$$

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - 2\sigma_2) = \frac{1-\nu}{E} \sigma_1 = \frac{1-0.29}{200 \times 10^9} (8 \times 10^6) = 28.4 \mu$$

$$(b) \Delta d = d \epsilon_1 = (3000)(28.4 \times 10^{-6}) = 85.2 \times 10^{-3} \text{ mm} = 0.0852 \text{ mm}$$

PROBLEM 7.104

7.104 When filled to capacity, the unpressurized storage tank shown contains water to a height of 15.5 m above its base. Knowing that the lower portion of the tank has a wall thickness of 16 mm, determine the maximum normal stress and the maximum shearing stress in the tank. (Density of water = 1000 kg/m<sup>3</sup>)



SOLUTION

$$p = \rho gh = (1000)(9.81)(15.5) = 152.06 \times 10^3 \text{ Pa}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(8) - 16 \times 10^{-3} = 3.984 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(152.06 \times 10^3)(3.984)}{16 \times 10^{-3}} = 37.9 \times 10^6 \text{ Pa} \\ = 37.9 \text{ MPa}$$

$$\tau_{max}(\text{out of plane}) = \frac{1}{2} \sigma_1 = 18.93 \text{ MPa}$$

PROBLEM 7.105

7.105 Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft diameter and  $\frac{5}{8}$ -in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

SOLUTION

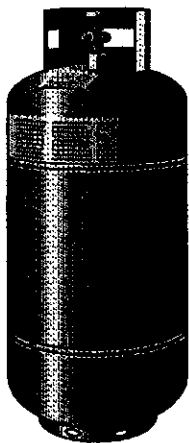
$$\sigma_1 = \frac{\sigma_u}{F.S.} = \frac{65}{5.0} = 13 \text{ ksi} \quad d = 5.5 \text{ ft} = 66 \text{ in}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(66) - 0.625 = 32.375 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} \quad p = \frac{\sigma_1 t}{r} = \frac{(13)(0.625)}{32.375} = 0.251 \text{ ksi} = 251 \text{ psi} \quad \blacktriangleleft$$

PROBLEM 7.106

7.106 The storage tank shown contains liquified propane under a pressure of 210 psi at a temperature of 100° F. Knowing that the tank has a diameter of 12.6 in. and a wall thickness of 0.11 in, determine the maximum normal stress and the maximum shearing stress in the tank.



SOLUTION

$$p = 210 \text{ psi}, \quad r = \frac{1}{2}d - t = \frac{1}{2}(12.6) - 0.11 = 6.19 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(210)(6.19)}{0.11} = 11.82 \times 10^3 \text{ psi} = 11.82 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{\max} (\text{out-of-plane}) = \frac{1}{2}\sigma_1 = 5.91 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 7.107

7.107 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.3 m and a wall thickness of 18 mm. At a time when the internal pressure of the tank is 1.5 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

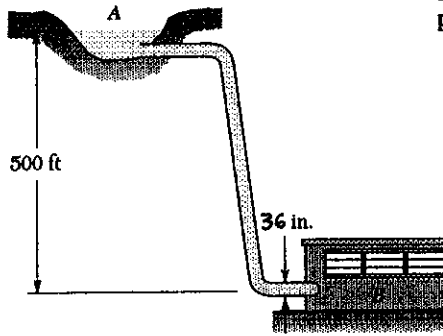
SOLUTION

$$d = 3.3 \text{ m}, \quad t = 18 \times 10^{-3} \text{ m} \quad r = \frac{1}{2}d - t = 1.632 \text{ m}$$

$$p = 1.5 \text{ MPa} \quad \sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6)(1.632)}{18 \times 10^{-3}} = 136 \times 10^6 \text{ Pa} = 136 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{\max} (\text{out-of-plane}) = \frac{1}{2}\sigma_1 = 68 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.108



7.108 A 36-in.-diameter penstock has a 0.5-in wall thickness and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft<sup>3</sup>, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

SOLUTION

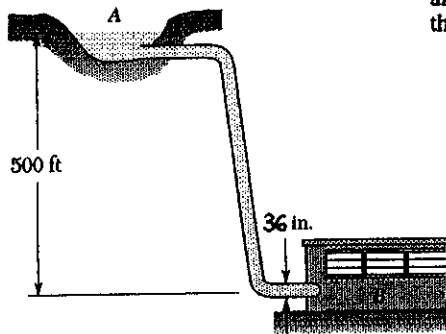
$$r = \frac{1}{2}d - t = \frac{1}{2}(30) - 0.5 = 1.5 \text{ in.}$$

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2 = 216.67 \text{ psi}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(216.67)(17.5)}{0.5} = 7583 \text{ psi} = 7.58 \text{ ksi}$$

$$\tau_{\max} (\text{out-of-plane}) = \frac{1}{2}\sigma_1 = 3.79 \text{ ksi}$$

PROBLEM 7.109



7.109 A 36-in.-diameter steel penstock connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft<sup>3</sup> and that the allowable normal stress in the steel is 12.5 ksi, determine the smallest wall thickness that can be used for the penstock.

SOLUTION

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2 = 216.67 \text{ psi}$$

$$\sigma_1 = 12.5 \text{ ksi} = 12.5 \times 10^3 \text{ psi}$$

$$r = \frac{1}{2}d - t = 18 - t$$

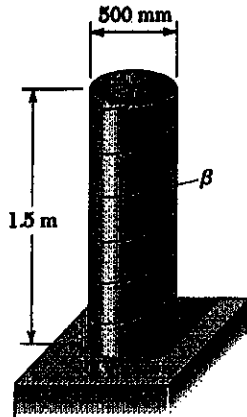
$$\frac{18-t}{t} = \frac{12.5 \times 10^3}{216.67} = 57.692$$

$$\sigma_1 = \frac{pr}{t}, \quad \frac{r}{t} = \frac{\sigma_1}{p}$$

$$\frac{18}{t} = 58.692$$

$$t = 0.307 \text{ in}$$

**PROBLEM 7.110**



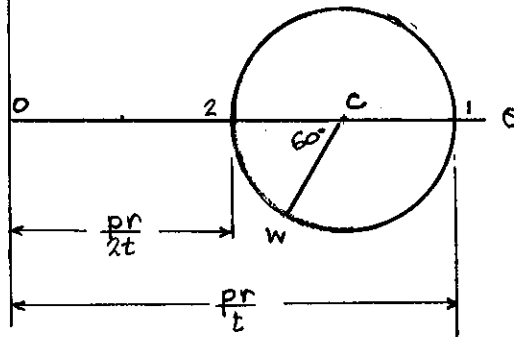
7.110 The cylindrical portion of the compressed air tank shown is fabricated of 6-mm-thick plate welded along a helix forming an angle  $\beta = 30^\circ$  with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

**SOLUTION**

$$r = \frac{1}{2}d - t = \frac{1}{2}(500) - 6 = 244 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{1}{2} \frac{pr}{t}$$

$\tau$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

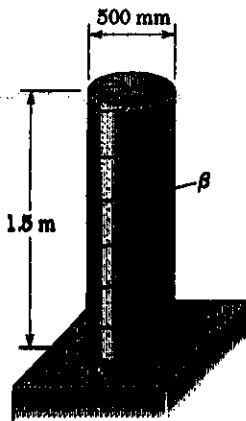
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} + R \cos 60^\circ = \frac{5}{8} \frac{pr}{t}$$

$$p = \frac{8}{5} \frac{\sigma_w t}{r}$$

$$p = \frac{8}{5} \frac{(75)(6)}{244} = 2.95 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 7.111**



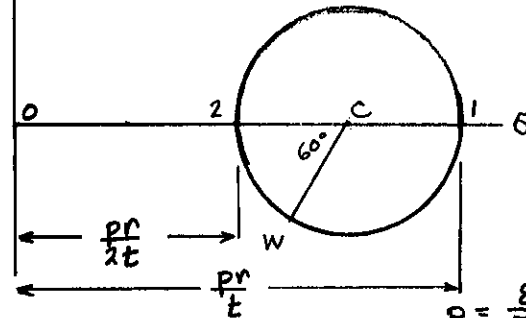
7.111 The cylindrical portion of the compressed air tank shown is fabricated of 6-mm-thick plate welded along a helix forming an angle  $\beta = 30^\circ$  with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

**SOLUTION**

$$r = \frac{1}{2}d - t = \frac{1}{2}(500) - 6 = 244 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}, \quad \sigma_2 = \frac{1}{2} \frac{pr}{t}$$

$\tau$



$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

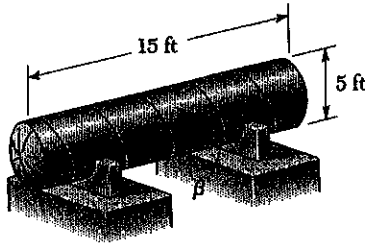
$$\tau_w = R \sin 60^\circ = \frac{\sqrt{3}}{8} \frac{pr}{t}$$

$$p = \frac{8}{\sqrt{3}} \frac{\tau_w t}{r}$$

$$p = \frac{8}{\sqrt{3}} \frac{(30)(6)}{244} = 3.41 \text{ MPa} \quad \blacktriangleleft$$



PROBLEM 7.112



7.112 The pressure tank shown has a  $\frac{3}{8}$ -in. wall thickness and butt-welded seams forming an angle  $\beta = 20^\circ$  with a transverse plane. For a gage pressure of 85 psi, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

SOLUTION

$$d = 5 \text{ ft} = 60 \text{ in.} \quad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$$

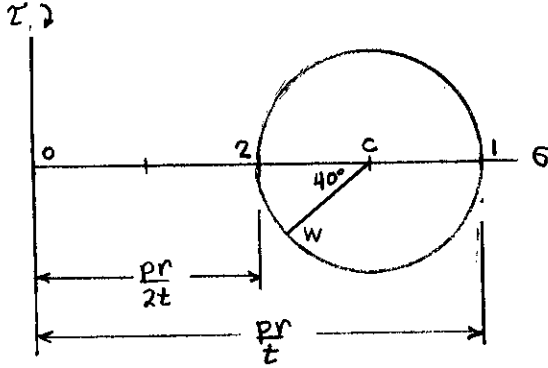
$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 5036.25 \text{ psi}$$

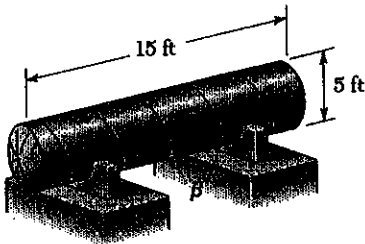
$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75 \text{ psi}$$

$$(a) \sigma_w = \sigma_{ave} - R \cos 40^\circ = 3750 \text{ psi}$$

$$(b) \tau_w = R \sin 40^\circ = 1079 \text{ psi}$$



PROBLEM 7.113



7.113 The pressure tank shown has a  $\frac{3}{8}$ -in. wall thickness and butt-welded seams forming an angle  $\beta$  with a transverse plane. Determine the range of values of  $\beta$  that can be used if the shearing stress parallel to the weld is not to exceed 1350 psi when the gage pressure is 85 psi.

SOLUTION

$$d = 5 \text{ ft} = 60 \text{ in.} \quad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75$$

$$\tau_w = R \sin 2\beta = \tau_{all}$$

$$\sin 2\beta_a = \frac{\tau_w}{R} = \frac{1350}{1678.75} = 0.80417$$

$$2\beta_a = -53.53^\circ$$

$$2\beta_b = +53.53^\circ$$

$$2\beta_c = -53.53^\circ + 180^\circ = 126.47^\circ$$

$$2\beta_d = 53.53^\circ + 180^\circ = 233.53^\circ$$

$$\beta_a = -26.8^\circ$$

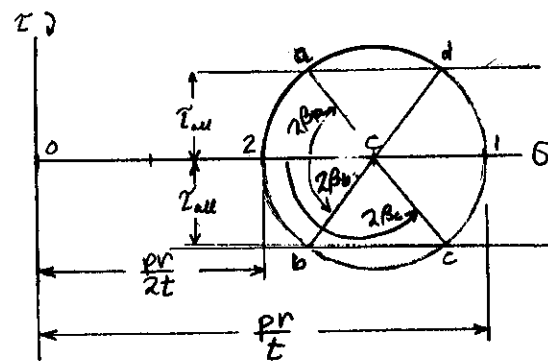
$$\beta_b = 26.8^\circ$$

$$\beta_c = 63.2^\circ$$

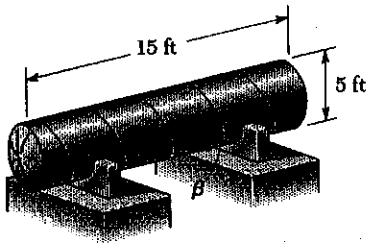
$$\beta_d = 116.8^\circ$$

$$-26.8^\circ \leq \beta \leq 26.8^\circ$$

$$63.2^\circ \leq \beta \leq 116.8^\circ$$



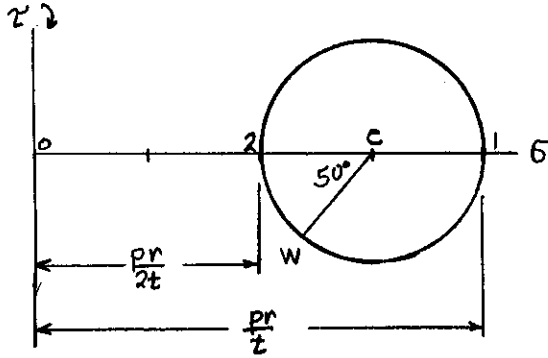
PROBLEM 7.114



7.114 The pressure tank shown has a  $\frac{3}{8}$ -in. wall thickness and butt-welded seams forming an angle  $\beta = 25^\circ$  with a transverse plane. Determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 18 ksi and the allowable shearing stress parallel to the weld is 10 ksi.

SOLUTION

$$d = 5 \text{ ft} = 60 \text{ in.} \quad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$$



$$\sigma_1 = \frac{Pr}{t}$$

$$\sigma_2 = \frac{Pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{Pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{Pr}{t}$$

$$\sigma_w = \sigma_{ave} - R \cos 50^\circ$$

$$= \left( \frac{3}{4} - \frac{1}{4} \cos 50^\circ \right) \frac{Pr}{t}$$

$$= 0.5893 \frac{Pr}{t}$$

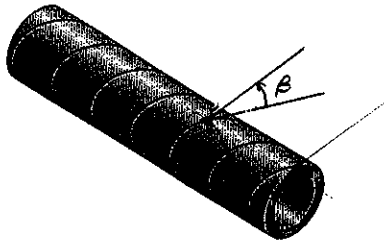
$$P = \frac{\sigma_w t}{0.5893 r} = \frac{(18)(0.375)}{(0.5893)(29.625)} = 0.387 \text{ ksi} = 387 \text{ psi}$$

$$\tau_w = R \sin 50^\circ = 0.19151 \frac{Pr}{t}$$

$$P = \frac{\tau_w t}{0.19151 r} = \frac{(10)(0.375)}{(0.19151)(29.625)} = 0.661 \text{ ksi} = 661 \text{ psi}$$

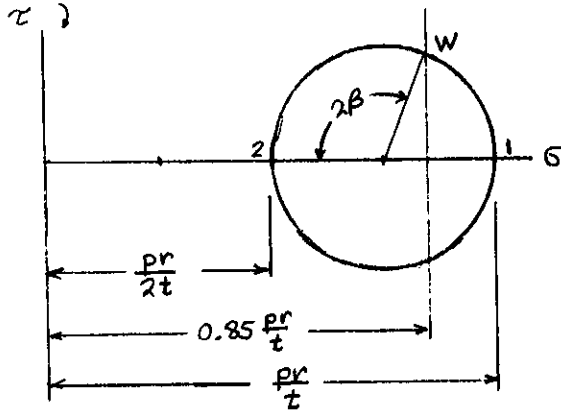
Allowable gage pressure is the smaller value  $p = 387 \text{ psi}$   $\blacktriangleleft$

PROBLEM 7.115



7.115 The pipe shown was fabricated by welding strips of plate along a helix forming an angle  $\beta$  with a transverse plane. Determine the largest value of  $\beta$  that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the pipe.

SOLUTION



$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

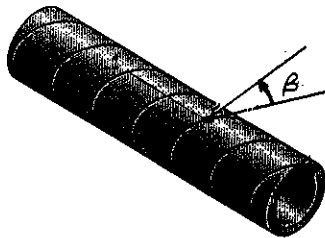
$$\sigma_w = \sigma_{ave} - R \cos 2\beta$$

$$0.85 \frac{pr}{t} = \left( \frac{3}{4} - \frac{1}{4} \cos 2\beta \right) \frac{pr}{t}$$

$$\cos 2\beta = -4 \left( 0.85 - \frac{3}{4} \right) = -0.4$$

$$2\beta = 113.6^\circ \quad \beta = 56.8^\circ$$

PROBLEM 7.116



7.116 The pipe shown has a diameter of 600 mm and was fabricated by welding strips of 10-mm-thick plate along a helix forming an angle  $\beta = 25^\circ$  with a transverse plane. Knowing that the ultimate normal stress perpendicular to the weld is 450 MPa and that a factor of safety of 6.0 is desired, determine the largest allowable gage pressure that can be used.

SOLUTION

$$t = 10 \text{ mm}$$

$$r = \frac{1}{2}d - t = 300 - 10 = 290 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

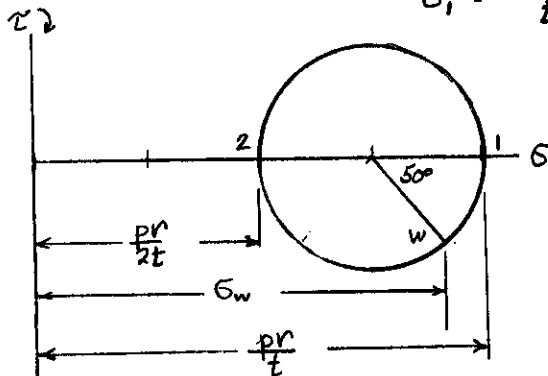
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} + R \cos 50^\circ = 0.9107 \frac{pr}{t}$$

$$\sigma_{w,all} = \frac{\sigma_u}{F.S} = \frac{450}{6} = 75 \text{ MPa}$$

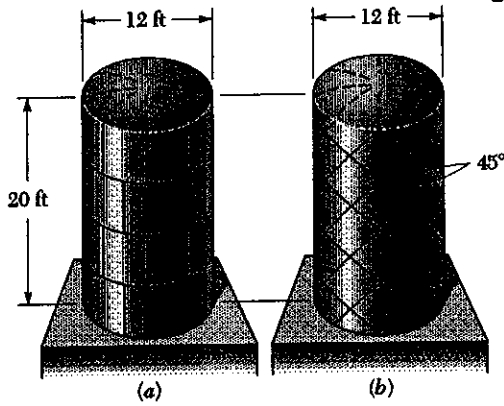
$$0.9107 \frac{pr}{t} = 75$$

$$P = \frac{(75)(10)}{(0.9107)(290)} = 2.84 \text{ MPa}$$



PROBLEM 7.117

7.117 Square plates, each of 0.5-in. thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed-air tank. Knowing that the allowable normal stress perpendicular to the weld is 12 ksi, determine the largest allowable gage pressure in each case.



SOLUTION

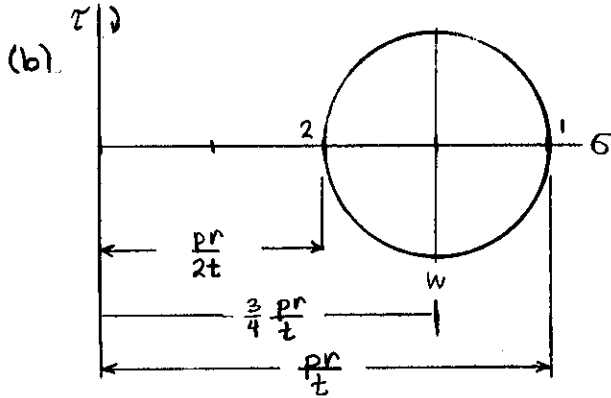
$$d = 12 \text{ ft} = 144 \text{ in} \quad r = \frac{1}{2}d - t = 71.5 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

(a)  $\sigma_1 = 12 \text{ ksi}$

$$p = \frac{\sigma_1 t}{r} = \frac{(12)(0.5)}{71.5} = 0.0839 \text{ ksi} = 83.9 \text{ psi}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4} \frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4} \frac{pr}{t}$$

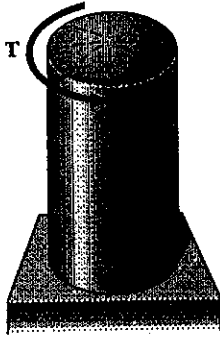
$$\beta = \pm 45^\circ$$

$$\begin{aligned} \sigma_w &= \sigma_{ave} + R \cos \beta \\ &= \frac{3}{4} \frac{pr}{t} \end{aligned}$$

$$p = \frac{4}{3} \frac{\sigma_w t}{r} = \frac{4}{3} \cdot \frac{(12)(0.5)}{71.5} = 0.1119 \text{ ksi} = 111.9 \text{ psi}$$

PROBLEM 7.118

7.118 A torque of magnitude  $T = 12 \text{ kN} \cdot \text{m}$  is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inside diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.



SOLUTION

$d = 180 \text{ mm}$      $r = \frac{1}{2}d = 90 \text{ mm}$      $t = 12 \text{ mm}$

Torsion:  $C_1 = 90 \text{ mm}$      $C_2 = 90 + 12 = 102 \text{ mm}$

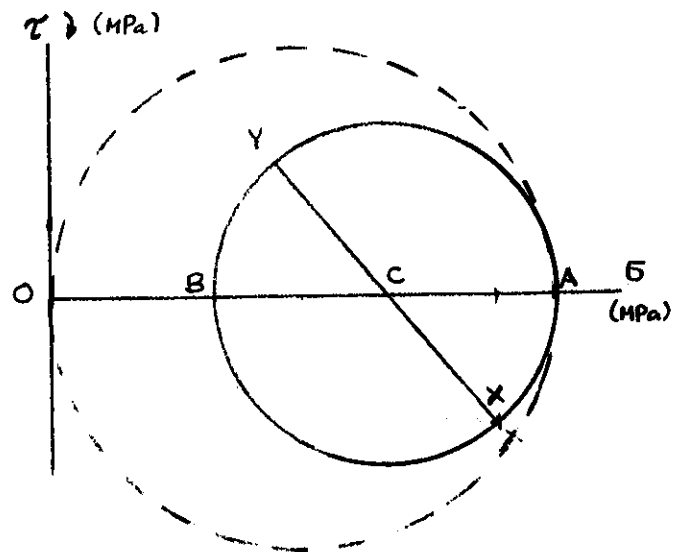
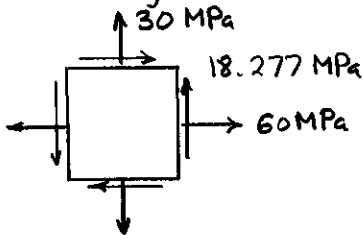
$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$

$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277 \text{ MPa}$

Pressure:  $\sigma_1 = \frac{Pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa}$      $\sigma_2 = \frac{Pr}{2t} = 30 \text{ MPa}$

Summary of stresses

$\sigma_x = 60 \text{ MPa}$ ,  $\sigma_y = 30 \text{ MPa}$ ,  $\tau_{xy} = 18.277 \text{ MPa}$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = 68.64 \text{ MPa}$

$\sigma_b = \sigma_{ave} - R = 21.36 \text{ MPa}$

$\sigma_c \approx 0$

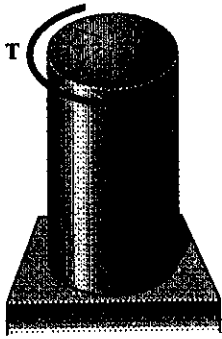
$\sigma_{max} = 68.64 \text{ MPa}$      $\blacktriangleleft$

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 34.32 \text{ MPa}$      $\blacktriangleleft$

PROBLEM 7.119

7.119 The tank shown has a 180-mm inside diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude  $T$  of the applied torque for which the maximum normal stress in the tank is 75 MPa.



SOLUTION

$$r = \frac{1}{2}d = \left(\frac{1}{2}\right)(180) = 90 \text{ mm} \quad t = 12 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60 \text{ MPa} \quad \sigma_2 = \frac{pr}{2t} = 30 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 45 \text{ MPa}$$

$$\sigma_{max} = 75 \text{ MPa} \quad R = \sigma_{max} - \sigma_{ave} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$$

$$\begin{aligned} \tau_{xy} &= \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98 \text{ MPa} \\ &= 25.98 \times 10^6 \text{ Pa} \end{aligned}$$

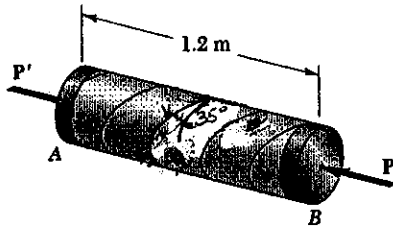
Torsion:  $c_1 = 90 \text{ mm} \quad c_2 = 90 + 12 = 102 \text{ mm}$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\begin{aligned} \tau_{xy} &= \frac{Tc}{J} \quad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N}\cdot\text{m} \\ &= 17.06 \text{ kN}\cdot\text{m} \end{aligned}$$

PROBLEM 7.120

7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe  $AB$  and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces  $P$  and  $P'$  are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.



SOLUTION

$$r = \frac{1}{2}d = 125 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

$$\sigma_2 = \frac{Pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$$

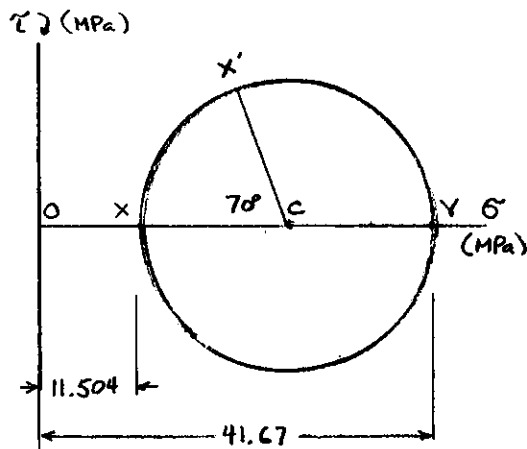
$$r_o = r + t = 125 + 6 = 131 \text{ mm}$$

$$A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P'}{A} = -\frac{45 \times 10^3}{4.825 \times 10^{-3}} = -9.326 \times 10^6 \text{ Pa} = -9.326 \text{ MPa}$$

Total stresses: Longitudinal  $\sigma_x = 20.83 - 9.326 = 11.504 \text{ MPa}$

Circumferential  $\sigma_y = 41.67 \text{ MPa}$



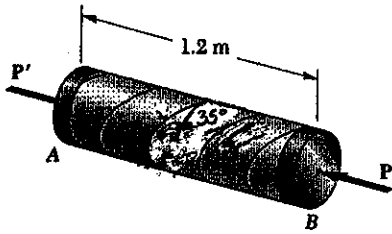
$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 26.585 \text{ MPa}$$

$$R = \frac{\sigma_y - \sigma_x}{2} = 15.081$$

$$\begin{aligned} \text{(a) } \sigma_{x'} &= \sigma_{ave} + R \cos 70^\circ \\ &= 26.585 - 15.081 \cos 70^\circ \\ &= 21.4 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{(b) } \tau_{x'y} &= R \sin 70^\circ = 15.081 \sin 70^\circ \\ &= 14.17 \text{ MPa} \end{aligned}$$

PROBLEM 7.121



7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe  $AB$  and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces  $P$  and  $P'$  are applied to the end plates. Determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

7.121 Solve Prob. 7.120, assuming that the magnitude  $P$  of the two forces is increased to 120 kN.

SOLUTION

$$r = \frac{1}{2}d = 125 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{Pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

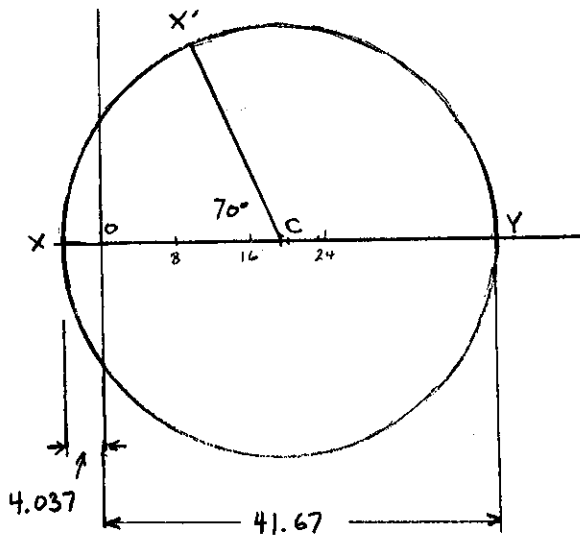
$$\sigma_2 = \frac{Pr}{2t} = 20.833 \text{ MPa}$$

$$r_o = r + t = 125 + 6 = 131 \text{ mm} \quad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{120 \times 10^3}{4.825 \times 10^{-3}} = -24.870 \times 10^6 \text{ Pa} = -24.870 \text{ MPa}$$

Total stresses: Longitudinal  $\sigma_x = 20.833 - 24.870 = -4.037 \text{ MPa}$

Circumferential  $\sigma_y = 41.67 \text{ MPa}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 18.815 \text{ MPa}$$

$$R = \left| \frac{\sigma_x - \sigma_y}{2} \right| = 22.852 \text{ MPa}$$

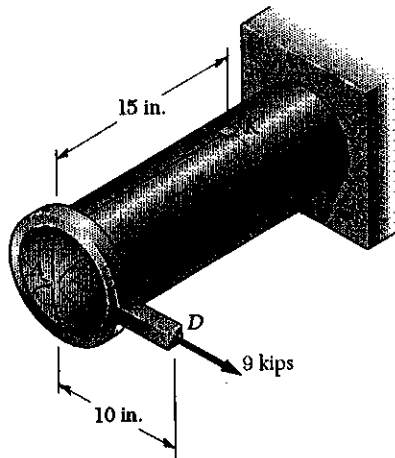
$$\begin{aligned} (a) \sigma_{x'} &= \sigma_{ave} - R \cos 70^\circ \\ &= 18.815 - 22.852 \cos 70^\circ \\ &= 11.00 \text{ MPa} \end{aligned}$$

$$\begin{aligned} (b) \tau_{xy} &= R \sin 70^\circ = 22.852 \sin 70^\circ \\ &= 21.5 \text{ MPa} \end{aligned}$$



PROBLEM 7.122

7.122 The cylindrical tank AB has an 8-in. inside diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point K.



SOLUTION

$$r_i = \frac{d_i}{2} = 4 \text{ in} \quad r_o = r_i + t = 4.32 \text{ in.}$$

$$\sigma_1 = \frac{pr_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$$

Torsion: No applied torque

Bending: Point K lies on neutral axis.

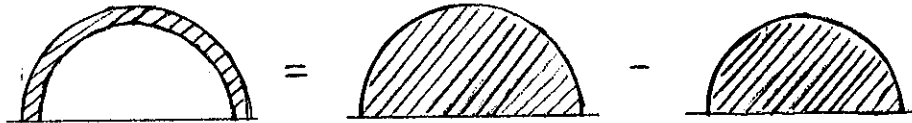
Transverse shear:  $V = 9 \text{ kips}$

For semicircle

$$A = \frac{\pi}{2}r^2$$

$$\bar{y} = \frac{4r}{3\pi}$$

$$Q = \frac{2}{3}r^3$$



$$Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = \frac{2}{3}(4.32^3 - 4^3) = 11.081 \text{ in}^3$$

$$t = (2)(0.32) = 0.64 \text{ in}$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(4.32^4 - 4^4) = 72.481 \text{ in}^4$$

$$\tau = \frac{VQ}{It} = \frac{(9)(11.081)}{(72.481)(0.64)} = 2.15 \text{ ksi}$$

Summary of stresses:

Longitudinal

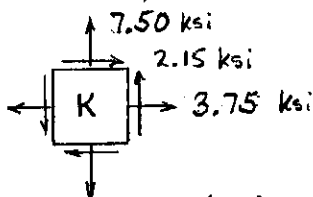
$$\sigma_x = \sigma_1 = 3.75 \text{ ksi}$$

Circumferential

$$\sigma_y = \sigma_2 = 7.50 \text{ ksi}$$

Shear

$$\tau_{xy} = 2.15 \text{ ksi}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.625 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 2.853 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 8.48 \text{ ksi}$$

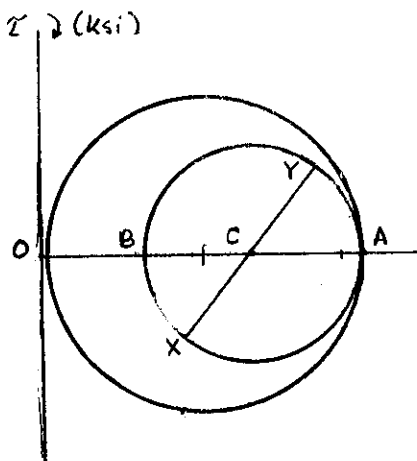
$$\sigma_b = \sigma_{ave} - R = 2.77 \text{ ksi}$$

$$\sigma_z = 0$$

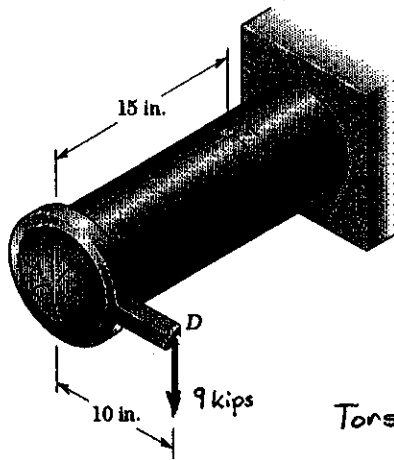
$$\sigma_{max} = 8.48 \text{ ksi}$$

$$\sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 4.24 \text{ ksi}$$



PROBLEM 7.123



7.122 The cylindrical tank  $AB$  has an 8-in. inside diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point  $K$ .

7.123 Solve Prob. 7.122, assuming that the 9-kip force applied at point  $D$  is directed vertically downward.

SOLUTION

$$r_i = \frac{d_i}{2} = 4 \text{ in.} \quad r_o = r_i + t = 4.32 \text{ in}$$

$$\sigma_1 = \frac{pr_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$$

$$\sigma_2 = \frac{1}{2} \sigma_1 = 3.75 \text{ ksi}$$

Torsion:  $J = \frac{\pi}{2}(r_o^4 - r_i^4) = 144.96 \text{ in}^4 \quad c = r_o = 4.32 \text{ in}$

$$T = (9)(10) = 90 \text{ kip}\cdot\text{in}$$

$$\tau = \frac{Tc}{J} = \frac{(90)(4.32)}{144.96} = 2.68 \text{ ksi}$$

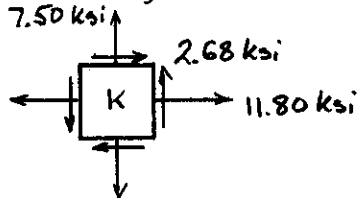
Bending:  $I = \frac{1}{2} J = 72.48 \text{ in}^4 \quad c = r_o = 4.32 \text{ in}$

$$M = (9)(15) = 135 \text{ kip}\cdot\text{in}$$

$$\sigma_m = \frac{Mc}{I} = \frac{(135)(4.32)}{72.48} = 8.05 \text{ ksi}$$

Transverse shear: At point  $K$ ,  $VQ/It = 0$

Summary of stresses:



longitudinal  $\sigma_x = \sigma_1 = 3.75 + 8.05 = 11.80 \text{ ksi}$

Circumferential  $\sigma_y = \sigma_2 = 7.50 \text{ ksi}$

Shear  $\tau_{xy} = 2.68 \text{ ksi}$

$$\sigma_{ave} = \frac{1}{2}(11.80 + 7.50) = 9.65 \text{ ksi}$$

$$R = \sqrt{\left(\frac{11.80 - 7.50}{2}\right)^2 + (2.68)^2} = 3.44 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 13.09 \text{ ksi}$$

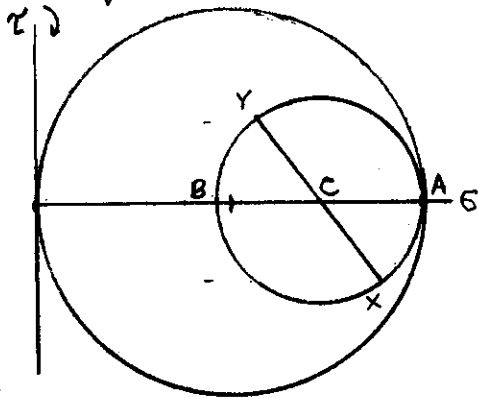
$$\sigma_b = \sigma_{ave} - R = 6.21 \text{ ksi}$$

$$\sigma_2 = 0$$

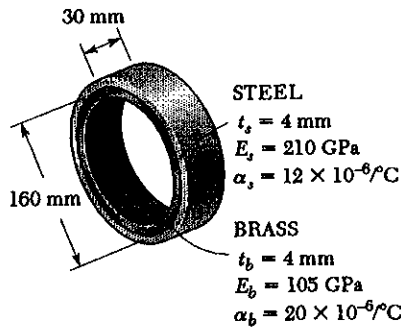
$$\sigma_{max} = 13.09 \text{ ksi}$$

$$\sigma_{min} = 0$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.54 \text{ ksi}$$



PROBLEM 7.124



7.124 A brass ring of 160-mm outside diameter fits exactly inside a steel ring of 160-mm inside diameter when the temperature of both rings is 5° C. Knowing that the temperature of the rings is then raised to 55° C, determine (a) the tensile stress in the steel ring, (b) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let  $p$  be the contact pressure between the rings. Subscript  $s$  refers to the steel ring. Subscript  $b$  refers to the brass ring.

Steel ring: Internal pressure  $p$ ,  $\sigma_s = \frac{pr}{t_s}$  (1)

Corresponding strain  $\epsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$   
 strain due to temperature change  $\epsilon_{sT} = \alpha_s \Delta T$   
 Total strain  $\epsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference  
 $\Delta L_s = 2\pi r \epsilon_s = 2\pi r \left( \frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$

Brass ring: External pressure  $p$ ,  $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains  $\epsilon_{bp} = -\frac{pr}{E_b t_b}$ ,  $\epsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference  
 $\Delta L_b = 2\pi r \epsilon_b = 2\pi r \left( -\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$

Equating  $\Delta L_s$  to  $\Delta L_b$   $\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$

$\left( \frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T$  (2)

Data  $\Delta T = 55^\circ C - 5^\circ = 50^\circ C$

$r = \frac{1}{2} d = 80 \text{ mm}$

From eq. (2)  $\left\{ \frac{80 \times 10^{-3}}{(210 \times 10^9)(4 \times 10^{-3})} + \frac{80 \times 10^{-3}}{(105 \times 10^9)(4 \times 10^{-3})} \right\} p = (8 \times 10^{-6})(50)$

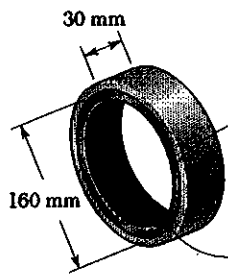
$285.71 \times 10^{12} p = 400 \times 10^{-6}$ ,  $p = 1.4 \times 10^6 \text{ Pa}$

From eq. (1)  $\sigma_s = \frac{pr}{t_s} = \frac{(1.4 \times 10^6)(80 \times 10^{-3})}{4 \times 10^{-3}} = 28 \times 10^6 \text{ Pa}$

(a)  $\sigma_s = 28.0 \text{ MPa}$

(b)  $p = 1.400 \text{ MPa}$

PROBLEM 7.125



STEEL

$$t_s = 4 \text{ mm}$$

$$E_s = 210 \text{ GPa}$$

$$\alpha_s = 12 \times 10^{-6} / ^\circ\text{C}$$

BRASS

$$t_b = 4 \text{ mm}$$

$$E_b = 105 \text{ GPa}$$

$$\alpha_b = 20 \times 10^{-6} / ^\circ\text{C}$$

7.124 A brass ring of 160-mm outside diameter fits exactly inside a steel ring of 160-mm inside diameter when the temperature of both rings is 5° C. Knowing that the temperature of the rings is then raised to 55° C, determine (a) the tensile stress in the steel ring, (b) the corresponding pressure exerted by the brass ring on the steel ring.

7.125 Solve Prob. 7.124, assuming that the thickness of the brass ring is  $t_b = 6$  mm.

SOLUTION

Let  $p$  be the contact pressure between the rings. Subscript  $s$  refers to the steel ring. Subscript  $b$  refers to the brass ring.

Steel ring: Internal pressure  $p$ ,  $\sigma_s = \frac{pr}{t_s}$  (1)

Corresponding strain  $\epsilon_s = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

strain due to temperature change  $\epsilon_{sT} = \alpha_s \Delta T$

Total strain  $\epsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \epsilon_s = 2\pi r \left( \frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure  $p$ ,  $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains  $\epsilon_{bp} = -\frac{pr}{E_b t_b}$ ,  $\epsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \epsilon_b = 2\pi r \left( -\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating  $\Delta L_s$  to  $\Delta L_b$   $\frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left( \frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \quad (2)$$

Data:  $\Delta T = 55^\circ\text{C} - 5^\circ\text{C} = 50^\circ\text{C}$   $t_b = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

$r = \frac{1}{2}d = 80 \text{ mm}$

From eq. (2)  $\left\{ \frac{(80 \times 10^3)}{(210 \times 10^9)(4 \times 10^{-3})} + \frac{(80 \times 10^{-3})}{(105 \times 10^9)(6 \times 10^{-3})} \right\} p = (8 \times 10^{-6})(50)$

$$222.22 \times 10^{-12} p = 400 \times 10^{-6}; \quad p = 1.8 \times 10^6 \text{ Pa}$$

From eq. (1)  $\sigma_s = \frac{pr}{t_s} = \frac{(1.8 \times 10^6)(80 \times 10^{-3})}{4 \times 10^{-3}} = 36 \times 10^6 \text{ Pa}$

$$\sigma_s = 36.0 \text{ MPa}$$

$$p = 1.800 \text{ MPa}$$

PROBLEM 7.126

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the given angle  $\theta$ .

$$\epsilon_x = -720 \mu, \quad \epsilon_y = 0, \quad \gamma_{xy} = +300 \mu, \quad \theta = -30^\circ$$

SOLUTION

$$\frac{\epsilon_x + \epsilon_y}{2} = -360 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -360 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -360 - 360 \cos(-60^\circ) + \frac{300}{2} \sin(-60^\circ) \right\} \mu = -670 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -360 - (-360) \cos(-60^\circ) - \frac{300}{2} \sin(-60^\circ) \right\} \mu = -50 \mu \end{aligned}$$

$$\begin{aligned} \gamma_{x'y'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(-720 - 0) \sin(-60^\circ) + 300 \cos(-60^\circ) \right\} \mu = -474 \mu \end{aligned}$$

PROBLEM 7.127

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the given angle  $\theta$ .

$$\epsilon_x = 0 \quad \epsilon_y = +320 \mu \quad \gamma_{xy} = -100 \mu \quad \theta = 30^\circ$$

SOLUTION

$$\frac{\epsilon_x + \epsilon_y}{2} = 160 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -160 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 160 - 160 \cos 60^\circ - \frac{100}{2} \sin 60^\circ \right\} \mu = +36.7 \mu \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ 160 + 160 \cos 60^\circ + \frac{100}{2} \sin 60^\circ \right\} \mu = +283 \mu \end{aligned}$$

$$\begin{aligned} \gamma_{x'y'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(0 - 320) \sin 60^\circ - 100 \cos 60^\circ \right\} \mu = +227 \mu \end{aligned}$$

## PROBLEM 7.128

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes  $x$  and  $y'$  rotated through the given angle  $\theta$

$$\epsilon_x = -800 \mu, \quad \epsilon_y = +450 \mu, \quad \gamma_{xy} = +200 \mu, \quad \theta = -25^\circ$$

## SOLUTION

$$\frac{\epsilon_x + \epsilon_y}{2} = -175 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -625 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -175 - 625 \cos(-50^\circ) + \frac{200}{2} \sin(-50^\circ) \right\} \mu = -653 \mu \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -175 + 625 \cos(-50^\circ) - \frac{200}{2} \sin(-50^\circ) \right\} \mu = +303 \mu \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \gamma_{x'y'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(-800 - 450) \sin(-50^\circ) + 200 \cos(-50^\circ) \right\} \mu = -829 \mu \quad \blacktriangleleft \end{aligned}$$

## PROBLEM 7.129

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the given angle  $\theta$

$$\epsilon_x = -500 \mu, \quad \epsilon_y = +250 \mu, \quad \gamma_{xy} = 0, \quad \theta = 15^\circ$$

## SOLUTION

$$\frac{\epsilon_x + \epsilon_y}{2} = -125 \mu \quad \frac{\epsilon_x - \epsilon_y}{2} = -375 \mu$$

$$\begin{aligned} \epsilon_{x'} &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -125 - 375 \cos 30^\circ + 0 \right\} \mu = -450 \mu \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} \epsilon_{y'} &= \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \\ &= \left\{ -125 + 375 \cos 30^\circ - 0 \right\} \mu = +200 \mu \quad \blacktriangleleft \end{aligned}$$

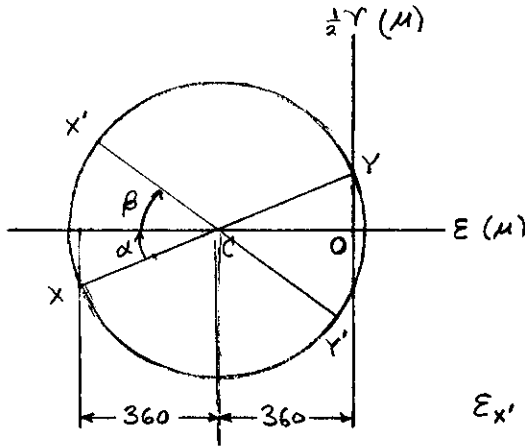
$$\begin{aligned} \gamma_{x'y'} &= -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta \\ &= \left\{ -(-500 - 250) \sin 30^\circ + 0 \right\} \mu = +375 \mu \quad \blacktriangleleft \end{aligned}$$

**PROBLEM 7.130**

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the angle  $\theta$

$E_x = -720 \mu, E_y = 0, \gamma_{xy} = +300 \mu, \theta = -30^\circ$

**SOLUTION**



Plotted points

$X: (-720 \mu, -150 \mu)$

$Y: (0, 150 \mu)$

$C: (-360 \mu, 0)$

$\tan \alpha = \frac{150 \mu}{360 \mu} \quad \alpha = 22.62^\circ$

$R = \sqrt{(360 \mu)^2 + (150 \mu)^2} = 390 \mu$

$\beta = 2\theta - \alpha = 60^\circ - 22.62^\circ = 37.38^\circ$

$E_{x'} = E_{ave} - R \cos \beta = -360 \mu - 390 \mu \cos 37.38^\circ = -670 \mu$

$E_{y'} = E_{ave} + R \cos \beta = -360 \mu + 390 \mu \cos 37.38^\circ = -50 \mu$

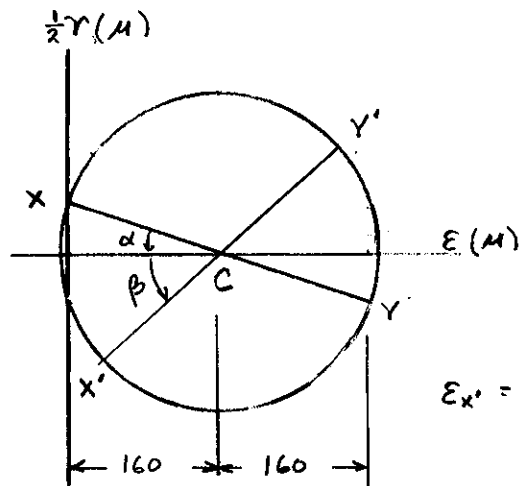
$\frac{\gamma_{x'y'}}{2} = -R \sin \beta = -390 \mu \sin 37.38^\circ$   
 $\gamma_{x'y'} = -474 \mu$

**PROBLEM 7.131**

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the angle  $\theta$

$E_x = 0, E_y = +320 \mu, \gamma_{xy} = -100 \mu, \theta = 30^\circ$

**SOLUTION**



Plotted points

$X: (0, 50 \mu)$

$Y: (320 \mu, -50 \mu)$

$C: (160 \mu, 0)$

$\tan \alpha = \frac{50}{160} \quad \alpha = 17.35^\circ$

$R = \sqrt{(160 \mu)^2 + (50 \mu)^2} = 167.63 \mu$

$\beta = 2\theta - \alpha = 60^\circ - 17.35^\circ = 42.65^\circ$

$E_{x'} = E_{ave} - R \cos \beta = 160 \mu - 167.63 \mu \cos 42.65^\circ = -36.7 \mu$

$E_{y'} = E_{ave} + R \cos \beta = 160 \mu + 167.63 \mu \cos 42.65^\circ = 283 \mu$

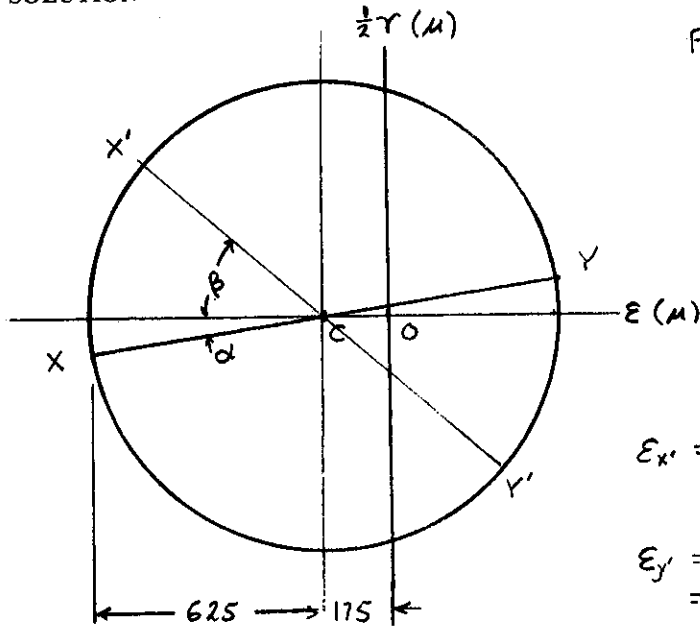
$\frac{\gamma_{x'y'}}{2} = R \sin \beta = 167.63 \mu \sin 42.65^\circ$   
 $\gamma_{x'y'} = 227 \mu$

PROBLEM 7.132

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the angle  $\theta$ .

$E_x = -800 \mu$     $E_y = 450 \mu$     $\gamma_{xy} = +200 \mu$     $\theta = -25^\circ$

SOLUTION



Plotted points

$X: (-800 \mu, -100 \mu)$

$Y: (+450 \mu, +100 \mu)$

$C: (-175 \mu, 0)$

$\tan \alpha = \frac{100}{625}$     $\alpha = 9.09^\circ$

$R = \sqrt{(625 \mu)^2 + (100 \mu)^2} = 632.95 \mu$

$\beta = 2\theta - \alpha = 50^\circ - 9.09^\circ = 40.91^\circ$

$E_{x'} = E_{ave} - R \cos \beta = -175 \mu - 632.95 \mu \cos 40.91^\circ$   
 $= -653 \mu$     $\blacktriangleleft$

$E_{y'} = E_{ave} + R \cos \beta = -175$   
 $= +303 \mu$     $\blacktriangleleft$

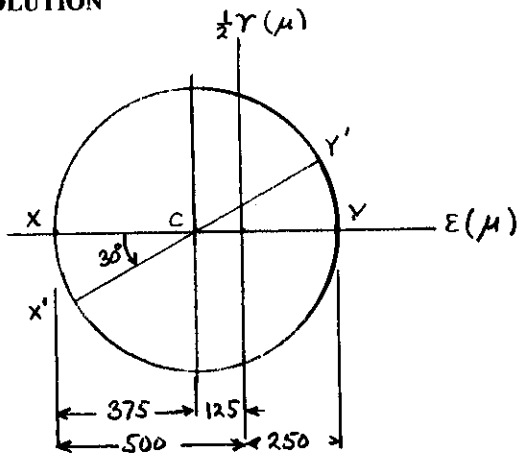
$\frac{\gamma_{x'y'}}{2} = -R \sin \beta = -632.95 \mu \sin 40.91^\circ$     $\gamma_{x'y'} = -829 \mu$     $\blacktriangleleft$

PROBLEM 7.133

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes  $x'$  and  $y'$  rotated through the angle  $\theta$ .

$E_x = -500 \mu$ ,    $E_y = +250 \mu$ ,    $\gamma_{xy} = 0$ ,    $\theta = 15^\circ$

SOLUTION



Plotted points

$X: (-500 \mu, 0)$

$Y: (+250 \mu, 0)$

$C: (-125 \mu, 0)$

$R = 375 \mu$

$E_{x'} = E_{ave} - R \cos 2\theta = -125 - 375 \cos 30^\circ$   
 $= -450 \mu$     $\blacktriangleleft$

$E_{y'} = E_{ave} + R \cos 2\theta = -125 + 375 \cos 30^\circ$   
 $= 200 \mu$     $\blacktriangleleft$

$\frac{1}{2} \gamma_{xy} = R \sin 2\theta = 375 \sin 30^\circ$

$\gamma_{x'y'} = 375 \mu$     $\blacktriangleleft$



PROBLEM 7.134

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use  $\nu = \frac{1}{3}$ )

$$\epsilon_x = +160 \mu \quad \epsilon_y = -480 \mu \quad \gamma_{xy} = -600 \mu$$

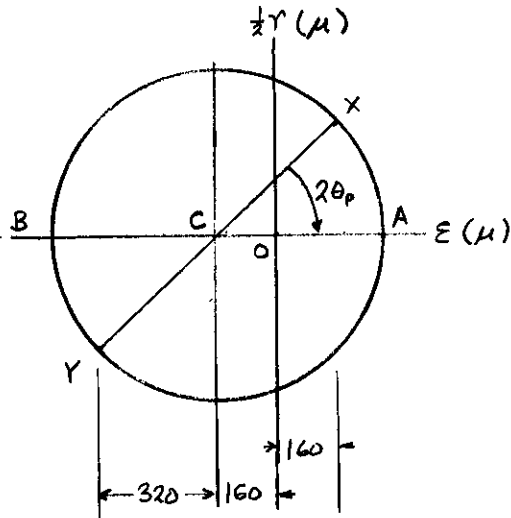
SOLUTION

(a) For Mohr's circle of strain, plot points

$$X: (160 \mu, 300 \mu)$$

$$Y: (-480 \mu, -300 \mu)$$

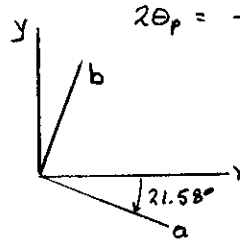
$$C: (-160 \mu, 0)$$



$$(a) \tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-300}{320} = -0.9375$$

$$2\theta_p = -43.15^\circ \quad \theta_p = -21.58^\circ$$

and  $-21.58 + 90 = 68.42^\circ$



$$\theta_a = -21.58^\circ$$

$$\theta_b = 68.42^\circ$$

$$R = \sqrt{(320 \mu)^2 + (300 \mu)^2} = 438.6 \mu$$

$$\epsilon_a = \epsilon_{ave} + R = -160 \mu + 438.6 \mu = +278.6 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -160 \mu - 438.6 \mu = -598.6 \mu$$

$$(b) \frac{1}{2} \gamma_{(\max, \text{in-plane})} = R \quad \gamma_{(\max, \text{in-plane})} = 2R = 877 \mu$$

$$(c) \epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (160 \mu - 480 \mu) = 160 \mu$$

$$\epsilon_{\max} = 278.6 \mu \quad \epsilon_{\min} = -598.6 \mu$$

$$\gamma_{\max} = \epsilon_{\max} - \epsilon_{\min} = 278.6 \mu + 598.6 \mu = 877 \mu$$

PROBLEM 7.135

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use  $\nu = \frac{1}{3}$ )

$$\epsilon_x = -260 \mu \quad \epsilon_y = -60 \mu \quad \gamma_{xy} = +480 \mu$$

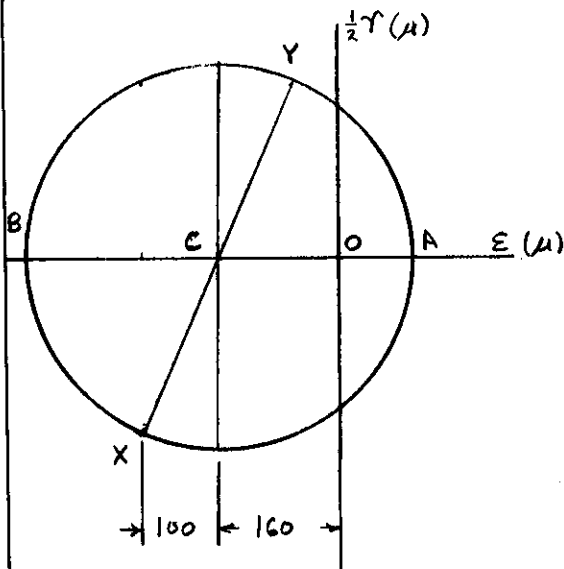
SOLUTION

For Mohr's circle of strain plot points

$$X: (-260 \mu, -240 \mu)$$

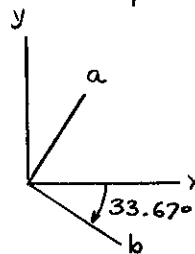
$$Y: (-60 \mu, 240 \mu)$$

$$C: (-160 \mu, 0)$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{480}{-260 - 60} = -2.4$$

$$2\theta_p = -67.38^\circ \quad \theta_b = -33.67^\circ \quad \theta_a = 56.31^\circ$$



$$R = \sqrt{(100 \mu)^2 + (240 \mu)^2}$$

$$R = 260 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = -160 \mu + 260 \mu = 100 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -160 \mu - 260 \mu = -420 \mu$$

$$(b) \quad \frac{1}{2} \gamma_{max, (in-plane)} = R \quad \gamma_{max, (in-plane)} = 2R = 520 \mu$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (-260 - 60) = 160 \mu$$

$$\epsilon_{max} = 160 \mu \quad \epsilon_{min} = -420 \mu$$

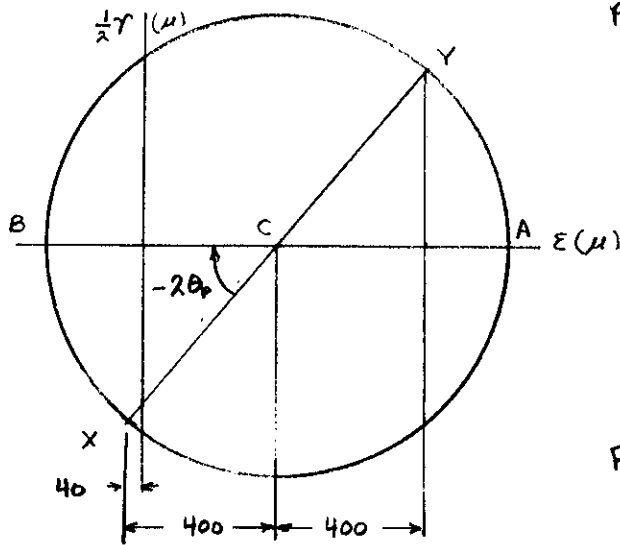
$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 160 \mu + 420 \mu = 580 \mu$$

PROBLEM 7.136

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use  $\nu = \frac{1}{3}$ )

$$\epsilon_x = -40 \mu \quad \epsilon_y = 760 \mu \quad \gamma_{xy} = +960 \mu$$

SOLUTION



Plotted points

$$X: (-40 \mu, -480 \mu)$$

$$Y: (760 \mu, +480 \mu)$$

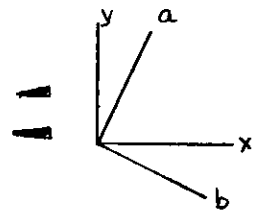
$$C: (360 \mu, 0)$$

$$\tan 2\theta_p = \frac{-480}{400} = -1.2$$

$$2\theta_p = -50.19^\circ$$

$$\theta_b = -25.10^\circ$$

$$\theta_a = 64.90^\circ$$



$$R = \sqrt{(400 \mu)^2 + (480 \mu)^2} = 624.8 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 360 \mu + 624.8 \mu = 985 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = 360 \mu - 624.8 \mu = -265 \mu$$

$$(b) \quad \gamma_{max(in-plane)} = 2R = 1250 \mu$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (-40 \mu + 760 \mu) = -360 \mu$$

$$\epsilon_{max} = 985 \mu \quad \epsilon_{min} = -265 \mu$$

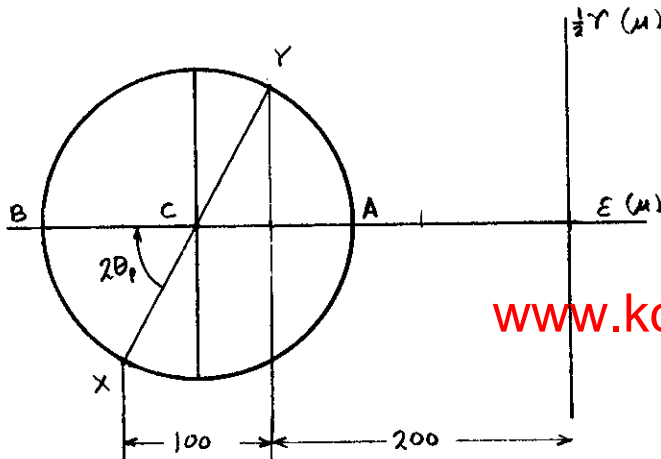
$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 985 + 265 = 1250 \mu$$

PROBLEM 7.137

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain. (Use  $\nu = \frac{1}{3}$ )

$$\epsilon_x = -300 \mu \quad \epsilon_y = -200 \mu \quad \gamma_{xy} = +175 \mu$$

SOLUTION



Plotted points.

$$X: (-300 \mu, -87.5 \mu)$$

$$Y: (-200 \mu, +87.5 \mu)$$

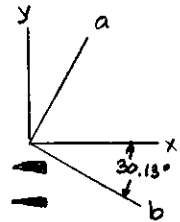
$$C: (-250 \mu, 0)$$

$$\tan 2\theta_p = -\frac{87.5}{50}$$

$$2\theta_p = -60.26$$

$$\theta_b = -30.13^\circ$$

$$\theta_a = 59.87^\circ$$



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$$R = \sqrt{(50 \mu)^2 + (87.5 \mu)^2} = 100.8$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = -250 \mu + 100.8 \mu = -149.2 \mu$$

$$\epsilon_b = \epsilon_{ave} - R = -250 \mu - 100.8 \mu = -351 \mu$$

$$(b) \quad \gamma_{max (in-plane)} = 2R = 201.6 \mu$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu} (\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3} (-300 \mu - 200 \mu) = +250 \mu$$

$$\epsilon_{max} = 250 \mu \quad \epsilon_{min} = -351 \mu$$

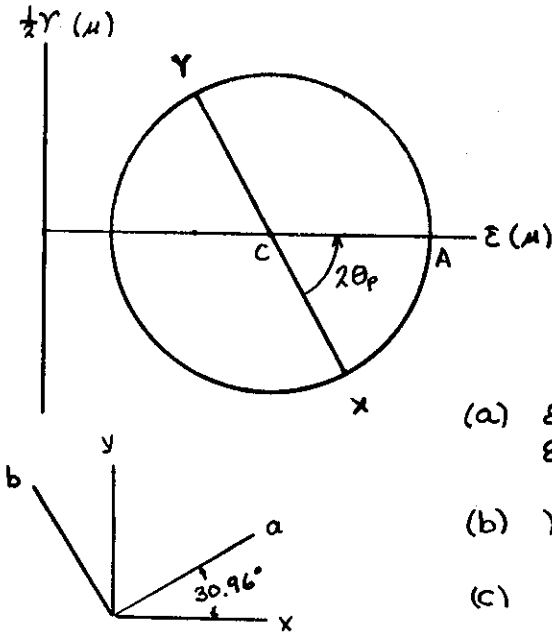
$$(c) \quad \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 250 \mu + 351 \mu = 601 \mu$$

PROBLEM 7.140

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$\epsilon_x = +400 \mu \quad \epsilon_y = +200 \mu \quad \gamma_{xy} = +375 \mu$$

SOLUTION



Plotted points

$$X: (+400\mu, -187.5\mu)$$

$$Y: (+200\mu, +187.5\mu)$$

$$C: (+300\mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{375}{400 - 200} = 1.875$$

$$2\theta_p = 61.93^\circ \quad \theta_a = 30.96^\circ, \theta_b = 120.96^\circ$$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 300\mu + 212.5\mu = 512.5\mu \quad \blacktriangleleft$$

$$\epsilon_b = \epsilon_{ave} - R = 300\mu - 212.5\mu = 87.5\mu \quad \blacktriangleleft$$

$$(b) \quad \gamma_{max (in-plane)} = 2R = 425\mu \quad \blacktriangleleft$$

$$(c) \quad \epsilon_c = 0 \quad \epsilon_{max} = 512.5\mu \quad \epsilon_{min} = 0$$

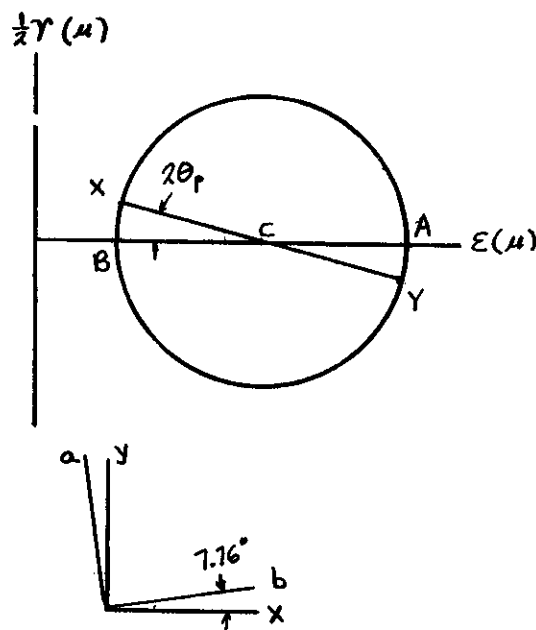
$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 512.5\mu \quad \blacktriangleleft$$

PROBLEM 7.141

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$\epsilon_x = +60 \mu \quad \epsilon_y = +240 \mu \quad \gamma_{xy} = -50 \mu$$

SOLUTION



Plotted points

$$X: (60\mu, 25\mu) \quad C: (150\mu, 0)$$

$$Y: (240\mu, -25\mu)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-50}{60 - 240} = 0.277778$$

$$2\theta_p = 15.52^\circ \quad \theta_b = 7.76^\circ, \theta_a = 97.76^\circ$$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4 \mu$$

$$(a) \quad \epsilon_a = \epsilon_{ave} + R = 150\mu + 93.4\mu = 243.4\mu \quad \blacktriangleleft$$

$$\epsilon_b = \epsilon_{ave} - R = 150\mu - 93.4\mu = 56.6\mu \quad \blacktriangleleft$$

$$(b) \quad \gamma_{max (in-plane)} = 2R = 186.8 \mu \quad \blacktriangleleft$$

$$(c) \quad \epsilon_c = 0 \quad \epsilon_{max} = 243.4\mu \quad \epsilon_{min} = 0$$

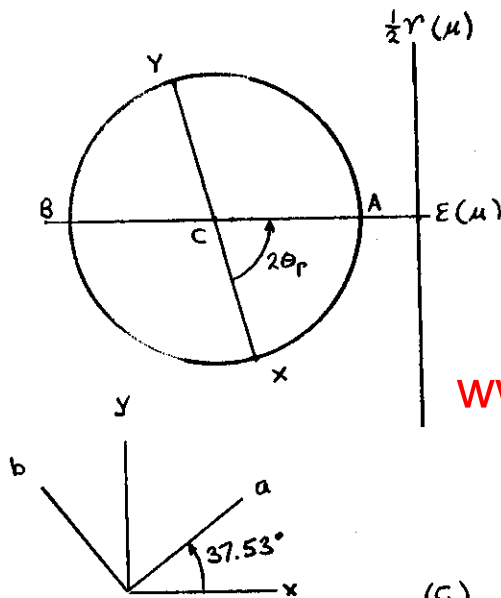
$$\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 243.4 \quad \blacktriangleleft$$

**PROBLEM 7.138**

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$E_x = -90 \mu \quad E_y = -130 \mu \quad \gamma_{xy} = +150 \mu$$

SOLUTION



Plot points

$$X: (-90 \mu, -75 \mu) \quad Y: (-130 \mu, 75 \mu) \\ C: (-110 \mu, 0)$$

$$(a) \tan 2\theta_p = \frac{\gamma_{xy}}{E_x - E_y} = \frac{150}{40} = 3.75$$

$$2\theta_p = 75.07^\circ \quad \theta_a = 37.53^\circ \quad \theta_b = 127.53^\circ$$

$$R = \sqrt{(20 \mu)^2 + (75 \mu)^2} = 77.6 \mu$$

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$$E_a = E_{ave} + R = -110 \mu + 77.6 \mu = -32.4 \mu$$

$$E_b = E_{ave} - R = -110 \mu - 77.6 \mu = -187.6 \mu$$

$$(b) \gamma_{max(in-plane)} = 2R = 155.2 \mu$$

$$(c) E_c = 0 \quad E_{max} = 0, \quad E_{min} = -187.6 \mu$$

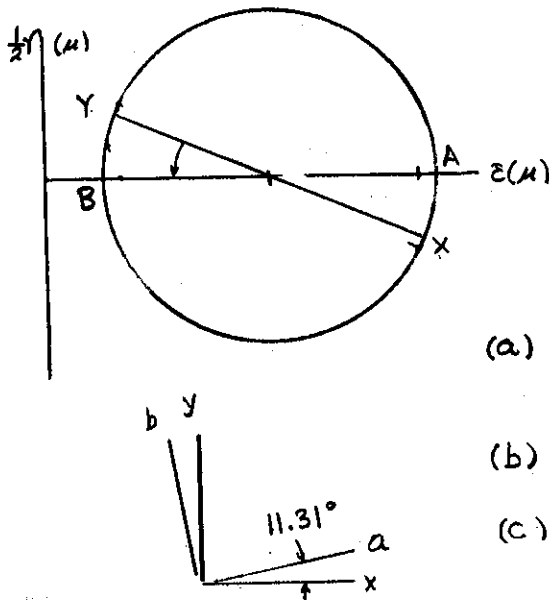
$$\gamma_{max} = E_{max} - E_{min} = 0 + 187.6 \mu = 187.6 \mu$$

**PROBLEM 7.139**

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane strain, (c) the maximum shearing strain.

$$E_x = +375 \mu \quad E_y = +75 \mu \quad \gamma_{xy} = +125 \mu$$

SOLUTION



$$X: (375 \mu, -62.5 \mu) \quad Y: (75 \mu, 62.5 \mu) \\ C: (225 \mu, 0)$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{E_x - E_y} = \frac{125}{375 - 75} = 22.62^\circ$$

$$\theta_a = 11.31^\circ \quad \theta_b = 101.31^\circ$$

$$R = \sqrt{(150 \mu)^2 + (62.5 \mu)^2} = 162.5 \mu$$

$$(a) E_a = E_{ave} + R = 225 \mu + 162.5 \mu = 387.5 \mu$$

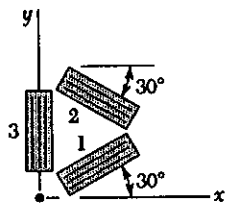
$$E_b = E_{ave} - R = 225 \mu - 162.5 \mu = 62.5 \mu$$

$$(b) \gamma_{max(in-plane)} = 2R = 325 \mu$$

$$(c) E_c = 0 \quad E_{max} = 387.5 \mu \quad E_{min} = 0$$

$$\gamma_{max} = E_{max} - E_{min} = 387.5 \mu$$

PROBLEM 7.142



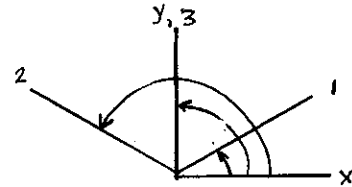
7.142 The strains determined by use of the rosette shown during the test of a rocker arm are:

$$\epsilon_1 = +600 \mu \quad \epsilon_2 = +450 \mu \quad \epsilon_3 = -75 \mu$$

Determine (a) the in-plane principal strains, (b) the in-plane maximum shearing strain.

SOLUTION

$$\begin{aligned} \theta_1 &= 30^\circ \\ \theta_2 &= 150^\circ \\ \theta_3 &= 90^\circ \end{aligned}$$



$$\begin{aligned} \epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 &= \epsilon_1 \\ 0.75 \epsilon_x + 0.25 \epsilon_y + 0.43301 \gamma_{xy} &= 600 \mu \quad (1) \end{aligned}$$

$$\begin{aligned} \epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 &= \epsilon_2 \\ 0.75 \epsilon_x + 0.25 \epsilon_y - 0.43301 \gamma_{xy} &= 450 \mu \quad (2) \end{aligned}$$

$$\begin{aligned} \epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 &= \epsilon_3 \\ 0 + \epsilon_y + 0 &= -75 \mu \quad (3) \end{aligned}$$

Solving (1), (2), and (3) simultaneously

$$\epsilon_x = 725 \mu, \quad \epsilon_y = -75 \mu, \quad \gamma_{xy} = 173.21 \mu$$

$$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 325 \mu$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 + 75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3 \mu$$

(a)  $\epsilon_a = \epsilon_{ave} + R = 734 \mu$

$\epsilon_b = \epsilon_{ave} - R = -84.3 \mu$

(b)  $\gamma_{max(in-plane)} = 2R = 819 \mu$

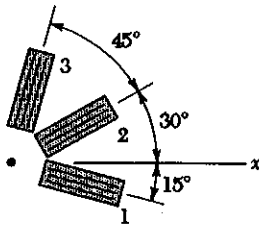
PROBLEM 7.143

7.143 Determine the strain  $\epsilon_x$ , knowing that the following strains have been determined by use of the rosette shown:

$$\epsilon_1 = +720 \times 10^{-6} \text{ in./in.}$$

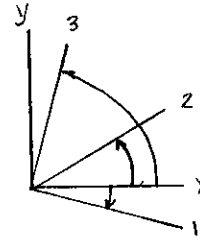
$$\epsilon_2 = -180 \times 10^{-6} \text{ in./in.}$$

$$\epsilon_3 = +120 \times 10^{-6} \text{ in./in.}$$



SOLUTION

$$\begin{aligned} \theta_1 &= -15^\circ \\ \theta_2 &= 30^\circ \\ \theta_3 &= 75^\circ \end{aligned}$$



$$\begin{aligned} \epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 &= \epsilon_1 \\ 0.9330 \epsilon_x + 0.06699 \epsilon_y - 0.25 \gamma_{xy} &= 720 \times 10^{-6} \quad (1) \end{aligned}$$

$$\begin{aligned} \epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 &= \epsilon_2 \\ 0.75 \epsilon_x + 0.25 \epsilon_y + 0.4330 \gamma_{xy} &= -180 \times 10^{-6} \quad (2) \end{aligned}$$

$$\begin{aligned} \epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 &= \epsilon_3 \\ 0.06699 \epsilon_x + 0.9330 \epsilon_y + 0.25 \gamma_{xy} &= 120 \times 10^{-6} \quad (3) \end{aligned}$$

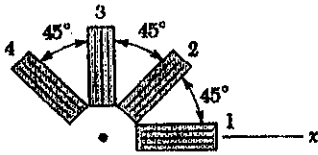
Solving (1), (2), and (3) simultaneously

$$\epsilon_x = 380 \times 10^{-6} \text{ in./in.}, \quad \epsilon_y = 460 \times 10^{-6} \text{ in./in.}, \quad \gamma_{xy} = -1339 \times 10^{-6} \text{ in./in.}$$

$$\epsilon_x = 380 \times 10^{-6} \text{ in./in.}$$



PROBLEM 7.144



7.144 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$$\epsilon_1 = +420 \mu \quad \epsilon_2 = -45 \mu \quad \epsilon_4 = +165 \mu$$

(a) What should be the reading of gage 3? (b) Determine the principal strains and the maximum in-plane shearing strain.

SOLUTION

(a) Gages 2 and 4 are  $90^\circ$  apart  $E_{ave} = \frac{1}{2} (\epsilon_2 + \epsilon_4)$

$$E_{ave} = \frac{1}{2} (-45 \mu + 165 \mu) = 60 \mu$$

Gages 1 and 3 are also  $90^\circ$  apart  $E_{ave} = \frac{1}{2} (\epsilon_1 + \epsilon_3)$

$$\epsilon_3 = 2E_{ave} - \epsilon_1 = (2)(60 \mu) - 420 \mu = -300 \mu$$

(b)  $E_x = \epsilon_1 = 420 \mu$   $E_y = \epsilon_3 = -300 \mu$

$$\begin{aligned} \gamma_{xy} &= 2\epsilon_2 - \epsilon_x - \epsilon_y = (2)(-45 \mu) - 420 \mu + 300 \mu \\ &= -210 \mu \end{aligned}$$

$$\begin{aligned} R &= \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420 \mu + 300 \mu}{2}\right)^2 + \left(\frac{-210 \mu}{2}\right)^2} \\ &= 375 \mu \end{aligned}$$

$$E_a = E_{ave} + R = 60 \mu + 375 \mu = 435 \mu$$

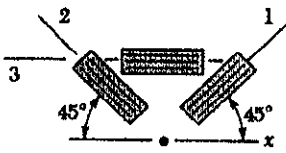
$$E_b = E_{ave} - R = 60 \mu - 375 \mu = -315 \mu$$

$$\tau_{max(in-plane)} = 2R = 750 \mu$$

PROBLEM 7.145

7.145 Determine the largest in-plane normal strain, knowing that the following strains have been obtained by use of the rosette shown:

$\epsilon_1 = -50 \times 10^{-6}$  in./in.     $\epsilon_2 = +360 \times 10^{-6}$  in./in.  
 $\epsilon_3 = +315 \times 10^{-6}$  in./in.



SOLUTION

$\theta_1 = 45^\circ$      $\theta_2 = -45^\circ$      $\theta_3 = 0$

$E_x \cos^2 \theta_1 + E_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \epsilon_1$   
 $0.5 E_x + 0.5 E_y + 0.5 \gamma_{xy} = -50 \times 10^{-6}$     (1)

$E_x \cos^2 \theta_2 + E_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \epsilon_2$   
 $0.5 E_x + 0.5 E_y - 0.5 \gamma_{xy} = 360 \times 10^{-6}$     (2)

$E_x \cos^2 \theta_3 + E_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \epsilon_3$   
 $E_x + 0 + 0 = 315 \times 10^{-6}$     (3)

From (3)     $E_x = 315 \times 10^{-6}$  in./in.

Eq. (1) - Eq. (2)     $\gamma_{xy} = -50 \times 10^{-6} - 360 \times 10^{-6} = -410 \times 10^{-6}$  in./in.

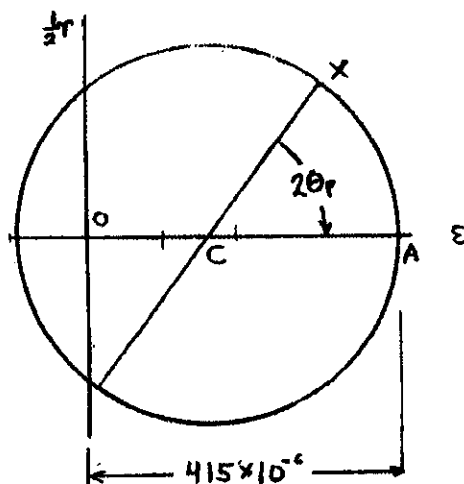
Eq. (1) + Eq. (2)     $E_x + E_y = \epsilon_1 + \epsilon_2$

$E_y = \epsilon_1 + \epsilon_2 - E_x = -50 \times 10^{-6} + 360 \times 10^{-6} - 315 \times 10^{-6} = -5 \times 10^{-6}$  in./in.

$E_{ave} = \frac{1}{2}(E_x + E_y) = 155 \times 10^{-6}$  in./in.

$R = \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{315 \times 10^{-6} + 5 \times 10^{-6}}{2}\right)^2 + \left(\frac{-410 \times 10^{-6}}{2}\right)^2}$   
 $= 260 \times 10^{-6}$  in./in.

$E_{max} = E_{ave} + R = 415 \times 10^{-6}$  in./in.

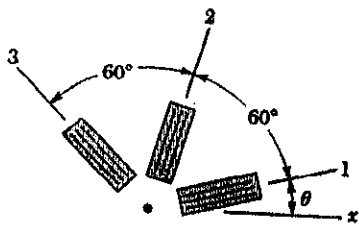


$\tan 2\theta_p = \frac{\gamma_{xy}}{E_x - E_y} = -1.323$

$2\theta_p = -52.0^\circ$

$\theta_p = -26.0^\circ$

PROBLEM 7.146



7.146 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{ave}$$

where  $\epsilon_{ave}$  is the abscissa of the center of the corresponding Mohr's circle for strain.

SOLUTION

$$\epsilon_1 = \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \quad (1)$$

$$\begin{aligned} \epsilon_2 &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos (2\theta + 120^\circ) + \frac{\gamma_{xy}}{2} \sin (2\theta + 120^\circ) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} (\cos 120^\circ \cos 2\theta - \sin 120^\circ \sin 2\theta) \\ &\quad + \frac{\gamma_{xy}}{2} (\cos 120^\circ \sin 2\theta + \sin 120^\circ \cos 2\theta) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \left( -\frac{1}{2} \cos 2\theta - \frac{\sqrt{3}}{2} \sin 2\theta \right) \\ &\quad + \frac{\gamma_{xy}}{2} \left( -\frac{1}{2} \sin 2\theta + \frac{\sqrt{3}}{2} \cos 2\theta \right) \quad (2) \end{aligned}$$

$$\begin{aligned} \epsilon_3 &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \cos (2\theta + 240^\circ) + \frac{\gamma_{xy}}{2} \sin (2\theta + 240^\circ) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} (\cos 240^\circ \cos 2\theta - \sin 240^\circ \sin 2\theta) \\ &\quad + \frac{\gamma_{xy}}{2} (\cos 240^\circ \sin 2\theta + \sin 240^\circ \cos 2\theta) \\ &= \epsilon_{ave} + \frac{\epsilon_x - \epsilon_y}{2} \left( -\frac{1}{2} \cos 2\theta + \frac{\sqrt{3}}{2} \sin 2\theta \right) \\ &\quad + \frac{\gamma_{xy}}{2} \left( -\frac{1}{2} \sin 2\theta - \frac{\sqrt{3}}{2} \cos 2\theta \right) \quad (3) \end{aligned}$$

Adding (1), (2), and (3)

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{ave} + 0 + 0$$

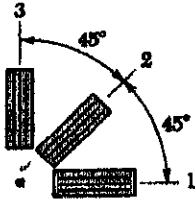
$$3\epsilon_{ave} = \epsilon_1 + \epsilon_2 + \epsilon_3$$

PROBLEM 7.147

7.147 Using a 45° rosette, the strains  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  have been determined at a given point. Using Mohr's circle, show that the principal strains are

$$\epsilon_{\max, \min} = \frac{1}{2}(\epsilon_1 + \epsilon_3) \pm \frac{1}{\sqrt{2}} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]^{1/2}$$

(Hint: The shaded triangles are congruent.)



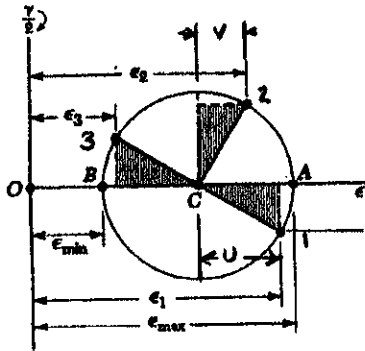
SOLUTION

Since gage directions 1 and 3 are 90° apart

$$E_{ave} = \frac{1}{2}(\epsilon_1 + \epsilon_3)$$

$$\text{Let } U = \epsilon_1 - E_{ave} = \frac{1}{2}(\epsilon_1 - \epsilon_3)$$

$$V = \epsilon_2 - E_{ave} = \epsilon_2 - \frac{1}{2}(\epsilon_1 + \epsilon_3)$$



$$R^2 = U^2 + V^2$$

$$= \frac{1}{4}(\epsilon_1 - \epsilon_3)^2 + \epsilon_2^2 - \epsilon_2(\epsilon_1 + \epsilon_3) + \frac{1}{4}(\epsilon_1 + \epsilon_3)^2$$

$$= \frac{1}{4} \epsilon_1^2 - \frac{1}{2} \epsilon_1 \epsilon_3 + \frac{1}{4} \epsilon_3^2 + \epsilon_2^2 - \epsilon_2 \epsilon_1 - \epsilon_2 \epsilon_3 + \frac{1}{4} \epsilon_1^2 + \frac{1}{2} \epsilon_1 \epsilon_3 + \frac{1}{4} \epsilon_3^2$$

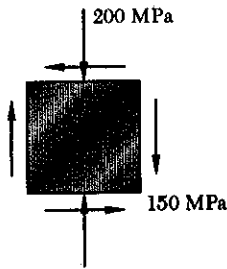
$$= \frac{1}{2} \epsilon_1^2 - \epsilon_2 \epsilon_1 + \epsilon_2^2 - \epsilon_2 \epsilon_3 + \frac{1}{2} \epsilon_3^2$$

$$= \frac{1}{2} (\epsilon_1 - \epsilon_2)^2 + \frac{1}{2} (\epsilon_2 - \epsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2]$$

$$\epsilon_{\max, \min} = E_{ave} \pm R \quad \text{gives the required formula.}$$

PROBLEM 7.148



7.148 The given state of plane stress is known to exist on the surface of a machine component. Knowing that  $E = 200 \text{ GPa}$  and  $G = 77 \text{ GPa}$ , determine the direction and magnitude of the three principal strains (a) by determining the corresponding state of strain [use Eq. (2.43), page 94, and Eq. (2.38) page 91] and then using Mohr's circle for strain, (b) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

SOLUTION

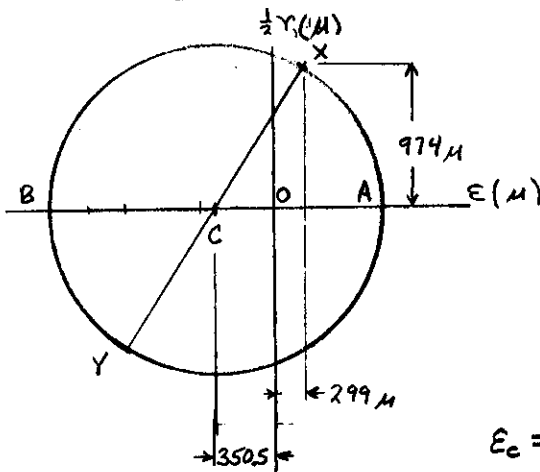
(a)  $\sigma_x = 0, \sigma_y = -200 \times 10^6 \text{ Pa}, \tau_{xy} = -150 \times 10^6 \text{ Pa}$   
 $E = 200 \times 10^9 \text{ Pa} \quad G = 77 \times 10^9 \text{ Pa}$

$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = 0.2987$

$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200 \times 10^9} [0 + (0.2987)(200 \times 10^6)] = 299 \mu$

$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200 \times 10^9} [(-200 \times 10^6) - 0] = -1000 \mu$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-150 \times 10^6}{77 \times 10^9} = -1948 \mu \quad \frac{\gamma_{xy}}{2} = -974 \mu$



$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = -350.5 \mu$

$\epsilon_x - \epsilon_y = 1299 \mu$

$\tan 2\theta_a = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-1948}{1299} = -1.4996$

$2\theta_a = -56.3^\circ \quad \theta_a = -28.15^\circ$

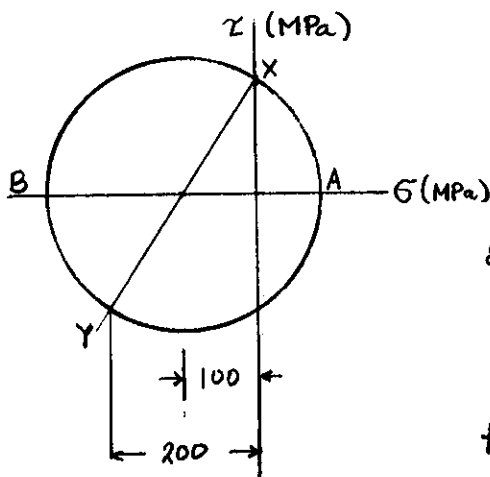
$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 1171 \mu$

$\epsilon_a = \epsilon_{ave} + R = 820 \mu$

$\epsilon_b = \epsilon_{ave} - R = -1521 \mu$

$\epsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 200 \times 10^6)}{200 \times 10^9} = -299 \mu$

(b)



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 100 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 + 200}{2}\right)^2 + 150^2} = 180.28 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = 80.3 \text{ MPa}$

$\sigma_b = \sigma_{ave} - R = -280.3 \text{ MPa}$

$\epsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b)$   
 $= \frac{1}{200 \times 10^9} [80.3 \times 10^6 - (0.2987)(-280.3 \times 10^6)]$   
 $= 820 \times 10^{-6} = 820 \mu$

$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} = -1.5 \quad 2\theta_a = -56.3^\circ$   
 $\theta_a = -28.15^\circ$

PROBLEM 7.149

7.149 The following state of strain has been determined on the surface of a cast-iron machine element:

$$\epsilon_1 = -720 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = -400 \times 10^{-6} \text{ in./in.}$$

$$\gamma = +660 \times 10^{-6} \text{ rad}$$

Knowing that  $E = 10 \times 10^6$  psi and  $G = 4 \times 10^6$  psi, determine the principal planes and the principal stresses (a) by determining the corresponding state of plane stress [use Eq. 2.36, page 94; Eq. 2.43; page 91; and the first two equations of Prob. 2.75, page 90] and then using Mohr's circle for stress, (b) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determining the corresponding stresses.

SOLUTION

$$G = \frac{E}{2(1+\nu)} \quad \nu = \frac{E}{2G} - 1 = \frac{10}{(2)(4)} - 1 = 0.25$$

Note that the 3rd principal stress

$$\frac{E}{1-\nu^2} = \frac{10 \times 10^6}{1-0.25^2} = 10.667 \times 10^6 \text{ psi}$$

$$\sigma_c = 0$$

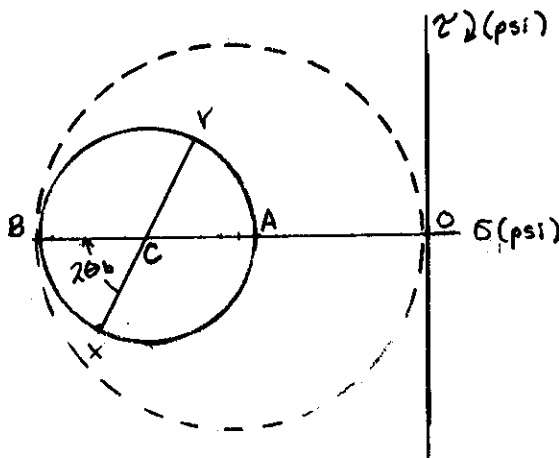
$$(a) \quad \sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu \epsilon_2) = 10.667 \times 10^6 [-720 \times 10^{-6} + (0.25)(-400 \times 10^{-6})]$$

$$= -8746.7 \text{ psi}$$

$$\sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu \epsilon_1) = 10.667 \times 10^6 [-400 \times 10^{-6} + (0.25)(-720 \times 10^{-6})]$$

$$= -6186.7 \text{ psi}$$

$$\tau = G\gamma = (4 \times 10^6)(660 \times 10^{-6}) = 2640 \text{ psi}$$



$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = -7466.7 \text{ psi}$$

$$\tan 2\theta_b = \frac{2\tau}{\sigma_1 - \sigma_2} = -2.0625$$

$$2\theta_b = -64.1^\circ \quad \theta_b = -32.1^\circ \quad \theta_a = 57.9^\circ$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = 2934 \text{ psi}$$

$$\sigma_a = \sigma_{ave} + R = -4533 \text{ psi}$$

$$\sigma_b = \sigma_{ave} - R = -10400 \text{ psi}$$

$$(b) \quad \epsilon_{ave} = \frac{1}{2}(\epsilon_1 + \epsilon_2) = -560 \times 10^{-6}$$

$$\tan 2\theta_b = \frac{\gamma}{\epsilon_1 - \epsilon_2} = \frac{660}{-720 + 400} = -2.0625$$

$$2\theta_b = -64.1^\circ \quad \theta_b = -32.1^\circ \quad \theta_a = 57.9^\circ$$

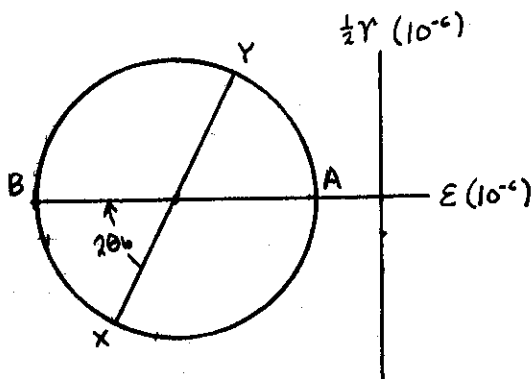
$$R = \sqrt{\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} = 366.74 \times 10^{-6}$$

$$\epsilon_a = \epsilon_{ave} + R = -193.26 \times 10^{-6}$$

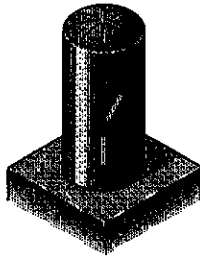
$$\epsilon_b = \epsilon_{ave} - R = -926.74 \times 10^{-6}$$

$$\sigma_a = \frac{E}{1-\nu^2} (\epsilon_a + \nu \epsilon_b) = -4533 \text{ psi}$$

$$\sigma_b = \frac{E}{1-\nu^2} (\epsilon_b + \nu \epsilon_a) = -10400 \text{ psi}$$



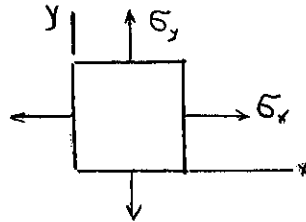
PROBLEM 7.150



7.150 A single strain gage forming an angle  $\beta = 30^\circ$  with the vertical is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is  $\frac{3}{8}$  in. thick, has a 36-in. inside diameter, and is made of a steel with  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ . Determine the pressure in the tank corresponding to a gage reading of  $220 \times 10^{-6}$  in./in.

SOLUTION

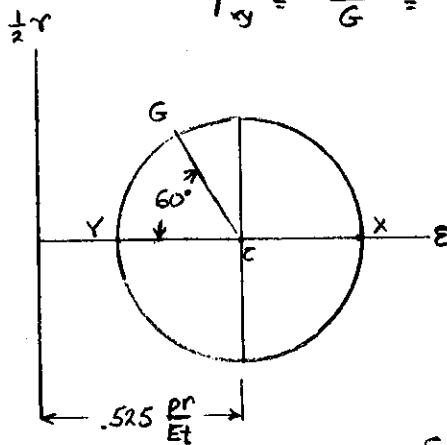
Stresses in the tank wall



$$\begin{aligned} \sigma_x &= \frac{pr}{t} \\ \sigma_y &= \frac{pr}{2t} \\ \tau_{xy} &= 0 \end{aligned}$$

Strains

$$\begin{aligned} \epsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}\left(\frac{pr}{t} - \nu\frac{pr}{2t}\right) = \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right) = 0.85 \frac{pr}{Et} \\ \epsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right) = \frac{pr}{Et}\left(\frac{1}{2} - \nu\right) = 0.20 \frac{pr}{Et} \\ \tau_{xy} &= \frac{\tau_{xy}}{G} = 0 \end{aligned}$$



$$E_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0.525 \frac{pr}{Et}$$

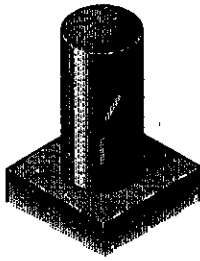
$$R = \frac{1}{2}(\epsilon_x - \epsilon_y) = 0.325$$

$$\begin{aligned} \epsilon_g &= E_{ave} - R \cos 60^\circ \\ &= 0.525 \frac{pr}{Et} - 0.325 \frac{pr}{Et} \cos 60^\circ \\ &= 0.3625 \frac{pr}{Et} \end{aligned}$$

Solving for  $p$        $p = \frac{Et \epsilon_g}{0.3625 r}$

$$p = \frac{(29 \times 10^6) \left(\frac{3}{8}\right) (220 \times 10^{-6})}{(0.3625)(36/2)} = 367 \text{ psi}$$

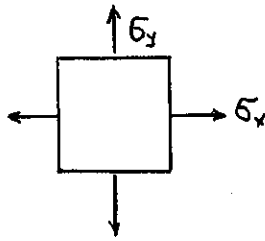
PROBLEM 7.151



7.150 A single strain gage forming an angle  $\beta = 30^\circ$  with the vertical is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is  $\frac{3}{8}$  in. thick, has a 36-in. inside diameter, and is made of a steel with  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ . Determine the pressure in the tank corresponding to a gage reading of  $220 \times 10^{-6}$  in./in.

7.151 Solve Prob. 7.150, assuming that the gage forms an angle  $\beta = 60^\circ$  with the vertical.

SOLUTION



Stresses:  $\sigma_x = \frac{pr}{t}$      $\sigma_y = \frac{pr}{2t}$      $\tau_{xy} = 0$

Strains:  $\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}(\frac{pr}{t} - \nu\frac{pr}{2t})$   
 $= \frac{pr}{Et}(1 - \frac{\nu}{2}) = 0.85 \frac{pr}{Et}$

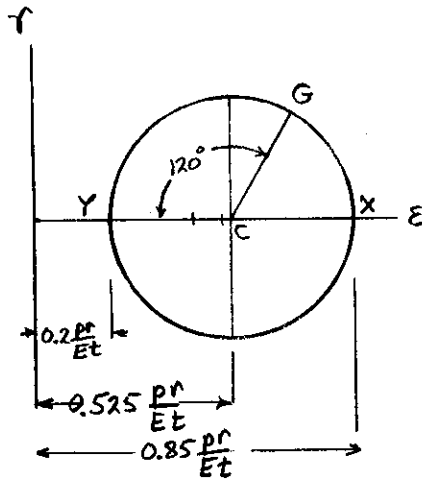
$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}(\frac{pr}{2t} - \nu\frac{pr}{t})$   
 $= (\frac{1}{2} - \nu) \frac{pr}{Et} = 0.20 \frac{pr}{Et}$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$

$\epsilon_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0.525 \frac{pr}{Et}$

$R = \frac{1}{2}(\epsilon_x - \epsilon_y) = 0.325 \frac{pr}{Et}$

$\epsilon_g = \epsilon_{ave} + R \cos 60^\circ$   
 $= 0.525 \frac{pr}{Et} + 0.325 \frac{pr}{Et} \cos 60^\circ$   
 $= 0.6875 \frac{pr}{Et}$

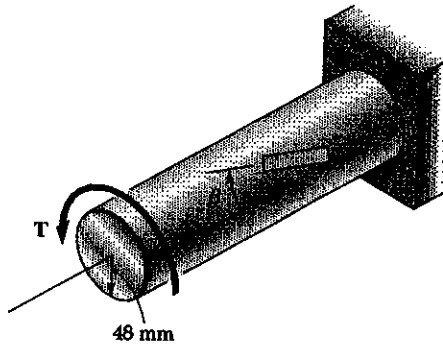


Solving for p     $p = \frac{Et\epsilon_g}{0.6875r} = \frac{(29 \times 10^6)(\frac{3}{8})(220 \times 10^{-6})}{(0.6875)(36/2)} = 193.3 \text{ psi}$

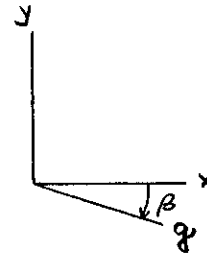
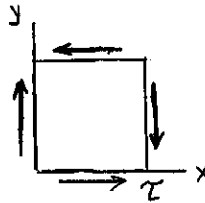


PROBLEM 7.152

7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle  $\beta = 20^\circ$  with a line parallel to the axis of the shaft. Knowing that  $G = 27$  GPa, determine the torque  $T$  corresponding to a gage reading of  $400 \mu$ .



SOLUTION



$$\gamma = \frac{Tc}{J}$$

$$J = \frac{\pi}{2} c^3$$

$$\gamma = \frac{\tau}{G}$$

$$\sigma_x = \sigma_y = 0$$

$$\epsilon_x = \epsilon_y = 0$$

Sketch Mohr's circle for strain.

Gage direction is  $\beta$  clockwise from  $x$

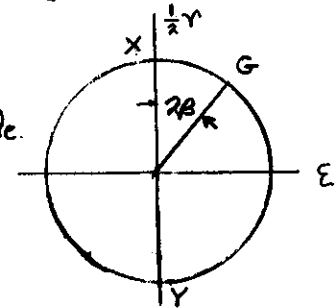
Point  $G$  is  $2\beta$  clockwise from  $X$  on Mohr's circle.

$$E_{ave} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0$$

$$R = \frac{1}{2} \gamma_{xy}$$

$$E_g = E_{ave} + R \sin 2\beta = \frac{1}{2} \gamma_{xy} \sin 2\beta = \frac{\tau_{xy}}{2G} \sin 2\beta$$

$$= \frac{Tc}{2GJ} \sin 2\beta =$$



Solving for  $T$   $T = \frac{2GJ E_g}{c \sin 2\beta} = \frac{\pi G c^3 E_g}{\sin 2\beta}$

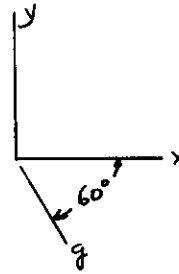
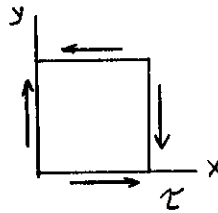
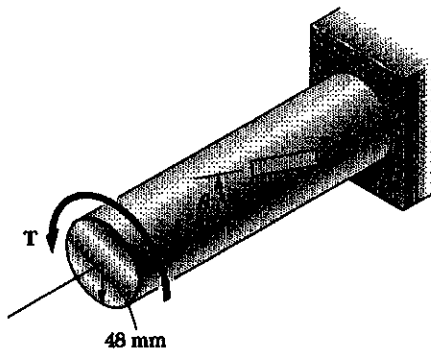
$$T = \frac{\pi (27 \times 10^9) (48 \times 10^{-3})^3 (400 \times 10^{-6})}{\sin 40^\circ} = 5.84 \times 10^3 \text{ N}\cdot\text{m} = 5.84 \text{ kN}\cdot\text{m}$$

PROBLEM 7.153

7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle  $\beta = 20^\circ$  with a line parallel to the axis of the shaft. Knowing that  $G = 27$  GPa, determine the torque  $T$  corresponding to a gage reading of  $400 \mu$ .

7.153 Solve Prob. 7.152, assuming that the gage forms an angle  $\beta = 60^\circ$  with a line parallel to the axis of the shaft.

SOLUTION



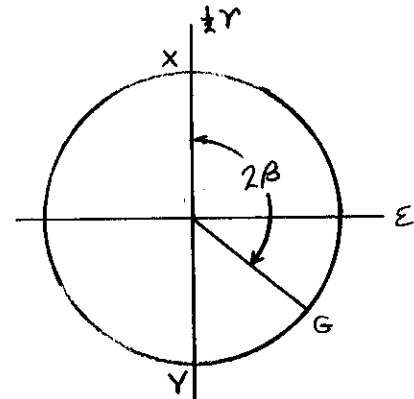
$$\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4 \quad \gamma = \frac{\tau}{G}$$

$$\sigma_x = \sigma_y = 0 \quad \epsilon_x = \epsilon_y = 0$$

Sketch Mohr's circle for strain.

Gage direction  $g$  is  $\beta = 60^\circ$  clockwise from  $X$ .

Point  $G$  is  $2\beta = 120^\circ$  clockwise from point  $X$  on Mohr's circle.



$$\epsilon_{\text{ave}} = \frac{1}{2}(\epsilon_x + \epsilon_y) = 0$$

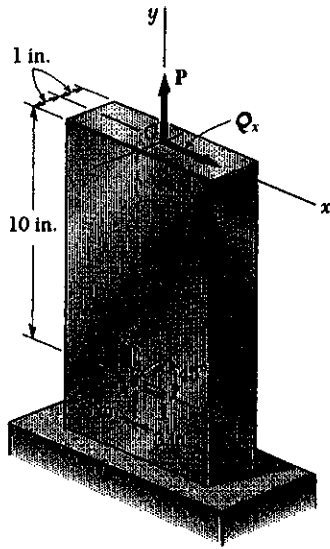
$$R = \frac{1}{2}\gamma$$

$$\begin{aligned} \epsilon_g &= \epsilon_{\text{ave}} + R \sin 2\beta = \frac{1}{2}\gamma \sin 2\beta = \frac{\tau}{2G} \sin 2\beta \\ &= \frac{Tc}{2GJ} \sin 2\beta \end{aligned}$$

Solving for  $T$  
$$T = \frac{2GJ\epsilon_g}{c \sin 2\beta} = \frac{\pi Gc^3 \epsilon_g}{\sin 2\beta}$$

$$T = \frac{\pi (27 \times 10^9) (48 \times 10^{-3})^3 (400 \times 10^{-6})}{\sin 120^\circ} = 4.33 \times 10^3 \text{ N}\cdot\text{m} = 4.33 \text{ kN}\cdot\text{m}$$

PROBLEM 7.154



7.154 A centric axial force  $P$  and a horizontal force  $Q_x$  are both applied at point  $C$  of the rectangular bar shown. A  $45^\circ$  strain rosette on the surface of the bar at point  $A$  indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$  in./in.  $\epsilon_2 = +300 \times 10^{-6}$  in./in.  $\epsilon_3 = +250 \times 10^{-6}$  in./in.  
 Knowing that  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ , determine the magnitudes of  $P$  and  $Q_x$ .

SOLUTION

$$\epsilon_x = \epsilon_1 = -75 \times 10^{-6} \qquad \epsilon_y = \epsilon_3 = 250 \times 10^{-6}$$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_1 - \epsilon_3 = 425 \times 10^{-6}$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu\epsilon_y) = \frac{29}{1-0.3^2} [-75 + (0.3)(250)]$$

$$= 0$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu\epsilon_x) = \frac{29}{1-0.3^2} [250 + (0.3)(-75)]$$

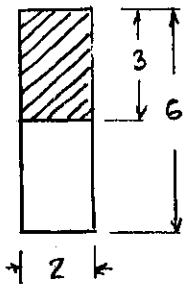
$$= 7.25 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(7.25 \times 10^3)$$

$$= 87.0 \times 10^3 \text{ lb} = 87.0 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.154)(425) = 4.740 \times 10^3 \text{ psi}$$



$$I = \frac{1}{12} bh^3 = \frac{1}{12} (2)(6)^3 = 36 \text{ in}^4$$

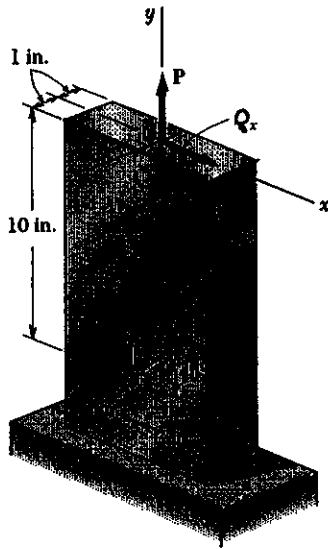
$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in}$$

$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(4.74 \times 10^3)}{9} = 37.9 \times 10^3 \text{ lb}$$

$$Q_x = V = 37.9 \times 10^3 \text{ lb} = 37.9 \text{ kips}$$

PROBLEM 7.155



7.154 A centric axial force  $P$  and a horizontal force  $Q$ , are both applied at point  $C$  of the rectangular bar shown. A  $45^\circ$  strain rosette on the surface of the bar at point  $A$  indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$  in./in.     $\epsilon_2 = +300 \times 10^{-6}$  in./in.     $\epsilon_3 = +250 \times 10^{-6}$  in./in.  
 Knowing that  $E = 29 \times 10^6$  psi and  $\nu = 0.30$ , determine the magnitudes of  $P$  and  $Q$ .

7.155 Solve Prob. 7.154, assuming that the rosette at point  $A$  indicates the following strains:

$\epsilon_1 = -60 \times 10^{-6}$  in./in.     $\epsilon_2 = +410 \times 10^{-6}$  in./in.     $\epsilon_3 = +200 \times 10^{-6}$  in./in.

SOLUTION

$$\epsilon_x = \epsilon_1 = -60 \times 10^{-6} \qquad \epsilon_y = \epsilon_3 = 200 \times 10^{-6}$$

$$\gamma_{xy} = 2\epsilon_2 - \epsilon_1 - \epsilon_3 = 680 \times 10^{-6}$$

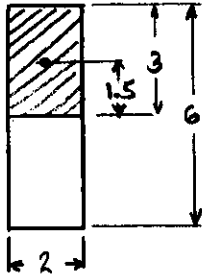
$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu\epsilon_y) = \frac{29}{1-0.3^2} [-60 + (0.3)(200)] = 0$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu\epsilon_x) = \frac{29}{1-0.3^2} [200 + (0.3)(-60)] = 5.800 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(5.800 \times 10^3) = 69.6 \times 10^3 \text{ lb} = 69.6 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.154)(680) = 7.585 \times 10^3 \text{ psi}$$



$$I = \frac{1}{12} b h^3 = \frac{1}{12} (2)(6)^3 = 36 \text{ in}^4$$

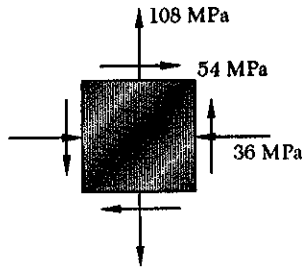
$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in.}$$

$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(7.585 \times 10^3)}{9} = 60.7 \times 10^3 \text{ lb.}$$

$$Q_x = V = 60.7 \times 10^3 \text{ lb.} = 60.7 \text{ kips}$$

PROBLEM 7.156



7.156 The state of stress shown occurs in a steel member made of a grade of steel with a tensile yield strength of 270 MPa. Determine the factor of safety with respect to yield strength, using (a) the maximum-shearing-stress criterion, (b) the maximum distortion-strength criterion.

SOLUTION

$$\sigma_x = -36 \text{ MPa}, \quad \sigma_y = 108 \text{ MPa}, \quad \tau_{xy} = 54 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 36 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 90 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 126 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -54 \text{ MPa}, \quad \sigma_2 = 0$$

(a)  $\sigma_{max} = 126 \text{ MPa}, \quad \sigma_{min} = -54 \text{ MPa}$

$$2\tau_{max} = \sigma_{max} - \sigma_{min} = 180 \text{ MPa} < 270 \text{ MPa} \quad (\text{No yielding})$$

$$F.S. = \frac{\sigma_y}{2\tau_{max}} = \frac{270}{180} = 1.500$$

(b)  $\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b} = 159.99 \text{ MPa} < 270 \text{ MPa} \quad (\text{No yielding})$

$$F.S. = \frac{\sigma_y}{\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a \sigma_b}} = \frac{270}{155.99} = 1.688$$

PROBLEM 7.157

7.157 A spherical pressure tank has 1.2-m outer diameter and a uniform wall thickness of 10 mm. Knowing that the gage pressure is 1.25 MPa in the tank, determine (a) the maximum normal stress, (b) the maximum shearing stress, (c) the normal strain on the surface of the tank. (Use  $E = 200 \text{ GPa}$  and  $\nu = 0.30$ .)

SOLUTION

$$t = 10 \times 10^{-3} \text{ m}, \quad r = \frac{1}{2}d - t = \frac{1}{2}(1.2) - 10 \times 10^{-3} = 0.590 \text{ m}, \quad p = 1.25 \text{ MPa}$$

For a spherical tank under internal pressure

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.25)(0.590)}{(2)(10 \times 10^{-3})} = 36.9 \text{ MPa}$$

$$\sigma_3 \approx 0$$

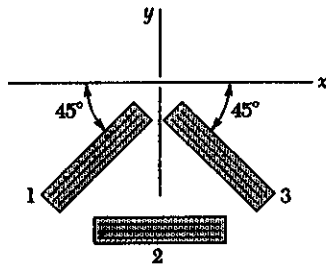
(a)  $\sigma_{max} = 36.9 \text{ MPa}$

(b)  $\sigma_{min} = 0 \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 18.44 \text{ MPa}$

(c) 
$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) = \frac{1}{200 \times 10^9} [36.9 \times 10^6 - (0.3)(36.9 \times 10^6) - 0]$$
  

$$= 129 \times 10^{-6} = 129 \mu$$

**PROBLEM 7.158**

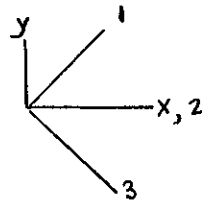


7.158 The strains determined by the use of a rosette attached as shown to the surface of a structural member are:

$$\epsilon_1 = 220 \times 10^{-6} \text{ in./in.} \quad \epsilon_2 = 425 \times 10^{-6} \text{ in./in.} \quad \epsilon_3 = 480 \times 10^{-6} \text{ in./in.}$$

Determine (a) the orientation and magnitude of the principal strains in the plane of the rosette, (b) the maximum in-plane shearing strain.

**SOLUTION**



$$\theta_1 = 45^\circ$$

$$\theta_2 = 0$$

$$\theta_3 = -45^\circ$$

$$E_x \cos^2 \theta_1 + E_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 \cos \theta_1 = \epsilon_1$$

$$\frac{1}{2} E_x + \frac{1}{2} E_y + \frac{1}{2} \gamma_{xy} = 220 \times 10^{-6} \text{ in./in.} \quad (1)$$

$$E_x \cos^2 \theta_2 + E_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 \cos \theta_2 = \epsilon_2$$

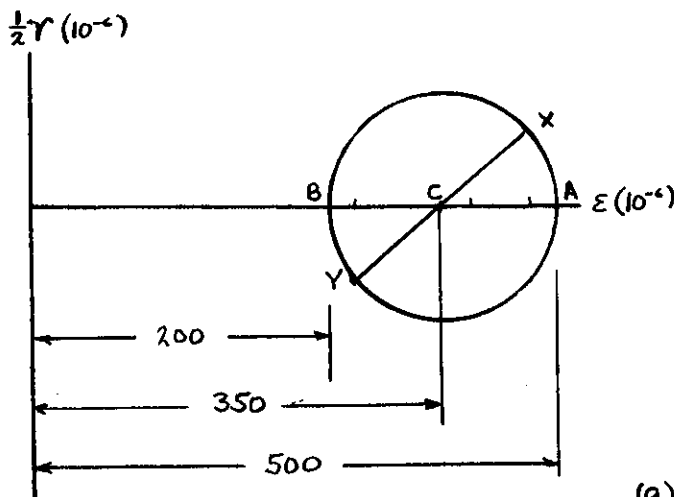
$$E_x + 0 + 0 = 425 \times 10^{-6} \text{ in./in.} \quad (2)$$

$$E_x \cos^2 \theta_3 + E_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 \cos \theta_3 = \epsilon_3$$

$$\frac{1}{2} E_x + \frac{1}{2} E_y - \frac{1}{2} \gamma_{xy} = 480 \times 10^{-6} \text{ in./in.} \quad (3)$$

Solving (1), (2) and (3) simultaneously gives

$$E_x = 425 \times 10^{-6} \text{ in./in.}, \quad E_y = 275 \times 10^{-6} \text{ in./in.}, \quad \gamma_{xy} = -260 \times 10^{-6} \text{ in./in.}$$



$$E_{ave} = \frac{1}{2}(E_x + E_y) = 350 \times 10^{-6} \text{ in./in.}$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{E_x - E_y} = \frac{-260}{425 - 275}$$

$$= -1.7333$$

$$2\theta_a = -60^\circ \quad \theta_a = -30^\circ \quad \blacktriangleleft$$

$$\theta_b = 60^\circ \quad \blacktriangleright$$

$$R = \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= 150 \times 10^{-6} \text{ in./in.}$$

$$(a) \quad E_a = E_{ave} + R = 500 \times 10^{-6} \text{ in./in.} \quad \blacktriangleleft$$

$$E_b = E_{ave} - R = 200 \times 10^{-6} \text{ in./in.} \quad \blacktriangleleft$$

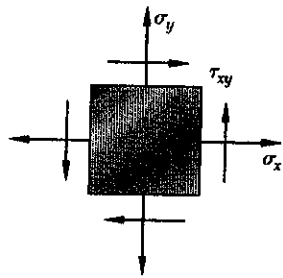
$$(b) \quad \gamma_{max(in\ plane)} = 2R$$

$$= 300 \times 10^{-6} \text{ in./in.} \quad \blacktriangleleft$$

PROBLEM 7.159

7.159 For a state of plane stress it is known that the normal and shearing stresses are directed as shown and that  $\sigma_x = 5$  ksi,  $\sigma_y = 12$  ksi, and  $\sigma_{max} = 18$  ksi. Determine (a) the orientation of the principal planes, (b) the maximum in-plane shearing stress.

SOLUTION



$$(a) \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(5 + 12) = 8.5 \text{ ksi}$$

$$R = \sigma_{max} - \sigma_{ave} = 18 - 8.5 = 9.5 \text{ ksi}$$

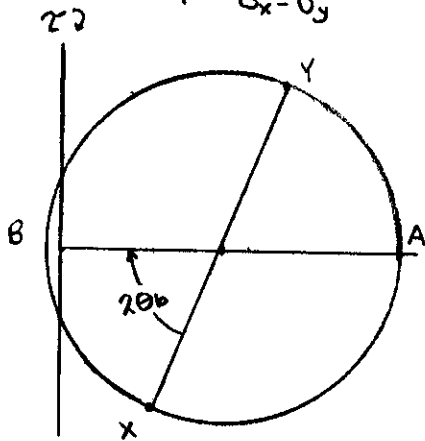
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{9.5^2 - \left(\frac{5-12}{2}\right)^2}$$

$$\pm 8.83 \text{ ksi}$$

In the sketch  $\tau_{xy}$  is shown positive; hence  $\tau_{xy} = +8.83$  ksi

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.523 \quad 2\theta_p = -68.4^\circ$$



$$\theta_b = -34.2^\circ, \quad \theta_c = 55.8^\circ$$

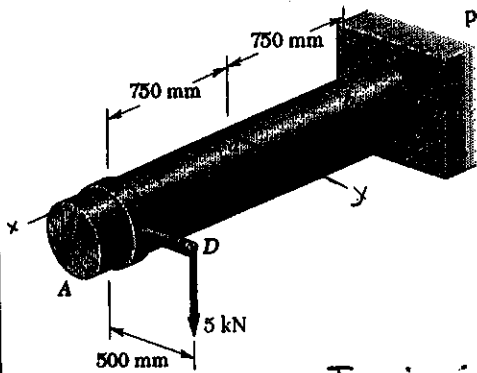
$$\sigma_a = \sigma_{ave} + R = \sigma_{max} = 18 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = \sigma_{min} = -1 \text{ ksi}$$

$$(b) \tau_{max(in-plane)} = R = 9.5 \text{ ksi}$$

**PROBLEM 7.160**

7.160 The compressed-air tank *AB* has an inside diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at points *a* and *b* on the top of the tank.



**SOLUTION**

$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$$

Torsion:  $C_1 = 225 \text{ mm}$ ,  $C_2 = 225 + 6 = 231 \text{ mm}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N}\cdot\text{m}$$

$$\tau = \frac{TC}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear:  $\tau = 0$  at points *a* and *b*.

Bending:  $I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4$ ,  $c = 231 \times 10^{-3} \text{ m}$ .

Point a

$$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88 \text{ MPa}$$

Total stresses (MPa)

longitudinal  $\sigma_x = 22.5 + 3.88 = 26.38$

Circumferential  $\sigma_y = 45$

Shear  $\tau_{xy} = 1.292$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max}(\text{in-plane}) = R = 9.40 \text{ MPa} \quad \blacktriangleleft$$

Point b

$$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{Mc}{I} = 7.75 \text{ MPa}$$

Total stresses (MPa)

$$\sigma_x = 22.5 + 7.75 = 30.25$$

$$\sigma_y = 45$$

$$\tau_{xy} = 1.292$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa} \quad \blacktriangleleft$$

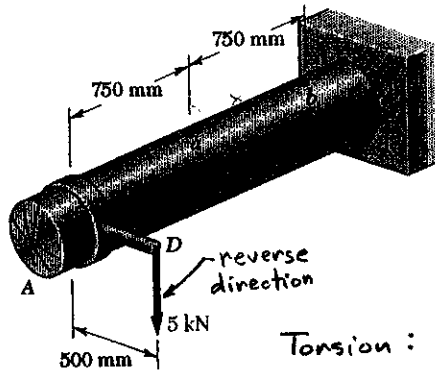
$$\tau_{max}(\text{in-plane}) = R = 7.49 \text{ MPa} \quad \blacktriangleleft$$



**PROBLEM 7.161**

7.160 The compressed-air tank  $AB$  has an inside diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at points  $a$  and  $b$  on the top of the tank.

7.161 Solve Prob. 7.160, assuming that the 5-kN force applied at  $D$  is directed vertically upward.



**SOLUTION**

$$r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$$

Torsion:  $C_1 = 225 \text{ mm}$   $C_2 = 225 + 6 = 231 \text{ mm}$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear:  $\tau = 0$  at points  $a$  and  $b$ .

Bending:  $I = \frac{1}{2}J = 223.45 \times 10^6 \text{ m}^4$ ,  $c = 231 \times 10^{-3} \text{ m}$

Point a

$$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^6} = 3.88 \text{ MPa}$$

Total stresses (MPa)

Longitudinal  $\sigma_x = 22.5 - 3.88 = 18.62 \text{ MPa}$

Circumferential  $\sigma_y = 45 \text{ MPa}$

Shear  $\tau_{xy} = -1.292 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 31.81 \text{ MPa}$$

$$R = \sqrt{\frac{\sigma_x - \sigma_y}{2}^2 + \tau_{xy}^2} = 13.25 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max(in-plane)} = R = 13.25 \text{ MPa} \quad \blacktriangleleft$$

Point b

$$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{Mc}{I} = 7.75 \text{ MPa}$$

Total stresses (MPa)

$\sigma_x = 22.5 - 7.75 = 14.75$

$\sigma_y = 45$

$\tau_{xy} = -1.292$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 29.875 \text{ MPa}$$

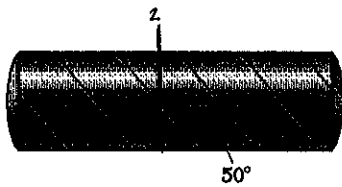
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 15.18 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max(in-plane)} = R = 15.18 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.162

7.162 The steel pressure tank shown has a 30-in. inside diameter and a  $\frac{3}{8}$ -in. wall thickness. Knowing that the butt-welded seams form an angle of  $50^\circ$  with the longitudinal axis of the tank and that the gage pressure in the tank is 200 psi, determine (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.



SOLUTION

$$r = \frac{1}{2}d = 15 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(200)(15)}{0.375} = 8000 \text{ psi}$$

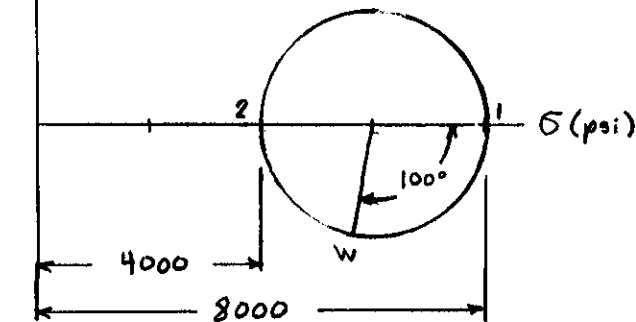
$$\sigma_2 = \frac{1}{2}\sigma_1 = 4000 \text{ psi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 6000 \text{ psi}$$

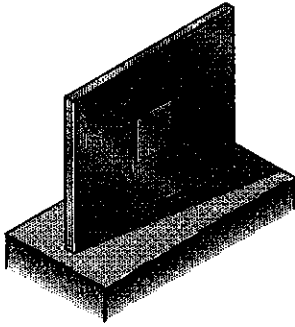
$$R = \frac{\sigma_1 - \sigma_2}{2} = 2000 \text{ psi}$$

$$(a) \quad \sigma_w = \sigma_{ave} + R \cos 100^\circ = 5652 \text{ psi}$$

$$(b) \quad \tau_w = R \sin 100^\circ = 1970 \text{ psi}$$

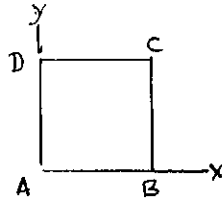


PROBLEM 7.163



7.163 A square  $ABCD$  of 2.4-in. side is scribed on the surface of a thin plate while the plate is unloaded. After the plate is loaded, the lengths of sides  $AB$  and  $AD$  are observed to have increased, respectively, by  $540 \times 10^{-6}$  in. and  $900 \times 10^{-6}$  in., while the angle  $DAB$  is observed to have decreased by  $360 \times 10^{-6}$  rad. Knowing that  $\nu = \frac{1}{3}$ , determine (a) the orientation and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum shearing strain.

SOLUTION



$$\begin{aligned} \epsilon_x &= \frac{\Delta l_x}{\Delta x} = \frac{\Delta \overline{AB}}{\overline{AB}} \\ &= \frac{540 \times 10^{-6}}{2.4} = 225 \times 10^{-6} \\ \epsilon_y &= \frac{\Delta l_y}{\Delta y} = \frac{\Delta \overline{AD}}{\overline{AD}} \\ &= \frac{900 \times 10^{-6}}{2.4} = 375 \times 10^{-6} \end{aligned}$$

$$\gamma_{xy} = \text{decrease in right angle } DAB = 360 \times 10^{-6} \text{ rad} = 360 \times 10^{-6}$$

$$\epsilon_{ave} = \frac{1}{2} \epsilon_x + \epsilon_y = 300 \times 10^{-6}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{360}{225 - 375} = -2.4$$

$$2\theta_p = -67.38^\circ \quad \theta_p = -33.7^\circ$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 195 \times 10^{-6}$$

$$\epsilon_a = \epsilon_{ave} + R = 495 \times 10^{-6} \quad \blacktriangleleft$$

$$\epsilon_b = \epsilon_{ave} - R = 105 \times 10^{-6} \quad \blacktriangleleft$$

$$(b) \gamma_{max(in-plane)} = \epsilon_a - \epsilon_b = 390 \times 10^{-6} \quad \blacktriangleleft$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b)$$

$$= -\frac{(1/3)}{(2/3)} (495 \times 10^{-6} + 105 \times 10^{-6}) = -300 \times 10^{-6}$$

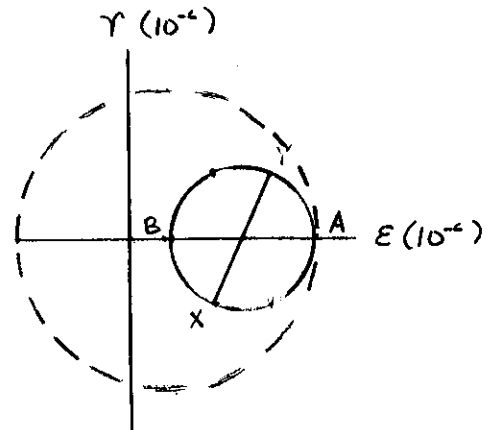
$$\epsilon_{max} = 495 \times 10^{-6} \quad \epsilon_{min} = -300 \times 10^{-6}$$

$$(c) \gamma_{max} = \epsilon_{max} - \epsilon_{min} = 795 \times 10^{-6} \quad \blacktriangleleft$$

$$\epsilon_{ave} = \frac{1}{2} (\epsilon_{max} + \epsilon_{min}) = 97.5 \times 10^{-6}$$

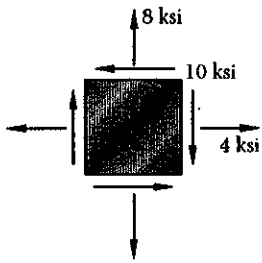
$$R = \frac{1}{2} \gamma_{max} = 397.5 \times 10^{-6}$$

For dotted Mohr's circle.



PROBLEM 7.164

7.164 For the state of plane stress shown, determine (a) the principal planes, (b) the principal stresses, (c) the maximum shearing stress.



SOLUTION

$$\sigma_x = 4 \text{ ksi}, \quad \sigma_y = 8 \text{ ksi}, \quad \tau_{xy} = -10 \text{ ksi}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 6 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 10.198 \text{ ksi}$$

(a)  $\tan 2\theta_b = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 5.00$

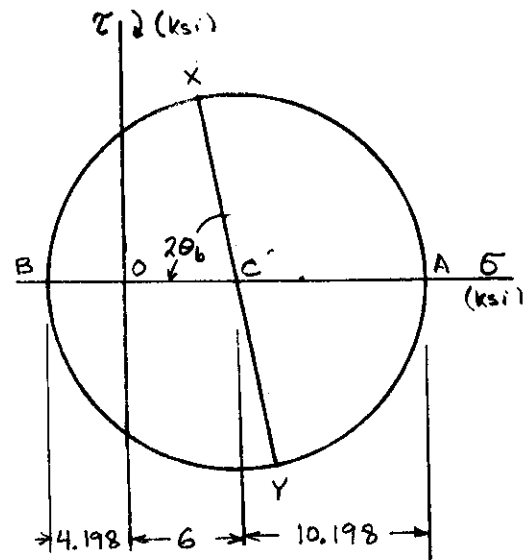
$$2\theta_b = 78.69^\circ \quad \theta_b = 39.345^\circ \quad \blacktriangleleft$$

$$\theta_a = \theta_b - 90^\circ = -50.655^\circ \quad \blacktriangleleft$$

(b)  $\sigma_a = \sigma_{ave} + R = 16.198 \text{ ksi} \quad \blacktriangleleft$

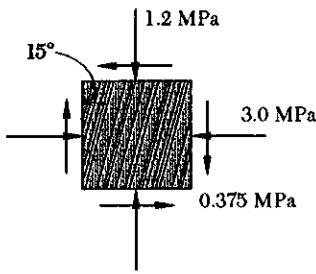
$$\sigma_b = \sigma_{ave} - R = -4.198 \text{ ksi} \quad \blacktriangleleft$$

(c)  $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = R = 10.198 \text{ ksi} \quad \blacktriangleleft$



**PROBLEM 7.165**

7.165 The grain a wooden member forms an angle of  $15^\circ$  with the vertical. For the state of plane stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



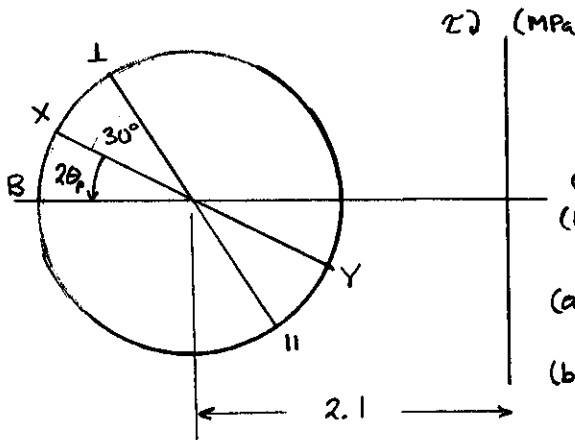
**SOLUTION**

$$\sigma_x = -3.0 \text{ MPa}, \quad \sigma_y = -1.2 \text{ MPa}$$

$$\tau_{xy} = -0.375 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -2.10 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0.975 \text{ MPa}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.41667$$

$$2\theta_p = 22.62^\circ$$

$$2\theta_p + 30^\circ = 52.62^\circ$$

$$(a) \tau_{L(\text{in-plane})} = R \sin 52.5^\circ = 0.775 \text{ MPa}$$

$$(b) \sigma_{\perp} = \sigma_{ave} - R \cos 52.52^\circ = -2.10 - 0.592 = -2.692 \text{ MPa}$$

**PROBLEM 7.166**

7.166 A cylindrical steel pressure tank has a 26-in. inside diameter and a uniform  $\frac{1}{4}$ -in. wall thickness. Knowing that the ultimate stress of the steel used is 65 ksi, determine the maximum allowable gage pressure if a factor of safety of 5.0 must be maintained.

**SOLUTION**

$$r = \frac{1}{2}d = \frac{1}{2}(26) = 13 \text{ in.} \quad t = 0.25 \text{ in.}$$

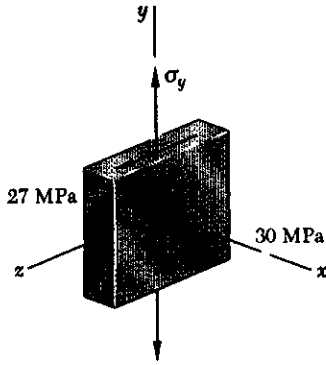
$$\sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{65}{5} = 13 \text{ ksi.} \quad \sigma_i = \frac{pr}{t}$$

$$p = \frac{\sigma_i t}{r} = \frac{(13)(0.25)}{13} = 0.25 \text{ ksi} = 250 \text{ psi}$$

PROBLEM 7.167

7.167 For the state of stress shown, determine the maximum shearing stress when (a)  $\sigma_y = +72 \text{ MPa}$ , (b)  $\sigma_y = -72 \text{ MPa}$ ,

SOLUTION



$$\sigma_x = -30 \text{ MPa}$$

$$\tau_{yz} = 27 \text{ MPa}, \sigma_2 = 0$$

(a)  $\sigma_y = +72 \text{ MPa}$

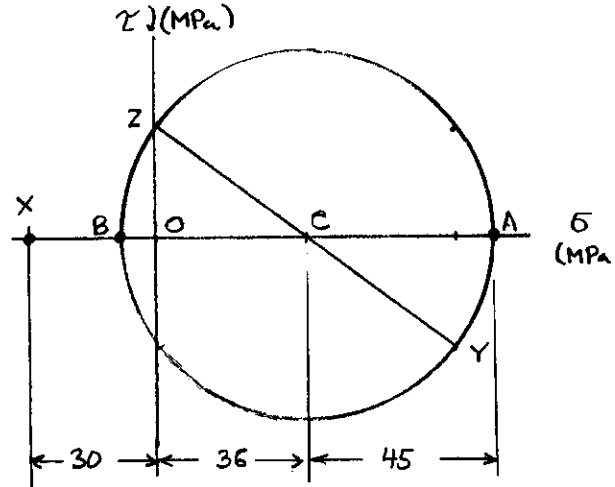
$$\sigma_{ave} = \frac{1}{2} (\sigma_y + \sigma_z) = 36 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 45 \text{ MPa}$$

$$\sigma_{max} = \sigma_a = \sigma_{ave} + R = 81 \text{ MPa}$$

$$\sigma_{min} = \sigma_x = -30 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min}) = 55.5 \text{ MPa} \rightarrow$$



(b)  $\sigma_y = -72 \text{ MPa}$

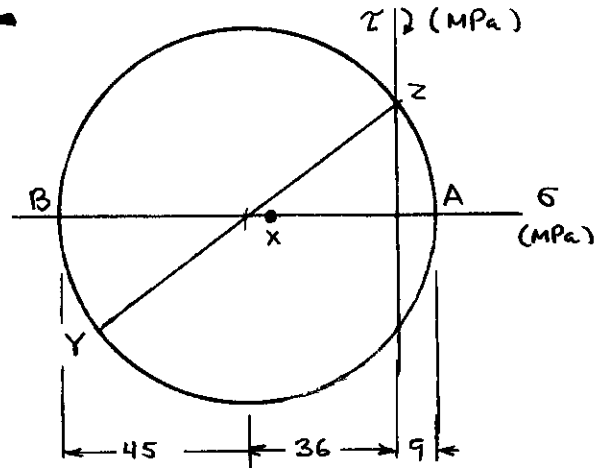
$$\sigma_{ave} = \frac{1}{2} (\sigma_y + \sigma_z) = -36 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 45 \text{ MPa}$$

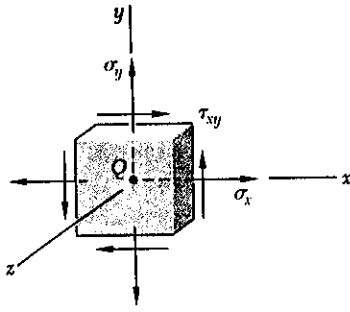
$$\sigma_{max} = \sigma_a = \sigma_{ave} + R = 9 \text{ MPa}$$

$$\sigma_{min} = \sigma_b = \sigma_{ave} - R = -81 \text{ MPa}$$

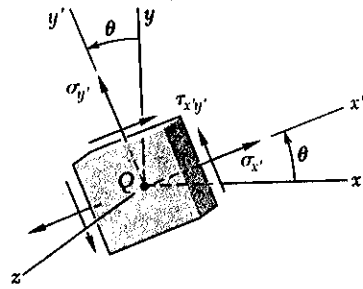
$$\tau_{max} = \frac{1}{2} (\sigma_{max} - \sigma_{min}) = 45 \text{ MPa} \rightarrow$$



**PROBLEM 7.C1**



(a)



(b)

**7.C1** A state of plane stress is defined by the stress components  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  associated with the element shown in Fig. P7.C1a. (a) Write a computer program that can be used to calculate the stress components  $\sigma_{x'}$ ,  $\sigma_{y'}$ , and  $\tau_{x'y'}$  associated with the element after it has rotated through an angle  $\theta$  about the  $z$  axis (Fig. P7.C1b). (b) Use this program to solve Probs. 7.13 through 7.16.

**SOLUTION** PROGRAM FOLLOWING EQUATIONS

$$\text{EQ (7.5), p 427: } \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\text{EQ (7.7), p 427: } \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\text{EQ (7.6), p 427: } \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

ENTER  $\sigma_x, \sigma_y, \tau_{xy}$  AND  $\theta$

PRINT VALUES OBTAINED FOR  $\tau_{x'}$ ,  $\sigma_{y'}$ , AND  $\tau_{x'y'}$

Problem 7.13a

Sigma x = -40 MPa  
Sigma y = 60 MPa  
Tau xy = 20 MPa

Rotation of element  
(+ counterclockwise)  
theta = -25 degrees

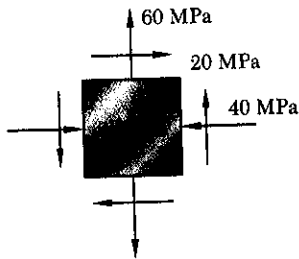
Sigma x' = -37.46 MPa  
Sigma y' = 57.46 MPa  
Tau x'y' = -25.45 MPa

Problem 7.13b

Sigma x = -40 MPa  
Sigma y = 60 MPa  
Tau xy = 20 MPa

Rotation of element  
(+ counterclockwise)  
theta = 10 degrees

Sigma x' = -30.14 MPa  
Sigma y' = 50.14 MPa  
Tau x'y' = 35.89 MPa



Problem 7.14a

Sigma x = 0 MPa  
Sigma y = -80 MPa  
Tau xy = -50 MPa

Rotation of element  
(+ counterclockwise)  
theta = -25 degrees

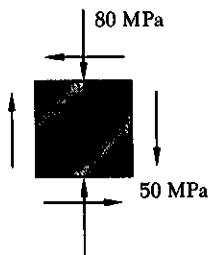
Sigma x' = 24.01 MPa  
Sigma y' = -104.01 MPa  
Tau x'y' = -1.50 MPa

Problem 7.14b

Sigma x = 0 MPa  
Sigma y = -80 MPa  
Tau xy = -50 MPa

Rotation of element  
(+ counterclockwise)  
theta = 10 degrees

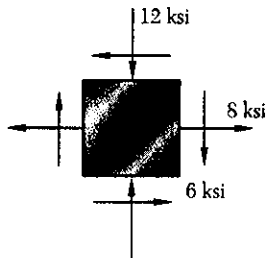
Sigma x' = -19.51 MPa  
Sigma y' = -60.49 MPa  
Tau x'y' = -60.67 MPa



CONTINUED

PROBLEM 7.C1 - CONTINUED

PROGRAM OUTPUT



Problem 7.15a

Sigma x = 8 ksi  
 Sigma y = -12 ksi  
 Tau xy = -6 ksi

Rotation of element  
 (+ counterclockwise)  
 theta = -25 degrees

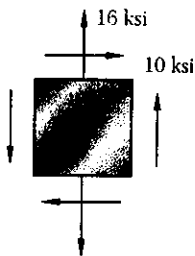
Sigma x' = 9.02 ksi  
 Sigma y' = -13.02 ksi  
 Tau x'y' = 3.80 ksi

Problem 7.15b

Sigma x = 8 ksi  
 Sigma y = -12 ksi  
 Tau xy = -6 ksi

Rotation of element  
 (+ counterclockwise)  
 theta = 10 degrees

Sigma x' = 5.34 ksi  
 Sigma y' = -9.34 ksi  
 Tau x'y' = -9.06 ksi



Problem 7.16a

Sigma x = 0 ksi  
 Sigma y = 16 ksi  
 Tau xy = 10 ksi

Rotation of element  
 (+ counterclockwise)  
 theta = -25 degrees

Sigma x' = -4.80 ksi  
 Sigma y' = 20.80 ksi  
 Tau x'y' = 0.30 ksi

Problem 7.16b

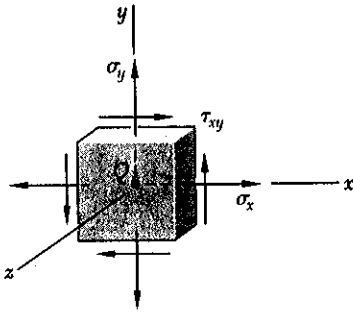
Sigma x = 0 ksi  
 Sigma y = 16 ksi  
 Tau xy = 10 ksi

Rotation of element  
 (+ counterclockwise)  
 theta = 10 degrees

Sigma x' = 3.90 ksi  
 Sigma y' = 12.10 ksi  
 Tau x'y' = 12.13 ksi



**PROBLEM 7.C2**



7.C2 A state of plane stress is defined by the stress components  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  associated with the element shown in Fig. P7.C1a. (a) Write a computer program that can be used to determine the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (b) Use this program to solve Probs. 7.7, 7.11, 7.66, and 7.67.

**SOLUTION** PROGRAM FOLLOWING EQUATIONS

EQ.(7.10)  $\tau_{ave} = \frac{\tau_x + \tau_y}{2} ; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

EQ.(7.14)  $\tau_{max} = \tau_{ave} + R$

$\tau_{min} = \tau_{ave} - R$

EQ.(7.12)  $\theta_p = \tan^{-1} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

EQ.(7.15)  $\theta_s = \tan^{-1} - \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$

SHEARING STRESS IF  $\tau_{max} > 0$  and  $\tau_{min} < 0$ :

THEN  $\tau_{max}(in-plane) = R ; \tau_{max}(out-of-plane) = R$

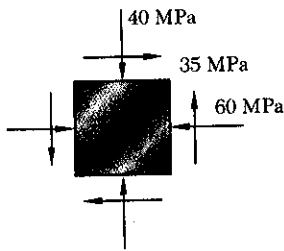
IF  $\tau_{max} > 0$  and  $\tau_{min} > 0$ :

THEN  $\tau_{max}(in-plane) = R ; \tau_{max}(out-of-plane) = \frac{1}{2} \tau_{max}$

IF  $\tau_{max} < 0$  and  $\tau_{min} < 0$ :

THEN  $\tau_{max}(in-plane) = R ; \tau_{max}(out-of-plane) = \frac{1}{2} |\tau_{min}|$

PROGRAM OUTPUT



Problems 7.7 AND 7.11

Sigma x = -60.00 MPa  
Sigma y = -40.00 MPa  
Tau xy = 35.00 MPa

Angle between xy axes and principal axes  
( + counterclockwise )

Theta p = -37.03 deg. and 52.97 deg.

Sigma max = -13.60 MPa

Sigma min = -86.40 MPa

Angle between xy axis and planes of maximum in-plane shearing stress  
( + counterclockwise )

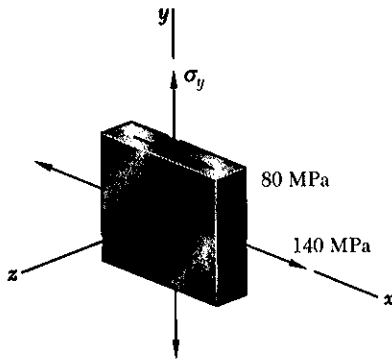
Theta s = 7.97 deg. and 97.97 deg.

Tau max (in plane) = 36.40 MPa

Tau max = 43.20 MPa

CONTINUED

**PROBLEM 7.C2 - CONTINUED**



**Fig. P7.66 and P7.67**

Problem 7.66a:  $\sigma_x = 140.00$  MPa  
 $\sigma_y = 20.00$  MPa  
 $\tau_{xy} = 80.00$  MPa

Angle between xy axes and principal axes  
 ( + counterclockwise )  
 $\theta_p = 26.57$  deg. and  $116.57$  deg.  
 $\sigma_{max} = 180.00$  MPa  
 $\sigma_{min} = -20.00$  MPa  
 Angle between xy axis and planes of maximum in-plane  
 in-plane shearing stress ( + counterclockwise )  
 $\theta_s = 71.57$  deg. and  $161.57$  deg.  
 $\tau_{max} \text{ (in-plane)} = 100.00$  MPa  
 $\tau_{max} \text{ (out-of-plane)} = 100.00$  MPa

Problem 7.66b:  $\sigma_x = 140.00$  MPa  
 $\sigma_y = 140.00$  MPa  
 $\tau_{xy} = 80.00$  MPa

Angle between xy axes and principal axes  
 ( + counterclockwise )  
 $\theta_p = 45.00$  deg. and  $135.00$  deg.  
 $\sigma_{max} = 220.00$  MPa  
 $\sigma_{min} = 60.00$  MPa  
 Angle between xy axis and planes of maximum in-plane  
 in-plane shearing stress ( + counterclockwise )  
 $\theta_s = 90.00$  deg. and  $180.00$  deg.  
 $\tau_{max} \text{ (in-plane)} = 80.00$  MPa  
 $\tau_{max} \text{ (out-of-plane)} = 110.00$  MPa

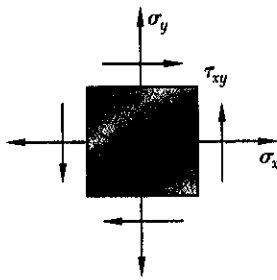
Problem 7.67a:  $\sigma_x = 140.00$  MPa  
 $\sigma_y = 40.00$  MPa  
 $\tau_{xy} = 80.00$  MPa

Angle between xy axes and principal axes  
 ( + counterclockwise )  
 $\theta_p = 29.00$  deg. and  $119.00$  deg.  
 $\sigma_{max} = 184.34$  MPa  
 $\sigma_{min} = -4.34$  MPa  
 Angle between xy axis and planes of maximum in-plane  
 in-plane shearing stress ( + counterclockwise )  
 $\theta_s = 74.00$  deg. and  $164.00$  deg.  
 $\tau_{max} \text{ (in-plane)} = 94.34$  MPa  
 $\tau_{max} \text{ (out-of-plane)} = 94.34$  MPa

Problem 7.67b:  $\sigma_x = 140.00$  MPa  
 $\sigma_y = 120.00$  MPa  
 $\tau_{xy} = 80.00$  MPa

Angle between xy axes and principal axes  
 ( + counterclockwise )  
 $\theta_p = 41.44$  deg. and  $131.44$  deg.  
 $\sigma_{max} = 210.62$  MPa  
 $\sigma_{min} = 49.38$  MPa  
 Angle between xy axis and planes of maximum in-plane  
 in-plane shearing stress ( + counterclockwise )  
 $\theta_s = 86.44$  deg. and  $176.44$  deg.  
 $\tau_{max} \text{ (in-plane)} = 80.62$  MPa  
 $\tau_{max} \text{ (out-of-plane)} = 105.31$  MPa

**PROBLEM 7.C3**



7.C3 (a) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum-shearing-stress criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (b) Use this program to solve Probs. 7.81 through 7.84.

**SOLUTION**

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} ; R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MAXIMUM-SHEARING-STRESS CRITERION  $\tau_y = \frac{1}{2} \sigma_y$

IF  $\sigma_a$  AND  $\sigma_b$  HAVE SAME SIGN,  $\tau_{max} = \frac{1}{2} \sigma_a$

IF  $\tau_{max} > \tau_y$ , YIELDING OCCURS

IF  $\tau_{max} < \tau_y$ , NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_y}{\tau_{max}}$$

MAXIMUM-DISTORTION-ENERGY CRITERION

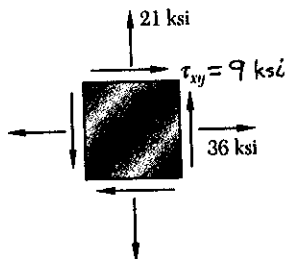
COMPUTE RADICAL =  $\sqrt{\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2}$

IF RADICAL  $> \sigma_y$ , YIELDING OCCURS

IF RADICAL  $< \sigma_y$ , NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\sigma_y}{\text{RADICAL}}$$

PROGRAM OUTPUT



Problems 7.81a and 7.82a

Sigma x = 36.00 ksi  
 Sigma y = 21.00 ksi  
 Tau xy = 9.00 ksi

Sigmax = 40.22 ksi

Sigmin = 16.78 ksi

Using the maximum-shearing-stress criterion:

Material will not yield

F.S. = 1.119

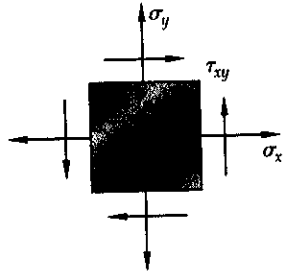
Using the maximum-distortion-energy criterion:

Material will not yield

F.S. = 1.286

CONTINUED

**PROBLEM 7.C4**



7.C4 (a) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and in compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (b) Use this program to solve Probs. 7.91 and 7.92 and to check the answers given for Probs. 7.93 and 7.94.

**SOLUTION**

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MOHR'S FRACTURE CRITERION

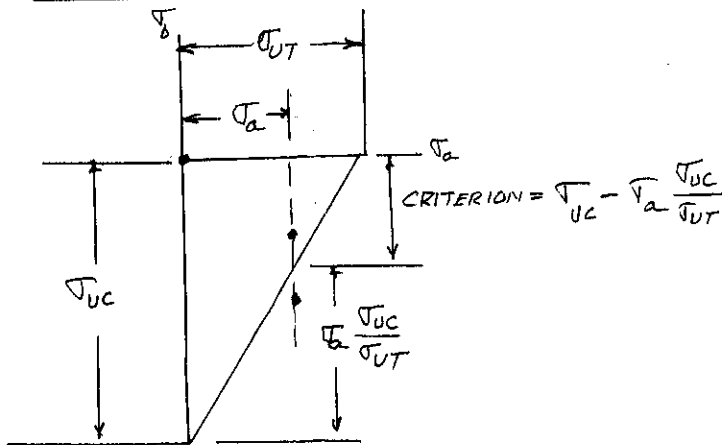
IF  $\sigma_a$  AND  $\sigma_b$  HAVE SAME SIGN, AND

$\sigma_a < \sigma_{UT}$  AND  $\sigma_b < \sigma_{UC}$ , NO FAILURE

$\sigma_a > \sigma_{UT}$  OR  $\sigma_b > \sigma_{UC}$ , FAILURE

IF  $\sigma_a > 0$  AND  $\sigma_b < 0$ :

CONSIDER FOURTH QUADRANT OF FIG. 7.47



FOR NO RUPTURE TO OCCUR:

POINT  $(\sigma_a, \tau_b)$  MUST LIE WITHIN MOHR'S ENVELOPE (FIG. 7.47)

IF  $\tau_b >$  CRITERION,  
THEN RUPTURE OCCURS

IF  $\tau_b <$  CRITERION,  
THEN NO RUPTURE OCCURS

PROGRAM OUTPUT

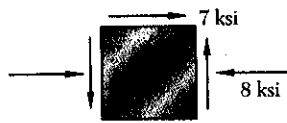


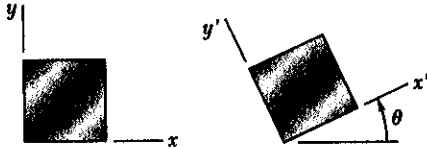
Fig. P7.91

Problem 7.91       $\sigma_x = -8.00$  ksi  
                           $\sigma_y = 0.00$  ksi  
                           $\tau_{xy} = 7.00$  ksi  
 Ultimate strength in tension = 10 ksi  
 Ultimate strength in compression = 30 ksi

$\sigma_{max} = \sigma_a = 4.06$  ksi  
 $\sigma_{min} = \sigma_b = -12.06$  ksi  
 Rupture will not occur

CONTINUED

**PROBLEM 7.C5**



**7.C5** A state of plane strain is defined by the strain components  $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$  associated with the x and y axes. (a) Write a computer program that can be used to calculate the strain components  $\epsilon_{x'}$ ,  $\epsilon_{y'}$ , and  $\gamma_{x'y'}$  associated with the frame of reference  $x'y'$  obtained by rotating the x and y axes through an angle  $\theta$ . (b) Use this program to solve Probs. 7.126 through 7.129.

**SOLUTION** PROGRAM FOLLOWING EQUATIONS

$$\text{EQ. (7.44)} \quad \epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{1}{2} \gamma_{xy} \sin 2\theta$$

$$\text{EQ. (7.45)} \quad \epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta - \frac{1}{2} \gamma_{xy} \cos 2\theta$$

$$\text{EQ. (7.46)} \quad \gamma_{x'y'} = -(\epsilon_x - \epsilon_y) \sin 2\theta + \gamma_{xy} \cos 2\theta$$

ENTER  $\epsilon_x$ ,  $\epsilon_y$ ,  $\gamma_{xy}$ , AND  $\theta$

PRINT VALUES OBTAINED FOR  $\epsilon_{x'}$ ,  $\epsilon_{y'}$ , AND  $\gamma_{x'y'}$

PROGRAM OUTPUT

Problem 7.126      Epsilon x = -720 micro meters  
                          Epsilon y =    0 micro meters  
                          Gamma xy =  300 micro radians  
 Rotation of element, in degrees ( + counterclockwise )  
                          Theta = -30 degrees  
  
                          Epsilon x' = -669.90 micro meters  
                          Epsilon y' =  -50.10 micro meters  
                          Gamma x'y' = -473.54 micro radians

Problem 7.127      Epsilon x =    0 micro meters  
                          Epsilon y =  320 micro meters  
                          Gamma xy = -100 micro radians  
 Rotation of element, in degrees ( + counterclockwise )  
                          Theta =  30 degrees  
  
                          Epsilon x' =   36.70 micro meters  
                          Epsilon y' =  283.30 micro meters  
                          Gamma x'y' =  227.13 micro radians

Problem 7.128      Epsilon x = -800 micro meters  
                          Epsilon y =  450 micro meters  
                          Gamma xy =  200 micro radians  
 Rotation of element, in degrees ( + counterclockwise )  
                          Theta = -25 degrees  
  
                          Epsilon x' = -653.35 micro meters  
                          Epsilon y' =  303.35 micro meters  
                          Gamma x'y' = -829.00 micro radians

Problem 7.129      Epsilon x = -500 micro meters  
                          Epsilon y =  250 micro meters  
                          Gamma xy =    0 micro radians  
 Rotation of element, in degrees ( + counterclockwise )  
                          Theta =  15 degrees  
  
                          Epsilon x' = -449.76 micro meters  
                          Epsilon y' =  199.76 micro meters  
                          Gamma x'y' =  375.00 micro radians

**PROBLEM 7.C6**

**7.C6** A state of strain is defined by the strain components  $\epsilon_x$ ,  $\epsilon_y$  and  $\gamma_{xy}$  associated with the x and y axes. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.134 through 7.137.

**SOLUTION** PROGRAM FOLLOWING EQUATIONS

EQ.(7.50)  $E_{ave} = \frac{\epsilon_x + \epsilon_y}{2}$   $R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

EQ.(7.51)  $E_{max} = E_{ave} + R$   $E_{min} = E_{ave} - R$

EQ.(7.52)  $\theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$\gamma_{max(in-plane)} = 2R$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET  $E_a = E_{max}$   
 $E_b = E_{min}$

CALCULATE  $E_c = -\frac{\nu}{1-\nu}(E_a + E_b)$

IF  $E_a > E_b > E_c$  :  $\gamma_{out-of-plane} = E_a - E_c$

IF  $E_a > E_c > E_b$  :  $\gamma_{out-of-plane} = E_a - E_b = 2R$

IF  $E_c > E_a > E_b$  :  $\gamma_{out-of-plane} = E_c - E_b$

PROGRAM PRINTOUT

Problem 7.134

Epsilon x = 160 micro meters  
Epsilon y = -480 micro meters  
Gamma xy = -600 micro radians  
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = -21.58 degrees  
Epsilon a = 278.63 micro meters  
Epsilon b = -598.63 micro meters  
Epsilon c = 159.98 micro meters

Gamma max (in plane) = 877.27 micro radians  
Gamma max = 877.27 micro radians

CONTINUED

PROBLEM 7.C6 - CONTINUED

Problem 7.135      Epsilon x = -260 micro meters  
Epsilon y = -60 micro meters  
Gamma xy = 480 micro radians  
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)  
Theta p = -33.69 degrees  
Epsilon a = 100.00 micro meters  
Epsilon b = -420.00 micro meters  
Epsilon c = 159.98 micro meters  
  
Gamma max (in plane) = 520.00 micro radians  
Gamma max = 579.98 micro radians

Problem 7.136      Epsilon x = -40 micro meters  
Epsilon y = 760 micro meters  
Gamma xy = 960 micro radians  
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)  
Theta p = -25.10 degrees  
Epsilon a = 984.82 micro meters  
Epsilon b = -264.82 micro meters  
Epsilon c = -359.95 micro meters  
  
Gamma max (in plane) = 1249.64 micro radians  
Gamma max = 1344.77 micro radians

Problem 7.137      Epsilon x = -300 micro meters  
Epsilon y = -200 micro meters  
Gamma xy = 175 micro radians  
nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)  
Theta p = -30.13 degrees  
Epsilon a = -149.22 micro meters  
Epsilon b = -350.78 micro meters  
Epsilon c = 250.00 micro meters  
  
Gamma max (in plane) = 201.56 micro radians  
Gamma max = 600.77 micro radians

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components  $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$  measured at a point. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.138 through 7.141.

SOLUTION

PROGRAM FOLLOWING EQUATIONS

EQ(7.50)  $E_{ave} = \frac{\epsilon_x + \epsilon_y}{2}$       $R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

EQ(7.51)  $E_{max} = E_{ave} + R$       $E_{min} = E_{ave} - R$

EQ(7.52)  $\theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$\gamma_{xy}(\text{in-plane}) = 2R$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET  $E_a = E_{max}$

$E_b = E_{min}$

$E_c = 0$  (PLAIN STRAIN)

IF  $E_a > E_b > E_c$ :  $\gamma_{\text{OUT-OF-PLANE}} = E_a - E_c$

IF  $E_a > E_c > E_b$ :  $\gamma_{\text{OUT-OF-PLANE}} = E_a - E_b = 2R$

IF  $E_c > E_a > E_b$ :  $\gamma_{\text{OUT-OF-PLANE}} = E_c - E_b$

PROGRAM PRINTOUT

Problem 7.138

Epsilon x = -90

Epsilon y = -130

Gamma xy = 150

Angle between xy axes and principal axes (+ = counterclockwise)

Theta p = 37.53 and -52.47 degrees

Epsilon a = -32.38 micro meters at 37.53 degrees

Epsilon b = -187.62 micro meters at -52.47 degrees

Epsilon c = 0.00 micro meters

Gamma max (in plane) = 155.24 micro radians

Gamma max = 187.62 micro radians

CONTINUED



**PROBLEM 7.C7 - CONTINUED**

Problem 7.139      Epsilon x = 375  
                         Epsilon y = 75  
                         Gamma xy = 125

Angle between xy axes and principal axes (+ = counterclockwise)  
Theta p = 11.31 and -78.69 degrees  
Epsilon a = 387.50 micro meters at 11.31 degrees  
Epsilon b = 62.50 micro meters at -78.69 degrees  
Epsilon c = 0.00 micro meters  
  
Gamma max (in plane) = 325.00 micro radians  
Gamma max = 387.50 micro radians

Problem 7.140      Epsilon x = 400  
                         Epsilon y = 200  
                         Gamma xy = 375

Angle between xy axes and principal axes (+ = counterclockwise)  
Theta p = 30.96 and -59.04 degrees  
Epsilon a = 512.50 micro meters at 30.96 degrees  
Epsilon b = 87.50 micro meters at -59.04 degrees  
Epsilon c = 0.00 micro meters  
  
Gamma max (in plane) = 425.00 micro radians  
Gamma max = 512.50 micro radians

Problem 7.141      Epsilon x = 60  
                         Epsilon y = 240  
                         Gamma xy = -50

Angle between xy axes and principal axes (+ = counterclockwise)  
Theta p = 7.76 and -82.24 degrees  
Epsilon a = 243.41 micro meters at 7.76 degrees  
Epsilon b = 56.59 micro meters at 97.76 degrees  
Epsilon c = 0.00 micro meters  
  
Gamma max (in plane) = 186.82 micro radians  
Gamma max = 243.41 micro radians

## PROBLEM 7.C8

**7.C8** A rosette consisting of three gages forming, respectively, angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  with the  $x$  axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio  $\nu$ . (a) Write a computer program that, for given readings  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  of the gages, can be used to calculate the strain components associated with the  $x$  and  $y$  axes and to determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.142 through 7.145.

## SOLUTION

FOR  $n=1$  TO 3, ENTER  $\theta_n$  and  $\epsilon_n$

ENTER:  $\nu = \nu$

SOLVE EQS. (7.60) FOR  $\epsilon_x$ ,  $\epsilon_y$ , AND  $\gamma_{xy}$  USING METHOD OF DETERMINATES OR ANY OTHER METHOD,

$$\text{ENTER } \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2}; \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \gamma_{xy}^2}$$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{ave} - R$$

$$\epsilon_c = -\frac{\nu}{1-\nu} (\epsilon_a + \epsilon_b)$$

$$\theta_p = \frac{1}{2} \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINSMAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(in-plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN,  
AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN,

$$\text{IF } \epsilon_c < \epsilon_b : \gamma_{OUT-OF-PLANE} = \epsilon_c - \epsilon_b$$

$$\text{IF } \epsilon_c > \epsilon_a : \gamma_{OUT-OF-PLANE} = \epsilon_c - \epsilon_a$$

$$\text{OTHERWISE: } \gamma_{OUT-OF-PLANE} = 2R$$

PROGRAM OUTPUTProblem 7.142

Gage	theta degrees	epsilon micro meters
1	30	600
2	-30	450
3	90	-75

Epsilon x = 725.000 micro meters  
Epsilon y = -75.000 micro meters  
Gamma xy = 173.205 micro radians

Epsilon a = 734.268 micro meters  
Epsilon b = -84.268 micro meters  
Gamma max (in plane) = 818.535 micro radians

CONTINUED

PROBLEM 7.C8 - CONTINUED

Problem 7.143

Gage	theta degrees	epsilon in./in.
1	-15	720
2	30	-180
3	75	120

Epsilon x = 379.808 in./in. ←  
 Epsilon y = 460.192 in./in.  
 Gamma xy = -1339.230 micro radians

Epsilon a = 1090.820 in./in.  
 Epsilon b = -250.820 in./in.  
 Gamma max (in plane) = 1341.641 micro radians

Problem 7.144

OBSERVE THAT GAGE 3 IS ORIENTATED ALONG THE  $u$  AXIS. THEREFORE

ENTER  $\epsilon_4$  AND  $\epsilon_4$  AS  $\epsilon_3$  AND  $\epsilon_3$ ,  
 THE VALUE OF  $\epsilon_u$  THAT IS OBTAINED IS ALSO THE EXPECTED READING OF GAGE 3.

Gage	theta degrees	epsilon micro meters
1	0	420
2	45	-45
4 → 3	135	165

Epsilon x = 420.000 micro meters  
 Epsilon y = -300.000 micro meters ←  
 Gamma xy = -210.000 micro radians

Epsilon a = 435.000 micro meters  
 Epsilon b = -315.000 micro meters  
 Gamma max (in plane) = 750.000 micro radians

Problem 7.145

Gage	theta degrees	epsilon in./in.
1	45	-50
2	-45	360
3	0	315

Epsilon x = 315.000 in./in.  
 Epsilon y = -5.000 in./in.  
 Gamma xy = -410.000 micro radians

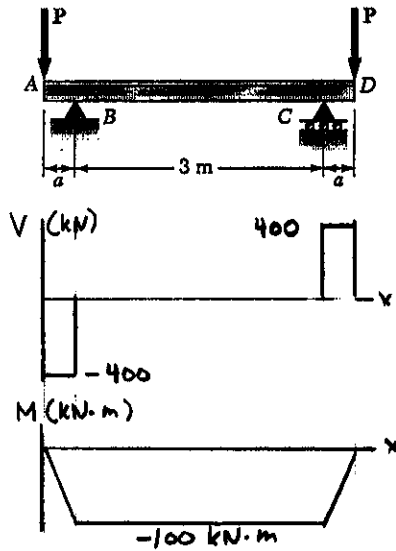
Epsilon a = 415.048 in./in. ←  
 Epsilon b = -105.048 in./in.  
 Gamma max (in plane) = 520.096 micro radians

# CHAPTER 8

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PROBLEM 8.1

8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that  $P = 400$  kN,  $a = 0.25$  m, and  $\sigma_{all} = 250$  MPa, determine (a) the maximum value of the normal stress  $\sigma_n$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.



$$|V|_{max} = 400 \text{ kN} = 400 \times 10^3 \text{ N}$$

$$|M|_{max} = (400 \times 10^3)(0.25) = 100 \times 10^3 \text{ N}\cdot\text{m}$$

For W 250 × 58 rolled steel section

$$d = 252 \text{ mm} \quad b_f = 203 \text{ mm} \quad t_f = 13.5 \text{ mm}$$

$$t_w = 8.0 \text{ mm} \quad I_x = 87.3 \times 10^6 \text{ mm}^4 \quad S_x = 693 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 126 \text{ mm} \quad y_b = c - t_f = 112.5 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S_x} = \frac{100 \times 10^3}{693 \times 10^3} = 144.3 \times 10^6 \text{ Pa} = 144.3 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{112.5}{126} (144.3) = 128.84 \text{ MPa}$$

$$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.25 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$$

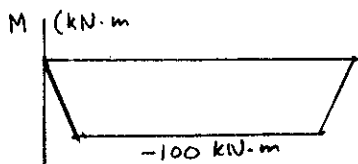
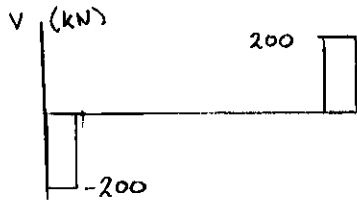
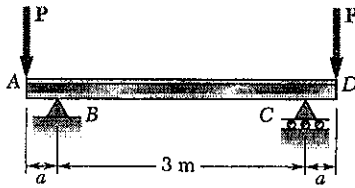
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(400 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^{-6})(8 \times 10^{-3})} = 187.2 \times 10^6 \text{ Pa} = 187.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 197.97 \text{ MPa}$$

$$(b) \quad \sigma_{max} = \frac{\sigma_b}{2} + R = 262 \text{ MPa}$$

(c) Since  $\sigma_{max} > 250$  MPa, W250 × 58 is not acceptable.

PROBLEM 8.2



8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that  $P = 400$  kN,  $a = 0.25$  m, and  $\sigma_{all} = 250$  MPa, determine (a) the maximum value of the normal stress  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of flange and the web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

8.2 Solve Prob. 8.1, assuming that  $P = 200$  kN and  $a = 0.5$  m.

$$|V|_{max} = 200 \text{ kN} = 200 \times 10^3 \text{ N}$$

$$|M|_{max} = (200 \times 10^3)(0.5) = 100 \times 10^3 \text{ N}\cdot\text{m}$$

For W250 × 58 rolled steel section

$$d = 252 \text{ mm} \quad b_f = 203 \text{ mm} \quad t_f = 13.5 \text{ mm}$$

$$t_w = 8.0 \text{ mm} \quad I_x = 87.3 \times 10^6 \text{ mm}^4 \quad S_x = 693 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 126 \text{ mm} \quad y_b = c - t_f = 112.5 \text{ mm}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S_x} = \frac{100 \times 10^3}{693 \times 10^3} = 144.3 \times 10^6 \text{ Pa} = 144.3 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{112.5}{126} (144.3) = 128.84 \text{ MPa}$$

$$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.75 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(200 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^6)(8 \times 10^{-3})} = 93.6 \times 10^6 \text{ Pa} = 93.6 \text{ MPa}$$

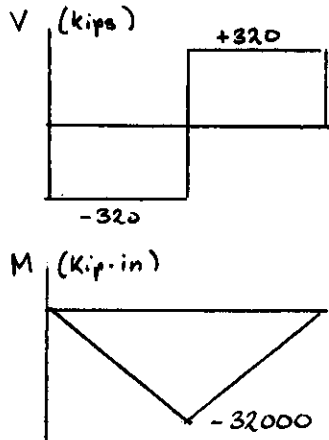
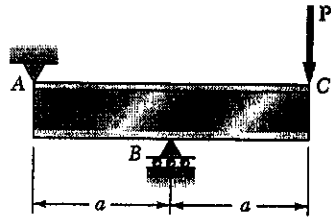
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 113.63 \text{ MPa}$$

$$(b) \quad \sigma_{max} = \frac{\sigma_b}{2} + R = 178.0 \text{ MPa}$$

(c) Since  $\sigma_{max} < 250$  MPa, W 250 × 58 is acceptable.

PROBLEM 8.3

8.3 An overhanging W36 × 300 rolled-steel beam supports a load P as shown. Knowing that  $P = 320$  kips,  $a = 100$  in., and  $\sigma_{all} = 29$  ksi, determine (a) the maximum value of the normal stress  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.



$$|V|_{max} = 320 \text{ kips}$$

$$|M|_{max} = (320)(100) = 32000 \text{ kip}\cdot\text{in.}$$

For W36 × 300 rolled steel beam

$$d = 36.74 \text{ in.} \quad b_f = 16.655 \text{ in.} \quad t_f = 1.680 \text{ in.}$$

$$t_w = 0.945 \text{ in.} \quad I_x = 20300 \text{ in}^4 \quad S_x = 1110 \text{ in}^3$$

$$c = \frac{1}{2}d = 18.37 \text{ in.} \quad y_b = c - t_f = 16.69 \text{ in.}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S_x} = \frac{32000}{1110} = 28.8 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{16.69}{18.37}\right)(28.8) = 26.2 \text{ ksi}$$

$$A_f = b_f t_f = 27.98 \text{ in}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 17.53 \text{ in.}$$

$$Q_b = A_f \bar{y}_f = 490.49 \text{ in}^3$$

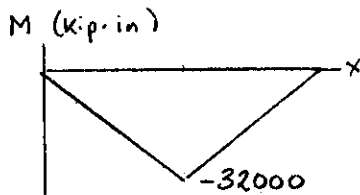
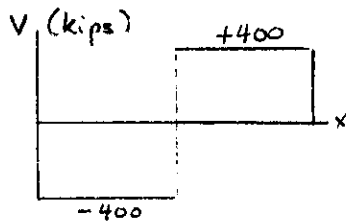
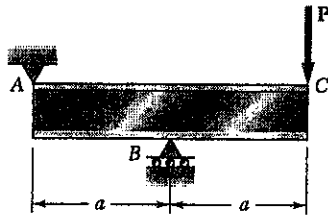
$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(320)(490.49)}{(20300)(0.945)} = 8.18 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{26.2}{2}\right)^2 + (8.18)^2} = 15.44 \text{ ksi}$$

$$(b) \quad \sigma_{max} = \frac{\sigma_b}{2} + R = 28.5 \text{ ksi} \quad \blacktriangleleft$$

$$(c) \quad \text{Since } 28.5 \text{ ksi} < \sigma_{all}, \text{ W36} \times 300 \text{ is acceptable.} \quad \blacktriangleleft$$

PROBLEM 8.4



8.3 An overhanging W36 × 300 rolled-steel beam supports a load P as shown. Knowing that  $P = 320$  kips,  $a = 100$  in., and  $\sigma_{all} = 29$  ksi, determine (a) the maximum value of the normal stress  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

8.4 Solve Prob. 8.3, assuming that  $P = 400$  kips and  $a = 80$  in.

$$|V|_{max} = 400 \text{ kips}$$

$$|M|_{max} = (400)(80) = 32000 \text{ kip}\cdot\text{in.}$$

For W36 × 300 rolled steel section

$$d = 36.74 \text{ in} \quad b_f = 16.655 \text{ in} \quad t_f = 1.680 \text{ in}$$

$$t_w = 0.945 \text{ in} \quad I_x = 20300 \text{ in}^4 \quad S_x = 1110 \text{ in}^3$$

$$c = \frac{1}{2}d = 18.37 \text{ in} \quad y_b = c - t_f = 16.69 \text{ in}$$

$$(a) \quad \sigma_m = \frac{|M|_{max}}{S_x} = \frac{32000}{1110} = 28.8 \text{ ksi}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{(16.69)}{(18.37)} (28.8) = 26.2 \text{ ksi}$$

$$A_f = b_f t_f = 27.98 \text{ in}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 17.53 \text{ in}$$

$$Q_b = A_f \bar{y}_f = 490.49 \text{ in}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(400)(490.49)}{(20300)(0.945)} = 10.23 \text{ ksi}$$

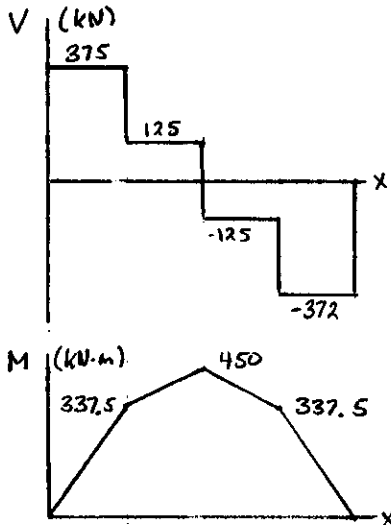
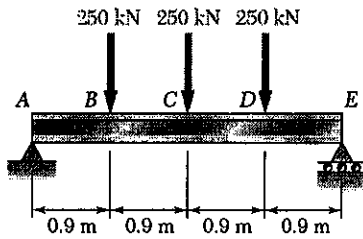
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(13.1)^2 + (10.23)^2} = 16.62 \text{ ksi}$$

$$(b) \quad \sigma_{max} = \frac{\sigma_b}{2} + R = 29.7 \text{ ksi}$$

(c) Since  $29.7 \text{ ksi} > \sigma_{all}$  W36 × 300 is not acceptable



**PROBLEM 8.5**



8.5 and 8.6 (a) Knowing that  $\sigma_{all} = 160 \text{ MPa}$  and  $\tau_{all} = 100 \text{ MPa}$ , select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for  $\sigma_m$ ,  $\tau_m$ , and the principal stress  $\sigma_{max}$  at the junction of a flange and the web of the selected beam.

Reactions:  $R_A = 375 \text{ kN } \uparrow$ ,  $R_E = 375 \text{ kN } \uparrow$

$|V|_{max} = 375 \text{ kN}$

$|M|_{max} = 450 \text{ kN}\cdot\text{m}$

$|V|$  at point C  $125 \text{ kN}$

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450 \times 10^3}{160 \times 10^6} = 2.8125 \times 10^{-3} \text{ m}^3 = 2812.5 \times 10^3 \text{ mm}^3$$

Shape	$S_x (10^3 \text{ mm}^3)$
W 840 x 176	5890
W 760 x 147	4410
W 690 x 125	3510
W 610 x 155	4220
W 530 x 150	3720
W 460 x 158	3340
W 360 x 216	3800

(a) Use  
 W 690 x 125  
 $d = 678 \text{ mm}$   
 $t_f = 16.30 \text{ mm}$   
 $t_w = 11.7 \text{ mm}$

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450 \times 10^3}{3510 \times 10^{-6}} = 128.2 \times 10^6 \text{ Pa} = 128.2 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d t_w} = \frac{375 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 47.3 \times 10^6 \text{ Pa} = 47.3 \text{ MPa}$$

At point C  $\tau_w = \frac{V}{A_w} = \frac{125 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 15.76 \times 10^6 \text{ Pa} = 15.76 \text{ MPa}$

$$c = \frac{1}{2}d = \frac{678}{2} = 339 \text{ mm} \quad y_b = c - t_f = 339 - 16.30 = 322.7 \text{ mm}$$

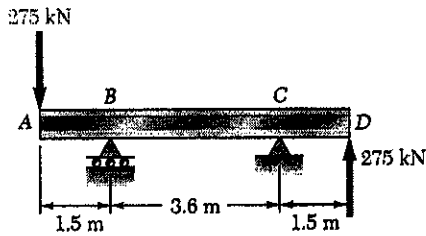
$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 = 124.0 \text{ MPa}$$

**PROBLEM 8.6**

8.5 and 8.6 (a) Knowing that  $\sigma_{all} = 160 \text{ MPa}$  and  $\tau_{all} = 100 \text{ MPa}$ , select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for  $\sigma_m$ ,  $\tau_m$ , and the principal stress  $\sigma_{max}$  at the junction of a flange and the web of the selected beam.

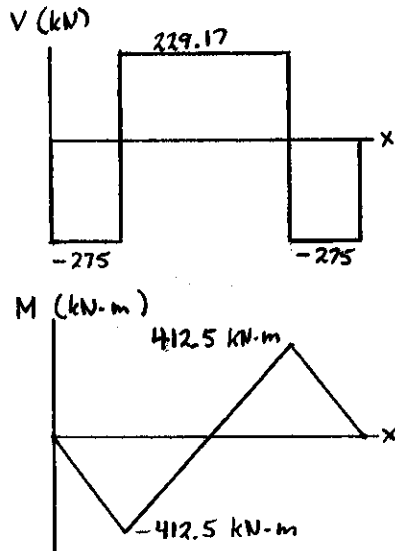


$$R_B = 504.17 \text{ kN } \uparrow \quad R_C = 504.17 \text{ kN } \downarrow$$

$$|V|_{max} = 275 \text{ kN}$$

$$|M|_{max} = 412.5 \text{ kN}\cdot\text{m}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5 \times 10^3}{160 \times 10^6} = 2578 \times 10^{-6} \text{ m}^3 = 2578 \times 10^3 \text{ mm}^3$$



Shape	$S_x (10^3 \text{ mm}^3)$
W 760 x 147	4410
W 690 x 125	3510
W 530 x 150	3720
W 460 x 158	3340
W 360 x 216	3800

(a) Use  
 W 690 x 125  
 $d = 678 \text{ mm}$   
 $t_f = 16.30 \text{ mm}$   
 $t_w = 11.7 \text{ mm}$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5 \times 10^3}{3150 \times 10^3} = 117.5 \times 10^6 \text{ Pa} = 117.5 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d t_w} = \frac{275 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 34.7 \times 10^6 \text{ Pa} = 34.7 \text{ MPa}$$

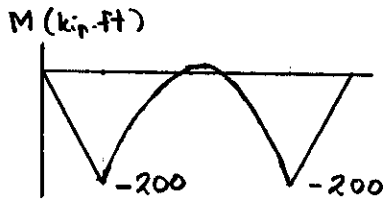
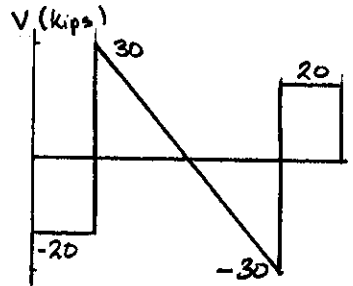
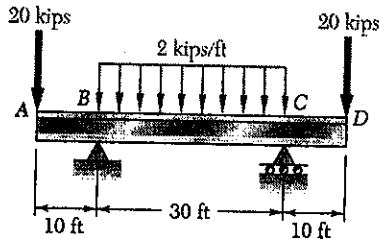
$$c = \frac{1}{2}d = \frac{678}{2} = 339 \text{ mm} \quad t_f = 16.30 \text{ mm} \quad y_b = c - t_f = 339 - 16.30 = 322.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{322.7}{339}\right)(117.5) = 111.85 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(55.925)^2 + (34.7)^2} = 65.815 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 55.925 + 65.815 = 121.7 \text{ MPa}$$

**PROBLEM 8.7**



8.7 and 8.8 (a) Knowing that  $\sigma_{all} = 24$  ksi and  $\tau_{all} = 14.5$  ksi, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for  $\sigma_m$ ,  $\tau_m$ , and the principal stress  $\sigma_{max}$  at the junction of a flange and the web of the selected beam.

$R_A = 50$  kips  $\uparrow$        $R_D = 50$  kips  $\uparrow$

$|V|_{max} = 30$  kips

$|M|_{max} = 200$  kip-ft = 2400 kip-in.

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100$  in<sup>3</sup>

Shape	S (in <sup>3</sup> )
W 24 x 68	154
→ W 21 x 62	127
W 18 x 76	146
W 16 x 77	134
W 12 x 96	103
W 10 x 112	131

(a) Use

W 21 x 62

$d = 20.99$  in.

$t_f = 0.615$  in.

$t_w = 0.400$  in.

$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.90$  ksi

$\tau_m = \frac{|V|_{max}}{d t_w} = \frac{30}{(20.99)(0.400)} = 3.57$  ksi

$y_b = c - t_f = 10.495 - 0.615 = 9.88$  in.

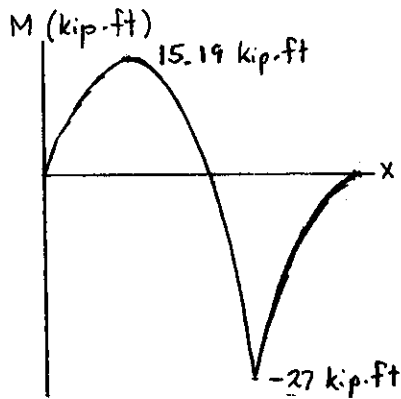
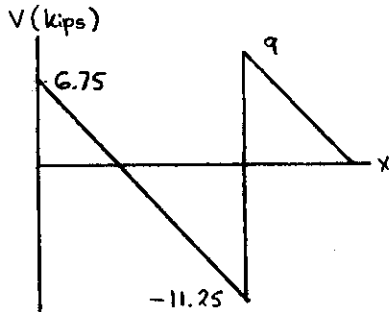
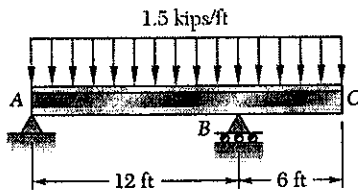
$c = \frac{1}{2}d = \frac{20.99}{2} = 10.495$  in.

$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{9.88}{10.495}\right)(18.90) = 17.79$  ksi

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.896)^2 + (3.57)^2} = 9.586$  ksi

$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.896 + 9.586 = 18.48$  ksi

**PROBLEM 8.8**



8.7 and 8.8 (a) Knowing that  $\sigma_{all} = 24$  ksi and  $\tau_{all} = 14.5$  ksi, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for  $\sigma_m$ ,  $\tau_m$ , and the principal stress  $\sigma_{max}$  at the junction of a flange and the web of the selected beam.

$$\uparrow \Sigma M_B = 0 \quad -12R_A + (1.5)(18)(3) = 0 \quad R_A = 6.75 \text{ kips } \uparrow$$

$$\uparrow \Sigma M_A = 0 \quad 12R_B + (1.5)(18)(9) = 0 \quad R_B = 20.25 \text{ kips } \uparrow$$

$$|V|_{max} = 11.25 \text{ kips}$$

$$|M|_{max} = 27 \text{ kip-ft} = 324 \text{ kip-in}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

Shape	S (in <sup>3</sup> )
W 12 x 16	17.1
W 10 x 15	13.8
W 8 x 18	15.2
W 6 x 20	13.4

(a) Use

W 10 x 15

$$d = 9.99 \text{ in.}$$

$$t_f = 0.270 \text{ in.}$$

$$t_w = 0.230 \text{ in.}$$

$$(b) \quad \sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.5 \text{ ksi}$$

$$\tau_m = \frac{|V|_{max}}{d t_w} = \frac{11.25}{(9.99)(0.230)} = 4.90 \text{ ksi}$$

$$c = \frac{1}{2}d = \frac{9.99}{2} = 4.995 \text{ in.}$$

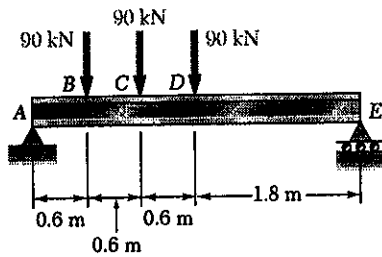
$$y_b = c - t_f = 4.995 - 0.270 = 4.725 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{4.725}{4.995}\right)(23.5) = 22.2 \text{ ksi}$$

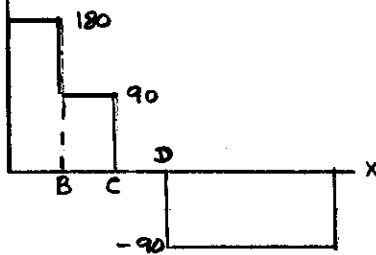
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.2}{2}\right)^2 + (4.90)^2} = 12.1 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{22.2}{2} + 12.1 = 23.2 \text{ ksi}$$

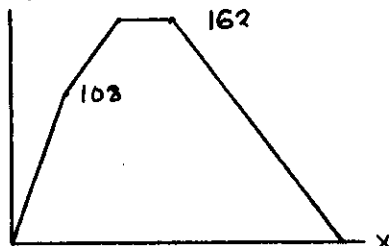
**PROBLEM 8.9**



$V$  (kN)



$M$  (kN·m)



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.

8.9 Loading of Prob. 5.81 and selected W410 × 60 shape.

From Problem 5.81  $\sigma_{all} = 160 \text{ MPa}$

$$|M|_{max} = 162 \text{ kN}\cdot\text{m at C and D}$$

$$|V| = 90 \text{ kN at C and D}$$

For W 410 × 60 rolled steel section

$$d = 407 \text{ mm}, b_f = 178 \text{ mm}, t_f = 12.80 \text{ mm}$$

$$t_w = 7.7 \text{ mm}, I_z = 216 \times 10^6 \text{ mm}^4, S_z = 1060 \times 10^3 \text{ mm}^3$$

$$c = \frac{1}{2}d = 203.5 \text{ mm}$$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{162 \times 10^3}{1060 \times 10^3} = 152.8 \text{ MPa}$$

$$y_b = c - t_f = 190.7 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 143.2 \text{ MPa}$$

$$A_f = b_f t_f = 2278 \text{ mm}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 197.1 \text{ mm}$$

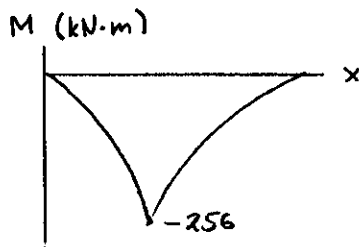
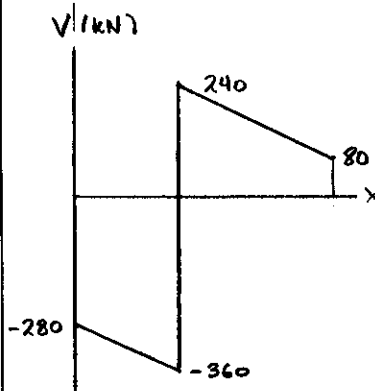
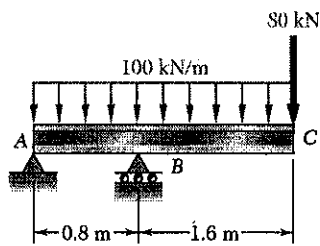
$$Q = A_f \bar{y} = (2278)(197.1) = 449 \times 10^3 \text{ mm}^3$$

$$\tau_b = \frac{VQ}{I t_w} = \frac{(90 \times 10^3)(449 \times 10^3)}{(216 \times 10^6)(7.7 \times 10^{-3})} = 24.3 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{71.6^2 + 24.3^2} = 75.6 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = \sqrt{71.6 + 75.6} = 147.2 \text{ MPa}$$

PROBLEM 8.10



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.

8.10 Loading of Prob. 5.86 and selected S 510 × 98.3 shape.

From Problem 5.86  $\sigma_{all} = 160 \text{ MPa}$

$M_{max} = 256 \text{ kN}\cdot\text{m}$  at point B

$|V| = 360 \text{ kN}$  at B

For S 510 × 98.3 rolled steel section

$d = 508 \text{ mm}, b_f = 159 \text{ mm}, t_f = 20.2 \text{ mm}$

$t_w = 12.8 \text{ mm}, I_x = 495 \times 10^6 \text{ mm}^4, S_x = 1950 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 254 \text{ mm}$

$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{256 \times 10^3}{1950 \times 10^3} = 131.3 \text{ MPa}$

$y_b = c - t_f = 233.8$

$\sigma_b = \frac{y_b}{c} \sigma_m = 120.9 \text{ MPa}$        $\frac{\sigma_b}{2} = 60.45 \text{ MPa}$

$A_f = b_f t_f = 3212 \text{ mm}^2$

$\bar{y} = \frac{1}{2} (c + y_b) = 243.9 \text{ mm}$

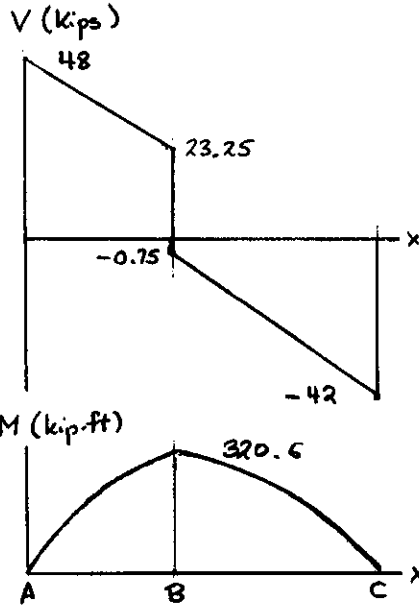
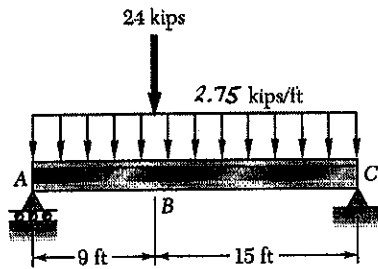
$Q = A_f \bar{y} = 783.4 \times 10^3 \text{ mm}^3$

$\tau_b = \frac{VQ}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^3)}{(495 \times 10^6)(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$

$\sigma_{max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$

**PROBLEM 8.11**



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.

8.11 Loading of Prob. 5.83 and selected W27 × 84 shape.

From Problem 5.83  $\sigma_{all} = 24 \text{ ksi}$

$$|M|_{max} = 320.6 \text{ kip}\cdot\text{ft} = 3847 \text{ kip}\cdot\text{in}$$

$$\text{At B- } |V| = 23.25 \text{ kips}$$

For W 27 × 84 rolled steel section

$$d = 26.71 \text{ in}, \quad b_f = 9.960 \text{ in}, \quad t_f = 0.640 \text{ in}$$

$$t_w = 0.460 \text{ in}, \quad I_z = 2850 \text{ in}^4, \quad S_z = 213 \text{ in}^3$$

$$c = \frac{1}{2}d = 13.355 \text{ in.}$$

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{3847}{213} = 18.06 \text{ ksi}$$

$$y_b = c - t_f = 12.715 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 17.20 \text{ ksi} \quad \frac{\sigma_b}{2} = 8.60 \text{ ksi}$$

$$A_f = b_f t_f = (9.960)(0.640) = 6.3744 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 13.305 \text{ in}^2$$

$$Q = A_f \bar{y} = 83.09 \text{ in}^3$$

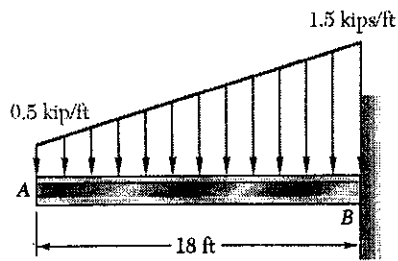
$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(23.25)(83.09)}{(2850)(0.460)} = 1.47 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{(8.60)^2 + (1.47)^2} = 8.72 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.60 + 8.72 \text{ ksi} = 17.32 \text{ ksi}$$

PROBLEM 8.12

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.



8.12 Loading of Prob. 5.84 and selected W18 × 50 shape.

From Problem 5.84  $\sigma_{all} = 24 \text{ ksi}$

$|M|_{max} = 135 \text{ kip}\cdot\text{ft} = 1620 \text{ kip}\cdot\text{in}$  at B

$|V|_{max} = 18 \text{ kips}$  at B

For W18 × 50 shape  $d = 17.99 \text{ in}$ ,  $b_f = 7.495 \text{ in}$ ,  $t_f = 0.570 \text{ in}$ .

$t_w = 0.355 \text{ in}$ ,  $I_z = 800 \text{ in}^4$ ,  $S_z = 88.9 \text{ in}^3$ ,  $c = \frac{1}{2}d = 8.995 \text{ in}$

$\sigma_m = \frac{|M|_{max}}{S_z} = 18.22 \text{ ksi}$

$y_b = c - t_f = 8.425 \text{ in}$

$\sigma_b = \frac{y_b}{c} \sigma_m = 17.07 \text{ ksi}$

$\frac{\sigma_b}{2} = 8.535 \text{ ksi}$

$A_f = b_f t_f = 4.272 \text{ in}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 8.71 \text{ in}$

$Q = A_f \bar{y} = 37.21 \text{ in}^3$

$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(37.21)}{(800)(0.355)} = 2.36 \text{ ksi}$

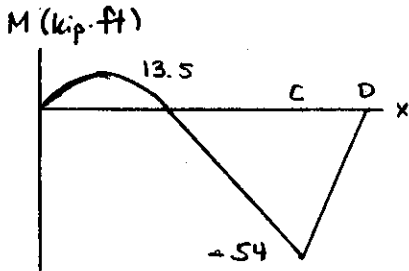
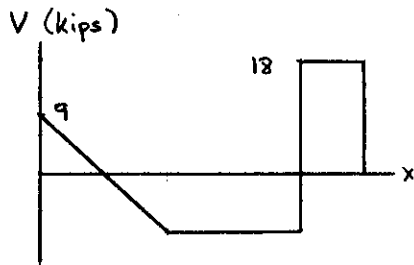
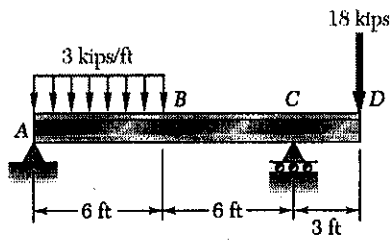
$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.535^2 + 2.36^2} = 8.855 \text{ ksi}$

$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.535 + 8.855 = 17.39 \text{ ksi}$



**PROBLEM 8.13**

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.



8.13 Loading of Prob. 5.87 and selected S12 x 31.8 shape.

From Problem 5.87  $\sigma_{all} = 24 \text{ ksi}$

$$|M|_{max} = 54 \text{ kip}\cdot\text{ft} = 648 \text{ kip}\cdot\text{in} \quad \text{at C}$$

$$\text{At C} \quad |V| = 18 \text{ kips}$$

For S12 x 31.8

$$d = 12.00 \text{ in}, \quad b_f = 5.00 \text{ in}, \quad t_f = 0.544 \text{ in}$$

$$t_w = 0.350 \text{ in}, \quad I_2 = 218 \text{ in}^4, \quad S_2 = 36.4 \text{ in}^3$$

$$c = \frac{1}{2}d = 6.00 \text{ in}$$

$$\sigma_m = \frac{|M|}{S_2} = \frac{648}{36.4} = 17.80 \text{ ksi}$$

$$y_b = c - t_f = 5.456 \text{ in}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = 16.186 \text{ ksi} \quad \frac{\sigma_b}{2} = 8.093 \text{ ksi}$$

$$A_f = b_f t_f = 2.72 \text{ in}^2$$

$$\bar{y} = \frac{1}{2}(c + y_b) = 5.728 \text{ in}$$

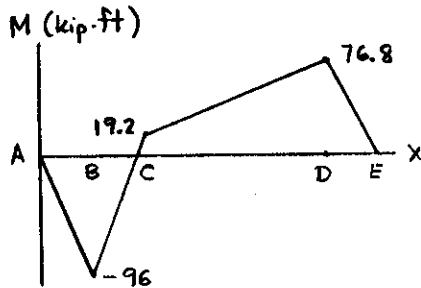
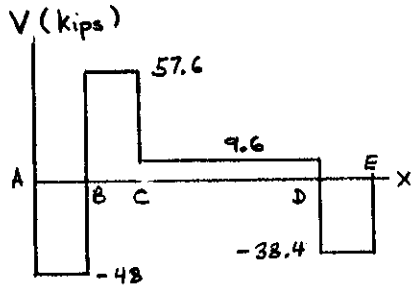
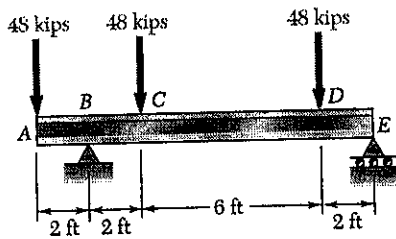
$$Q = A_f \bar{y} = 15.58 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_2 t_w} = \frac{(18)(15.58)}{(218)(0.350)} = 3.675 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.093^2 + 3.675^2} = 8.889 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.093 + 8.889 = 16.98 \text{ ksi}$$

**PROBLEM 8.14**



8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam, (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.

8.14 Loading of Prob. 5.88 and selected S15 x 42.9 shape.

From Problem 5.88  $\sigma_{all} = 24 \text{ ksi}$

$|M|_{max} = 96 \text{ kip}\cdot\text{ft} = 1152 \text{ kip}\cdot\text{in}$  at D

At D  $|V| = 38.4 \text{ kips}$ .

For S15 x 42.9 shape

$d = 15.00 \text{ in}, b_f = 5.501 \text{ in}, t_f = 0.622 \text{ in}$

$t_w = 0.411 \text{ in}, I_2 = 447 \text{ in}^4, S_2 = 59.6 \text{ in}^3$

$c = \frac{1}{2}d = 7.5 \text{ in}$

$\sigma_m = \frac{|M|}{S} = \frac{1152}{59.6} = 19.33 \text{ ksi}$

$y_b = c - t_f = 6.878 \text{ in}$

$\sigma_b = \frac{y_b}{c} \sigma_m = 17.73 \text{ ksi}$

$\frac{\sigma_b}{2} = 8.86 \text{ ksi}$

$A_f = b_f t_f = 3.4216 \text{ in}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 7.189 \text{ in}$

$Q = A_f \bar{y} = 24.60 \text{ in}^3$

$\tau_b = \frac{VQ}{I_2 t_w} = \frac{(57.6)(24.60)}{(447)(0.411)} = 7.71 \text{ ksi}$

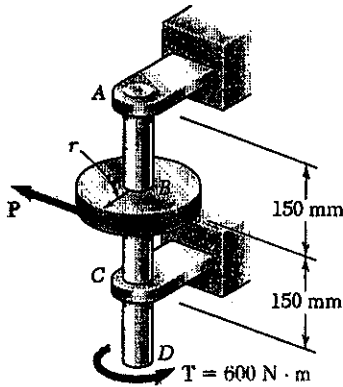
$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.86^2 + 7.71^2} = 11.74 \text{ ksi}$

$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.86 + 11.74 = 20.6 \text{ ksi}$

**PROBLEM 8.15**

8.15 Determine the smallest allowable diameter of the solid shaft *ABCD*, that  $\tau_{all} = 60 \text{ MPa}$  and that the radius of disk *B* is  $r = 80 \text{ mm}$ .

**SOLUTION**

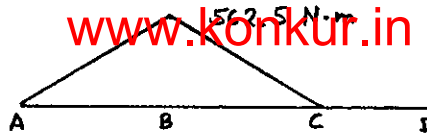
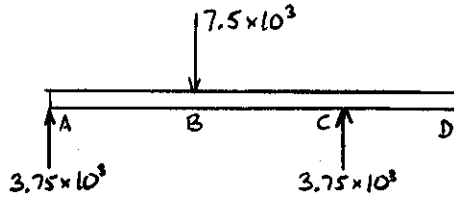


$$\sum M_{axis} = 0 \quad T - Pr = 0$$

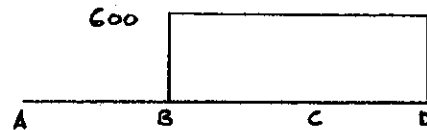
$$P = \frac{T}{r} = \frac{600}{80 \times 10^{-3}} = 7.5 \times 10^3 \text{ N}$$

$$R_A = R_C = \frac{1}{2}P = 3.75 \times 10^3 \text{ N}$$

$$M_B = (3.75 \times 10^3)(150 \times 10^{-3}) = 562.5 \text{ N}\cdot\text{m}$$



Bending moment



Torque

Critical section lies at point B

$$M = 562.5 \text{ N}\cdot\text{m}, \quad T = 600 \text{ N}\cdot\text{m}$$

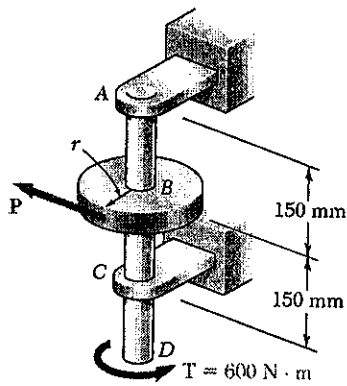
$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{(\sqrt{M^2 + T^2})_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi} \frac{\sqrt{(562.5)^2 + (600)^2}}{60 \times 10^6} = 8.726 \times 10^{-6} \text{ m}^3$$

$$c = 20.58 \times 10^{-3} \text{ m} \quad d = 2c = 41.2 \times 10^{-3} \text{ m} = 41.2 \text{ mm}$$

PROBLEM 8.16

8.16 Determine the smallest allowable diameter of the solid shaft ABCD, knowing that  $\tau_{all} = 60 \text{ MPa}$  and that the radius of disk B is  $r = 120 \text{ mm}$



SOLUTION

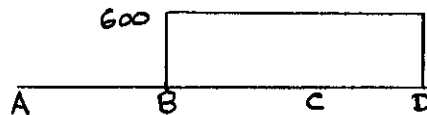
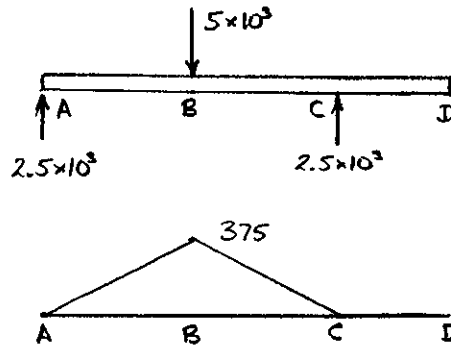
$$\sum M_{AD} = 0 \quad T - Pr = 0$$

$$P = \frac{T}{r} = \frac{600}{120 \times 10^{-3}} = 5 \times 10^3 \text{ N}$$

$$R_A = R_C = \frac{1}{2} P = 2.5 \times 10^3 \text{ N}$$

$$M_B = (2.5 \times 10^3)(0.150 \times 10^{-3}) = 375 \text{ N}\cdot\text{m}$$

Bending moment



Torque

Critical section lies at point B  $M = 375 \text{ N}\cdot\text{m}$ ,  $T = 600 \text{ N}\cdot\text{m}$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M^2 + T^2})_{max}}{\tau_{all}}$$

$$C^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi} \frac{\sqrt{375^2 + 600^2}}{60 \times 10^6} = 7.507 \times 10^{-6} \text{ m}^3$$

$$C = 19.58 \times 10^{-3} \text{ m} \quad d = 2C = 39.2 \times 10^{-3} \text{ m} = 39.2 \text{ mm}$$

PROBLEM 8.17

8.17 Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a cylindrical shaft can be expressed as

$$\sigma_{\max} = \frac{c}{J} \left[ (M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}} \right]_{\max}$$

SOLUTION

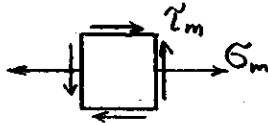
Maximum bending stress

$$\sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2} c}{I}$$

Maximum torsional stress

$$\tau_m = \frac{Tc}{J}$$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2} c}{2I} = \frac{c}{J} \sqrt{M_y^2 + M_z^2}$$



Using Mohr's circle  
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$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2} (M_y^2 + M_z^2) + \frac{T^2 c^2}{J^2}}$$

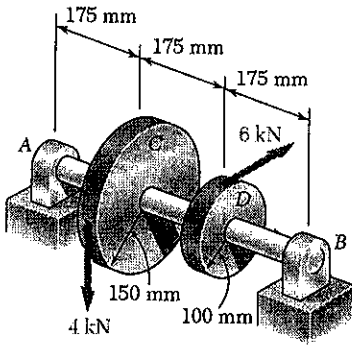
$$= \frac{c}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{\max} = \frac{\sigma_m}{2} + R = \frac{c}{J} \sqrt{M_y^2 + M_z^2} + \frac{c}{J} \sqrt{M_y^2 + M_z^2 + T^2}$$

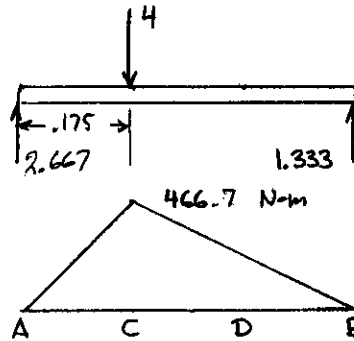
$$= \frac{c}{J} \left[ (M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}} \right]$$

**PROBLEM 8.18**

8.18 Use the expression given in Prob. 8.17 to determine the maximum normal stress in the solid shaft  $AB$ , knowing that its diameter is 36 mm.



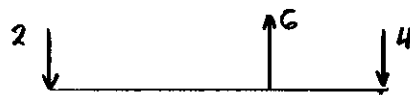
**SOLUTION**



Vertical forces, kN

Bending moment  $M_2$

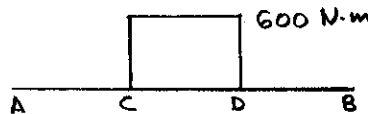
$$M_{2C} = (0.175)(2.667 \times 10^3) = 466.7 \text{ N}\cdot\text{m}$$



Horizontal forces, kN

Bending moment  $M_y$

$$M_{yD} = (0.175)(4 \times 10^3) = 700 \text{ N}\cdot\text{m}$$



Torque

$$T = (6 \times 10^3)(100 \times 10^{-3}) = 600 \text{ N}\cdot\text{m}$$

At point C  $\sqrt{M_y^2 + M_2^2} = \sqrt{350^2 + 466.7^2} = 583.3 \text{ N}\cdot\text{m}$

At point D  $\sqrt{M_y^2 + M_2^2} = \sqrt{700^2 + 233.3^2} = 737.9 \text{ N}\cdot\text{m}$

Point D is critical

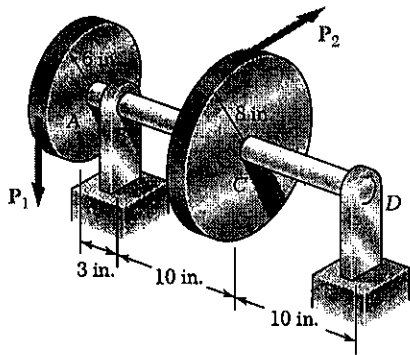
$$c = \frac{1}{2}d = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$J = \frac{\pi}{2}c^4 = 164.90 \times 10^3 \text{ mm}^4 = 164.90 \times 10^{-9} \text{ m}^4$$

$$\begin{aligned} \sigma_{max} &= \frac{c}{J} \left[ \sqrt{M_y^2 + M_2^2} + \sqrt{M_y^2 + M_2^2 + T^2} \right] \\ &= \frac{18 \times 10^{-3}}{164.90 \times 10^{-9}} \left[ 737.9 + \sqrt{737.9^2 + 600^2} \right] = 184.4 \times 10^6 \text{ Pa} \\ &= 184.4 \text{ MPa} \end{aligned}$$

**PROBLEM 8.19**

8.19 The vertical force  $P_1$  and the horizontal force  $P_2$  are applied as shown to disks welded to the solid shaft  $AD$ . Knowing that the diameter of the shaft is 1.75 in. and that  $\tau_{all} = 8$  ksi, determine the largest permissible magnitude of the force  $P_2$ .

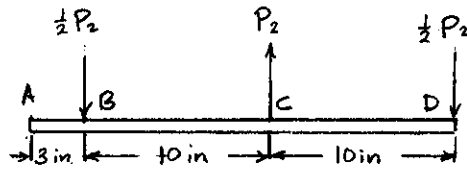


**SOLUTION**

Let  $P_2$  be in kips.

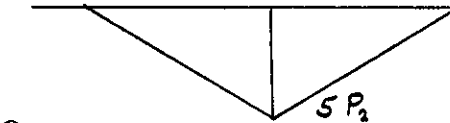
$$\sum M_{shaft} = 0 \quad 6P_1 - 8P_2 = 0 \quad P_1 = \frac{4}{3}P_2$$

Torque over portion ABC  $T = 8P_2$

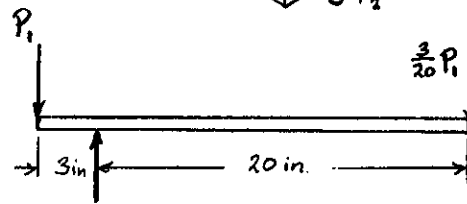


$$M_{Cy} = 10 \cdot \frac{1}{2}P_2 = 5P_2$$

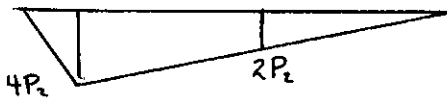
Bending in horizontal plane



Bending in vertical plane



$$M_{Bz} = 3P_1 = 3 \cdot \frac{4}{3}P_2 = 4P_2$$



Critical point is just to the left of point C.

$$T = 8P_2 \quad M_y = 5P_2 \quad M_z = 2P_2$$

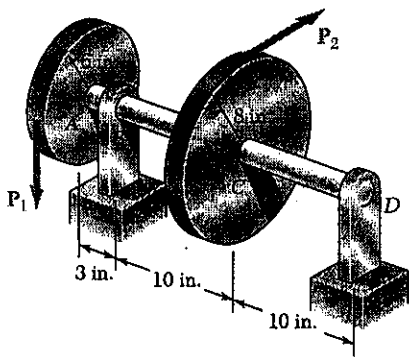
$$d = 1.75 \text{ in} \quad c = \frac{1}{2}d = 0.875 \text{ in} \quad J = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$$

$$\tau_{all} = \frac{c}{J} \sqrt{T^2 + M_y^2 + M_z^2}$$

$$8 = \frac{0.875}{0.92077} \sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 9.164 P_2$$

$$P_2 = 0.873 \text{ kips} = 873 \text{ lb.}$$

PROBLEM 8.20



8.19 The vertical force  $P_1$  and the horizontal force  $P_2$  are applied as shown to disks welded to the solid shaft  $AD$ . Knowing that the diameter of the shaft is 1.75 in. and that  $\tau_{all} = 8$  ksi, determine the largest permissible magnitude of the force  $P_2$ .

8.20 Solve Prob. 8.19, assuming that the solid shaft  $AD$  has been replaced by a hollow shaft of the same material and of inner diameter 1.50 in. and outer diameter 1.75 in.

SOLUTION

Let  $P_2$  be in kips

$$\sum M_{shaft} = 0 \quad 6P_1 - 8P_2 = 0$$

$$P_1 = \frac{4}{3}P_2$$

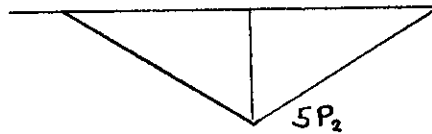
Torque over portion ABC

$$T = 8P_2$$

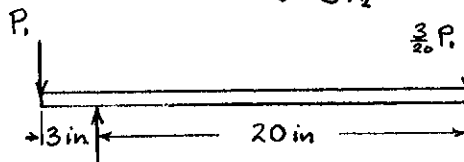
Bending in horizontal plane.



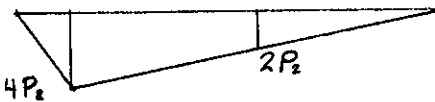
$$M_{cy} = 10 \cdot \frac{1}{2}P_2 = 5P_2$$



Bending in vertical plane.



$$M_{Bz} = 3P_1 = 3 \cdot \frac{4}{3}P_2 = 4P_2$$



Critical point is just to the left of point C

$$T = 8P_2 \quad M_y = 5P_2 \quad M_z = 2P_2$$

$$c_o = \frac{1}{2}d_o = 0.875 \text{ in.} \quad c_i = \frac{1}{2}d_i = 0.750 \text{ in.}$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 0.42376 \text{ in}^4$$

$$\tau_{all} = \frac{c_o}{J} \sqrt{T^2 + M_y^2 + M_z^2}$$

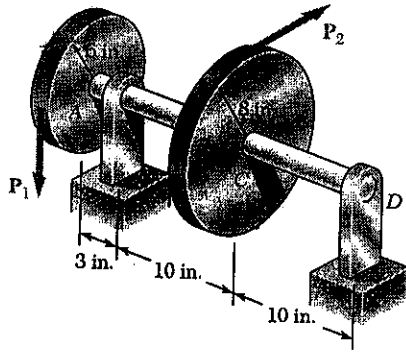
$$8 = \frac{0.875}{0.42376} \sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 19.913 P_2$$

$$P_2 = 0.402 \text{ kips} = 402 \text{ lb.}$$

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**PROBLEM 8.22**



8.22 Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.19 are, respectively,  $P_1 = 1080$  lb and  $P_2 = 810$  lb, and using the expressions given in Prob. 8.21, determine the values of  $\tau_H$  and  $\tau_K$  in a section (a) just to the left of B, (b) just to the left of C.

**SOLUTION**

From Prob. 8.19, shaft diameter = 1.75 in.

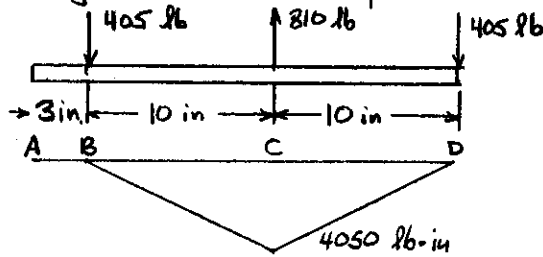
$$c = \frac{1}{2}d = 0.875 \text{ in}$$

$$J = \frac{\pi}{2}c^4 = 0.92077$$

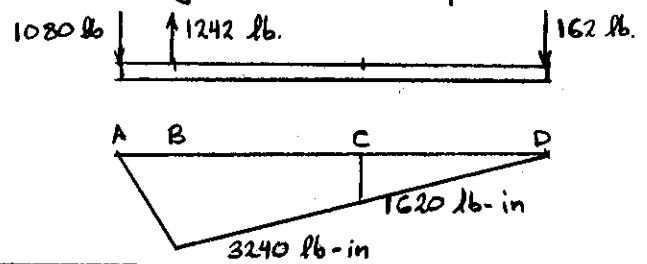
Torque over portion ABC

$$T = (6)(1080) = (8)(810) = 6480 \text{ lb-in}$$

Bending in horizontal plane



Bending in vertical plane



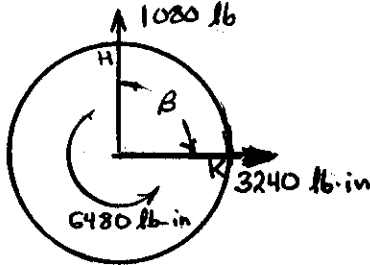
(a) Just to the left of point B

$$V = 1080 \text{ lb.}$$

$$M = 3240 \text{ lb-in}$$

$$\beta = 90^\circ$$

$$T = 6480 \text{ lb-in}$$



$$\tau_H = \frac{c}{J} \sqrt{M^2 + T^2} = \frac{0.875}{0.92077} \sqrt{(3240)^2 + (6480)^2} = 6880 \text{ psi}$$

$$\tau_K = \frac{c}{J} \sqrt{(M \cos \beta)^2 + \left(\frac{2}{3}Vc + T\right)^2} = \frac{c}{J} \left[\frac{2}{3}Vc + T\right] = \frac{0.875}{0.92077} \left[\left(\frac{2}{3}\right)(1080)(0.875) + 6480\right] = 6780 \text{ psi}$$

(b) Just to the left of point C

$$V = \sqrt{(162)^2 + (405)^2} = 436.2 \text{ lb.}$$

$$\alpha = \tan^{-1} \frac{162}{405} = 21.8^\circ$$

$$M = \sqrt{(1620)^2 + (4050)^2} = 4362 \text{ lb-in}$$

$$\gamma = \tan^{-1} \frac{1620}{4050} = 21.8^\circ$$

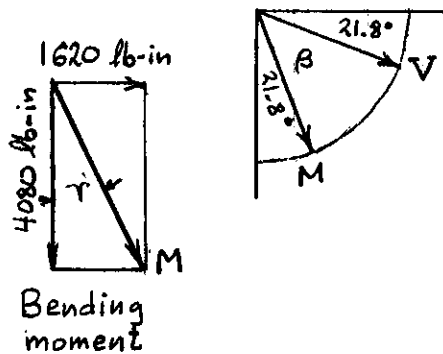
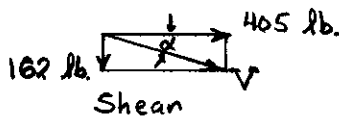
$$\beta = 90^\circ - 21.8^\circ - 21.8^\circ = 46.4^\circ$$

$$\tau_H = \frac{0.875}{0.92077} \sqrt{(6480)^2 + (4362)^2} = 7420 \text{ psi}$$

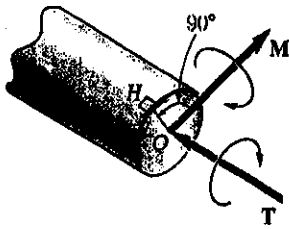
$$\frac{2}{3}Vc + T = \left(\frac{2}{3}\right)(436.2)(0.875) + 6480 = 6734 \text{ lb-in}$$

$$M \cos \beta = 4362 \cos 46.4^\circ = 3008 \text{ lb-in}$$

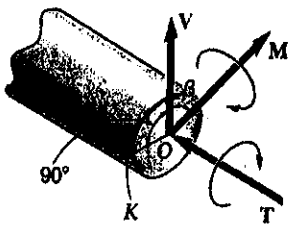
$$\tau_K = \frac{0.875}{0.92077} \sqrt{(3008)^2 + (6734)^2} = 7010 \text{ psi}$$



**PROBLEM 8.21**



(a)



(b)

8.21 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.21a) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J} \sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.21b), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J} \sqrt{(M \cos \beta)^2 + \left(\frac{2}{3}cV + T\right)^2}$$

where  $\beta$  is the angle between the vectors V and M. It is clear that the effect of the shear V cannot be ignored when  $\tau_K \geq \tau_H$ . (Hint. Only the component of M along V contributes to the shearing stress at K.)

**SOLUTION**

Shearing stress at point k

Due to V: For a semicircle  $Q = \frac{2}{3}c^3$   
 For a circle cut across its diameter  $t = d = 2c$   
 For a circular section  $I = \frac{1}{2}J$   

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3}c^3)}{(\frac{1}{2}J)(2c)} = \frac{2}{3} \frac{Vc^2}{J}$$

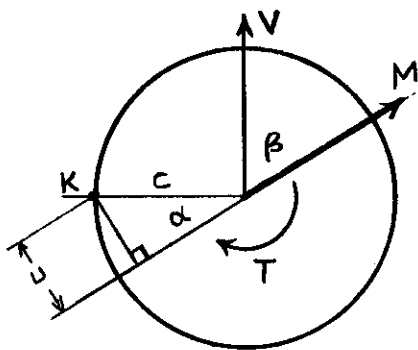
Due to T 
$$\tau_{xy} = \frac{Tc}{J}$$

Since these shearing stresses have the same orientation

$$\tau_{xy} = \frac{c}{J} \left( \frac{2}{3}Vc + T \right)$$

Bending stress at point K. 
$$\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$$

where u is distance between point K and the neutral axis,



cross-section

$$u = c \sin \alpha = c \sin \left( \frac{\pi}{2} - \beta \right) = c \cos \beta$$

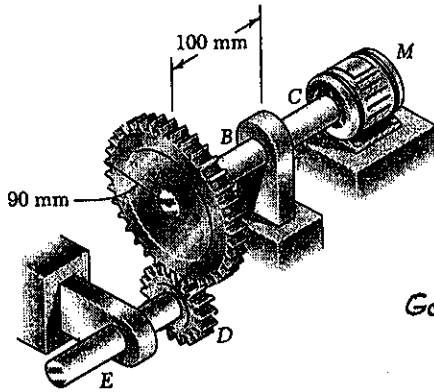
$$\sigma_x = \frac{2Mc \cos \beta}{J}$$

Using Mohr's circle

$$\begin{aligned} \tau_K &= R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{c}{J} \sqrt{(M \cos \beta)^2 + \left(\frac{2}{3}Vc + T\right)^2} \end{aligned}$$

**PROBLEM 8.23**

8.23 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that  $\tau_{all} = 60$  MPa, determine the smallest permissible diameter of shaft *ABC*.

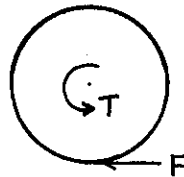


**SOLUTION**

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B

$$M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

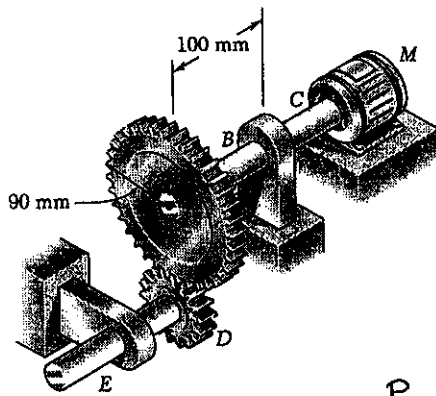
$$C^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{(2) \sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$$C = 18.479 \times 10^{-3} \text{ m}$$

$$d = 2C = 37.0 \times 10^{-3} \text{ m} = 37.0 \text{ mm}$$

PROBLEM 8.24

8.24 Assuming that shaft ABC of Prob. 8.23 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.



SOLUTION

From Prob. 8.23

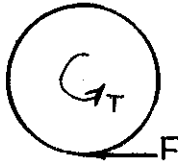
Power transmitted  $P = 10 \text{ kW}$

Motor speed =  $240 \text{ rpm} = 4 \text{ Hz}$

$\tau_{all} = 60 \text{ MPa}$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A



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$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B

$$M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C_o}{J} \sqrt{M^2 + T^2}$$

$$C_o = \frac{1}{2} d_o = 25 \times 10^{-3} \text{ m}$$

$$\frac{J}{C_o} = \frac{\pi}{2} \frac{(C_o^4 - C_i^4)}{C_o} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$C_i^4 = C_o^4 - \frac{2C_o \sqrt{M^2 + T^2}}{\pi \tau_{all}} = (25 \times 10^{-3})^4 - \frac{(2)(25 \times 10^{-3}) \sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)}$$

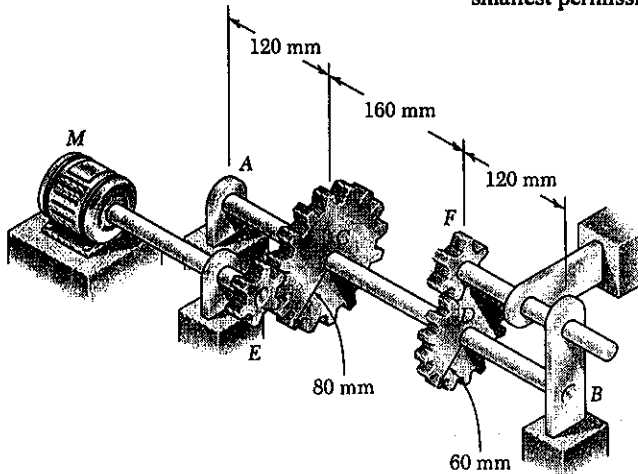
$$= 390.625 \times 10^{-9} - 157.772 \times 10^{-9} = 232.85 \times 10^{-9}$$

$$C_i = 21.967 \times 10^{-3} \text{ m}$$

$$d_i = 2C_i = 43.93 \times 10^{-3} \text{ m} = 43.9 \text{ mm} \quad \blacktriangleleft$$

**PROBLEM 8.25**

8.25 The solid shaft  $AB$  rotates at 600 rpm and transmits 80 kW from the motor  $M$  to a machine tool connected to gear  $F$ . Knowing that  $\tau_{all} = 60$  MPa, determine the smallest permissible diameter of shaft  $AB$ .



**SOLUTION**

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(10)} = 1273.2 \text{ N}\cdot\text{m}$$

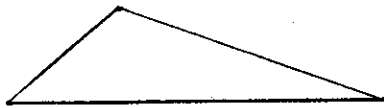
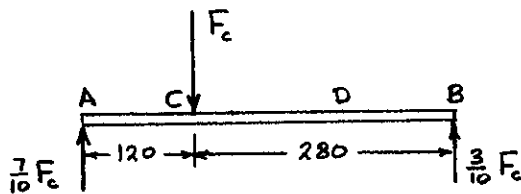
Gear C  $F_c = \frac{T}{r_c}$

$$F_c = \frac{1273.2}{80 \times 10^{-3}} = 15.913 \times 10^3 \text{ N}$$

Gear D  $F_D = \frac{T}{r_D}$

$$F_D = \frac{1273.2}{60 \times 10^{-3}} = 21.221 \times 10^3 \text{ N}$$

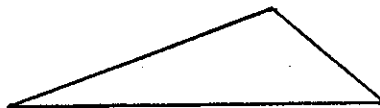
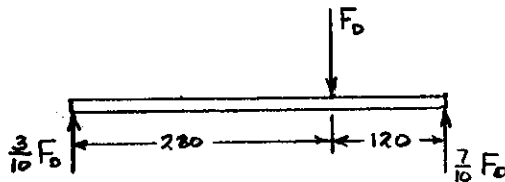
Forces in vertical plane



$$M_{Cz} = (120 \times 10^{-3}) \left(\frac{7}{10} F_c\right) = 1336.7 \text{ N}\cdot\text{m}$$

$$M_{Dz} = \frac{120}{280} M_{Cz} = 572.9 \text{ N}\cdot\text{m}$$

Forces in horizontal plane



$$M_{Dy} = (120 \times 10^{-3}) \left(\frac{7}{10} F_D\right) = 1782.6 \text{ N}\cdot\text{m}$$

$$M_{Cy} = \frac{120}{280} M_{Dy} = 764.0 \text{ N}\cdot\text{m}$$

$$\text{At C: } \sqrt{M_y^2 + M_z^2 + T^2} = 1997.9 \text{ N}\cdot\text{m}$$

$$\text{At D: } \sqrt{M_y^2 + M_z^2 + T^2} = 2264.3 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C}{J} \left( \sqrt{M_y^2 + M_z^2 + T^2} \right)_{max}$$

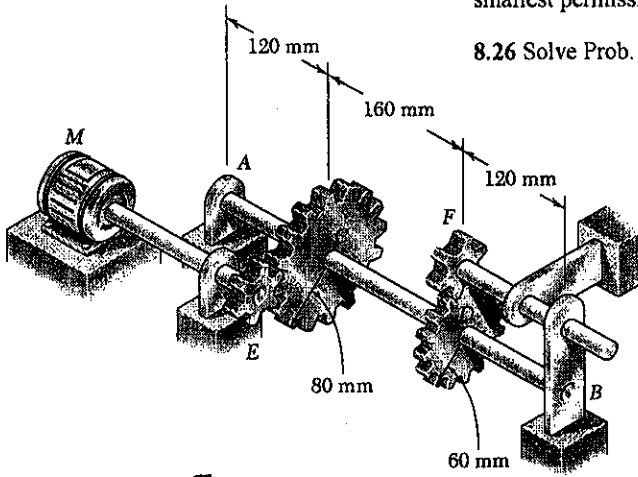
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\left( \sqrt{M_y^2 + M_z^2 + T^2} \right)_{max}}{\tau_{all}} = \frac{2264.3}{60 \times 10^6} = 37.738 \times 10^{-6} \text{ m}^3$$

$$C = 28.85 \times 10^{-3} \text{ m} \quad d = 2C = 57.7 \times 10^{-3} \text{ m} = 57.7 \text{ mm}$$

**PROBLEM 8.26**

8.25 The solid shaft  $AB$  rotates at 600 rpm and transmits 80 kW from the motor  $M$  to a machine tool connected to gear  $F$ . Knowing that  $\tau_{all} = 60$  MPa, determine the smallest permissible diameter of shaft  $AB$ .

8.26 Solve Prob. 8.25, assuming that shaft  $AB$  rotates at 720 rpm



**SOLUTION**

$$f = \frac{720 \text{ rpm}}{60 \text{ sec/min}} = 12 \text{ Hz}$$

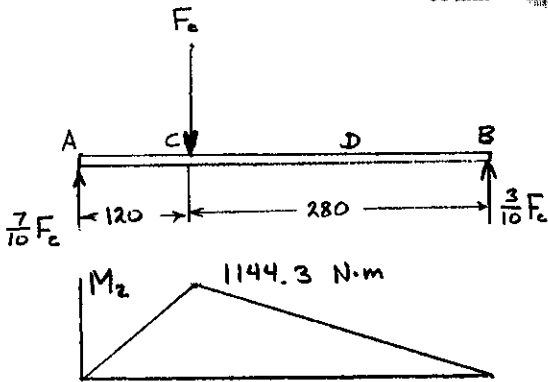
$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(12)} = 1061.0 \text{ N}\cdot\text{m}$$

Gear C  $F_c = \frac{T}{r_c}$

$$F_c = \frac{1061.0}{80 \times 10^{-3}} = 13.262 \times 10^3 \text{ N}$$

Gear D  $F_D = \frac{T}{r_D}$

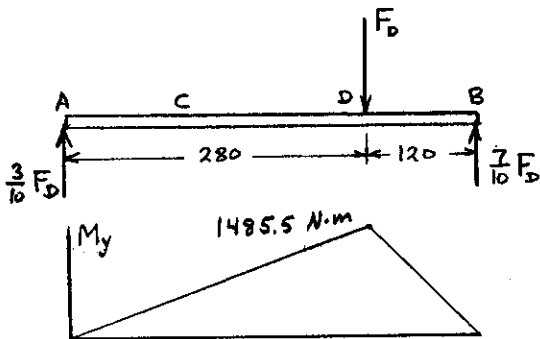
$$F_D = \frac{1061.0}{60 \times 10^{-3}} = 17.684 \times 10^3 \text{ N}$$



Forces in vertical plane

$$M_{Cz} = (120 \times 10^{-3}) \left(\frac{7}{10} F_c\right) = 1114.0 \text{ N}\cdot\text{m}$$

$$M_{Dz} = \frac{120}{280} M_{Cz} = 477.4 \text{ N}\cdot\text{m}$$



Forces in horizontal plane

$$M_{Dy} = (120 \times 10^{-3}) \left(\frac{7}{10} F_D\right) = 1485.5 \text{ N}\cdot\text{m}$$

$$M_{Cy} = \frac{120}{280} M_{Dy} = 636.6 \text{ N}\cdot\text{m}$$

At C:  $\sqrt{M_y^2 + M_z^2 + T^2} = 1664.9 \text{ N}\cdot\text{m}$

At D:  $\sqrt{M_y^2 + M_z^2 + T^2} = 1886.9 \text{ N}\cdot\text{m}$

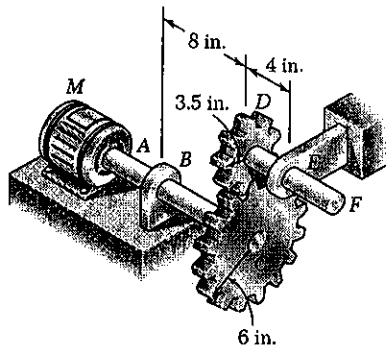
$$\tau_{all} = \frac{C}{J} (\sqrt{M_y^2 + M_z^2 + T^2})_{max}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M_y^2 + M_z^2 + T^2})_{max}}{\tau_{all}} = \frac{1886.9}{60 \times 10^6} = 31.448 \times 10^{-6} \text{ m}^3$$

$$C = 27.15 \times 10^{-3} \text{ m} \quad d = 2C = 54.3 \times 10^{-3} \text{ m} = 54.3 \text{ mm}$$

PROBLEM 8.27

8.27 The solid shafts  $ABC$  and  $DEF$  and the gears shown are used to transmit 20 hp from the motor  $M$  to a machine tool connected to shaft  $DEF$ . Knowing that the motor rotates at 240 rpm and that  $\tau_{all} = 7.5$  ksi, determine the smallest permissible diameter of (a) shaft  $ABC$ , (b) shaft  $DEF$ .



SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(a) Shaft  $ABC$       $T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ in}\cdot\text{lb}$

Gear  $C$       $F_{CD} = \frac{T}{r_c} = \frac{5252}{6} = 875.4 \text{ lb}$

Bending moment at  $B$       $M_B = (8)(875.4) = 7003 \text{ in}\cdot\text{lb}$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi C^3}{2} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$$C = 0.9057 \text{ in} \quad d = 2c = 1.811 \text{ in}$$

(b) Shaft  $DEF$       $T = r_d F_{CD} = (3.5)(875.4) = 3064 \text{ in}\cdot\text{lb}$

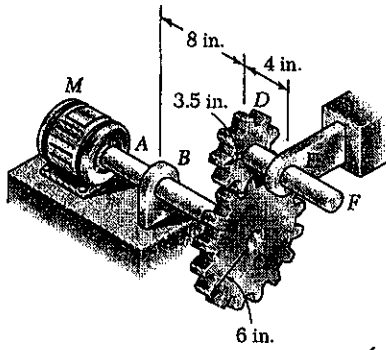
Bending moment at  $E$       $M_E = (4)(875.4) = 3502 \text{ in}\cdot\text{lb}$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi C^3}{2} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$$C = 0.7337 \text{ in} \quad d = 2c = 1.467 \text{ in}$$

PROBLEM 8.28



8.27 The solid shafts  $ABC$  and  $DEF$  and the gears shown are used to transmit 20 hp from the motor  $M$  to a machine tool connected to shaft  $DEF$ . Knowing that the motor rotates at 240 rpm and that  $\tau_{all} = 7.5$  ksi, determine the smallest permissible diameter of (a) shaft  $ABC$ , (b) shaft  $DEF$ .

8.28 Solve Prob. 8.27, assuming that the motor rotates at 360 rpm.

SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb}/\text{s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(a) Shaft  $ABC$   $T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ in}\cdot\text{lb}$

Gear  $C$   $F_{C0} = \frac{T}{r_c} = \frac{3501}{6} = 583.6 \text{ lb}$

Bending moment at  $B$   $M_B = (8)(583.6) = 4669 \text{ in}\cdot\text{lb}$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$$C = 0.791 \text{ in} \quad d = 2c = 1.582 \text{ in}$$

(b) Shaft  $DEF$   $T = r_b F_{C0} = (3.5)(583.6) = 2043 \text{ in}\cdot\text{lb}$

Bending moment at  $E$   $M_E = (4)(583.6) = 2334 \text{ in}\cdot\text{lb}$

$$\tau_{all} = \frac{C}{J} \sqrt{M^2 + T^2}$$

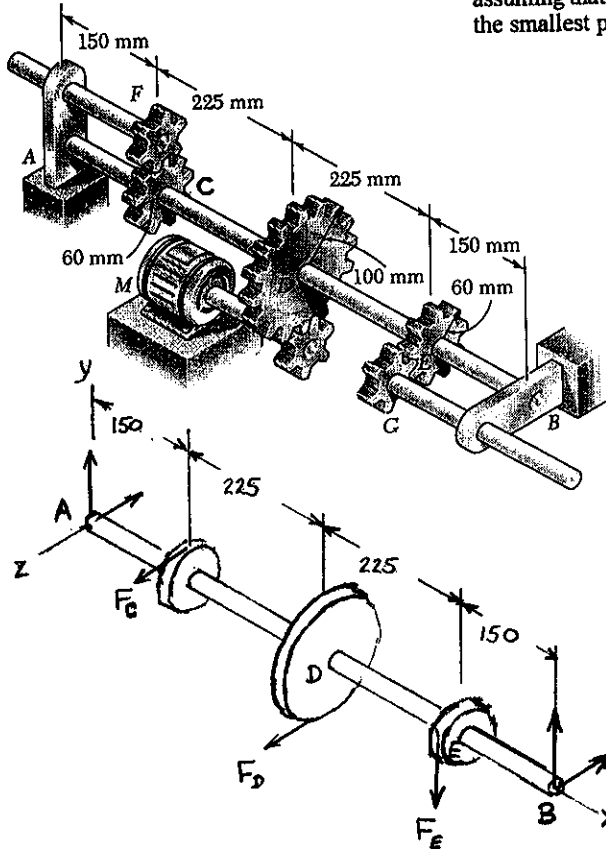
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{2334^2 + 2043^2}}{7500} = 0.41362 \text{ in}^3$$

$$C = 0.6410 \text{ in} \quad d = 2c = 1.282 \text{ in}$$



**PROBLEM 8.29**

8.29 The solid shaft  $AB$  rotates at 450 rpm and transmits 20 kW from the motor  $M$  to machine tools connected to gears  $F$  and  $G$ . Knowing that  $\tau_{all} = 55$  MPa and assuming that 8 kW is taken off at gear  $F$  and 12 kW is taken off at gear  $G$ , determine the smallest permissible diameter of shaft  $AB$ .



**SOLUTION**

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torques on gears C and E

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N}\cdot\text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N}\cdot\text{m}$$

Forces on gears

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

Torques in various parts

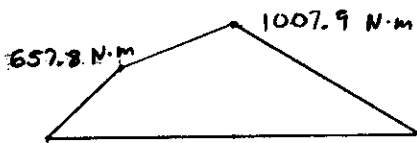
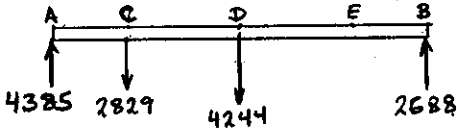
AC:  $T = 0$

CD:  $T = 169.76 \text{ N}\cdot\text{m}$

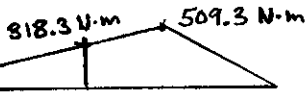
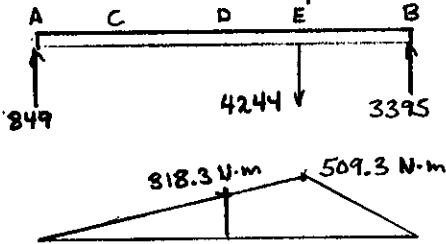
DE:  $T = 254.65 \text{ N}\cdot\text{m}$

EB:  $T = 0$

Forces in horizontal plane:



Forces in vertical plane



Critical point lies just the right of D

$$T = 254.65 \text{ N}\cdot\text{m}$$

$$M_y = 1007.9 \text{ N}\cdot\text{m}$$

$$M_z = 318.3 \text{ N}\cdot\text{m}$$

$$(\sqrt{M_y^2 + M_z^2 + T^2})_{max} = 1087.2 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C}{J} (\sqrt{M_y^2 + M_z^2 + T^2})_{max}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M_y^2 + M_z^2 + T^2})_{max}}{\tau_{all}} = \frac{1087.2}{55 \times 10^6} = 19.767 \times 10^{-3} \text{ m}$$

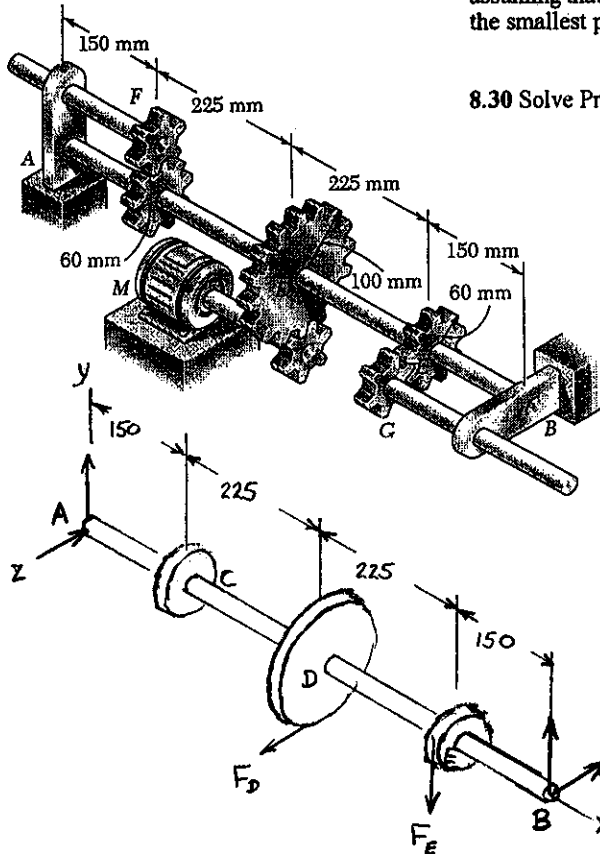
$$C = 23.26 \times 10^{-3} \text{ m}$$

$$d = 2C = 46.5 \times 10^{-3} \text{ m} = 46.5 \text{ mm}$$

**PROBLEM 8.30**

8.29 The solid shaft  $AB$  rotates at 450 rpm and transmits 20 kW from the motor  $M$  to machine tools connected to gears  $F$  and  $G$ . Knowing that  $\tau_{all} = 55 \text{ MPa}$  and assuming that 8 kW is taken off at gear  $F$  and 12 kW is taken off at gear  $G$ , determine the smallest permissible diameter of shaft  $AB$ .

8.30 Solve Prob. 8.29, assuming that the entire 20 kW is taken off at gear  $G$ .



**SOLUTION**

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torque on gear E

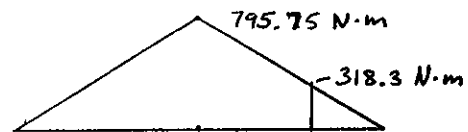
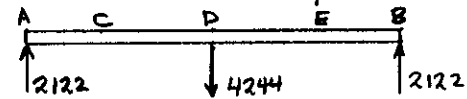
$$T_E = T_D = 424.41 \text{ N}\cdot\text{m}$$

Forces on gears D and E

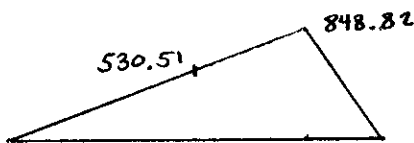
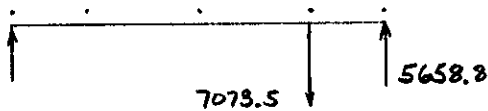
$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{424.41}{60 \times 10^{-3}} = 7073.5 \text{ N}$$

Forces in horizontal plane



Forces in vertical plane



Bending moments

$$M_D = \sqrt{530.51^2 + 795.75^2} = 956.4 \text{ N}\cdot\text{m}$$

$$M_E = \sqrt{848.82^2 + 318.3^2} = 906.5 \text{ N}\cdot\text{m}$$

$$(\sqrt{M^2 + T^2})_{max} = \sqrt{956.4^2 + 424.41^2} = 1046.3 \text{ N}\cdot\text{m}$$

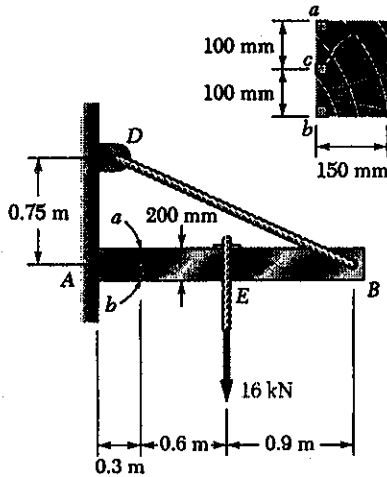
$$\tau_{all} = \frac{c}{J} (\sqrt{M^2 + T^2})_{max}$$

$$\frac{J}{c} = \frac{\pi c^3}{2} = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{1046.3}{55 \times 10^6} = 19.024 \times 10^{-6} \text{ m}^3$$

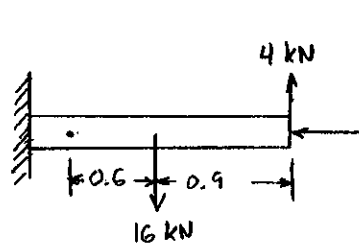
$$c = 22.96 \times 10^{-3} \text{ m} \quad d = 2c = 45.9 \times 10^{-3} \text{ m} = 45.9 \text{ mm}$$

**PROBLEM 8.31**

8.31 The cantilever beam  $AB$  has a rectangular cross section of  $150 \times 200$  mm. Knowing that the tension in cable  $BD$  is  $10.4$  kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.



**SOLUTION**



$$DB = \sqrt{.75^2 + 1.8^2} = 1.95 \text{ m}$$

Vertical component of  $T_{DB}$   
 $(\frac{0.75}{1.95})(10.4) = 4 \text{ kN}$

Horizontal component of  $T_{DB}$   $(\frac{1.8}{1.95})(10.4) = 9.6 \text{ kN}$

At section containing points  $a, b,$  and  $c$

$$P = -9.6 \text{ kN} \qquad 16 - 4 = 12 \text{ kN}$$

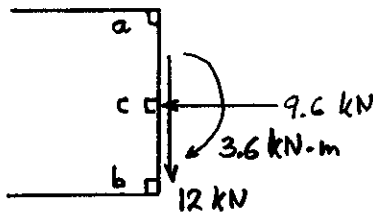
$$M = (1.5)(4) - (0.6)(16) = -3.6 \text{ kN}\cdot\text{m}$$

Section properties

$$A = (0.150)(0.200) = 0.030 \text{ m}^2$$

$$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^4$$

$$c = 0.100 \text{ m}$$



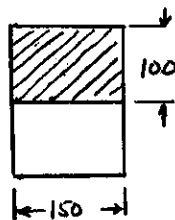
At point  $a$   $\sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} + \frac{(3.6 \times 10^3)(0.100)}{100 \times 10^{-6}} = 3.28 \text{ MPa}$   $\blacktriangleleft$

$\tau_{xy} = 0$   $\blacktriangleleft$

At point  $b$   $\sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} - \frac{(3.6 \times 10^3)(0.100)}{100 \times 10^{-6}} = -3.92 \text{ MPa}$   $\blacktriangleleft$

$\tau_{xy} = 0$   $\blacktriangleleft$

At point  $c$   $\sigma_x = -\frac{P}{A} = -\frac{9.6 \times 10^3}{0.030} = -0.320 \text{ MPa}$   $\blacktriangleleft$

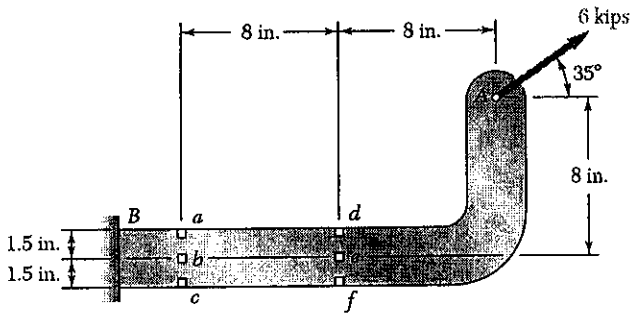


$$Q = (150)(100)(50) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$$

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12 \times 10^3)(750 \times 10^{-6})}{(100 \times 10^{-6})(0.150)} = -0.600 \text{ MPa}$   $\blacktriangleleft$

PROBLEM 8.32

8.32 A 6-kip force is applied to the machine element AB as shown. Determine the normal and shearing stresses at (a) point a, (b) point b, (c) point c.



SOLUTION

thickness = 0.8 in.

At the section containing points a, b, and c

$$P = 6 \cos 35^\circ = 4.9149 \text{ kips}$$

$$V = 6 \sin 35^\circ = 3.4415 \text{ kips}$$

$$M = (6 \sin 35^\circ)(16) - (6 \cos 35^\circ)(8) = 15.744 \text{ kip}\cdot\text{in.}$$

$$A = (0.8)(3.0) = 2.4 \text{ in}^2$$

$$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$$

(a) At point a  $\sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(15.744)(1.5)}{1.80} = -11.07 \text{ ksi}$   $\blacktriangleleft$

$$\tau_{xy} = 0$$

(b) At point b  $\sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05 \text{ ksi}$   $\blacktriangleright$

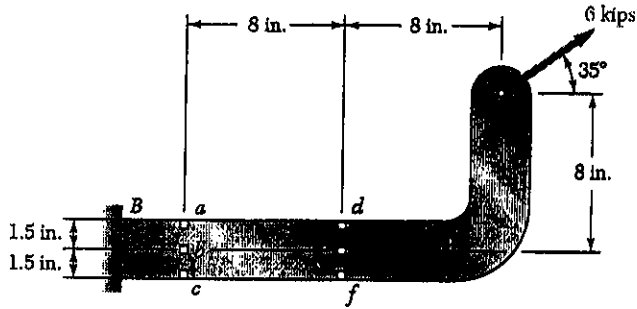
$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \cdot \frac{3.4415}{2.4} = 2.15 \text{ ksi}$$

(c) At point c  $\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(15.744)(1.5)}{1.80} = 15.17 \text{ ksi}$   $\blacktriangleright$

$$\tau_{xy} = 0$$

PROBLEM 8.33

8.33 A 6-kip force is applied to the machine element AB as shown. Determine the normal and shearing stresses at (a) point d, (b) point e, (c) point f



SOLUTION

thickness = 0.8 in

At the section containing points d, e, and f

$$P = 6 \cos 35^\circ = 4.9149 \text{ kips} \quad V = 6 \sin 35^\circ = 3.4415 \text{ kips}$$

$$M = (6 \sin 35^\circ)(8) - (6 \cos 35^\circ)(8) = -11.788 \text{ kip}\cdot\text{in.}$$

$$A = (0.8)(3.0) = 2.4 \text{ in}^2$$

$$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$$

(a) At point d  $\sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(11.788)(1.5)}{1.8} = 11.87 \text{ ksi}$  ▲

$\tau_{xy} = 0$  ▲

(b) At point e  $\sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05 \text{ ksi}$  ▲

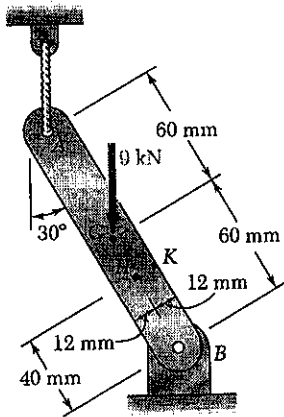
$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \cdot \frac{3.4415}{2.4} = 2.15 \text{ ksi}$  ▲

(c) At point f  $\sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(11.788)(1.5)}{1.8} = -7.78 \text{ ksi}$  ▲

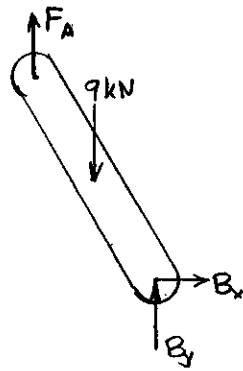
$\tau_{xy} = 0$  ▲

PROBLEM 8.34

8.34 through 8.36 Member  $AB$  has a uniform rectangular cross section of  $10 \times 24$  mm. For the loading shown, determine the normal and shearing stresses at (a) point  $H$ , (b) point  $K$ .



SOLUTION



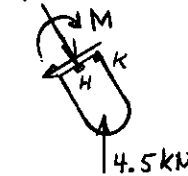
$$\sum F_x = 0$$

$$B_x = 0$$

$$\sum M_A = 0$$

$$B_y (120 \sin 30^\circ) - 9(60 \sin 30^\circ) = 0$$

$$B_y = 4.5 \text{ kN}$$



At the section containing points  $H$  and  $K$

$$P = 4.5 \cos 30^\circ = 3.897 \text{ kN}$$

$$V = 4.5 \sin 30^\circ = 2.25 \text{ kN}$$

$$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point  $H$  
$$\sigma_x = -\frac{P}{A} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} = -16.24 \text{ MPa}$$

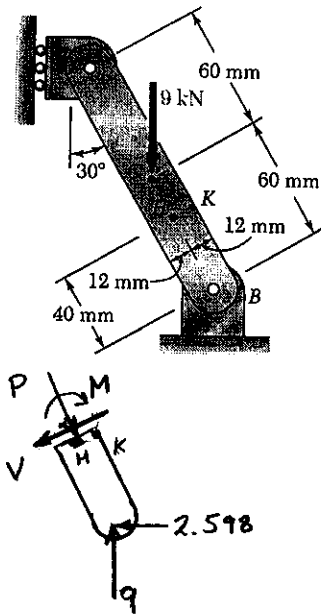
$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$

(b) At point  $K$  
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{3.897 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}} = -110.0 \text{ MPa}$$

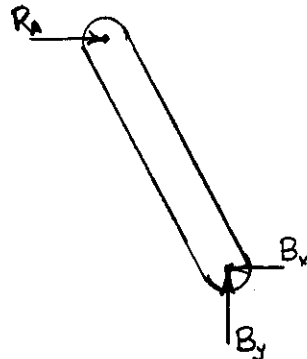
$$\tau_{xy} = 0$$

PROBLEM 8.35

8.34 through 8.36 Member *AB* has a uniform rectangular cross section of  $10 \times 24$  mm. For the loading shown, determine the normal and shearing stresses at (a) point *H*, (b) point *K*.



SOLUTION



$$\sum M_B = 0$$

$$(120 \cos 30^\circ) R_A - (60 \sin 30^\circ)(9) = 0$$

$$R_A = 2.598 \text{ kN}$$

$$\uparrow \sum F_y = 0 \quad B_y - 9 = 0 \quad B_y = 9 \text{ kN} \uparrow$$

$$\rightarrow \sum F_x = 0 \quad 2.598 - B_x = 0 \quad B_x = 2.598 \text{ kN} \leftarrow$$

At the section containing points *H* and *K*

$$P = 9 \cos 30^\circ + 2.598 \sin 30^\circ = 9.093 \text{ kN}$$

$$V = 9 \sin 30^\circ - 2.598 \cos 30^\circ = 2.25 \text{ kN}$$

$$M = (9 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point *H*  $\sigma_x = -\frac{P}{A} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} = -37.9 \text{ MPa}$  ▶

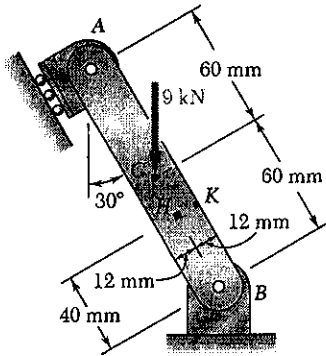
$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$
 ▶

(b) At point *K*  $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{9.093 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}} = -131.6 \text{ MPa}$  ▶

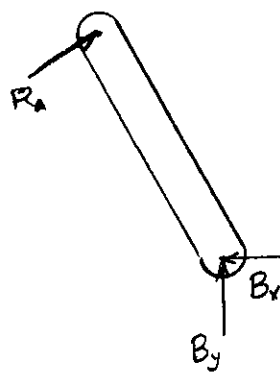
$$\tau_{xy} = 0$$
 ▶

PROBLEM 8.36

8.34 through 8.36 Member AB has a uniform rectangular cross section of  $10 \times 24$  mm. For the loading shown, determine the normal and shearing stresses at (a) point H, (b) point K.



SOLUTION



$$\rightarrow \Sigma M_B = 0$$

$$(9)(60 \sin 30^\circ) - 120 R_A = 0$$

$$R_A = 2.25 \text{ kN}$$

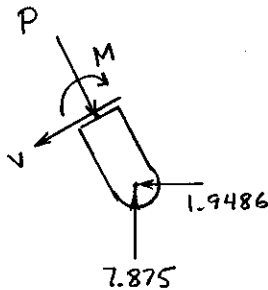
$$\rightarrow \Sigma F_x = 0 \quad 2.25 \cos 30^\circ - B_x = 0$$

$$B_x = 1.9486 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0$$

$$2.25 \sin 30^\circ - 9 + B_y = 0$$

$$B_y = 7.875 \text{ kN} \uparrow$$



At the section containing points H and K

$$P = 7.875 \cos 30^\circ + 1.9486 \sin 30^\circ = 7.794 \text{ kN}$$

$$V = 7.875 \sin 30^\circ - 1.9486 \cos 30^\circ = 2.25 \text{ kN}$$

$$M = (7.875 \times 10^3)(40 \times 10^{-3} \sin 30^\circ) - (1.9486 \times 10^3)(40 \times 10^{-3} \cos 30^\circ) = 90 \text{ N}\cdot\text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point H  $\sigma_x = -\frac{P}{A} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} = -32.5 \text{ MPa}$

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$

(b) At point K  $\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

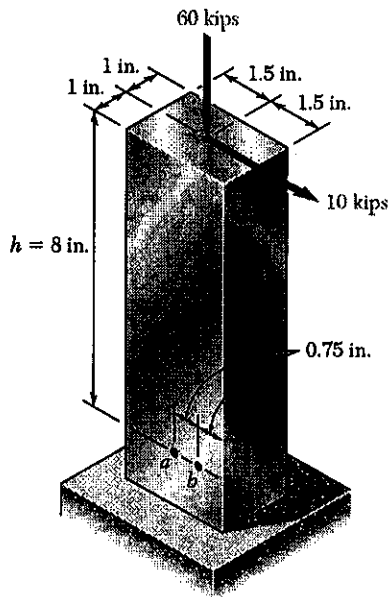
$$= -126.2 \text{ MPa}$$

$$\tau_{xy} = 0$$



**PROBLEM 8.37**

8.37 Two forces are applied to the bar shown. At point *a*, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



**SOLUTION**

At the section containing point *a* and *b*.

$$V = 10 \text{ kips} \quad P = 60 \text{ kips (compression)}$$

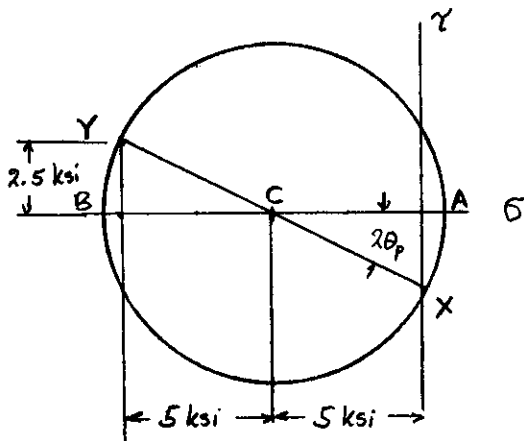
$$M = (8)(10) = 80 \text{ kip}\cdot\text{in}$$

$$A = (2)(3) = 6 \text{ in}^2$$

$$I = \frac{1}{12}(2)(3)^3 = 4.5 \text{ kip}\cdot\text{in}$$

At point *a*  $\sigma_y = -\frac{P}{A} = -\frac{60}{6} = -10 \text{ ksi}$

$$\tau = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{(10)}{6} = 2.5 \text{ ksi}, \quad \sigma_x = 0$$



Use Mohr's circle

$$\sigma_c = -5 \text{ ksi}$$

$$R = \sqrt{5^2 + 2.5^2} = 5.590 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 0.590 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -10.59 \text{ ksi}$$

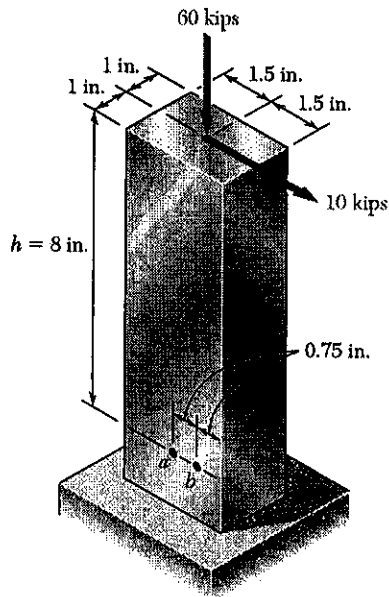
$$\tan 2\theta_p = \frac{2.5}{5} = 0.5$$

$$\theta_p = 13.3^\circ, 103.3^\circ$$

$$\tau_{max} = R = 5.59 \text{ ksi}$$

**PROBLEM 8.38**

**8.38** Two forces are applied to the bar shown. At point *b*, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



**SOLUTION**

At the section containing points *a* and *b*

$$V = 10 \text{ kips}, \quad P = 60 \text{ kips (compression)}$$

$$M = (8)(10) = 80 \text{ kip-in.}$$

$$A = (2)(3) = 6 \text{ in}^2$$

$$I = \frac{1}{12}(2)(3)^3 = 4.5 \text{ in}^4$$

At point *b*  $\sigma_x = 0$

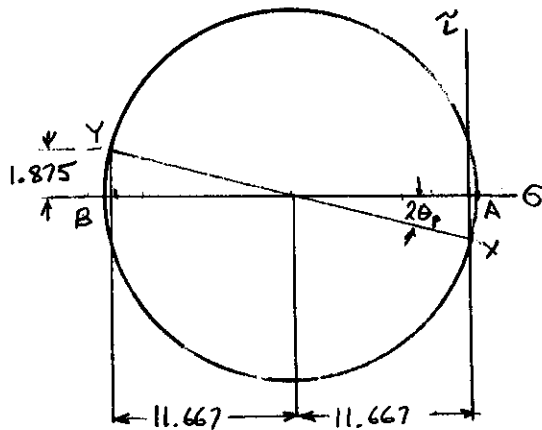
$$\sigma_y = -\frac{P}{A} - \frac{Mx}{I} = -\frac{60}{6} - \frac{(80)(0.75)}{4.5} = -23.33 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(2)(0.75)(1.125)}{(4.5)(2)} = 1.875 \text{ ksi}$$

Use Mohr's circle

$$\sigma_c = -11.667 \text{ ksi}$$

$$R = \sqrt{11.667^2 + 1.875^2} = 11.8164 \text{ ksi}$$



$$\sigma_a = \sigma_c + R = 0.150 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -23.5 \text{ ksi}$$

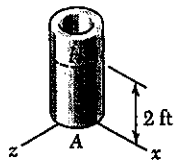
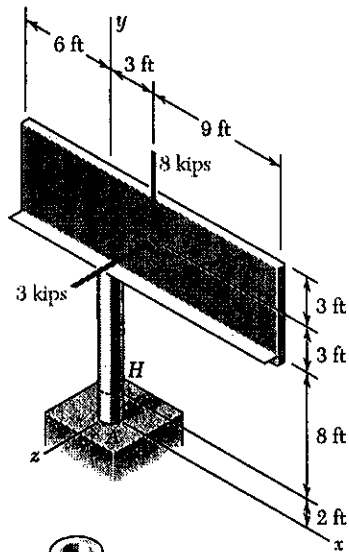
$$\tan 2\theta_p = \frac{1.875}{11.667} = 0.16071$$

$$\theta_p = 4.6^\circ, \quad 94.6^\circ$$

$$\tau_{max} = R = 11.82 \text{ ksi}$$

**PROBLEM 8.39**

8.39 The billboard shown weighs 8,000 lb and is supported by a structural tube that has a 15-in. outer diameter and a 0.5-in. wall thickness. At a time when the resultant of the wind pressure is 3 kips located at the center *C* of the billboard, determine the normal and shearing stresses at point *H*.



**SOLUTION**

At section containing point *H*

$$P = 8 \text{ kips (compression)}$$

$$T = (3)(3) = 9 \text{ kip}\cdot\text{ft} = 108 \text{ kip}\cdot\text{in}$$

$$M_x = -(11)(3) = -33 \text{ kip}\cdot\text{ft} = -396 \text{ kip}\cdot\text{in}$$

$$M_z = -(3)(8) = -24 \text{ kip}\cdot\text{ft} = -288 \text{ kip}\cdot\text{in}$$

$$V = 3 \text{ kip}$$

Section properties.

$$d_o = 15 \text{ in.} \quad c_o = \frac{1}{2}d_o = 7.5 \text{ in.} \quad c_i = c_o - t = 7.0 \text{ in.}$$

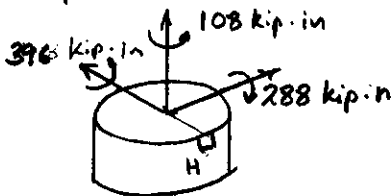
$$A = \pi(c_o^2 - c_i^2) = 22.777 \text{ in}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 599.31 \text{ in}^4$$

$$J = 2I = 1198.62 \text{ in}^4$$

$$Q = \frac{2}{3}(c_o^3 - c_i^3) = 52.583 \text{ in}^3$$

Couples

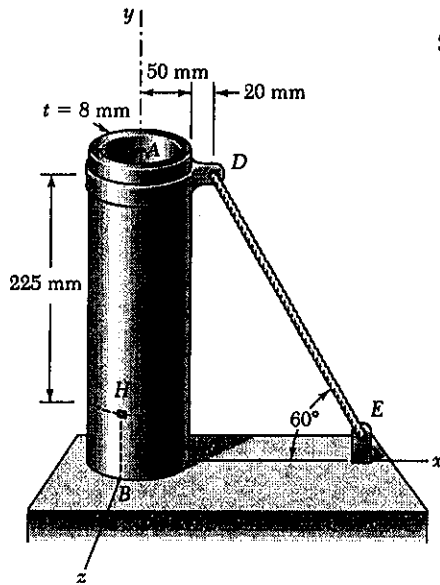


$$\sigma = -\frac{P}{A} - \frac{M_z}{I} = -\frac{8}{22.777} - \frac{(288)(7.5)}{599.31} = -0.351 - 3.604 = -3.96 \text{ ksi} \quad \leftarrow$$

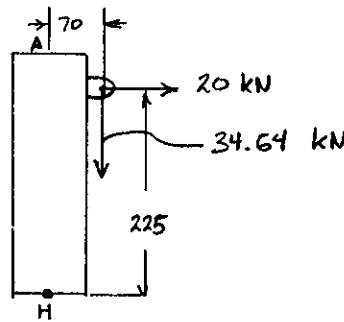
$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(108)(7.5)}{1198.62} + \frac{(3)(52.583)}{(599.31)(1.0)} = 0.675 + 0.268 = 0.938 \text{ ksi} \quad \leftarrow$$

PROBLEM 8.40

8.40 The steel pipe  $AB$  has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point  $H$ .



SOLUTION



Vertical force  
 $40 \cos 30^\circ = 34.64 \text{ kN}$

Horizontal force  
 $40 \sin 30^\circ = 20 \text{ kN}$

Point  $H$  lies on neutral axis of bending

Section properties

$$d_o = 100 \text{ mm} \quad c_o = \frac{1}{2} d_o = 50 \text{ mm} \quad c_i = c_o - t = 42 \text{ mm}$$

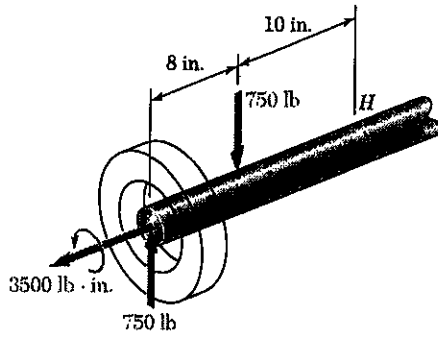
$$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3 \text{ mm}^2 = 2.312 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-3}} = -14.98 \text{ MPa}$$

$$\text{For thin pipe} \quad \tau = 2 \frac{V}{A} = \frac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}} = 17.29 \text{ MPa}$$

PROBLEM 8.41

8.41 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point *H* located on the top of the axle.



SOLUTION

The bending moment causing normal stress at point *H* is

$$M = (8)(750) = 6000 \text{ lb}\cdot\text{in.}$$

$$c = \frac{1}{2}d = 0.71 \text{ in.}$$

$$I = \frac{\pi}{4}c^4 = 0.19958 \text{ in}^4, \quad J = 2I = 0.39916 \text{ in}^4$$

Normal stress at *H* 
$$\sigma_H = -\frac{Mc}{I} = -\frac{(6000)(0.71)}{0.19958} = -21.3 \times 10^3 \text{ psi}$$
  

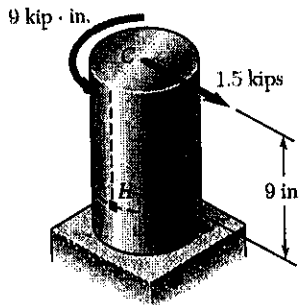
$$= -21.3 \text{ ksi}$$

At the section containing point *H*  $V = 0, T = 3500 \text{ lb}\cdot\text{in}$

$$\tau_H = \frac{Tc}{J} = \frac{(3500)(0.71)}{0.39916} = 6.23 \text{ ksi}$$

PROBLEM 8.42

8.42 A 1.5-kip force and a 9-kip-in. couple are applied at the top of the cast-iron post shown. Determine the normal and shearing stresses at (a) point *H*, (b) point *K*.



SOLUTION

diameter = 2.5 in.

At the section containing points *H* and *K*.

$$P = 0 \quad V = 1.5 \text{ kips}$$

$$T = 9 \text{ kip}\cdot\text{in} \quad M = (1.5)(9) = 13.5 \text{ kip}\cdot\text{in}$$

$$d = 2.5 \text{ in} \quad c = \frac{1}{2}d = 1.25 \text{ in}$$

$$A = \pi c^2 = 4.909 \text{ in}^2 \quad I = \frac{\pi}{4}c^4 = 1.9175 \text{ in}^4 \quad J = 2I = 3.835 \text{ in}^4$$

For a semicircle  $Q = \frac{2}{3}c^3 = 1.3021 \text{ in}^3$

(a) At point *H*  $\sigma_H = 0$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407$$
  

$$= 3.34 \text{ ksi}$$

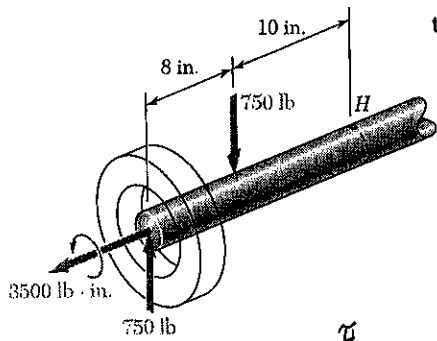
(b) At point *K*  $\sigma_K = -\frac{Mc}{I} = -\frac{(13.5)(1.25)}{1.9175} = -8.80 \text{ ksi}$

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi}$$

**PROBLEM 8.43**

**8.41** The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point *H* located on the top of the axle.

**8.43** For the truck axle and loading of Prob. 8.41, determine the principal stresses and the maximum shearing stress at point *H*.

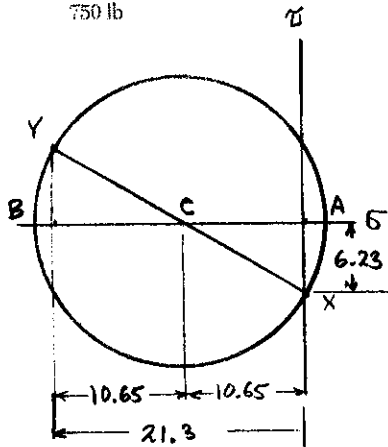
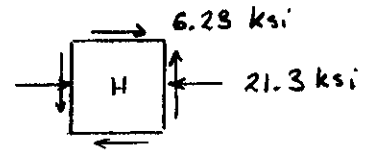


**SOLUTION**

From the solution of Prob. 8.41

$$\sigma_H = -21.3 \text{ ksi}$$

$$\tau_H = 6.23 \text{ ksi}$$



$$\sigma_c = -\frac{21.3}{2} = -10.65 \text{ ksi}$$

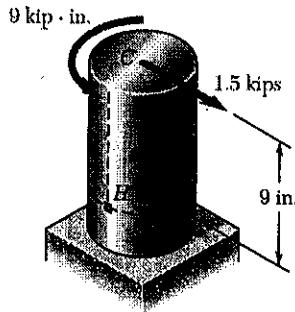
$$R = \sqrt{\left(\frac{21.3}{2}\right)^2 + (6.23)^2} = 12.34 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 1.69 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -23.0 \text{ ksi}$$

$$\tau_{max} = R = 12.34 \text{ ksi}$$

PROBLEM 8.44



8.42 A 1.5-kip force and a 9-kip-in. couple are allied at the top of the cast-iron post shown. Determine the normal and shearing stresses at (a) point H, (b) point K.

8.44 For the post and loading of Prob. 8.42, determine the principal stresses and the maximum shearing stress at (a) point H, (b) point K.

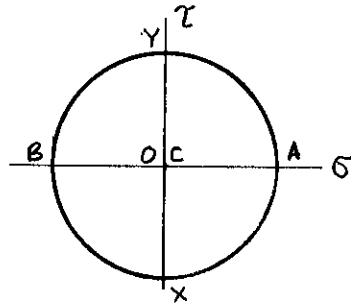
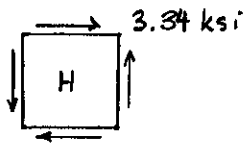
SOLUTION

From the solution of Prob. 8.42

(a)  $\sigma_H = 0$ ,  $\tau_H = 3.34 \text{ ksi}$

(b)  $\sigma_K = -8.80 \text{ ksi}$ ,  $\tau_K = 2.93 \text{ ksi}$

(a) Point H



$\sigma_c = 0$

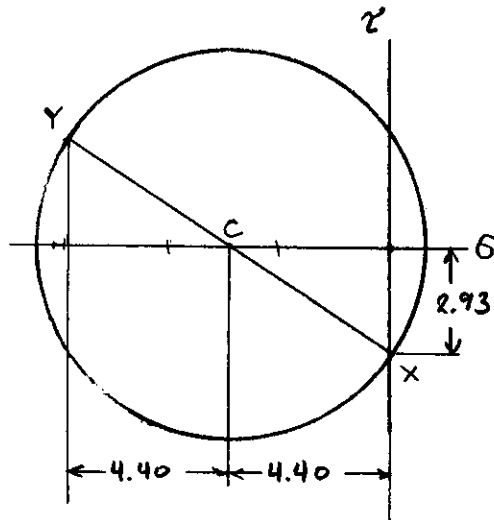
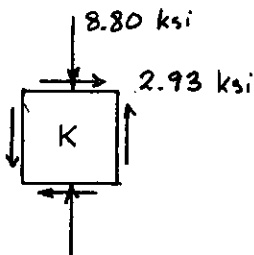
$R = 3.34 \text{ ksi}$

$\sigma_a = \sigma_c + R = 3.34 \text{ ksi}$

$\sigma_b = \sigma_c - R = -3.34 \text{ ksi}$

$\tau_{max} = R = 3.34 \text{ ksi}$

(b) Point K



$\sigma_c = -\frac{8.80}{2} = -4.40 \text{ ksi}$

$R = \sqrt{\left(\frac{8.80}{2}\right)^2 + 2.93^2} = 5.29 \text{ ksi}$

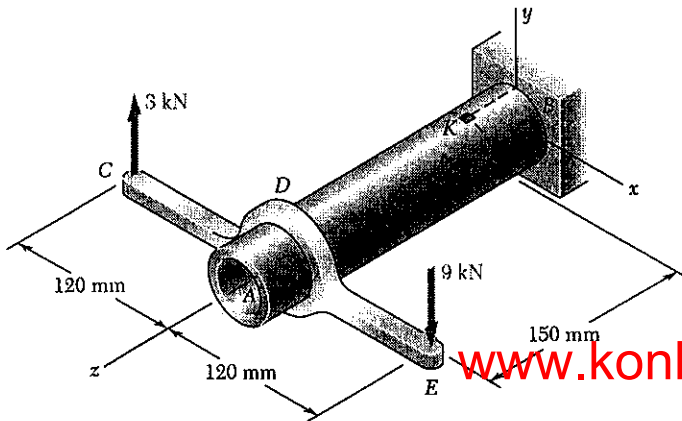
$\sigma_a = \sigma_c + R = 0.89 \text{ ksi}$

$\sigma_b = \sigma_c - R = -9.69 \text{ ksi}$

$\tau_{max} = R = 5.29 \text{ ksi}$

**PROBLEM 8.45**

**8.45** The steel pipe  $AB$  has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that arm  $CDE$  is rigidly attached to the pipe, determine the principal stresses, principal planes, and maximum shearing stress at point  $H$ .

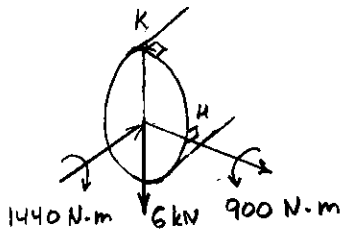


**SOLUTION**

Replace the forces at  $C$  and  $E$  by an equivalent force-couple system at  $D$ .

$$F_D = 9 - 3 = 6 \text{ kN} \downarrow$$

$$T_D = (9 \times 10^3)(120 \times 10^{-3}) + (3 \times 10^3)(120 \times 10^{-3}) = 1440 \text{ N}\cdot\text{m}$$



At the section containing points  $H$  and  $K$

$$P = 0, \quad V = 6 \text{ kN}, \quad T = 1440 \text{ N}\cdot\text{m}$$

$$M = (6 \times 10^3)(150 \times 10^{-3}) = 900 \text{ N}\cdot\text{m}$$

Section properties:  $d_o = 72 \text{ mm}$      $c_o = \frac{1}{2} d_o = 36 \text{ mm}$      $c_i = c_o - t = 31 \text{ mm}$

$$A = \pi(c_o^2 - c_i^2) = 1.0524 \times 10^3 \text{ mm}^2 = 1.0524 \times 10^{-3} \text{ m}^2$$

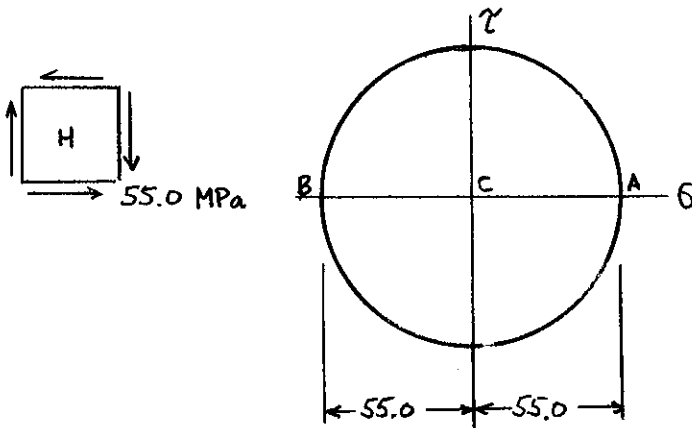
$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84 \times 10^{-3} \text{ mm}^4 = 593.84 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 1.1877 \times 10^{-6} \text{ m}^4$$

$$\text{For half-pipe} \quad Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243 \times 10^3 \text{ mm}^3 = 11.243 \times 10^{-6} \text{ m}^3$$

At point  $H$     Point  $H$  lies on the neutral axis of bending.  $\sigma_H = 0$ .

$$\tau_H = \frac{TC}{J} + \frac{VQ}{It} = \frac{(1440)(36 \times 10^{-3})}{1.1877 \times 10^{-6}} + \frac{(6 \times 10^3)(11.243 \times 10^{-6})}{(593.84 \times 10^{-9})(5 \times 10^{-3})} = 55.0 \text{ MPa}$$



$$\sigma_c = 0$$

$$R = 55.0 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 55.0 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -55.0 \text{ MPa}$$

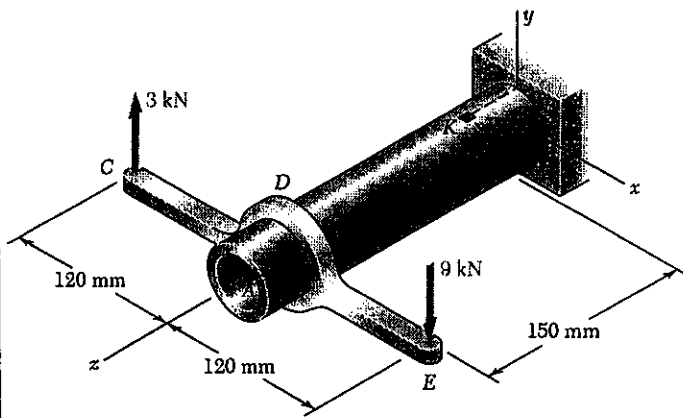
$$\theta_a = -45^\circ, \quad \theta_b = +45^\circ$$

$$\tau_{\max} = R = 55.0 \text{ MPa}$$



**PROBLEM 8.46**

8.46 The steel pipe  $AB$  has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that arm  $CDE$  is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point  $K$ .



**SOLUTION**

Replace the forces at  $C$  and  $E$  by an equivalent force-couple system at  $D$ .

$$F_D = 9 - 3 = 6 \text{ kN} \downarrow$$

$$T_D = (9 \times 10^3)(120 \times 10^{-3}) + (3 \times 10^3)(120 \times 10^{-3}) = 1440 \text{ N}\cdot\text{m}$$

At the section containing points  $H$  and  $K$

$$P = 0, \quad V = 6 \text{ kN} \quad T = 1440 \text{ N}\cdot\text{m}$$

$$M = (6 \times 10^3)(150 \times 10^{-3}) = 900 \text{ N}\cdot\text{m}$$



Section properties:  $d_o = 72 \text{ mm}$   $c_o = \frac{1}{2} d_o = 36 \text{ mm}$   $c_i = c_o - t = 31 \text{ mm}$

$$A = \pi(c_o^2 - c_i^2) = 1.0524 \times 10^3 \text{ mm}^2 = 1.0524 \times 10^{-3} \text{ m}^2$$

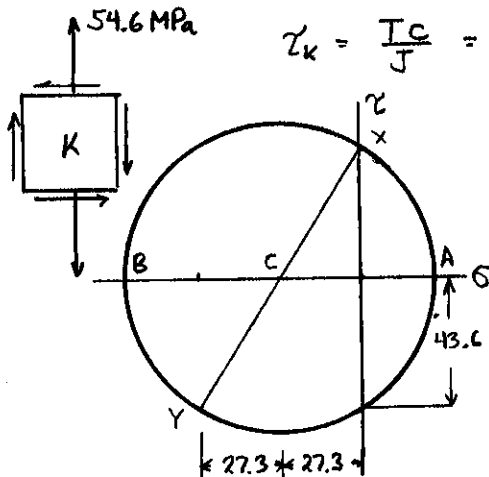
$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84 \times 10^{-3} \text{ mm}^4 = 593.84 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 1.1877 \times 10^{-6} \text{ m}^4$$

$$\text{For half-pipe } Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243 \times 10^3 \text{ mm}^3 = 11.243 \times 10^{-6} \text{ m}^3$$

At point  $K$  
$$\sigma_K = \frac{Mc}{I} = \frac{(900)(36 \times 10^{-3})}{(593.84 \times 10^{-9})} = 54.6 \text{ MPa}$$

$$\tau_K = \frac{Tc}{J} = \frac{(1440)(36 \times 10^{-3})}{1.1877 \times 10^{-6}} = 43.6 \text{ MPa}$$



$$\sigma_c = -\frac{54.6}{2} = -27.3 \text{ MPa}$$

$$R = \sqrt{\left(\frac{54.6}{2}\right)^2 + 43.6^2} = 51.4 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 24.1 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -78.7 \text{ MPa}$$

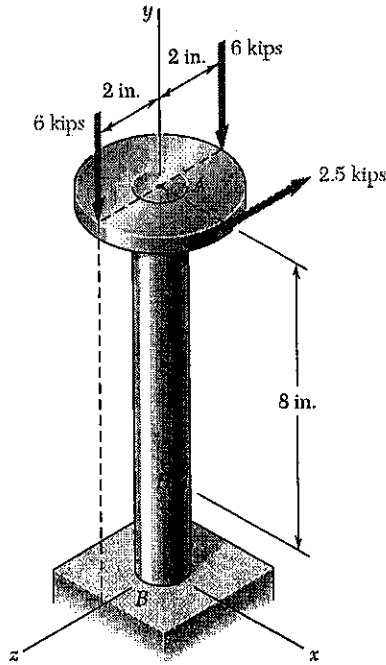
$$\tan 2\theta_p = \frac{43.6}{27.3} = 1.597$$

$$\theta_a = 57.9^\circ, \quad \theta_b = -32.1^\circ$$

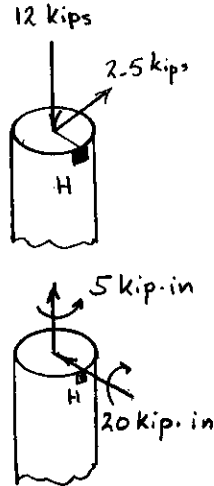
$$\tau_{\max} = R = 51.4 \text{ MPa}$$

**PROBLEM 8.47**

8.47 Three forces are applied to 4-in.-diameter plate that is attached to the solid 1.8-in.-diameter shaft AB. At point H, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



**SOLUTION**



At the section containing point H

$P = 12 \text{ kips (compression)}$

$V = 2.5 \text{ kips}$

$T = (2)(2.5) = 5 \text{ kip}\cdot\text{in}$

$M = (8)(2.5) = 20 \text{ kip}\cdot\text{in}$

$d = 1.8 \text{ in} \quad c = \frac{1}{2}d = 0.9 \text{ in}$

$A = \pi c^2 = 2.545 \text{ in}^2$

$I = \frac{\pi}{4} c^4 = 0.5153 \text{ in}^4$

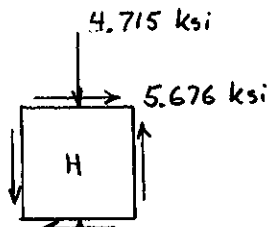
$J = 2I = 1.0306 \text{ in}^4$

For a semicircle

$Q = \frac{2}{3} c^3 = 0.486 \text{ in}^3$

Point H lies on neutral axis of bending  $\sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715 \text{ ksi}$

$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)} = 5.676 \text{ ksi}$



$\sigma_c = \frac{1}{2}(-4.715) = -2.3575 \text{ ksi}$

$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2} = 6.1461$

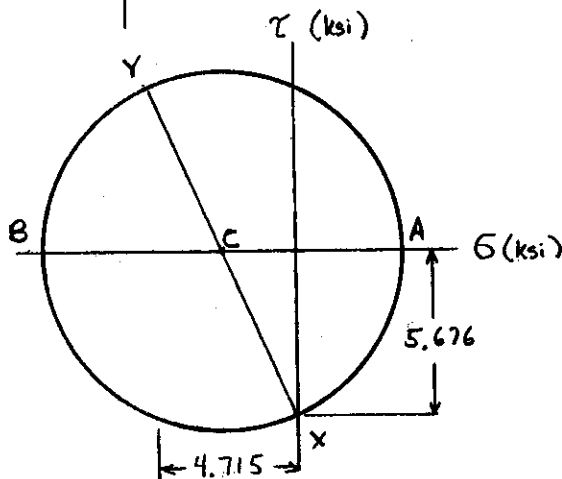
(a)  $\sigma_a = \sigma_c + R = 3.79 \text{ ksi}$

$\sigma_b = \sigma_c - R = -8.50 \text{ ksi}$

$\tan 2\theta_p = \frac{2(5.676)}{4.715} = 2.408$

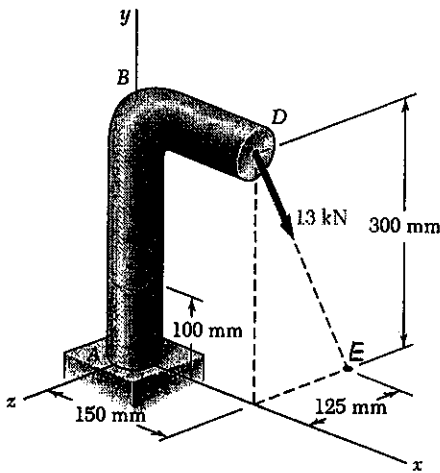
$\theta_a = 33.7^\circ \quad \theta_b = 123.7^\circ$

(b)  $\tau_{max} = R = 6.15 \text{ ksi}$



**PROBLEM 8.48**

**8.48** A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



**SOLUTION**

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

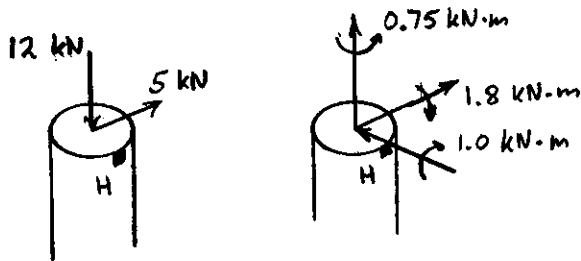
At point D  $F_x = 0$

$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system about the centroid of the section containing point *H*

$$\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00 \hat{i} + 0.75 \hat{j} - 1.8 \hat{k} \text{ kN}\cdot\text{m}$$



Section properties

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$$

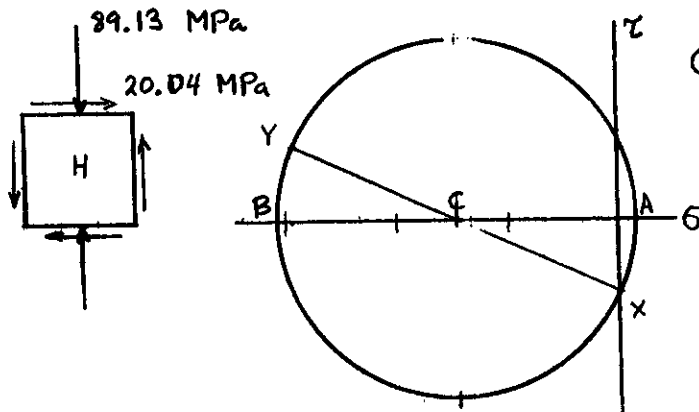
$$I = \frac{\pi}{4} c^4 = 636.17 \times 10^3 \text{ mm}^4$$

$$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$$

For a semicircle  $Q = \frac{2}{3} c^3 = 18.00 \times 10^3 \text{ mm}^3$

At point *H*  $\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12 \times 10^3}{2.8274 \times 10^3} - \frac{(1.8 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^3} = -89.13 \text{ MPa}$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75 \times 10^3)(30 \times 10^{-3})}{1.2723 \times 10^6} + \frac{(5 \times 10^3)(18.00 \times 10^3)}{(636.17 \times 10^3)(60 \times 10^{-3})} = 20.04 \text{ MPa}$$



(a)  $\sigma_c = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$$

$$\sigma_a = \sigma_c + R = 4.3 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_c - R = -93.4 \text{ MPa} \quad \blacktriangleleft$$

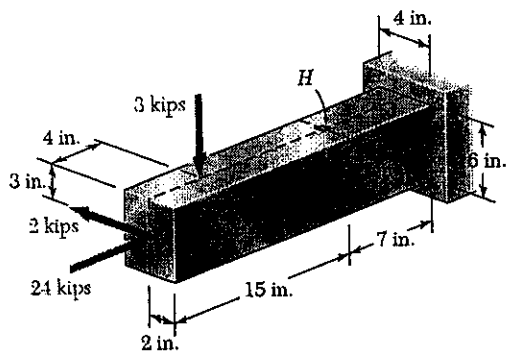
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = 0.4497$$

$$\theta_a = 12.1^\circ, \theta_b = 102.1^\circ \quad \blacktriangleleft$$

(b)  $\tau_{max} = R = 48.9 \text{ MPa} \quad \blacktriangleleft$

**PROBLEM 8.49**

**8.49** Three forces are applied to the cantilever beam shown. Determine the normal and shearing stresses at point *H*.



**SOLUTION**

At the section containing points *H* and *K*, the axial and shearing forces are:

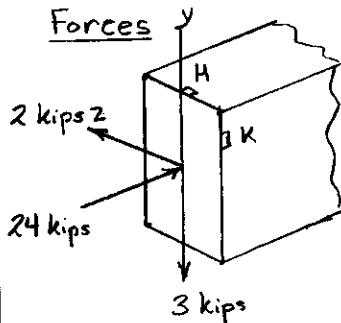
$$P = 24 \text{ kips}, \quad V = \begin{matrix} 3 \text{ kips vertical} \\ 2 \text{ kips horizontal} \end{matrix}$$

The bending moment components are:

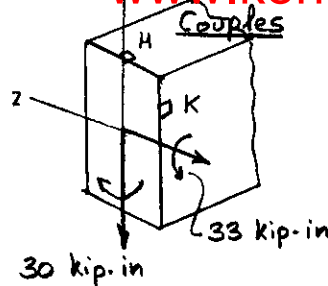
about horizontal axis =  $M = (15 - 7)(3) = 33 \text{ kip}\cdot\text{in}$

about vertical axis =  $M = (15)(2) = 30 \text{ kip}\cdot\text{in}$

Forces



[www.konkur.in](http://www.konkur.in)



Section properties

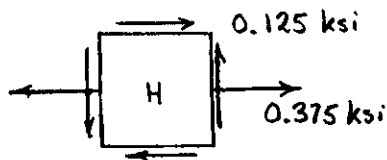
$$A = (4)(6) = 24 \text{ in}^2$$

$$I_z = \frac{1}{12}(4)(6)^3 = 72 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)(4)^3 = 32 \text{ in}^4$$

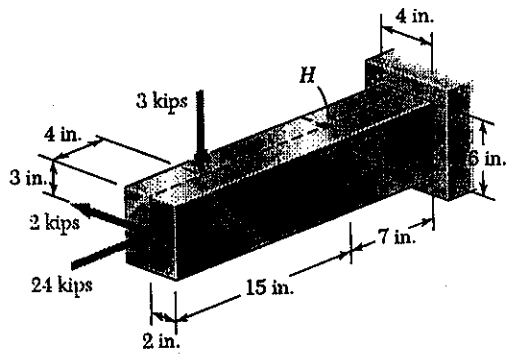
At point *H*  $\sigma_H = -\frac{P}{A} + \frac{Mc}{I} = -\frac{24}{24} + \frac{(33)(3)}{72} = 0.375 \text{ ksi}$  ▶

$$\tau_H = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{2}{24} = 0.125 \text{ ksi}$$
 ▶



**PROBLEM 8.50**

**8.50** Three forces are applied to the cantilever beam shown. Determine the normal and shearing stresses at point *K*.



**SOLUTION**

At the section containing points *H* and *K* the axial and shearing forces are

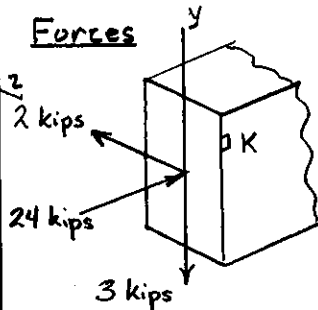
$$P = 24 \text{ kips}, \quad V = \begin{matrix} 3 \text{ kips vertical} \\ 2 \text{ kips horizontal} \end{matrix}$$

The bending moment components are

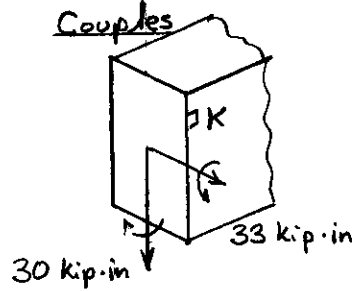
about horizontal axis:  $M = (15-7)(3) = 33 \text{ kip}\cdot\text{in}$

about vertical axis:  $M = (15)(2) = 30 \text{ kip}\cdot\text{in}$

**Forces**



**Couples**



**Section properties**

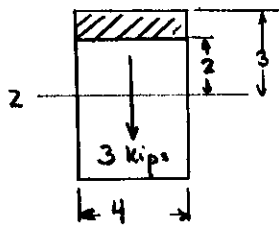
$$A = (4)(6) = 24 \text{ in}^2$$

$$I_z = \frac{1}{12}(4)(6)^3 = 72 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)(4)^3 = 32 \text{ in}^4$$

At point *K*

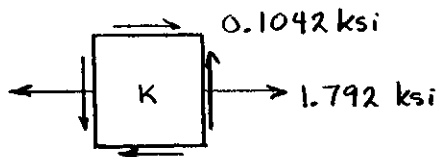
$$\sigma_K = -\frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{24}{24} - \frac{(-33)(2)}{72} + \frac{(-30)(-2)}{32} = 1.792 \text{ ksi} \quad \blacktriangleleft$$



$$A^* = (1)(4) = 4 \text{ in}^2 \quad \bar{y} = 2.5 \text{ in.}$$

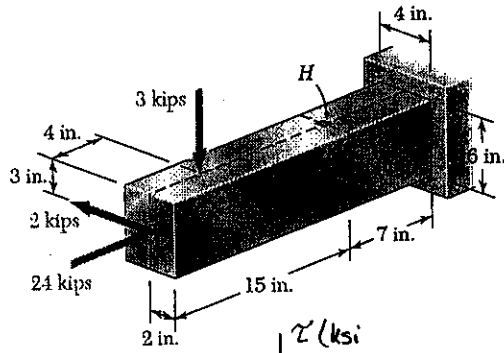
$$Q = A^* \bar{y} = (4)(2.5) = 10 \text{ in}^3$$

$$\tau_K = \frac{VQ}{It} = \frac{(3)(10)}{(72)(4)} = 0.1042 \text{ ksi} \quad \blacktriangleleft$$



**PROBLEM 8.51**

8.51 For the beam and loading of Prob. 8.49, determine the principal stresses and the maximum shearing stress at point H.

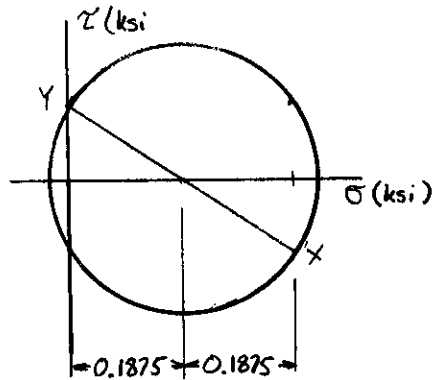
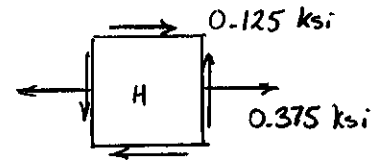


**SOLUTION**

From the solution of Prob. 8.49

$$\sigma_H = 0.375 \text{ ksi}$$

$$\tau_H = 0.125 \text{ ksi}$$



$$\sigma_c = \frac{0.375}{2} = 0.1875 \text{ ksi}$$

$$R = \sqrt{\left(\frac{0.375}{2}\right)^2 + (0.125)^2} = 0.2253 \text{ ksi}$$

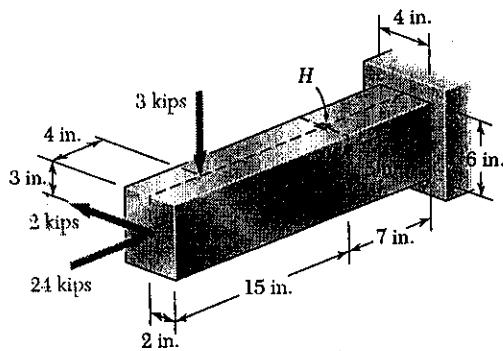
$$\sigma_a = \sigma_c + R = 0.413 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_c - R = -0.0378 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{max} = R = 0.225 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 8.52**

8.52 For the beam and loading of Prob. 8.50, determine the principal stresses and the maximum shearing stress at point K.

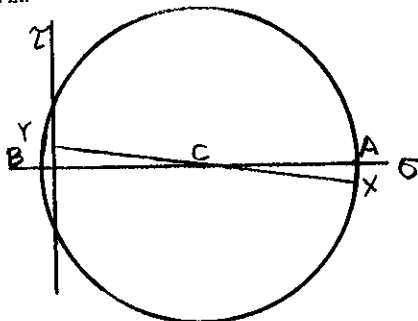
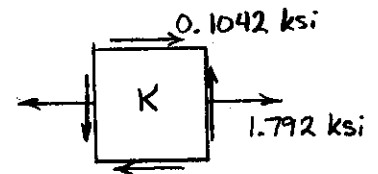


**SOLUTION**

From the solution of Prob. 8.50

$$\sigma_H = 1.792 \text{ ksi}$$

$$\tau_H = 0.1042 \text{ ksi}$$



$$\sigma_c = \frac{1.792}{2} = 0.896 \text{ ksi}$$

$$R = \sqrt{\left(\frac{1.792}{2}\right)^2 + (0.1042)^2} = 0.902 \text{ ksi}$$

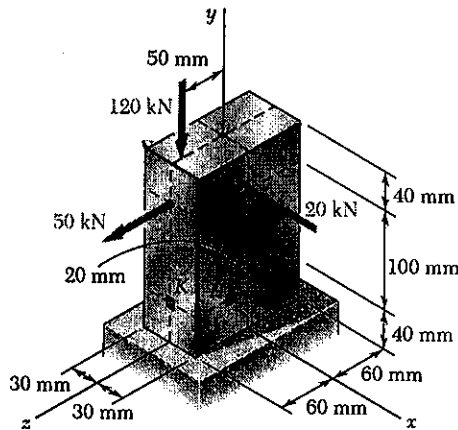
$$\sigma_a = \sigma_c + R = 1.798 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_c - R = -0.006 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_{max} = R = 0.902 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 8.53

8.53 Three forces are applied to a steel post as shown. Determine the normal and shearing stresses at point H.



SOLUTION

$$A = (120)(60) = 7.2 \times 10^3 \text{ mm}^2 = 7.2 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12}(60)(120)^3 = 8.64 \times 10^6 \text{ mm}^4 = 8.64 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(120)(60)^3 = 2.16 \times 10^6 \text{ mm}^4 = 2.16 \times 10^{-6} \text{ m}^4$$

At the section containing points H and K.

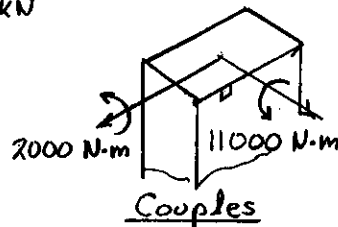
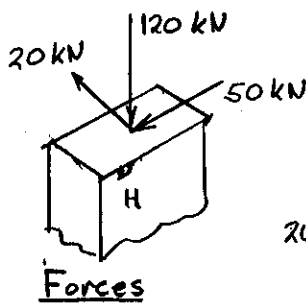
$$P = 120 \text{ kN (compression)}$$

$$V_x = -20 \text{ kN}$$

$$V_z = 50 \text{ kN}$$

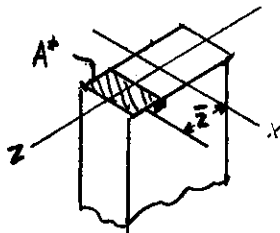
$$M_z = (20 \times 10^3)(100 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

$$M_x = (120 \times 10^3)(50 \times 10^{-3}) + (50 \times 10^3)(100 \times 10^{-3}) = 11000 \text{ N}\cdot\text{m}$$



Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_z z}{I_x} + \frac{M_x x}{I_z} = -\frac{120 \times 10^3}{7.2 \times 10^3} - \frac{(11000)(20 \times 10^{-3})}{8.64 \times 10^{-6}} + \frac{(2000)(30 \times 10^{-3})}{2.16 \times 10^{-6}} = -16.67 \text{ MPa} - 25.46 \text{ MPa} + 27.78 \text{ MPa} = -14.35 \text{ MPa}$$

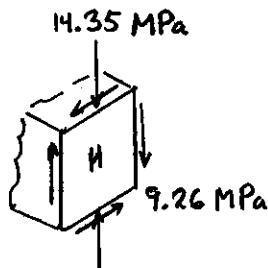


$$A^* = (60)(60 - 20) = 2.4 \times 10^3 \text{ mm}^2$$

$$\bar{z} = (20 + \frac{40}{2}) = 40 \text{ mm}$$

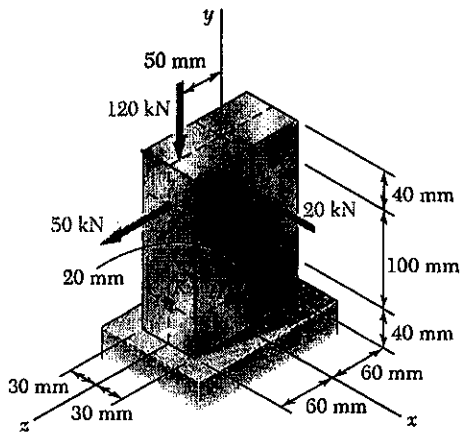
$$\bar{Q}_x = A^* \bar{z} = 96 \times 10^3 \text{ mm}^3 = 96 \times 10^{-6} \text{ m}^3$$

$$\tau_H = \frac{V_z Q_x}{I_x t} = \frac{(50 \times 10^3)(96 \times 10^{-6})}{(8.64 \times 10^{-6})(60 \times 10^{-3})} = 9.26 \text{ MPa}$$



PROBLEM 8.54

8.54 Three forces are applied to a steel post as shown. Determine the normal and shearing stresses at point K.



SOLUTION

$$A = (120)(60) = 7.2 \times 10^3 \text{ mm}^2 = 7.2 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12}(60)(120)^3 = 8.64 \times 10^6 \text{ mm}^4 = 8.64 \times 10^{-6} \text{ m}^4$$

$$I_z = \frac{1}{12}(120)(60)^3 = 2.16 \times 10^6 \text{ mm}^4 = 2.16 \times 10^{-6} \text{ m}^4$$

At the section containing points H and K.

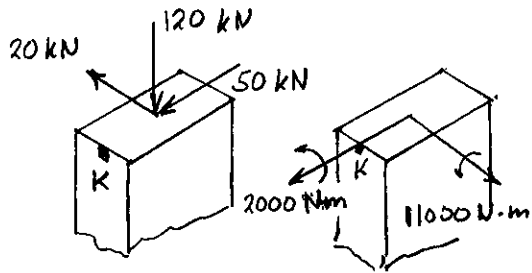
$$P = 120 \text{ kN (compression)}$$

$$V_x = -20 \text{ kN}$$

$$V_z = 50 \text{ kN}$$

$$M_z = (20 \times 10^3)(100 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$$

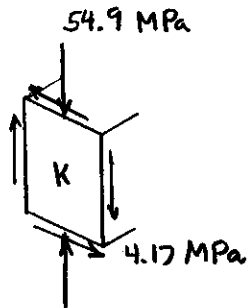
$$M_x = (120 \times 10^3)(50 \times 10^{-3}) + (50 \times 10^3)(100 \times 10^{-3}) = 11000 \text{ N}\cdot\text{m}$$



Stresses at point K

$$\begin{aligned} \sigma_K &= -\frac{P}{A} - \frac{M_z z}{I_x} + \frac{M_x x}{I_z} = -\frac{120 \times 10^3}{7.2 \times 10^{-3}} - \frac{(2000)(60 \times 10^{-3})}{8.64 \times 10^{-6}} + 0 \\ &= -16.67 \text{ MPa} - 36.39 \text{ MPa} + 0 = -93.1 \text{ MPa} \end{aligned}$$

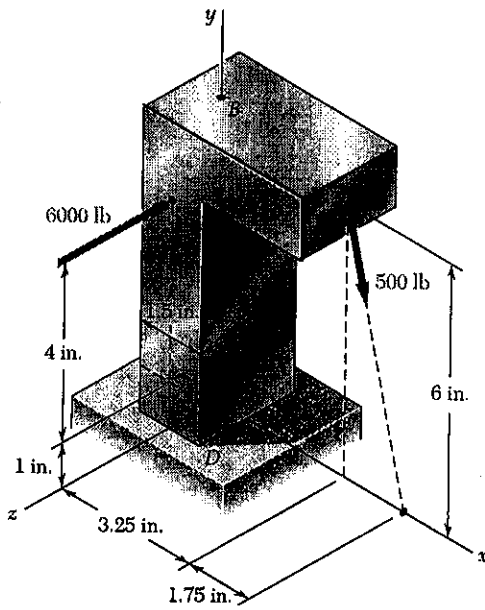
$$\tau_K = \frac{3}{2} \frac{|V_x|}{A} = \frac{3}{2} \frac{20 \times 10^3}{7.2 \times 10^{-3}} = 4.17 \text{ MPa}$$





PROBLEM 8.55

8.55 Two forces are applied to the small post  $BD$  as shown. Knowing that the vertical portion of the post has a cross section of  $1.5 \times 2.4$  in., determine the principal stresses, principal planes, and maximum shearing stress at point  $H$ .



SOLUTION

Components of 500 lb force.

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb.}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb force

$$\vec{r} = 3.25 \vec{i} + (6-1) \vec{j}$$

Moment of 500 lb force

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260 \vec{k} \text{ lb}\cdot\text{in}$$

At the section containing point  $H$ :

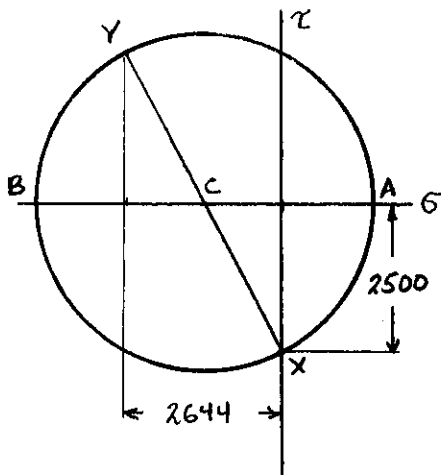
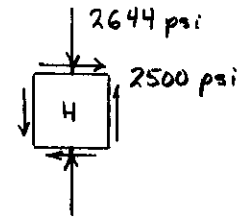
$$P = -480 \text{ lb.} \quad V_x = 140 \text{ lb.}$$

$$V_z = -6000 \text{ lb.}, \quad M_z = -2260 \text{ lb}\cdot\text{in}, \quad M_x = -(4)(6000) = -24000 \text{ lb}\cdot\text{in.}$$

$$A = (1.5)(2.4) = 3.6 \text{ in}^2 \quad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2} \frac{V_z}{A} = \frac{3}{2} \frac{6000}{3.6} = 2500 \text{ psi}$$



$$\sigma_c = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 1506 \text{ psi} \quad \leftarrow$$

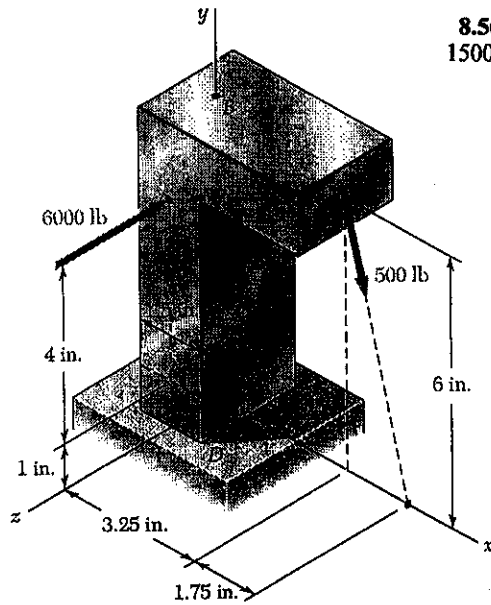
$$\sigma_b = \sigma_c - R = -4150 \text{ psi} \quad \leftarrow$$

$$\tan 2\theta_p = \frac{2\tau_x}{\sigma_x} = \frac{2(2500)}{0} = 1.891$$

$$\theta_a = 31.1^\circ, \quad \theta_b = 121.1^\circ \quad \leftarrow$$

$$\tau_{max} = R = 2828 \text{ psi} \quad \leftarrow$$

PROBLEM 8.56



8.55 Two forces are applied to the small post  $BD$  as shown. Knowing that the vertical portion of the post has a cross section of  $1.5 \times 2.4$  in., determine the principal stresses, principal planes, and maximum shearing stress at point  $H$ .

8.56 Solve Prob 8.55, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.

SOLUTION

Components of 500 lb. force

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb}$$

Moment arm of 500 lb. force

$$\vec{r} = 3.25 \vec{i} + (6-1) \vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260 \vec{k} \text{ lb}\cdot\text{in}$$

At the section containing point  $H$ :  $P = -480 \text{ lb}$   $V_x = 140 \text{ lb}$

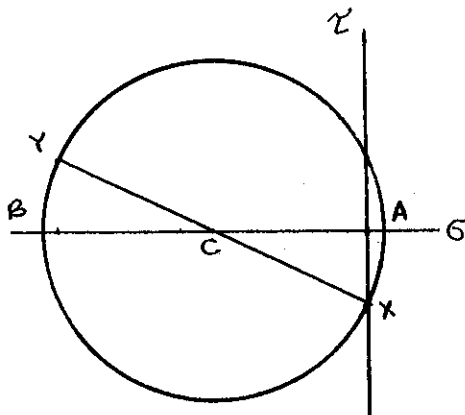
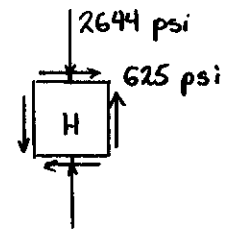
$V_z = -1500 \text{ lb}$ ,  $M_z = -2260 \text{ lb}\cdot\text{in}$ ,  $M_x = -(4)(1500) = -6000 \text{ lb}\cdot\text{in}$

$$A = (1.5)(2.4) = 3.6 \text{ in}^2$$

$$I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2} \frac{V_x}{A} = \frac{3}{2} \frac{1500}{3.6} = 625 \text{ psi}$$



$$\sigma_c = \frac{1}{2} \sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 140 \text{ psi}$$

$$\sigma_b = \sigma_c - R = -2784 \text{ psi}$$

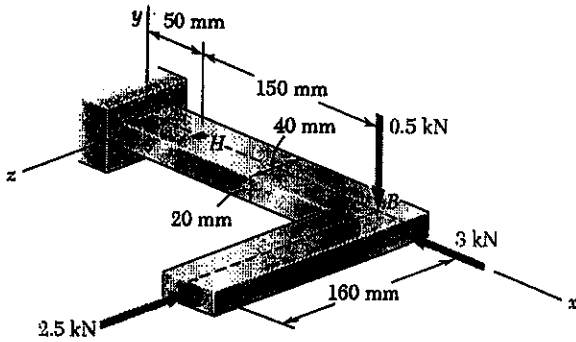
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(625)}{2644} = 0.4728$$

$$\theta_a = 12.7^\circ \quad \theta_b = 102.7^\circ$$

$$\tau_{max} = R = 1462 \text{ psi}$$

**PROBLEM 8.57**

8.57 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a  $20 \times 40$ -mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.



**SOLUTION**

Equivalent force-couple system at section containing point *H*

$$F_x = -3 \text{ kN}, \quad F_y = -0.5 \text{ kN}, \quad F_z = -2.5 \text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(2500) = 375 \text{ N}\cdot\text{m}$$

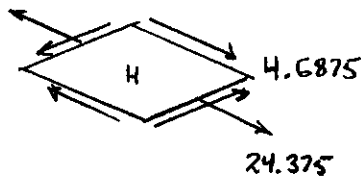
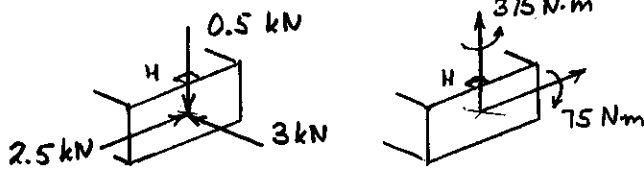
$$M_z = -(0.150)(500) = -75 \text{ N}\cdot\text{m}$$

$$A = (20)(40) = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3 \text{ mm}^4 = 26.667 \times 10^{-9} \text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{|V_z|}{A} = \frac{3}{2} \frac{2500}{800 \times 10^{-6}} = 4.6875 \text{ MPa}$$



$$\sigma_c = \frac{1}{2} \sigma_H = 12.1875 \text{ MPa}$$

$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579 \text{ MPa}$$

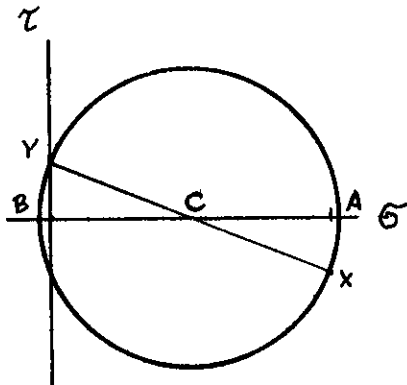
$$\sigma_a = \sigma_c + R = 25.2 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -0.87 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$$

$$\theta_a = 10.5^\circ, \quad \theta_b = 100.5^\circ$$

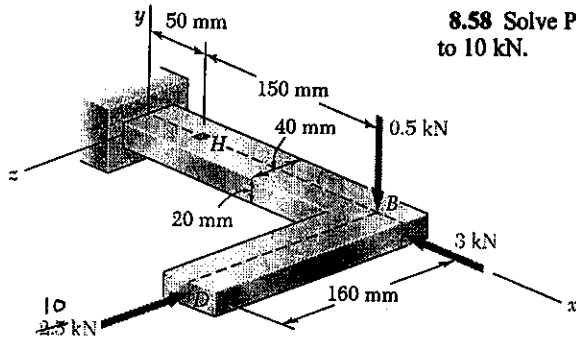
$$\tau_{\max} = R = 13.06 \text{ MPa}$$



**PROBLEM 8.58**

**8.57** Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a  $20 \times 40$ -mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

**8.58** Solve Prob. 8.57, assuming that the magnitude of the 2.5-kN force is increased to 10 kN.



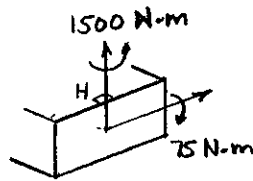
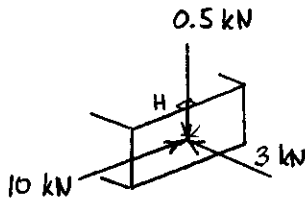
**SOLUTION**

Equivalent force-couple system at section containing point *H*.

$$F_x = -3 \text{ kN}, \quad F_y = -0.5 \text{ kN}, \quad F_z = -10 \text{ kN}$$

$$M_x = 0, \quad M_y = (0.150)(10000) = 1500 \text{ N}\cdot\text{m}$$

$$M_z = -(0.150)(500) = -75 \text{ N}\cdot\text{m}$$



$$A = (20)(40) = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$$

$$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3 \text{ mm}^4 = 26.667 \times 10^{-9} \text{ m}^4$$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{10000}{800 \times 10^{-6}} = 18.75 \text{ MPa}$$

$$\sigma_c = \frac{1}{2} \sigma_H = 12.1875 \text{ MPa}$$

$$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (18.75)^2} = 22.363 \text{ MPa}$$

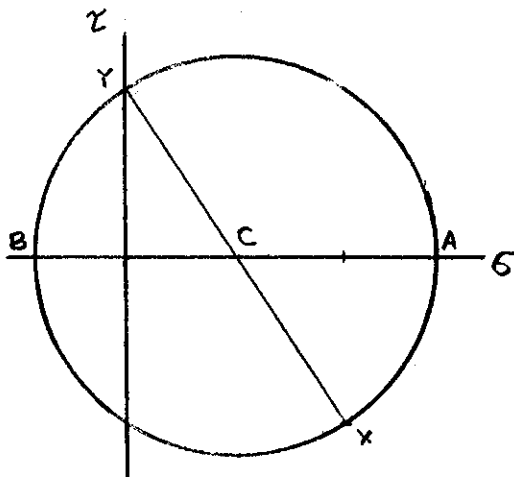
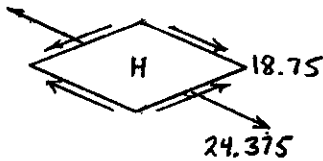
$$\sigma_a = \sigma_c + R = 34.6 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -10.18 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(18.75)}{24.375} = 1.5385$$

$$\theta_a = 28.5^\circ, \quad \theta_b = 118.5^\circ$$

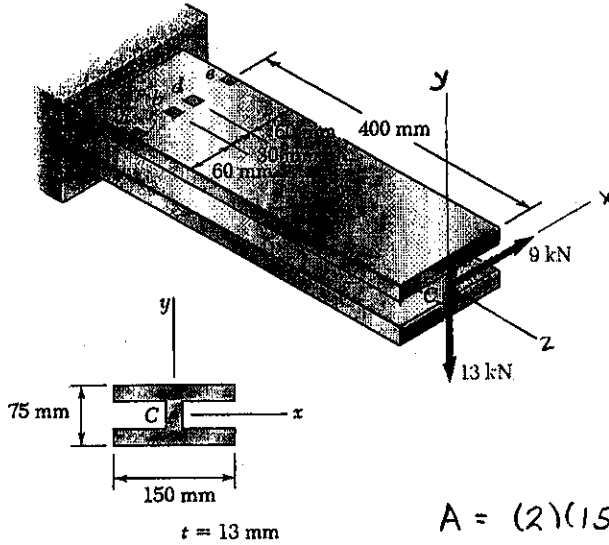
$$\tau_{max} = R = 22.4 \text{ MPa}$$



PROBLEM 8.59

8.59 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points a and b.

SOLUTION



Equivalent force-couple system at section containing points a and b.

$$F_x = 9 \text{ kN}, F_y = -13 \text{ kN}, F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

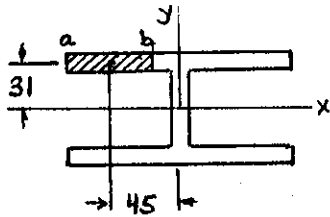
$$M_y = 0.400(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26) = 4537 \text{ mm}^2 = 4537 \times 10^{-6} \text{ m}^2$$

$$I_x = 2 \left[ \frac{1}{12}(150)(13)^3 + (150)(13)(37.5 - 6.5)^2 \right] + \frac{1}{12}(13)(75 - 26)^3 = 3.9303 \times 10^6 \text{ mm}^4 = 3.9303 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \cdot \frac{1}{12}(13)(150)^3 + \frac{1}{12}(75 - 26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$

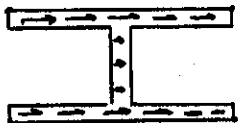


For point a  $Q_x = 0$   $Q_y = 0$

For point b  $A^* = (60)(13) = 780 \text{ mm}^2$   
 $\bar{x} = -45 \text{ mm}$   $\bar{y} = 31 \text{ mm}$

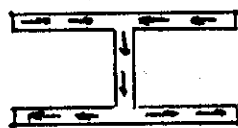
$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = -35.1 \times 10^3 \text{ mm}^3 = -35.1 \times 10^{-6} \text{ m}^3$$



At point a  $\sigma_a = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$   
 $= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-75 \times 10^{-3})}{7.3215 \times 10^{-6}} = 86.5 \text{ MPa}$

$$\tau_a = 0$$



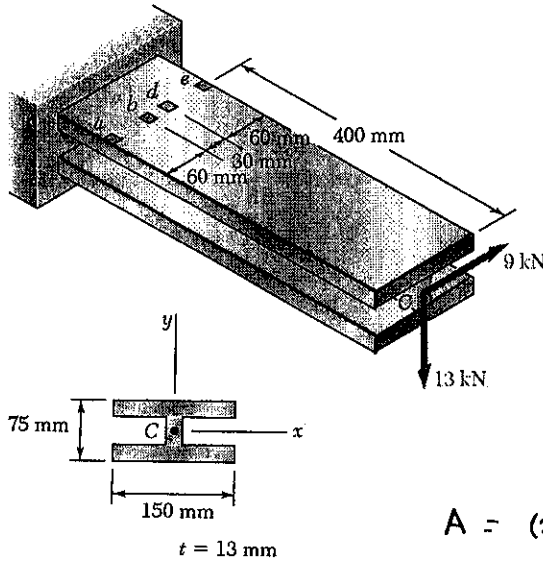
At point b  $\sigma_b = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$   
 $= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(-15 \times 10^{-3})}{7.3215 \times 10^{-6}} = 57.0 \text{ MPa}$

$$\tau_b = \frac{V_x |Q_y|}{I_y t} + \frac{V_y |Q_x|}{I_x t} = \frac{(9 \times 10^3)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} + \frac{(13 \times 10^3)(24.18 \times 10^{-6})}{(3.9303 \times 10^{-6})(13 \times 10^{-3})}$$

$$= 3.32 \text{ MPa} + 6.15 \text{ MPa} = 9.47 \text{ MPa}$$

PROBLEM 8.60

8.60 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.



SOLUTION

Equivalent force-couple system at section containing points *a* and *b*.

$$F_x = 9 \text{ kN}, F_y = -13 \text{ kN}, F_z = 0$$

$$M_x = (0.400)(13 \times 10^3) = 5200 \text{ N}\cdot\text{m}$$

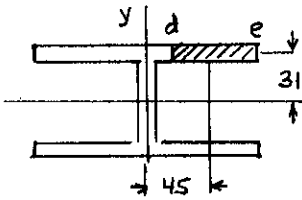
$$M_y = (0.400)(9 \times 10^3) = 3600 \text{ N}\cdot\text{m}$$

$$M_z = 0$$

$$A = (2)(150)(13) + (13)(75 - 26) = 4537 \text{ mm}^2 = 4537 \times 10^{-6} \text{ m}^2$$

$$I_x = 2 \left[ \frac{1}{12} (150)(13)^3 + (150)(13)(37.5 - 6.5)^2 \right] + \frac{1}{12} (13)(75 - 26)^3 = 3.9303 \times 10^6 \text{ mm}^4 = 3.9303 \times 10^{-6} \text{ m}^4$$

$$I_y = 2 \left[ \frac{1}{12} (13)(150)^3 \right] + \frac{1}{12} (75 - 26)(13)^3 = 7.3215 \times 10^6 \text{ mm}^4 = 7.3215 \times 10^{-6} \text{ m}^4$$

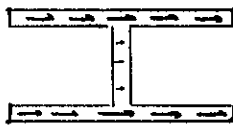


For point *d*  $A^* = (60)(13) = 780 \text{ mm}^2$   
 $\bar{x} = 45 \text{ mm}$   $\bar{y} = 31 \text{ mm}$

$$Q_x = A^* \bar{y} = 24.18 \times 10^3 \text{ mm}^3 = 24.18 \times 10^{-6} \text{ m}^3$$

$$Q_y = A^* \bar{x} = 35.1 \times 10^3 \text{ mm}^3 = 35.1 \times 10^{-6} \text{ m}^3$$

For point *e*  $Q_x = 0$  ,  $Q_y = 0$



At point *d*  $\sigma_d = \frac{M_x y}{I_x} - \frac{M_y x}{I_y}$   
 $= \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(15 \times 10^{-3})}{7.3215 \times 10^{-6}} = 42.2 \text{ MPa}$

Due to  $V_x$   $\tau_d = \frac{V_x Q_y}{I_y t} = \frac{(9000)(35.1 \times 10^{-6})}{(7.3215 \times 10^{-6})(13 \times 10^{-3})} = 3.32 \text{ MPa}$

Due to  $V_y$   $\tau_d = \frac{V_y Q_x}{I_x t} = \frac{(13000)(24.18 \times 10^{-6})}{(3.9303 \times 10^{-6})(13 \times 10^{-3})} = 6.15 \text{ MPa}$

Net  $\tau_d = 2.83 \text{ MPa}$

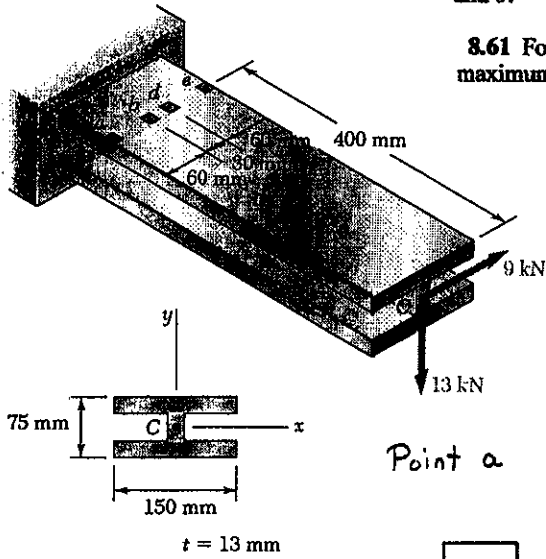
At point *e*  $\sigma_e = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(5200)(37.5 \times 10^{-3})}{3.9303 \times 10^{-6}} - \frac{(3600)(75 \times 10^{-3})}{7.3215 \times 10^{-6}} = 12.74 \text{ MPa}$

$\tau_e = 0$

**PROBLEM 8.61**

**8.59** Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

**8.61** For the beam and loading of Prob. 8.59, determine the principal stresses and the maximum shearing stress at points *a* and *b*.



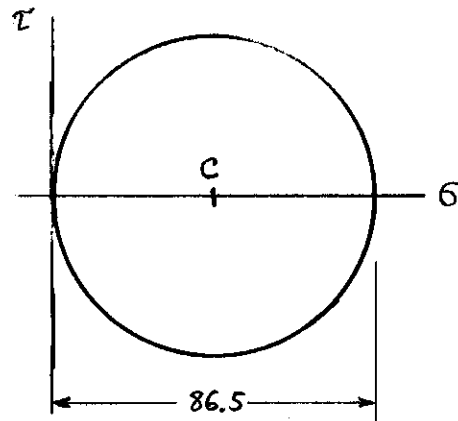
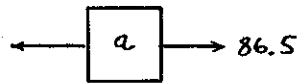
**SOLUTION**

From the solution of Prob. 8.59

$$\sigma_a = 86.5 \text{ MPa} \quad \tau_a = 0$$

$$\sigma_b = 57.0 \text{ MPa} \quad \tau_b = 9.47 \text{ MPa}$$

Point *a*



$$\sigma_c = \frac{86.5}{2} \text{ MPa} = 43.25 \text{ MPa}$$

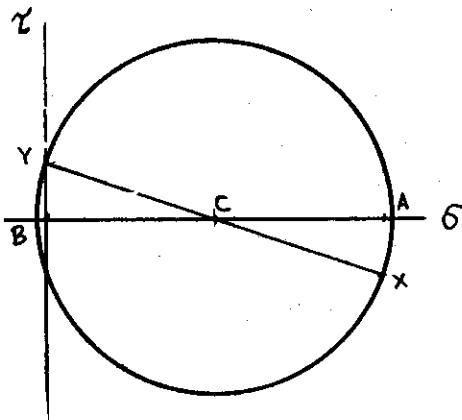
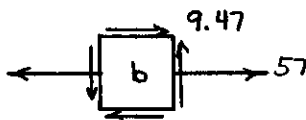
$$R = \frac{86.5}{2} \text{ MPa} = 43.25 \text{ MPa}$$

$$\sigma_{max} = \sigma_c + R = 86.5 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_c - R = 0$$

$$\tau_{max} = R = 43.3 \text{ MPa} \quad \blacktriangleleft$$

Point *b*



$$\sigma_c = \frac{57.0}{2} = 28.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{57.0}{2}\right)^2 + (9.47)^2} = 30.03 \text{ MPa}$$

$$\sigma_{max} = \sigma_c + R = 58.5 \text{ MPa} \quad \blacktriangleleft$$

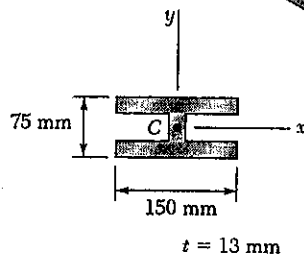
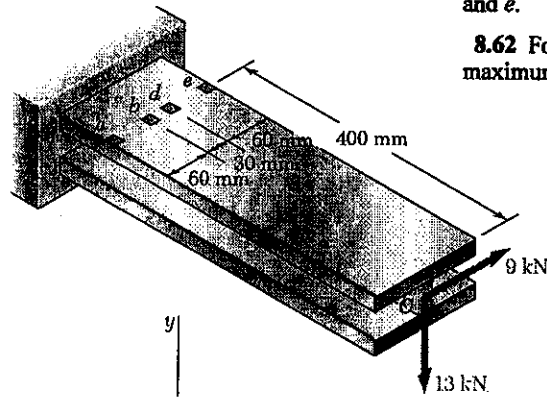
$$\sigma_{min} = \sigma_c - R = -1.53 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max} = R = 30.0 \text{ MPa} \quad \blacktriangleleft$$

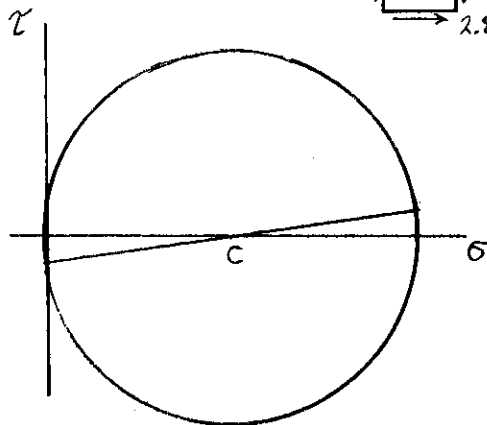
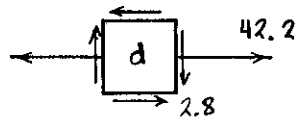
**PROBLEM 8.62**

**8.60** Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.

**8.62** For the beam and loading of Prob. 8.60, determine the principal stresses and the maximum shearing stress at points *d* and *e*.



Point *d*



**SOLUTION**

From the solution of Prob 8.60

$$\sigma_d = 42.2 \text{ MPa} \quad \tau_d = 2.83 \text{ MPa}$$

$$\sigma_e = 12.74 \text{ MPa} \quad \tau_e = 0$$

$$\sigma_c = \frac{42.2}{2} = 42.2 \text{ MPa}$$

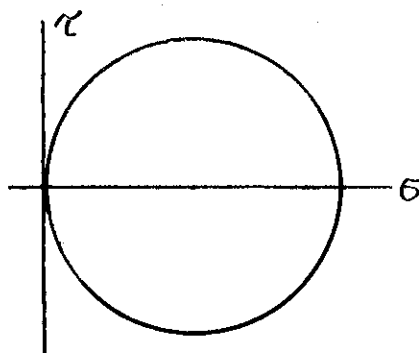
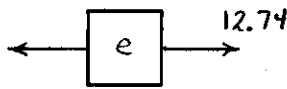
$$R = \sqrt{\left(\frac{42.2}{2}\right)^2 + (2.83)^2} = 21.29 \text{ MPa}$$

$$\sigma_{\max} = \sigma_c + R = 42.4 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{\min} = \sigma_c - R = -0.19 \text{ MPa} \quad \blacktriangleright$$

$$\tau_{\max} = R = 21.3 \text{ MPa} \quad \blacktriangleleft$$

Point *e*



$$\sigma_c = \frac{12.74}{2} = 6.37 \text{ MPa}$$

$$R = \frac{12.74}{2} = 6.37 \text{ MPa}$$

$$\sigma_{\max} = \sigma_c + R = 12.74 \text{ MPa} \quad \blacktriangleleft$$

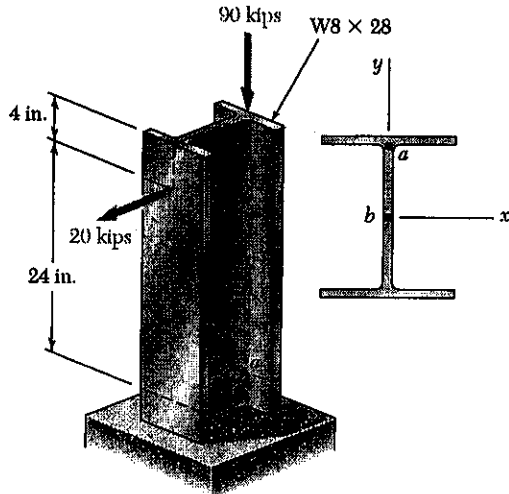
$$\sigma_{\min} = \sigma_c - R = 0 \quad \blacktriangleright$$

$$\tau_{\max} = R = 6.37 \text{ MPa} \quad \blacktriangleleft$$



**PROBLEM 8.63**

**8.63** Two forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses, principal planes, and maximum shearing stress at point *a*.



**SOLUTION**

For W8 × 28 rolled steel section

$$A = 8.25 \text{ in}^2, \quad d = 8.06 \text{ in}, \quad b_f = 6.535 \text{ in}$$

$$t_f = 0.465 \text{ in}, \quad t_w = 0.285 \text{ in}, \quad I_x = 98.0 \text{ in}^4$$

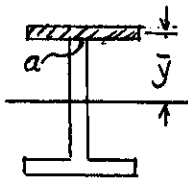
At the section containing points *a* and *b*.

$$P = -90 \text{ kips}, \quad V = 20 \text{ kips}$$

$$M = (20)(24) - (4.03)(90) = 117.3 \text{ kip}\cdot\text{in}$$

At point *a*  $y = \frac{1}{2}d - t_f = 4.03 - 0.465 = 3.565 \text{ in}$

$$\sigma = \frac{P}{A} + \frac{My}{I} = -\frac{90}{8.25} - \frac{(-117.3)(3.565)}{98.0} = -6.642 \text{ ksi}$$

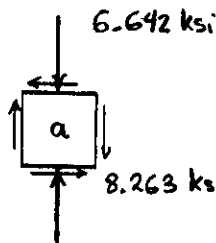


$$\bar{y} = \frac{1}{2}d - \frac{1}{2}t_f = 4.03 - 0.2325 = 3.7975 \text{ in}$$

$$A_f = b_f t_f = (6.535)(0.465) = 3.0388 \text{ in}^2$$

$$Q_a = A_f \bar{y} = 11.540 \text{ in}^3$$

$$\tau = \frac{VQ_a}{I t_w} = \frac{(20)(11.540)}{(98.0)(0.285)} = 8.263 \text{ ksi}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-8.263)}{0 + 6.642} = -2.4881$$

$$\theta_a = -34.1^\circ, \quad \theta_b = 55.9^\circ$$

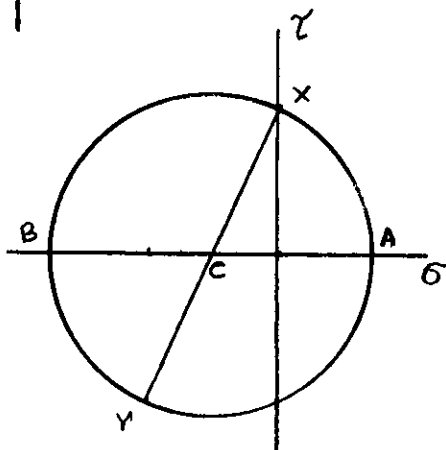
$$\sigma_c = -\frac{6.642}{2} = -3.321 \text{ ksi}$$

$$R = \sqrt{\left(\frac{6.642}{2}\right)^2 + (8.263)^2} = 8.905 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 5.58 \text{ ksi}$$

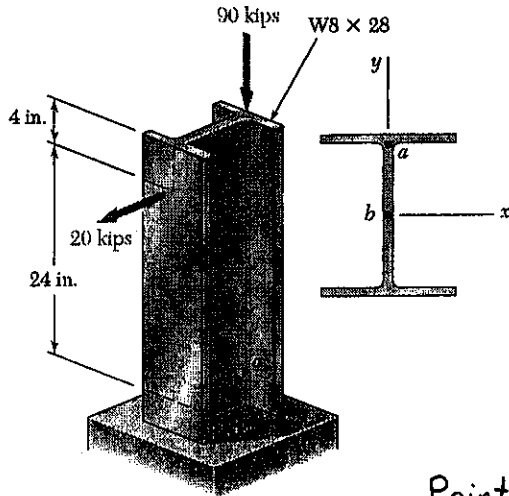
$$\sigma_b = \sigma_c - R = -12.23 \text{ ksi}$$

$$\tau_{max} = R = 8.91 \text{ ksi}$$



PROBLEM 8.64

8.64 Two forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses, principal planes, and maximum shearing stress at point b.



SOLUTION

For W8 × 28 rolled steel section

$$A = 8.25 \text{ in}^2, \quad d = 8.06 \text{ in}, \quad b_f = 6.535 \text{ in}$$

$$t_f = 0.465 \text{ in}, \quad t_w = 0.285 \text{ in}, \quad I_x = 98.0 \text{ in}^4$$

At the section containing points a and b.

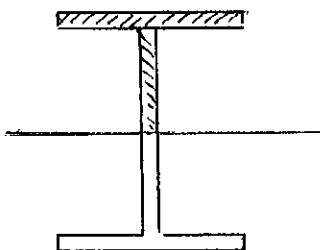
$$P = -90 \text{ kips}, \quad V = 20 \text{ kips}$$

$$M = (20)(24) - (4.03)(90) = -117.3 \text{ kip}\cdot\text{in.}$$

Point b lies on the neutral axis of bending

At point b

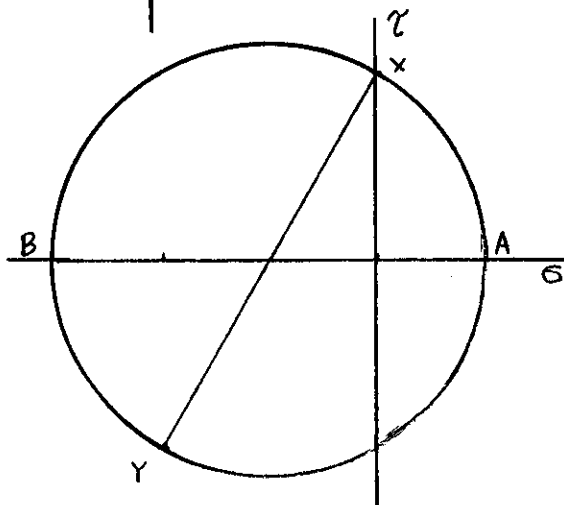
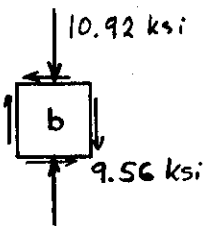
$$\sigma = \frac{P}{A} = \frac{-90}{8.25} = -10.92 \text{ ksi}$$



Part	A (in <sup>2</sup> )	$\bar{y}$ (in)	$A\bar{y}$ (in <sup>3</sup> )
Flange	3.0388	3.7975	11.540
Half-web	1.0161	1.7825	1.811
$\Sigma$			13.351

$$Q_b = 13.351 \text{ in}^3$$

$$\tau = \frac{VQ_b}{It_w} = \frac{(20)(13.351)}{(98.0)(0.285)} = 9.56 \text{ ksi}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-9.56)}{0 + 10.92} = -1.7509$$

$$\theta_a = -30.1^\circ \quad \theta_b = 59.9^\circ$$

$$\sigma_c = -\frac{10.92}{2} = -5.46 \text{ ksi}$$

$$R = \sqrt{\left(\frac{10.92}{2}\right)^2 + (9.56)^2} = 11.01 \text{ ksi}$$

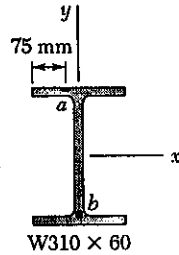
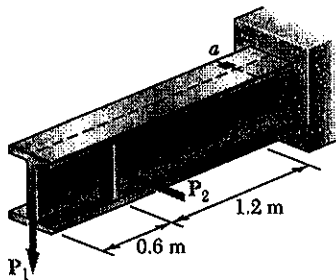
$$\sigma_{max} = \sigma_c + R = 5.55 \text{ ksi}$$

$$\sigma_{min} = \sigma_c - R = -16.47 \text{ ksi}$$

$$\tau_{max} = R = 11.01 \text{ ksi}$$

PROBLEM 8.65

8.65 Two forces  $P_1$  and  $P_2$  are applied as shown in directions perpendicular to the longitudinal axis of a  $W310 \times 60$  beam. Knowing that  $P_1 = 25$  kN and  $P_2 = 24$  kN, determine the principal stresses and the maximum shearing stress at point  $a$ .



SOLUTION

At the section containing points  $a$  and  $b$

$$M_x = (1.8)(25) = 45 \text{ kN}\cdot\text{m}$$

$$M_y = -(1.2)(24) = -28.8 \text{ kN}\cdot\text{m}$$

$$V_x = -24 \text{ kN} \quad V_y = -25 \text{ kN}$$

For  $W310 \times 60$  rolled steel section

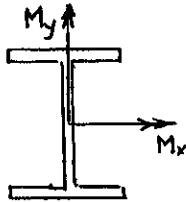
$$d = 303 \text{ mm}, \quad bf = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.5 \text{ mm}$$

$$I_x = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4, \quad I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$$

Normal stress at point  $a$

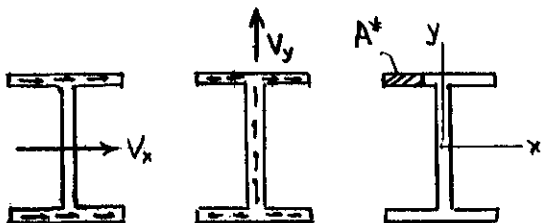
$$x = -\frac{bf}{2} + 75 = -26.5 \text{ mm}$$

$$y = \frac{1}{2}d = 151.5 \text{ mm}$$



$$\begin{aligned} \sigma_z &= \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(151.5 \times 10^{-3})}{129 \times 10^{-6}} - \frac{(-28.8 \times 10^3)(-26.5 \times 10^{-3})}{18.3 \times 10^{-6}} \\ &= 52.849 \text{ MPa} - 41.705 \text{ MPa} = 11.144 \text{ MPa} \end{aligned}$$

Shearing stress at point  $a$



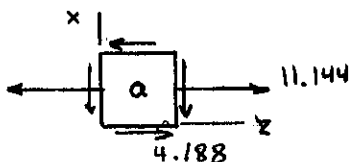
$$\tau_{xz} = -\frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

$$A^* = (75 \times 10^{-3})(13.1 \times 10^{-3}) = 982.5 \times 10^{-6} \text{ m}^2$$

$$\bar{x} = -\frac{bf}{2} + \frac{75}{2} = -64 \text{ mm}$$

$$\bar{y} = \frac{d}{2} - \frac{t_f}{2} = 144.95 \text{ mm}$$

$$\begin{aligned} \tau_{xz} &= -\frac{(-24 \times 10^3)(982.5 \times 10^{-6})(-64 \times 10^{-3})}{(18.3 \times 10^{-6})(13.1 \times 10^{-3})} - \frac{(-25 \times 10^3)(982.5 \times 10^{-6})(144.95 \times 10^{-3})}{(129 \times 10^{-6})(13.1 \times 10^{-3})} \\ &= -6.295 \text{ MPa} + 2.107 \text{ MPa} = -4.188 \text{ MPa} \end{aligned}$$



$$\sigma_{ave} = \frac{11.144}{2} = 5.572 \text{ MPa}$$

$$R = \sqrt{\left(\frac{11.144}{2}\right)^2 + (4.188)^2} = 6.970 \text{ MPa}$$

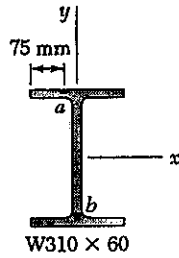
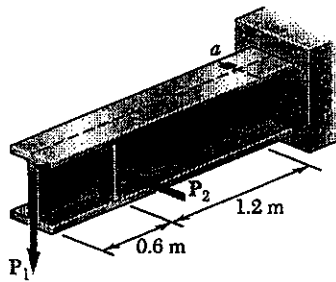
$$\sigma_{max} = \sigma_{ave} + R = 12.54 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -1.40 \text{ MPa}$$

$$\tau_{max} = R = 6.97 \text{ MPa}$$

PROBLEM 8.66

8.66 Two forces  $P_1$  and  $P_2$  are applied as shown in directions perpendicular to the longitudinal axis of a  $W310 \times 60$  beam. Knowing that  $P_1 = 25$  kN and  $P_2 = 24$  kN, determine the principal stresses and the maximum shearing stress at point  $b$ .



SOLUTION

At the section containing points  $a$  and  $b$

$$M_x = (1.8)(25) = 45 \text{ kN}\cdot\text{m}$$

$$M_y = -(1.2)(24) = -28.8 \text{ kN}\cdot\text{m}$$

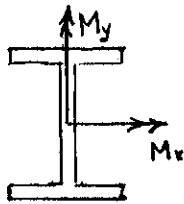
$$V_x = -24 \text{ kN}, \quad V_y = -25 \text{ kN}$$

For  $W310 \times 60$  rolled steel section

$$d = 303 \text{ mm}, \quad b_f = 203 \text{ mm}, \quad t_f = 13.1 \text{ mm}, \quad t_w = 7.5 \text{ mm}$$

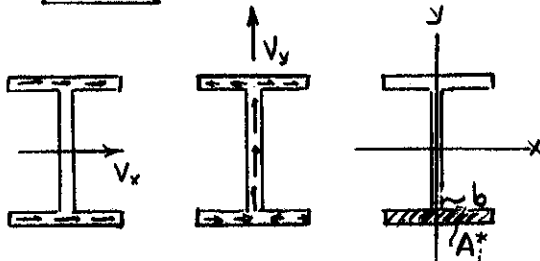
$$I_x = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4, \quad I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$$

Normal stress at point  $b$   $x \approx 0, y = -\frac{1}{2}d + t_f = -138.4 \text{ mm}$ .



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(-138.4 \times 10^{-3})}{129 \times 10^{-6}} - 0 = -48.28 \text{ MPa}$$

Shearing stress at point  $b$ .

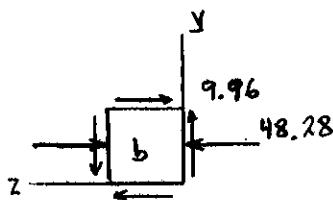


$$\tau_{yz} = -\frac{V_y A^* \bar{y}}{I_x t_w}$$

$$A^* = A_f = b_f t_f = 2659 \text{ mm}^2 = 2659 \times 10^{-6} \text{ m}^2$$

$$\bar{x} = 0, \quad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.95 \text{ mm}$$

$$\tau_{yz} = -\frac{(-25 \times 10^3)(2659 \times 10^{-6})(-144.95 \times 10^{-3})}{(129 \times 10^{-6})(7.5 \times 10^{-3})} = -9.96 \text{ MPa}$$



$$\sigma_{ave} = -\frac{48.28}{2} = -24.14 \text{ MPa}$$

$$R = \sqrt{\left(\frac{48.28}{2}\right)^2 + (9.96)^2} = 26.11$$

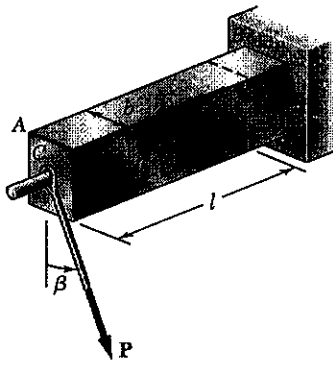
$$\sigma_{max} = \sigma_{ave} + R = 1.97 \text{ MPa}$$

$$\sigma_{min} = \sigma_{ave} - R = -50.3 \text{ MPa}$$

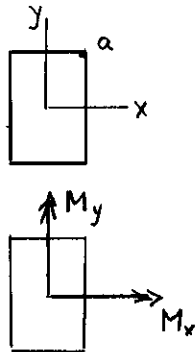
$$\tau_{max} = R = 26.1 \text{ MPa}$$

PROBLEM 8.67

8.67 A force  $P$  is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that  $P$  acts in a direction perpendicular to the longitudinal axis of the beam, determine (a) the normal stress at point  $a$  in terms of  $P$ ,  $b$ ,  $h$ ,  $l$ , and  $\beta$ , (b) the values of  $\beta$  for which the normal stress at  $a$  is zero.



SOLUTION



$$I_x = \frac{1}{12} b h^3 \quad I_y = \frac{1}{12} h b^3$$

$$\begin{aligned} \sigma &= \frac{M_x (h/2)}{I_x} - \frac{M_y (b/2)}{I_y} \\ &= \frac{6 M_x}{b h^2} - \frac{6 M_y}{h b^2} \end{aligned}$$

$$\vec{P} = P \sin \beta \vec{i} - P \cos \beta \vec{j} \quad \vec{r} = l \vec{k}$$

$$\vec{M} = \vec{r} \times \vec{P} = l \vec{k} \times (P \sin \beta \vec{i} - P \cos \beta \vec{j}) = P l \cos \beta \vec{i} + P l \sin \beta \vec{j}$$

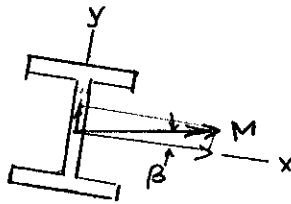
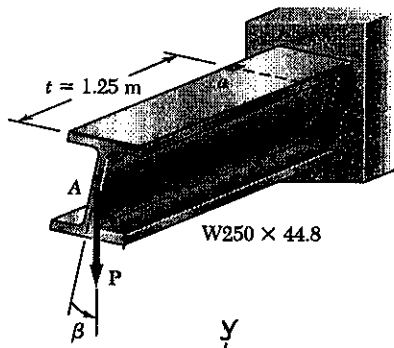
$$M_x = P l \cos \beta \quad M_y = P l \sin \beta$$

$$(a) \quad \sigma = \frac{6 P l \cos \beta}{b h^2} - \frac{6 P l \sin \beta}{h b^2} = \frac{6 P l}{b h} \left[ \frac{\cos \beta}{h} - \frac{\sin \beta}{b} \right]$$

$$(b) \quad \sigma = 0 \quad \frac{\cos \beta}{h} - \frac{\sin \beta}{b} = 0 \quad \tan \beta = \frac{b}{h}$$

$$\beta = \tan^{-1} \left( \frac{b}{h} \right)$$

PROBLEM 8.68



8.68 A vertical force  $P$  is applied at the center of the free end of cantilever beam  $AB$ . (a) If the beam is installed with the web vertical ( $\beta = 0$ ) and with its longitudinal axis  $AB$  horizontal, determine the magnitude of the force  $P$  for which the normal stress at point  $a$  is  $+120$  MPa. (b) Solve part  $a$ , assuming that the beam is installed with  $\beta = 3^\circ$ .

SOLUTION

For  $W250 \times 44.8$  rolled steel section

$$S_x = 535 \times 10^3 \text{ mm}^3 = 535 \times 10^{-6} \text{ m}^3$$

$$S_y = 95.0 \times 10^3 \text{ mm}^3 = 95.0 \times 10^{-6} \text{ m}^3$$

At the section containing point  $a$

$$M_x = Pl \cos \beta, \quad M_y = Pl \sin \beta$$

Stress at  $a$

$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl \cos \beta}{S_x} + \frac{Pl \sin \beta}{S_y}$$

Allowable load

$$P_{all} = \frac{\sigma_{all}}{l} \left[ \frac{\cos \beta}{S_x} + \frac{\sin \beta}{S_y} \right]^{-1}$$

(a)  $\beta = 0$

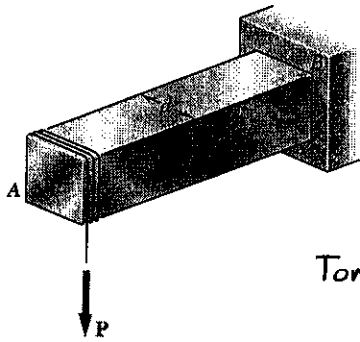
$$P_{all} = \frac{120 \times 10^6}{1.25} \left[ \frac{1}{535 \times 10^{-6}} + 0 \right]^{-1} = 51.4 \times 10^3 \text{ N} = 51.4 \text{ kN} \quad \blacktriangleleft$$

(b)  $\beta = 3^\circ$

$$P_{all} = \frac{120 \times 10^6}{1.25} \left[ \frac{\cos 3^\circ}{535 \times 10^{-6}} + \frac{\sin 3^\circ}{95.0 \times 10^{-6}} \right]^{-1} = 39.7 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 8.69

\*8.69 A 500-lb force  $P$  is applied to a wire that is wrapped around the bar  $AB$  as shown. Knowing that the cross section of the bar is a square of side  $d = 0.75$  in., determine the principal stresses and the maximum shearing stress at point  $a$ .



SOLUTION

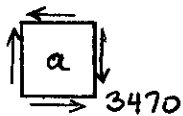
Bending: Point  $a$  lies on the neutral axis.  
 $\sigma = 0$

Torsion:  $\tau = \frac{T}{c_1 ab^2}$  where  $a = b = d$  and  $c_1 = 0.208$  for a square section.

Since  $T = \frac{Pd}{2}$   $\tau = \frac{P}{0.416 d^2} = 2.404 \frac{P}{d^2}$

Transverse shear:  $V = P$   $\tau = \frac{3}{2} \frac{V}{A} = 1.5 \frac{P}{d^2}$

Using superposition:  $\tau = 3.904 \frac{P}{d^2} = 3.904 \frac{500}{(0.75)^2} = 3470$  psi



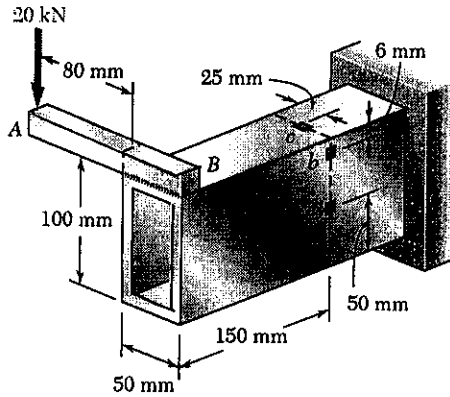
$\sigma_{max} = 3470$  psi  $\blacktriangleleft$

$\sigma_{min} = -3470$  psi  $\blacktriangleleft$

$\tau_{max} = 3470$  psi  $\blacktriangleleft$

PROBLEM 8.70

\*8.70 A vertical 20-kN force is applied to end A of the bar AB, which is welded to an extruded aluminum tube. Knowing that the tube has a uniform wall thickness of 6 mm, determine the shearing stress at points a, b, and c.



SOLUTION

$$I = \frac{1}{12} (50)(100)^3 - \frac{1}{12} (38)(88)^3 = 2.0087 \times 10^6 \text{ mm}^4 = 2.0087 \times 10^{-6} \text{ m}^4$$

Torsion:  $T = (20 \times 10^3)(80 + 25)(10^3) = 2100 \text{ N}\cdot\text{m}$

$$Q = (44)(94) = 4.136 \times 10^3 \text{ mm}^2 = 4.136 \times 10^{-3} \text{ m}^2$$

For points a, b, and c

$$\tau = \frac{T}{2tQ} = \frac{2100}{(2)(6 \times 10^{-3})(4.136 \times 10^{-3})} = 42.31 \text{ MPa}$$

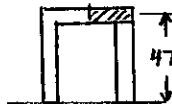


Transverse shear:  $V = 20 \times 10^3 \text{ N}$

Point c - on symmetry axis  $\tau = 0$



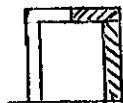
Point b



$$Q_b = (25)(6)(47) = 7.05 \times 10^3 \text{ mm}^3 = 7.05 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(7.05 \times 10^{-6})}{(2.0087 \times 10^{-6})(6 \times 10^{-3})} = 11.70 \text{ MPa}$$

Point a



$$Q_a = Q_b + (6)(44)(22) = 12.858 \times 10^3 \text{ mm}^3 = 12.858 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(12.858 \times 10^{-6})}{(2.0087 \times 10^{-6})(6 \times 10^{-3})} = 21.34 \text{ MPa}$$

Net shearing stress:

Point a  $\tau = 42.31 - 0 = 42.3 \text{ MPa}$   $\blacktriangleleft$

Point b  $\tau = 42.31 - 11.70 = 30.6 \text{ MPa}$   $\blacktriangleleft$

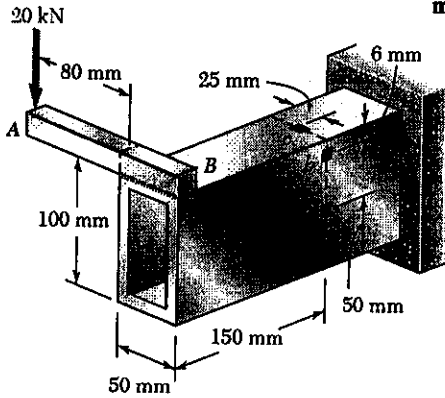
Point c  $\tau = 42.31 - 21.34 = 21.0 \text{ MPa}$   $\blacktriangleleft$



PROBLEM 8.71

\*8.70 A vertical 20-kN force is applied to end A of the bar AB, which is welded to an extruded aluminum tube. Knowing that the tube has a uniform wall thickness of 6 mm, determine the shearing stress at points a, b, and c.

\*8.71 For the tube and loading of Prob. 8.70, determine the principal stresses and the maximum shearing stress at point b.



SOLUTION

Bending:  $M = (20 \times 10^3)(150 \times 10^{-3}) = 3000 \text{ N}\cdot\text{m}$

$$I = \frac{1}{12}(50)(100)^3 - \frac{1}{12}(38)(88)^3 = 2.0087 \times 10^6 \text{ mm}^4 = 2.0087 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{My}{I} = \frac{(3000)(44 \times 10^{-3})}{2.0087 \times 10^{-6}} = 65.7 \text{ MPa}$$

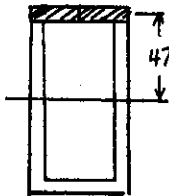
Torsion:  $T = (20 \times 10^3)(80 + 25)(10^{-3}) = 2100 \text{ N}\cdot\text{m}$

$$Q = (44)(94) = 4.136 \times 10^3 \text{ mm}^2 = 4.136 \times 10^{-3} \text{ m}^2$$

$$\tau = \frac{T}{2tQ} = \frac{2100}{(2)(6 \times 10^{-3})(4.136 \times 10^{-3})} = 42.31 \text{ MPa}$$

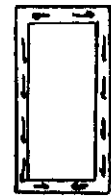


Transverse shear:  $V = 20 \times 10^3 \text{ N}$

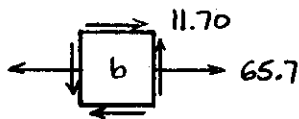


$$Q = (50)(6)(47) = 14.1 \times 10^3 \text{ mm}^3 = 14.1 \times 10^{-6} \text{ m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20 \times 10^3)(14.1 \times 10^{-6})}{(2.0087 \times 10^{-6})(12 \times 10^{-3})} = 11.70 \text{ MPa}$$



Net shearing stress  $\tau = 42.31 - 11.70 = 30.6 \text{ MPa}$



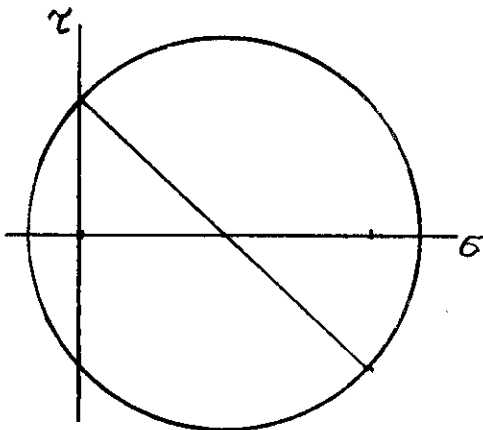
$$\sigma_c = \frac{1}{2}\sigma = 32.85 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{65.7}{2}\right)^2 + (30.6)^2} = 44.89 \text{ MPa}$$

$$\sigma_{max} = \sigma_c + R = 77.7 \text{ MPa}$$

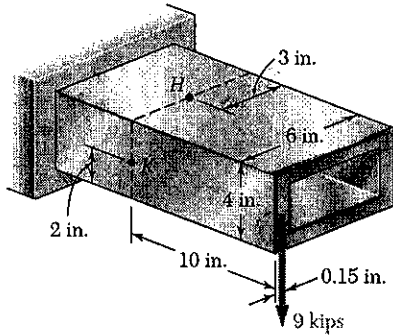
$$\sigma_{min} = \sigma_c - R = -12.04 \text{ MPa}$$

$$\tau_{max} = R = 44.9 \text{ MPa}$$



PROBLEM 8.72

\*8.72 Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (a) point H, (b) point K.



SOLUTION

At the section containing points H and K

$$V = 9 \text{ kips} \quad M = (9)(10) = 90 \text{ kip}\cdot\text{in.}$$

$$T = (9)(3 - 0.15) = 25.65 \text{ kip}\cdot\text{in.}$$

Torsion:

$$Q = (5.7)(3.7) = 21.09 \text{ in}^2$$

$$\gamma = \frac{T}{2tQ} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027 \text{ ksi}$$

Transverse shear:

$$Q_H = 0$$

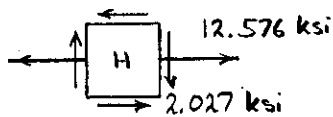
$$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85) = 2.0985 \text{ in}^3$$

$$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132 \text{ in}^4$$

$$\gamma_H = 0 \quad \gamma_K = \frac{VQ_K}{I t} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398 \text{ ksi}$$

Bending:  $\sigma_H = \frac{Mc}{I} = \frac{(90)(2)}{14.3132} = 12.576 \text{ ksi}$ ,  $\sigma_K = 0$

(a) Point H:



$$\sigma_c = \frac{12.576}{2} = 6.288 \text{ ksi}$$

$$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607 \text{ ksi}$$

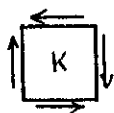
$$\sigma_{max} = \sigma_c + R = 12.90 \text{ ksi}$$

$$\sigma_{min} = \sigma_c - R = -0.32 \text{ ksi}$$

$$\tan 2\theta_p = \frac{2\gamma}{\sigma} = -0.3224 \quad \theta_p = -8.9^\circ, 81.1^\circ$$

$$\gamma_{max} = R = 6.61 \text{ ksi}$$

(b) Point K:



$$\sigma = 0 \quad \gamma = 2.027 + 4.398 = 6.425 \text{ ksi}$$

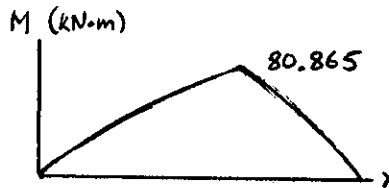
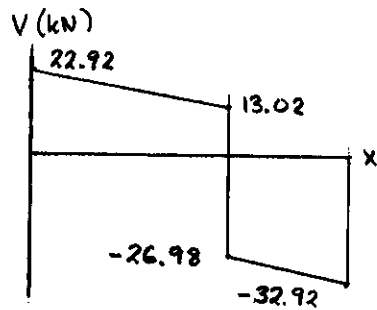
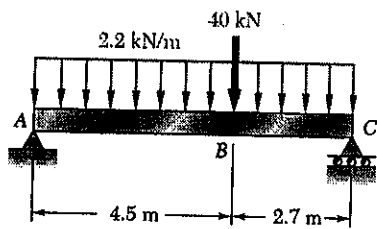
$$\sigma_{max} = 6.43 \text{ ksi}$$

$$\sigma_{min} = -6.43 \text{ ksi}$$

$$\theta_p = \pm 45^\circ$$

$$\gamma_{max} = 6.43 \text{ ksi}$$

PROBLEM 8.73



Shape	S (10 <sup>3</sup> mm <sup>3</sup> )
W 360 x 39	578
W 310 x 38.7	549 ←
W 250 x 44.8	535
W 200 x 52	512

8.73 (a) Knowing that  $\sigma_{all} = 165 \text{ MPa}$  and  $\tau_{all} = 100 \text{ MPa}$ , select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for  $\sigma_m$ ,  $\tau_m$ , and the principal stress  $\sigma_{max}$  at the junction of a flange and the web of the selected beam.

$$+\Sigma M_c = 0$$

$$-7.2 R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$$

$$R_A = 22.92 \text{ kN}$$

$$V_A = R_A = 22.92 \text{ kN}$$

$$V_B^- = 22.92 - (2.2)(4.5) = 13.02 \text{ kN}$$

$$V_B^+ = 13.02 - 40 = -26.98 \text{ kN}$$

$$V_C = -26.98 - (2.2)(2.7) = -32.92 \text{ kN}$$

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$$M_B = 0 + \frac{1}{2}(22.92 + 13.02)(4.5) = 80.865 \text{ kN}\cdot\text{m}$$

$$M_C = 0$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{165 \times 10^6} = 490 \times 10^{-6} \text{ m}^3 = 490 \times 10^3 \text{ mm}^3$$

Try W 310 x 38.7

$$d = 310 \text{ mm} \quad t_f = 9.7 \text{ mm}$$

$$t_w = 5.8 \text{ mm}$$

$$\sigma_m = \frac{M_B}{S} = \frac{80.865 \times 10^3}{549 \times 10^{-6}} = 147.3 \text{ MPa}$$

$$\tau_m = \frac{|V|_{max}}{d t_w} = \frac{26.98 \times 10^3}{(310 \times 10^{-3})(5.8 \times 10^{-3})} = 18.31 \text{ MPa}$$

$$c = \frac{1}{2}d = 155 \text{ mm}$$

$$y_b = c - t_f = 155 - 9.7 = 145.3 \text{ mm}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \left(\frac{145.3}{155}\right)(147.3) = 138.1 \text{ MPa}$$

$$\text{At point B} \quad \tau_w = \frac{V}{d t_w} = \frac{(26.98 \times 10^3)}{(310 \times 10^{-3})(5.8 \times 10^{-3})} = 15.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.05)^2 + (15.0)^2} = 70.66 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 69.05 + 70.66 = 139.7 \text{ MPa}$$

PROBLEM 8.74

8.74 Knowing that the shear and bending moment in a given section of a W21 × 101 rolled-steel beam are, respectively, 120 kips and 300 kip · ft, determine the values in that section of (a) the maximum normal stress  $\sigma_m$ , (b) the principal stress  $\sigma_{max}$  at the junction of a flange and the web.

SOLUTION

$$M = 300 \text{ kip} \cdot \text{ft} = 3600 \text{ kip} \cdot \text{in} \quad V = 120 \text{ kips.}$$

For W21 × 101 shape  $d = 21.36 \text{ in}$   $b_f = 12.290 \text{ in}$   $t_f = 0.800 \text{ in}$ .

$$t_w = 0.500 \text{ in}, \quad I_z = 2420 \text{ in}^4, \quad S_z = 227 \text{ in}^3, \quad c = \frac{1}{2}d = 10.68 \text{ in}.$$

$$(a) \quad \sigma_m = \frac{M}{S} = \frac{3600}{227} = 15.86 \text{ ksi} \quad \blacktriangleleft$$

$$(b) \quad y_b = c - t_f = 9.88 \text{ in}$$

$$\sigma_b = \frac{y_b}{c} \sigma_m = \frac{9.88}{10.68} \cdot 15.86 = 7.336 \text{ ksi}$$

$$A_f = b_f t_f = 9.832 \text{ in}^2 \quad \bar{y} = \frac{1}{2}(c + y_b) = 10.28 \text{ in}$$

$$Q = A_f \bar{y} = 101.07 \text{ in}^3$$

$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(120)(101.07)}{(2420)(0.500)} = 10.024 \text{ ksi}$$

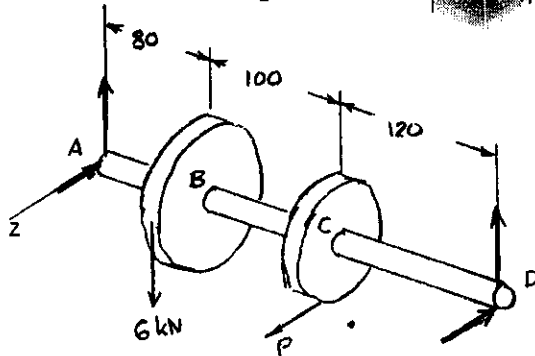
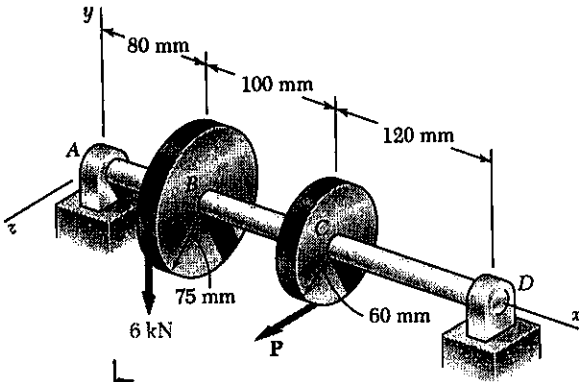
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{7.336^2 + 10.024^2} = 12.421 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 7.336 + 10.421 = 19.76 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 8.75**

8.75 The 6-kN force is vertical and the force  $P$  is parallel to the  $z$  axis. Knowing that  $\tau_{all} = 60$  MPa, determine the smallest permissible diameter of the solid shaft  $AD$ .

**SOLUTION**



$$\sum M_x = 0$$

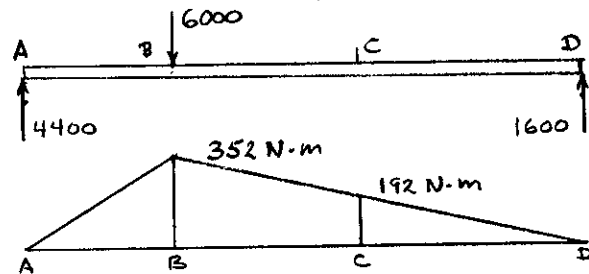
$$(6 \times 10^3)(75 \times 10^{-3}) - (60 \times 10^{-3})P = 0$$

$$P = 7.5 \times 10^3 \text{ N}$$

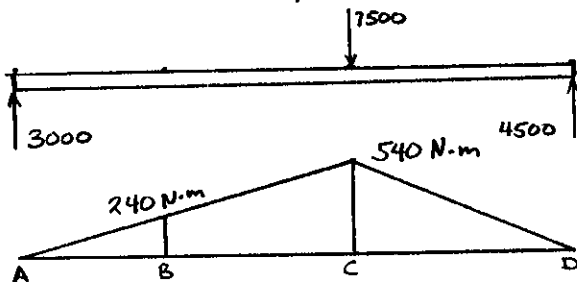
Over portion BC

$$T = (6 \times 10^3)(75 \times 10^{-3}) = 450 \text{ N}\cdot\text{m}$$

Forces in vertical plane



Forces in horizontal plane



Bending moments

At B  $M = \sqrt{352^2 + 240^2}$   
 $= 426.0 \text{ N}\cdot\text{m}$

At C  $M = \sqrt{540^2 + 192^2}$   
 $= 573.1 \text{ N}\cdot\text{m}$

Critical section is just to the left of gear C

$$M = 573.1 \text{ N}\cdot\text{m} \quad T = 450 \text{ N}\cdot\text{m} \quad \sqrt{M^2 + T^2} = 728.67 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C}{J}(\sqrt{M^2 + T^2})_{max}$$

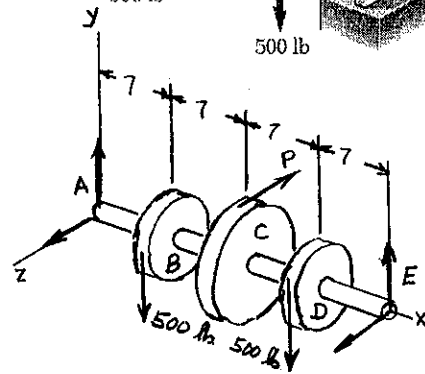
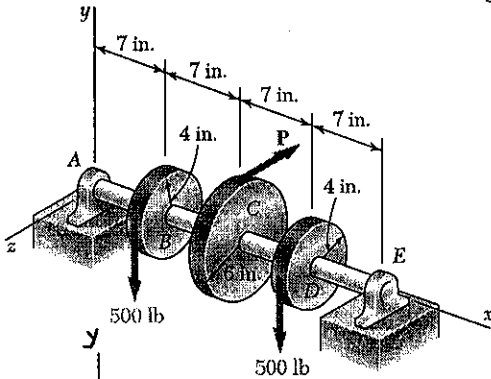
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{(\sqrt{M^2 + T^2})_{max}}{\tau_{all}} = \frac{728.67}{60 \times 10^6} = 12.145 \times 10^{-6} \text{ m}^3$$

$$C = 19.77 \times 10^{-3} \text{ m}$$

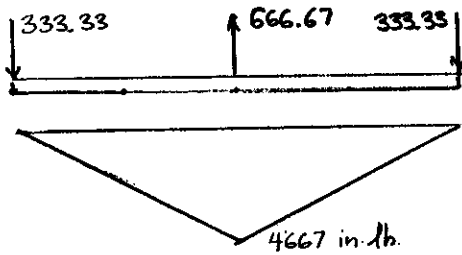
$$d = 2C = 39.5 \times 10^{-3} \text{ m} = 39.5 \text{ mm}$$

**PROBLEM 8.76**

8.76 The two 500-lb forces are vertical and the force  $P$  is parallel to the  $z$  axis. Knowing that  $\tau_{all} = 8$  ksi, determine the smallest permissible diameter of the solid shaft  $AE$ .



Forces in horizontal plane



**SOLUTION**

$$\sum M_x = 0 \quad (4)(500) - 6P + (4)(500) = 0$$

$$P = 666.67 \text{ lb}$$

Torques:

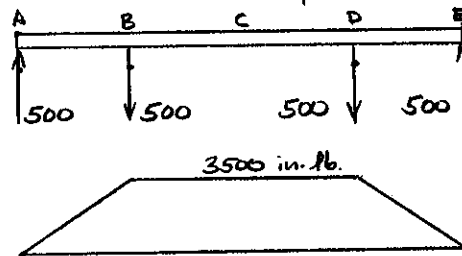
AB:  $T = 0$

BC:  $T = -(4)(500) = -2000 \text{ in}\cdot\text{lb}$

CD:  $T = (4)(500) = 2000 \text{ in}\cdot\text{lb}$

DE:  $T = 0$

Forces in vertical plane



Critical sections are either side of disk C

$$T = 2000 \text{ in}\cdot\text{lb} \quad M_2 = 3500 \text{ in}\cdot\text{lb}$$

$$M_y = 4667 \text{ in}\cdot\text{lb}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M_y^2 + M_2^2 + T^2}$$

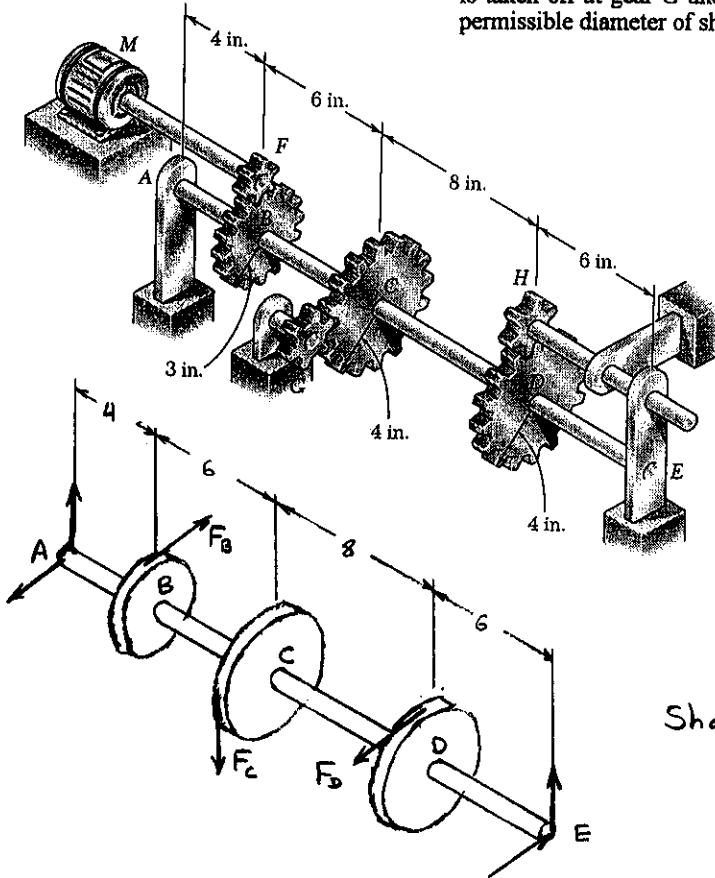
$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M_y^2 + M_2^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

$$C = 0.789 \text{ in.}$$

$$d = 2C = 1.578 \text{ in.}$$

**PROBLEM 8.77**

8.77 The solid shaft *AE* rotates at 600 rpm and transmits 60 hp from the motor *M* to machine tools connected to gears *G* and *H*. Knowing that  $\tau_{all} = 8$  ksi and that 40 hp is taken off at gear *G* and 20 hp is taken off at gear *H*, determine the smallest permissible diameter of shaft *AE*.



**SOLUTION**

$$60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ in}\cdot\text{lb}/\text{sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec}/\text{min}} = 10 \text{ Hz}$$

Torque on gear B

$$T_B = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(10)} = 6302.5 \text{ in}\cdot\text{lb}$$

Torques on gears C and D

$$T_C = \frac{40}{60} T_B = 4201.7 \text{ in}\cdot\text{lb}$$

$$T_D = \frac{20}{60} T_B = 2100.8 \text{ in}\cdot\text{lb}$$

Shaft torques

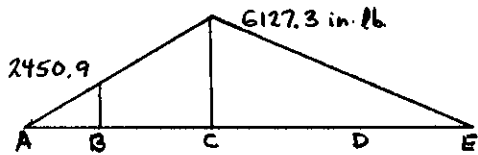
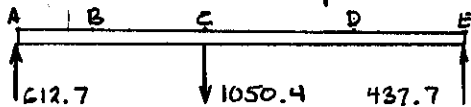
$$AB: T_{AB} = 0$$

$$BC: T_{BC} = 6302.5 \text{ in}\cdot\text{lb}$$

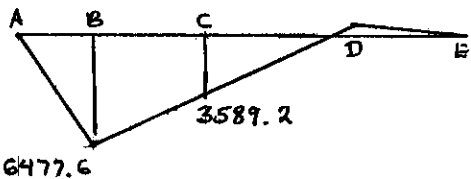
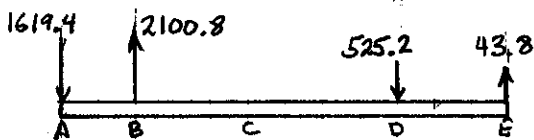
$$CD: T_{CD} = 2100.8 \text{ in}\cdot\text{lb}$$

$$DE: T_{DE} = 0$$

Forces in vertical plane



Forces in horizontal plane



Gear forces

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb}$$

$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb}$$

$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb}$$

$$\begin{aligned} \text{At } B^+ \quad & \sqrt{M_x^2 + M_y^2 + T^2} \\ & = \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2} \\ & = 9364 \text{ in}\cdot\text{lb} \end{aligned}$$

$$\begin{aligned} \text{At } C^- \quad & \sqrt{M_x^2 + M_y^2 + T^2} \\ & = \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2} \\ & = 9495 \text{ in}\cdot\text{lb} \text{ (maximum)} \end{aligned}$$

$$\tau_{all} = \frac{c}{J} (\sqrt{M_x^2 + M_y^2 + T^2})_{max}$$

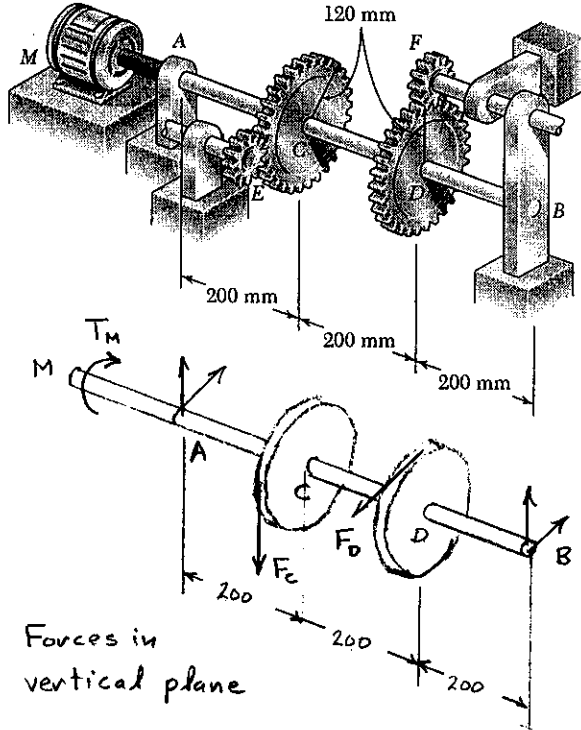
$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{(\sqrt{M_x^2 + M_y^2 + T^2})_{max}}{\tau_{all}} = \frac{9495}{8 \times 10^3} = 1.1868 \text{ in}^3$$

$$c = 0.911 \text{ in}$$

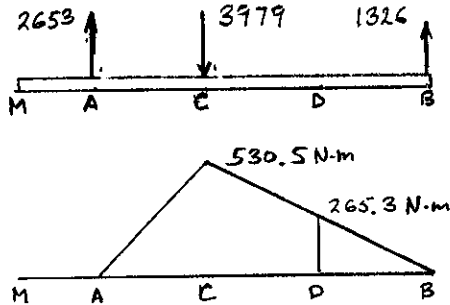
$$d = 2c = 1.822 \text{ in}$$

**PROBLEM 8.78**

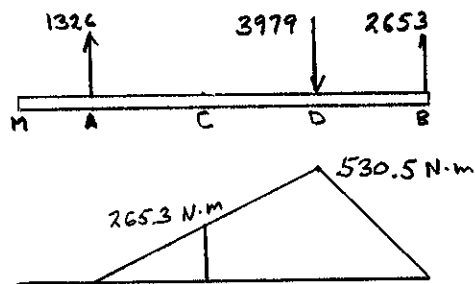
8.78 The motor  $M$  rotates at 300 rpm and transmits 30 kW to the solid shaft  $AB$  through a flexible connection. Half of this power is transferred to a machine tool connected to gear  $E$  and the other half to a machine tool connected to gear  $F$ . Knowing that  $\tau_{all} = 60$  MPa, determine the smallest permissible diameter of shaft  $AB$ .



Forces in vertical plane



Forces in horizontal plane



**SOLUTION**

$$300 \text{ rpm} = \frac{300}{60} = 5 \text{ Hz}$$

$$T_m = \frac{P}{2\pi f} = \frac{30 \times 10^3}{(2\pi)(5)} = 954.9 \text{ N}\cdot\text{m}$$

Torques on gears C and D

$$T_c = T_D = \frac{1}{2} T_m = 477.5 \text{ N}\cdot\text{m}$$

Shaft torques.

$$MA: T_{MA} = 954.9 \text{ N}\cdot\text{m}$$

$$AC: T_{AC} = 954.9 \text{ N}\cdot\text{m}$$

$$CD: T_{CD} = 477.5 \text{ N}\cdot\text{m}$$

$$DB: T_{DB} = 0$$

Gear forces

$$F_c = \frac{T_c}{r_c} = \frac{477.5}{120 \times 10^{-3}} = 3979 \text{ N}$$

$$F_D = \frac{T_D}{r_D} = \frac{477.5}{120 \times 10^{-3}} = 3979 \text{ N}$$

Critical point is just to the left of gear C

$$T_{AC} = 954.9 \text{ N}\cdot\text{m}$$

$$M_{Cz} = 530.5 \text{ N}\cdot\text{m}$$

$$M_{Cy} = 265.3 \text{ N}\cdot\text{m}$$

$$\sqrt{M_z^2 + M_y^2 + T^2} = 1124.1 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{C}{J} \sqrt{M_z^2 + M_y^2 + T^2}$$

$$\frac{J}{C} = \frac{\pi}{2} C^3 = \frac{\sqrt{M_z^2 + M_y^2 + T^2}}{\tau_{all}}$$

$$= \frac{1124.1}{60 \times 10^6} = 18.735 \times 10^{-6} \text{ m}^3$$

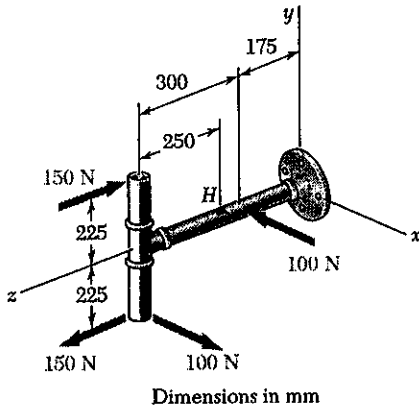
$$C = 22.85 \times 10^{-3} \text{ m}$$

$$d = 2C = 45.7 \times 10^{-3} \text{ m} = 45.7 \text{ mm}$$



**PROBLEM 8.79**

8.79 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters respectively equal to 36 mm and 42 mm, determine the normal and shearing stresses at point *H* located at the top of the outer surface of the pipe.



**SOLUTION**

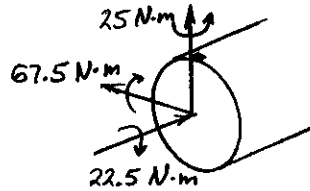
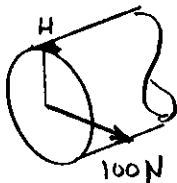
At the section containing point *H*

$$P = 0, \quad V_x = 100 \text{ N}, \quad V_y = 0$$

$$M_x = -(0.450)(150) = -67.5 \text{ N}\cdot\text{m}$$

$$M_y = (0.250)(100) = 25 \text{ N}\cdot\text{m}$$

$$M_z = -(0.225)(100) = -22.5 \text{ N}\cdot\text{m}$$



$$d_o = 42 \text{ mm} \quad d_i = 36 \text{ mm}$$

$$c_o = 21 \text{ mm} \quad c_i = 18 \text{ mm}$$

$$t = c_o - c_i = 3 \text{ mm}$$

$$A = \pi(c_o^2 - c_i^2) = 367.57 \text{ mm}^2 = 367.57 \times 10^{-6} \text{ m}^2$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30 \times 10^3 \text{ mm}^4 = 70.30 \times 10^{-9} \text{ m}^4, \quad J = 2I = 140.59 \times 10^{-9} \text{ m}^4$$

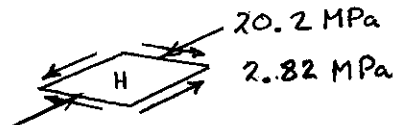
For half-pipe  $Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286 \times 10^3 \text{ mm}^3 = 2.286 \times 10^{-6} \text{ m}^3$

$$\sigma_H = \frac{M_x y}{I_x} = \frac{(-67.5)(21 \times 10^{-3})}{70.30 \times 10^{-9}} = -20.2 \text{ MPa}$$

Due to torque  $(\tau_H)_T = \frac{Tc}{J} = \frac{(22.5)(21 \times 10^{-3})}{140.59 \times 10^{-9}} = 3.36 \text{ MPa}$

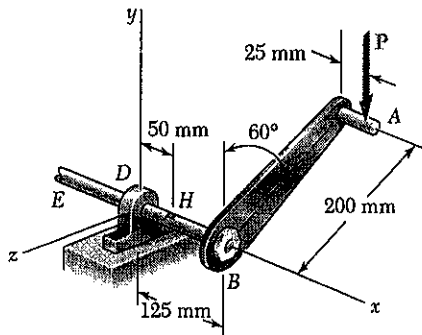
Due to shear  $(\tau_H)_V = \frac{VQ}{It} = \frac{(100)(2.286 \times 10^{-6})}{(70.30 \times 10^{-9})(6 \times 10^{-3})} = 0.54 \text{ MPa}$

Net  $\tau_H = 3.36 - 0.54 = 2.82 \text{ MPa}$



**PROBLEM 8.80**

8.80 A vertical force  $P$  of magnitude 250 N is applied to the crank at point  $A$ . Knowing that the shaft  $BDE$  has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point  $H$  located at the top of the shaft, 50 mm to the right of support  $D$ .



**SOLUTION**

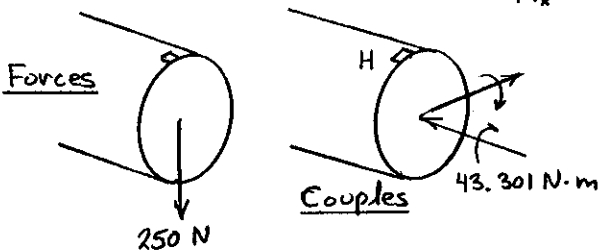
Force-couple system at the centroid of the section containing point  $H$ .

$$F_x = 0, \quad V_y = -250 \text{ N}, \quad V_z = 0$$

$$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25 \text{ N}\cdot\text{m}$$

[www.konkur.in](http://www.konkur.in)

$$M_x = -(200 \sin 60^\circ)(10^{-3})(250) = -43.301 \text{ N}\cdot\text{m}$$



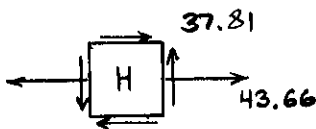
$$d = 18 \text{ mm} \quad c = \frac{1}{2}d = 9 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$J = 2I = 10.306 \times 10^{-9} \text{ m}^4$$

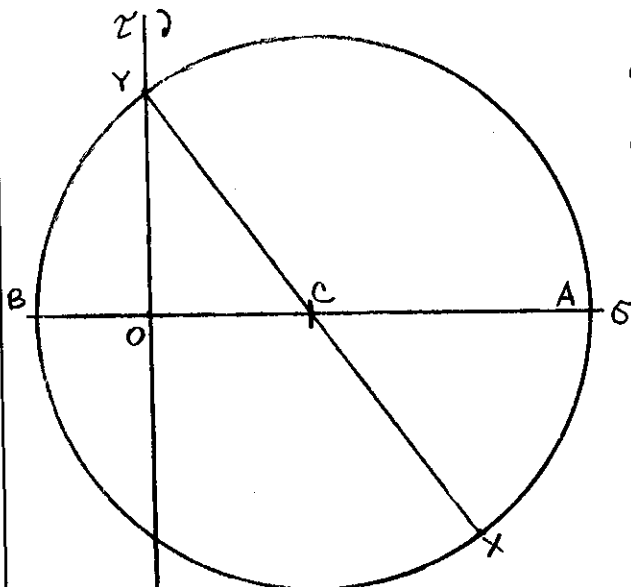
At point  $H$  
$$\sigma_H = -\frac{M_z y}{I_x} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-9}} = 37.81 \text{ MPa}$$



$$\sigma_c = \frac{1}{2}\sigma_H = 21.83 \text{ MPa}$$

$$R = \sqrt{\left(\frac{43.66}{2}\right)^2 + (37.81)^2} = 43.66 \text{ MPa}$$



$$\sigma_a = \sigma_c + R = 65.5 \text{ MPa} \quad \blacktriangleright$$

$$\sigma_b = \sigma_c - R = -21.8 \text{ MPa} \quad \blacktriangleright$$

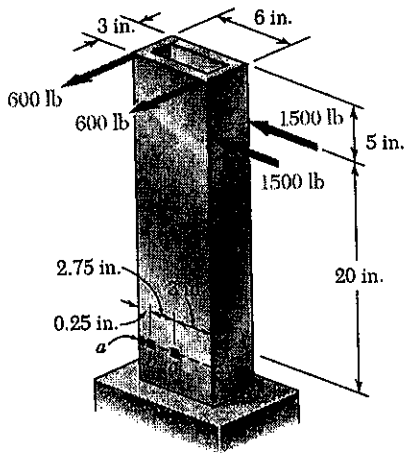
$$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$$

$$\theta_a = 30^\circ, \quad \theta_b = 120^\circ$$

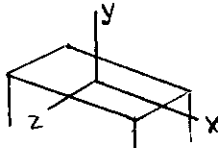
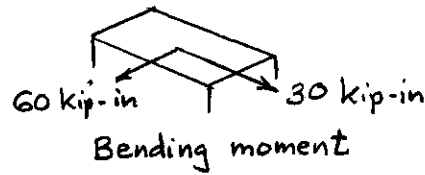
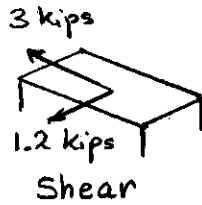
$$\tau_{\max} = R = 43.7 \text{ MPa} \quad \blacktriangleright$$

**PROBLEM 8.81**

8.81 Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.



**SOLUTION**



$$b_o = 6 \text{ in.} \quad b_i = b_o - 2t = 5.5 \text{ in.}$$

$$h_o = 3 \text{ in.} \quad h_i = h_o - 2t = 2.5 \text{ in.}$$

$$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.3385 \text{ in}^4$$

$$I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385 \text{ in}^4$$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

(a)  $\frac{(60)(-3)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -16.41 \text{ ksi}$   $\blacktriangleleft$

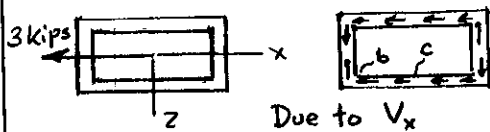
(b)  $\frac{(60)(-2.75)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -15.63 \text{ ksi}$   $\blacktriangleleft$

(c)  $\frac{(60)(0)}{19.3385} - \frac{(30)(1.5)}{6.3385} = -7.10 \text{ ksi}$   $\blacktriangleleft$

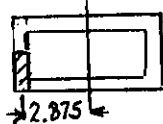
Shearing stresses

(a) Point a is an outside corner;  $\tau_a = 0$   $\blacktriangleleft$

Direction of shearing stresses



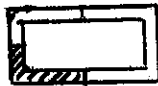
At point b



$$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781 \text{ in}^3$$

$$\tau_{b, V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(1.0781)}{(19.3385)(0.25)} = 0.669 \text{ ksi}$$

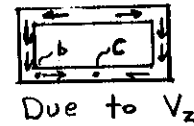
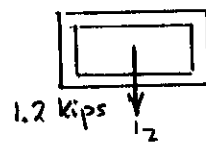
At point c



$$Q_{zc} = Q_{zb} + (2.75)(0.25)\left(\frac{2.75}{2}\right)$$

$$= 2.0234 \text{ in}^3$$

$$\tau_{c, V_x} = \frac{V_x Q_z}{I_z t} = \frac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256 \text{ ksi}$$



At point b



$$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453 \text{ in}^3$$

$$\tau_{b, V_z} = \frac{V_z Q_x}{I_x t} = \frac{(1.2)(0.9453)}{(6.3385)(0.25)} = 0.716 \text{ ksi}$$

At point c

(symmetry axis)  
 $\tau_{c, V_z} = 0$

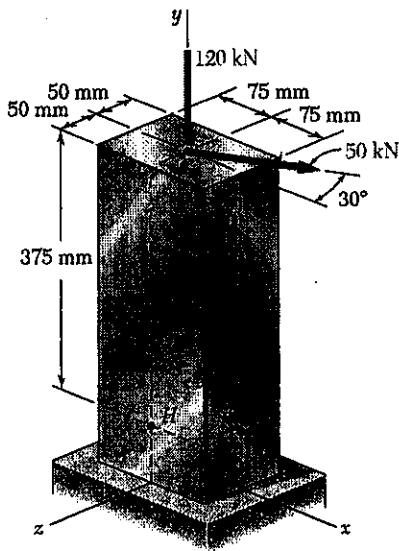
Net shearing stress at points b and c

$$\tau_b = 0.716 - 0.669 = 0.047 \text{ ksi} \quad \blacktriangleleft$$

$$\tau_c = 1.256 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 8.82**

**8.82** For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point H.



**SOLUTION**

Components of force at point C.

$$F_x = 50 \cos 30^\circ = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30^\circ = -25 \text{ kN}, \quad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K.

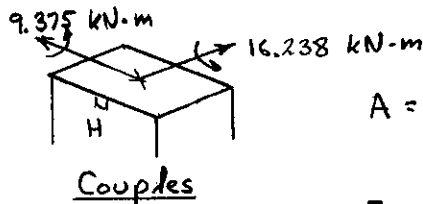
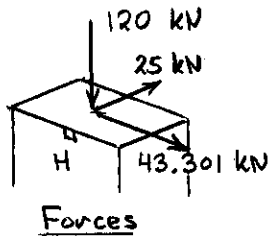
$$P = 120 \text{ kN (compression)}$$

$$V_x = 43.301 \text{ kN}, \quad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN}\cdot\text{m}$$

$$M_y = 0$$

$$M_z = -(43.301)(0.375) = -16.238 \text{ kN}\cdot\text{m}$$



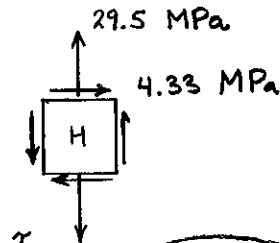
$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12} (150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$$

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120 \times 10^3)}{15 \times 10^{-3}} - \frac{(-9.375 \times 10^3)(50 \times 10^{-3})}{12.5 \times 10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2} \frac{V_x}{A} = \frac{3}{2} \frac{43.301 \times 10^3}{15 \times 10^{-3}} = 4.33 \text{ MPa}$$



$$\sigma_c = \frac{1}{2} \sigma_H = 14.75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37 \text{ MPa}$$

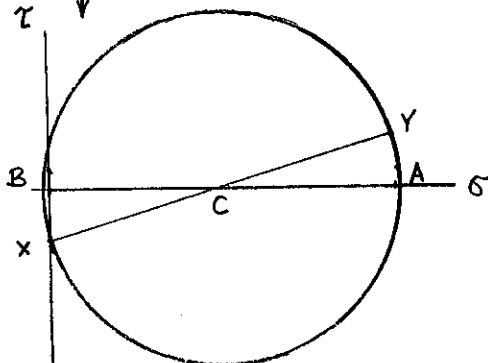
$$\sigma_a = \sigma_c + R = 30.1 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -0.62 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$$

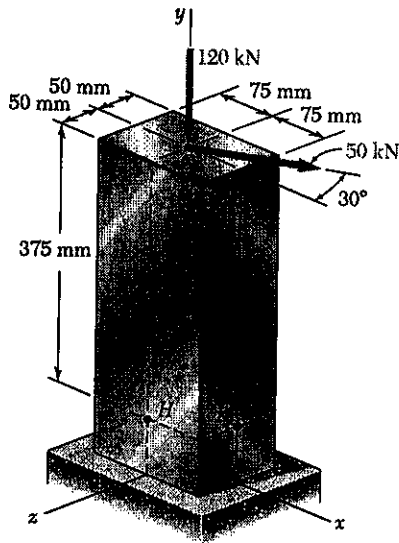
$$\theta_a = -8.2^\circ \quad \theta_b = 81.8^\circ$$

$$\tau_{\max} = R = 15.37 \text{ MPa}$$



**PROBLEM 8.83**

**8.83** For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point K.



**SOLUTION**

Components of force at point C

$$F_x = 50 \cos 30^\circ = 43.301 \text{ kN}$$

$$F_z = -50 \sin 30^\circ = -25 \text{ kN} \quad F_y = -120 \text{ kN}$$

Section forces and couples at the section containing points H and K.

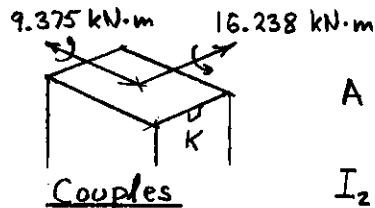
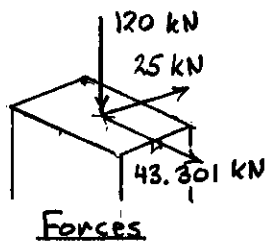
$$P = 120 \text{ kN (compression)}$$

$$V_x = 43.301 \text{ kN}, \quad V_z = -25 \text{ kN}$$

$$M_x = -(25)(0.375) = -9.375 \text{ kN}\cdot\text{m}$$

$$M_y = 0$$

$$M_z = -(43.301)(0.375) = -16.238 \text{ kN}\cdot\text{m}$$



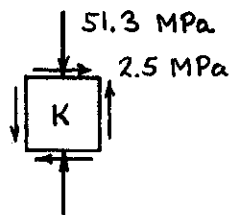
$$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$$

$$I_z = \frac{1}{12} (100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4 = 28.125 \times 10^{-6} \text{ m}^4$$

Stresses at point K

$$\sigma_K = -\frac{P}{A} + \frac{M_z x}{I_z} = -\frac{120 \times 10^3}{15 \times 10^{-3}} + \frac{(-16.238 \times 10^3)(75 \times 10^{-3})}{28.125 \times 10^{-6}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2} \frac{V_z}{A} = \frac{3}{2} \frac{25 \times 10^3}{15 \times 10^{-3}} = 2.5 \text{ MPa}$$



$$\sigma_c = \frac{1}{2} \sigma_K = -25.65 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_K}{2}\right)^2 + (\tau_K)^2} = 25.77 \text{ MPa}$$

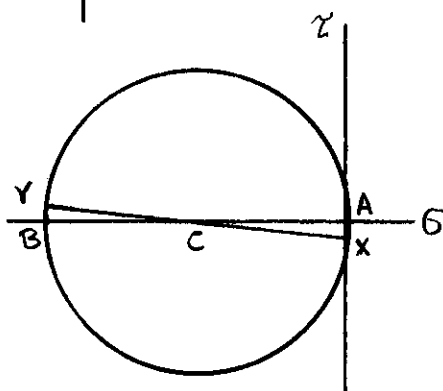
$$\sigma_a = \sigma_c + R = 0.12 \text{ MPa}$$

$$\sigma_b = \sigma_c - R = -51.4 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$$

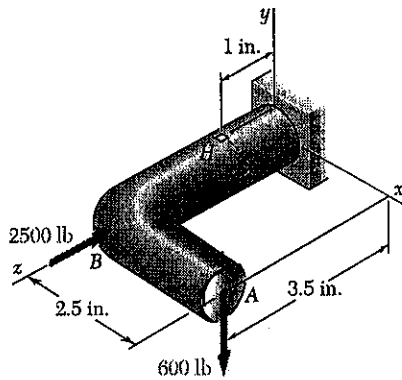
$$\theta_a = 2.8^\circ \quad \theta_b = 92.8^\circ$$

$$\tau_{\max} = R = 25.8 \text{ MPa}$$



PROBLEM 8.84

8.84 Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress (a) at point H, (b) at point K.



SOLUTION

At the section containing points H and K

$$P = 2500 \text{ lb (compression)}$$

$$V_y = -600 \text{ lb} \quad V_x = 0$$

$$M_x = (3.5 - 1)(600) = 1500 \text{ lb}\cdot\text{in}$$

$$M_y = 0 \quad M_z = -(2.5)(600) = -1500 \text{ lb}\cdot\text{in}$$

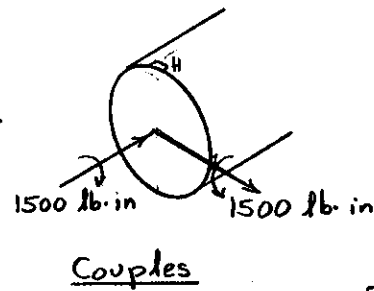
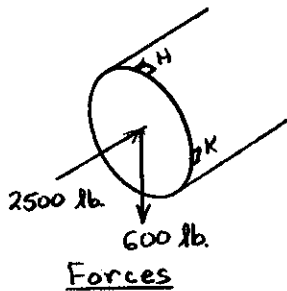
$$c = \frac{1}{2}d = 0.4 \text{ in}$$

$$A = \pi c^2 = 0.50265 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 20.106 \times 10^{-3} \text{ in}^4$$

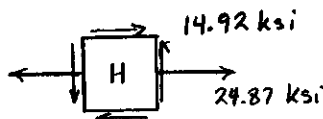
$$J = 2I = 40.212 \times 10^{-3} \text{ in}^4$$

For semi-circle  $Q = \frac{2}{3}c^3 = 42.667 \times 10^{-3} \text{ in}^3$



(a) At point H:  $\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106 \times 10^{-3}} = 24.87 \times 10^3 \text{ psi}$

$$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} = 14.92 \times 10^3 \text{ psi}$$



$$\sigma_{ave} = \frac{24.87}{2} = 12.435 \text{ ksi}$$

$$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423 \text{ ksi}$$

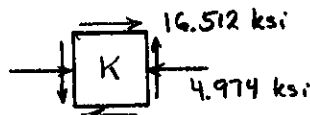
$$\sigma_{max} = \sigma_{ave} + R = 31.9 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -6.99 \text{ ksi} \quad \blacktriangleright$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 19.42 \text{ ksi} \quad \blacktriangleleft$$

(b) At point K:  $\sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974 \times 10^3 \text{ psi}$

$$\tau_K = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} + \frac{(600)(42.667 \times 10^{-3})}{(20.106 \times 10^{-3})(0.8)} = 16.512 \times 10^3 \text{ psi}$$



$$\sigma_{ave} = -\frac{4.974}{2} = -2.487 \text{ ksi}$$

$$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698 \text{ ksi}$$

$$\sigma_{max} = \sigma_{ave} + R = 14.21 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -19.18 \text{ ksi} \quad \blacktriangleright$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 16.70 \text{ ksi} \quad \blacktriangleleft$$

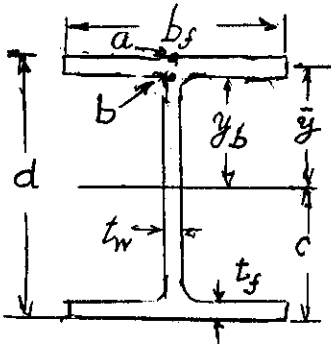
**PROBLEM 8.C1**

**8.C1** Let us assume that the shear  $V$  and the bending moment  $M$  have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (a) the maximum normal stress  $\sigma_m$ , (b) the principal stress  $\sigma_{max}$  at the junction of a flange and the web. Use this program to solve parts a and b of the following problems:

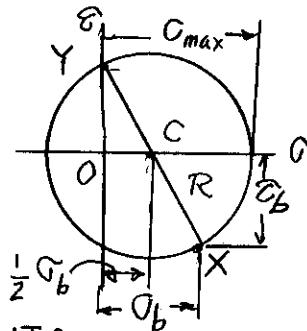
- (1) Prob. 8.1 (Use  $V = 400$  kN and  $M = 100$  kN · m)
- (2) Prob. 8.2 (Use  $V = 200$  kN and  $M = 100$  kN · m)
- (3) Prob. 8.3 (Use  $V = 320$  kips and  $M = 32 \times 10^3$  kip · in.)
- (4) Prob. 8.74.

**SOLUTION**

We enter the given values of  $V$  and  $M$  obtain from Appendix C the values of  $d$ ,  $b_f$ ,  $t_f$ ,  $t_w$ ,  $I$ , and  $S$  for the given WF shape.



We compute  $c = d/2$ ,  $y_b = c - t_f$   
 $\bar{y} = c - \frac{1}{2}t_f$ ,  $\sigma_a = M/S$ ,  $\sigma_b = \sigma_a (y_b/c)$   
 $Q = b_f t_f \bar{y}$ ,  $\tau_b = \frac{VQ}{It_w}$



From Mohr's circle:

$$\sigma_{max} = \frac{1}{2} \sigma_b + R$$

$$\sigma_{max} = \frac{1}{2} \sigma_b + \sqrt{\left(\frac{1}{2} \sigma_b\right)^2 + \tau_b^2}$$

**PROGRAM OUTPUTS**

Prob. 8.1

Given Data:  
 $V = 400$  kN,  $M = 100$  kN.m  
 $d = 252$  mm,  $b_f = 203$  mm  
 $t_f = 13.5$  mm,  $t_w = 8.6$  mm  
 $I = 87.30$  ( $10^6$  mm<sup>4</sup>)  
 $S = 693.0$  ( $10^3$  mm<sup>3</sup>)

Answers:  
 (a) SIGA = 144.3 MPa   
 (b) SIGM = 250.1 MPa

Prob. 8.2

Given Data:  
 $V = 200$  kN,  $M = 100$  kN.m  
 $d = 252$  mm,  $b_f = 203$  mm  
 $t_f = 13.5$  mm,  $t_w = 8.6$  mm  
 $I = 87.30$  ( $10^6$  mm<sup>4</sup>)  
 $S = 693.0$  ( $10^3$  mm<sup>3</sup>)

Answers:  
 (a) SIGA = 144.3 MPa   
 (b) SIGM = 172.7 MPa

Prob. 8.3

Given Data:  
 $V = 320$  kips,  $M = 32000$  kip.in.  
 $d = 36.74$  in.,  $b_f = 16.655$  in.  
 $t_f = 1.680$  in.,  $t_w = 0.945$  in.  
 $I = 20300$  in<sup>4</sup>,  $S = 1110$  in<sup>3</sup>

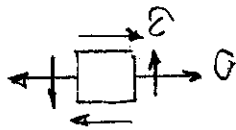
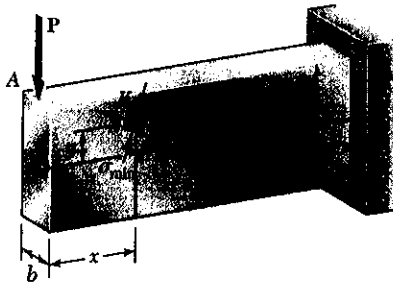
Answers:  
 (a) SIGA = 28.8 ksi   
 (b) SIGM = 28.5 ksi

Prob. 8.74

Given Data:  
 $V = 120$  kips,  $M = 3600$  kip.in.  
 $d = 21.36$  in.,  $b_f = 12.290$  in.  
 $t_f = 0.800$  in.,  $t_w = 0.500$  in.  
 $I = 2420$  in<sup>4</sup>,  $S = 227$  in<sup>3</sup>

Answers:  
 (a) SIGA = 15.86 ksi   
 (b) SIGM = 19.76 ksi

**PROBLEM 8.C2**



**8.C2** A cantilever beam  $AB$  with a rectangular cross section of width  $b$  and depth  $2c$  supports a single concentrated load  $P$  at its end  $A$ . Write a computer program to calculate, for any values of  $x/c$  and  $y/c$ , (a) the ratios  $\sigma_{\max}/\sigma_m$  and  $\sigma_{\min}/\sigma_m$ , where  $\sigma_{\max}$  and  $\sigma_{\min}$  are the principal stresses at point  $K(x, y)$  and  $\sigma_m$  the maximum normal stress in the same transverse section, (b) the angle  $\theta_p$  that the principal planes at  $K$  form with a transverse and a horizontal plane through  $K$ . Use this program to check the values shown in Fig. 8.8 and to verify that  $\sigma_{\max}$  exceeds  $\sigma_m$  if  $x \leq 0.544c$ , as indicated in the second footnote on page 499.

**SOLUTION**

Since the distribution of the normal stresses is linear, we have  $\sigma = \sigma_m (y/c)$  (1)

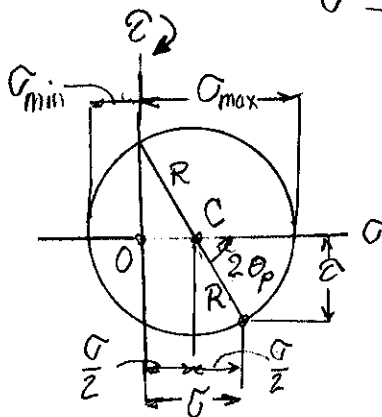
where  $\sigma_m = \frac{Mc}{I} = \frac{Px c}{I}$  (2)

We use Eq. (8.4), page 498:  $\tau = \frac{3}{2} \frac{P}{A} \left(1 - \frac{y^2}{c^2}\right)$  (3)

Dividing (3) by (2):  $\frac{\tau}{\sigma_m} = \frac{3}{2} \frac{I}{A} \frac{1 - (y/c)^2}{xc}$

or, since  $\frac{I}{A} = \frac{\frac{1}{12} b (2c)^3}{b (2c)} = \frac{1}{3} c^2$ ;  $\frac{\tau}{\sigma_m} = \frac{1}{2} \frac{1 - (y/c)^2}{x/c}$  (4)

Letting  $X = x/c$  and  $Y = y/c$ , Eqs. (1) and (4) yield  $\sigma = \sigma_m Y$   $\tau = \sigma_m \frac{1 - Y^2}{2X}$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\frac{1}{2}\sigma\right)^2 + \tau^2}$$

$$= \frac{1}{2} \sigma_m \sqrt{Y^2 + \left(\frac{1 - Y^2}{X}\right)^2}$$

$$\frac{\sigma_{\max}}{\sigma_m} = \frac{1}{2} Y + R \quad \frac{\sigma_{\min}}{\sigma_m} = \frac{1}{2} Y - R$$

$$\tan 2\theta_p = \frac{\tau}{\sigma/2} = \frac{1 - Y^2}{2X(Y/2)} = \frac{1 - Y^2}{XY} \quad \theta_p = \frac{1}{2} \tan^{-1}\left(\frac{1 - Y^2}{XY}\right)$$

**NOTE**

For  $y > 0$ , the angle  $\theta_p$  is  $\curvearrowright$ , which is opposite to what was arbitrarily assumed in Fig. P8.C2.

(CONTINUED)



PROBLEM 8.C2 CONTINUED

PROGRAM OUTPUTS

For  $x/c = 2$  :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta $^{\circ}$
1.0	0.000	1.000	0.00
0.8	-0.010	0.810	6.34
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.160	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	-33.69
-0.4	-0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	-0.810	0.010	-6.34
-1.0	-1.000	0.000	-0.00

For  $x/c = 8$  :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta $^{\circ}$
1.0	0.000	1.000	0.00
0.8	-0.001	0.801	1.61
0.6	-0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.48
-0.4	-0.407	0.007	-7.35
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that  $\sigma_{max} > \sigma_m$  if  $x \leq 0.544c$ , we run the program for  $x/c = 0.544$  and for  $x/c = 0.545$  and observe that  $\sigma_{max}/\sigma_m$  exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For  $x/c = 0.544$  :

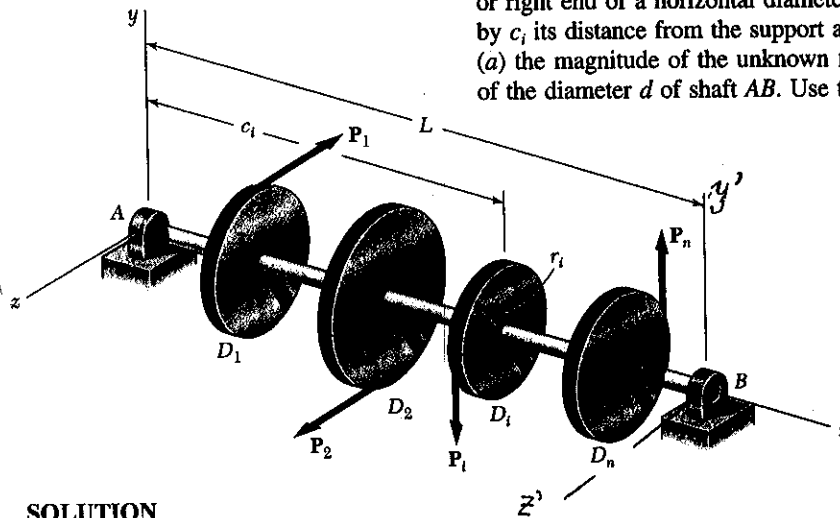
y/c	Sigmin/Sigm	Sigmax/Sigm	Theta $^{\circ}$
0.30	-0.700	0.9997	39.92
0.31	-0.690	1.0001	39.72
0.32	-0.680	1.0004	39.51
0.33	-0.670	1.0005	39.30
0.34	-0.660	1.0005	39.09
0.35	-0.650	1.0003	38.88
0.36	-0.640	1.0000	38.66
0.37	-0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.608	0.9983	37.98
0.40	-0.598	0.9975	37.74

For  $x/c = 0.545$  :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta $^{\circ}$
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9988	38.87
0.36	-0.639	0.9986	38.65
0.37	-0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

**PROBLEM 8.C3**

**8.C3** Disks  $D_1, D_2, \dots, D_n$  are attached as shown in Fig. P8.C3 to the solid shaft  $AB$  of length  $L$ , uniform diameter  $d$ , and allowable shearing stress  $\tau_{all}$ . Forces  $P_1, P_2, \dots, P_n$  of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of a vertical diameter, or at the left or right end of a horizontal diameter. Denoting by  $r_i$  the radius of disk  $D_i$  and by  $c_i$  its distance from the support at  $A$ , write a computer program to calculate (a) the magnitude of the unknown force  $P_i$ , (b) the smallest permissible value of the diameter  $d$  of shaft  $AB$ . Use this program to solve Probs. 8.75 and 8.76.



**SOLUTION**

1. Determine the unknown force  $P_i$  by equating to zero the sum of their torques  $T_i$  about the  $x$  axis.

2. Determine the components  $(F_y)_i$  and  $(F_z)_i$  of all forces.

3. Determine the components  $A_y$  and  $A_z$  of reaction at  $A$  by summing moments about axes  $Bz' // z$  and  $By' // y$ :

$$\sum M_{z'} = 0: -A_y L - \sum (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L} \sum (F_y)_i (L - c_i)$$

$$\sum M_{y'} = 0: A_z L + \sum (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L} \sum (F_z)_i (L - c_i)$$

4. Determine  $(M_y)_i, (M_z)_i$ , and torque  $T_i$  just to the left of disk  $D_i$ :

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle'$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle'$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$


Where  $\langle \rangle$  indicates a singularity function.

5. The minimum diameter  $d$  required to the left of  $D_i$  is obtained by first computing  $(J/c)_i$  from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

(CONTINUED)

PROBLEM 8.C3 CONTINUED


6 Recalling that  $J = \frac{1}{2} \pi c^4$  and, thus, that  $\left(\frac{J}{c}\right)_i = \frac{1}{2} \pi c_i^3$ ,  
 we have  $c_i = \frac{2}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$  and  $d_i = \frac{4}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$  


This is the required diameter just to the left of disk  $D_i$

7. The required diameter just to the right of disk  $D_i$  is obtained by replacing  $T_i$  with  $T_{i+1}$  in the above computation.

8. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for  $d_i$

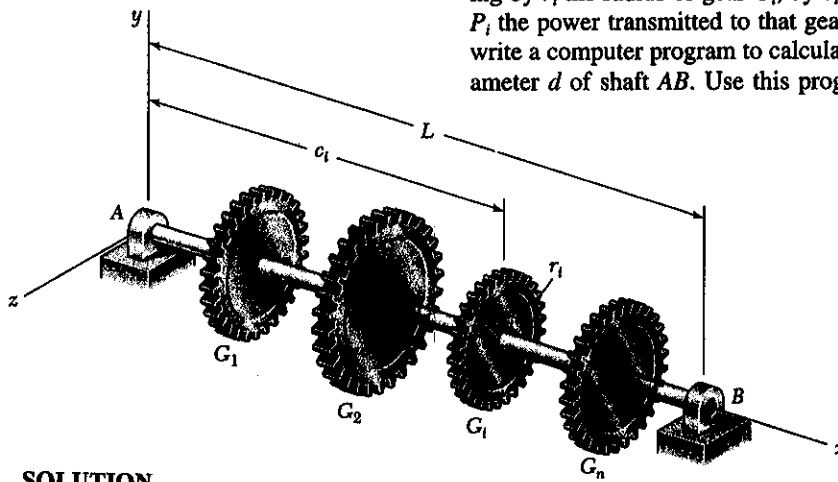
PROGRAM OUTPUTS

Prob. 8.75  
 Length of shaft = 300 mm  
 TAU = 60 MPa  
 For Disk 1  
 Force = 6.000 kN  
 Radius of disk = 75 mm  
 Distance from A in mm = 80  
 For Disk 2  
 Force = 0.000 kN  
 Radius of disk = 60 mm  
 Distance from A in mm = 180  
 Unknown force = -7.500 kN  
 AY = 4.400 kN, AZ = -3.000 kN  
 BY = 1.600 kN, BZ = -4.500 kN  
 Just to the left of Disk 1  
 MY = -240.00 Nm  
 MZ = -352.00 Nm  
 T = 0.00 Nm  
 Diameter must be at least 33.07 mm  
 Just to the right of Disk 1  
 T = 450.00 Nm  
 Diameter must be at least 37.47 mm  
 Just to the left of Disk 2  
 MY = -540.00 Nm  
 MZ = -192.00 Nm  
 T = 450.00 Nm  
 Diameter must be at least 39.55 mm   
 Just to the right of Disk 2  
 T = 0.00 Nm  
 Diameter must be at least 36.51 mm

Prob. 8.76  
 Length of shaft = 28 in.  
 TAU (ksi) = 8  
 For Disk 1  
 Force = 0.500 kips  
 Radius of disk = 4.0 in.  
 Distance from A = 7.0 in.  
 For Disk 2  
 Force = 0.000 kips  
 Radius of disk = 6.0 in.  
 Distance from A = 14.0 in.  
 For Disk 3  
 Force = 0.500 kips  
 Radius of disk = 4.0 in.  
 Distance from A = 21.0 in.  
 Unknown force = -0.667 kips  
 AY = 0.500 kips, AZ = 0.333 kips  
 BY = 0.500 kips, BZ = 0.333 kips  
 Just to the left of Disk 1  
 MY = 2.3333 kip.in.  
 MZ = -3.5000 kip.in.  
 T = 0.0000 kip.in.  
 Diameter must be at least 1.389 in.  
 Just to the right of Disk 1  
 T = 2.00 kip.in.  
 Diameter must be at least 1.437 in.  
 Just to the left of Disk 2  
 MY = 4.6667 kip.in.  
 MZ = -3.5000 kip.in.  
 T = 2.0000 kip.in.  
 Diameter must be at least 1.578 in.   
 Just to the right of Disk 2  
 T = -2.00 kip.in.  
 Diameter must be at least 1.578 in.  
 Just to the left of Disk 3  
 MY = 2.3333 kip.in.  
 MZ = -3.5000 kip.in.  
 T = -2.0000 kip.in.  
 Diameter must be at least 1.437 in.  
 Just to the right of Disk 3  
 T = 0.00 kip.in.  
 Diameter must be at least 1.389 in.

**PROBLEM 8.C4**

**8.C4** The solid shaft  $AB$  of length  $L$ , uniform diameter  $d$ , and allowable shearing stress  $\tau_{all}$  rotates at a given speed expressed in rpm (Fig. P8.C4). Gears  $G_1, G_2, \dots, G_n$  are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these other gears is connected to a motor and the rest of them to various machine tools. Denoting by  $r_i$  the radius of gear  $G_i$ , by  $c_i$  its distance from the support at  $A$ , and by  $P_i$  the power transmitted to that gear (+ sign) or taken off that gear (- sign), write a computer program to calculate the smallest permissible value of the diameter  $d$  of shaft  $AB$ . Use this program to solve Probs. 8.25, 8.29, and 8.77.



**SOLUTION**

1. Enter  $\omega$  in rpm and determine frequency  $f = \omega/60$ .
2. For each gear, determine the torque  $T_i = P_i / 2\pi f$ , where  $P_i$  is the power input (+) or output (-) at the gear.
3. For each gear, determine the force  $F_i = T_i / r_i$  exerted on the gear and its components  $(F_y)_i$  and  $(F_z)_i$ .
4. Determine the components  $A_y$  and  $A_z$  of reaction at  $A$  by summing moments about axes  $Bz' \parallel z$  and  $By' \parallel y$ :  
 $\sum M_{z'} = 0: -A_y L - \sum (F_y)_i (L - c_i) = 0, A_y = -\frac{1}{L} \sum (F_y)_i (L - c_i)$   
 $\sum M_{y'} = 0: A_z L + \sum (F_z)_i (L - c_i) = 0, A_z = -\frac{1}{L} \sum (F_z)_i (L - c_i)$
5. Determine  $(M_y)_i, (M_z)_i$ , and torque  $T_i$  just to the left of gear  $G_i$ :

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle'$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle'$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where  $\langle \rangle$  indicates a singularity function.

(CONTINUED)

PROBLEM 8.C4 CONTINUED

6. The minimum diameter  $d$  required to the left of  $G_i$  is obtained by first computing  $(J/c)_i$  from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{\text{all}}}$$

7. Recalling that  $J = \frac{1}{2} \pi c^4$  and, thus, that  $\left(\frac{J}{c}\right)_i = \frac{1}{2} \pi c_i^3$

we have  $c_i = \frac{2}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$  and  $d = \frac{4}{\pi} \left(\frac{J}{c}\right)_i^{1/3}$  

This is the required diameter just to the left of gear  $G_i$ .

8. The required diameter just to the right of gear  $G_i$  is obtained by replacing  $T_i$  with  $T_{i+1}$  in the above computation.

9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for  $d_i$ .

PROGRAM OUTPUTS

```

Prob. 8.25
Omega = 600 rpm
Number of Gears: 2
Length of shaft = 400 mm
Tau = 60 MPa
For Gear 1
Power input = 80.00 kW
Radius of gear = 80 mm
Distance from A in mm = 120
For Gear 2
Power input = -80.00 kW
Radius of gear = 60 mm
Distance from A in mm = 280
AY = 11.141 kN, AZ = 6.366
BY = 4.775 kN, BZ = 14.854
Just to the left of Gear 1
MY = 763.94 Nm
MZ = -1336.90 Nm
T = 0.00 Nm
Diameter must be at least 50.75 mm
Just to the right of Gear 1
T = 1273.24 Nm
Diameter must be at least 55.35 mm
Just to the left of Gear 2
MY = 1782.54 Nm
MZ = -572.96 Nm
T = 1273.24 Nm
Diameter must be at least 57.71 mm
Just to the right of Gear 2
T = 0.00 Nm
Diameter must be at least 54.17 mm
    
```

(CONTINUED)

**PROBLEM 8.C4 CONTINUED**

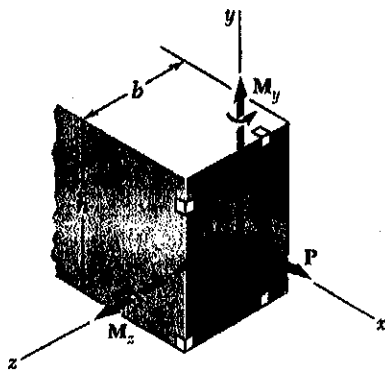
Prob. 8.29

Omega = 450 rpm  
 Number of Gears: 3  
 Length of shaft = 750 mm  
 Tau = 55 MPa  
 For Gear 1  
 Power input = -8.00 kW  
 Radius of gear = 60 mm  
 Distance from A in mm = 150  
 For Gear 2  
 Power input = 20.00 kW  
 Radius of gear = 100 mm  
 Distance from A in mm = 375  
 For Gear 3  
 Power input = -12.00 kW  
 Radius of gear = 60 mm  
 Distance from A in mm = 600  
 AY = -0.849 kN, AZ = 4.386  
 BY = -3.395 kN, BZ = 2.688  
 Just to the left of Gear 1  
 MY = 657.84 Nm  
 MZ = 127.32 Nm  
 T = 0.00 Nm  
 Diameter must be at least 39.59 mm  
 Just to the right of Gear 1  
 T = -169.77 Nm  
 Diameter must be at least 40.00 mm  
 Just to the left of Gear 2  
 MY = 1007.98 Nm  
 MZ = 318.31 Nm  
 T = -169.77 Nm  
 Diameter must be at least 46.28 mm  
 Just to the right of Gear 2  
 T = 254.65 Nm  
 Diameter must be at least 46.52 mm  
 Just to the left of Gear 3  
 MY = 403.19 Nm  
 MZ = 509.30 Nm  
 T = 254.65 Nm  
 Diameter must be at least 40.13 mm  
 Just to the right of Gear 3  
 T = 0.00 Nm  
 Diameter must be at least 39.18 mm

Prob. 8.77

Omega = 600 rpm  
 Number of Gears: 3  
 Length of shaft = 24 in.  
 Tau = 8 ksi  
 For Gear 1  
 Power input = 60.00 hp  
 Radius of gear = 3.00 in.  
 Distance from A in inches = 4.0  
 FY = 0  
 FZ = 2.100845  
 For Gear 2  
 Power input = -40.00 hp  
 Radius of gear = 4.00 in.  
 Distance from A in inches = 10.0  
 FY = 1.050423  
 FZ = 0  
 For Gear 3  
 Power input = -20.00 hp  
 Radius of gear = 4.00 in.  
 Distance from A in inches = 18.0  
 FY = 0  
 FZ = -0.5252113  
 AY = -0.6127 kips, AZ = -1.6194 kips  
 BY = -0.4377 kips, BZ = 0.0438 kips  
 Just to the left of Gear 1  
 MY = -6.478 kip.in.  
 MZ = 2.451 kip.in.  
 T = 0.000 kip.in.  
 Diameter must be at least 1.640 in.  
 Just to the right of Gear 1  
 T = 6.3025 kip.in.  
 Diameter must be at least 1.813 in.  
 Just to the left of Gear 2  
 MY = -3.589 kip.in.  
 MZ = 6.127 kip.in.  
 T = 6.303 kip.in.  
 Diameter must be at least 1.822 in.  
 Just to the right of Gear 2  
 T = 2.1008 kip.in.  
 Diameter must be at least 1.677 in.  
 Just to the left of Gear 3  
 MY = 0.263 kip.in.  
 MZ = 2.626 kip.in.  
 T = 2.101 kip.in.  
 Diameter must be at least 1.290 in.  
 Just to the right of Gear 3  
 T = 0.0000 kip.in.  
 Diameter must be at least 1.189 in.

**PROBLEM 8.C5**



**8.C5** Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates  $y$  and  $z$  located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (a) Prob. 8.50, (b) Prob. 8.53.

**SOLUTION**

ENTER:  $b$  and  $h$

PROGRAM:  $A = bh$   $I_y = b^3 h / 12$   $I_z = b h^3 / 12$

FOR POINT ON SURFACE, ENTER  $y$  AND  $z$

NOTE  $y$  AND  $z$  MUST SATISFY ONE OF FOLLOWING:

$$y^2 = h^2/4 \text{ AND } z^2 \leq b^2/4 \quad (1)$$

$$\text{OR } z^2 = b^2/4 \text{ AND } y^2 \leq h^2/4 \quad (2)$$

IF EITHER (1) OR (2) ARE SATISFIED, COMPUTE

$$\sigma = \frac{P}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

IF  $z^2 = b^2/4$ , THEN POINT IS ON VERTICAL SURFACE AND

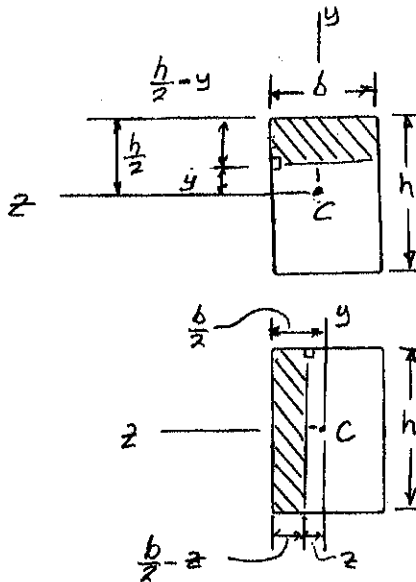
$$Q_z = b \left( \frac{h}{2} - y \right) \left( \frac{h}{2} + y \right) \frac{1}{2} = b \left( \frac{h^2}{8} - \frac{y^2}{2} \right)$$

$$\tau = \frac{V_y Q_z}{I_z b}$$

IF  $y^2 = h^2/4$ , THE POINT IS ON HORIZONTAL SURFACE, AND

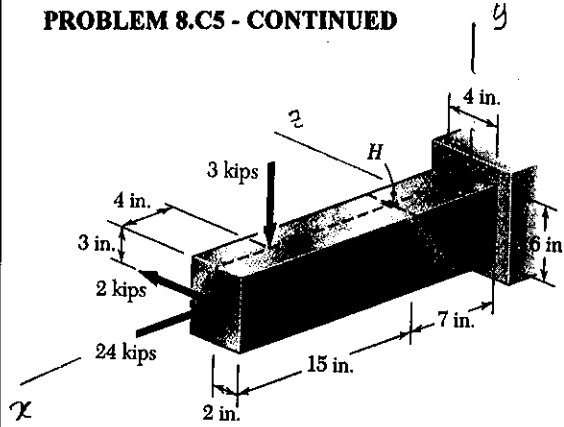
$$Q_y = h \left( \frac{b}{2} - z \right) \left( \frac{b}{2} + z \right) \frac{1}{2} = h \left( \frac{b^2}{8} - \frac{z^2}{2} \right)$$

$$\tau = \frac{V_z Q_y}{I_y h}$$

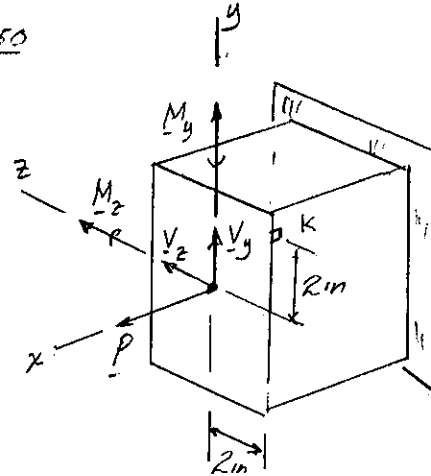


CONTINUED

**PROBLEM 8.C5 - CONTINUED**



PROBLEM 8.50  
POINT K



FORCE-COUPLE SYSTEM

$$P = 24 \text{ kips} \quad V_y = -3 \text{ kips} \quad V_z = 2 \text{ kips}$$

$$M_y = -(2 \text{ kips})(15 \text{ in.}) = -30 \text{ kip}\cdot\text{in.} \quad M_z = -(3 \text{ kips})(15 \text{ in.} - 4 \text{ in.}) = -33 \text{ kip}\cdot\text{in.}$$

POINT K  $y = 2 \text{ in.}$   $z = -2 \text{ in.}$

Problem 8.50

Force-Couple at Centroid

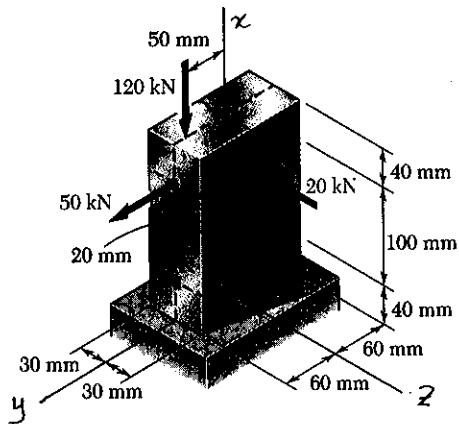
$P = -24.000 \text{ kips}$	$M_Z = -33.000 \text{ kip}\cdot\text{in.}$
$M_Y = -30.000 \text{ kip}\cdot\text{in.}$	$V_Z = 2.000 \text{ kips}$
$V_Y = 3.000 \text{ kips}$	

+++++

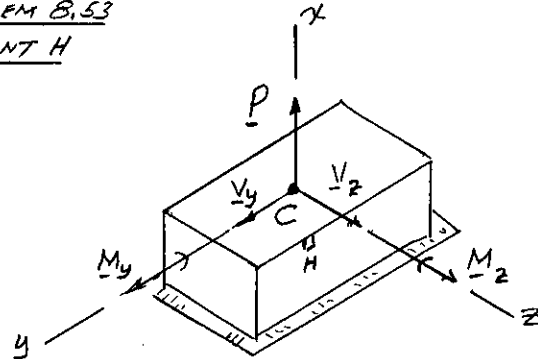
At point of coordinates:  $y = 2.000 \text{ in.}$   $z = -2.000 \text{ in.}$

$\sigma = 1.792 \text{ ksi}$

$\tau = 0.104 \text{ ksi}$



PROBLEM 8.53  
POINT H



FORCE-COUPLE SYSTEM

$$P = -120 \text{ kN} \quad V_y = 50 \text{ kN} \quad V_z = -20 \text{ kN}$$

$$M_y = (20 \text{ kN})(0.1 \text{ m}) = 2 \text{ kN}\cdot\text{m}$$

$$M_z = (120 \text{ kN})(0.05 \text{ m}) + (50 \text{ kN})(0.1 \text{ m}) = 11 \text{ kN}\cdot\text{m}$$

POINT H  $y = 20 \text{ mm}$   $z = 30 \text{ mm}$

Problem 8.53

Force-Couple at Centroid

$P = -120000.00 \text{ N}$	$M_Z = 11000.00 \text{ N}\cdot\text{m}$
$M_Y = 2000.00 \text{ N}\cdot\text{m}$	$V_Z = -20000.00 \text{ N}$
$V_Y = 50000.00 \text{ N}$	

+++++

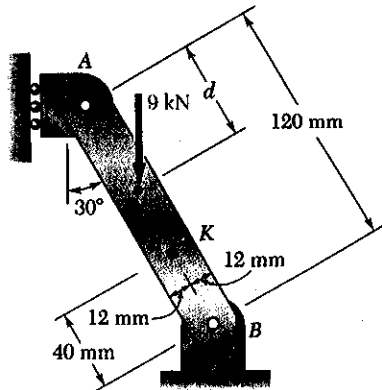
At point of coordinates:  $y = 20.00 \text{ mm}$   $z = 30.00 \text{ mm}$

$\sigma = -14.352 \text{ MPa}$

$\tau = 9.259 \text{ MPa}$

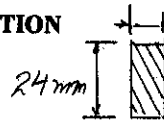


**PROBLEM 8.C6**



**8.C6** Member AB has a rectangular cross section of 10 × 24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at points H and K for values of d from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.35.

**SOLUTION**



CROSS SECTION

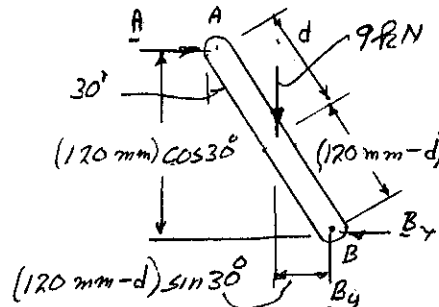
ENTER

$$A = (0.010 \text{ m})(0.024 \text{ m}) = 240 \times 10^{-6} \text{ m}^2$$

$$I = (0.010 \text{ m})(0.024 \text{ m})^3 / 12 = 138.24 \times 10^{-9} \text{ m}^4$$

$$c = 0.5(0.024 \text{ m}) = 12 \times 10^{-3} \text{ m}$$

COMPUTE REACTION AT A.

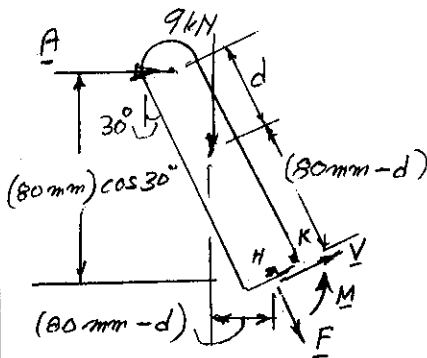


$$\sum M_B = 0:$$

$$(9 \text{ kN})(120 - d) \sin 30^\circ - A(120) \cos 30^\circ = 0$$

$$A = (9 \text{ kN}) \frac{(120 \text{ mm} - d)}{120 \text{ mm}} \tan 30^\circ$$

FREE BODY FROM A TO SECTION CONTAINING POINTS H AND K.



DEFINE: IF  $d < 80 \text{ mm}$  THEN  $STP = 1$  ELSE  $STP = 0$

PROGRAM FORCE-COUPLE SYSTEM

$$F = -A \sin 30^\circ - (9 \text{ kN}) \cos 30^\circ (STP)$$

$$V = -A \cos 30^\circ + (9 \text{ kN}) \sin 30^\circ (STP)$$

$$M = A(80 \text{ mm}) \cos 30^\circ - (9 \text{ kN})(80 \text{ mm} - d) \sin 30^\circ (STP)$$

AT POINT H:

$$\sigma_H = +F/A \quad \tau_H = \frac{3}{2} V/A$$

AT POINT K:

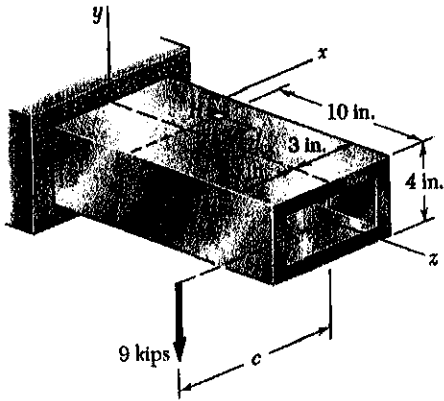
$$\sigma_K = +F/A - Mc/I \quad \tau_K = 0$$

PROGRAM OUTPUT

Problem 8.35

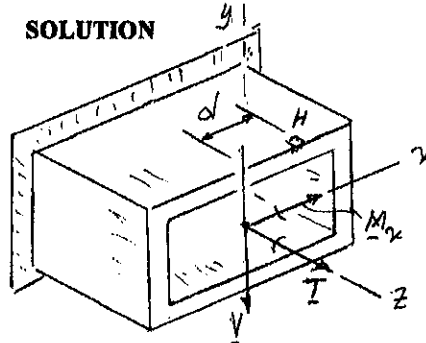
d mm	Stresses in MPa			
	SigmaH	TauH	SigmaK	TauK
0.0	-43.30	0.00	-43.30	0.00
15.0	-41.95	3.52	-65.39	0.00
30.0	-40.59	7.03	-87.47	0.00
45.0	-39.24	10.55	-109.55	0.00
60.0	-37.89	14.06	-131.64	0.00
75.0	-36.54	17.58	-153.72	0.00
90.0	-2.71	-7.03	-96.46	0.00
105.0	-1.35	-3.52	-48.23	0.00
120.0	0.00	0.00	0.00	0.00

**PROBLEM 8.C7**



**\*8.C7** The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied to a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of  $c$ , the principal stresses, principal planes, and maximum shearing stress at point  $H$  for values of  $d$  from  $-3$  in. to  $3$  in., using one-inch increments. Use this program to solve Prob. 8.72a.

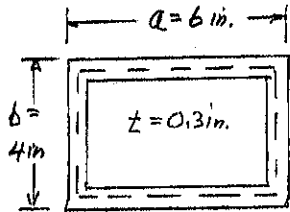
**SOLUTION**



FORCE-COUPLE SYSTEM

ENTER:

$V = 9 \text{ kips} \downarrow$   
 $M_x = (9 \text{ kips})(10 \text{ in.}) = 90 \text{ kip} \cdot \text{in.}$   
 $T = (9 \text{ kips})c \leftarrow$

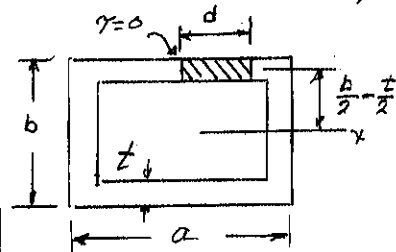
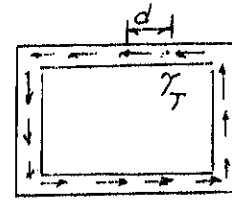


AREA ENCLOSED

$A = (a - t)(b - t)$

$\tau = \frac{T}{A} = \frac{9c}{2tA}$

$\tau_T =$  SHEARING STRESS DUE TO TORSION

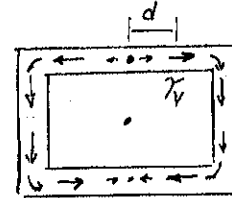


$Q = dt \left( \frac{b}{2} - \frac{t}{2} \right)$

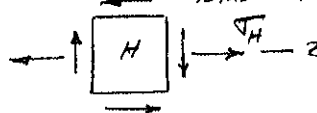
$I = \frac{ab^3}{12} - \frac{(a-2t)(b-2t)^3}{12}$

$\tau = \frac{VQ}{It}$

$\tau_V =$  SHEARING STRESS DUE TO V



$\tau_{TOTAL} = \tau_T + \tau_V$



BENDING:  $\sigma_H = \frac{M_x \left( \frac{b}{2} \right)}{I}$

PRINCIPAL STRESSES

$\sigma_{ave} = \frac{1}{2} \sigma_H$ ;  $R = \sqrt{\left( \frac{\sigma_H}{2} \right)^2 + \tau_{TOTAL}^2}$

$\sigma_{max} = \sigma_{ave} + R$ ;  $\sigma_{min} = \sigma_{ave} - R$ ;  $\theta_p = \frac{1}{2} \tan^{-1} \left( \frac{\tau_{TOTAL}}{\sigma_{ave}} \right)$ ;  $\tau_{max} = \sqrt{\left( \frac{\sigma_H}{2} \right)^2 + \tau_{TOTAL}^2}$

Rectangular tube of uniform thickness  $t = 0.3$  in,  
 Outside dimensions  
 Horizontal width  $a = 6$  in.  
 Vertical depth  $b = 4$  in.  
 Vertical load  $P = 9$  kips; line of action at  $x = -c$   
 Find normal and shearing stresses at  
 Point  $H$  ( $x = d$ ,  $y = b/2$ )

**Problem 8.72** Program Output for Value of  $c = 2.85$  in.

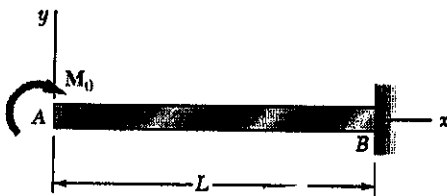
d in.	sigma ksi	tauV ksi	tauT ksi	tauTotal ksi	sigmaMax ksi	sigmaMin ksi	tauMax ksi	theta p degrees
-3.00	12.58	-3.49	-2.03	-5.52	14.65	-2.08	8.36	-18.49
-2.00	12.58	-2.33	-2.03	-4.35	13.94	-1.36	7.65	-16.00
-1.00	12.58	-1.16	-2.03	-3.19	13.34	-0.76	7.05	-12.78
0.00	12.58	0.00	-2.03	-2.03	12.89	-0.32	6.61	-8.73
1.00	12.58	1.16	-2.03	-0.86	12.63	-0.06	6.35	-3.89
2.00	12.58	2.33	-2.03	0.30	12.58	-0.01	6.30	1.36
3.00	12.58	3.49	-2.03	1.46	12.74	-0.17	6.46	6.46

# CHAPTER 9

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**PROBLEM 9.1**

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.



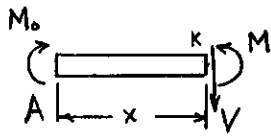
**SOLUTION**

$$\sum M_k = 0 \quad -M_0 + M = 0$$

$$M = M_0$$

$$[x=L, y=0] \quad EI \frac{d^2y}{dx^2} = M = M_0$$

$$[x=L, \frac{dy}{dx}=0] \quad EI \frac{dy}{dx} = M_0x + C_1$$



$$[x=L, \frac{dy}{dx}=0] \quad 0 = M_0L + C_1 \quad C_1 = -M_0L$$

$$EIy = \frac{1}{2}M_0x^2 + C_1x + C_2$$

$$[x=L, y=0] \quad 0 = \frac{1}{2}M_0L^2 - M_0L^2 + C_2$$

$$C_2 = \frac{1}{2}M_0L^2$$

(a) Elastic curve

$$y = \frac{M_0}{2EI} (x^2 - 2Lx + L^2)$$

$$= \frac{M_0}{2EI} (L-x)^2$$

(b)  $y$  @  $x=0$

$$y_A = \frac{M_0}{2EI} (L-0)^2 = \frac{M_0L^2}{2EI} \uparrow$$

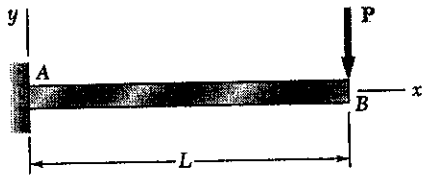
(c)  $\frac{dy}{dx}$  @  $x=0$

$$\frac{dy}{dx} = -\frac{M_0}{EI} (L-x) = -\frac{M_0}{EI} (L-0) = -\frac{M_0L}{EI}$$

$$\theta_A = \frac{M_0L}{EI} \nabla$$

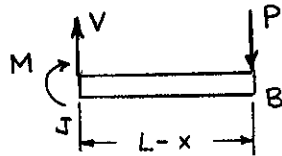
PROBLEM 9.2

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.



$[x=0, y=0]$

$[x=0, \frac{dy}{dx} = 0]$



$[x=0, y=0]$

(a) Elastic curve

(b)  $y$  @  $x=L$

(c)  $\frac{dy}{dx}$  @  $x=L$

SOLUTION

$$\sum M_J = 0 \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$

$$EI \frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI \frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$$

$$[x=0, \frac{dy}{dx} = 0] \quad 0 = -0 + 0 + C_1$$

$$C_1 = 0$$

$$EIy = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$$

$$0 = -0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$y = -\frac{Px^2}{6EI} (3L-x)$$

$$\frac{dy}{dx} = -\frac{Px}{2EI} (2L-x)$$

$$y_B = -\frac{PL^2}{6EI} (3L-L) = -\frac{PL^3}{3EI}$$

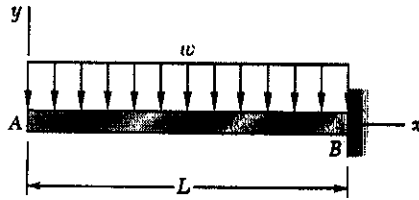
$$y_B = \frac{PL^3}{3EI} \downarrow$$

$$\left. \frac{dy}{dx} \right|_B = -\frac{PL}{2EI} (2L-L) = -\frac{PL^2}{2EI}$$

$$\theta_B = \frac{PL^2}{2EI} \searrow$$

PROBLEM 9.3

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.



SOLUTION

$$\sum M_J = 0 \quad (wx) \frac{x}{2} + M = 0$$

$$M = -\frac{1}{2} wx^2$$

$[x=L, y=0]$   
 $[x=L, \frac{dy}{dx} = 0]$

$$EI \frac{d^2 y}{dx^2} = M = -\frac{1}{2} wx^2$$

$$EI \frac{dy}{dx} = -\frac{1}{6} wx^3 + C_1$$

$[x=L, \frac{dy}{dx} = 0]$        $0 = -\frac{1}{6} wL^3 + C_1$

$$C_1 = \frac{1}{6} wL^3$$

$$EI \frac{dy}{dx} = -\frac{1}{6} wx^3 + \frac{1}{6} wL^3$$

$$EI y = -\frac{1}{24} wx^4 + \frac{1}{6} wL^3 x + C_2$$

$[x=L, y=0]$

$$0 = -\frac{1}{24} wL^4 + \frac{1}{6} wL^4 + C_2 = 0$$

$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right) wL^4 = -\frac{3}{24} wL^4$$

(a) Elastic curve

$$y = -\frac{w}{24 EI} (x^4 - 4L^3 x + 3L^4)$$

(b)  $y$  @  $x=0$

$$y_A = -\frac{3wL^4}{24 EI} = -\frac{wL^4}{8 EI}$$

$$y_A = \frac{wL^4}{8 EI} \downarrow$$

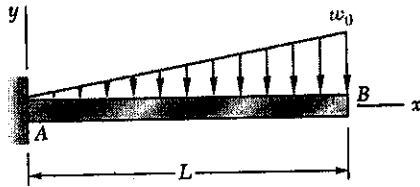
(c)  $\frac{dy}{dx}$  @  $x=0$

$$\left. \frac{dy}{dx} \right|_A = \frac{wL^3}{6 EI}$$

$$\theta_A = \frac{wL^3}{6 EI} \triangleleft$$

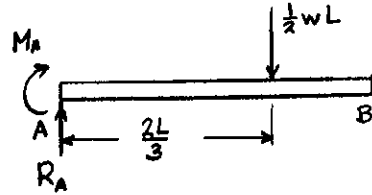
PROBLEM 9.4

9.1 through 9.4 For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.



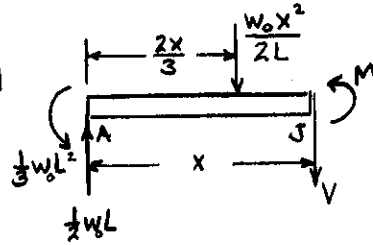
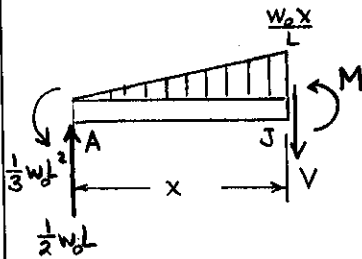
$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$

SOLUTION



$\uparrow \sum F_y = 0$   
 $R_A - \frac{1}{2} wL = 0$   
 $R_A = \frac{1}{2} wL$

$\curvearrowright \sum M_A = 0$   
 $-M_A - \frac{2L}{3} \cdot \frac{wL}{2} = 0$   
 $M_A = -\frac{1}{3} wL^2$



$\curvearrowright \sum M_J = 0$        $\frac{1}{3} wL^2 - \frac{1}{2} wLx + \frac{w_0 x^2}{2L} \cdot \frac{x}{3} + M = 0$

$M = -\frac{1}{3} w_0 L^2 + \frac{1}{2} w_0 Lx - \frac{w_0 x^3}{6L}$

$EI \frac{d^2 y}{dx^2} = -\frac{1}{3} w_0 L^2 + \frac{1}{2} w_0 Lx - \frac{w_0 x^3}{6L}$

$EI \frac{dy}{dx} = -\frac{1}{3} w_0 L^2 x + \frac{1}{4} w_0 Lx^2 - \frac{w_0 x^4}{24L} + C_1$

$[x=0, \frac{dy}{dx} = 0]$        $0 = -0 + 0 - 0 + C_1$        $C_1 = 0$

$EI y = -\frac{1}{6} w_0 L^2 x^2 + \frac{1}{12} w_0 Lx^3 - \frac{w_0 x^5}{120L} + C_2$

$[x=0, y=0]$        $0 = -0 + 0 - 0 + 0 + C_2$        $C_2 = 0$

(a) Elastic curve       $y = -\frac{w_0}{EIL} \left( \frac{1}{6} L^3 x^2 - \frac{1}{12} Lx^3 + \frac{1}{120} x^5 \right)$

(b)  $y @ x=L$        $y_B = -\frac{w_0 L^4}{EI} \left( \frac{1}{6} - \frac{1}{12} + \frac{1}{120} \right) = -\frac{11}{120} \frac{w_0 L^4}{EI}$

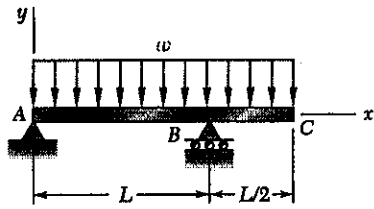
$y_B = \frac{11}{120} \frac{w_0 L^4}{EI} \downarrow$

(c)  $\frac{dy}{dx} @ x=L$        $\frac{dy}{dx} \Big|_B = -\frac{w_0 L^3}{EI} \left( \frac{1}{3} - \frac{1}{4} + \frac{1}{24} \right) = -\frac{1}{8} \frac{w_0 L^3}{EI}$

$\theta_B = \frac{1}{8} \frac{w_0 L^3}{EI} \triangleleft$

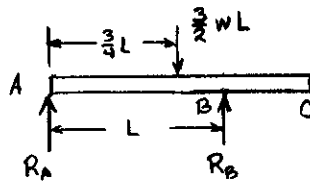
**PROBLEM 9.5**

9.5 For the beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the slope at A, (c) the slope at B.



$[x=0, y=0]$     $[x=L, y=0]$

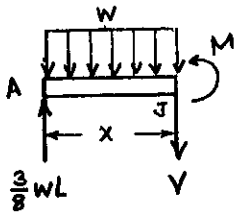
**SOLUTION**



$\rightarrow \sum M_B = 0$

$-R_A L + (\frac{3}{2} wL)(\frac{1}{4} L) = 0$

$R_A = \frac{3}{8} wL$



For portion AB only ( $0 \leq x < L$ )

$\rightarrow \sum M_J = 0 \quad -\frac{3}{8} wLx + (wx)\frac{x}{2} + M = 0$

$M = \frac{3}{8} wLx - \frac{1}{2} wx^2$

$EI \frac{d^2 y}{dx^2} = \frac{3}{8} wLx - \frac{1}{2} wx^2$

$EI \frac{dy}{dx} = \frac{3}{16} wLx^2 - \frac{1}{6} wx^3 + C_1$

$EI y = \frac{1}{16} wLx^3 - \frac{1}{24} wx^4 + C_1 x + C_2$

$[x=0, y=0]$

$0 = 0 - 0 + 0 + C_2$

$C_2 = 0$

$[x=L, y=0]$

$0 = \frac{1}{16} wL^3 - \frac{1}{24} wL^4 + C_1 L$

$C_1 = -\frac{1}{48} wL^3$

(a) Elastic curve

$y = \frac{w}{EI} \left( \frac{1}{16} Lx^3 - \frac{1}{24} x^4 - \frac{1}{48} L^3 x \right)$

$\frac{dy}{dx} = \frac{w}{EI} \left( \frac{3}{16} Lx^2 - \frac{1}{6} x^3 - \frac{1}{48} L^3 \right)$

(b)  $\frac{dy}{dx} @ x=0$

$\frac{dy}{dx} \Big|_A = \frac{w}{EI} \left( 0 - 0 - \frac{1}{48} L^3 \right) = -\frac{wL^3}{48EI}$

$\theta_A = \frac{wL^3}{48EI} \triangleleft$

(c)  $\frac{dy}{dx} @ x=L$

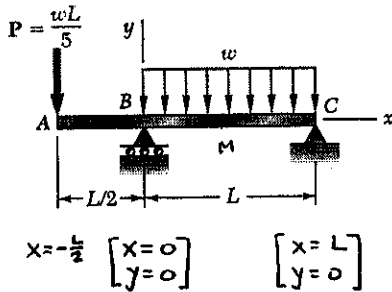
$\frac{dy}{dx} \Big|_B = \frac{w}{EI} \left( \frac{3}{16} L^3 - \frac{1}{6} L^3 - \frac{1}{48} L^3 \right) = 0$

$\theta_B = 0$



**PROBLEM 9.6**

9.6 For the beam and loading shown, determine (a) the equation of the elastic curve for portion BC of the beam, (b) the deflection at midspan, (c) the slope at B.

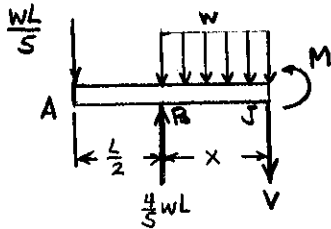


**SOLUTION**

Using ABC as a free body

$$\sum M_C = 0 \quad \left(\frac{wL}{5}\right)\left(\frac{3L}{2}\right) - R_B L + (wL)\left(\frac{L}{2}\right) = 0$$

$$R_B = \frac{4}{5}wL$$



For portion BC only  $0 < x < L$

$$\sum M_J = 0 \quad \frac{wL}{5}\left(\frac{L}{2} + x\right) - \frac{4}{5}wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI \frac{d^2y}{dx^2} = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI \frac{dy}{dx} = \frac{3}{10}wLx^2 - \frac{1}{6}wx^3 - \frac{1}{10}wL^2x + C_1$$

$$EI y = \frac{1}{10}wLx^3 - \frac{1}{24}wx^4 - \frac{1}{20}wL^2x^2 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \left(\frac{1}{10} - \frac{1}{24} - \frac{1}{20}\right)wL^4 + C_1L + 0 \quad C_1 = -\frac{1}{120}wL^3$$

(a) Elastic curve

$$y = \frac{w}{EI} \left( \frac{1}{10}Lx^3 - \frac{1}{24}x^4 - \frac{1}{20}L^2x^2 - \frac{1}{120}L^3x \right)$$

$$\frac{dy}{dx} = \frac{w}{EI} \left( \frac{3}{10}Lx^2 - \frac{1}{6}x^3 - \frac{1}{10}L^2x - \frac{1}{120}L^3 \right)$$

(b)  $y$  @  $x = \frac{L}{2}$

$$y_M = \frac{w}{EI} \left\{ \frac{1}{10}L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120}L^3\left(\frac{L}{2}\right) \right\}$$

$$= \frac{wL^4}{EI} \left\{ \frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240} \right\} = -\frac{13wL^4}{1920EI}$$

$$y_M = \frac{13wL^4}{1920EI} \downarrow$$

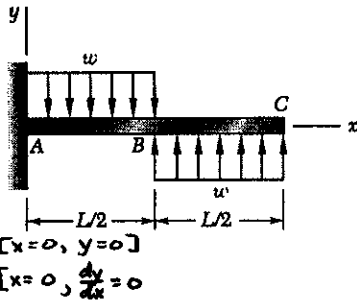
(c)  $\frac{dy}{dx}$  @  $x=0$

$$\left. \frac{dy}{dx} \right|_B = \frac{w}{EI} \left( 0 - 0 - 0 - \frac{1}{120}L^3 \right) = -\frac{wL^3}{120EI}$$

$$\theta_B = \frac{wL^3}{120EI} \triangleleft$$

**PROBLEM 9.7**

9.7 For the cantilever beam and loading shown, determine (a) the equation of the elastic curve for portion AB of the beam, (b) the deflection at B, (c) the slope at B.



**SOLUTION**

Using ABC as a free body

$$\uparrow \Sigma F_y = 0 \quad R_A - \frac{wL}{2} + \frac{wL}{2} = 0 \quad R_A = 0$$

$$\curvearrowright \Sigma M_A = 0 \quad -M_A + \left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = 0 \quad M_A = \frac{wL^2}{4}$$

Using AJ as a free body (Portion AB only)

$$\curvearrowright \Sigma M_J = 0 \quad -\frac{wL^2}{4} + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{4}wL^2 - \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{4}wL^2x - \frac{1}{6}wx^3 + C_1$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 - 0 + C_1 \quad C_1 = 0$$

$$EI y = \frac{1}{8}wL^2x^2 - \frac{1}{24}wx^4 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

(a) Elastic curve

$$y = \frac{w}{EI} \left( \frac{1}{8}L^2x^2 - \frac{1}{24}x^4 \right)$$

$$\frac{dy}{dx} = \frac{w}{EI} \left( \frac{1}{4}L^2x - \frac{1}{6}x^3 \right)$$

(b)  $y$  at  $x = \frac{L}{2}$

$$y_B = \frac{w}{EI} \left\{ \frac{1}{8}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{24}\left(\frac{L}{2}\right)^4 \right\} = \frac{wL^4}{EI} \left\{ \frac{1}{32} - \frac{1}{384} \right\}$$

$$= \frac{11wL^4}{384EI}$$

$$y_B = \frac{11wL^4}{384EI} \uparrow$$

(c)  $\frac{dy}{dx}$  at  $x = \frac{L}{2}$

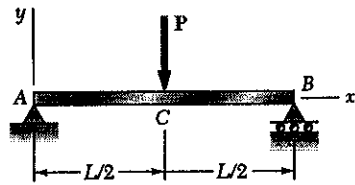
$$\theta_B = \frac{w}{EI} \left\{ \frac{1}{4}L^2\left(\frac{L}{2}\right) - \frac{1}{6}\left(\frac{L}{2}\right)^3 \right\} = \frac{wL^3}{EI} \left\{ \frac{1}{8} - \frac{1}{48} \right\}$$

$$= \frac{5wL^3}{48EI}$$

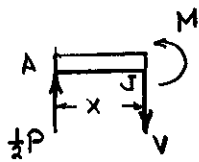
$$\theta_B = \frac{5wL^3}{48EI} \nearrow$$

**PROBLEM 9.8**

9.8 For the beam shown with load  $P$ , determine (a) the equation of the elastic curve for portion  $AC$  of the beam, (b) the slope at  $A$ , (c) the deflection at  $C$ .



$[x=0, y=0]$   
 $[x=L/2, \frac{dy}{dx}=0]$



**SOLUTION**

Because of symmetry  $\frac{dy}{dx} = 0$  at  $x = \frac{L}{2}$ .

Reaction at A  $R_A = \frac{1}{2}P$

For portion  $AC$  only, using free body  $AJ$

$\sum M_J = 0 \quad -\frac{1}{2}Px + M = 0$

$M = \frac{1}{2}Px$

$EI \frac{d^2y}{dx^2} = \frac{1}{2}Px$

$EI \frac{dy}{dx} = \frac{1}{4}Px^2 + C_1$

$[x = \frac{L}{2}, \frac{dy}{dx} = 0] \quad 0 = \frac{1}{4}P(\frac{L}{2})^2 + C_1, \quad C_1 = -\frac{1}{16}PL^2$

$EIy = \frac{1}{12}Px^3 + C_1x + C_2$

$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$

(a) Elastic curve

$y = \frac{P}{EI} \left( \frac{1}{12}x^3 - \frac{1}{16}L^2x \right)$

$\frac{dy}{dx} = \frac{P}{EI} \left( \frac{1}{4}x^2 - \frac{1}{16}L^2 \right)$

(b)  $\frac{dy}{dx}$  at  $x=0$

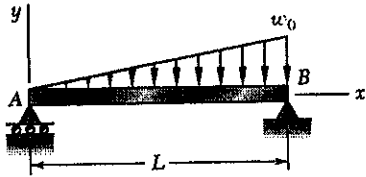
$\theta_A = \frac{P}{EI} \left( 0 - \frac{1}{16}L^2 \right) = -\frac{PL^2}{16EI}$  or  $\frac{PL^2}{16EI}$

(c)  $y$  at  $x = \frac{L}{2}$

$y_C = \frac{P}{EI} \left\{ \frac{1}{12} \left( \frac{L}{2} \right)^3 - \frac{1}{16} L^2 \left( \frac{L}{2} \right) \right\} = \frac{PL^3}{EI} \left\{ \frac{1}{96} - \frac{1}{32} \right\}$   
 $= -\frac{PL^3}{48EI}$  or  $\frac{PL^3}{48EI} \downarrow$

PROBLEM 9.9

9.9 and 9.10 For the beam and loading shown, (a) express the magnitude and location of the maximum deflection in terms of  $w_0$ ,  $L$ ,  $E$ , and  $I$ . (b) Calculate the value of the maximum deflection, assuming that beam  $AB$  is a W18  $\times$  50 rolled shape and that  $w_0 = 4.5$  kips/ft,  $L = 18$  ft, and  $E = 29 \times 10^6$  psi.



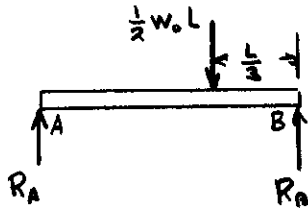
SOLUTION

$[x=0, y=0]$        $[x=L, y=0]$

Using entire beam as a free body

$$\uparrow \sum M_B = 0 \quad -R_A L + \left(\frac{1}{2} w_0 L\right) \left(\frac{L}{3}\right) = 0$$

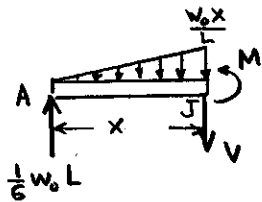
$$R_A = \frac{1}{6} w_0 L$$



Using AJ as a free body  $\uparrow \sum M_J = 0$

$$-\frac{1}{6} w_0 L x + \left(\frac{1}{2} \frac{w_0 x^2}{L}\right) \left(\frac{x}{3}\right) + M = 0$$

$$M = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$$



$$EI \frac{d^2 y}{dx^2} = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3$$

$$EI \frac{dy}{dx} = \frac{1}{12} w_0 L x^2 - \frac{1}{24} \frac{w_0}{L} x^4 + C_1$$

$$EI y = \frac{1}{36} w_0 L x^3 - \frac{1}{120} \frac{w_0}{L} x^5 + C_1 x + C_2$$

$[x=0, y=0]$        $0 = 0 - 0 + 0 + C_2$        $C_2 = 0$

$[x=L, y=0]$        $0 = \frac{1}{36} w_0 L^4 - \frac{1}{120} w_0 L^4 + C_1 L + 0$        $C_1 = -\frac{7 w_0 L^3}{360}$

Elastic curve       $y = \frac{w_0}{EI} \left\{ \frac{1}{36} L x^3 - \frac{1}{120} \frac{x^5}{L} - \frac{7}{360} L^3 x \right\}$

$$\frac{dy}{dx} = \frac{w_0}{EI} \left\{ \frac{1}{12} L x^2 - \frac{1}{24} \frac{x^4}{L} - \frac{7}{360} L^3 \right\}$$

To find location of maximum deflection set  $\frac{dy}{dx} = 0$

$$-15 x_m^4 - 30 L^2 x_m^2 + 7 L^4 = 0 \quad x_m^2 = \frac{30 L^2 - \sqrt{900 L^4 - 420 L^4}}{30}$$

$$x_m^2 = \left(1 - \sqrt{\frac{8}{15}}\right) L^2 = 0.2697 L^2 \quad x_m = 0.5193 L$$

$$y_m = \frac{w_0}{EI} \left\{ \frac{1}{36} L (0.5193 L)^3 - \frac{1}{120} \frac{(0.5193 L)^5}{L} - \frac{7}{360} L^3 (0.5193 L) \right\}$$

$$= -0.00652 \frac{w_0 L^4}{EI} \quad \text{or} \quad 0.00652 \frac{w_0 L^4}{EI} \downarrow$$

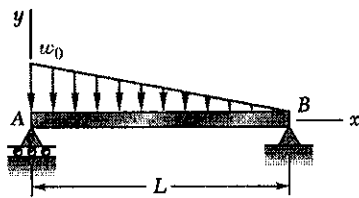
Data:  $w_0 = 4.5$  kips/ft  $= \frac{4500}{12} = 375$  lb/in,       $L = 18$  ft  $= 216$  in

$I = 800$  in<sup>4</sup> for W18  $\times$  50

$$y_m = \frac{(0.00652)(375)(216)^4}{(29 \times 10^6)(800)} = 0.229 \text{ in.}$$

PROBLEM 9.10

9.9 and 9.10 For the beam and loading shown, (a) express the magnitude and location of the maximum deflection in terms of  $w_0$ ,  $L$ ,  $E$ , and  $I$ . (b) Calculate the value of the maximum deflection, assuming that beam  $AB$  is a W18  $\times$  50 rolled shape and that  $w_0 = 4.5$  kips/ft,  $L = 18$  ft, and  $E = 29 \times 10^6$  psi.



SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L-x)$$

$$V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + C_v = \frac{dM}{dx}$$

$[x=0, y=0]$        $[x=L, y=0]$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_v x + C_m$$

$$[x=0, M=0] \quad 0 = 0 + 0 + 0 + C_m \quad C_m = 0$$

$$[x=L, M=0] \quad 0 = -\frac{w_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{6}L^3\right) + C_v L \quad C_v = \frac{1}{3}w_0 L$$

$$EI \frac{d^2y}{dx^2} = M = \frac{w_0}{L}\left(\frac{1}{3}L^2x - \frac{1}{2}Lx^2 + \frac{1}{6}x^3\right)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{6}L^2x^2 - \frac{1}{6}Lx^3 + \frac{1}{24}x^4\right) + C_1$$

$$EI y = \frac{w_0}{L}\left(\frac{1}{18}L^2x^3 - \frac{1}{24}Lx^4 + \frac{1}{120}x^5\right) + C_1x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{w_0}{L}\left\{\frac{1}{18}L^5 - \frac{1}{24}L^5 + \frac{1}{120}L^5\right\} + C_1L + 0 \quad C_1 = -\frac{1}{45}w_0L^3$$

$$y = \frac{w_0}{EIL}\left(\frac{1}{18}L^2x^3 - \frac{1}{24}Lx^4 + \frac{1}{120}x^5 - \frac{1}{45}L^4x\right)$$

$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{6}L^2x^2 - \frac{1}{6}Lx^3 + \frac{1}{24}x^4 - \frac{1}{45}L^4\right)$$

To find location of maximum deflection set  $\frac{dy}{dx} = 0$ .

$$f = \frac{1}{6}L^2x_m^2 - \frac{1}{6}Lx_m^3 + \frac{1}{24}x_m^4 - \frac{1}{45}L^4 = 0$$

$$\text{Let } z = \frac{x_m}{L} \quad f(z) = \frac{1}{6}z^2 - \frac{1}{6}z^3 + \frac{1}{24}z^4 - \frac{1}{45}$$

$$\frac{df}{dz} = \frac{1}{3}z - \frac{1}{2}z^2 + \frac{1}{6}z^3$$

$$\text{By Newton-Raphson method } z = z_0 - \frac{f(z_0)}{df/dz}$$

$$z = 0.5, 0.4805, 0.4807, 0.4807 \quad x_m = 0.4807 L$$

$$y_m = \frac{w_0 L^4}{EI} \left\{ \frac{1}{18}(0.4807)^3 - \frac{1}{24}(0.4807)^4 + \frac{1}{120}(0.4807)^5 - \frac{1}{45}(0.4807) \right\}$$

$$= -0.00652 \frac{w_0 L^4}{EI} \quad y_m = 0.00652 \frac{w_0 L^4}{EI} \downarrow$$

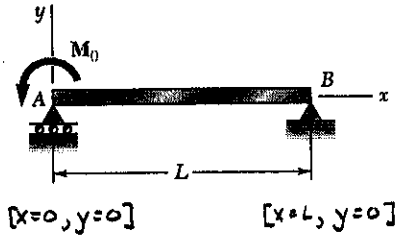
Data:  $w_0 = 4.5 \text{ kips/ft} = \frac{4500}{12} = 375 \text{ lb/in}$ ,  $L = 18 \text{ ft} = 216 \text{ in}$ .

$I = 800 \text{ in}^4$  for W18 $\times$ 50

$$y_m = \frac{(0.00652)(375)(216)^4}{(29 \times 10^6)(800)} = 0.229 \text{ in}$$

PROBLEM 9.11

9.11 (a) Determine the location and magnitude of the maximum deflection of beam AB. (b) Assuming that beam AB is a W360 × 64,  $L = 3.5$  m and  $E = 200$  GPa, calculate the maximum allowable value of the applied moment  $M_0$  if the maximum deflection is not to exceed 1 mm.



SOLUTION

Using entire beam as a free body

$$\sum M_B = 0 \quad M_0 - R_A L = 0 \quad R_A = \frac{M_0}{L}$$

Using portion AJ

$$\sum M_J = 0 \quad M_0 - \frac{M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(x - L)$$

$$EI \frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2 - Lx) + C_1$$

$$EI y = \frac{M_0}{L}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2) + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{M_0}{L}(\frac{1}{6}L^3 - \frac{1}{2}L^3) + C_1 L + 0 \quad C_1 = \frac{1}{3}M_0 L$$

$$y = \frac{M_0}{EIL}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2 + \frac{1}{3}L^2x) \quad \frac{dy}{dx} = \frac{M_0}{EIL}(\frac{1}{2}x^2 - Lx + \frac{1}{3}L^2)$$

To find location of maximum deflection set  $\frac{dy}{dx} = 0$

$$\frac{1}{2}x_m^2 - Lx_m + \frac{1}{3}L^2 = 0 \quad x_m = L - \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)} = (1 - \sqrt{\frac{1}{3}})L = 0.42265L$$

$$y_m = \frac{M_0 L^2}{EI} \left\{ \frac{1}{6}(0.42265)^3 - \frac{1}{2}(0.42265)^2 + \frac{1}{3}(0.42265) \right\} = 0.06415 \frac{M_0 L^2}{EI} \uparrow$$

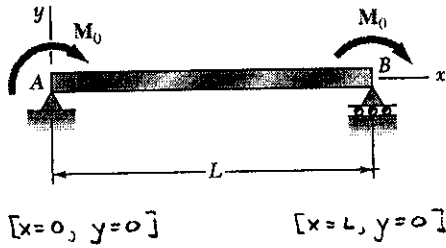
Solving for  $M_0$   $M_0 = \frac{EI y_m}{0.06415 L^2}$

Data:  $E = 200 \times 10^9$  Pa,  $I = 178 \times 10^6$  mm<sup>4</sup> =  $178 \times 10^{-6}$  m<sup>4</sup>  
 $L = 3.5$  m,  $y_m = 1$  mm =  $10^{-3}$  m

$$M_0 = \frac{(200 \times 10^9)(178 \times 10^{-6})(10^{-3})}{(0.06415)(3.5)^2} = 45.3 \times 10^3 \text{ N}\cdot\text{m} = 45.3 \text{ kN}\cdot\text{m}$$

PROBLEM 9.12

9.12 (a) Determine the location and magnitude of the maximum absolute deflection in  $AB$  between  $A$  and the center of the beam. (b) Assuming that beam  $AB$  is a  $W460 \times 113$ ,  $M_0 = 224 \text{ kN}\cdot\text{m}$  and  $E = 200 \text{ GPa}$ , determine the maximum allowable length  $L$  of the beam if the maximum deflection is not to exceed  $1.2 \text{ mm}$ .



SOLUTION

Using  $AB$  as a free body

$$\sum M_B = 0 \quad -2M_0 - R_A L = 0$$

$$R_A = -\frac{2M_0}{L}$$

Using portion  $AJ$  as a free body

$$\sum M_J = 0 \quad -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$

$$EI \frac{d^2 y}{dx^2} = \frac{M_0}{L}(L - 2x)$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EI y = \frac{M_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3\right) + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad 0 = \frac{M_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{3}L^3\right) + C_1 L + 0 \quad C_1 = -\frac{1}{6}M_0 L^2$$

$$y = \frac{M_0}{EIL} \left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3 - \frac{1}{6}L^2 x\right) \quad \frac{dy}{dx} = \frac{M_0}{EIL} (Lx - x^2 - \frac{1}{6}L^2)$$

To find location of maximum deflection set  $\frac{dy}{dx} = 0$

$$x_m^2 - Lx_m - \frac{1}{6}L^2 = 0 \quad x_m = \frac{L - \sqrt{L^2 - (4)(\frac{1}{6}L^2)}}{2} = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right)L = 0.21132 L$$

$$y_m = \frac{M_0 L^2}{EI} \left\{ \left(\frac{1}{2}\right)(0.21132)^2 - \left(\frac{1}{3}\right)(0.21132)^3 - \left(\frac{1}{6}\right)(0.21132) \right\} = -0.0160375 \frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375 \frac{M_0 L^2}{EI}$$

Solving for  $L$  
$$L = \left\{ \frac{EI |y_m|}{0.0160375 M_0} \right\}^{1/2}$$

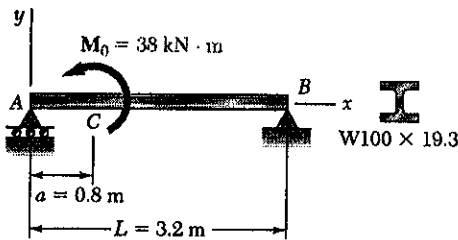
Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 556 \times 10^6 \text{ mm}^4 = 556 \times 10^{-6} \text{ m}^4$

$$|y_m| = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}, \quad M_0 = 224 \times 10^3 \text{ N}\cdot\text{m}$$

$$L = \left\{ \frac{(200 \times 10^9)(556 \times 10^{-6})(1.2 \times 10^{-3})}{(0.0160375)(224 \times 10^3)} \right\}^{1/2} = 6.09 \text{ m}$$

PROBLEM 9.13

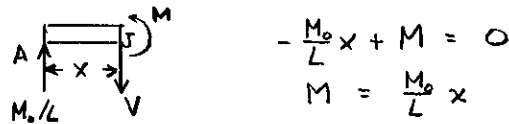
9.13 and 9.14 For the beam and loading shown, determine the deflection at point C. Use  $E = 200 \text{ GPa}$ .



SOLUTION

Reactions:  $R_A = M_0/L \uparrow$ ,  $R_B = M_0/L \downarrow$

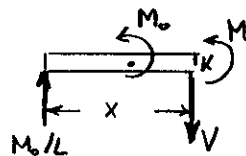
$0 < x < a$   $\curvearrowright \Sigma M_J = 0$



$$-\frac{M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}x$$

$a < x < L$   $\curvearrowright M_K = 0$



$$-\frac{M_0}{L}x + M_0 + M = 0$$

$$M = \frac{M_0}{L}(x - L)$$

$0 < x < a$

$$EI \frac{d^2y}{dx^2} = \frac{M_0}{L}x$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2) + C_1 \quad (1)$$

$$EI y = \frac{M_0}{L}(\frac{1}{6}x^3) + C_1x + C_2 \quad (2)$$

$a < x < L$

$$EI \frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$

$$EI \frac{dy}{dx} = \frac{M_0}{L}(\frac{1}{2}x^2 - Lx) + C_3 \quad (3)$$

$$EI y = \frac{M_0}{L}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2) + C_3x + C_4 \quad (4)$$

$[x=0, y=0]$  Eq.(2):  $0 = 0 + 0 + C_2$   $C_2 = 0$

$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$  Eqs.(1) & (3):  $\frac{M_0}{L}(\frac{1}{2}a^2) + C_1 = \frac{M_0}{L}(\frac{1}{2}a^2 - La) + C_3$

$$C_3 = C_1 + M_0a$$

$[x=a, y=y]$  Eqs.(2) & (4):  $\frac{M_0}{L}(\frac{1}{6}a^3) + C_1a = \frac{M_0}{L}(\frac{1}{6}a^3 - \frac{1}{2}La^2) + (C_1 + M_0a)a + C_4$

$$C_4 = -\frac{1}{2}M_0a^2$$

$[x=L, y=0]$  Eq.(4):  $\frac{M_0}{L}(\frac{1}{6}L^3 - \frac{1}{2}L^3) + (C_1 + M_0a)L - \frac{1}{2}M_0a^2 = 0$

$$C_1 = \frac{M_0}{L}(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)$$

Elastic curve for  $0 < x < a$   $y = \frac{M_0}{EIL} [\frac{1}{6}x^3 + (\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL)x]$

Make  $x = a$   $y_c = \frac{M_0}{EIL} [\frac{1}{6}a^3 + \frac{1}{3}L^2a + \frac{1}{2}a^3 - a^2L] = \frac{M_0}{EIL} [\frac{2}{3}a^3 + \frac{1}{3}L^2a - La^2]$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6} \text{ m}^4$ ,  $M_0 = 38 \times 10^3 \text{ N}\cdot\text{m}$

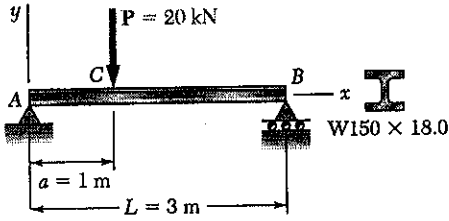
$$y_c = \frac{(38 \times 10^3)}{(200 \times 10^9)(4.77 \times 10^{-6})(3.2)} [(2)(0.8)^3/3 + (3.2)^2(0.8)/3 - (3.2)(0.8)^2] = 12.75 \times 10^{-3} \text{ m}$$

$$= 12.75 \text{ mm}$$



PROBLEM 9.14

9.13 and 9.14 For the beam and loading shown, determine the deflection at point C. Use  $E = 200 \text{ GPa}$ .



SOLUTION

Let  $b = L - a$

Reactions:  $R_A = \frac{Pb}{L} \uparrow$ ,  $R_B = \frac{Pa}{L} \uparrow$

Bending moments

$0 < x < a$   $M = \frac{Pb}{L} x$

$a < x < L$   $M = \frac{P}{L} [bx - L(x - a)]$

$[x=0, y=0]$   $[x=L, y=0]$   
 $[x=a, y=y]$   
 $[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$

$0 < x < a$

$EI \frac{d^2y}{dx^2} = \frac{P}{L} (bx)$

$EI \frac{dy}{dx} = \frac{P}{L} (\frac{1}{2}bx^2) + C_1$  (1)

$EI y = \frac{P}{L} (\frac{1}{6}bx^3) + C_1 x + C_2$  (2)

$a < x < L$

$EI \frac{d^2y}{dx^2} = \frac{P}{L} [bx - L(x - a)]$

$EI \frac{dy}{dx} = \frac{P}{L} [\frac{1}{2}bx^2 - \frac{1}{2}L(x - a)^2] + C_3$  (3)

$EI y = \frac{P}{L} [\frac{1}{6}bx^3 - \frac{1}{6}L(x - a)^3] + C_3 x + C_4$  (4)

$[x=0, y=0]$  Eq (2)  $0 = 0 + 0 + C_2$   $C_2 = 0$

$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$  Eqs. (1) and (3)  $\frac{P}{L} (\frac{1}{2}ba^2) + C_1 = \frac{P}{L} [\frac{1}{2}ba^2 + 0] + C_3 \therefore C_3 = C_1$

$[x=a, y=y]$  Eqs (2) and (4)  $\frac{P}{L} (\frac{1}{6}ba^3) + C_1 a + C_2 = \frac{P}{L} [\frac{1}{6}ba^3 + 0] + C_1 a + C_4$   $C_4 = C_2 = 0$

$[x=L, y=0]$  Eq. (4)  $\frac{P}{L} [\frac{1}{6}bL^3 - \frac{1}{6}L(L - a)^3] + C_3 L = 0$

$C_1 = C_3 = \frac{P}{L} [\frac{1}{6}(L - a)^3 - \frac{1}{6}bL^2] = \frac{P}{L} (\frac{1}{6}b^3 - \frac{1}{6}bL^2)$

Make  $x = a$  in Eq (2)

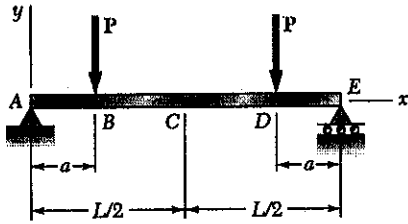
$y_c = \frac{P}{EIL} [\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$

Data:  $P = 20 \times 10^3 \text{ N}$ ,  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 9.17 \times 10^6 \text{ mm}^4 = 9.17 \times 10^{-6} \text{ m}^4$   
 $L = 3 \text{ m}$ ,  $a = 1 \text{ m}$ ,  $b = 2 \text{ m}$

$y_c = \frac{(20 \times 10^3) [(2)(1)^3 + (2)^3(1) - (3)^2(1)(2)]}{(6)(200 \times 10^9)(9.17 \times 10^{-6})(3)} = -4.85 \times 10^{-3} \text{ m}$   
 i.e.  $4.85 \text{ mm} \downarrow$

PROBLEM 9.15

9.15 Knowing that beam  $AE$  is an S200  $\times$  27.4 rolled shape and that  $P = 17.5$  kN,  $L = 2.5$  m,  $a = 0.8$  m and  $E = 200$  GPa, determine (a) the equation of the elastic curve for portion  $BD$ , (b) the deflection at the center  $C$  of the beam.



SOLUTION

Consider portion  $ABC$  only, and consider symmetry about  $C$ .

Reactions  $R_A = R_E = P$

Boundary conditions:  $[x=0, y=0]$ ,  $[x=a, y=y]$ ,  $[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$ ,  $[x=\frac{L}{2}, \frac{dy}{dx} = 0]$

$0 < x < a$

$$EI \frac{d^2y}{dx^2} = M = Px$$

$$EI \frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \quad (1)$$

$$EI y = \frac{1}{6}Px^3 + C_1x + C_2 \quad (2)$$

$$[x=0, y=0] \rightarrow C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \quad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$$

$$[x=\frac{L}{2}, y=y]$$

$$\frac{1}{6}Pa^3 + (\frac{1}{2}Pa^2 - \frac{1}{2}PaL)a = \frac{1}{2}Pa^3 - \frac{1}{2}Pa^2L + C_4$$

$$C_4 = \frac{1}{6}Pa^3$$

$a < x < L-a$

$$EI \frac{d^2y}{dx^2} = M = Pa$$

$$EI \frac{dy}{dx} = Pax + C_3$$

$$EI y = \frac{1}{2}Pax^2 + C_3x + C_4$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = 0] \rightarrow C_3 = -\frac{1}{2}PaL$$

(a) Elastic curve for portion  $BD$

$$y = \frac{1}{EI} (\frac{1}{2}Pax^2 + C_3x + C_4)$$

$$= \frac{P}{EI} (\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3)$$

For deflection at  $C$  set  $x = \frac{L}{2}$

$$y_c = \frac{P}{EI} (\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3) = -\frac{Pa}{EI} (\frac{1}{8}L^2 - \frac{1}{6}a^2)$$

Data:  $I = 23.9 \times 10^6 \text{ mm}^4 = 23.9 \times 10^{-6} \text{ m}^4$ ,  $E = 200 \times 10^9 \text{ Pa}$

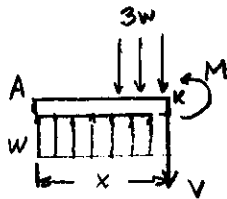
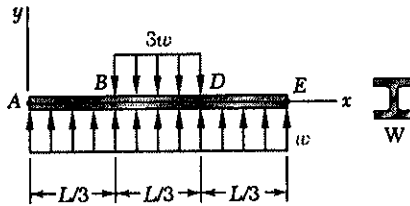
$P = 17.5 \times 10^3 \text{ N}$ ,  $L = 2.5 \text{ m}$ ,  $a = 0.8 \text{ m}$

$$(b) \quad y_c = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^6)} \left\{ \frac{2.5^2}{8} - \frac{0.8^2}{6} \right\} = -1.976 \times 10^{-3} \text{ m}$$

$$y_c = 1.976 \text{ mm } \downarrow$$

PROBLEM 9.16

9.16 Uniformly distributed loads are applied to beam  $AE$  as shown. (a) Selecting the  $x$  axis through the centers  $A$  and  $E$  of the end sections of the beam, determine the equation of the elastic curve for portion  $AB$  of the beam. (b) Knowing that the beam is a  $W200 \times 35.9$  rolled shape and that  $L = 3$  m,  $w = 5$  kN/m, and  $E = 200$  GPa, determine the distance of the center of the beam from the  $x$  axis.



SOLUTION

$$0 < x < \frac{L}{3} \quad \sum M_f = 0$$

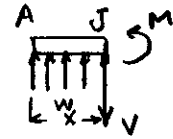
$$-(wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{1}{2}wx^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = \frac{1}{6}wx^3 + C_1$$

$$EI y = \frac{1}{24}wx^4 + C_1x + C_2$$



$$\frac{L}{3} \leq x \leq \frac{2L}{3}$$

$$\sum M_k = 0$$

$$-(wx)\left(\frac{x}{2}\right) + 3w\left(x - \frac{L}{3}\right)\left(\frac{x - \frac{L}{3}}{2}\right) + M = 0$$

$$M = \frac{1}{2}wx^2 - \frac{3}{2}w\left(x - \frac{L}{3}\right)^2$$

$$EI \frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{3}{2}w\left(x - \frac{L}{3}\right)^2$$

$$EI \frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{1}{2}w\left(x - \frac{L}{3}\right)^3 + C_3$$

$$EI y = \frac{1}{24}wx^4 - \frac{1}{8}w\left(x - \frac{L}{3}\right)^4 + C_3x + C_4$$

$$[x=0, y=0]$$

$$0 = 0 + 0 + C_2$$

$$C_2 = 0$$

$$\left[x = \frac{L}{3}, \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$\frac{1}{6}w\left(\frac{L}{3}\right)^3 + C_1 = \frac{1}{6}w\left(\frac{L}{3}\right)^3 + 0 + C_3$$

$$C_1 = C_3$$

$$\left[x = \frac{L}{3}, y = y\right]$$

$$\frac{1}{24}w\left(\frac{L}{3}\right)^4 + C_1\left(\frac{L}{3}\right) + C_2 = \frac{1}{24}w\left(\frac{L}{3}\right)^4 + 0 + C_3\left(\frac{L}{3}\right) + C_4$$

$$C_4 = C_2 = 0$$

Symmetry boundary condition  $\left[x = \frac{L}{2}, \frac{dy}{dx} = 0\right]$

$$\frac{1}{6}w\left(\frac{L}{2}\right)^3 - \frac{1}{2}w\left(\frac{L}{2} - \frac{L}{3}\right)^3 + C_3 = 0$$

$$C_3 = -\left(\frac{1}{48} - \frac{1}{432}\right)wL^3 = -\frac{1}{54}wL^3$$

(a) Elastic curve for portion AB

$$y = \frac{1}{EI} \left\{ \frac{1}{24}wx^4 + C_1x + C_2 \right\} = \frac{w}{EI} \left( \frac{1}{24}x^4 - \frac{1}{54}L^3x \right)$$

(b) Deflection at center

$$y_c = \frac{1}{EI} \left\{ \frac{1}{24}w\left(\frac{L}{2}\right)^4 - \frac{1}{8}w\left(\frac{L}{2} - \frac{L}{3}\right)^4 - \frac{1}{54}wL^3\left(\frac{L}{2}\right) + 0 \right\}$$

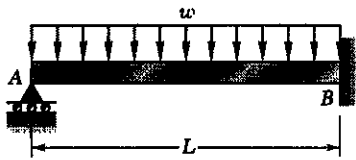
$$= \frac{wL^4}{EI} \left\{ \frac{1}{384} - \frac{1}{10368} - \frac{1}{108} \right\} = -\frac{35}{5184} \frac{wL^4}{EI}$$

Data:  $I = 34.4 \times 10^6 \text{ mm}^4 = 34.4 \times 10^{-6} \text{ m}^4$ ,  $E = 200 \times 10^9 \text{ Pa}$ ,  $L = 3$  m  
 $w = 5 \times 10^3 \text{ N/m}$

$$y_c = -\frac{35}{5184} \frac{(5 \times 10^3)(3)^4}{(200 \times 10^9)(34.4 \times 10^{-6})} = -397 \times 10^{-6} \text{ m}, \text{ ie } 0.397 \text{ mm } \downarrow$$

PROBLEM 9.17

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Reactions are statically indeterminate.  
Boundary conditions are shown at left.

$[x=0, y=0]$

$[x=L, y=0]$

$[x=L, \frac{dy}{dx}=0]$

$\sum M_J = 0 \quad -R_A x + (wx) \frac{x}{2} + M = 0$

$M = -\frac{1}{2} wx^2 + R_A x$

$EI \frac{d^2 y}{dx^2} = -\frac{1}{2} wx^2 + R_A x$

$EI \frac{dy}{dx} = -\frac{1}{6} wx^3 + \frac{1}{2} R_A x^2 + C_1$

$EI y = -\frac{1}{24} wx^4 + \frac{1}{6} R_A x^3 + C_1 x + C_2$

$[x=0, y=0]$

$0 = -0 + 0 + 0 + C_2$

$C_2 = 0$

$[x=L, \frac{dy}{dx}=0]$

$0 = -\frac{1}{6} wL^3 + \frac{1}{2} R_A L^2 + C_1$

$C_1 = \frac{1}{6} wL^3 - \frac{1}{2} R_A L^2$

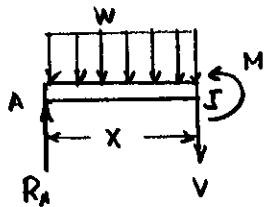
$[x=L, y=0]$

$0 = -\frac{1}{24} wL^4 + \frac{1}{6} R_A L^3 + (\frac{1}{6} wL^3 - \frac{1}{2} R_A L^2)L + 0$

$(\frac{1}{2} - \frac{1}{6}) R_A = (\frac{1}{6} - \frac{1}{24}) wL$

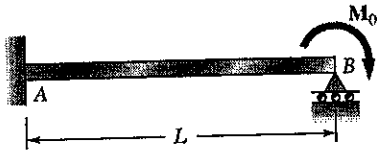
$\frac{1}{3} R_A = \frac{1}{8} wL$

$R_A = \frac{3}{8} wL \uparrow$



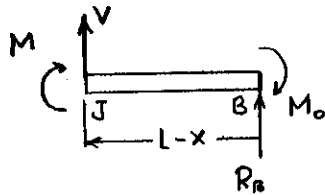
PROBLEM 9.18

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$

$[x=L, y=0]$



SOLUTION

Reactions are statically indeterminate.  
 Boundary condition are shown at left.

Using free body JB

$$\sum M_J = 0 \quad -M + R_B(L-x) - M_0 = 0$$

$$M = -M_0 + R_B(L-x)$$

$$EI \frac{d^2y}{dx^2} = -M_0 + R_B(L-x)$$

$$EI \frac{dy}{dx} = -M_0x + R_B(Lx - \frac{1}{2}x^2) + C_1$$

$$EI y = -\frac{1}{2}M_0x^2 + R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + C_1x + C_2$$

$[x=0, y=0]$

$$0 = -0 + 0 - 0 + 0 + C_2$$

$C_2 = 0$

$[x=0, \frac{dy}{dx}=0]$

$$0 = -0 + 0 - 0 + C_1$$

$C_1 = 0$

$[x=L, y=0]$

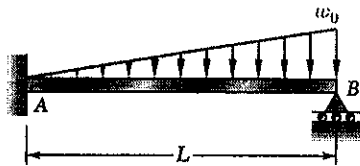
$$0 = -\frac{1}{2}M_0L^2 + R_B(\frac{1}{2}L^3 - \frac{1}{6}L^3)$$

$$\frac{1}{3}R_B = \frac{1}{2}\frac{M_0}{L}$$

$$R_B = \frac{3}{2}\frac{M_0}{L} \uparrow$$

PROBLEM 9.19

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$

$[x=L, y=0]$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown at left.

Using free body JB  $\sum M_J = 0$

$$-M + R_B(L-x) + \frac{1}{2}w_0(L-x)\frac{2}{3}(L-x) + \frac{1}{2}\frac{w_0x}{L}(L-x)\frac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L} [2L(L-x)^2 + x(L-x)^2]$$

$$= R_B(L-x) - \frac{w_0}{6L} [2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3]$$

$$= R_B(L-x) - \frac{w_0}{6L} (x^3 - 3L^2x + 2L^3)$$

$$EI \frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L} (x^3 - 3L^2x + 2L^3)$$

$$EI \frac{dy}{dx} = R_B(Lx - \frac{1}{2}x^2) - \frac{w_0}{6L} (\frac{1}{4}x^4 - \frac{3}{2}L^2x^2 + 2L^2x) + C_1$$

$$EI y = R_B(\frac{1}{2}Lx^2 - \frac{1}{6}x^3) - \frac{w_0}{6L} (\frac{1}{20}x^5 - \frac{1}{2}L^2x^3 + L^2x^2) + C_1x + C_2$$

$[x=0, y=0] \rightarrow C_2 = 0$

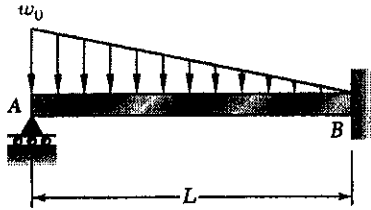
$[x=0, \frac{dy}{dx}=0] \rightarrow C_1 = 0$

$[x=L, y=0] \quad 0 = R_B L^3 (\frac{1}{2} - \frac{1}{6}) - \frac{w_0 L^4}{6} (\frac{1}{20} - \frac{1}{2} + 1)$

$$\frac{1}{3} R_B = (\frac{1}{6}) (\frac{11}{20}) w_0 L \quad R_B = \frac{11}{40} w_0 L \uparrow$$

PROBLEM 9.20

9.17 through 9.20 For the beam and loading shown, determine the reaction at the roller support.



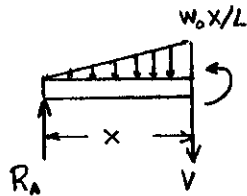
SOLUTION

Reactions are statically indeterminate.  
Boundary conditions are shown at left.

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$



$$w = \frac{w_0}{L} (L-x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L} (L-x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L} (Lx - \frac{1}{2}x^2) + R_A$$

$$M = -\frac{w_0}{L} (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L} (\frac{1}{2}Lx^2 - \frac{1}{6}x^3) + R_A x$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L} (\frac{1}{6}Lx^3 - \frac{1}{24}x^4) + \frac{1}{2}R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L} (\frac{1}{24}Lx^3 - \frac{1}{120}x^5) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$[x=0, y=0]$$

$$0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$-\frac{w_0}{L} (\frac{1}{6}L^4 - \frac{1}{24}L^4) + \frac{1}{2}R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$$[x=L, y=0]$$

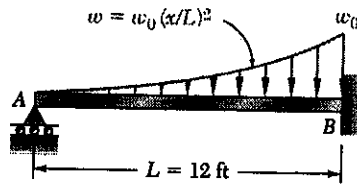
$$-\frac{w_0}{L} (\frac{1}{24}L^4 - \frac{1}{120}L^4) + \frac{1}{6}R_A L^3 + (\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2)L = 0$$

$$(\frac{1}{2} - \frac{1}{6})R_A = (\frac{1}{8} - \frac{1}{24} + \frac{1}{120})w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L$$

$$R_A = \frac{11}{40}w_0 L \uparrow$$

**PROBLEM 9.21**



9.21 For the beam shown, determine the reaction at the roller support when  $w_0 = 6$  kips/ft.

**SOLUTION**

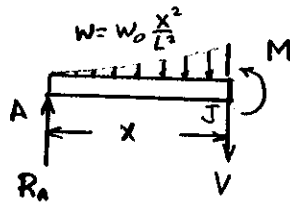
Reactions are statically indeterminate.

Boundary conditions are shown at left

$[x=0, y=0]$

$[x=L, y=0]$

$[x=L, \frac{dy}{dx}=0]$



$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2} \frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2} \frac{x^4}{12} + R_A x$$

$$EI \frac{d^2y}{dx^2} = -\frac{w_0}{L^2} \frac{x^2}{12} + R_A$$

$$EI \frac{dy}{dx} = -\frac{w_0}{L^2} \frac{x^3}{60} + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = -\frac{w_0}{L^2} \frac{x^4}{360} + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$[x=0, y=0]$

$$0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$[x=L, \frac{dy}{dx} = 0]$

$$-\frac{1}{60} w_0 L^3 + \frac{1}{2} R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2$$

$[x=L, y=0]$

$$-\frac{1}{360} w_0 L^4 + \frac{1}{6} R_A L^3 + (\frac{1}{60} w_0 L^3 - \frac{1}{2} R_A L^2) L = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A = (\frac{1}{60} - \frac{1}{360}) w_0 L$$

$$\frac{1}{3} R_A = \frac{1}{72} w_0 L$$

$$R_A = \frac{1}{18} w_0 L$$

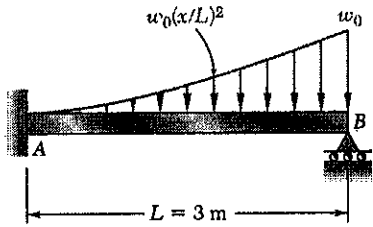
Data:  $w_0 = 6$  kips/ft,  $L = 12$  ft

$$R_A = \frac{1}{18} (6)(12) = 3 \text{ kips } \uparrow$$



PROBLEM 9.22

9.22 For the beam shown, determine the reaction at the roller support when  $w_0 = 15$  kN/m.



SOLUTION

Reactions are statically indeterminate. Boundary conditions are shown at left.

Using free body JB  $\sum M_J = 0$

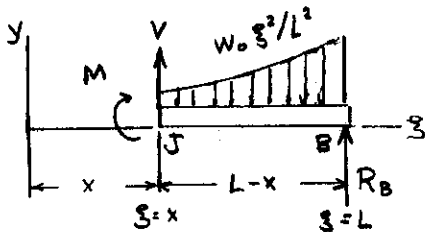
$$-M + \int_x^L \frac{w_0}{L^2} \xi^2 (\xi - x) d\xi + R_B(L-x) = 0$$

$$M = \frac{w_0}{L^2} \int_x^L \xi^2 (\xi - x) d\xi - R_B(L-x)$$

$$= \frac{w_0}{L^2} \left( \frac{1}{4} \xi^4 - \frac{1}{3} x \xi^3 \right) \Big|_x^L - R_B(L-x)$$

$$= \frac{w_0}{L^2} \left( \frac{1}{4} L^4 - \frac{1}{3} x L^3 + \frac{1}{12} x^4 \right) - R_B(L-x)$$

$[x=0, y=0]$   $[x=L, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$



$$EI \frac{d^2 y}{dx^2} = \frac{w_0}{L^2} \left( \frac{1}{4} L^4 - \frac{1}{3} L^3 x + \frac{1}{12} x^4 \right) - R_B(L-x)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L^2} \left( \frac{1}{4} L^4 x - \frac{1}{6} L^3 x^2 + \frac{1}{60} x^5 \right) - R_B(Lx - \frac{1}{2} x^2) + C_1$$

$$EI y = \frac{w_0}{L^2} \left( \frac{1}{8} L^4 x^2 - \frac{1}{18} L^3 x^3 + \frac{1}{360} x^6 \right) - R_B \left( \frac{1}{2} Lx^2 - \frac{1}{6} x^3 \right) + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx} = 0] \quad 0 = 0 + 0 + C_1 \quad C_1 = 0$$

$$[x=0, y = 0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=L, y=0] \quad \left( \frac{1}{8} - \frac{1}{18} + \frac{1}{360} \right) w_0 L^4 - \left( \frac{1}{2} - \frac{1}{6} \right) R_B L^3 = 0$$

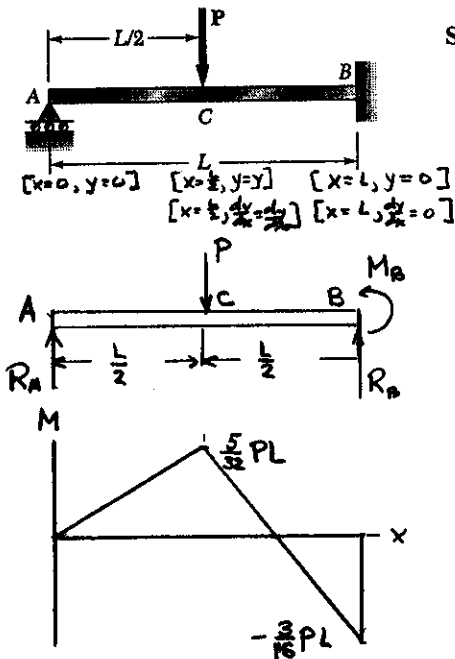
$$\frac{13}{180} w_0 L^4 - \frac{1}{3} R_B L^3 = 0 \quad R_B = \frac{13}{60} w_0 L$$

Data:  $w_0 = 15 \times 10^3$  N/m  $L = 3$  m

$$R_B = \frac{13}{60} (15 \times 10^3)(3) = 9.75 \times 10^3 \text{ N} = 9.75 \text{ kN} \uparrow$$

**PROBLEM 9.23**

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



**SOLUTION**

Reactions are statically indeterminate

$$0 < x < \frac{L}{2}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x \quad (1)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1 \quad (2)$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2 \quad (3)$$

$$\frac{L}{2} < x < L$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - P(x - \frac{L}{2}) \quad (4)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P(x - \frac{L}{2})^2 + C_3 \quad (5)$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} P(x - \frac{L}{2})^3 + C_3 x + C_4 \quad (6)$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2} R_A (\frac{L}{2})^2 + C_1 = \frac{1}{2} R_A (\frac{L}{2})^2 + 0 + C_3 \quad C_3 = C_1$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{6} R_A (\frac{L}{2})^3 + C_1 \frac{L}{2} + C_2 = \frac{1}{6} R_A (\frac{L}{2})^3 + 0 + C_3 \frac{L}{2} + C_4 \quad C_4 = 0$$

$$[x=L, \frac{dy}{dx} = 0] \quad \frac{1}{2} R_A L^2 - \frac{1}{2} P(\frac{L}{2})^2 + C_3 = 0 \quad C_3 = \frac{1}{8} PL^2 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{6} P(\frac{L}{2})^3 + (\frac{1}{8} PL^2 - \frac{1}{2} R_A L^2)L + 0 = 0$$

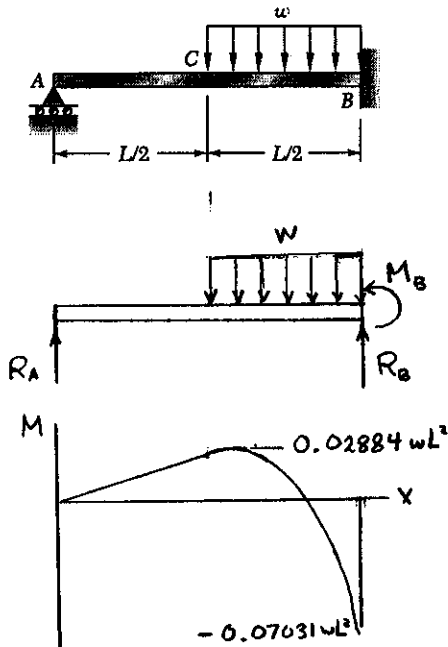
$$(\frac{1}{2} - \frac{1}{6}) R_A L^3 = (\frac{1}{8} - \frac{1}{48}) PL^3 \quad \frac{1}{3} R_A = \frac{\sqrt{5}}{48} P \quad R_A = \frac{\sqrt{5}}{16} P \uparrow$$

$$\text{From (1), with } x = \frac{L}{2} \quad M_C = R_A \frac{L}{2} = \frac{\sqrt{5}}{32} PL$$

$$\text{From (4), with } x = L \quad M_B = R_A L - \frac{1}{2} PL = -\frac{3}{16} PL$$

PROBLEM 9.24

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x \quad (1)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1 \quad (2)$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2 \quad (3)$$

$$\frac{L}{2} < x < L$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2} w \left(x - \frac{L}{2}\right)^2 \quad (4)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w \left(x - \frac{L}{2}\right)^3 + C_3 \quad (5)$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} w \left(x - \frac{L}{2}\right)^4 + C_3 x + C_4 \quad (6)$$

$$\begin{aligned} [x=0, y=0] & \quad 0 = 0 + 0 + C_2 & \quad C_2 = 0 \\ [x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] & \quad \frac{1}{2} R_A \left(\frac{L}{2}\right)^2 + C_1 = \frac{1}{2} R_A \left(\frac{L}{2}\right)^2 + 0 + C_3 & \quad C_1 = C_3 \\ [x=\frac{L}{2}, y=y] & \quad \frac{1}{6} R_A \left(\frac{L}{2}\right)^3 + C_1 \left(\frac{L}{2}\right) + C_2 = \frac{1}{6} R_A \left(\frac{L}{2}\right)^3 - 0 + C_3 \left(\frac{L}{2}\right) + C_4 & \quad C_2 = C_4 = 0 \\ [x=L, \frac{dy}{dx} = 0] & \quad \frac{1}{2} R_A L^2 - \frac{1}{6} w \left(\frac{L}{2}\right)^3 + C_3 = 0 & \quad C_3 = \frac{1}{48} w L^3 - \frac{1}{2} R_A L^2 \\ [x=L, y=0] & \quad \frac{1}{6} R_A L^3 - \frac{1}{24} w \left(\frac{L}{2}\right)^4 + \left(\frac{1}{48} w L^3 - \frac{1}{2} R_A L^2\right) L + 0 = 0 \end{aligned}$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_A L^3 = \left(\frac{1}{48} - \frac{1}{384}\right) w L^4 \quad \frac{1}{3} R_A = \frac{7}{384} w L \quad R_A = \frac{7}{128} w L \uparrow \quad \blacktriangleleft$$

$$\text{From (1), with } x = \frac{L}{2} \quad M_C = R_A \left(\frac{L}{2}\right) = \frac{7}{256} w L^2 = 0.02734 w L^2 \quad \blacktriangleleft$$

$$\begin{aligned} \text{From (4), with } x = L \quad M_B &= R_A L - \frac{1}{2} w \left(\frac{L}{2}\right)^2 = \left(\frac{7}{128} - \frac{1}{8}\right) w L - \frac{9}{128} w L^2 \\ &= -0.07031 w L \quad \blacktriangleleft \end{aligned}$$

Location of maximum positive M

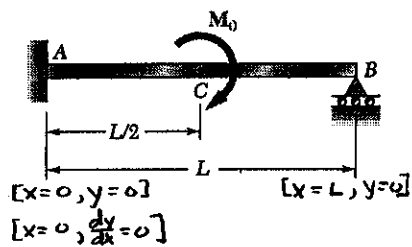
$$\frac{L}{2} < x < L \quad V_m = R_A - w \left(x_m - \frac{L}{2}\right) = 0 \quad x_m - \frac{L}{2} = \frac{R_A}{w} = \frac{7}{128} L$$

$$x_m = \frac{L}{2} + \frac{7}{128} L = \frac{71}{128} L$$

$$\begin{aligned} \text{From (4), with } x = x_m \quad M_m &= R_A x_m - \frac{1}{2} w \left(x_m - \frac{L}{2}\right)^2 \\ &= \left(\frac{7}{128} w L\right) \left(\frac{71}{128} L\right) - \frac{1}{2} w \left(\frac{7}{128} L\right)^2 = 0.02884 w L^2 \quad \blacktriangleleft \end{aligned}$$

PROBLEM 9.26

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



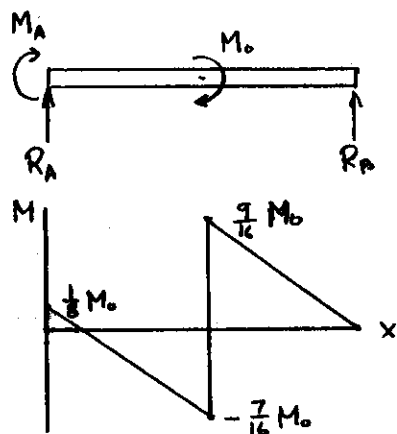
SOLUTION

Reactions are statically indeterminate.

$$\uparrow \sum F_y = 0 \quad R_A + R_B = 0 \quad R_A = -R_B$$

$$\circlearrowleft \sum M_A = 0 \quad -M_A - M_0 + R_B L = 0$$

$$M_A = R_B L - M_0$$



$$0 < x < \frac{L}{2}$$

$$M = R_A x + M_A = -M_0 + R_B L - R_A x$$

$$EI \frac{d^2 y}{dx^2} = -M_0 + R_B (L-x)$$

$$EI \frac{dy}{dx} = -M_0 x + R_B (Lx - \frac{1}{2} x^2) + C_1$$

$$EI y = -\frac{1}{2} M_0 x^2 + R_B (\frac{1}{2} Lx^2 - \frac{1}{6} x^3) + C_1 x + C_2$$

$$\frac{L}{2} < x < L \quad M = R_B (L-x)$$

$$EI \frac{d^2 y}{dx^2} = R_B (L-x)$$

$$EI \frac{dy}{dx} = R_B (Lx - \frac{1}{2} x^2) + C_3$$

$$EI y = R_B (\frac{1}{2} Lx^2 - \frac{1}{6} x^3) + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0]$$

$$0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$[x=0, y=0]$$

$$0 + 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$-M_0 \frac{L}{2} + R_B (\frac{1}{2} L^2 - \frac{1}{6} L^2) = R_B (\frac{1}{2} L^2 - \frac{1}{6} L^2) + C_3$$

$$C_3 = -\frac{M_0 L}{2}$$

$$[x=\frac{L}{2}, y=y]$$

$$-\frac{1}{2} M_0 (\frac{L}{2})^2 + R_B (\frac{1}{8} L^3 - \frac{1}{48} L^3) = R_B (\frac{1}{8} L^3 - \frac{1}{48} L^3) + C_3 \frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{8} M_0 L^2 - \frac{1}{2} C_3 L = (-\frac{1}{8} + \frac{1}{4}) M_0 L^2 = \frac{1}{8} M_0 L^2$$

$$[x=L, y=0]$$

$$R_B (\frac{1}{2} L^3 - \frac{1}{6} L^3) + \frac{M_0 L}{2} L + \frac{1}{8} M_0 L^2 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_B L^3 = (\frac{1}{2} - \frac{1}{8}) M_0 L^2 \quad \frac{1}{3} R_B = \frac{3}{8} \frac{M_0}{L}$$

$$R_B = \frac{9}{8} \frac{M_0}{L} \uparrow$$

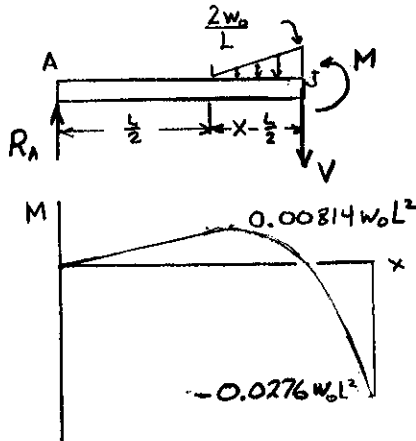
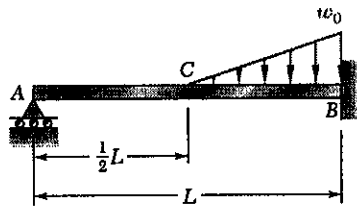
$$M_A = \frac{9}{8} M_0 - M_0 = \frac{1}{8} M_0$$

$$M_C = -M_0 + \frac{9}{8} \frac{M_0}{L} \frac{L}{2} = -\frac{7}{16} M_0$$

$$M_{C+} = R_B (L - \frac{L}{2}) = \frac{9}{8} \frac{M_0}{L} (\frac{L}{2}) = \frac{9}{16} M_0$$

PROBLEM 9.25

9.23 through 9.26 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Reactions are statically indeterminate.

$$0 \leq x \leq \frac{L}{2}$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x \quad (1)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1 \quad (2)$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2 \quad (3)$$

$$\frac{L}{2} \leq x \leq L \quad \sum M_J = 0$$

$$-R_A x + \frac{1}{2} \frac{2w_0}{L} (x - \frac{L}{2}) \frac{1}{3} (x - \frac{L}{2}) + M = 0$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x - \frac{1}{3} \frac{w_0}{L} (x - \frac{L}{2})^3 \quad (4)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{12} \frac{w_0}{L} (x - \frac{L}{2})^4 + C_3 \quad (5)$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{60} \frac{w_0}{L} (x - \frac{L}{2})^5 + C_3 x + C_4 \quad (6)$$

$$[x=0, y=0] \quad 0 = 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}] \quad \frac{1}{2} R_A (\frac{L}{2})^2 + C_1 = \frac{1}{2} R_A (\frac{L}{2})^2 - 0 + C_3 \quad C_1 = C_3$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{6} R_A (\frac{L}{2})^3 + C_1 \frac{L}{2} + C_2 = \frac{1}{6} R_A (\frac{L}{2})^3 - 0 + C_3 \frac{L}{2} + C_4 \quad C_4 = C_2 = 0$$

$$[x=L, \frac{dy}{dx} = 0] \quad \frac{1}{2} R_A L^2 - \frac{1}{12} \frac{w_0}{L} (\frac{L}{2})^4 + C_3 = 0 \quad C_3 = \frac{1}{192} w_0 L^3 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{60} \frac{w_0}{L} (\frac{L}{2})^5 + \frac{1}{2} w_0 L^4 - \frac{1}{2} R_A L^3 + 0 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A L^3 = (\frac{1}{192} - \frac{1}{1920}) w_0 L^4 \quad \frac{1}{3} R_A = \frac{3}{640} w_0 L \quad R_A = \frac{9}{640} w_0 L \quad \blacktriangleleft$$

$$\text{From (1), with } x = \frac{L}{2} \quad M_C = R_A \frac{L}{2} = \frac{9}{1280} w_0 L^2 = 0.007031 w_0 L^2 \quad \blacktriangleleft$$

$$\text{From (4), with } x = L \quad M_B = \frac{9}{640} w_0 L^2 - \frac{1}{3} \frac{w_0}{L} (\frac{L}{2})^3 = -\frac{53}{1920} w_0 L^2 = -0.02761 w_0 L^2 \quad \blacktriangleleft$$

Location of maximum positive M in portion CB.

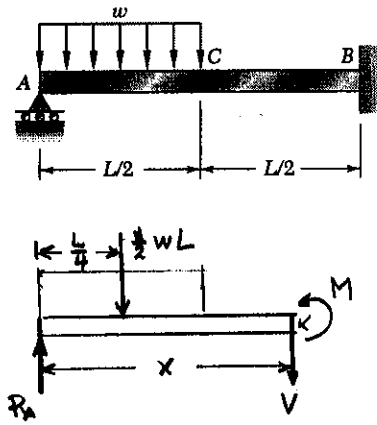
$$\frac{dM}{dx} = R_A - \frac{w_0}{L} (x_m - \frac{L}{2})^2 = 0 \quad x_m - \frac{L}{2} = \sqrt{\frac{R_A L}{w_0}} = \sqrt{\frac{9}{640}} L = 0.1186 L$$

$$x_m = 0.5L + 0.1186 L = 0.6186 L$$

$$\text{From (4), with } x = x_m \quad M_m = R_A (0.6186 L) - \frac{1}{3} \frac{w_0}{L} (0.1186 L)^3 = 0.008143 w_0 L^2 \quad \blacktriangleleft$$

PROBLEM 9.27

9.27 and 9.28 Determine the reaction at the roller support and the deflection at point C.



SOLUTION

Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2} w x^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} w x^3 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} w x^4 + C_1 x + C_2$$

$$\frac{L}{2} < x < L \quad (\text{See free body diagram})$$

$$\sum M_K = 0 \quad -R_A x + \frac{1}{2} w L (x - \frac{L}{4}) + M = 0$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2} w L (x - \frac{L}{4})$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{4} w L (x - \frac{L}{4})^2 + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{12} w L (x - \frac{L}{4})^3 + C_3 x + C_4$$

$$[x=0, y=0]$$

$$0 - 0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x = \frac{L}{2}, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\frac{1}{2} R_A (\frac{L}{2})^2 - \frac{1}{6} w (\frac{L}{2})^3 + C_1 = \frac{1}{2} R_A (\frac{L}{2})^2 - \frac{1}{4} w L (\frac{L}{4})^2 + C_3$$

$$C_1 = C_3 + \frac{1}{48} w L^3 - \frac{1}{64} w L^3 = C_3 + \frac{1}{192} w L^3$$

$$[x = \frac{L}{2}, y = y]$$

$$\frac{1}{6} R_A (\frac{L}{2})^3 - \frac{1}{24} w (\frac{L}{2})^4 + (C_3 + \frac{1}{192} w L^3) \frac{L}{2} = \frac{1}{6} R_A (\frac{L}{2})^3 - \frac{1}{12} w L (\frac{L}{4})^3 + C_3 \frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{384} w L^4 + \frac{1}{384} w L^4 + \frac{1}{768} w L^4 = \frac{1}{768} w L^4$$

$$[x=L, \frac{dy}{dx} = 0]$$

$$\frac{1}{2} R_A L^2 - \frac{1}{4} w L (\frac{3L}{4})^2 + C_3 = 0$$

$$C_3 = \frac{9}{64} w L^3 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0]$$

$$\frac{1}{6} R_A L^3 - \frac{1}{12} w L (\frac{3L}{4})^3 + (\frac{9}{64} w L^3 - \frac{1}{2} R_A L^2) L + \frac{1}{768} w L^4 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_A L^3 = (\frac{9}{64} - \frac{27}{768} + \frac{1}{768}) w L^4$$

$$\frac{1}{3} R_A = \frac{41}{384} w L$$

$$R_A = \frac{41}{128} w L \uparrow$$

$$C_3 = \frac{9}{64} w L^3 - \frac{1}{2} \cdot \frac{41}{128} w L^3 = -\frac{5}{256} w L^3$$

$$C_1 = -\frac{5}{256} w L^3 + \frac{1}{192} w L^3 = -\frac{11}{768} w L^3$$

Deflection at C (y at  $x = \frac{L}{2}$ )

$$y_c = \frac{w L^4}{EI} \left\{ \frac{1}{6} \cdot \frac{41}{128} (\frac{L}{2})^3 - \frac{1}{24} \cdot (\frac{L}{2})^4 - \frac{11}{768} \cdot \frac{L}{2} + 0 \right\}$$

$$= \left( \frac{41}{6144} - \frac{1}{384} - \frac{11}{1536} \right) \frac{w L^4}{EI} = -\frac{19}{6144} \frac{w L^4}{EI}$$

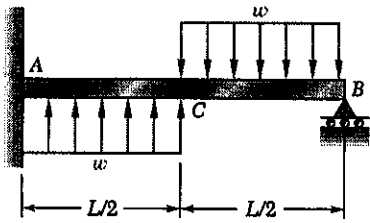
$$y_c = \frac{19}{6144} \frac{w L^4}{EI} \downarrow$$

$$\text{or } y_c = \frac{w L^4}{EI} \left\{ \frac{1}{6} \cdot \frac{41}{128} (\frac{L}{2})^3 - \frac{1}{12} \cdot (\frac{L}{4})^3 + \frac{5}{256} \cdot \frac{L}{2} + \frac{1}{768} \right\}$$

$$= \left( \frac{41}{6144} - \frac{1}{768} - \frac{5}{512} + \frac{1}{768} \right) \frac{w L^4}{EI} = -\frac{19}{6144} \frac{w L^4}{EI}$$

PROBLEM 9.28

9.27 and 9.28 Determine the reaction at the roller support and the deflection at point C.



SOLUTION

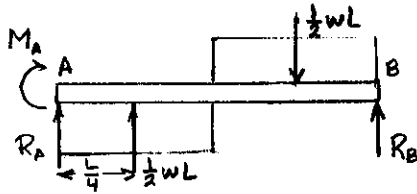
Reactions are statically indeterminate.

$$+\uparrow \sum F_y = 0 \quad R_A + \frac{1}{2}WL - \frac{1}{2}WL + R_B = 0 \quad R_A = -R_B$$

$$\circlearrowleft \sum M_A = 0 \quad -M_A - (\frac{1}{2}WL)\frac{L}{2} + R_B L = 0$$

$$M_A = R_B L - \frac{1}{4}WL^2$$

$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$   
 $[x=L, y=0]$   
 $[x=\frac{L}{2}, y=y]$   
 $[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}]$



From A to C  $0 < x \leq \frac{L}{2}$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$$

From C to B  $\frac{L}{2} \leq x < L$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}WL(x - \frac{L}{4}) - \frac{1}{2}w(x - \frac{L}{2})^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}WL(x - \frac{L}{4}) - \frac{1}{6}w(x - \frac{L}{2})^3 + C_3$$

$$EI y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{12}WL(x - \frac{L}{4})^2 - \frac{1}{24}(x - \frac{L}{2})^4 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=\frac{L}{2}, \frac{dy}{dx}=\frac{dy}{dx}] \quad M_A \frac{L}{2} - \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{6}w(\frac{L}{2})^3 = M_A \frac{L}{2} + \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{4}WL(\frac{L}{4})^2 - 0 + C_3$$

$$C_3 = (\frac{1}{48} - \frac{1}{64})WL^3 = \frac{1}{192}WL^3$$

$$[x=\frac{L}{2}, y=y] \quad \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4 = \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{2}R_A (\frac{L}{2})^2 + \frac{1}{12}WL(\frac{L}{4})^3 - 0 + \frac{1}{192}WL^3(\frac{L}{2}) + C_4$$

$$C_4 = (\frac{1}{384} - \frac{1}{768} + \frac{1}{384})WL^4 = -\frac{1}{768}WL^4$$

$$[x=L, y=0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 + \frac{1}{12}WL(\frac{3L}{4})^3 - \frac{1}{24}w(\frac{L}{2})^4 + \frac{1}{192}WL^3(L) - \frac{1}{768}WL^4 = 0$$

$$\frac{1}{2}(R_B L - \frac{1}{4}WL^2)L^2 + \frac{1}{6}(-R_B)L^3 + (\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768})WL^4 = 0$$

$$(\frac{1}{2} - \frac{1}{6})R_B L^3 = -(\frac{1}{8} - \frac{7}{192})WL^4 \quad \frac{1}{3}R_B = \frac{17}{192}WL \quad R_B = \frac{17}{64}WL \uparrow$$

$$R_A = -R_B = -\frac{17}{64}WL$$

$$M_A = R_B L - \frac{1}{4}WL^2 = (\frac{17}{64} - \frac{1}{4})WL^2 = \frac{1}{64}WL^2$$

(b) Deflection at C ( $y$  at  $x = \frac{L}{2}$ )

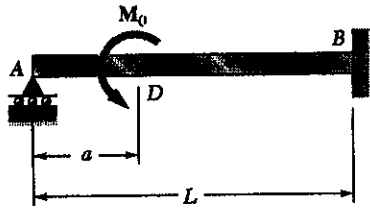
$$EI y_c = \frac{1}{2}M_A (\frac{L}{2})^2 + \frac{1}{6}R_A (\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4 = \frac{1}{2}(\frac{1}{64}WL^2)(\frac{L}{2})^2 + \frac{1}{6}(-\frac{17}{64}WL)(\frac{L}{2})^3 + \frac{1}{24}w(\frac{L}{2})^4$$

$$= (\frac{1}{512} - \frac{17}{3072} + \frac{1}{384})WL^4 = -\frac{1}{1024}WL^4$$

$$y_c = \frac{1}{1024} \frac{WL^4}{EI} \downarrow$$

PROBLEM 9.29

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point D, knowing that a is equal to L/3.



SOLUTION

Reactions are statically indeterminate.

$$0 < x < a \quad M = R_A x$$

$$EI \frac{d^2 y}{dx^2} = M = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$a < x < L \quad M = R_A x - M_0$$

$$EI \frac{d^2 y}{dx^2} = R_A x - M_0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0(x-a) + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0(x-a)^2 + C_3 x + C_4$$

$$[x=0, y=0]$$

$$[x=L, y=0]$$

$$[x=L, \frac{dy}{dx}=0]$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$[x=a, y=y]$$

$$[x=0, y=0]$$

$$0 + 0 + C_2 = 0$$

$$C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}]$$

$$\frac{1}{2} R_A a^2 + C_1 = \frac{1}{2} R_A a^2 - 0 + C_3$$

$$C_1 = C_3$$

$$[x=a, y=y]$$

$$\frac{1}{6} R_A a^3 + C_1 a + C_2 = \frac{1}{6} R_A a^3 + 0 + C_3 a + C_4$$

$$C_2 = C_4 = 0$$

$$[x=L, \frac{dy}{dx}=0]$$

$$\frac{1}{2} R_A L^2 - M_0(L-a) + C_3 = 0$$

$$C_3 = M_0(L-a) - \frac{1}{2} R_A L^2$$

$$[x=L, y=0]$$

$$\frac{1}{6} R_A L^3 - \frac{1}{2} M_0(L-a)^2 + [M_0(L-a) - \frac{1}{2} R_A L^2] L + 0 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_A L^3 = M_0 [(L-a)L - \frac{1}{2}(L-a)^2]$$

$$\frac{1}{3} R_A L^3 = M_0 [L^2 - aL - \frac{1}{2}L^2 + La - \frac{1}{2}a^2] = \frac{1}{2} M_0 (L^2 - a^2)$$

$$R_A = \frac{3}{2} \frac{M_0}{L^3} (L^2 - a^2) = \frac{3}{2} \frac{M_0}{L^3} \left[L^2 - \left(\frac{L}{3}\right)^2\right] = \frac{4}{3} \frac{M_0}{L} \uparrow$$

Deflection at D (y at  $x = a = \frac{L}{3}$ )

$$y_0 = \frac{1}{EI} \left\{ \frac{1}{6} R_A \left(\frac{L}{3}\right)^3 + C_1 \left(\frac{L}{3}\right) \right\} = \frac{1}{EI} \left\{ \frac{1}{6} \left(\frac{4}{3} \frac{M_0}{L}\right) \left(\frac{L}{3}\right)^3 + C_3 \left(\frac{L}{3}\right) \right\}$$

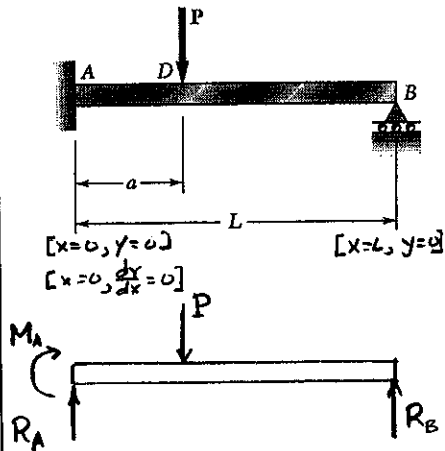
$$= \frac{1}{EI} \left\{ \frac{4}{486} M_0 L^2 + [M_0(L - \frac{L}{3}) - \frac{1}{2} \cdot \frac{4}{3} \frac{M_0}{L} L^2] \frac{L}{3} \right\}$$

$$= \frac{M_0 L^2}{EI} \left( \frac{4}{486} + \frac{2}{9} - \frac{4}{18} \right) = \frac{2}{243} \frac{M_0 L^2}{EI} \uparrow$$



PROBLEM 9.30

9.29 and 9.30 Determine the reaction at the roller support and the deflection at point D, knowing that  $a$  is equal to  $L/3$ .



SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0 \quad -M_A - Pa - R_B L = 0$$

$$M_A = R_B L - Pa$$

$$0 < x < a \quad M = M_A + R_A x$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 + C_1 x + C_2$$

$$a < x < L \quad M = M_A + R_A x - P(x-a)$$

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x - P(x-a)$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P(x-a)^2 + C_3$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P(x-a)^3 + C_3 x + C_4$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=a, \frac{dy}{dx} = \frac{dy}{dx}] \quad M_A a + \frac{1}{2} R_A a^2 + C_1 = M_A a + \frac{1}{2} R_A a^2 - 0 + C_3 \quad C_3 = C_1 = 0$$

$$[x=a, y=y] \quad \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 + C_1 a + C_2 = \frac{1}{2} M_A a^2 + \frac{1}{6} R_A a^3 - 0 + C_3 a + C_4 \quad C_4 = C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P(L-a)^3 + 0 + 0 = 0$$

$$\frac{1}{2} (R_B L - Pa) L^2 + \frac{1}{6} (P - R_B) L^3 - \frac{1}{6} P(L-a)^3 = 0$$

$$(\frac{1}{2} - \frac{1}{6}) R_B L^3 = P [\frac{1}{2} a L^2 - \frac{1}{6} L^3 + \frac{1}{6} (L-a)^3]$$

$$\frac{1}{3} R_B L^3 = P [\frac{1}{2} a L^2 - \frac{1}{6} L^3 + \frac{1}{6} L^3 - \frac{1}{2} L^2 a + \frac{1}{2} L a^2 - \frac{1}{6} a^3]$$

$$= P a^2 (\frac{1}{2} L - \frac{1}{6} a)$$

$$R_B = \frac{P a^2}{2 L^3} (3L - a) = \frac{P (L/3)^2}{2 L^3} (3L - \frac{L}{3}) = \frac{4}{27} P \uparrow$$

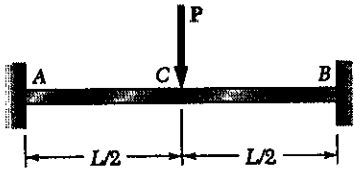
Deflection at D ( $y$  at  $x = a = \frac{L}{3}$ )

$$y_D = \frac{1}{EI} \left\{ \frac{1}{2} M_A \left(\frac{L}{3}\right)^2 + \frac{1}{6} R_A \left(\frac{L}{3}\right)^3 \right\} = \frac{1}{EI} \left\{ \frac{1}{18} (R_B L - P \frac{L}{3}) L^2 + \frac{1}{162} (P - R_B) L^3 \right\}$$

$$= \frac{P L^3}{EI} \left\{ \frac{1}{18} \left(\frac{4}{27} - \frac{1}{3}\right) + \frac{1}{162} \left(1 - \frac{4}{27}\right) \right\} = -\frac{11}{2187} \frac{P L^3}{EI}, \quad y_D = \frac{11}{2187} \frac{P L^3}{EI} \downarrow$$

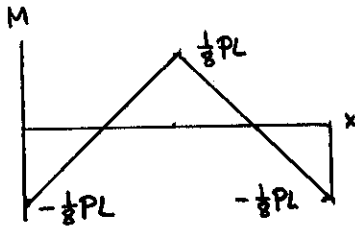
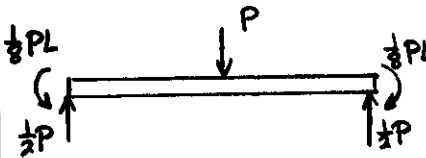
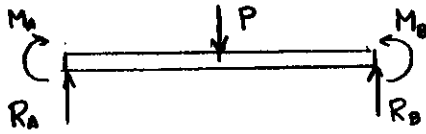
PROBLEM 9.31

9.31 and 9.32 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$

$[x=\frac{L}{2}, \frac{dy}{dx}=0]$



SOLUTION

By symmetry,  $R_A = R_B$  and  $\frac{dy}{dx} = 0$  at  $x = \frac{L}{2}$ .

$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - P = 0 \quad R_A = R_B = \frac{1}{2}P \quad \blacktriangleleft$

Moment reaction is statically indeterminate.

$0 < x < \frac{L}{2} \quad M = M_A + R_A x = M_A + \frac{1}{2}Px$

$EI \frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$

$EI \frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$

$[x=0, \frac{dy}{dx}=0] \quad 0 - 0 + C_1 = 0 \quad C_1 = 0$

$[x=\frac{L}{2}, \frac{dy}{dx}=0] \quad M_A \frac{L}{2} + \frac{1}{4}P(\frac{L}{2})^2 + 0 = 0$

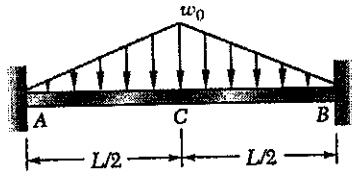
$M_A = -\frac{1}{8}PL \quad M_A = \frac{1}{8}PL \quad \blacktriangleleft$

By symmetry  $M_B = M_A = \frac{1}{8}PL \quad \blacktriangleleft$

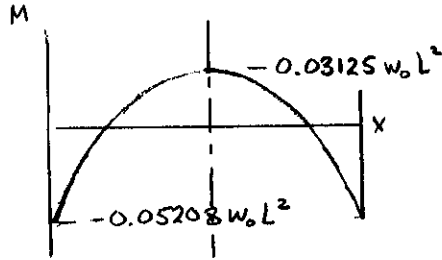
$M_C = M_A + \frac{1}{2}P \frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL \quad \blacktriangleleft$

PROBLEM 9.32

9.31 and 9.32 Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.



$$\begin{array}{l|l} [x=0, y=0] & [x=L, y=0] \\ [x=0, \frac{dy}{dx}=0] & [x=L, \frac{dy}{dx}=0] \\ [x=\frac{L}{2}, V=0] & \\ [x=\frac{L}{2}, \frac{d^2y}{dx^2}=0] & \end{array}$$



SOLUTION

Reactions are statically indeterminate.

Because of symmetry  $\frac{dy}{dx} = 0$  and  $V = 0$  at  $x = \frac{L}{2}$ .

Use portion AC of beam ( $0 < x \leq \frac{L}{2}$ )

$$\frac{dV}{dx} = -w = -2 \frac{w_0}{L} x$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L} x^2 + R_A \quad (1)$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{3} \frac{w_0}{L} x^3 + R_A x + M_A \quad (2)$$

$$EI \frac{dy}{dx} = -\frac{1}{12} \frac{w_0}{L} x^4 + \frac{1}{2} R_A x^2 + M_A x + C_1 \quad (3)$$

$$EI y = -\frac{1}{60} \frac{w_0}{L} x^5 + \frac{1}{6} R_A x^3 + \frac{1}{2} M_A x^2 + C_1 x + C_2 \quad (4)$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 + 0 + 0 + C_1 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 = 0 + 0 + 0 + 0 + C_2 \quad C_2 = 0$$

$$[x=\frac{L}{2}, V=0] \quad -\frac{w_0}{L} \left(\frac{L}{2}\right)^2 + R_A = 0 \quad R_A = \frac{1}{4} w_0 L \quad \blacktriangleleft$$

$$[x=\frac{L}{2}, \frac{d^2y}{dx^2}=0] \quad -\frac{1}{12} \frac{w_0}{L} \left(\frac{L}{2}\right)^4 + \frac{1}{2} \left(\frac{1}{4} w_0 L\right) \left(\frac{L}{2}\right)^2 + M_A \frac{L}{2} + 0 = 0$$

$$M_A = -2 \left( \frac{1}{32} - \frac{1}{192} \right) w_0 L^2 = -\frac{5}{96} w_0 L^2 = -0.05208 w_0 L^2 \quad \blacktriangleleft$$

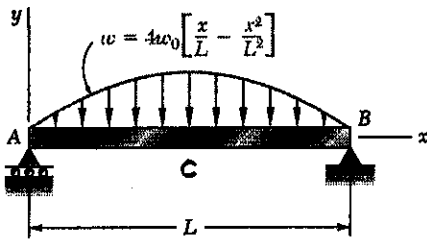
From (2), with  $x = \frac{L}{2}$

$$M_C = -\frac{1}{3} \frac{w_0}{L} \left(\frac{L}{2}\right)^3 + \left(\frac{1}{4} w_0 L\right) \left(\frac{L}{2}\right) - \frac{5}{96} w_0 L^2$$

$$= \left( -\frac{1}{24} + \frac{1}{8} - \frac{5}{96} \right) w_0 L^2 = \frac{1}{32} w_0 L^2 = 0.03125 w_0 L^2 \quad \blacktriangleleft$$

PROBLEM 9.33

9.33 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection at the midpoint of the span.



$[x=0, M=0]$                        $[x=L, M=0]$   
 $[x=0, y=0]$                          $[x=L, y=0]$

SOLUTION

Boundary conditions at A and B are noted.

$$w = \frac{w_0}{L^2} (4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} (4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2} \left( \frac{4}{3}x^3 - 2Lx^2 \right) + C_1$$

$$M = \frac{w_0}{L^2} \left( \frac{1}{3}x^4 - \frac{2}{3}Lx^3 \right) + C_1x + C_2$$

$[x=0, M=0]$                        $0 = 0 + 0 + 0 + C_2$                        $C_2 = 0$   
 $[x=L, M=0]$                        $0 = \frac{w_0}{L^2} \left( \frac{1}{3}L^4 - \frac{2}{3}L^4 \right) + C_1L + 0$                        $C_1 = \frac{1}{3}w_0L^2$

$$EI \frac{d^2y}{dx^2} = M = \frac{w_0}{L^2} \left( \frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x \right)$$

$$EI \frac{dy}{dx} = \frac{w_0}{L^2} \left( \frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^2x^2 \right) + C_3$$

$$EI y = \frac{w_0}{L^2} \left( \frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^2x^3 \right) + C_3x + C_4$$

$[x=0, y=0]$                        $0 = 0 + 0 + 0 + 0 + C_4$                        $C_4 = 0$   
 $[x=L, y=0]$                        $0 = \frac{w_0}{L^2} \left( \frac{1}{90}L^6 - \frac{1}{30}L^6 + \frac{1}{18}L^6 \right) + C_3L + 0$                        $C_3 = -\frac{1}{30}w_0L^3$

(a) Elastic curve:  $y = \frac{w_0}{EIL^2} \left( \frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^2x^3 - \frac{1}{30}L^3x \right)$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2} \left( \frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^2x^2 - \frac{1}{30}L^3 \right)$$

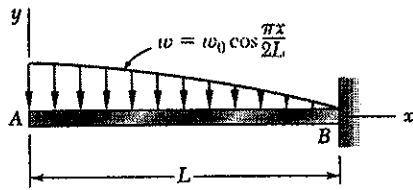
(b) Slope at end A. Set  $x=0$  in  $\frac{dy}{dx}$                        $\left. \frac{dy}{dx} \right|_A = -\frac{1}{30} \frac{w_0L^3}{EI}$   
 $\theta_A = \frac{1}{30} \frac{w_0L^3}{EI}$

(c) Deflection at midpoint. Set  $x = \frac{L}{2}$  in  $y$

$$\begin{aligned}
 y_c &= \frac{w_0L^4}{EI} \left\{ \left( \frac{1}{90} \right) \left( \frac{1}{2} \right)^6 - \left( \frac{1}{30} \right) \left( \frac{1}{2} \right)^5 + \frac{1}{18} \left( \frac{1}{2} \right)^3 - \frac{1}{30} \left( \frac{1}{2} \right) \right\} \\
 &= \frac{w_0L^4}{EI} \left\{ \frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60} \right\} = -\frac{61}{5760} \frac{w_0L^4}{EI} \\
 y_c &= \frac{61}{5760} \frac{w_0L^4}{EI} \downarrow
 \end{aligned}$$

PROBLEM 9.34

9.34 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at the free end, (c) the deflection at the free end.



SOLUTION

$[x=0, V=0]$   
 $[x=0, M=0]$

$[x=L, \frac{dy}{dx}=0]$   
 $[x=L, y=0]$

$$\frac{dV}{dx} = -W = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\frac{2w_0 L}{\pi} \sin \frac{\pi x}{2L} + C_1$$

$[x=0, V=0] \quad 0 = 0 + C_1 \quad C_1 = 0$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi} \sin \frac{\pi x}{2L}$$

$$M = \frac{4w_0 L^2}{\pi^2} \cos \frac{\pi x}{2L} + C_2$$

$[x=0, M=0] \quad C_2 = -\frac{4w_0 L^2}{\pi^2}$

$$EI \frac{d^2 y}{dx^2} = M = \frac{4w_0 L^2}{\pi^2} (\cos \frac{\pi x}{2L} - 1)$$

$$EI \frac{dy}{dx} = \frac{4w_0 L^2}{\pi^2} \left( \frac{2L}{\pi} \sin \frac{\pi x}{2L} - x \right) + C_3$$

$[x=L, \frac{dy}{dx}=0] \quad \frac{4w_0 L^2}{\pi^2} \left( \frac{2L}{\pi} - L \right) + C_3 = 0 \quad C_3 = \frac{4w_0 L^3}{\pi^3} (\pi - 2)$

$$EI y = \frac{4w_0 L^2}{\pi^2} \left[ -\frac{4L^2}{\pi^2} \cos \frac{\pi x}{2L} - \frac{1}{2} x^2 \right] + C_3 x + C_4$$

$[x=L, y=0] \quad \frac{4w_0 L^2}{\pi^2} \left( -\frac{1}{2} L^2 \right) + C_3 L + C_4 = 0$

$$C_4 = \frac{2w_0 L^4}{\pi^2} - C_3 L$$

(a) Elastic curve  $y = \frac{w_0}{EI} \left\{ -\frac{16L^4}{\pi^4} \cos \frac{\pi x}{2L} - \frac{2L^2 x^2}{\pi^2} + \frac{4L^3}{\pi^3} (\pi - 2)(x - L) + \frac{2L^4}{\pi^2} \right\}$

$$y = \frac{2w_0 L^4}{\pi^4 EI} \left\{ -8 \cos \frac{\pi x}{2L} - \pi^2 \frac{x^2}{L^2} + 2\pi(\pi - 2) \frac{x}{L} + \pi(4 - \pi) \right\}$$

(b) Slope at free end ( $x=0$ )

$$EI \frac{dy}{dx} \Big|_{x=0} = C_3 = \frac{4(\pi - 2)}{\pi^3} w_0 L^3$$

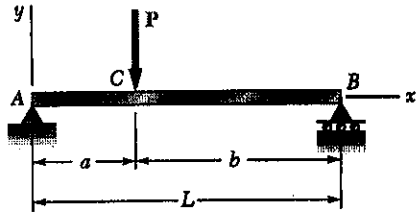
$$\frac{dy}{dx} \Big|_A = \frac{4(\pi - 2)}{\pi^3} \frac{w_0 L^3}{EI} = 0.14727 \frac{w_0 L^3}{EI}$$

(c) Deflection at free end ( $x=0$ )

$$y_A = \frac{2w_0 L^4}{\pi^4 EI} \left\{ -8 + \pi(4 - \pi) \right\} = -0.10889 \frac{w_0 L^4}{EI}$$

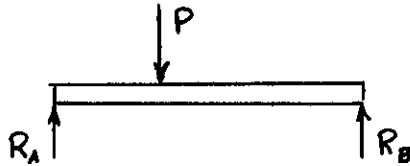
PROBLEM 9.35

9.35 through 9.38 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.



$[x=0, M=0]$   
 $[x=0, y=0]$

$[x=L, M=0]$   
 $[x=L, y=0]$



SOLUTION

$\rightarrow \sum M_B = 0 \quad -R_A L + P b = 0 \quad R_A = \frac{P b}{L}$

$\frac{dM}{dx} = V = R_A - P \langle x-a \rangle^0 = \frac{P b}{L} - P \langle x-a \rangle^0$

$M = \frac{P b}{L} x - P \langle x-a \rangle^1 + M_A$

$EI \frac{d^2 y}{dx^2} = \frac{P b}{L} x - P \langle x-a \rangle^0$

$EI \frac{dy}{dx} = \frac{P b}{2L} x^2 - \frac{1}{2} P \langle x-a \rangle^2 + C_1$

$EI y = \frac{P b}{6L} x^3 - \frac{1}{6} P \langle x-a \rangle^3 + C_1 x + C_2$

$[x=0, y=0] \quad C_2 = 0$

$[x=L, y=0] \quad \frac{P b}{6L} L^3 - \frac{1}{6} P (L-a)^3 + C_1 L = 0$

$C_1 = -\frac{1}{6L} P (bL^2 - b^3) = -\frac{1}{6} \frac{P b}{L} (L^2 - b^2)$

(a) Elastic curve  $y = \frac{P}{EI} \left\{ \frac{b}{6L} x^3 - \frac{1}{6} \langle x-a \rangle^3 - \frac{1}{6} \frac{b}{L} (L^2 - b^2) x \right\}$

$y_c = \frac{P}{6EIL} \left\{ b x^3 - L \langle x-a \rangle^3 - b (L^2 - b^2) \right\}$

(b) Slope at end A.

$EI \left. \frac{dy}{dx} \right|_{x=0} = C_1 = -\frac{P b}{6L} (L^2 - b^2)$

$\theta_A = -\frac{P b}{6EIL} (L^2 - b^2)$

(c) Deflection at C

$EI y_c = \frac{P b}{6L} a^3 + C_1 a = \frac{P b a^3}{6L} - \frac{P b (L^2 - b^2) a}{6L}$

$= \frac{P b a}{6L} (a^2 - L^2 + b^2)$

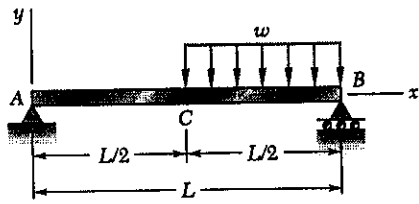
$y_c = -\frac{P a b}{6EIL} (L^2 - a^2 - b^2) = -\frac{P a b}{6EIL} \{ a^2 + 2ab + b^2 - a^2 - b^2 \}$

$= -\frac{P a^2 b^2}{3EIL}$

PROBLEM 9.36

9.35 through 9.38 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.

SOLUTION



$[x=0, M=0]$   
 $[x=0, y=0]$

$[x=L, M=0]$   
 $[x=L, y=0]$

$$\frac{dV}{dx} = -w \langle x - \frac{L}{2} \rangle^0$$

$$\frac{dM}{dx} = V = R_A - w \langle x - \frac{L}{2} \rangle^1$$

$$M = M_A + R_A x - \frac{1}{2} w \langle x - \frac{L}{2} \rangle^2$$

$[x=L, M=0]$   $R_A L - \frac{1}{2} w (\frac{L}{2})^2 = 0$

$$R_A = \frac{1}{8} w L$$

$$EI \frac{d^2 y}{dx^2} = \frac{1}{8} w L x - \frac{1}{2} w \langle x - \frac{L}{2} \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{16} w L x^2 - \frac{1}{6} w \langle x - \frac{L}{2} \rangle^3 + C_1$$

$$EI y = \frac{1}{48} w L x^3 - \frac{1}{24} w \langle x - \frac{L}{2} \rangle^4 + C_1 x + C_2$$

$[x=0, y=0]$

$$0 = 0 + 0 + 0 + C_2$$

$$C_2 = 0$$

$[x=L, y=0]$

$$\frac{1}{48} w L^4 - \frac{1}{24} w (\frac{L}{2})^4 + C_1 L + 0 = 0$$

$$C_1 = -(\frac{1}{48} - \frac{1}{24} \cdot \frac{1}{16}) w L^3 = -\frac{7}{384} w L^3$$

(a) Elastic curve

$$EI y = \frac{1}{48} w L x^3 - \frac{1}{24} w \langle x - \frac{L}{2} \rangle^4 - \frac{7}{384} w L^3 x$$

$$y = \frac{w}{EI} \left\{ \frac{1}{48} L x^3 - \frac{1}{24} \langle x - \frac{L}{2} \rangle^4 - \frac{7}{384} L^3 x \right\}$$

$$\frac{dy}{dx} = \frac{w}{EI} \left\{ \frac{1}{16} L x^2 - \frac{1}{6} \langle x - \frac{L}{2} \rangle^3 - \frac{7}{384} L^3 \right\}$$

(b) Slope at A ( $x=0$  in  $\frac{dy}{dx}$ )

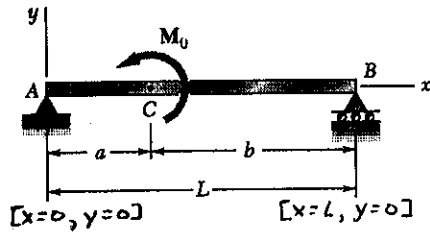
$$\theta_A = -\frac{7}{384} \frac{w L^3}{EI}$$

(c) Deflection at C ( $x = \frac{L}{2}$  in  $y$ )

$$y_C = \frac{w L^4}{EI} \left\{ \frac{1}{48} \cdot \frac{1}{8} - \frac{7}{384} \cdot \frac{1}{2} \right\} = \left( \frac{1}{384} - \frac{7}{768} \right) \frac{w L^4}{EI} = -\frac{5}{768} \frac{w L^4}{EI}$$

PROBLEM 9.37

9.35 through 9.38 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.



SOLUTION

Reactions  $R_A = \frac{M_0}{L} \uparrow$ ,  $R_B = \frac{M_0}{L} \downarrow$

$0 < x < a$   $M = R_A x$

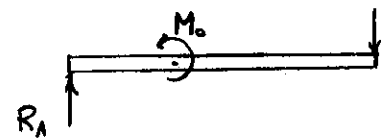
$a < x < L$   $M = R_A x - M_0$

Using singularity functions

$$EI \frac{d^2 y}{dx^2} = M = R_A x - M_0 \langle x-a \rangle^0$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - M_0 \langle x-a \rangle' + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 \langle x-a \rangle^2 + C_1 x + C_2$$



$[x=0, y=0]$   $0 = 0 - 0 + 0 + C_2$   $C_2 = 0$

$[x=L, y=0]$   $\frac{1}{6} R_A L^3 - \frac{1}{2} M_0 (L-a)^2 + C_1 L + 0 = 0$

$$C_1 L = -\frac{1}{6} \frac{M_0}{L} L^3 + \frac{1}{2} M_0 b^2 \quad C_1 = \frac{M_0}{6L} (3b^2 - L^2)$$

(a) Elastic curve  $y = \frac{1}{EI} \left\{ \frac{1}{6} \frac{M_0}{L} x^3 - \frac{1}{2} M_0 \langle x-a \rangle^2 + \frac{M_0}{6L} (3b^2 - L^2) x \right\}$

$$= \frac{M_0}{6EIL} \left\{ x^3 - 3L \langle x-a \rangle^2 + (3b^2 - L^2) x \right\}$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL} \left\{ 3x^2 - 6L \langle x-a \rangle' + (3b^2 - L^2) \right\}$$

(b) Slope at A  $\left( \frac{dy}{dx} \text{ at } x=0 \right)$

$$\theta_A = \frac{M_0}{6EIL} \left\{ 0 - 0 + 3Lb^2 - L^3 \right\} = \frac{M_0}{6EIL} (3b^2 - L^2)$$

(c) Deflection at C  $(y \text{ at } x=a)$

$$y_c = \frac{M_0}{6EIL} \left\{ a^3 - 0 + (3b^2 - L^2) a \right\} = \frac{M_0 a}{6EIL} \left\{ a^2 + 3b^2 - (a+b)^2 \right\}$$

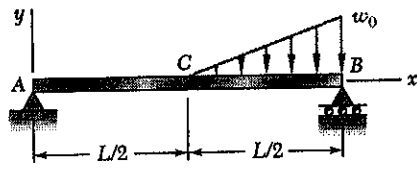
$$= \frac{M_0 a}{6EIL} \left\{ a^2 + 3b^2 - a^2 - 2ab - b^2 \right\} = \frac{M_0 a}{6EIL} \left\{ 2b^2 - 2ab \right\}$$

$$= \frac{M_0 ab}{3EIL} (b-a) \uparrow$$



PROBLEM 9.38

9.35 through 9.38 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the slope at end A, (c) the deflection of point C.



SOLUTION

$[x=0, y=0]$   
 $[x=0, M=0]$

$[x=L, y=0]$   
 $[x=L, M=0]$

$$w = \frac{2w_0}{L} \left\langle x - \frac{L}{2} \right\rangle'$$

$$\frac{dV}{dx} = -w = -\frac{2w_0}{L} \left\langle x - \frac{L}{2} \right\rangle'$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^2 + R_A$$

$$M = -\frac{1}{3} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^3 + R_A x + M_A$$

$[x=0, M=0] \quad M_A = 0$

$[x=L, M=0] \quad -\frac{1}{3} \frac{w_0}{L} \left(\frac{L}{2}\right)^3 + R_A L + 0 = 0 \quad R_A = \frac{1}{24} w_0 L$

$EI \frac{d^2y}{dx^2} = M = -\frac{1}{3} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^3 + \frac{1}{24} w_0 L x$

$EI \frac{dy}{dx} = -\frac{1}{12} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^4 + \frac{1}{48} w_0 L x^2 + C_1$

$EI y = -\frac{1}{60} \frac{w_0}{L} \left\langle x - \frac{L}{2} \right\rangle^5 + \frac{1}{144} w_0 L x^3 + C_1 x + C_2$

$[x=0, y=0] \quad 0 = 0 + 0 + 0 + C_2 \quad C_2 = 0$

$[x=L, y=0] \quad -\frac{1}{60} \frac{w_0}{L} \left(\frac{L}{2}\right)^5 + \frac{1}{144} w_0 L^4 + C_1 L + 0 = 0$

$C_1 = -\left(\frac{1}{144} - \frac{1}{1920}\right) w_0 L^3 = -\frac{37}{5760} w_0 L^3$

(a) Elastic curve  $y = \frac{w_0}{EIL} \left\{ -\frac{1}{60} \left\langle x - \frac{L}{2} \right\rangle^5 + \frac{1}{144} L^2 x^3 - \frac{37}{5760} L^4 x \right\}$

$\frac{dy}{dx} = \frac{w_0}{EIL} \left\{ -\frac{1}{12} \left\langle x - \frac{L}{2} \right\rangle^4 + \frac{1}{48} L^2 x^2 - \frac{37}{5760} L^4 \right\}$

(b) Slope at A  $\left(\frac{dy}{dx} \text{ at } x=0\right)$

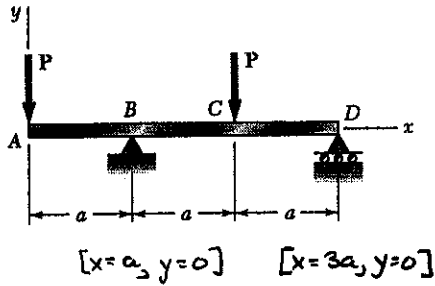
$\theta_A = \frac{w_0}{EIL} \left\{ 0 + 0 - \frac{37}{5760} L^4 \right\} = -\frac{37}{5760} \frac{w_0 L^3}{EI}, \quad \theta_A = \frac{37}{5760} \frac{w_0 L^3}{EI}$

(c) Deflection at C  $(y \text{ at } x = \frac{L}{2})$

$y_C = \frac{w_0}{EIL} \left\{ 0 + \frac{1}{144} L^2 \left(\frac{L}{2}\right)^3 - \frac{37}{5760} L^4 \left(\frac{L}{2}\right) \right\}$   
 $= \left(\frac{1}{1152} - \frac{37}{11520}\right) \frac{w_0 L^4}{EI} = -\frac{3}{1280} \frac{w_0 L^4}{EI}$

PROBLEM 9.39

9.39 and 9.40 For the beam and loading shown, determine (a) the deflection at end A, (b) the deflection point C, (c) the slope at end D.



SOLUTION

Reactions:  $R_B = 2P \uparrow, R_D = 0$

$$\begin{aligned} 0 < x < a & V = -P \\ a < x < 2a & V = -P + 2P \\ 2a < x < 3a & V = -P + 2P - P \end{aligned}$$

Using singularity functions

$$\begin{aligned} \frac{dM}{dx} = V &= -P + 2P\langle x-a \rangle^0 - P\langle x-2a \rangle^0 \\ M &= -Px + 2P\langle x-a \rangle' - P\langle x-2a \rangle' + M_A \end{aligned}$$

But  $M = 0$  at  $x=0$   $M_A = 0$

$$EI \frac{d^2y}{dx^2} = M = -Px + 2P\langle x-a \rangle' - P\langle x-2a \rangle' \tag{1}$$

$$EI \frac{dy}{dx} = -\frac{1}{2}Px^2 + P\langle x-a \rangle^2 - \frac{1}{2}P\langle x-2a \rangle^2 + C_1 \tag{2}$$

$$EI y = -\frac{1}{6}Px^3 + \frac{1}{3}P\langle x-a \rangle^3 - \frac{1}{6}P\langle x-2a \rangle^3 + C_1x + C_2 \tag{3}$$

$$[x=a, y=0] \quad -\frac{1}{6}Pa^3 + 0 - 0 + C_1a + C_2 = 0 \quad aC_1 + C_2 = \frac{1}{6}Pa^3 \tag{4}$$

$$[x=3a, y=0] \quad -\frac{1}{6}P(3a)^3 + \frac{1}{3}P(2a)^3 - \frac{1}{6}Pa^3 + C_1(3a) + C_2 = 0 \quad 3aC_1 + C_2 = 2Pa^2 \tag{5}$$

$$Eq(5) - Eq(4) \quad 2Ca = \frac{11}{6}Pa^2 \quad C_1 = \frac{11}{12}Pa^2$$

$$C_2 = \frac{1}{6}Pa^2 - aC_1 = -\frac{3}{4}Pa^3$$

$$y = \frac{P}{EI} \left\{ -\frac{1}{6}x^3 + \frac{1}{3}\langle x-a \rangle^3 - \frac{1}{6}\langle x-2a \rangle^3 + \frac{11}{12}a^2x - \frac{3}{4}a^3 \right\}$$

$$\frac{dy}{dx} = \frac{P}{EI} \left\{ -\frac{1}{2}x^2 + \langle x-a \rangle^2 - \frac{1}{2}\langle x-2a \rangle^2 + \frac{11}{12}a^2 \right\}$$

(a) Deflection at A ( $y$  at  $x=0$ )

$$y_A = \frac{Pa^3}{EI} \left\{ 0 + 0 - 0 + 0 - \frac{3}{4} \right\} = -\frac{3}{4} \frac{Pa^3}{EI} \quad y_A = \frac{3}{4} \frac{Pa^3}{EI} \downarrow$$

(b) Deflection at C ( $y$  at  $x=2a$ )

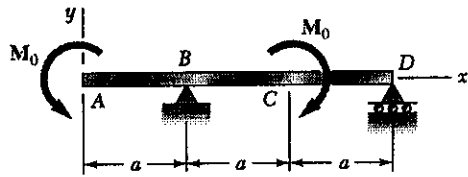
$$y_C = \frac{Pa^3}{EI} \left\{ -\frac{1}{6}(2)^3 + \frac{1}{3}(1)^3 - 0 + \frac{11}{12}(2) - \frac{3}{4} \right\} = \frac{1}{12} \frac{Pa^3}{EI} \uparrow$$

(c) Slope at D ( $\frac{dy}{dx}$  at  $x=3a$ )

$$\theta_D = \frac{Pa^2}{EI} \left\{ -\frac{1}{2}(3)^2 + (2)^2 - \frac{1}{2}(1)^2 + \frac{11}{12} \right\} = -\frac{1}{12} \frac{Pa^2}{EI} \quad \theta_D = \frac{1}{12} \frac{Pa^2}{EI} \leftarrow$$

PROBLEM 9.40

9.39 and 9.40 For the beam and loading shown, determine (a) the deflection at end A, (b) the deflection at point C, (c) the slope at end D.



$[x=a, y=0]$      $[x=2a, y=0]$

SOLUTION

Since loads self equilibrate

$R_B = 0$      $R_D = 0$

$0 < x < 2a$      $M = -M_0$

$2a < x < 3a$      $M = -M_0 + M_0 = 0$

Using singularity functions

$EI \frac{d^2y}{dx^2} = M = -M_0 + M_0 \langle x-2a \rangle^0$

$EI \frac{dy}{dx} = -M_0 x + M_0 \langle x-2a \rangle^1 + C_1$

$EI y = -\frac{1}{2} M_0 x^2 + \frac{1}{2} M_0 \langle x-2a \rangle^2 + C_1 x + C_2$

$[x=3a, y=0]$      $-\frac{1}{2} M_0 (3a)^2 + \frac{1}{2} M_0 a^2 + C_1 (3a) + C_2 = 0$

$3aC_1 + C_2 = 4M_0 a^2$

$[x=a, y=0]$      $-\frac{1}{2} M_0 a^2 + 0 + C_1 a + C_2 = 0$

$aC_1 + C_2 = \frac{1}{2} M_0 a^2$

Subtracting     $2aC_1 = \frac{7}{2} M_0 a^2$      $C_1 = \frac{7}{4} M_0 a$

$C_2 = \frac{1}{2} M_0 a^2 - aC_1 = -\frac{5}{4} M_0 a^2$

$y = \frac{M_0}{EI} \left\{ -\frac{1}{2} x^2 + \frac{1}{2} \langle x-2a \rangle^2 + \frac{7}{4} ax - \frac{5}{4} a^2 \right\}$

$\frac{dy}{dx} = \frac{M_0}{EI} \left\{ -x + \langle x-a \rangle^1 + \frac{7}{4} a \right\}$

(a) Deflection at A (y at  $x=0$ )

$y_A = \frac{M_0 a^2}{EI} \left\{ -0 + 0 + 0 - \frac{5}{4} \right\} = -\frac{5}{4} \frac{M_0 a^2}{EI}$ ,     $y_A = \frac{5}{4} \frac{M_0 a^2}{EI} \downarrow$

(b) Deflection at C (y at  $x=2a$ )

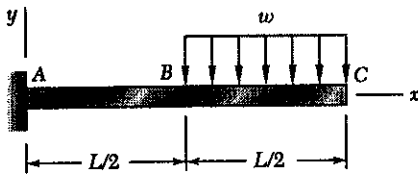
$y_C = \frac{M_0 a^2}{EI} \left\{ -\frac{1}{2} (2)^2 + 0 + \frac{7}{4} (2) - \frac{5}{4} \right\} = \frac{1}{4} \frac{M_0 a^2}{EI} \uparrow$

(c) Slope at D ( $\frac{dy}{dx}$  at  $x=3a$ )

$\theta_D = \frac{M_0 a}{EI} \left\{ -3 + 1 + \frac{7}{4} \right\} = -\frac{1}{4} \frac{M_0 a}{EI}$ ,     $\theta_D = \frac{1}{4} \frac{M_0 a}{EI} \swarrow$

PROBLEM 9.41

9.41 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point B, (c) the deflection at point C.



$[x=0, \frac{dy}{dx}=0]$   
 $[x=0, y=0]$

SOLUTION

$$\uparrow \sum F_y = 0 \quad R_A - \frac{1}{2}wL = 0 \quad R_A = \frac{1}{2}wL$$

$$\circlearrowleft \sum M_A = 0 \quad -M_A - (\frac{1}{2}wL)(\frac{3}{4}L) = 0$$

$$M_A = -\frac{3}{8}wL^2$$

$0 < x < \frac{L}{2} \quad M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx$

$\frac{L}{2} < x < L$  (See free body diagram.)

$\circlearrowleft \sum M_k = 0$

$$\frac{3}{8}wL^2 - \frac{1}{2}wLx + \frac{1}{2}w(x-\frac{L}{2})^2 + M = 0$$

$$M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w(x-\frac{L}{2})^2$$

Using singularity functions

$$EI \frac{d^2y}{dx^2} = M = -\frac{3}{8}wL^2 + \frac{1}{2}wLx - \frac{1}{2}w(x-\frac{L}{2})^2$$

$$EI \frac{dy}{dx} = -\frac{3}{8}wL^2x + \frac{1}{4}wLx^2 - \frac{1}{6}w(x-\frac{L}{2})^3 + C_1$$

$[x=0, \frac{dy}{dx}=0] \quad -0 + 0 - 0 + C_1 = 0 \quad C_1 = 0$

$$EI y = -\frac{3}{16}wL^2x^2 + \frac{1}{12}wLx^3 - \frac{1}{24}w(x-\frac{L}{2})^4 + C_1x + C_2$$

$[x=0, y=0] \quad -0 + 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$

(a) Elastic curve  $y = \frac{w}{EI} \left\{ -\frac{3}{16}L^2x^2 + \frac{1}{12}Lx^3 - \frac{1}{24}(x-\frac{L}{2})^4 \right\}$

$$\frac{dy}{dx} = \frac{w}{EI} \left\{ -\frac{3}{8}L^2x + \frac{1}{4}Lx^2 - \frac{1}{6}(x-\frac{L}{2})^3 \right\}$$

(b) Deflection at B (y at  $x = \frac{L}{2}$ )

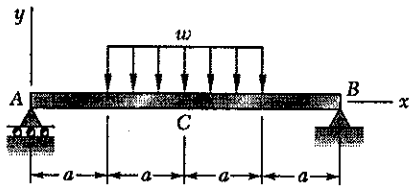
$$y_B = \frac{wL^4}{EI} \left\{ -\frac{3}{16}(\frac{1}{2})^2 + \frac{1}{12}(\frac{1}{2})^3 - 0 \right\} = -\frac{7}{192} \frac{wL^4}{EI}, \quad y_B = \frac{7}{192} \frac{wL^4}{EI} \downarrow$$

(c) Deflection at C (y at  $x = L$ )

$$y_C = \frac{wL^4}{EI} \left\{ -\frac{3}{16}(1)^2 + \frac{1}{12}(1)^3 - \frac{1}{24}(\frac{1}{2})^4 \right\} = -\frac{41}{384} \frac{wL^4}{EI}, \quad y_C = \frac{41}{384} \frac{wL^4}{EI} \downarrow$$

PROBLEM 9.42

9.42 and 9.43 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the midpoint C.



SOLUTION

By symmetry  $R_A = R_B$   
 $\uparrow \Sigma F_y = 0 \quad R_A + R_B - 2wa = 0 \quad R_A = wa$

$[x=0, y=0]$   
 $[x=0, M=0]$

$[x=4a, y=0]$   
 $[x=4a, M=0]$

$$w(x) = w\langle x-a \rangle^0 - w\langle x-3a \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a \rangle^0 + w\langle x-3a \rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x-a \rangle^1 + w\langle x-3a \rangle^1$$

$$M = M_A + R_A x - \frac{1}{2}w\langle x-a \rangle^2 + \frac{1}{2}w\langle x-3a \rangle^2 \quad \text{with } M_A = 0$$

$$EI \frac{d^2y}{dx^2} = M = wax - \frac{1}{2}w\langle x-a \rangle^2 + \frac{1}{2}w\langle x-3a \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}w\langle x-a \rangle^3 + \frac{1}{6}w\langle x-3a \rangle^3 + C_1$$

$$EI y = \frac{1}{6}wax^3 - \frac{1}{24}w\langle x-a \rangle^4 + \frac{1}{24}w\langle x-3a \rangle^4 + C_1 x + C_2$$

$[x=0, y=0] \quad 0 - 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x=4a, y=0] \quad \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(3a)^4 + \frac{1}{24}w(a)^4 + C_1(4a) = 0$

$$4C_1 = wa^3 \left\{ -\frac{64}{6} + \frac{81}{24} - \frac{1}{24} \right\} = -\frac{22}{3}wa^3 \quad C_1 = -\frac{11}{6}wa^3$$

(a) Equation of elastic curve

$$y = \frac{W}{EI} \left\{ \frac{1}{6}ax^3 - \frac{1}{24}\langle x-a \rangle^4 + \frac{1}{24}\langle x-3a \rangle^4 - \frac{11}{6}a^3x \right\}$$

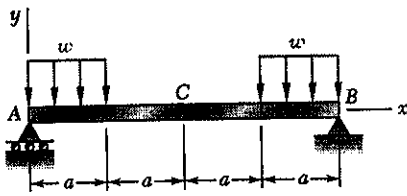
(b) Deflection at C (y at  $x=2a$ )

$$y_c = \frac{wa^4}{EI} \left\{ \frac{1}{6}(2)^3 - \frac{1}{24}(1)^4 + 0 - \frac{11}{6}(2) \right\} = -\frac{19}{8} \frac{wa^4}{EI}$$

$$y_c = \frac{19}{8} \frac{wa^4}{EI} \downarrow$$

PROBLEM 9.43

9.42 and 9.43 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at the midpoint C.



$[x=0, y=0]$   
 $[x=0, M=0]$

$[x=4a, y=0]$   
 $[x=4a, M=0]$

SOLUTION

By symmetry  $R_A = R_B$

$\uparrow \Sigma F_y = 0 \quad R_A + R_B - 2wa = 0 \quad R_A = wa$

$w(x) = w - w\langle x-a \rangle^0 + w\langle x-3a \rangle^0$

$\frac{dV}{dx} = -w(x) = -w + w\langle x-a \rangle^0 - w\langle x-3a \rangle^0$

$\frac{dM}{dx} = V = R_A - wx + w\langle x-a \rangle^1 - w\langle x-3a \rangle^1$

$M = M_A + R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a \rangle^2 - \frac{1}{2}w\langle x-3a \rangle^2$  with  $M_A = 0$

$EI \frac{d^2y}{dx^2} = M = wax - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a \rangle^2 - \frac{1}{2}w\langle x-3a \rangle^2$

$EI \frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}wx^3 + \frac{1}{6}w\langle x-a \rangle^3 - \frac{1}{6}w\langle x-3a \rangle^3 + C_1$

$EI y = \frac{1}{6}wax^3 - \frac{1}{24}wx^4 + \frac{1}{24}w\langle x-a \rangle^4 - \frac{1}{24}w\langle x-3a \rangle^4 + C_1 x + C_2$

$[x=0, y=0] \quad 0 - 0 + 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x=4a, y=0] \quad \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(4a)^4 + \frac{1}{24}w(3a)^4 - \frac{1}{24}wa^4 + C_1(4a) = 0$

$4C_1 = wa^3 \left\{ -\frac{64}{6} + \frac{256}{24} - \frac{81}{24} + \frac{1}{24} \right\} = -\frac{10}{3}wa^3 \quad C_1 = -\frac{5}{6}wa^3$

(a) Equation of elastic curve

$y = \frac{W}{EI} \left\{ \frac{1}{6}ax^3 - \frac{1}{24}x^4 + \frac{1}{24}\langle x-a \rangle^4 - \frac{1}{24}\langle x-3a \rangle^4 - \frac{5}{6}a^3x \right\}$

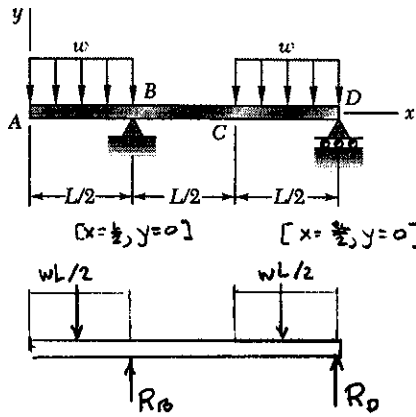
(b) Deflection at C (y at  $x=2a$ )

$y_c = \frac{wa^4}{EI} \left\{ \frac{1}{6}(2)^3 - \frac{1}{24}(2)^4 + \frac{1}{24}(1)^4 + 0 - \frac{5}{6}(2) \right\} = -\frac{23}{24} \frac{wa^4}{EI}$

$y_c = \frac{23}{24} \frac{wa^4}{EI} \downarrow$

PROBLEM 9.44

9.44 For the beam and loading shown, determine (a) the equation of the elastic curve, (b) the deflection at point A, (c) the deflection at point C.



SOLUTION

$$\begin{aligned} \sum M_D = 0 \quad \frac{wL}{2} \cdot \frac{5L}{4} - R_B L + \frac{wL}{2} \cdot \frac{L}{4} &= 0 \\ R_B &= \frac{3}{4} wL \end{aligned}$$

$$w(x) = w - w \langle x - \frac{L}{2} \rangle^0 + w \langle x - L \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w \langle x - \frac{L}{2} \rangle^0 - w \langle x - L \rangle^0$$

$$\frac{dM}{dx} = V = -wx + R_B \langle x - \frac{L}{2} \rangle^0 + w \langle x - \frac{L}{2} \rangle^1 - w \langle x - L \rangle^1$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{2} wx^2 + \frac{3}{4} wL \langle x - \frac{L}{2} \rangle^1 + \frac{1}{2} w \langle x - \frac{L}{2} \rangle^2 - \frac{1}{2} w \langle x - L \rangle^2$$

$$EI \frac{dy}{dx} = -\frac{1}{6} wx^3 + \frac{3}{8} wL \langle x - \frac{L}{2} \rangle^2 + \frac{1}{6} w \langle x - \frac{L}{2} \rangle^3 - \frac{1}{6} w \langle x - L \rangle^3 + C_1$$

$$EI y = -\frac{1}{24} wx^4 + \frac{1}{8} wL \langle x - \frac{L}{2} \rangle^3 + \frac{1}{24} w \langle x - \frac{L}{2} \rangle^4 - \frac{1}{24} w \langle x - L \rangle^4 + C_1 x + C_2$$

$$[x = \frac{L}{2}, y = 0] \quad -\frac{1}{24} w \left(\frac{L}{2}\right)^4 + 0 + 0 - 0 + C_1 \frac{L}{2} + C_2 = 0$$

$$C_2 = \frac{1}{384} wL^4 - C_1 \frac{L}{2}$$

$$[x = \frac{3L}{2}, y = 0] \quad -\frac{1}{24} w \left(\frac{3L}{2}\right)^4 + \frac{1}{8} wL L^3 + \frac{1}{24} wL^4 - \frac{1}{24} w \left(\frac{L}{2}\right)^4 + C_1 \frac{3L}{2} + \left(\frac{1}{384} wL^4 - C_1 \frac{L}{2}\right) = 0$$

$$\left(\frac{3}{2} - \frac{1}{2}\right) C_1 L = \left(\frac{1}{24} \cdot \frac{81}{16} - \frac{1}{8} - \frac{1}{24} + \frac{1}{24} \cdot \frac{1}{16} - \frac{1}{384}\right) wL^4 \quad C_1 = \frac{17}{384} wL^3$$

$$C_2 = \left(\frac{1}{384} - \frac{17}{768}\right) wL^4 = -\frac{5}{256} wL^4$$

$$(a) \quad y = \frac{w}{EI} \left\{ -\frac{1}{24} x^4 + \frac{1}{8} L \langle x - \frac{L}{2} \rangle^3 + \frac{1}{24} \langle x - \frac{L}{2} \rangle^4 - \frac{1}{24} \langle x - L \rangle^4 + \frac{17}{384} L^3 x - \frac{5}{256} L^4 \right\}$$

(b) Deflection at A (y at x = 0)

$$y_A = \frac{w}{EI} \left\{ 0 + 0 + 0 + 0 + 0 - \frac{5}{256} L^4 \right\} = -\frac{5}{256} \frac{wL^4}{EI}$$

$$y_A = \frac{5}{256} \frac{wL^4}{EI} \downarrow$$

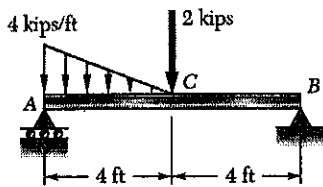
(c) Deflection at C (y at x = L)

$$y_C = \frac{w}{EI} \left\{ -\frac{1}{24} L^4 + \frac{1}{8} L \frac{L^3}{8} + \frac{1}{24} \frac{L^4}{16} - 0 + \frac{17}{384} L^3 L - \frac{5}{256} L^4 \right\}$$

$$= \frac{1}{768} \frac{wL^4}{EI} \uparrow$$

PROBLEM 9.45

9.45 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use  $E = 29 \times 10^6$  psi.



SOLUTION

Distributed loads: ①  $w_1(x) = w_0 - kx$   
 ②  $w_2(x) = kx$

Data:  $a = 4$  ft,  $w_0 = 4$  kips/ft,  $k = 1$  kip/ft<sup>2</sup>  
 $P = 2$  kips.

$$+\circlearrowleft \sum M_B = 0 \quad -8R_A + (8)(6\frac{2}{3}) + (2)(4) = 0 \quad R_A = \frac{23}{3} \text{ kips}$$

$$w(x) = w_0 - kx + k\langle x-4 \rangle$$

$$= 4 - x + \langle x-4 \rangle$$

$$\frac{dV}{dx} = -w = -4 + x - \langle x-4 \rangle$$

$$\frac{dM}{dx} = V = \frac{23}{3} - 4x + \frac{1}{2}x^2 - \frac{1}{2}\langle x-4 \rangle^2 - 2\langle x-4 \rangle$$

$$EI \frac{d^2y}{dx^2} = M = \frac{23}{3}x - 2x^2 + \frac{1}{6}x^3 - \frac{1}{6}\langle x-4 \rangle^3 - 2\langle x-4 \rangle \quad \text{kip}\cdot\text{ft}$$

$$EI \frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}\langle x-4 \rangle^4 - \langle x-4 \rangle^2 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI y = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}\langle x-4 \rangle^5 - \frac{1}{3}\langle x-4 \rangle^3 + C_1x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$$[x=0, y=0] \quad 0 - 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=8, y=0] \quad \left(\frac{23}{18}\right)(8)^3 - \frac{1}{6}(8)^4 + \frac{1}{120}(8)^5 - \frac{1}{120}(4)^5 - \frac{1}{3}(4)^3 + C_1(8) = 0$$

$$C_1 = -26.844 \quad \text{kip}\cdot\text{ft}^2$$

Data:  $E = 29 \times 10^6$  psi =  $29 \times 10^3$  ksi       $I = 22.1$  in<sup>4</sup>  
 $EI = (29 \times 10^3)(22.1) = 640.9 \times 10^3$  kip·in<sup>2</sup> =  $4451$  kip·ft<sup>2</sup>

(a) Slope at A ( $\frac{dy}{dx}$  at  $x=0$ )

$$EI \theta_A = 0 + 0 + 0 + 0 + 0 - 26.844 \quad \text{kip}\cdot\text{ft}^2$$

$$\theta_A = -\frac{26.844}{4451} = -6.03 \times 10^{-5} \text{ rad}$$

(b) Deflection at C ( $y$  at  $x=4$  ft.)

$$EI y_C = \frac{23}{18}(4)^3 - \frac{1}{6}(4)^4 + \frac{1}{120}(4)^5 - 0 - 0 - (26.844)(4) + 0$$

$$= -59.73 \quad \text{kip}\cdot\text{ft}^3$$

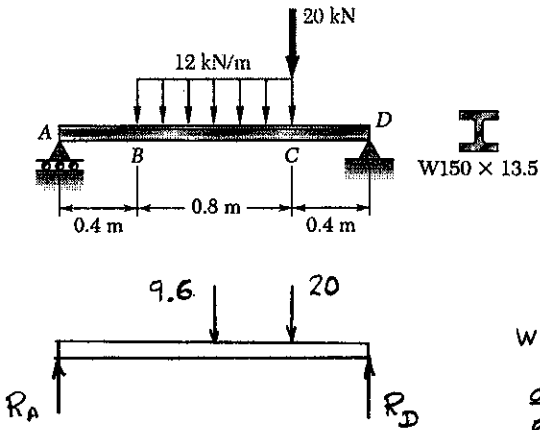
$$y_C = -\frac{59.73}{4451} = -13.42 \times 10^{-5} \text{ ft}$$

$$= 0.1610 \text{ in. } \downarrow$$



PROBLEM 9.46

9.46 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use  $E = 200 \text{ GPa}$ .



SOLUTION

Units: Forces in kN, lengths in m.

$$+\circlearrowleft M_D = 0$$

$$-1.6 R_A + (9.6)(0.8) + (20)(0.4) = 0$$

$$R_A = 9.8 \text{ kN}$$

$$w(x) = 12 \langle x - 0.4 \rangle^0 - 12 \langle x - 1.2 \rangle^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w(x) = -12 \langle x - 0.4 \rangle^0 + 12 \langle x - 1.2 \rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = 9.8 - 12 \langle x - 0.4 \rangle^1 + 12 \langle x - 1.2 \rangle^1 - 20 \langle x - 1.2 \rangle^0 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = 9.8x - 6 \langle x - 0.4 \rangle^2 + 6 \langle x - 1.2 \rangle^2 - 20 \langle x - 1.2 \rangle^1 \text{ kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 4.9x^2 - 2 \langle x - 0.4 \rangle^3 + 2 \langle x - 1.2 \rangle^3 - 10 \langle x - 1.2 \rangle^2 + C_1 \text{ kN}\cdot\text{m}^2$$

$$EI y = 1.63333x^3 - \frac{1}{2} \langle x - 0.4 \rangle^4 + \frac{1}{2} \langle x - 1.2 \rangle^4 - \frac{10}{3} \langle x - 1.2 \rangle^3 + C_1 x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x=0, y=0] \quad 0 - 0 + 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=1.6, y=0] \quad (1.63333)(1.6)^3 - \frac{1}{2}(1.2)^4 + \frac{1}{2}(0.4)^4 - \frac{10}{3}(0.4)^3 + C_1(1.6) + 0 = 0$$

$$C_1 = -3.4080 \text{ kN}\cdot\text{m}^2$$

Data:  $E = 200 \times 10^9 \text{ Pa}$        $I = 6.87 \times 10^6 \text{ mm}^4 = 6.87 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(6.87 \times 10^{-6}) = 1.374 \times 10^6 \text{ N}\cdot\text{m}^2 = 1374 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A ( $\frac{dy}{dx}$  at  $x=0$ )

$$EI \frac{dy}{dx} = 0 - 0 + 0 - 0 - 3.4080 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{3.4080}{1374} = -2.48 \times 10^{-3} \text{ rad} = 2.48 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at C ( $y$  at  $x=1.2 \text{ m}$ )

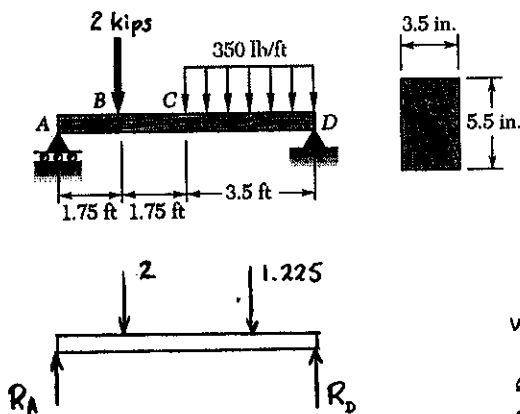
$$EI y_c = (1.63333)(1.2)^3 - \frac{1}{2}(0.8)^4 + 0 - 0 - (3.4080)(1.2) + 0$$

$$= -1.4720 \text{ kN}\cdot\text{m}^3$$

$$y_c = -\frac{1.4720}{1374} = -1.071 \times 10^{-3} \text{ m} = 1.071 \text{ mm} \downarrow \quad \blacktriangleleft$$

**PROBLEM 9.47**

9.47 For the timber beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use  $E = 1.6 \times 10^6$  psi.



**SOLUTION**

Units: Forces in kips, lengths in ft.

$$\rightarrow \sum M_b = 0$$

$$-7R_A + (2)(5.25) + (1.225)(1.75) = 0$$

$$R_A = 1.80625 \text{ kips}$$

$$w(x) = 0.350 \langle x - 3.5 \rangle^0$$

$$\frac{dV}{dx} = -w = -0.35 \langle x - 3.5 \rangle^0$$

$$\frac{dM}{dx} = V = 1.80625 - 0.35 \langle x - 1.75 \rangle^0 - 0.35 \langle x - 3.5 \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = 1.80625x - 2 \langle x - 1.75 \rangle^1 - 0.175 \langle x - 3.5 \rangle^2 \quad \text{kip} \cdot \text{ft}$$

$$EI \frac{dy}{dx} = 0.903125x^2 - 1 \langle x - 1.75 \rangle^2 - 0.05833 \langle x - 3.5 \rangle^3 + C_1 \quad \text{kip} \cdot \text{ft}^2$$

$$EI y = 0.301042x^3 - \frac{1}{3} \langle x - 1.75 \rangle^3 - 0.014583 \langle x - 3.5 \rangle^4 + C_1 x + C_2 \quad \text{kip} \cdot \text{ft}^3$$

$$[x=0, y=0] \quad C_2 = 0$$

$$[x=7, y=0] \quad (0.301042)(7)^3 - \frac{1}{3}(5.25)^3 - 0.014583(3.5)^4 + C_1(7) + 0 = 0$$

$$C_1 = -7.54779 \text{ kip} \cdot \text{ft}^2$$

Data:  $E = 1.6 \times 10^6 \text{ psi} = 1.6 \times 10^3 \text{ ksi}$

$$I = \frac{1}{12}(3.5)(5.5)^3 = 48.526 \text{ in}^3$$

$$EI = (1.6 \times 10^3)(48.526) = 77.6417 \text{ kip} \cdot \text{in}^2 = 539.18 \text{ kip} \cdot \text{ft}^2$$

(a) Slope at A ( $\frac{dy}{dx}$  at  $x=0$ )

$$EI \frac{dy}{dx} = 0 - 0 - 0 - 7.54779 \text{ kip} \cdot \text{ft}^2$$

$$\theta_A = -\frac{7.54779}{539.18} = -14.00 \times 10^{-3} \text{ rad} = 14.00 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at C ( $y$  at  $x=3.5$  ft.)

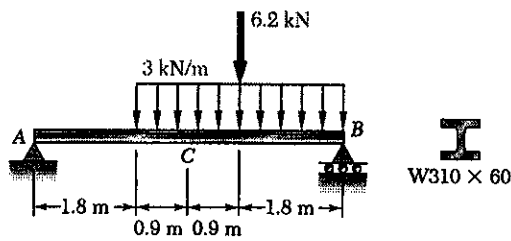
$$EI y_c = (0.301042)(3.5)^3 - \frac{1}{3}(1.75)^3 - 0 - (7.54779)(3.5) + 0$$

$$= -15.297 \text{ kip} \cdot \text{ft}^3$$

$$y_c = -\frac{15.297}{539.18} = -28.37 \times 10^{-3} \text{ ft} = 0.340 \text{ in} \downarrow \quad \blacktriangleleft$$

**PROBLEM 9.48**

9.48 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C. Use  $E = 200 \text{ GPa}$ .



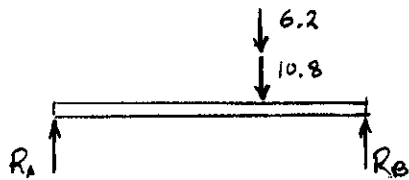
**SOLUTION**

Units: Forces in kN, lengths in meters.

$$+\circlearrowleft \sum M_B = 0$$

$$-5.4 R_A - (1.8)(6.2 + 10.8) = 0$$

$$R_A = 5.6667 \text{ kN}$$



$$w(x) = 3 \langle x - 1.8 \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -3 \langle x - 1.8 \rangle^0$$

$$\frac{dM}{dx} = V = 5.6667 - 3 \langle x - 1.8 \rangle^1 - 6.2 \langle x - 3.6 \rangle^0$$

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$$EI \frac{d^2y}{dx^2} = M = 5.6667x - \frac{3}{2} \langle x - 1.8 \rangle^2 - 6.2 \langle x - 3.6 \rangle^1 \quad \text{kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 2.8333x^2 - \frac{1}{2} \langle x - 1.8 \rangle^3 - 3.1 \langle x - 3.6 \rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = 0.9444x^3 - \frac{1}{8} \langle x - 1.8 \rangle^4 - 1.0333 \langle x - 3.6 \rangle^3 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x=0, y=0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=5.4, y=0] \quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0333(1.8)^3 + C_1(5.4) + 0 = 0$$

$$C_1 = -22.535 \text{ kN}\cdot\text{m}^2$$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N}\cdot\text{m}^2 = 25.8 \times 10^3 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A ( $\frac{dy}{dx}$  at  $x=0$ )

$$EI \frac{dy}{dx} = 0 - 0 - 0 - 22.535 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{22.535}{25.8 \times 10^3} = -873 \times 10^{-6} = 0.873 \times 10^{-3} \text{ rad} \quad \leftarrow$$

(b) Deflection at C ( $y$  at  $x=2.7 \text{ m}$ )

$$EI y_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 + (22.535)(2.7) + 0$$

$$= -42.337 \text{ kN}\cdot\text{m}^3$$

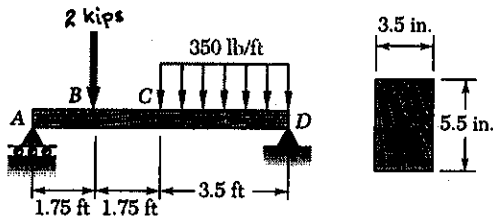
$$y_C = -\frac{42.337}{25.8 \times 10^3} = -1.641 \times 10^{-3} \text{ m}$$

$$y_C = 1.641 \text{ mm} \downarrow$$

PROBLEM 9.49

9.49 and 9.50 For the beam and loading indicated, write a computer program and use it to calculate the slope and deflection of the beam at intervals  $\Delta L$ , starting at point A and ending at the right-hand support.

9.49 Beam and loading of Prob. 9.47 with  $\Delta L = 3.0$  in.



SOLUTION

See solution to Prob. 9.47 for the derivation of the equations used in the following.

$$EI = 539.18 \text{ kip}\cdot\text{ft}^2$$

$$EI \frac{dy}{dx} = 0.903125 x^2 - 1 \langle x - 1.75 \rangle^2 - 0.05833 \langle x - 3.5 \rangle^3 - 7.54779 \text{ kip}\cdot\text{ft}^2$$

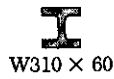
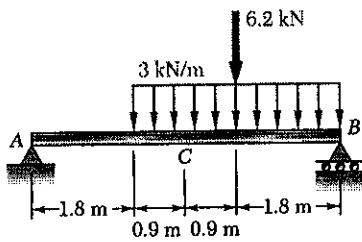
$$EI y = 0.301042 x^3 - \frac{1}{3} \langle x - 1.75 \rangle^3 - 0.014583 \langle x - 3.5 \rangle^4 - 7.54779 x \text{ kip}\cdot\text{ft}^3$$

X (in)	X (ft)	$\theta$ ( $10^{-3}$ rad)	y ( $10^{-3}$ ft)	y (in)
0	0	-14.00	0	0
3	0.25	-13.89	-3.49	-0.042
6	0.5	-13.58	-6.93	-0.083
9	0.75	-13.06	-10.26	-0.123
12	1.0	-12.32	-13.44	-0.161
15	1.25	-11.38	-16.41	-0.197
18	1.5	-10.23	-19.11	-0.229
→ 21	1.75	- 8.87	- 21.51	-0.258
24	2.0	- 7.41	- 23.54	-0.282
27	2.25	- 5.98	- 25.21	-0.303
30	2.5	- 4.57	- 26.53	-0.318
33	2.75	- 3.19	- 27.50	-0.330
36	3.0	- 1.82	- 28.13	-0.338
39	3.25	- 0.48	- 28.42	-0.341
→ 42	3.5	0.84	- 28.37	-0.340
45	3.75	2.14	- 28.00	-0.336
48	4.0	3.40	- 27.30	-0.328
51	4.25	4.62	- 26.30	-0.316
54	4.5	5.79	- 25.00	-0.300
57	4.75	6.89	- 23.41	-0.281
60	5.0	7.92	- 21.56	-0.259
63	5.25	8.87	- 19.46	-0.234
66	5.5	9.72	- 17.13	-0.206
69	5.75	10.47	- 14.61	-0.175
72	6.0	11.11	- 11.91	-0.143
75	6.25	11.62	- 9.06	-0.109
78	6.5	12.00	- 6.11	-0.073
81	6.75	12.24	- 3.07	-0.037
84	7.0	12.32	0	0

PROBLEM 9.50

9.49 and 9.50 For the beam and loading indicated, write a computer program and use it to calculate the slope and deflection of the beam at intervals  $\Delta L$ , starting at point A and ending at the right-hand support.

9.50 Beam and loading of Prob. 9.48 with  $\Delta L = 0.3$  m.



SOLUTION

See solution to Prob. 9.48 for the derivation of the equations used in the following.

$$EI = 25.8 \times 10^3 \text{ kN}\cdot\text{m}^2$$

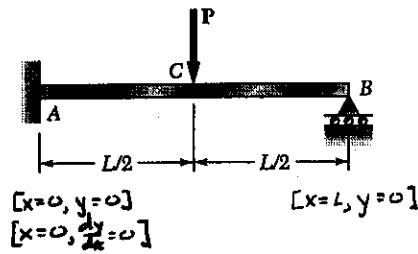
$$EI \frac{dy}{dx} = 2.8333 x^2 - \frac{1}{2} \langle x - 1.8 \rangle^3 - 3.1 \langle x - 3.6 \rangle^2 - 22.535 \text{ kN}\cdot\text{m}^2$$

$$EI y = 0.9444 x^3 - \frac{1}{8} \langle x - 1.8 \rangle^4 - 1.03333 \langle x - 3.6 \rangle^3 - 22.535 x \text{ kN}\cdot\text{m}^3$$

x (m)	$\Theta$ ( $10^{-6}$ rad)	y (mm)
0	-873	0
0.3	-864	-0.261
0.6	-834	-0.516
0.9	-784	-0.759
1.2	-715	-0.985
1.5	-626	-1.187
→ 1.8	-518	-1.359
2.1	-390	-1.495
2.4	-245	-1.591
2.7	-87	-1.641
3.0	81	-1.642
3.3	257	-1.591
→ 3.6	437	-1.487
3.9	606	-1.330
4.2	753	-1.126
4.5	872	-0.882
4.8	960	-0.606
5.1	1016	-0.309
5.4	1035	0

PROBLEM 9.51

9.51 through 9.54 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



SOLUTION

$$\begin{aligned} \uparrow \sum F_y = 0 \quad R_A + R_B - P &= 0 & R_A &= P - R_B \\ \circlearrowleft \sum M_A = 0 \quad -M_A - P\left(\frac{L}{2}\right) + R_B L &= 0 & M_A &= R_B L - \frac{1}{2} PL \end{aligned}$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - P\left(x - \frac{L}{2}\right)'$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - P\left(x - \frac{L}{2}\right)'$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{2} P\left(x - \frac{L}{2}\right)^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{6} P\left(x - \frac{L}{2}\right)^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{6} P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$$

$$\frac{1}{2} (R_B L - \frac{1}{2} PL) L^2 + \frac{1}{6} (P - R_B) L^3 - \frac{1}{48} PL^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_B L^3 = \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{48}\right) PL^3 \quad \frac{1}{3} R_B = \frac{5}{48} P \quad R_B = \frac{5}{16} P \uparrow$$

$$R_A = P - \frac{5}{16} P = \frac{11}{16} P$$

$$M_A = \frac{5}{16} PL - \frac{1}{2} PL = -\frac{3}{16} PL$$

(b) Deflection at C (y at  $x = \frac{L}{2}$ )

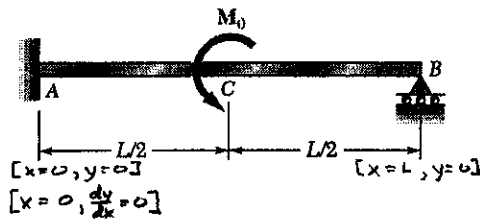
$$y_C = \frac{1}{EI} \left\{ \frac{1}{2} M_A \left(\frac{L}{2}\right)^2 + \frac{1}{6} R_A \left(\frac{L}{2}\right)^3 + 0 + 0 + 0 \right\}$$

$$= \frac{PL^3}{EI} \left\{ \left(\frac{1}{2}\right) \left(-\frac{3}{16}\right) \left(\frac{1}{4}\right) + \left(\frac{1}{6}\right) \left(\frac{11}{16}\right) \left(\frac{1}{8}\right) \right\} = -\frac{7}{168} \frac{PL^3}{EI}$$

$$y_C = \frac{7}{168} \frac{PL^3}{EI} \downarrow$$

PROBLEM 9.52

9.51 through 9.54 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



SOLUTION

$$+\uparrow \Sigma F_y = 0 \quad R_A + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0 \quad -M_A + M_0 - R_B L = 0 \quad M_A = M_0 + R_B L$$

Reactions are statically indeterminate.

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x - M_0 \langle x - \frac{L}{2} \rangle^0$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - M_0 \langle x - \frac{L}{2} \rangle^1 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 \langle x - \frac{L}{2} \rangle^2 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, y=L] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{2} M_0 \left(\frac{L}{2}\right)^2 + 0 + 0 = 0$$

$$\frac{1}{2} (M_0 + R_B L) L^2 - \frac{1}{6} R_B L^3 + \frac{1}{8} M_0 L^2 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_B L^3 = \left(\frac{1}{8} - \frac{1}{2}\right) M_0 L^2 \quad \frac{1}{3} R_B = -\frac{3}{8} \frac{M_0}{L} \quad R_B = -\frac{9}{8} \frac{M_0}{L}$$

$$R_B = \frac{9}{8} \frac{M_0}{L} \downarrow$$

$$R_A = \frac{9}{8} \frac{M_0}{L}$$

$$M_A = M_0 - \frac{9}{8} \frac{M_0}{L} \cdot L = -\frac{1}{8} M_0$$

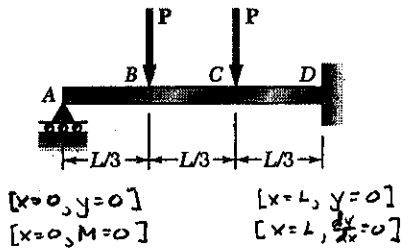
(b) Deflection at C (y at  $x = \frac{L}{2}$ )

$$y_C = \frac{1}{EI} \left\{ \frac{1}{2} M_A \left(\frac{L}{2}\right)^2 + \frac{1}{6} R_A \left(\frac{L}{2}\right)^3 \right\} = \frac{M_0 L^3}{EI} \left\{ \left(\frac{1}{2}\right) \left(-\frac{1}{8}\right) \left(\frac{1}{4}\right) + \left(\frac{1}{6}\right) \left(\frac{9}{8}\right) \left(\frac{1}{8}\right) \right\}$$

$$= \frac{1}{128} \frac{M_0 L^3}{EI} \uparrow$$

PROBLEM 9.53

9.51 through 9.54 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



SOLUTION

$$\frac{dM}{dx} = V = R_A - P\langle x - \frac{L}{3} \rangle^0 - P\langle x - \frac{2L}{3} \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - P\langle x - \frac{L}{3} \rangle^1 - P\langle x - \frac{2L}{3} \rangle^1$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P \langle x - \frac{L}{3} \rangle^2 - \frac{1}{2} P \langle x - \frac{2L}{3} \rangle^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} P \langle x - \frac{L}{3} \rangle^3 - \frac{1}{6} P \langle x - \frac{2L}{3} \rangle^3 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad \frac{1}{2} R_A L^2 - \frac{1}{2} P \left(\frac{2L}{3}\right)^2 - \frac{1}{2} P \left(\frac{L}{3}\right)^2 + C_1 + 0 = 0$$

$$C_1 = \frac{1}{2} \left[ \left(\frac{4}{9} + \frac{1}{9}\right) P - R_A \right] L^2 = \frac{1}{2} \left( \frac{5}{9} P - R_A \right) L^2$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{6} P \left(\frac{2L}{3}\right)^3 - \frac{1}{6} P \left(\frac{L}{3}\right)^3 + \frac{1}{2} \left( \frac{5}{9} P - R_A \right) L^2 L + 0 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_A L^3 = \left[ \left(\frac{1}{2}\right) \left(\frac{5}{9}\right) - \left(\frac{1}{6}\right) \left(\frac{8}{27}\right) - \left(\frac{1}{6}\right) \left(\frac{1}{27}\right) \right] PL^3, \quad \frac{1}{3} R_A = \frac{2}{9} P, \quad R_A = \frac{2}{3} P \quad \blacktriangleleft$$

$$C_1 = \frac{1}{2} \left( \frac{5}{9} P - \frac{2}{3} P \right) L^2 = -\frac{1}{18} PL^2$$

(b) Deflection at C ( $y$  at  $x = \frac{2L}{3}$ )

$$y_c = \frac{1}{EI} \left\{ \frac{1}{6} \left(\frac{2}{3} P\right) \left(\frac{2L}{3}\right)^3 - \frac{1}{6} P \left(\frac{L}{3}\right)^3 - 0 - \frac{1}{18} PL^2 \left(\frac{2L}{3}\right) \right\}$$

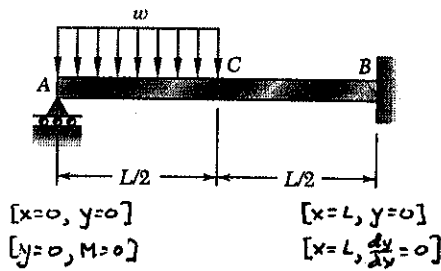
$$= \frac{PL^3}{EI} \left( \frac{16}{486} - \frac{1}{162} - \frac{2}{54} \right) = -\frac{5}{486} \frac{PL^3}{EI}$$

$$y_c = \frac{5}{486} \frac{PL^3}{EI} \downarrow \quad \blacktriangleleft$$



PROBLEM 9.54

9.51 through 9.54 For the beam and loading shown, determine (a) the reaction at the roller support, (b) the deflection at point C.



SOLUTION

$$w(x) = w - w \langle x - \frac{L}{2} \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w \langle x - \frac{L}{2} \rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w \langle x - \frac{L}{2} \rangle^1$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2} wx^2 + \frac{1}{2} w \langle x - \frac{L}{2} \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{6} wx^3 + \frac{1}{6} w \langle x - \frac{L}{2} \rangle^3 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{24} wx^4 + \frac{1}{24} w \langle x - \frac{L}{2} \rangle^4 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx}=0] \quad \frac{1}{2} R_A L^2 - \frac{1}{6} wL^3 + \frac{1}{6} w \left(\frac{L}{2}\right)^3 + C_1 = 0 \quad C_1 = \frac{7}{48} wL^3 - \frac{1}{2} R_A L^2$$

$$[x=L, y=0] \quad \frac{1}{6} R_A L^3 - \frac{1}{24} wL^4 + \frac{1}{24} w \left(\frac{L}{2}\right)^4 + \left(\frac{7}{48} wL^3 - \frac{1}{2} R_A L^2\right)L + 0 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right) R_A L^3 = \left(-\frac{1}{24} + \frac{1}{24} \left(\frac{1}{16}\right) + \frac{7}{48}\right) wL^4 \quad \frac{1}{3} R_A = \frac{41}{384} wL \quad R_A = \frac{41}{128} wL \uparrow$$

$$C_1 = \frac{7}{48} wL^3 - \frac{1}{2} \left(\frac{41}{128} wL\right) L^2 = -\frac{11}{768} wL^3$$

(b) Deflection at C (y at  $x = \frac{L}{2}$ )

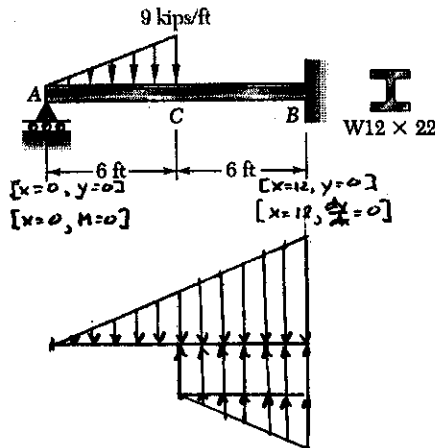
$$y_C = \frac{1}{EI} \left\{ \left(\frac{1}{6}\right) \left(\frac{41}{128} wL\right) \left(\frac{L}{2}\right)^3 - \frac{1}{24} w \left(\frac{L}{2}\right)^4 + 0 - \frac{11}{768} wL^3 \frac{L}{2} + 0 \right\}$$

$$= \frac{wL^4}{EI} \left( \frac{41}{6144} - \frac{1}{384} - \frac{11}{1536} \right) = -\frac{19}{6144} \frac{wL^4}{EI}$$

$$y_C = \frac{19}{6144} \frac{wL^4}{EI} \downarrow$$

**PROBLEM 9.55**

9.55 and 9.56 For the beam and loading shown, determine (a) the reaction at A, (b) the deflection at C. Use  $E = 29 \times 10^6$  psi.



**SOLUTION**

Units: Forces in kips, lengths in ft.

$$k = \frac{9 \text{ kips/ft}}{6 \text{ ft}} = 1.5 \text{ kips/ft}^2$$

$$w(x) = 1.5x - 9\langle x-6 \rangle^0 - 1.5\langle x-6 \rangle^1$$

$$\frac{dV}{dx} = -w(x) = -1.5x + 9\langle x-6 \rangle^0 + 1.5\langle x-6 \rangle^1$$

$$\frac{dM}{dx} = V = R_A - 0.75x^2 + 9\langle x-6 \rangle^1 + 0.75\langle x-6 \rangle^2$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - 0.25x^3 + 4.5\langle x-6 \rangle^2 + 0.25\langle x-6 \rangle^3$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - 0.0625 x^4 + 1.5\langle x-6 \rangle^3 + 0.0625\langle x-6 \rangle^4 + C_1 \quad \text{kip}\cdot\text{ft}^2$$

$$EI y = \frac{1}{6} R_A x^3 - 0.0125 x^5 + 0.375\langle x-6 \rangle^4 + 0.0125\langle x-6 \rangle^5 + C_1 x + C_2 \quad \text{kip}\cdot\text{ft}^3$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=12, \frac{dy}{dx}=0] \quad \frac{1}{2}(R_A)(12)^2 - (0.0625)(12)^4 + (1.5)(6)^3 + (0.0625)(6)^4 + C_1 = 0$$

$$C_1 = 891 - 72 R_A = 0 \quad \text{kip}\cdot\text{ft}^2$$

$$[x=12, y=0] \quad \frac{1}{6} R_A (12)^3 - (0.0125)(12)^5 + (0.375)(6)^4 + (0.0125)(6)^5 + (891 - 72 R_A)(12) + 0 = 0$$

$$(864 - 288) R_A = 8164.8 \quad R_A = 14.175 \text{ kips } \uparrow$$

$$C_1 = 891 - (72)(14.175) = -129.6 \text{ kip}\cdot\text{ft}^2$$

Data:  $E = 29 \times 10^6$  psi =  $29 \times 10^3$  ksi       $I = 156 \text{ in}^4$

$$EI = (29 \times 10^3)(156) = 4.524 \times 10^6 \text{ kip}\cdot\text{in}^2 = 31417 \text{ kip}\cdot\text{ft}^2$$

(b) Deflection at C (y at  $x = 6$ )

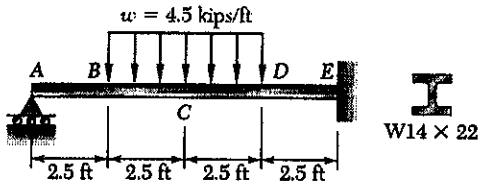
$$EI y_c = \frac{1}{6} (14.175)(6)^3 - (0.0125)(6)^5 + 0 + 0 - (129.6)(6) + 0 = -364.5 \text{ kip}\cdot\text{ft}^3$$

$$y_c = -\frac{364.5}{31417} = -11.60 \times 10^{-3} \text{ ft}$$

$$y_c = 0.1392 \text{ in } \downarrow$$

PROBLEM 9.56

9.55 and 9.56 For the beam and loading shown, determine (a) the reaction at A, (b) the deflection at C. Use  $E = 29 \times 10^6$  psi.



SOLUTION

Units: Forces in kips, lengths in ft.

$$w(x) = 4.5 \langle x-2.5 \rangle^0 - 4.5 \langle x-7.5 \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -4.5 \langle x-2.5 \rangle^0 + 4.5 \langle x-7.5 \rangle^0 \text{ kip}^2/\text{ft}$$

$[x=0, y=0]$   
 $[x=0, M=0]$

$[x=10, y=0]$   
 $[x=10, \frac{dy}{dx}=0]$

$$\frac{dM}{dx} = V = R_A + 4.5 \langle x-2.5 \rangle^1 + 4.5 \langle x-7.5 \rangle^1 \text{ kips}$$

$$EI \frac{d^2y}{dx^2} = M = R_A x - 2.25 \langle x-2.5 \rangle^2 + 2.25 \langle x-7.5 \rangle^2 \text{ kip}\cdot\text{ft}$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{2.25}{3} \langle x-2.5 \rangle^3 + \frac{2.25}{3} \langle x-7.5 \rangle^3 + C_1 \text{ kip}\cdot\text{ft}^2$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{2.25}{12} \langle x-2.5 \rangle^4 + \frac{2.25}{12} \langle x-7.5 \rangle^4 + C_1 x + C_2 \text{ kip}\cdot\text{ft}^3$$

$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x=10, \frac{dy}{dx}=0] \quad \frac{1}{2} R_A (10)^2 - \frac{2.25}{3} (7.5)^3 + \frac{2.25}{3} (2.5)^3 + C_1 = 0$

$$C_1 = 304.69 - 50 R_A \text{ kip}\cdot\text{ft}^2$$

$[x=10, y=0] \quad \frac{1}{6} R_A (10)^3 - \frac{2.25}{12} (7.5)^4 + \frac{2.25}{12} (2.5)^4 + (304.69 - 50 R_A)(10) + 0 = 0$

$$(500 - \frac{1000}{6}) R_A = 24609 \quad R_A = 7.3833 \text{ kips } \uparrow$$

$$C_1 = 304.69 - (50)(7.3833) = -64.45 \text{ kip}\cdot\text{ft}^2$$

Data:  $E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}, \quad I = 199 \text{ in}^4$

$$EI = (29 \times 10^3)(199) = 5.771 \times 10^6 \text{ kip}\cdot\text{in}^2 = 40076 \text{ kip}\cdot\text{ft}^2$$

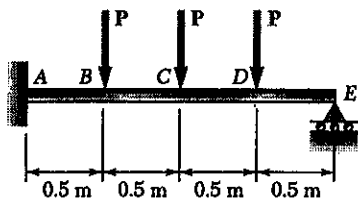
(b) Deflection at C ( $y$  at  $x = 5 \text{ ft}$ )

$$EI y_c = \frac{1}{6} (7.3833)(5)^3 - \frac{2.25}{12} (2.5)^4 + 0 - (64.45)(5) + 0 = -175.76 \text{ kip}\cdot\text{ft}^2$$

$$y_c = -\frac{175.76}{40076} = -4.3856 \times 10^{-3} \text{ ft} \quad y_c = -0.0526 \text{ in.}$$

**PROBLEM 9.57**

9.57 For the beam shown and knowing that  $P = 40$  kN, determine (a) the reaction at E, (b) the deflection at C. Use  $E = 200$  GPa.



**SOLUTION**

Units: Forces in kN, lengths in m.

$$\uparrow \Sigma F_y = 0 \quad R_A - 40 - 40 - 40 + R_E = 0$$

$$R_A = 120 - R_E \quad \text{kN}$$

$$\curvearrowright \Sigma M_A = 0 \quad -M_A - 20 - 40 - 60 + 2R_E = 0$$

$$M_A = 2R_E - 120 \quad \text{kN}\cdot\text{m}$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - 40\langle x-0.5 \rangle^0 - 40\langle x-1 \rangle^0 - 40\langle x-1.5 \rangle^0$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - 40\langle x-0.5 \rangle^1 - 40\langle x-1 \rangle^1 - 40\langle x-1.5 \rangle^1$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - 20\langle x-0.5 \rangle^2 - 20\langle x-1 \rangle^2 - 20\langle x-1.5 \rangle^2 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{20}{3} \langle x-0.5 \rangle^3 - \frac{20}{3} \langle x-1 \rangle^3 - \frac{20}{3} \langle x-1.5 \rangle^3 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=2, y=0] \quad \frac{1}{2} M_A (2)^2 + \frac{1}{6} R_A (2)^3 - \frac{20}{3} (1.5)^3 - \frac{20}{3} (1)^3 - \frac{20}{3} (0.5)^3 + 0 + 0 = 0$$

$$\frac{1}{2} (2R_E - 120)(2)^2 + \frac{1}{6} (120 - R_E)(2)^3 = 30$$

$$2.66667 R_E = 30 + 240 - 160 = 110 \quad R_E = 41.25 \text{ kN} \uparrow$$

$$M_A = (2)(41.25) - 120 = -37.5 \text{ kN}\cdot\text{m}$$

$$R_A = 120 - 41.25 = 78.75 \text{ kN}$$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(45.5 \times 10^{-6}) = 9.10 \times 10^6 \text{ N}\cdot\text{m}^2 = 9100 \text{ kN}\cdot\text{m}^2$$

(b) Deflection at C ( $y$  at  $x = 1 \text{ m}$ )

$$EI y_c = \frac{1}{2} (-37.5)(1)^2 + \frac{1}{6} (78.75)(1)^3 - \frac{20}{3} (0.5)^3 - 0 - 0 + 0 + 0$$

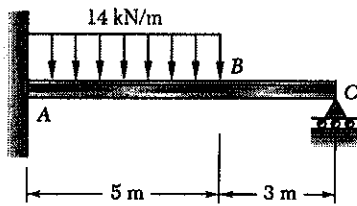
$$= -6.4583 \text{ kN}\cdot\text{m}^3$$

$$y_c = - \frac{6.4583}{9100} = -0.710 \times 10^{-3} \text{ m}$$

$$y_c = 0.710 \text{ mm} \downarrow$$

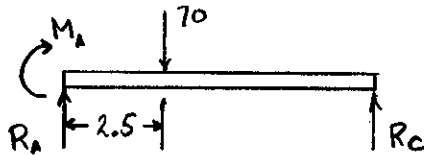
PROBLEM 9.58

9.58 For the beam and loading shown, determine (a) the reaction at C, (b) the deflection at B. Use  $E = 200$  GPa.



$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$

$[x=8, y=0]$



SOLUTION

Units: Forces in kN, lengths in m.

$\uparrow \Sigma F_y = 0 \quad R_A - 70 + R_C = 0$

$R_A = 70 - R_C \quad \text{kN}$

$\curvearrowright \Sigma M_A = 0 \quad -M_A - (70)(2.5) + 8R_C = 0$

$M_A = 8R_C - 175 \quad \text{kN}\cdot\text{m}$

Reactions are statically indeterminate.

$w(x) = 14 - 14\langle x-5 \rangle^0 \quad \text{kN/m}$

$\frac{dV}{dx} = -w = -14 + 14\langle x-5 \rangle^0 \quad \text{kN/m}$

$\frac{dM}{dx} = V = R_A - 14x + 14\langle x-5 \rangle^1 \quad \text{kN}$

$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x-5 \rangle^2 \quad \text{kN}\cdot\text{m}$

$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{7}{3} x^3 + \frac{7}{3} \langle x-5 \rangle^3 + C_1 \quad \text{kN}\cdot\text{m}^2$

$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{7}{12} x^4 + \frac{7}{12} \langle x-5 \rangle^4 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$

$[x=0, \frac{dy}{dx}=0] \quad 0 + 0 + 0 + 0 + C_1 = 0 \quad C_1 = 0$

$[x=0, y=0] \quad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$

$[x=8, y=0] \quad \frac{1}{2} M_A (8)^2 + \frac{1}{6} R_A (8)^3 - \frac{7}{12} (8)^4 + \frac{7}{12} (3)^4 + 0 + 0 = 0$

$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28105}{12} = 0$

$170.667 R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \quad R_C = 11.536 \text{ kN} \uparrow$

$M_A = (8)(11.536) - 175 = -82.715 \text{ kN}\cdot\text{m}$

$R_A = 70 - 11.536 = 58.464 \text{ kN}$

Data:  $E = 200 \times 10^9 \text{ Pa} \quad I = 216 \times 10^6 \text{ mm}^4 = 216 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(216 \times 10^{-6}) = 43.2 \times 10^6 \text{ N}\cdot\text{m}^2 = 43200 \text{ kN}\cdot\text{m}^2$

(b) Deflection at B (y at  $x=5\text{m}$ )

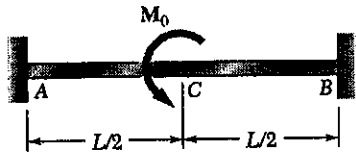
$EI y_B = \frac{1}{2}(-82.715)(5)^2 + \frac{1}{6}(58.464)(5)^3 - \frac{7}{12}(5)^4 = -180.52 \text{ kN}\cdot\text{m}^3$

$y_B = -\frac{180.52}{43200} = -4.18 \times 10^{-3} \text{ m}$

$y_B = 4.18 \text{ mm} \downarrow$

PROBLEM 9.59

9.59 For the beam and loading shown, determine (a) the reaction at A, (b) the slope at C.



SOLUTION

Reactions are statically indeterminate.

$$EI \frac{d^2 y}{dx^2} = M = M_A + R_A x - M_0 \langle x - \frac{L}{2} \rangle^0$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - M_0 \langle x - \frac{L}{2} \rangle^1 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{2} M_0 \langle x - \frac{L}{2} \rangle^2 + C_1 x + C_2$$

$$[x=0, \frac{dy}{dx} = 0] \quad C_1 = 0$$

$$[x=0, y = 0] \quad C_2 = 0$$

$$[x=L, \frac{dy}{dx} = 0] \quad M_A L + \frac{1}{2} R_A L^2 - M_0 \frac{L}{2} = 0 \quad M_A = \frac{1}{2} M_0 - \frac{1}{2} R_A L$$

$$[x=L, y = 0] \quad \frac{1}{2} M_A L^2 + \frac{1}{6} R_A L^3 - \frac{1}{2} M_0 \left(\frac{L}{2}\right)^2 = 0$$

$$\frac{1}{2} \left(\frac{1}{2} M_0 - \frac{1}{2} R_A L\right) L^2 + \frac{1}{6} R_A L^3 - \frac{1}{8} M_0 L^2 = 0$$

$$\left(\frac{1}{4} - \frac{1}{6}\right) R_A L^3 = \left(\frac{1}{4} - \frac{1}{8}\right) M_0 L^2 \quad R_A = \frac{3}{2} \frac{M_0}{L} \uparrow$$

$$M_A = \frac{1}{2} M_0 - \frac{1}{2} \frac{3}{2} \frac{M_0}{L} L = -\frac{1}{4} M_0$$

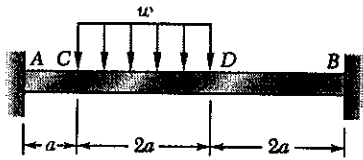
(b) Slope at C  $\frac{dy}{dx}$  at  $x = \frac{L}{2}$

$$\theta_c = \frac{1}{EI} \left\{ \left(-\frac{1}{4} M_0\right) \frac{L}{2} + \frac{1}{2} \left(\frac{3}{2} \frac{M_0}{L}\right) \left(\frac{L}{2}\right)^2 + 0 + 0 \right\} = -\frac{1}{16} \frac{M_0 L}{EI}$$

$$\theta_c = \frac{1}{16} \frac{M_0 L}{EI} \triangleleft$$

PROBLEM 9.60

9.60 For the beam and loading shown, determine (a) the reaction at A, (b) the deflection at D.



SOLUTION

$[x=0, y=0]$   
 $[x=0, \frac{dy}{dx}=0]$

$[x=5a, y=0]$   
 $[x=5a, \frac{dy}{dx}=0]$

$$w(x) = w \langle x-a \rangle^0 - w \langle x-3a \rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w \langle x-a \rangle^0 + w \langle x-3a \rangle^0$$

$$\frac{dM}{dx} = R_A - w \langle x-a \rangle^1 + w \langle x-3a \rangle^1$$

$$EI \frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2} w \langle x-a \rangle^2 + \frac{1}{2} w \langle x-3a \rangle^2$$

$$EI \frac{dy}{dx} = M_A x + \frac{1}{2} R_A x^2 - \frac{1}{6} w \langle x-a \rangle^3 + \frac{1}{6} w \langle x-3a \rangle^3 + C_1$$

$$EI y = \frac{1}{2} M_A x^2 + \frac{1}{6} R_A x^3 - \frac{1}{24} w \langle x-a \rangle^4 + \frac{1}{24} w \langle x-3a \rangle^4 + C_1 x + C_2$$

$[x=0, \frac{dy}{dx}=0]$        $0 + 0 + 0 + 0 + C_1 = 0$        $C_1 = 0$

$[x=0, y=0]$        $0 + 0 + 0 + 0 + 0 + C_2 = 0$        $C_2 = 0$

$[x=5a, \frac{dy}{dx}=0]$        $M_A(5a) + \frac{1}{2} R_A(5a)^2 - \frac{1}{6} w(4a)^3 + \frac{1}{6} w(2a)^3 + 0 = 0$

$$5M_A a + 12.5 R_A a^2 = 9.3333 w a^3 \quad (1)$$

$[x=5a, y=0]$        $\frac{1}{2} M_A(5a)^2 + \frac{1}{6} R_A(5a)^3 - \frac{1}{24} w(4a)^4 + \frac{1}{24} w(2a)^4 + 0 + 0 = 0$

$$12.5 M_A a^2 + 20.8333 R_A a^3 = 10 w a^4 \quad (2)$$

Solving (1) and (2) simultaneously       $M_A = -1.3333 w a^2$        $\blacktriangleleft$

$R_A = 1.280 w a$        $\blacktriangleleft$

(b) Deflection at D (y at  $x = 3a$ )

$$y_D = \frac{1}{EI} \left\{ \frac{1}{2} M_A (3a)^2 + \frac{1}{6} R_A (3a)^3 - \frac{1}{24} w (2a)^4 + 0 + 0 + 0 \right\}$$

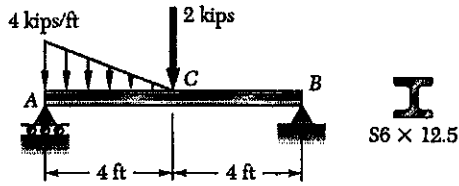
$$= \left[ \frac{9}{2} (-1.3333) + \frac{1}{6} (1.28) (27) - \frac{1}{24} (16) \right] \frac{w a^4}{EI} = -0.907 \frac{w a^4}{EI}$$

$y_D = 0.907 \frac{w a^4}{EI} \downarrow$        $\blacktriangleleft$

PROBLEM 9.61

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.61 Beam and loading of Prob. 9.45.



SOLUTION

See solution to Prob. 9.45 for the derivation of the equations used in the following.

$$EI = 4451 \text{ kip}\cdot\text{ft}^2$$

$$EI \frac{dy}{dx} = \frac{23}{6}x^2 - \frac{2}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{24}\langle x-4 \rangle^4 - \langle x-4 \rangle^2 - 26.844 \quad \text{kip}\cdot\text{ft}^2$$

$$EI y = \frac{23}{18}x^3 - \frac{1}{6}x^4 + \frac{1}{120}x^5 - \frac{1}{120}\langle x-4 \rangle^5 - \frac{1}{3}\langle x-4 \rangle^3 - 26.844x \quad \text{kip}\cdot\text{ft}^3$$

To find location of maximum  $|y|$ , set  $\frac{dy}{dx} = 0$ . Assume  $0 < x < 4$  ft.

$$EI \frac{dy}{dx} = \frac{23}{6}x_m^2 - \frac{2}{3}x_m^3 + \frac{1}{24}x_m^4 - 26.844 = 0 \quad f_1$$

Solve by iteration:  $x_m = 4.0 \quad 3.73 \quad 3.735 \quad x_m = 3.735 \text{ ft.} \blacktriangleleft$   
 $df/dx = 9.33 \quad 9.42$

$$EI y_m = \frac{23}{18}(3.735)^3 - \frac{1}{6}(3.735)^4 + \frac{1}{120}(3.735)^5 - (26.844)(3.735) = -60.06 \text{ kip}\cdot\text{ft}^3$$

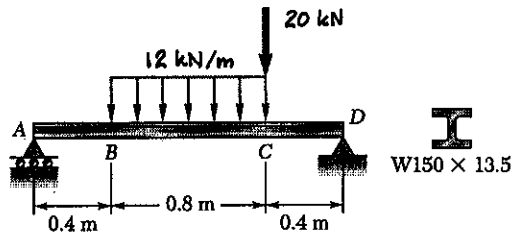
$$y_m = -\frac{60.06}{4451} = -13.49 \times 10^{-3} \text{ ft.} \quad y_m = 0.1619 \text{ in.} \blacktriangleleft$$



PROBLEM 9.62

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.62 Beam and loading of Prob. 9.46.



SOLUTION

See solution to Prob. 9.46 for the derivation of the equations used in the following.

$$EI = EI = 1374 \text{ kN}\cdot\text{m}^2$$

$$EI \frac{dy}{dx} = 4.9x^2 - 2(x-0.4)^3 + 2(x-1.2)^3 - 10(x-1.2)^2 - 3.4080 \quad \text{kN}\cdot\text{m}^2$$

$$EIy = 1.63333x^3 - \frac{1}{2}(x-0.4)^4 + \frac{1}{2}(x-1.2)^4 - \frac{10}{3}(x-1.2)^3 - 3.4080x \quad \text{kN}\cdot\text{m}^3$$

To find the location of maximum  $|y|$ , set  $\frac{dy}{dx} = 0$ . Assume  $0.4 < x_m < 1.2$

$$4.9x_m^2 - 2(x_m-0.4)^3 - 3.4080 = f(x_m) = 0$$

Solve by iteration

$x_m =$	0.8	0.858	0.857	0.8570	$x_m = 0.8570 \text{ m}$
$df/dx =$	6.88	7.123	7.145		

$$EIy = (1.63333)(0.8570)^3 - \frac{1}{2}(0.8570-0.4)^4 - (3.4080)(0.8570)$$

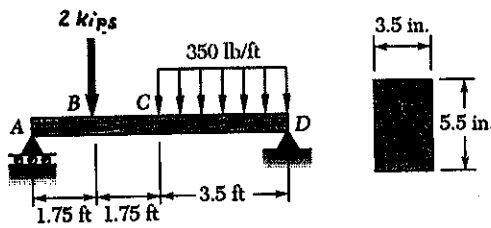
$$= -1.9144 \quad \text{kN}\cdot\text{m}$$

$$y = -\frac{1.9144}{1374} = -1.393 \times 10^{-3} \text{ m} = 1.393 \text{ mm} \downarrow$$

**PROBLEM 9.63**

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.63 Beam and loading of Prob. 9.47.



**SOLUTION**

See solution to Prob. 9.47 for the derivation of the equations used in the following.

$$EI = 539.18 \text{ kip}\cdot\text{ft}^2$$

$$EI \frac{dy}{dx} = 0.903125 x^2 - 1(x-1.75) - 0.05833(x-3.5)^2 - 7.54779 \quad \text{kip}\cdot\text{ft}^2$$

$$EI y = 0.301042 x^3 - \frac{1}{2}(x-1.75)^2 - 0.014583(x-3.5)^4 - 7.54779 x \quad \text{kip}\cdot\text{ft}^3$$

To find the location of maximum  $|y|$ , set  $\frac{dy}{dx} = 0$ . Assume  $1.75 < x_m < 3.5$

$$0.903125 x_m^2 - 1(x_m - 1.75) - 7.54779 = 0$$

$$0.096875 x_m^2 - 0.5 x_m + 10.61029 = 0$$

$$x_m = \frac{3.5 - \sqrt{(3.5)^2 - (4)(0.096875)(10.61029)}}{(2)(0.096875)} = 3.340 \text{ ft.} \quad \blacktriangleleft$$

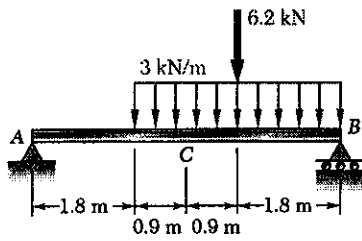
$$\begin{aligned} EI y &= (0.301042)(3.340)^3 - \frac{1}{2}(3.340 - 1.75)^2 - (7.54779)(3.340) \\ &= -15.3328 \quad \text{kip}\cdot\text{ft}^3 \end{aligned}$$

$$y = -\frac{15.3328}{539.18} = -12.44 \times 10^{-3} \text{ ft} = 0.341 \text{ in.} \downarrow \quad \blacktriangleleft$$

PROBLEM 9.64

9.61 through 9.64 For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

9.64 Beam and loading of Prob. 9.48.



SOLUTION

See solution to Prob. 9.48 for the derivation of the equations used in the following.

$$EI = 25.8 \times 10^3 \text{ kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}(x-1.8)^2 - 3.1(x-3.6)^2 - 22.535$$

$$EI y = 0.9444x^3 - \frac{1}{8}(x-1.8)^3 - 1.03333(x-3.6)^3 - 22.535x$$

To find location of maximum  $|y|$ , set  $\frac{dy}{dx} = 0$ . Assume  $1.8 < x_m < 3.6$

$$EI \frac{dy}{dx} = 2.8333x_m^2 - \frac{1}{2}(x_m-1.8)^2 - 22.535 = 0 \quad F_1$$

Solving by iteration:  $x_m = 3, 2.86, 2.855$        $x_m = 2.855 \text{ m}$   $\blacktriangleleft$   
 $df/dx = 15.8, 15.15$

$$EI y_m = 0.9444x_m^3 - \frac{1}{8}(x_m-1.8)^3 - 22.535x_m$$

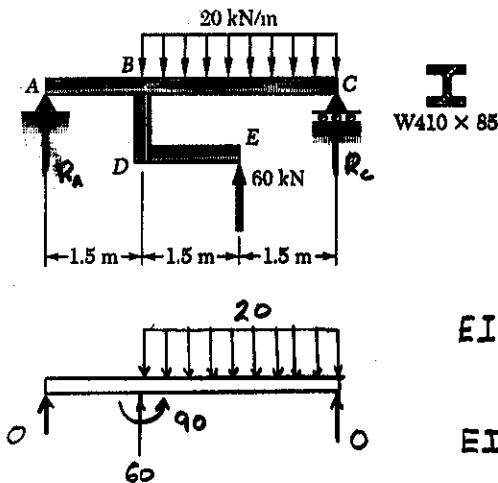
$$= (0.9444)(2.855)^3 - \frac{1}{8}(2.855-1.8)^3 - (22.535)(2.855) = -42.507 \text{ kN}\cdot\text{m}^3$$

$$y_m = -\frac{42.507}{25.8 \times 10^3} = -1.648 \times 10^{-3} \text{ m}$$

$$y_m = 1.648 \text{ mm} \downarrow \quad \blacktriangleleft$$

PROBLEM 9.65

9.65 The rigid bar  $BDE$  is welded at point  $B$  to the rolled steel beam  $AC$ . For the loading shown, determine (a) the slope at point  $A$ , (b) the deflection at point  $B$ . Use  $E = 200 \text{ GPa}$ .



SOLUTION

$$\begin{aligned} \circlearrowleft M_C &= 0 \\ -4.5 R_A + (20)(3)(1.5) - (60)(1.5) &= 0 \\ R_A &= 0 \end{aligned}$$

Units: Forces in kN, lengths in m

$$EI \frac{d^2y}{dx^2} = M = 60(x-1.5)' - 90(x-1.5)^0 - \frac{1}{2}(20)(x-1.5)^2$$

$$EI \frac{dy}{dx} = 30(x-1.5)^2 - 90(x-1.5)' - \frac{1}{6}(20)(x-1.5)^3 + C_1$$

$$EI y = 10(x-1.5)^3 - 45(x-1.5)^2 - \frac{1}{24}(20)(x-1.5)^4 + C_1 x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=4.5, y=0] \quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5 C_1 + 0 = 0$$

$$C_1 = 45 \text{ kN}\cdot\text{m}^2$$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 315 \times 10^6 \text{ mm}^4 = 315 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(315 \times 10^{-6}) = 63 \times 10^6 \text{ N}\cdot\text{m}^2 = 63000 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A  $\left(\frac{dy}{dx} \text{ at } x=0\right)$

$$EI \theta_A = C_1 = 45 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \frac{45}{63000} = 0.714 \times 10^{-3} \text{ rad} \quad \theta_A = 0.714 \times 10^{-3} \text{ rad} \quad \triangleleft$$

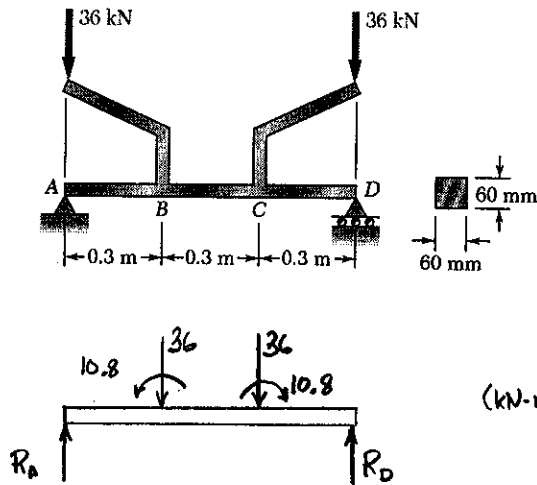
(b) Deflection at B  $(y \text{ at } x=1.5)$

$$EI y_B = (C_1)(1.5) = (45)(1.5) = 67.5 \text{ kN}\cdot\text{m}^3$$

$$y_B = \frac{67.5}{63000} = 1.071 \times 10^{-3} \text{ m} = 1.071 \text{ mm} \quad \uparrow$$

PROBLEM 9.66

9.66 Rigid bars are welded to the steel rod AD as shown. For the loading shown, determine (a) the deflection at point B, (b) the slope at end A. Use  $E = 200 \text{ GPa}$ .



SOLUTION

Units: Use kN for forces, m for lengths.

By symmetry  $R_A = R_D$

$$\uparrow \sum F_y = 0 \quad R_A + R_D - 36 - 36 = 0 \quad R_A = 36 \text{ kN}$$

$$EI \frac{d^2y}{dx^2} = M = 36x - 36\langle x-0.3 \rangle^1 - 36\langle x-0.6 \rangle^1 - 10.8\langle x-0.3 \rangle^0 + 10.8\langle x-0.6 \rangle^0$$

$$(\text{kN}\cdot\text{m}^2) EI \frac{dy}{dx} = 18x^2 - 18\langle x-0.3 \rangle^2 - 18\langle x-0.6 \rangle^2 - 10.8\langle x-0.3 \rangle^1 + 10.8\langle x-0.6 \rangle^1 + C_1$$

$$(\text{kN}\cdot\text{m}^3) EI y = 6x^3 - 6\langle x-0.3 \rangle^3 - 6\langle x-0.6 \rangle^3 - 5.4\langle x-0.3 \rangle^2 + 5.4\langle x-0.6 \rangle^2 + C_1x + C_2$$

$$[x=0, y=0] \quad 0 + 0 + 0 + 0 + 0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$[x=0.9, y=0] \quad (6)(0.9)^3 - (6)(0.6)^3 - (6)(0.3)^3 - (5.4)(0.6)^2 + (5.4)(0.3)^2 + 0.9C_1 + 0 = 0$$

$$C_1 = -1.62 \text{ kN}\cdot\text{m}^2$$

$$\text{Data: } E = 200 \times 10^9 \text{ Pa}, \quad I = bh^3 = \frac{1}{12}(60)(60)^3 = 1.08 \times 10^6 \text{ mm}^4 = 1.08 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(1.08 \times 10^{-6}) = 216 \times 10^3 \text{ N}\cdot\text{m}^2 = 216 \text{ kN}\cdot\text{m}^2$$

(a) Deflection at B (y at  $x = 0.3$ )

$$EI y_B = (6)(0.3)^3 - 0 - 0 - 0 + 0 - (1.62)(0.3) = -0.324 \text{ kN}\cdot\text{m}^2$$

$$y_B = -\frac{0.324}{216} = -1.500 \times 10^{-3} \text{ m}$$

$$y_B = 1.500 \text{ mm} \downarrow$$

(b) Slope at A ( $\frac{dy}{dx}$  at  $x=0$ )

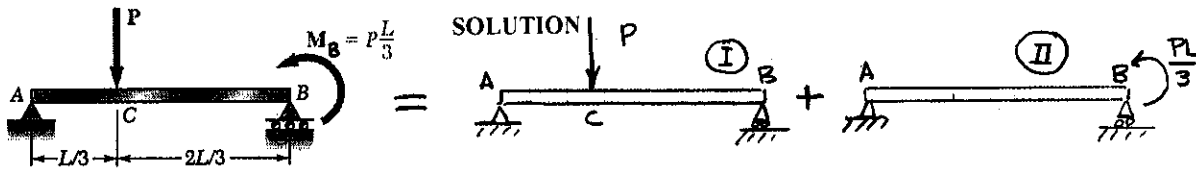
$$EI \theta_A = C_1 = -1.62 \text{ kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{1.62}{216} = -7.50 \times 10^{-3} \text{ rad}$$

$$\theta_A = 7.50 \times 10^{-3} \text{ rad} \leftarrow$$

PROBLEM 9.67

9.67 and 9.68 For the beam and loading shown, determine (a) the deflection at point C, (b) the slope at end A.



Loading I: Case 5  $a = \frac{L}{3}$ ,  $b = \frac{2L}{3}$ ,  $P = P$ ,  $x = a$

$$y_c = -\frac{Pa^2b^2}{6EIL} = -\frac{P}{6EIL}\left(\frac{L}{3}\right)^2\left(\frac{2L}{3}\right)^2 = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2-b^2)}{6EIL} = -\frac{P}{6EIL}\left(\frac{2L}{3}\right)\left[L^2-\left(\frac{2L}{3}\right)^2\right] = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Case 7  $M = -\frac{PL}{3}$   $x = \frac{L}{3}$

$$y_c = -\frac{M}{6EIL}(x^3 - L^2x) = +\frac{PL/3}{6EIL}\left\{\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right\} = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = +\frac{ML}{6EI} = \frac{-(PL/3)L}{6EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

(a) Deflection at C:  $y_c = -\frac{4}{243}\frac{PL^3}{EI} - \frac{4}{243}\frac{PL^3}{EI} = -\frac{8}{243}\frac{PL^3}{EI}$

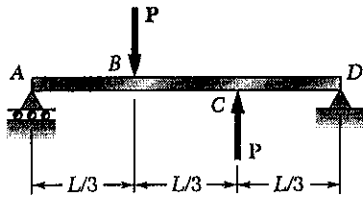
$$y_c = \frac{8}{243}\frac{PL^3}{EI} \downarrow$$

(b) Slope at A:  $\theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{1}{18}\frac{PL^2}{EI} = -\frac{19}{162}\frac{PL^2}{EI}$

$$\theta_A = \frac{19}{162}\frac{PL^2}{EI} \triangleleft$$

PROBLEM 9.68

9.67 and 9.68 For the beam and loading shown, determine (a) the deflection at point C, (b) the slope at end A.



SOLUTION

Loading I: Downward load P at B

Use Case 5 of Appendix D with

$$P = P, \quad a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad L = L, \quad x = \frac{2L}{3}$$

For  $x < a$ , given elastic curve is  $y = \frac{Pb}{EIL} [x^3 - (L^2 - b^2)x]$

To obtain elastic curve for  $x > a$  replace  $x$  by  $L - x$  and interchange  $a$  and  $b$  to get

$$y = \frac{Pa}{6EIL} [(L-x)^3 - (L^2 - a^2)(L-x)] \quad \text{with } x = \frac{2L}{3} \text{ at point C}$$

$$y_c = \frac{P(L/3)}{6EIL} \left[ \left(\frac{L}{3}\right)^3 - (L^2 - \left(\frac{L}{3}\right)^2)\left(\frac{L}{3}\right) \right] = -\frac{7}{486} \frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)[L^2 - (2L/3)^2]}{6EIL} = -\frac{5}{81} \frac{PL^2}{EI}$$

Loading II: Upward load at C. Use Case 5 of Appendix D with

$$P = -P, \quad a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad L = L, \quad x = a = \frac{2L}{3}$$

$$y_c = -\frac{(-P)(2L/3)^2(L/3)^2}{3EIL} = \frac{4}{243} \frac{PL^3}{EI}$$

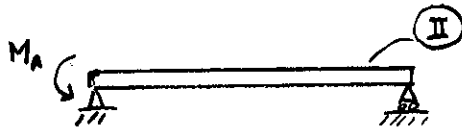
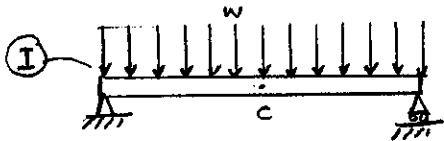
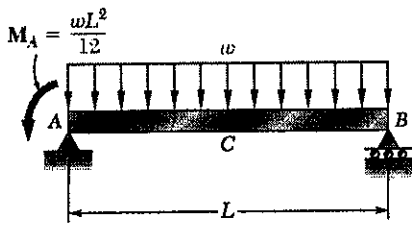
$$\theta_A = -\frac{(-P)(L/3)(L^2 - (L/3)^2)}{6EIL} = \frac{4}{81} \frac{PL^2}{EI}$$

(a) Deflection at C  $y_c = -\frac{7}{486} \frac{PL^3}{EI} + \frac{4}{243} \frac{PL^3}{EI} = \frac{1}{486} \frac{PL^3}{EI} \uparrow$

(b) Slope at A  $\theta_A = -\frac{5}{81} \frac{PL^2}{EI} + \frac{4}{81} \frac{PL^2}{EI} = \frac{1}{81} \frac{PL^2}{EI} \searrow$

PROBLEM 9.69

9.69 and 9.70 For the beam and loading shown, determine (a) the deflection at the midpoint C, (b) the slope at end A.



with  $M_A = \frac{wL^2}{12}$

SOLUTION

Loading I: Case 6 in Appendix D

$$y_C = -\frac{5}{384} \frac{wL^4}{EI}; \quad \theta_A = -\frac{1}{24} \frac{wL^3}{EI}$$

Loading II: Case 7 of Appendix D.

Note that center deflection is

$$y_C = -\frac{M_A}{6EIL} \left[ \left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right) \right]$$

$$= \frac{1}{16} \frac{M_A L}{EI}$$

$$\theta_A = \frac{M_A L}{3EI}$$

$$y_C = \frac{1}{192} \frac{wL^4}{EI}; \quad \theta_A = \frac{1}{36} \frac{wL^3}{EI}$$

(a) Deflection at C.

$$y_C = -\frac{5}{384} \frac{wL^4}{EI} + \frac{1}{192} \frac{wL^4}{EI} = -\frac{1}{128} \frac{wL^4}{EI}$$

$$y_C = \frac{1}{128} \frac{wL^4}{EI} \downarrow$$

(b) Slope at A.

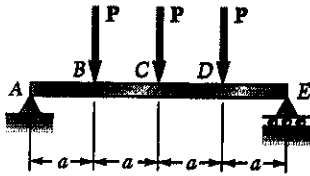
$$\theta_A = -\frac{1}{24} \frac{wL^3}{EI} + \frac{1}{36} \frac{wL^3}{EI} = -\frac{1}{72} \frac{wL^3}{EI}$$

$$\theta_A = \frac{1}{72} \frac{wL^3}{EI} \triangleleft$$



PROBLEM 9.70

9.69 and 9.70 For the beam and loading shown, determine (a) the deflection at the midpoint C, (b) the slope at end A.



SOLUTION

Loading I: Load at B

Case 5 in Appendix D.

$$L = 4a, \quad a = a, \quad b = 3a, \quad x = 2a$$

For  $x > a$ , replace  $x$  by  $L - x$  and interchange  $a$  and  $b$  in expression for elastic curve given.

$$y = \frac{Pa}{6EI} [(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_c = \frac{Pa}{6EI(4a)} [(2a)^3 - (16a^2 - a^2)(2a)] = -\frac{11}{12} \frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(3a)(16a^2 - 9a^2)}{6EI(4a)} = -\frac{7}{8} \frac{Pa^2}{EI}$$

Loading II load at C

Case 4 of Appendix D with  $L = 4a$

$$y_c = -\frac{PL^3}{48EI} = -\frac{P(4a)^3}{48EI} = -\frac{4}{3} \frac{Pa^3}{EI}$$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{P(4a)^2}{16EI} = -\frac{Pa^2}{EI}$$

Loading III load at D

Case 5 of Appendix D

$$L = 4a, \quad a = 3a, \quad b = a, \quad x = 2a \text{ at point C}$$

$$y_c = \frac{Pb}{6EIL} [x^3 - (L^2 - b^2)x] = \frac{Pa}{6EI(4a)} [(2a)^3 - (16a^2 - a^2)(2a)]$$

$$= -\frac{11}{12} \frac{Pa^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{Pa(16a^2 - a^2)}{6EI(4a)} = -\frac{5}{8} \frac{Pa^2}{EI}$$

(a) Deflection at C:  $y_c = -\frac{11}{12} \frac{Pa^3}{EI} - \frac{4}{3} \frac{Pa^3}{EI} - \frac{11}{12} \frac{Pa^3}{EI} = -\frac{19}{6} \frac{Pa^3}{EI}$

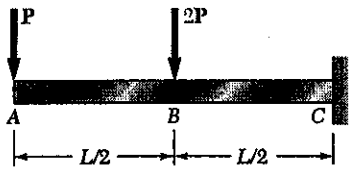
$$y_c = \frac{19}{6} \frac{Pa^3}{EI} \downarrow$$

(b) Slope at A:  $\theta_A = -\frac{7}{8} \frac{Pa^2}{EI} - \frac{Pa^2}{EI} - \frac{5}{8} \frac{Pa^2}{EI} = -\frac{5}{2} \frac{Pa^2}{EI}$

$$\theta_A = \frac{5}{2} \frac{Pa^2}{EI} \nearrow$$

**PROBLEM 9.71**

9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



**SOLUTION**

Loading I:  $2P$  downward at B.

Case 1 of Appendix D applied to portion BC.

$$\theta'_B = \frac{(2P)(L/2)^2}{2EI} = \frac{1}{4} \frac{PL^2}{EI}$$

$$y'_B = \frac{(2P)(L/2)^3}{3EI} = \frac{1}{12} \frac{PL^3}{EI}$$

AB remains straight.

$$\theta_A' = \theta_B' = \frac{1}{4} \frac{PL^2}{EI}$$

$$y_A' = y_B' - \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{12} \frac{PL^3}{EI} - \frac{1}{8} \frac{PL^3}{EI} = -\frac{5}{24} \frac{PL^3}{EI}$$

Loading II  $P$  downward at A. Case 1 of Appendix D.

$$\theta_A'' = \frac{PL^2}{2EI} \quad , \quad y_A'' = -\frac{PL^3}{3EI}$$

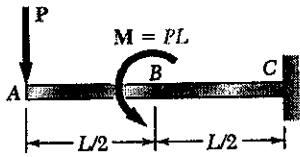
By superposition

$$\theta_A = \theta_A' + \theta_A'' = \frac{1}{4} \frac{PL^2}{EI} + \frac{1}{2} \frac{PL^2}{EI} = \frac{3}{4} \frac{PL^2}{EI}$$

$$y_A = y_A' + y_A'' = -\frac{5}{24} \frac{PL^3}{EI} - \frac{1}{3} \frac{PL^3}{EI} = -\frac{13}{24} \frac{PL^3}{EI}$$

PROBLEM 9.72

9.71 and 9.72 For the cantilever beam and loading shown, determine the slope and deflection at the free end.



SOLUTION

Loading I: Counterclockwise couple  $PL$  at B.

Case 3 of Appendix D applied to portion BC.

$$\theta_B' = \frac{(PL)(L/2)}{EI} = \frac{1}{2} \frac{PL^2}{EI}$$

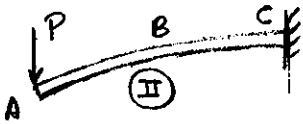
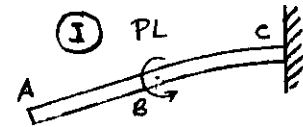
$$y_B' = \frac{(PL)(L/2)^2}{2EI} = \frac{1}{8} \frac{PL^3}{EI}$$

AB remains straight.

$$\theta_A' = \theta_B' = \frac{1}{2} \frac{PL^2}{EI}$$

[www.konkur.in](http://www.konkur.in)

$$y_A' = y_B' - \left(\frac{1}{2}\right)\theta_B' = -\frac{1}{8} \frac{PL^3}{EI} - \frac{1}{4} \frac{PL^3}{EI} = -\frac{3}{8} \frac{PL^3}{EI}$$



Loading II Case 1 of Appendix D.

$$\theta_A'' = \frac{PL^2}{2EI}, \quad y_A'' = -\frac{PL^3}{3EI}$$

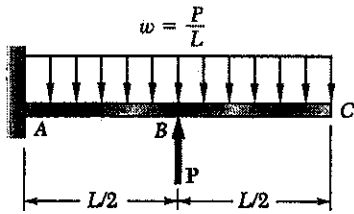
By superposition

$$\theta_A = \theta_A' + \theta_A'' = \frac{1}{2} \frac{PL^2}{EI} + \frac{1}{2} \frac{PL^2}{EI} = \frac{PL^2}{EI} \quad \triangleleft$$

$$y_A = y_A' + y_A'' = -\frac{3}{8} \frac{PL^3}{EI} - \frac{1}{3} \frac{PL^3}{EI} = -\frac{17}{24} \frac{PL^3}{EI} = \frac{17}{24} \frac{PL^3}{3EI} \quad \blacktriangledown$$

PROBLEM 9.73

9.73 and 9.74 For the cantilever beam and loading shown, determine the slope and deflection at point C.



SOLUTION

Loading I Uniformly distributed downward loading with  $w = P/L$ .

Case 2 of Appendix D.

$$\theta'_C = -\frac{(P/L)L^3}{6EI} = -\frac{1}{6} \frac{PL^2}{EI}$$

$$y'_C = -\frac{(P/L)L^4}{8EI} = -\frac{1}{8} \frac{PL^3}{EI}$$

Loading II Upward concentrated load at P.

Case 1 of Appendix D applied to portion AB.

$$\theta''_B = \frac{P(L/2)^2}{2EI} = \frac{1}{8} \frac{PL^2}{EI}$$

$$y''_B = \frac{P(L/2)^3}{3EI} = \frac{1}{24} \frac{PL^3}{EI}$$

Portion BC remains straight.

$$\theta''_C = \theta''_B = \frac{1}{8} \frac{PL^2}{EI}$$

$$y''_C = y''_B + \frac{L}{2} \theta''_B = \frac{1}{24} \frac{PL^3}{EI} + \frac{1}{16} \frac{PL^3}{EI} = \frac{5}{48} \frac{PL^3}{EI}$$

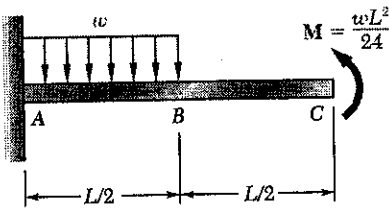
By superposition

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{6} \frac{PL^2}{EI} + \frac{1}{8} \frac{PL^2}{EI} = -\frac{1}{24} \frac{PL^2}{EI} \quad \triangleleft$$

$$y_C = y'_C + y''_C = -\frac{1}{8} \frac{PL^3}{EI} + \frac{5}{48} \frac{PL^3}{EI} = -\frac{1}{48} \frac{PL^3}{EI} \quad \downarrow$$

PROBLEM 9.74

9.73 and 9.74 For the cantilever beam and loading shown, determine the slope and deflection at point C.



SOLUTION

Loading I: Downward distributed load  $w$  applied to portion AB.

Case 2 of Appendix D applied to portion AB.

$$\theta_B' = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48} \frac{wL^3}{EI}$$

$$y_B' = -\frac{w(L/2)^4}{8EI} = -\frac{1}{128} \frac{wL^4}{EI}$$

Portion BC remains straight.

$$\theta_C' = \theta_B' = -\frac{1}{48} \frac{wL^3}{EI}$$

$$y_C' = y_B' + \left(\frac{L}{2}\right)\theta_B' = -\frac{1}{128} \frac{wL^4}{EI} - \frac{1}{96} \frac{wL^4}{EI} = -\frac{7}{384} \frac{wL^4}{EI}$$

Loading II: Counterclockwise couple  $\frac{wL^2}{24}$  applied at C

Case 3 of Appendix D

$$\theta_C'' = \frac{(wL^2/24)L}{EI} = \frac{1}{24} \frac{wL^3}{EI}$$

$$y_C'' = \frac{(wL^2/24)L^2}{2EI} = \frac{1}{48} \frac{wL^4}{EI}$$

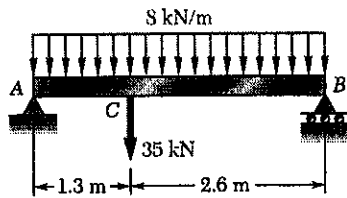
By superposition

$$\theta_C = \theta_C' + \theta_C'' = -\frac{1}{48} \frac{wL^3}{EI} + \frac{1}{24} \frac{wL^3}{EI} = \frac{1}{48} \frac{wL^3}{EI} \quad \blacktriangleleft$$

$$y_C = y_C' + y_C'' = -\frac{7}{384} \frac{wL^4}{EI} + \frac{1}{48} \frac{wL^4}{EI} = \frac{1}{384} \frac{wL^4}{EI} \quad \blacktriangleup$$

**PROBLEM 9.75**

9.75 For the W360 × 39 beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use  $E = 200 \text{ GPa}$ .



**SOLUTION**

Units: Forces in kN, lengths in m.

Loading I: 8 kN/m uniformly distributed.

Case 6:  $w = 8 \text{ kN/m}$ ,  $L = 3.9 \text{ m}$ ,  $x = 1.3 \text{ m}$

$$\theta_A = -\frac{wL^3}{24EI} = -\frac{(8)(3.9)^3}{24EI} = -\frac{19.773}{EI}$$

$$y_C = -\frac{w}{24EI} [x^4 - 2Lx^3 + L^3x] = -\frac{8}{24EI} [(1.3)^4 - (2)(3.9)(1.3)^3 + (3.9)^3(1.3)]$$

$$= -\frac{20.945}{EI}$$

Loading II 35 kN concentrated load at C. Case 5 of Appendix D

$P = 35 \text{ kN}$ ,  $L = 3.9 \text{ m}$ ,  $a = 1.3 \text{ m}$ ,  $b = 2.6 \text{ m}$ ,  $x = a = 1.3 \text{ m}$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(2.6)(3.9^2 - 2.6^2)}{6EI(3.9)} = -\frac{32.861}{EI}$$

$$y_C = -\frac{Pa^2b^2}{3EIL} = -\frac{(35)(1.3)^2(2.6)^2}{3EI(3.9)} = -\frac{34.176}{EI}$$

Data:  $E = 200 \times 10^9$ ,  $I = 102.0 \times 10^6 \text{ mm}^4 = 102.0 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(102.0 \times 10^{-6}) = 20.4 \times 10^6 \text{ N} \cdot \text{m}^2 = 20400 \text{ kN} \cdot \text{m}^2$$

(a) Slope at A  $\theta_A = -\frac{19.773 + 32.861}{20400} = -2.58 \times 10^{-3} \text{ rad}$

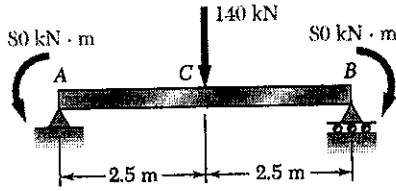
$$\theta_A = 2.58 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at C  $y_C = -\frac{20.934 + 34.176}{20400} = -2.70 \times 10^{-3} \text{ m}$

$$y_C = 2.70 \text{ mm} \quad \blacktriangledown$$

PROBLEM 9.76

9.76 For the W410 × 46.1 beam and loading shown, determine (a) the slope at end A, (b) the deflection at point C. Use  $E = 200$  GPa.



SOLUTION

Units: Forces in kN, lengths in m.

Loading I: Moment at B

Case 7 of Appendix D  $M = 80 \text{ kN}\cdot\text{m}$ ,  $L = 5.0 \text{ m}$ ,  $x = 2.5 \text{ m}$ .

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL} (x^3 - L^2x) = -\frac{80}{6EI(5.0)} [2.5^3 - (5.0)^2(2.5)] = \frac{125}{EI}$$

Loading II Moment at A Case 7 of Appendix D

$M = 80 \text{ kN}\cdot\text{m}$ ,  $L = 5.0 \text{ m}$ ,  $x = 2.5 \text{ m}$

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \quad (\text{Same as loading I})$$

Loading III 140 kN concentrated load at C  $P = 140 \text{ kN}$

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 156 \times 10^6 \text{ mm}^4 = 156 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(156 \times 10^{-6}) = 31.2 \times 10^6 \text{ N}\cdot\text{m}^2 = 31200 \text{ kN}\cdot\text{m}^2$$

(a) Slope at A  $\theta_A = \frac{66.667 + 133.333 - 218.75}{31200} = -0.601 \times 10^{-3} \text{ rad}$

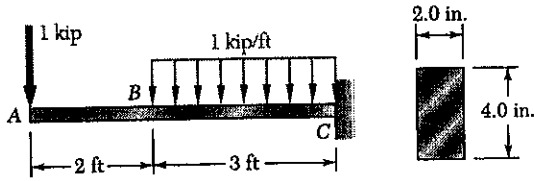
$$\theta_A = 0.601 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$$

(b) Deflection at C  $y_C = \frac{125 + 125 - 364.583}{31200} = -3.67 \times 10^{-3} \text{ m}$

$$y_C = 3.67 \text{ mm} \downarrow \quad \blacktriangleleft$$

PROBLEM 9.77

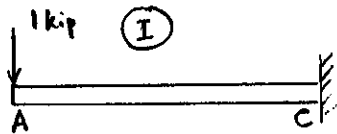
9.77 For the cantilever beam shown, determine the slope and deflection at end A. Use  $E = 29 \times 10^6$  psi.



SOLUTION

Units: Forces in kips, lengths in ft.

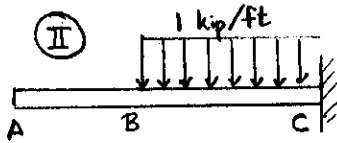
Loading I: Concentrated load at A



Case 1 of Appendix D.

$$\theta'_A = \frac{PL^2}{2EI} = \frac{(1)(5)^2}{2EI} = \frac{12.5}{EI}$$

$$y'_A = -\frac{PL^3}{3EI} = -\frac{(1)(5)^3}{3EI} = -\frac{41.667}{EI}$$



Loading II: Uniformly distributed load over portion BC.

Case 2 of Appendix D applied to portion BC

$$\theta''_B = \frac{WL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

$$y''_B = -\frac{WL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI}$$

Portion AB remains straight.  $\theta_A'' = \theta_B'' = \frac{4.5}{EI}$

$$y_A'' = y_B'' - 2\theta_B'' = -\frac{10.125}{EI} - (2)\left(\frac{4.5}{EI}\right) = -\frac{19.125}{EI}$$

By superposition

$$\theta_A = \theta'_A + \theta_A'' = \frac{12.5}{EI} + \frac{4.5}{EI} = \frac{17}{EI}$$

$$y_A = y'_A + y_A'' = -\frac{41.667}{EI} - \frac{19.125}{EI} = -\frac{60.792}{EI}$$

Data:  $E = 29 \times 10^6$  psi =  $29 \times 10^3$  ksi

$$I = \frac{1}{12}(2.0)(4.0)^3 = 10.667 \text{ in}^4$$

$$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3 \text{ kip}\cdot\text{in}^2 = 2148 \text{ kip}\cdot\text{ft}^2$$

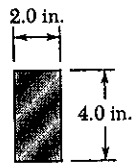
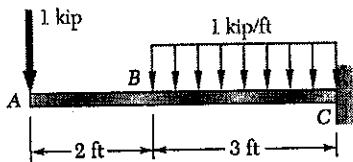
Slope at A  $\theta_A = \frac{17}{2148} = 7.91 \times 10^{-3} \text{ rad} \quad \blacktriangleleft$

Deflection at A  $y_A = -\frac{60.792}{2148} = -28.30 \times 10^{-3} \text{ ft}$   
 $= 0.340 \text{ in.} \quad \blacktriangledown$



PROBLEM 9.78

9.78 For the cantilever beam shown, determine the slope and deflection at point B. Use  $E = 29 \times 10^6$  psi.



SOLUTION

Units: Forces in kips, lengths in ft.

Loading I: Concentrated load at A.

Case 1 of Appendix D.

$$y = \frac{P}{6EI} [x^3 - 3Lx^2]$$

$$\frac{dy}{dx} = \frac{P}{6EI} [3x^2 - 6Lx]$$

with  $P = 1$  kip,  $L = 5$  ft,  $x = 3$  ft.

$$y_B' = \frac{1}{6EI} [(3)^3 - (3)(5)(3)^2] = -\frac{18}{EI}$$

$$\left. \frac{dy}{dx} \right|_B = \frac{1}{6EI} [(3)(3)^2 - (6)(5)(3)] = -\frac{10.5}{EI}$$

Adjusting the sign  $\theta_B' = \frac{10.5}{EI}$

Loading II Uniformly distributed load over portion BC.

Case 2 of Appendix D applied to portion BC

$$y_B'' = -\frac{WL^4}{8EI} = -\frac{(1)(3)^4}{8EI} = -\frac{10.125}{EI}$$

$$\theta_B'' = \frac{WL^3}{6EI} = \frac{(1)(3)^3}{6EI} = \frac{4.5}{EI}$$

By superposition

$$\theta_B = \theta_B' + \theta_B'' = \frac{10.5}{EI} + \frac{4.5}{EI} = \frac{15}{EI}$$

$$y_B = y_B' + y_B'' = -\frac{18}{EI} - \frac{10.125}{EI} = -\frac{28.125}{EI}$$

Data:  $E = 29 \times 10^6$  psi =  $29 \times 10^3$  ksi

$$I = \frac{1}{12} (2.0)(4.0)^3 = 10.667 \text{ in}^4$$

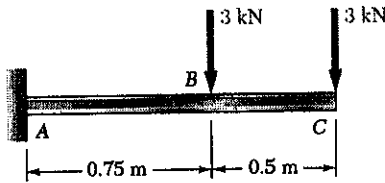
$$EI = (29 \times 10^3)(10.667) = 309.33 \times 10^3 \text{ kip}\cdot\text{in}^2 = 2148 \text{ kip}\cdot\text{ft}^2$$

Slope at B  $\theta_B = \frac{15}{2148} = 6.98 \times 10^{-3} \text{ rad. } \curvearrowright$

Deflection at B  $y_B = -\frac{28.125}{2148} = -13.09 \times 10^{-3} \text{ ft}$   
 $= 0.1571 \text{ in. } \downarrow$

PROBLEM 9.79

9.79 For the cantilever beam shown, determine the slope and deflection at end C. Use  $E = 200 \text{ GPa}$ .



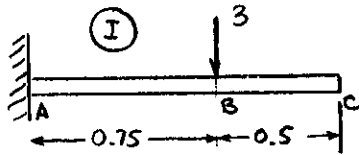
**I**  
S100 x 11.5

SOLUTION

Units: Forces in kN, lengths in m.

loading I: Concentrated load at B

Case 1 of Appendix D applied to portion AB.



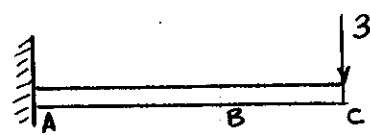
$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight

$$\theta'_C = \theta'_B = -\frac{0.84375}{EI}$$

$$y'_C = y'_B - (0.5)\theta'_B = -\frac{0.84375}{EI}$$



loading II: Concentrated load at C. Case 1 of Appendix D.

$$\theta''_A = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y''_A = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition:  $\theta_A = \theta'_A + \theta''_A = -\frac{3.1875}{EI}$

$$y_A = y'_A + y''_A = -\frac{2.796875}{EI}$$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

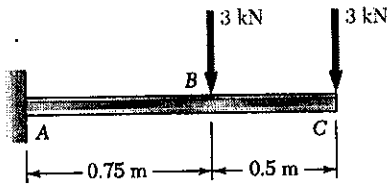
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at C  $\theta_C = -\frac{3.1875}{506} = -6.30 \times 10^{-3} \text{ rad} = 6.30 \times 10^{-3} \text{ rad} \blacktriangleleft$

Deflection at C  $y_C = -\frac{2.796875}{506} = -5.53 \times 10^{-3} \text{ m} = 5.53 \text{ mm} \downarrow$

PROBLEM 9.80

9.80 For the cantilever beam shown, determine the slope and deflection at point B. Use  $E = 200 \text{ GPa}$ .



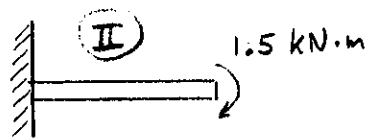
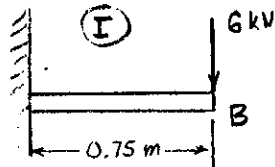
I  
S100 x 11.5

SOLUTION

Units: Forces in kN, lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.

Reducing the force at C to an equivalent force-couple system at B and adding the force already at B gives the loadings I and II shown.



Loading I: Case 1 of Appendix D

$$\theta'_B = \frac{PL^2}{2EI} = \frac{(6)(0.75)^2}{2EI} = \frac{1.6875}{EI}$$

$$y'_B = \frac{PL^3}{3EI} = \frac{(6)(0.75)^3}{3EI} = \frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D

$$\theta_B'' = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y_B'' = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{2EI} = -\frac{0.421875}{EI}$$

By superposition

$$\theta_B = \theta'_B + \theta_B'' = -\frac{2.8125}{EI}$$

$$y_B = y'_B + y_B'' = -\frac{1.265625}{EI}$$

Data:  $E = 200 \times 10^9 \text{ Pa}$ ,  $I = 2.53 \times 10^6 \text{ mm}^4 = 2.53 \times 10^{-6} \text{ m}^4$

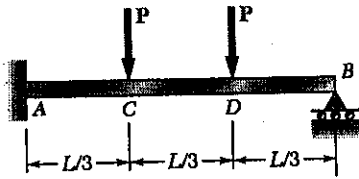
$$EI = (200 \times 10^9)(2.53 \times 10^{-6}) = 506 \times 10^3 \text{ N}\cdot\text{m}^2 = 506 \text{ kN}\cdot\text{m}^2$$

Slope at B  $\theta_B = -\frac{2.8125}{506} = -5.56 \times 10^{-3} \text{ rad} = 5.56 \text{ rad} \blacktriangleleft$

Deflection at B  $y_B = -\frac{1.265625}{506} = -2.50 \times 10^{-3} \text{ m} = 2.50 \text{ mm} \blacktriangledown$

PROBLEM 9.81

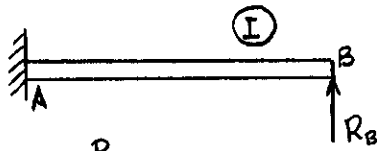
9.81 and 9.82 For the uniform beam shown, determine (a) the reaction at A, (b) the reaction at B.



SOLUTION

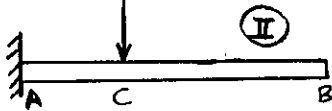
Consider  $R_B$  as redundant and replace loading system by I, II, and III

Loading I Case I of Appendix D applied to AB.



$$(y_B)_I = \frac{R_B L^3}{3EI}$$

Loading II Case I applied to portion AC.



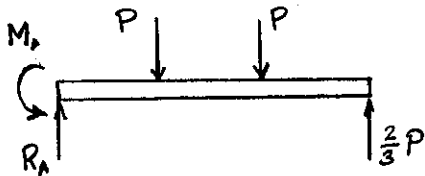
$$(\theta_C)_{II} = -\frac{P(L/3)^2}{2EI} = -\frac{1}{18} \frac{PL^2}{EI}$$

$$(y_C)_{II} = -\frac{P(L/3)^3}{3EI} = -\frac{1}{81} \frac{PL^3}{EI}$$

Portion CB remains straight

$$(y_B)_{II} = (y_C)_{II} + \frac{2L}{3}(\theta_C)_{II} = -\frac{4}{81} \frac{PL^3}{EI}$$

Loading III Case I applied to portion AD



$$(\theta_D)_{III} = \frac{P(2L/3)^2}{2EI} = -\frac{2}{9} \frac{PL^2}{EI}$$

$$(y_D)_{III} = \frac{P(2L/3)^3}{3EI} = -\frac{8}{81} \frac{PL^3}{EI}$$

Portion DB remains straight

$$(y_B)_{III} = (y_D)_{III} + \frac{L}{3}(\theta_D)_{III} = -\frac{14}{81} \frac{PL^3}{EI}$$

Super position and constraint.

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$\frac{1}{3} R_B L^3 - \frac{4}{81} \frac{PL^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = \frac{1}{3} \frac{R_B L^3}{EI} - \frac{2}{9} \frac{PL^3}{EI} = 0$$

$$R_B = \frac{2}{3} P \uparrow$$

Statics

$$+\uparrow \sum F_y = 0 \quad R_A - P - P + \frac{2}{3} P = 0$$

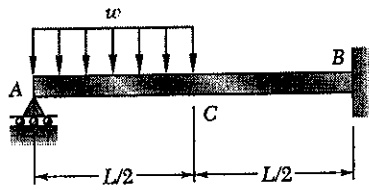
$$R_A = \frac{4}{3} P \uparrow$$

$$\curvearrowright \sum M_A = 0 \quad M_A - P\left(\frac{L}{3}\right) - P\left(\frac{2L}{3}\right) + \left(\frac{2}{3}P\right)(L) = 0$$

$$M_A = \frac{1}{3} PL \curvearrowright$$

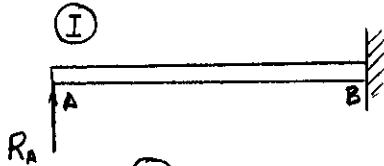
PROBLEM 9.82

9.81 and 9.82 For the uniform beam shown, determine (a) the reaction at A, (b) the reaction at B.



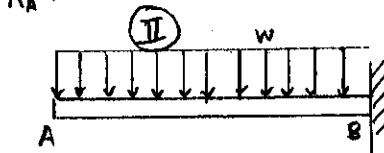
SOLUTION

Beam is indeterminate to first degree. Consider  $R_A$  as redundant and replace the given loading by loadings I, II, and III.



Loading I: Case 1 of Appendix D

$$(y_A)_I = \frac{R_A L^3}{3EI}$$



Loading II: Case 2 of Appendix D

$$(y_A)_{II} = -\frac{wL^4}{8EI}$$



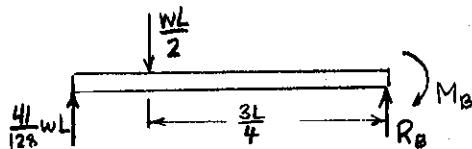
Loading III Case 2 of Appendix D (Portion CB)

$$(\theta_C)_{III} = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48} \frac{wL^3}{EI}$$

$$(y_C)_{III} = \frac{w(L/2)^4}{8EI} = \frac{1}{128} \frac{wL^4}{EI}$$

Portion AC remains straight

$$(y_A)_{III} = (y_C)_{III} + \frac{L}{2} (\theta_C)_{III} = \frac{7}{384} \frac{wL^4}{EI}$$



Superposition and constraint

$$y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III} = 0$$

$$\frac{1}{3} \frac{R_A L^3}{EI} - \frac{1}{8} \frac{wL^4}{EI} + \frac{7}{384} \frac{wL^4}{EI} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{41}{384} \frac{wL^4}{EI} = 0$$

$$R_A = \frac{41}{128} wL \uparrow$$

Statics

$$+\uparrow \sum F_y = 0 \quad \frac{41}{128} wL - \frac{1}{2} wL + R_B = 0$$

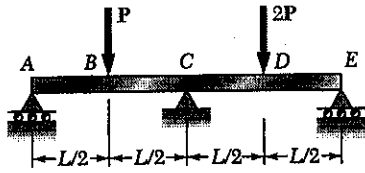
$$R_B = \frac{23}{128} wL \uparrow$$

$$+\circlearrowleft \sum M_B = 0 \quad -\left(\frac{41}{128} wL\right)L + \left(\frac{1}{2} wL\right)\left(\frac{3L}{4}\right) - M_B = 0$$

$$M_B = \frac{7}{128} wL^2 \curvearrowright$$

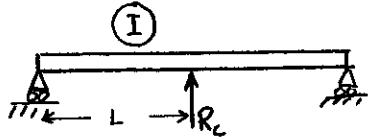
PROBLEM 9.83

9.83 and 9.84 For the uniform beam shown, determine the reaction at each of the three supports.



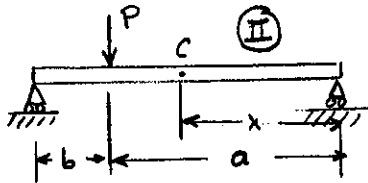
SOLUTION

Beam is indeterminate to first degree. Consider  $R_c$  to be the redundant reaction, and replace the loading by loadings I, II, and III.



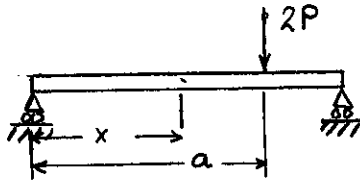
Loading I Case 4 of Appendix D.

$$(y_c)_I = \frac{R_c (2L)^3}{48EI} = \frac{1}{6} \frac{R_c L^3}{EI}$$



Loading II Case 5 of Appendix D.

$$\begin{aligned} (y_c)_{II} &= \frac{Pb}{6EI(2L)} [x^3 - \{(2L)^2 - b^2\}x] \\ &= \frac{P(L/2)}{12EIL} [L^3 - \{4L^2 - (\frac{1}{2})^2\}L] \\ &= -\frac{11}{48} \frac{PL^3}{EI} \end{aligned}$$



Loading III Case 5 of Appendix D.

$(y_c)_{III}$  = twice that of loading II

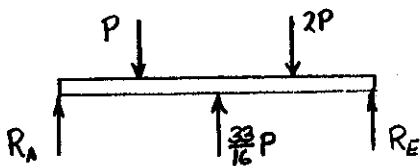
$$(y_c)_{III} = -\frac{11}{24} \frac{PL^3}{EI}$$

Superposition and constraint

$$y_c = (y_c)_I + (y_c)_{II} + (y_c)_{III} = 0$$

$$\frac{1}{6} \frac{R_c L^3}{EI} - \frac{11}{48} \frac{PL^3}{EI} - \frac{11}{24} \frac{PL^3}{EI} = \frac{1}{6} \frac{R_c L^3}{EI} - \frac{11}{16} \frac{PL^3}{EI} = 0 \quad R_c = \frac{33}{16} P \uparrow$$

Statics



$$\sum M_E = 0$$

$$-R_A(2L) + P(\frac{3L}{2}) - (\frac{33}{16}P)L + (2P)(\frac{1}{2}) = 0$$

$$R_A = \frac{7}{32} P \uparrow$$

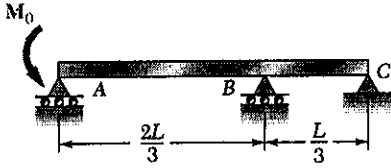
$$\sum F_y = 0$$

$$\frac{7}{32} P - P + \frac{33}{16} P - 2P + R_E = 0$$

$$R_E = \frac{23}{32} P \uparrow$$

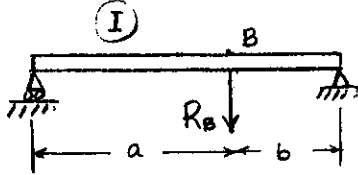
PROBLEM 9.84

9.83 and 9.84 For the uniform beam shown, determine the reaction at each of the three supports.



SOLUTION

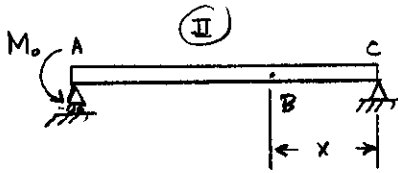
Beam is statically indeterminate to first degree. Consider  $R_B$  to be the redundant reaction, and replace the loading by loadings I and II.



Loading I: Case 5 of Appendix D.

$$(y_B)_I = -\frac{R_B a^2 b^2}{3EI L} = -\frac{R_B (2L/3)^2 (L/3)^2}{3EI L} = -\frac{4}{243} \frac{R_B L^3}{EI}$$

Loading II: Case 7 of Appendix D.

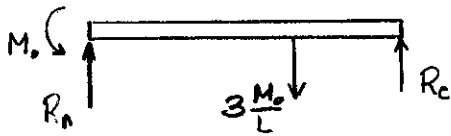


$$(y_B)_{II} = -\frac{M_0}{6EI L} (x^3 - L^2 x) = -\frac{M_0}{6EI L} \left[ \left(\frac{L}{3}\right)^3 - L^2 \left(\frac{L}{3}\right) \right] = \frac{4}{81} \frac{M_0 L^2}{EI}$$

Superposition and constraint.

$$y_B = (y_B)_I + (y_B)_{II} = 0$$

$$-\frac{4}{243} \frac{R_B L^3}{EI} + \frac{4}{81} \frac{M_0 L^2}{EI} = 0 \quad R_B = 3 \frac{M_0}{L} \downarrow$$



Statics

$$\sum M_C = 0$$

$$-R_A L + M_0 + 3 \frac{M_0}{L} \cdot \frac{L}{3} = 0 \quad R_A = 2 \frac{M_0}{L} \uparrow$$

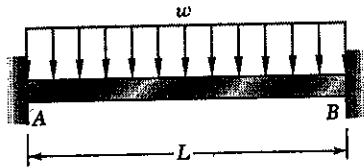
$$\sum F_y = 0$$

$$2 \frac{M_0}{L} - 3 \frac{M_0}{L} + R_C = 0 \quad R_C = \frac{M_0}{L} \uparrow$$

PROBLEM 9.85

9.85 and 9.86 For the beam shown, determine the reaction at B.

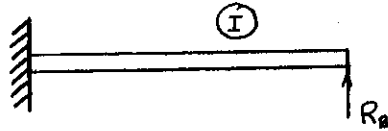
SOLUTION



Beam is second degree indeterminate. Choose  $R_B$  and  $M_B$  as redundant reactions.

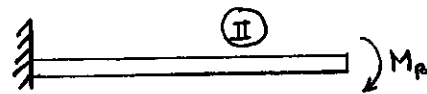
Loading I: Case 1 of Appendix D.

$$(y_B)_I = \frac{R_B L^3}{3EI} \quad (\theta_B)_I = \frac{R_B L^2}{2EI}$$



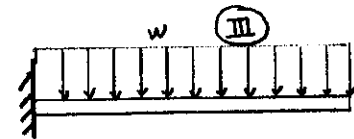
Loading II: Case 3 of Appendix D

$$(y_B)_{II} = -\frac{M_B L^2}{2EI} \quad (\theta_B)_{II} = -\frac{M_B L}{EI}$$



Loading III: Case 2 of Appendix D

$$(y_B)_{III} = -\frac{wL^4}{8EI} \quad (\theta_B)_{III} = -\frac{wL^3}{6EI}$$



Superposition and constraint

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$\frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B - \frac{wL^4}{8EI} = 0 \quad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$\frac{L^2}{2EI} R_B - \frac{L}{EI} M_B - \frac{wL^3}{6EI} = 0 \quad (2)$$

Solving (1) and (2) simultaneously

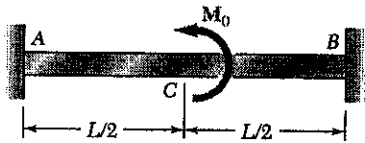
$$R_B = \frac{1}{2} wL \uparrow$$

$$M_B = \frac{1}{12} wL^2 \curvearrowright$$



PROBLEM 9.86

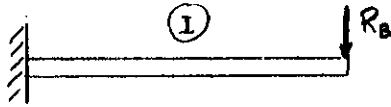
9.85 and 9.86 For the beam shown, determine the reaction at B.



SOLUTION

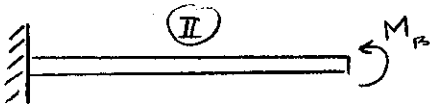
Beam is second degree indeterminate. Choose  $R_B$  and  $M_B$  as redundant reactions.

Loading I: Case 1 of Appendix D.



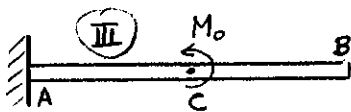
$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$

Loading II: Case 3 of Appendix D.



$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$

Loading III: Case 3 applied to portion AC.



$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8} \frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2} \frac{M_0 L}{EI}$$

Superposition and constraint

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI} R_B + \frac{L^2}{2EI} M_B + \frac{3}{8} \frac{M_0 L^2}{EI} = 0 \quad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI} R_B + \frac{L}{EI} M_B + \frac{1}{2} \frac{M_0 L}{EI} = 0 \quad (2)$$

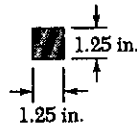
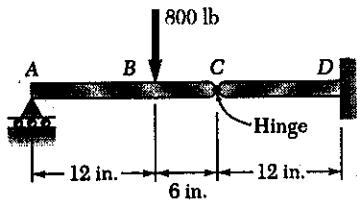
Solving (1) and (2) simultaneously

$$R_B = \frac{3}{2} \frac{M_0}{L} \downarrow$$

$$M_B = \frac{1}{4} M_0 \curvearrowright$$

**PROBLEM 9.87**

9.87 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (a) the slope at point A, (b) the deflection at point B. Use  $E = 29 \times 10^6$  psi.

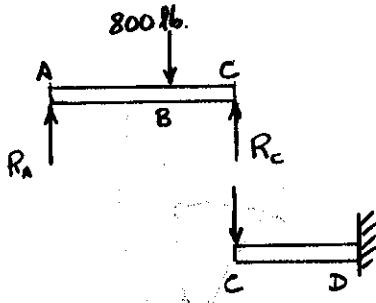


**SOLUTION**

Using free body ABC

$$\sum M_A = 0 \quad 18 R_C - (12)(800) = 0$$

$$R_C = 533.33 \text{ lb.}$$



$$E = 29 \times 10^6 \text{ psi}$$

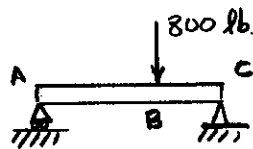
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (1.25)(1.25)^3 = 0.20345 \text{ in}^4$$

$$EI = (29 \times 10^6)(0.20345) = 5.900 \times 10^6 \text{ lb}\cdot\text{in}^2$$

Using cantilever beam CD with load  $R_C$

Case 1 of Appendix D

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(533.33)(12)^3}{(3)(5.900 \times 10^6)} = -52.067 \times 10^{-3} \text{ in.}$$



Calculation of  $\theta_A'$  and  $y_B'$  assuming that point C does not move.

Case 5 of Appendix D  $P = 800 \text{ lb.}$   $L = 18 \text{ in.}$   $a = 12 \text{ in.}$   $b = 6 \text{ in.}$

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(800)(6)(18^2 - 6^2)}{(6)(5.900 \times 10^6)(18)} = -2.1695 \times 10^{-3} \text{ rad.}$$

$$y_B' = -\frac{Pb^2 a^2}{3EIL} = -\frac{(800)(6)^2(12)^2}{(3)(5.900 \times 10^6)(18)} = -13.017 \times 10^{-3} \text{ in.}$$

Addition slope and deflection due to movement of point C

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{52.067 \times 10^{-3}}{18} = -2.8926 \times 10^{-3} \text{ rad.}$$

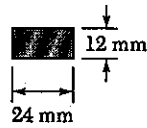
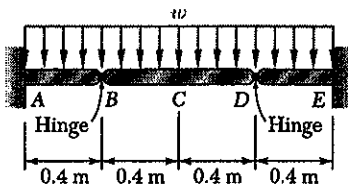
$$y_B'' = \frac{a}{L} y_C = -\frac{(12)(52.067 \times 10^{-3})}{18} = -34.711 \times 10^{-3} \text{ in.}$$

(a) Slope at A  $\theta_A = \theta_A' + \theta_A'' = -2.1695 \times 10^{-3} - 2.8926 \times 10^{-3}$   
 $= -5.06 \times 10^{-3} \text{ rad} = 5.06 \times 10^{-3} \text{ rad} \leftarrow$

(b) Deflection at B  $y_B = y_B' + y_B'' = -13.017 \times 10^{-3} - 34.711 \times 10^{-3}$   
 $= -47.7 \times 10^{-3} \text{ in.} = 47.7 \times 10^{-3} \text{ in.} \downarrow$

PROBLEM 9.88

9.88 A central beam  $BD$  is joined at hinges to two cantilever beams  $AB$  and  $DE$ . All beams have the cross section shown. For the loading shown, determine the largest allowable value of  $w$  if the deflection at  $C$  is not to exceed 3 mm. Use  $E = 200$  GPa.



SOLUTION

Let  $a = 0.4$  m

Cantilever beams  $AB$  and  $CD$ .

Cases 1 and 2 of Appendix D 
$$y_c = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{11}{24} \frac{wa^4}{EI}$$

Beam  $BCD$ , with  $L = 0.8$  m, assuming that points  $B$  and  $D$  do not move.

Case 6 of Appendix:

$$y_c' = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points  $B$  and  $D$ .

$$y_c'' = y_B = y_D = -\frac{11}{24} \frac{wa^4}{EI}$$

Total deflection at  $C$  
$$y_c = y_c' + y_c''$$

$$y_c = -\frac{w}{EI} \left\{ \frac{5L^4}{384} + \frac{11a^4}{24} \right\}$$

Data:  $E = 200 \times 10^9$  Pa,  $I = \frac{1}{12}(24)(12)^3 = 3.456 \times 10^{-3} \text{ mm}^4 = 3.456 \times 10^{-9} \text{ m}^4$

$$EI = (200 \times 10^9)(3.456 \times 10^{-9}) = 691.2 \text{ N}\cdot\text{m}^2$$

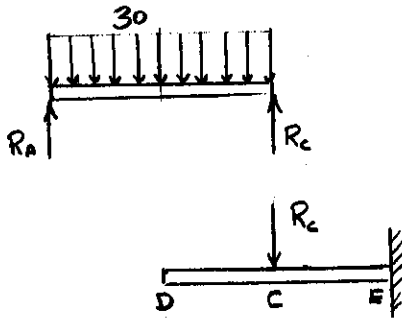
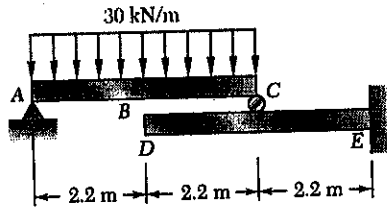
$$y_c = -3 \times 10^{-3} \text{ m}$$

$$-3 \times 10^{-3} = -\frac{w}{691.2} \left\{ \frac{5(0.8)^4}{384} + \frac{11(0.4)^4}{24} \right\} = -24.69 \times 10^{-6} w$$

$$w = 121.5 \text{ N/m}$$

PROBLEM 9.89

9.89 Beam AC rests on the cantilever beam DE, as shown. Knowing that a W410 × 38.8 rolled-steel shape is used for each beam, determine for the loading shown (a) the deflection at point B, (b) the deflection at point D. Use  $E = 200$  GPa.



SOLUTION

Units: Forces in kN, lengths in m.

Using free body ABC,  $\sum M_A = 0$

$$4.4 R_c - (4.4)(30)(2.2) = 0 \quad R_c = 66.0 \text{ kN}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 127 \times 10^6 \text{ mm}^4 = 127 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(127 \times 10^{-6}) = 25.4 \times 10^6 \text{ N} \cdot \text{m}^2 = 25400 \text{ kN} \cdot \text{m}^2$$

For slope and deflection at C, use Case 1, Appendix D applied to portion CE of beam DCE.

$$\theta_c = \frac{R_c L^2}{2EI} = \frac{(66.0)(2.2)^2}{(2)(25400)} = 6.2882 \times 10^{-3} \text{ rad.}$$

$$y_c = -\frac{R_c L^3}{3EI} = \frac{(66.0)(2.2)^3}{(3)(25400)} = -9.2227 \times 10^{-3} \text{ m}$$

Deflection at B assuming that point C does not move.

Use Case 6 of Appendix D.  $(y_B)_1 = -\frac{5wL^4}{384EI} = -\frac{(5)(30)(4.4)^4}{(384)(25400)} = -5.7642 \times 10^{-3}$

Additional deflection at B due to movement of point C

$$(y_B)_2 = \frac{1}{2} y_c = -4.6113 \times 10^{-3} \text{ m}$$

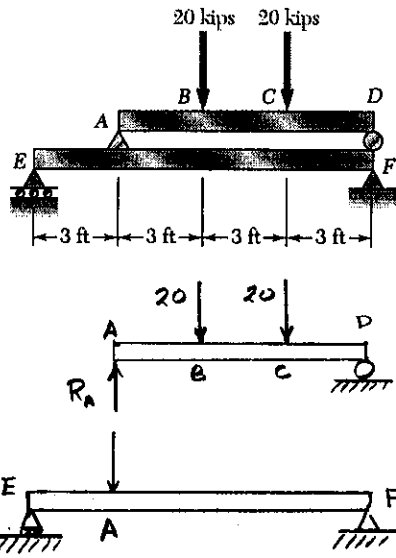
Total deflection at B  $y_B = (y_B)_1 + (y_B)_2 = -10.38 \times 10^{-3} \text{ m} = 10.38 \text{ mm} \downarrow$

Portion DC of beam DCB remains straight.

$$y_D = y_c - 2\theta_c = -9.2227 \times 10^{-3} - (2.2)(6.2882 \times 10^{-3}) = -23.1 \times 10^{-3} \text{ m} = 23.1 \text{ mm} \downarrow$$

PROBLEM 9.90

9.90 Beam  $AD$  rests on beam  $EF$  as shown. Knowing that a  $W12 \times 26$  rolled-steel shape is used for each beam, determine for the loading shown the deflection at points  $B$  and  $C$ . Use  $E = 29 \times 10^6$  psi.



SOLUTION

$$E = 29 \times 10^3 \text{ ksi}; \quad I = 204 \text{ in}^4$$

$$EI = (29 \times 10^3)(204) = 5.916 \times 10^6 \text{ kip}\cdot\text{in}^2 = 41083 \text{ kip}\cdot\text{ft}^2$$

For equilibrium of beam  $ABCD$   $R_A = 20$  kips.

Deflection at point  $A$  is due to bending of beam  $EAF$ . Using Case 5 of

$$y_A = -\frac{Pa^2b^2}{3EIL} = -\frac{(20)(3)^2(9)^2}{(3)(EI)(12)} = -\frac{405}{EI} \text{ ft}$$

Assuming that beam  $ABCD$  is rigid

$$y_B' = \frac{6}{9}y_A = -\frac{270}{EI} \text{ ft}, \quad y_C' = \frac{3}{9}y_A = -\frac{135}{EI} \text{ ft}$$

Additional deflection at  $B$  due to bending of beam  $ABCD$ . Using Case 5

$$y_B'' = -\frac{Pa^2b^2}{3EIL} + \frac{Pab}{6EIL} [x^3 - (L^2 - b^2)x]$$

$$= -\frac{(20)(3)^2(6)^2}{(3)(EI)(9)} + \frac{(20)(3)[(3)^3 - (9^2 - 3^2)(3)]}{(6)(EI)(9)} = -\frac{240}{EI} - \frac{210}{EI} = -\frac{450}{EI} \text{ ft}$$

Additional deflection at  $C$  due to bending of beam  $ABCD$ .

By symmetry  $y_C'' = y_B'' = -\frac{450}{EI} \text{ ft}$

Total deflection at  $B$

$$y_B = y_B' + y_B'' = -\frac{270}{EI} - \frac{450}{EI} = -\frac{720}{EI} = -\frac{720}{41083} = -17.525 \times 10^{-3} \text{ ft}$$

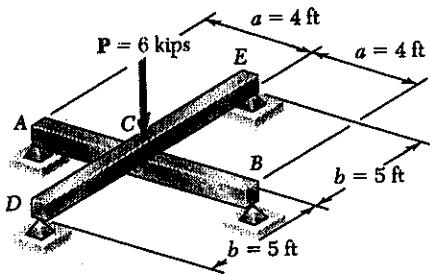
$$= 0.210 \text{ in.} \downarrow$$

$$y_C = y_C' + y_C'' = -\frac{135}{EI} - \frac{450}{EI} = -\frac{585}{EI} = -\frac{585}{41083} = -14.239 \times 10^{-3} \text{ ft}$$

$$= 0.171 \text{ in.} \downarrow$$

**PROBLEM 9.91**

9.91 For the loading shown, and knowing that beams  $AB$  and  $DE$  have the same flexural rigidity, determine the reaction (a) at  $B$ , (b) at  $E$ .



**SOLUTION**

Units: Forces in kips, lengths in ft.

For beam  $ACB$ , using Case 4 of Appendix D.

$$(y_c)_1 = -\frac{R_c(2a)^3}{48EI}$$

For beam  $DCE$ , using Case 4 of Appendix D.

$$(y_c)_2 = \frac{(R_c - P)(2b)^3}{48EI}$$

Matching deflections at  $C$

$$-\frac{R_c(2a)^3}{48EI} = \frac{(R_c - P)(2b)^3}{48EI}$$

$$R_c = \frac{Pb^3}{a^3 + b^3} = \frac{(6)(5)^3}{4^3 + 5^3} = 3.968 \text{ kips}$$

$$P - R_c = 6 - 3.968 = 2.032 \text{ kips}$$

Using free body  $ACB$   $\rightarrow M_A = 0$   $2aR_B - aR_c = 0$

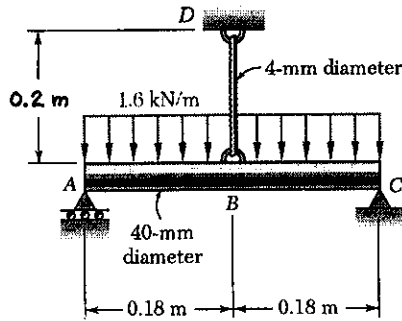
$$R_B = \frac{1}{2}R_c = 1.984 \text{ kips}$$

Using free body  $DCE$   $\rightarrow M_D = 0$   $2bR_E - b(P - R_c) = 0$

$$R_E = \frac{1}{2}(P - R_c) = 1.016 \text{ kips}$$

PROBLEM 9.92

9.92 Knowing that the rod  $ABC$  and the wire  $BD$  are both made of steel, determine (a) the deflection at  $B$ , (b) the reaction at  $A$ . Use  $E = 200$  GPa.



SOLUTION

Let  $F_{BD}$  be the tension in wire  $BD$ . The elongation of the wire is

$$S_{BD} = \frac{F_{BD} l}{EA}$$

Beam  $ABC$  is subjected to loads  $F_{BD}$  (I) and  $w$  (II.)

Loading I: Case 4 of Appendix D.

$$(y_B)_I = \frac{F_{BD} L^3}{48 EI}$$

Loading II: Case 6 of Appendix D.

$$(y_B)_{II} = -\frac{5}{384} \frac{WL^4}{EI}$$

Deflection at  $B$

$$-S_{BD} = y_B = (y_B)_I + (y_B)_{II}$$

$$-\frac{F_{BD} l}{EA} = \frac{F_{BD} L^3}{48 EI} - \frac{5}{384} \frac{WL^4}{EI}$$

$$\left( \frac{l}{EA} + \frac{L^3}{48 EI} \right) F_{BD} = \frac{5}{384} \frac{WL^4}{EI}$$

Data:  $l = 0.2$  m

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (4)^2 = 12.566 \text{ mm}^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$\frac{l}{EA} = 79.58 \times 10^{-9} \text{ m/N}$$

$$L = 0.36 \text{ m} \quad w = 1.6 \times 10^3 \text{ N/m}$$

$$I = \frac{\pi}{4} c^4 = \frac{\pi}{4} \left( \frac{40}{2} \right)^4 = 125.66 \times 10^3 \text{ mm}^4 = 125.66 \times 10^{-9} \text{ m}^4$$

$$EI = (200 \times 10^9)(125.66 \times 10^{-9}) = 25.132 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$\left[ 79.58 \times 10^{-9} + \frac{(0.36)^3}{(48)(25.132 \times 10^3)} \right] F_{BD} = \frac{(5)(1.6 \times 10^3)(0.36)^4}{(384)(25.132 \times 10^3)}$$

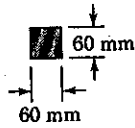
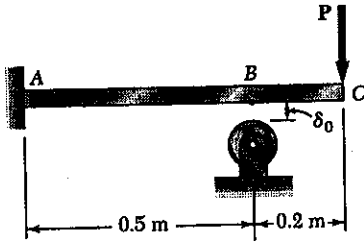
$$118.256 \times 10^9 F_{BD} = 13.923 \times 10^{-6} \quad F_{BD} = 117.74 \text{ N}$$

(a) Deflection at  $B$   $S_B = \frac{F_{BD} l}{EA} = (117.74)(79.58 \times 10^{-9}) = 9.374 \times 10^{-6} \text{ m} = 0.00937 \text{ mm} \downarrow$

(b)  $R_A = R_C = \frac{1}{2} [wL - F_{BD}] = \frac{1}{2} [(1600)(0.36) - 117.74] = 229 \text{ N} \uparrow$

PROBLEM 9.93

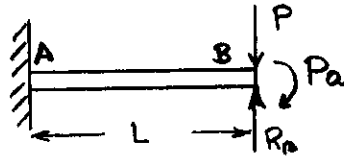
9.93 Before the load  $P$  was applied, a gap  $\delta_0 = 0.5$  mm existed between the cantilever beam  $AC$  and the support at  $B$ . Knowing that  $E = 200$  GPa, determine the magnitude of  $P$  for which the deflection at  $C$  is 1 mm.



SOLUTION

Let length  $AB = L = 0.5$  m  
length  $BC = a = 0.2$  m

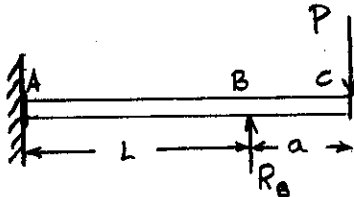
Consider portion  $AB$  of beam  $ABC$ .



The loading becomes forces  $P$  and  $R_B$  at  $B$  plus the couple  $Pa$ . The deflection at  $B$  is  $S_0$ . Using Cases 1 and 3 of Appendix D.

$$S_0 = \frac{(P - R_B)L^3}{3EI} + \frac{PaL^2}{2EI}$$

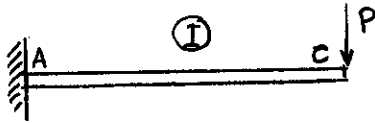
$$\left(\frac{L^3}{3} + \frac{L^2a}{2}\right)P - \frac{L^3}{3}R_B = EIS_0 \quad (1)$$



The deflection at  $C$  depends on the deformation of beam  $ABC$  subjected to loads  $P$  and  $R_B$ .

For loading I, using Case 1 of Appendix D

$$(S_c)_I = \frac{P(L+a)^3}{3EI}$$

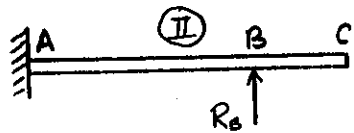


For loading II, using Case 1 of Appendix D

$$y_B = \frac{R_B L^3}{3EI} \quad \theta_B = \frac{R_B L^2}{2EI}$$

Portion  $BC$  remains straight

$$y_C = y_B + a\theta_B = \left(\frac{L^3}{3} + \frac{L^2a}{2}\right) \frac{R_B}{EI}$$



By superposition the downward deflection at  $C$  is

$$S_c = \frac{P(L+a)^3}{3EI} - \left(\frac{L^3}{3} + \frac{L^2a}{2}\right) \frac{R_B}{EI}$$

$$\frac{(L+a)^3}{3}P - \left(\frac{L^3}{3} + \frac{L^2a}{2}\right)R_B = EIS_c \quad (2)$$

Data:  $E = 200 \times 10^9$  Pa

$$I = \frac{1}{12}(60)(60)^3 = 1.08 \times 10^6 \text{ mm}^4 = 1.08 \times 10^{-6} \text{ m}^4$$

$$EI = 216 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$S_0 = 0.5 \times 10^{-3} \text{ m}$$

$$S_c = 1.0 \times 10^{-3} \text{ m}$$

Using the data, eqs (1) and (2) become

$$0.06667P - 0.04167R_B = 108 \quad (1)'$$

$$0.11433P - 0.06667R_B = 216 \quad (2)'$$

Solving simultaneously

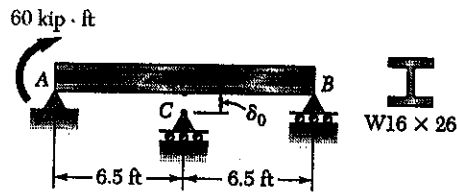
$$P = 5.63 \times 10^3 \text{ N} = 5.63 \text{ kN} \downarrow$$

$$R_B = 6.42 \times 10^3 \text{ N}$$



PROBLEM 9.94

9.94 Before the 60-kip·ft couple was applied, a gap,  $\delta_0 = 0.05$  in., existed between the W16 × 26 beam and the support at C. Knowing that  $E = 29 \times 10^6$  psi, determine the reaction at each support after the couple is applied.



SOLUTION

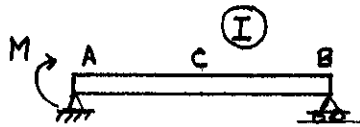
Units: Forces in kips, lengths in ft.

$$\delta_0 = 0.05 \text{ in} = 4.1667 \times 10^{-3} \text{ ft}$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 301 \text{ in}^4$$

$$EI = 8.729 \times 10^8 \text{ kip}\cdot\text{in}^2 = 60618 \text{ kip}\cdot\text{ft}^2$$

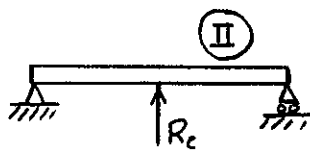


Loading I: Case 7 of Appendix D

$$y = -\frac{M}{6EIL} (x^3 - L^2x)$$

with  $M = 60 \text{ kip}\cdot\text{ft}$ ,  $L = 13 \text{ ft}$ ,  $x = 6.5 \text{ ft}$

$$(y_c)_1 = -\frac{(60)[6.5^3 - (13)^2(6.5)]}{(6)(60618)(13)} = -10.454 \times 10^{-3} \text{ ft}$$



Loading II: Case 4 of Appendix D

$$(y_c)_2 = \frac{R_c L^3}{48 EI} = \frac{(13)^3 R_c}{(48)(60618)}$$

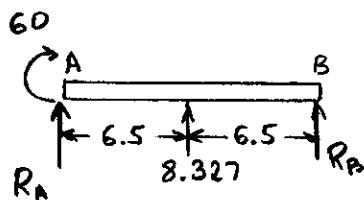
$$= 755.07 \times 10^{-6} R_c$$

Deflection at C

$$y_c = (y_c)_1 + (y_c)_2 = -\delta_0$$

$$-10.454 \times 10^{-3} + 755.07 \times 10^{-6} R_c = -4.1667 \times 10^{-3}$$

$$R_c = 8.327 \text{ kips } \uparrow$$



Statics:

$$\sum M_B = 0$$

$$-13 R_A - 60 - (6.5)(8.327) = 0$$

$$R_A = -8.779 \text{ kips}$$

$$R_A = 8.779 \text{ kips } \downarrow$$

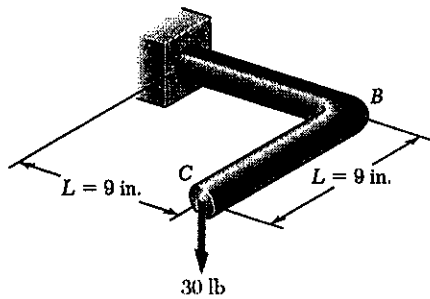
$$\sum M_A = 0$$

$$13 R_B - 60 + (6.5)(8.327) = 0$$

$$R_B = 0.452 \text{ kips } \uparrow$$

**PROBLEM 9.95**

9.95 A 5/8-inch-diameter rod  $ABC$  was been bent into the shape shown. Determine the deflection of end  $C$  after the 30-lb force is applied. Use  $E = 29 \times 10^6$  psi, and  $G = 11.2 \times 10^6$  psi.



**SOLUTION**

Let  $30 \text{ lb} = P$ .

Consider torsion of rod  $AB$ .

$$\phi_B = \frac{TL}{GJ} = \frac{(PL)L}{GJ} = \frac{PL^2}{GJ}$$

$$(y_C)_I = -L\phi_B = -\frac{PL^3}{GJ}$$

Consider bending of  $AB$  (Case I, App.D)

$$y_B = \frac{PL^3}{3EI}$$

$$(y_C)_II = y_B = -\frac{PL^3}{3EI}$$

Consider bending of  $BC$  (Case I, App.D)

$$(y_B)_III = -\frac{PL^3}{3EI}$$

Superposition

$$\begin{aligned} y_B &= (y_B)_I + (y_B)_II + (y_B)_III \\ &= -PL^3 \left( \frac{1}{GJ} + \frac{1}{3EI} + \frac{1}{3EI} \right) \\ &= -\frac{PL^3}{EI} \left( \frac{EI}{GJ} + \frac{2}{3} \right) \end{aligned}$$

Data:  $G = 11.2 \times 10^6$  psi,  $J = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} \left(\frac{5}{16}\right)^4 = 0.014980 \text{ in}^4$

$E = 29 \times 10^6$  psi,  $I = \frac{1}{2}J = 0.007490 \text{ in}^4$

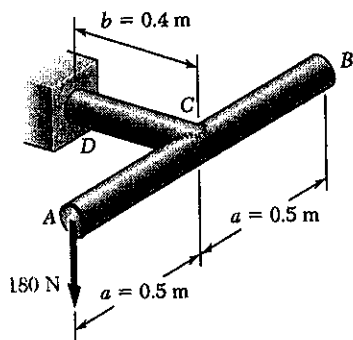
$EI = 217.21 \times 10^3 \text{ lb}\cdot\text{in}^2$        $GJ = 167.78 \times 10^3 \text{ lb}\cdot\text{in}^2$

$$y_B = -\frac{(30)(9)^3}{217.21 \times 10^3} \left( \frac{217.21 \times 10^3}{167.78 \times 10^3} + \frac{2}{3} \right) = -0.1975 \text{ in.}$$

$y_B = 0.1975 \text{ in.} \downarrow$

**PROBLEM 9.96**

9.96 Two 24-mm-diameter aluminum rods are welded together to form the T-shaped hanger shown. Knowing that  $E = 70 \text{ GPa}$  and  $G = 26 \text{ GPa}$ , determine the deflection at (a) end A, (b) end B.



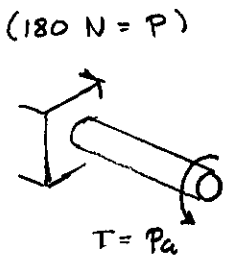
**SOLUTION**

Consider torsion of rod CD  $(180 \text{ N} = P)$

$$\phi_c = \frac{TL}{GJ} = \frac{(Pa)b}{GJ}$$

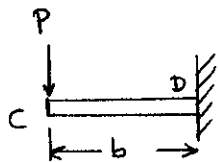
$$(y_A)_I = -a\phi_c = -\frac{Pa^2b}{GJ}$$

$$(y_B)_I = a\phi_c = \frac{Pa^2b}{GJ}$$



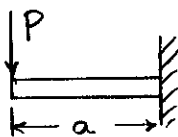
Consider bending of rod CD

$$(y_A)_{II} \quad (y_B)_{II} \quad (y_C)_{II} = -\frac{Pb^3}{3EI} \quad (\text{Case I, App D.})$$



Consider bending of rod portion AC

$$(y_A)_{III} = -\frac{Pa^3}{3EI}$$



By superposition.

$$y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III}$$

$$= P \left\{ -\frac{a^2b}{GJ} - \frac{b^3}{3EI} - \frac{a^3}{3EI} \right\}$$

$$y_B = (y_B)_I + (y_B)_{II}$$

$$= P \left\{ \frac{a^2b}{GJ} - \frac{b^3}{3EI} \right\}$$

Data:  $G = 26 \times 10^9 \text{ Pa}$ ,  $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$   
 $E = 70 \times 10^9 \text{ Pa}$ ,  $I = \frac{1}{2}J = 16.286 \times 10^{-9} \text{ m}^4$   
 $GJ = 846.87 \text{ N}\cdot\text{m}^2$   $EI = 1140.02 \text{ N}\cdot\text{m}^2$   
 $a = 0.5 \text{ m}$ ,  $b = 0.4 \text{ m}$

$$y_A = 180 \left\{ -\frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)} - \frac{(0.5)^3}{(3)(1140.02)} \right\} = -31.2 \times 10^{-3} \text{ m}$$

$$= 31.2 \text{ mm} \downarrow$$

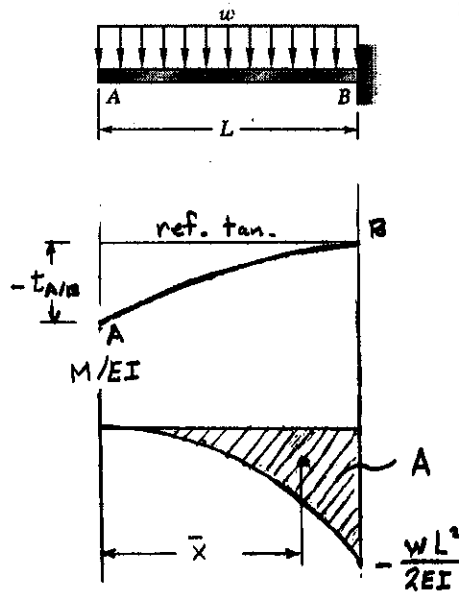
$$y_B = 180 \left\{ \frac{(0.5)^2(0.4)}{846.87} - \frac{(0.4)^3}{(3)(1140.02)} \right\} = 17.89 \times 10^{-3} \text{ m}$$

$$= 17.89 \text{ mm} \uparrow$$

PROBLEM 9.97

9.97 and 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.

SOLUTION



Place reference tangent at B.  $\theta_B = 0$

Draw  $\frac{M}{EI}$  curve as parabola.

$$A = -\frac{1}{3} \left( \frac{wL^2}{2EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$

By first moment-area theorem

$$\theta_{B/A} = A = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6} \frac{wL^3}{EI} = \frac{1}{6} \frac{wL^3}{EI}$$

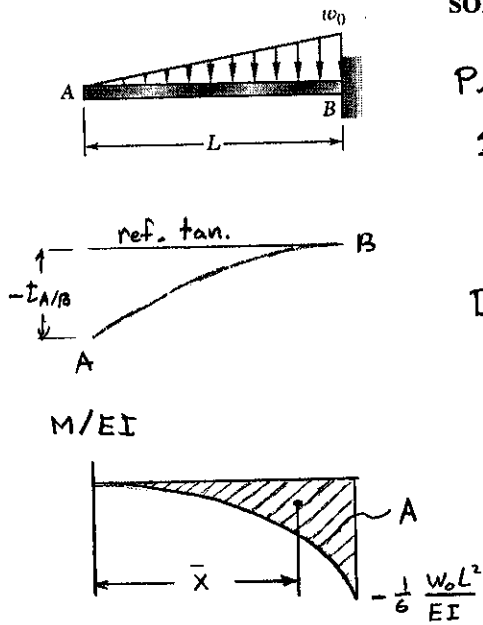
By second moment-area theorem

$$t_{A/B} = \bar{x} A = \left( \frac{3}{4}L \right) \left( -\frac{1}{6} \frac{wL^3}{EI} \right) = -\frac{1}{8} \frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8} \frac{wL^4}{EI}$$

PROBLEM 9.98

9.97 and 9.98 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



SOLUTION

Place reference tangent at B.  $\theta_B = 0$

$$\sum M_B = 0 \quad \left(\frac{1}{2}w_0L\right)\frac{L}{3} + M_B = 0$$

$$M_B = -\frac{1}{6}w_0L^2$$

Draw  $\frac{M}{EI}$  curve as cubic parabola.

$$A = -\frac{1}{4}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0L^3}{EI}$$

$$\bar{x} = L - \frac{1}{5}L = \frac{4}{5}L$$

By first moment-area theorem

$$\theta_{B/A} = A = -\frac{1}{24}\frac{w_0L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{24}\frac{w_0L^3}{EI} = \frac{1}{24}\frac{w_0L^3}{EI}$$

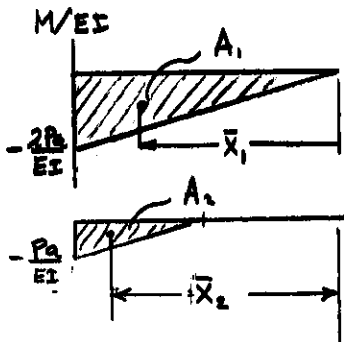
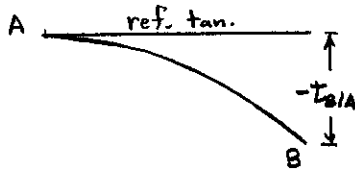
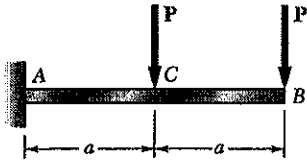
By second moment-area theorem

$$t_{A/B} = \bar{x}A = \left(\frac{4}{5}L\right)\left(-\frac{1}{24}\frac{w_0L^3}{EI}\right) = -\frac{1}{30}\frac{w_0L^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{30}\frac{w_0L^4}{EI}$$

PROBLEM 9.99

9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



SOLUTION

Place reference tangent at A.  $\theta_A = 0$ .

Draw  $\frac{M}{EI}$  diagram by parts (two triangles)

$$A_1 = \frac{1}{2} \left( -\frac{2Pa}{EI} \right) (2a) = -\frac{2Pa^2}{EI}$$

$$\bar{x}_1 = \frac{2}{3} (2a) = \frac{4}{3}a$$

$$A_2 = \frac{1}{2} \left( -\frac{Pa}{EI} \right) a = -\frac{1}{2} \frac{Pa^2}{EI}$$

$$\bar{x}_2 = a + \frac{2}{3}a = \frac{5}{3}a$$

By first moment-area theorem

$$\theta_{B/A} = A_1 + A_2 = -\frac{2Pa^2}{EI} - \frac{1}{2} \frac{Pa^2}{EI} = -\frac{5}{2} \frac{Pa^2}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{5}{2} \frac{Pa^2}{EI}$$

By second moment area theorem

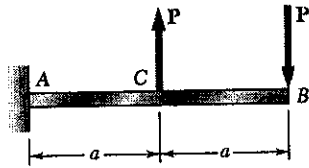
$$t_{B/A} = A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$= \left( -\frac{2Pa^2}{EI} \right) \left( \frac{4}{3}a \right) + \left( -\frac{1}{2} \frac{Pa^2}{EI} \right) \left( \frac{5}{3}a \right) = -\frac{7}{2} \frac{Pa^3}{EI}$$

$$y_B = t_{B/A} = -\frac{7}{2} \frac{Pa^3}{EI}$$

PROBLEM 9.100

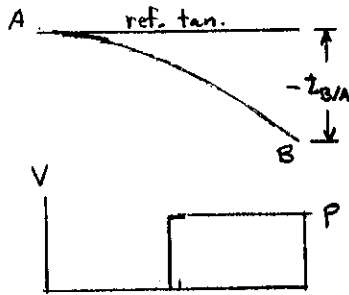
9.99 and 9.100 For the uniform cantilever beam and loading shown, determine (a) the slope at the free end, (b) the deflection at the free end.



SOLUTION

Place reference tangent at A.  $\theta_A = 0$

Draw V (shear) and  $\frac{M}{EI}$  diagrams.



$$A_1 = -\left(\frac{Pa}{EI}\right)(a) = -\frac{Pa^2}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{Pa}{EI}\right)(a) = -\frac{1}{2}\frac{Pa^2}{EI}$$

$$\bar{x}_1 = a + \frac{1}{2}a = \frac{3}{2}a$$

$$\bar{x}_2 = \frac{2}{3}a$$

By first moment-area theorem

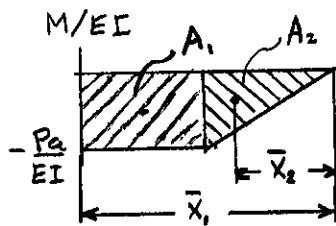
$$\theta_{B/A} = A_1 + A_2 = -\frac{Pa^2}{EI} - \frac{1}{2}\frac{Pa^2}{EI} = -\frac{3}{2}\frac{Pa^2}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{3}{2}\frac{Pa^2}{EI}$$

By second moment-area theorem

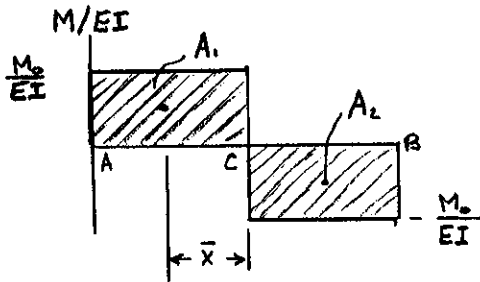
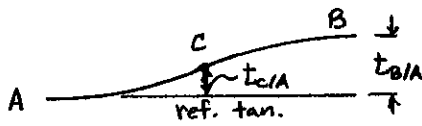
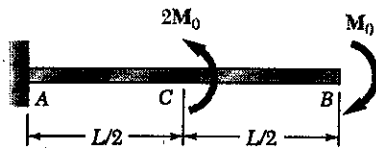
$$\begin{aligned} t_{B/A} &= A_1 \bar{x}_1 + A_2 \bar{x}_2 = \left(-\frac{Pa^2}{EI}\right)\left(\frac{3}{2}a\right) + \left(-\frac{1}{2}\frac{Pa^2}{EI}\right)\left(\frac{2}{3}a\right) \\ &= -\frac{11}{6}\frac{Pa^3}{EI} \end{aligned}$$

$$y_B = t_{B/A} = -\frac{11}{6}\frac{Pa^3}{EI}$$



PROBLEM 9.101

9.101 and 102 For the uniform cantilever beam and loading shown, determine (a) the slope at point B, (b) the deflection at C.



SOLUTION

Place reference tangent at A.  $\theta_A = 0$

Draw  $\frac{M}{EI}$  diagram.

$$A_1 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{2} \frac{M_0 L}{EI}$$

$$A_2 = \left(-\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{2} \frac{M_0 L}{EI}$$

By first moment-area theorem

$$\theta_{B/A} = A_1 + A_2 = \frac{1}{2} \frac{M_0 L}{EI} - \frac{1}{2} \frac{M_0 L}{EI} = 0$$

$$\theta_B = \theta_A + \theta_{B/A} = 0$$

Deflection at C.

By second moment-area theorem

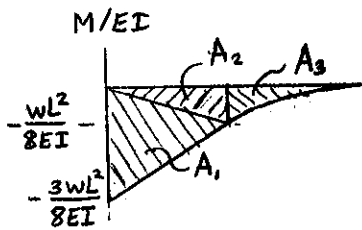
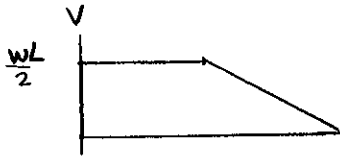
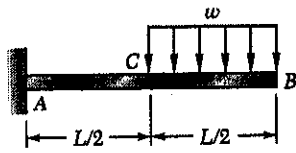
$$t_{C/A} = A_1 \bar{x} = \left(\frac{1}{2} \frac{M_0 L}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{8} \frac{M_0 L^2}{EI}$$

$$y_C = t_{C/A} = \frac{1}{8} \frac{M_0 L^2}{EI}$$



PROBLEM 9.102

9.101 and 102 For the uniform cantilever beam and loading shown, determine (a) the slope at point B, (b) the deflection at C.



SOLUTION

Place reference tangent at A.  $\theta_A = 0$

Draw V (shear) and  $\frac{M}{EI}$  diagrams.

(a) Slope at B

$$A_1 = -\frac{1}{2} \left( \frac{3wL^2}{8EI} \right) \left( \frac{L}{2} \right) = -\frac{3}{32} \frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{2} \left( \frac{wL^2}{8EI} \right) \left( \frac{L}{2} \right) = -\frac{1}{32} \frac{wL^3}{EI}$$

$$A_3 = -\frac{1}{3} \left( \frac{wL^2}{8EI} \right) \left( \frac{L}{2} \right) = -\frac{1}{48} \frac{wL^3}{EI}$$

$$\theta_{B/A} = A_1 + A_2 + A_3 = -\frac{7}{48} \frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{7}{48} \frac{wL^3}{EI}$$

(b) Deflection at C

$$\bar{x}_{1C} = \frac{2}{3} \cdot \frac{L}{2} = \frac{1}{3}L$$

$$\bar{x}_{2C} = \frac{1}{3} \cdot \frac{L}{2} = \frac{1}{6}L$$

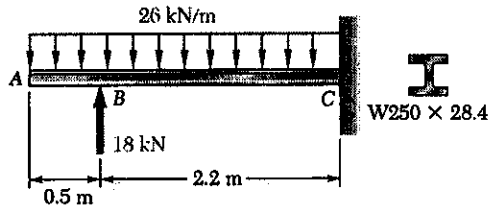
$$t_{C/A} = A_1 \bar{x}_{1C} + A_2 \bar{x}_{2C}$$

$$= \left( -\frac{3}{32} \frac{wL^3}{EI} \right) \left( \frac{1}{3}L \right) + \left( -\frac{1}{32} \frac{wL^3}{EI} \right) \left( \frac{1}{6}L \right) = -\frac{7}{192} \frac{wL^4}{EI}$$

$$y_C = t_{C/A} = -\frac{7}{192} \frac{wL^4}{EI}$$

**PROBLEM 9.103**

9.103 For the cantilever beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A. Use  $E = 200 \text{ GPa}$ .



**SOLUTION**

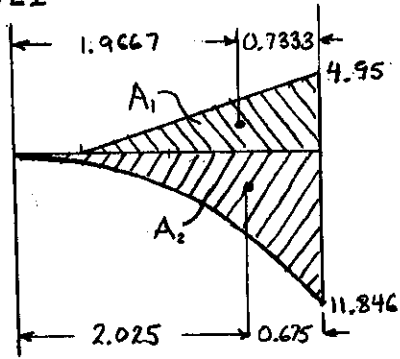
Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 40.0 \times 10^6 \text{ mm}^4 = 40.0 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(40.0 \times 10^{-6}) = 8.00 \times 10^6 \text{ N}\cdot\text{m}^2 = 8000 \text{ kN}\cdot\text{m}^2$$

$10^3 \text{ M/EI}$



Draw  $M/EI$  diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8000} = 4.95 \times 10^{-3} \text{ m}^{-1}$$

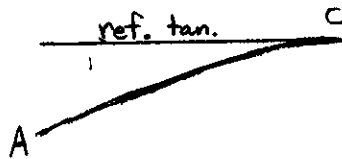
$$A_1 = \frac{1}{2}(4.95 \times 10^{-3})(2.2) = 5.445 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8000)} = -11.846 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-11.846 \times 10^{-3})(2.7) = -10.662 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$$



Draw reference tangent at C.

$$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$$

$$\theta_A = -A_1 - A_2 = -5.445 \times 10^{-3} + 10.662 \times 10^{-3} = 5.22 \times 10^{-3} \text{ rad}$$

$$\theta_A = 5.22 \times 10^{-3} \text{ } \triangleleft \quad \rightarrow$$

$$y_A = y_C - \theta_C L + t_{A/C}$$

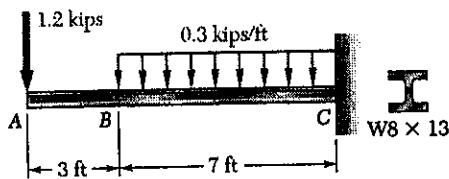
$$= 0 - 0 + A_1 \bar{x}_1 + A_2 \bar{x}_2$$

$$= 0 - 0 + (5.445 \times 10^{-3})(1.9667) - (10.662 \times 10^{-3})(2.025)$$

$$= -10.881 \times 10^{-3} \text{ m} = 10.88 \text{ mm } \downarrow$$

PROBLEM 9.104

9.104 For the cantilever beam and loading shown, determine (a) the slope at point A, (b) the deflection at point A. Use  $E = 29 \times 10^6$  psi.



SOLUTION

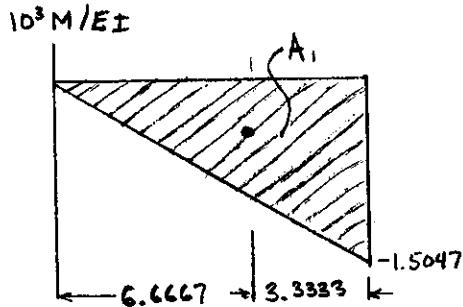
Units: Forces in kips, lengths in ft.

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 39.6 \text{ in}^4$$

$$EI = (29 \times 10^3)(39.6) = 1.1484 \times 10^6 \text{ kip}\cdot\text{in}^2 = 7975 \text{ kip}\cdot\text{ft}^2$$

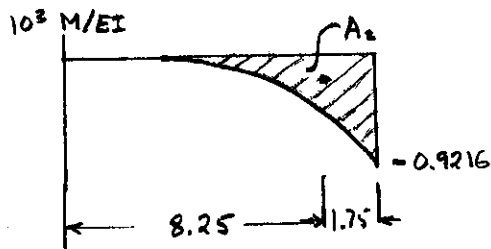
Draw  $\frac{M}{EI}$  diagram by parts.



$$\frac{M_1}{EI} = -\frac{(1.2)(10)}{7975} = -1.5047 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{2} (1.5047 \times 10^{-3})(10) = -7.5235 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3} (10) = 3.3333 \text{ ft}$$



$$\frac{M_2}{EI} = -\frac{(0.3)(7)^2}{(2)(7975)} = -0.9216 \times 10^{-3} \text{ ft}^{-1}$$

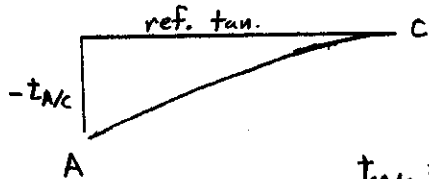
$$A_2 = \frac{1}{3} (-0.9216 \times 10^{-3})(7) = -2.1505 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4} (7) = 1.75 \text{ ft}$$

Place reference tangent at C.  $\theta_c = 0$

$$\theta_{C/A} = A_1 + A_2 = -9.67 \times 10^{-3}$$

$$\theta_A = \theta_c - \theta_{C/A} = 9.67 \times 10^{-3} \text{ rad}$$

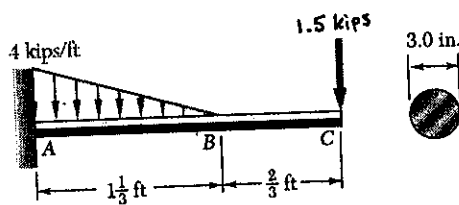


$$t_{A/C} = (6.667)(-7.5235 \times 10^{-3}) + (8.25)(-2.1505 \times 10^{-3}) = -67.90 \times 10^{-3} \text{ ft}$$

$$y_A = t_{A/C} = -67.90 \times 10^{-3} \text{ ft} = -0.814 \text{ in.}$$

PROBLEM 9.105

9.105 For the cantilever beam and loading shown, determine (a) the slope at point C, (b) the deflection at point C. Use  $E = 29 \times 10^6$  psi.



SOLUTION

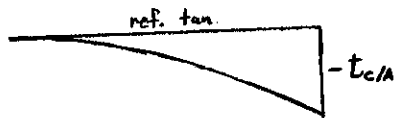
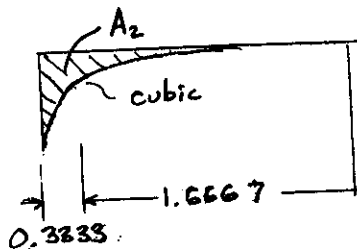
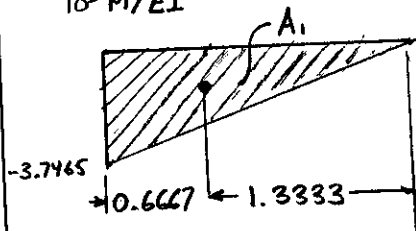
Units: Forces in kips, lengths in ft.

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = \frac{\pi}{4} \left(\frac{3}{2}\right)^4 = \frac{\pi}{4} (1.5)^4 = 3.97608 \text{ in}^4$$

$$EI = (29 \times 10^3)(3.97608) = 115.306 \text{ kip} \cdot \text{in}^2 = 800.74 \text{ kip} \cdot \text{ft}^2$$

$10^3 M/EI$



Draw  $\frac{M}{EI}$  diagram by parts

$$\frac{M_1}{EI} = -\frac{(1.5)(2)}{800.74} = -3.7465 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{2}(-3.7465 \times 10^{-3})(2) = -3.7465 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2) = 0.66667 \text{ ft}$$

$$\frac{M_2}{EI} = \frac{\frac{1}{2}(4)\left(\frac{4}{3}\right)\left(\frac{1}{3}-\frac{4}{3}\right)}{800.74} = -1.4801 \times 10^{-3} \text{ ft}^{-1}$$

$$A_2 = \frac{1}{4}(-1.4801 \times 10^{-3})\left(\frac{4}{3}\right) = -0.49337 \times 10^{-3}$$

$$\bar{x} = \frac{1}{4} \cdot \frac{4}{3} = 0.33333 \text{ ft}$$

Place reference tangent at A.  $\theta_A = 0$

$$\theta_{c/A} = A_1 + A_2 = -4.24 \times 10^{-3} \text{ rad}$$

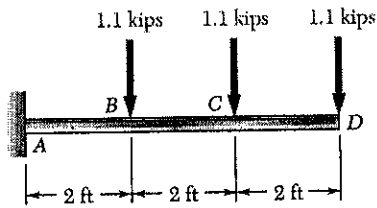
$$\theta_c = \theta_A + \theta_{c/A} = -4.24 \times 10^{-3} \text{ rad}$$

$$t_{c/A} = (1.3333)(-3.7465 \times 10^{-3}) + (1.6667)(-0.49337 \times 10^{-3}) = -6.71 \times 10^{-3} \text{ ft}$$

$$y_c = y_A + (2)(\theta_A) + t_{c/A} = 0 + 0 - 5.82 \times 10^{-3} = -5.82 \times 10^{-3} \text{ ft} = 0.0698 \text{ in.} \downarrow$$

PROBLEM 9.106

9.106 Two C 6 × 8.2 channels are welded back to back and loaded as shown. Knowing that  $E = 29 \times 10^6$  psi., determine (a) the slope at D, (b) the deflection at D.



SOLUTION

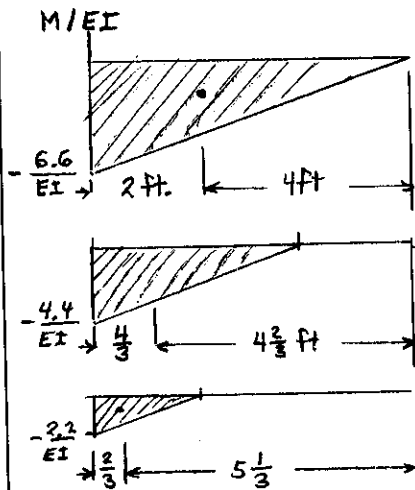
Units: Forces in kips, lengths in ft.

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = (2)(13.1) = 26.2 \text{ in}^4$$

$$EI = (29 \times 10^3)(26.2) = 759.8 \times 10^3 \text{ kip} \cdot \text{in}^2 = 5276 \text{ kip} \cdot \text{ft}^2$$

Draw  $\frac{M}{EI}$  diagram by parts.



$$\frac{M_1}{EI} = \frac{(1.1)(6)}{EI} = -\frac{6.6}{EI} \text{ ft}^{-1}$$

$$A_1 = \frac{1}{2} \left( \frac{6.6}{EI} \right) (6) = -\frac{19.8}{EI}$$

$$\bar{x}_1 = \frac{1}{3} (6) = 2 \text{ ft.}$$

$$\frac{M_2}{EI} = -\frac{(1.1)(4)}{EI} = -\frac{4.4}{EI} \text{ ft}^{-1}$$

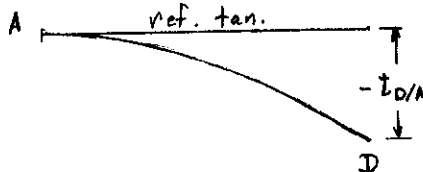
$$A_2 = \frac{1}{2} \left( -\frac{4.4}{EI} \right) (4) = -\frac{8.8}{EI}$$

$$\bar{x}_2 = \frac{1}{3} (4) = \frac{4}{3} \text{ ft}$$

$$\frac{M_3}{EI} = -\frac{(1.1)(2)}{EI} = -\frac{2.2}{EI} \text{ ft}^{-1}$$

$$A_3 = \frac{1}{2} \left( -\frac{2.2}{EI} \right) (2) = -\frac{2.2}{EI}$$

$$\bar{x}_3 = \frac{1}{3} (2) = \frac{2}{3} \text{ ft.}$$



Place reference tangent at A.  $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{30.8}{EI} = -\frac{30.8}{5276} = -5.84 \times 10^{-3} \text{ rad.}$$

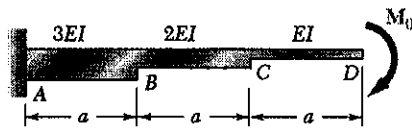
$$\theta_D = \theta_A + \theta_{D/A} = -5.84 \times 10^{-3} \text{ rad.}$$

$$t_{D/A} = \left( -\frac{19.8}{EI} \right) (4) + \left( -\frac{8.8}{EI} \right) \left( 4\frac{2}{3} \right) + \left( -\frac{2.2}{EI} \right) \left( 5\frac{1}{3} \right) = -\frac{132.0}{EI} = -\frac{132.0}{5276} = 25.02 \times 10^{-3} \text{ ft}$$

$$y_D = t_{D/A} = 25.02 \times 10^{-3} \text{ ft} = 0.300 \text{ in. } \downarrow$$

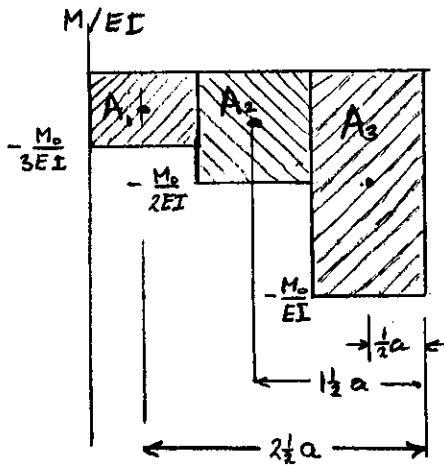
PROBLEM 9.107

9.107 For the cantilever beam and loading shown, determine the deflection and slope at end D caused by the couple  $M_0$ .



SOLUTION

Draw  $\frac{M}{EI}$  diagram.



$$A_1 = -\frac{M_0 a}{3EI}$$

$$A_2 = -\frac{M_0 a}{2EI}$$

$$A_3 = -\frac{M_0 a}{EI}$$

Place reference tangent at A.  $\theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3$$

$$= -\frac{11}{6} \frac{M_0 a}{EI}$$

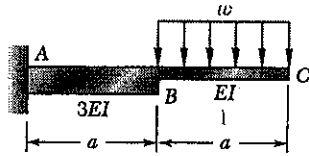
$$\theta_D = \theta_A + \theta_{D/A} = -\frac{11}{6} \frac{M_0 a}{EI}$$

$$t_{D/A} = -\left(\frac{M_0 a}{3EI}\right)\left(2\frac{1}{2}a\right) - \left(\frac{M_0 a}{2EI}\right)\left(1\frac{1}{2}a\right) - \left(\frac{M_0 a}{EI}\right)\left(\frac{1}{2}a\right) = -\frac{25}{12} \frac{M_0 a^2}{EI}$$

$$y_D = t_{D/A} = -\frac{25}{12} \frac{M_0 a^2}{EI}$$

PROBLEM 9.108

9.108 For the cantilever beam and loading shown, determine the deflection at (a) point B, (b) point C.



SOLUTION

Draw  $\frac{M}{EI}$  diagram

$$A_1 = \frac{1}{2} \left( -\frac{1}{6} \frac{wa^2}{EI} \right) a = -\frac{1}{12} \frac{wa^3}{EI}$$

$$A_2 = \frac{1}{2} \left( -\frac{1}{2} \frac{wa}{EI} \right) a = -\frac{1}{4} \frac{wa^3}{EI}$$

$$A_3 = \frac{1}{3} \left( -\frac{1}{2} \frac{wa^2}{EI} \right) a = -\frac{1}{6} \frac{wa^3}{EI}$$

Place reference tangent at A

(a) Deflection at B.

$$t_{B/A} = A_1 \left( \frac{1}{3}a \right) + A_2 \left( \frac{2}{3}a \right)$$

$$= \left( -\frac{1}{12} \frac{wa^3}{EI} \right) \left( \frac{1}{3}a \right) + \left( -\frac{1}{4} \frac{wa^3}{EI} \right) \left( \frac{2}{3}a \right)$$

$$= -\frac{7}{36} \frac{wa^4}{EI}$$

$$y_B = t_{B/A} = -\frac{7}{36} \frac{wa^4}{EI}$$

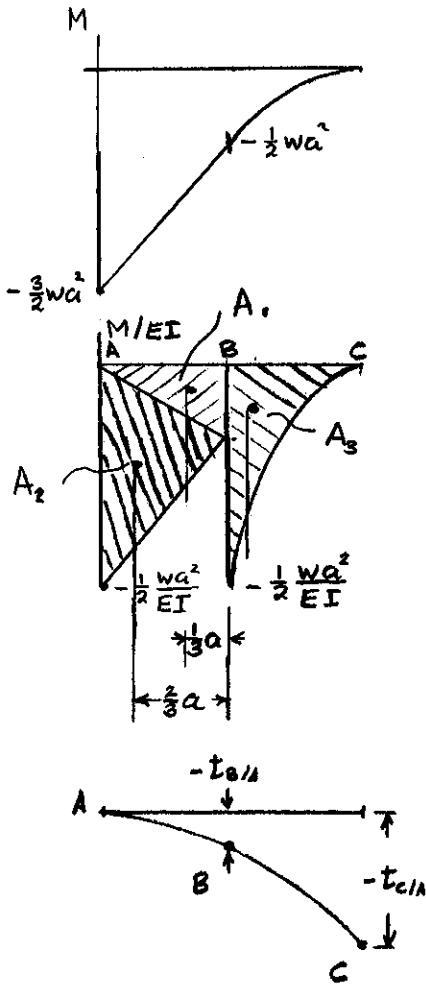
(b) Deflection at C.

$$t_{C/A} = A_1 \left( a + \frac{1}{3}a \right) + A_2 \left( a + \frac{2}{3}a \right) + A_3 \left( a - \frac{1}{4}a \right)$$

$$= \left( -\frac{1}{12} \frac{wa^3}{EI} \right) \left( \frac{4}{3}a \right) + \left( -\frac{1}{4} \frac{wa^3}{EI} \right) \left( \frac{5}{3}a \right)$$

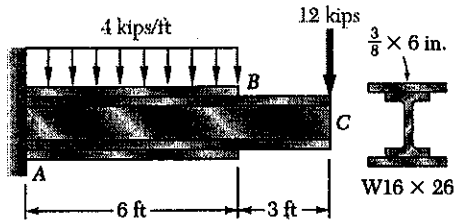
$$- \left( \frac{1}{6} \frac{wa^3}{EI} \right) \left( \frac{3}{4}a \right) = -\frac{47}{72} \frac{wa^4}{EI}$$

$$y_C = t_{C/A} = -\frac{47}{72} \frac{wa^4}{EI}$$



PROBLEM 9.109

9.109 Two cover plates are welded to the rolled-steel beam as shown. Using  $E = 29 \times 10^6$  psi., determine (a) the slope at end C, (b) the deflection at end C.



SOLUTION

For W16x26 rolled steel section

$$d = 15.69 \text{ in} \quad I = 301 \text{ in}^4$$

For the two cover plates

$$I = 2 \left[ \frac{1}{2}(6) \left(\frac{3}{8}\right)^3 + (6) \left(\frac{3}{8}\right) \left(\frac{15.69}{2} + \frac{3}{8}\right)^2 \right] = 290.4 \text{ in}^4$$

$$A \text{ to } B \quad EI_1 = (29 \times 10^3)(301 + 290.4) = 17.151 \times 10^6 \text{ kip}\cdot\text{in}^2 = 119101 \text{ kip}\cdot\text{ft}^2$$

$$B \text{ to } C \quad EI_2 = (29 \times 10^3)(301) = 8.729 \times 10^6 \text{ kip}\cdot\text{in}^2 = 60618 \text{ kip}\cdot\text{ft}^2$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$\frac{M_1}{EI_1} = -\frac{(12)(9)}{119101} = -0.90679 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = -\frac{1}{2}(0.90679 \times 10^{-3})(9) = -4.081 \times 10^{-3}$$

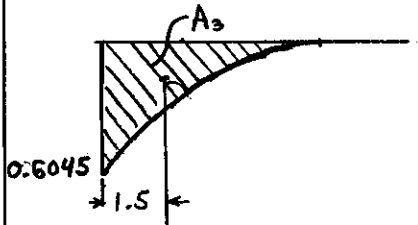
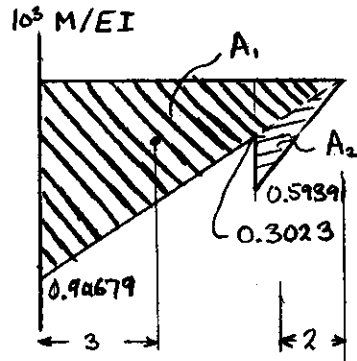
$$\frac{M_2}{EI_2} = -\frac{(12)(3)}{60618} = -0.3023 \times 10^{-3} \text{ ft}^{-1}$$

$$\frac{M_3}{EI_2} = -\frac{(12)(3)}{60618} = -0.5939 \times 10^{-3} \text{ ft}^{-1}$$

$$A_2 = -\frac{1}{2}(0.5939 - 0.3023)(10^{-3})(3) = -0.437 \times 10^{-3}$$

$$\frac{M_3}{EI_1} = -\frac{(4)(6)(6)}{(2)(119101)} = -0.6045 \times 10^{-3} \text{ ft}^{-1}$$

$$A_3 = -\frac{1}{3}(0.6045 \times 10^{-3})(6) = -1.209 \times 10^{-3}$$



Place reference tangent at A where  $y_A = 0$ ,  $\theta_A = 0$

$$(a) \quad \theta_c = \theta_A + \theta_{c/A} = 0 + A_1 + A_2 + A_3 = -5.73 \times 10^{-3} \text{ rad}$$

$$(b) \quad y_c = y_A + L\theta_A + t_{c/A}$$

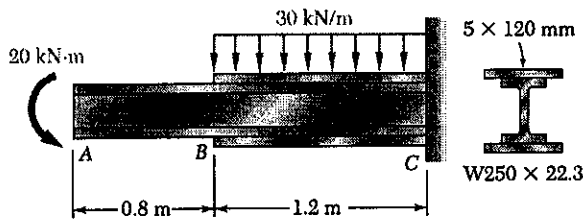
$$= 0 + 0 - (4.081 \times 10^{-3})(6) - (0.437 \times 10^{-3})(2) - (1.209 \times 10^{-3})(7.5)$$

$$= -34.43 \times 10^{-3} \text{ ft} = 0.413 \text{ in.} \downarrow$$



PROBLEM 9.110

9.110 Two cover plates are welded to the rolled-steel beam as shown. Using  $E = 200$  GPa, determine (a) the slope at end A, (b) the deflection at end A.



SOLUTION

Units: Forces in kN, lengths in m.

$E = 200 \times 10^9 \text{ Pa}$

From A to B  $I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6 \text{ N}\cdot\text{m}^2 = 5780 \text{ kN}\cdot\text{m}^2$

From B to C  $I = I_w + 2A_p d^2 + 2\bar{I}_p$

$A_p = 5 \times 120 = 600 \text{ mm}^2$

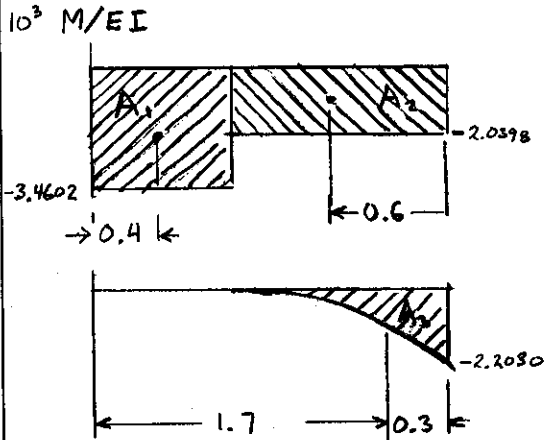
$d = \frac{254}{2} + \frac{5}{2} = 129.5 \text{ mm}$

$A_p d^2 = 10.062 \times 10^6 \text{ mm}^4$

$\bar{I}_p = \frac{1}{12}(120)(5)^3 = 0.00125 \times 10^6 \text{ mm}^4$

$I = [28.9 + (2)(10.062) + (2)(0.00125)] \times 10^6 \text{ mm}^4 = 49.03 \times 10^6 \text{ mm}^4 = 49.03 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(49.03 \times 10^{-6}) = 9.805 \times 10^6 \text{ N}\cdot\text{m}^2 = 9805 \text{ kN}\cdot\text{m}^2$



Draw M/EI diagram by parts

A to B  $\frac{M_1}{EI} = -\frac{20}{5780} = -3.4602 \times 10^{-3} \text{ m}^{-1}$

B to C  $\frac{M_2}{EI} = -\frac{20}{9805} = -2.0398 \times 10^{-3} \text{ m}^{-1}$

$\frac{M_3}{EI} = -\frac{(30)(1.2)^2}{(2)(9805)} = -2.2030 \times 10^{-3} \text{ m}^{-1}$

$A_1 = (-3.4602 \times 10^{-3})(0.8) = -2.7682 \times 10^{-3}$

$A_2 = (-2.0398 \times 10^{-3})(1.2) = -2.4478 \times 10^{-3}$

$A_3 = \frac{1}{3}(-2.2030 \times 10^{-3})(1.2) = -0.8812 \times 10^{-3}$

Place reference tangent at C.  $\theta_c = 0$

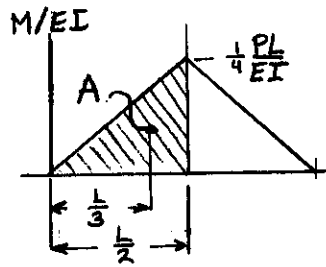
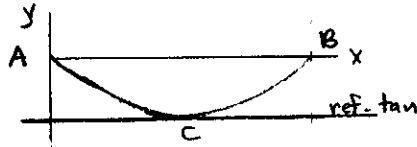
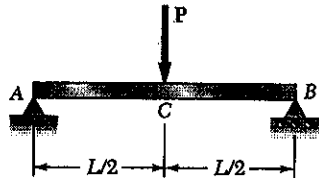
(a) Slope at A  $\theta_A = \theta_c - \theta_{A/C} = 0 - (A_1 + A_2 + A_3) = 6.10 \times 10^{-3} \text{ rad}$

(b) Deflection at A

$y_A = t_{A/C} = (-2.7682 \times 10^{-3})(0.4) + (-2.4478 \times 10^{-3})(1.4) + (-0.8812 \times 10^{-3})(1.7) = -5.03 \times 10^{-3} \text{ m} = 5.03 \text{ mm} \downarrow$

PROBLEM 9.111

9.111 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



SOLUTION

Symmetrical beam and loading.

Place reference tangent at C.

$$\theta_c = 0, \quad y_c = -t_{A/C}$$

$$\text{Reactions } R_A = R_B = \frac{1}{2}P$$

$$\text{Bending moment at C } M_c = \frac{1}{4}PL$$

$$A = \frac{1}{2} \left( \frac{1}{4} \frac{PL}{EI} \right) \left( \frac{L}{2} \right) = \frac{1}{16} \frac{PL^2}{EI}$$

$$(a) \text{ Slope at A: } \theta_A = \theta_c - \theta_{c/A}$$

$$\theta_A = 0 - \frac{1}{16} \frac{PL^2}{EI} = -\frac{1}{16} \frac{PL^2}{EI}$$

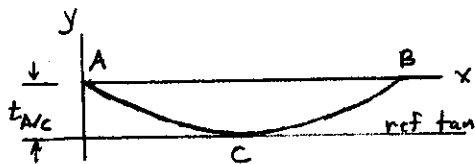
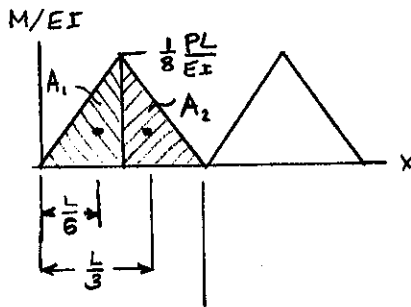
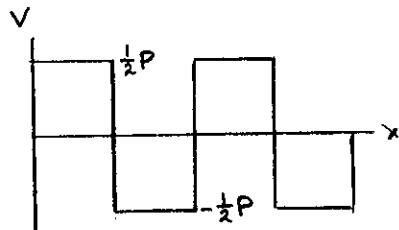
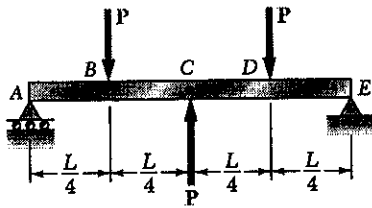
$$(b) \text{ Deflection at C}$$

$$y_c = -t_{A/C} = -A \left( \frac{L}{3} \right) = - \left( \frac{1}{16} \frac{PL^2}{EI} \right) \left( \frac{L}{3} \right)$$

$$y_c = \frac{1}{48} \frac{PL^3}{EI}$$

PROBLEM 9.112

9.111 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



SOLUTION

Symmetrical beam and loading.

Place reference tangent at C.  $\theta_c = 0$

Reactions  $R_A = R_E = \frac{1}{2}P$

Draw V (shear) and M/EI diagrams.

$$A_1 = A_2 = \frac{1}{2} \left( \frac{1}{8} \frac{PL}{EI} \right) \frac{L}{4} = \frac{1}{64} \frac{PL^2}{EI}$$

(a) Slope at A

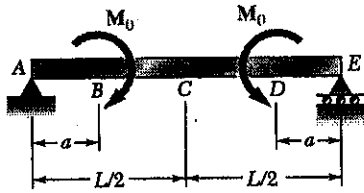
$$\begin{aligned} \theta_A &= \theta_C - \theta_{A/C} = 0 - A_1 - A_2 \\ &= -\frac{1}{32} \frac{PL^2}{EI} \end{aligned}$$

(b) Deflection at C

$$\begin{aligned} y_C &= -t_{A/C} = -\left( A_1 \frac{L}{6} + A_2 \frac{L}{3} \right) \\ &= -\left( \frac{1}{64} \frac{PL^3}{EI} \cdot \frac{L}{6} + \frac{1}{64} \frac{PL^3}{EI} \cdot \frac{L}{3} \right) \\ &= -\frac{1}{128} \frac{PL^3}{EI} \end{aligned}$$

PROBLEM 9.113

9.111 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.

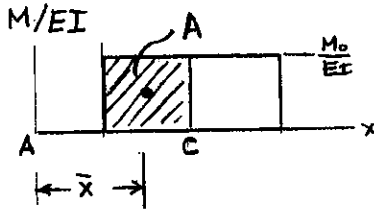


SOLUTION

Symmetrical beam and loading.

Place reference tangent at C.  $\theta_c = 0$ .

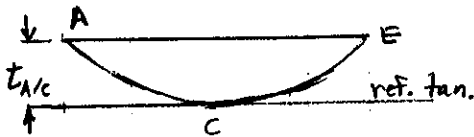
Draw  $\frac{M}{EI}$  diagram.



(a) Slope at A  $\theta_A$

$$A = \frac{M_0}{EI} \left( \frac{L}{2} - a \right) = \frac{1}{2} \frac{M_0}{EI} (L - 2a)$$

$$\begin{aligned} \theta_A &= \theta_c - \theta_{c/A} = 0 - A = \\ &= -\frac{1}{2} \frac{M_0}{EI} (L - 2a) \end{aligned}$$



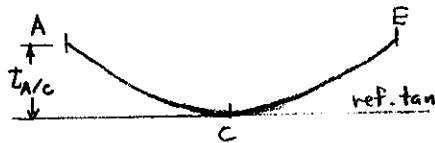
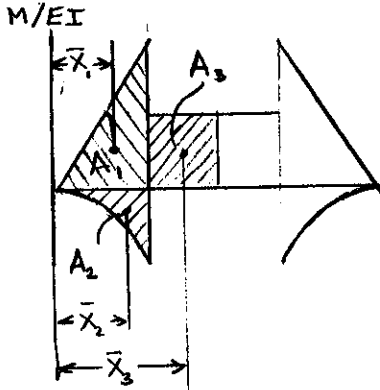
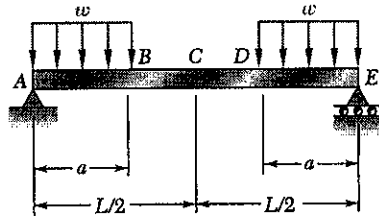
(b) Deflection at C

$$\bar{x} = a + \frac{1}{2} \left( \frac{L}{2} - a \right) = \frac{1}{4} (L + 2a)$$

$$\begin{aligned} y_c &= -t_{c/A} = A \bar{x} \\ &= -\frac{1}{2} \frac{M_0}{EI} (L - 2a) \frac{1}{4} (L + 2a) \\ &= -\frac{1}{8} \frac{M_0}{EI} (L^2 - 4a^2) \end{aligned}$$

PROBLEM 9.114

9.111 through 9.114 For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.



SOLUTION

Symmetric beam and loading.

Place reference tangent at C.  $\theta_c = 0$

Reactions  $R_A = R_E = wa$

Bending moment

Over AB  $M = wax - \frac{1}{2}wa^2$

Over BD  $M = \frac{1}{2}wa^2$

Draw  $\frac{M}{EI}$  diagram by parts

$$\frac{M_1}{EI} = \frac{wa^2}{EI} \quad \frac{M_2}{EI} = -\frac{1}{2} \frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2} \frac{wa^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} a = \frac{1}{2} \frac{wa^3}{EI}$$

$$A_2 = -\frac{1}{3} \frac{M_2}{EI} a = -\frac{1}{6} \frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI} \left(\frac{L}{2} - a\right) = \frac{1}{4} \frac{wa^2}{EI} (L - 2a)$$

(a) Slope at A.  $\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$

$$= -\frac{1}{2} \frac{wa^3}{EI} + \frac{1}{6} \frac{wa^3}{EI} - \frac{1}{4} \frac{wa^2}{EI} (L - 2a) = -\frac{wa^2}{EI} \left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12} \frac{wa^2}{EI} (3L - 2a)$$

(b) Deflection at C  $y_c = -t_{c/A}$

$$\bar{x}_1 = \frac{2}{3}a, \quad \bar{x}_2 = \frac{3}{4}a, \quad \bar{x}_3 = a + \frac{1}{2} \left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_c = -t_{c/A} = -A_1 \bar{x}_1 - A_2 \bar{x}_2 - A_3 \bar{x}_3$$

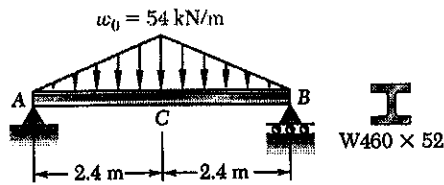
$$= -\left(\frac{1}{2} \frac{wa^3}{EI}\right) \left(\frac{2}{3}a\right) + \left(\frac{1}{6} \frac{wa^3}{EI}\right) \left(\frac{3}{4}a\right) - \frac{1}{4} \left(\frac{wa^2}{EI}\right) (L - 2a) \frac{1}{4}(L + 2a)$$

$$= -\frac{1}{3} \frac{wa^3}{EI} + \frac{1}{8} \frac{wa^3}{EI} - \frac{1}{16} \frac{wa^2}{EI} (L^2 - 4a^2)$$

$$= -\frac{wa^2}{EI} \left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48} \frac{wa^2}{EI} (3L^2 - 2a^2)$$

PROBLEM 9.115

9.115 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint of the beam. Use  $E = 200 \text{ GPa}$ .



SOLUTION

Symmetric beam and loading.  $R_A = R_B$   
Place reference tangent at C.  $\theta_c = 0$

Units: Force in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 212 \times 10^6 \text{ mm}^4 = 212 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{ N}\cdot\text{m}^2 = 42400 \text{ kN}\cdot\text{m}^2$$

$$+\uparrow \Sigma F_y = 0 \quad R_A + R_B - \frac{1}{2}(54)(4.8) = 0$$

$$R_A = 64.8 \text{ kN}$$

$$k = \frac{54}{2.4} = 22.5 \text{ kN/m}^2$$

For A to C  $M = R_A x - \frac{1}{6} kx^3$

At C

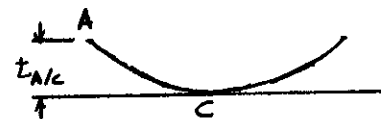
$$\frac{M}{EI} = \frac{(64.8)(2.4)}{42400} - \frac{(22.5)(2.4)^3}{(6)(42400)}$$

$$= 3.6679 \times 10^{-3} - 1.2226 \times 10^{-3} \text{ m}^{-1}$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$A_1 = \frac{1}{2}(3.6679 \times 10^{-3})(2.4) = 4.4015 \times 10^{-3}$$

$$A_2 = -\frac{1}{4}(1.2226 \times 10^{-3})(2.4) = -0.73356 \times 10^{-3}$$



(a) Slope at A  $\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2)$

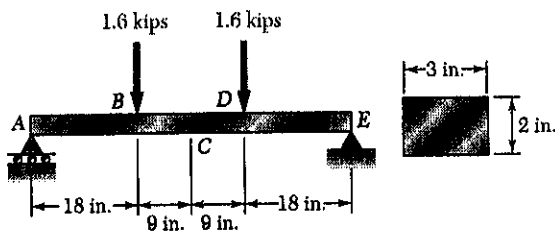
$$= -4.4015 \times 10^{-3} + 0.73356 \times 10^{-3} = -3.67 \times 10^{-3} \text{ rad.}$$

(b) Deflection at C  $y_c = -t_{A/C} = -[(4.4015 \times 10^{-3})(1.6) - (0.73356 \times 10^{-3})(1.92)]$

$$= -5.63 \times 10^{-3} \text{ m} = 5.63 \text{ mm} \downarrow$$

PROBLEM 9.116

9.116 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C of the beam. Use  $E = 29 \times 10^6$  psi.



SOLUTION

$$I = \frac{1}{12}(3)(2)^3 = 2.0 \text{ in}^4$$

$$E = 29 \times 10^3 \text{ ksi}$$

$$EI = (29 \times 10^3)(2.0) = 58 \times 10^3 \text{ kip} \cdot \text{in}^2$$

Symmetric beam and loading.

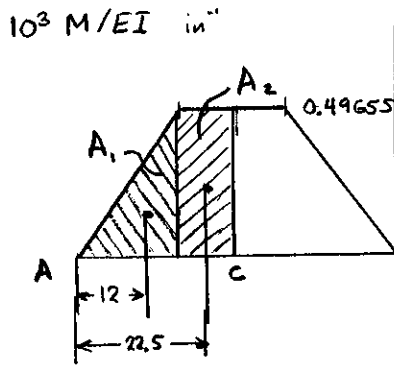
$$R_A = R_E = 1.6 \text{ kips}$$

Draw  $\frac{M}{EI}$  diagram.

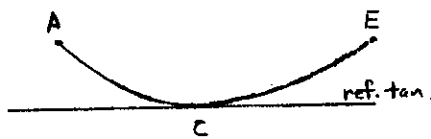
$$\frac{M_{max}}{EI} = \frac{(1.6)(18)}{58 \times 10^3} = 0.49655 \times 10^{-3} \text{ in}^{-1}$$

$$A_1 = \frac{1}{2}(0.49655 \times 10^{-3})(18) = 4.469 \times 10^{-3}$$

$$A_2 = (0.49655 \times 10^{-3})(9) = 4.469 \times 10^{-3}$$



Place reference tangent at C.  $\theta_c = 0$



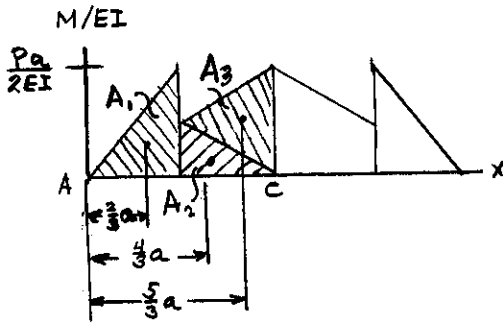
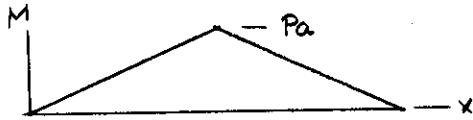
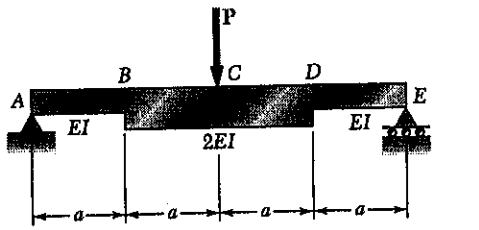
(a) Slope at A  $\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2) = -8.94 \times 10^{-3} \text{ rad}$

(b) Deflection at C  $|y_c| = t_{A/C} = (4.469 \times 10^{-3})(12) + (4.469 \times 10^{-3})(22.5) = 0.1542 \text{ in} \downarrow$

PROBLEM 9.117

9.117 and 9.118 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C of the beam.

SOLUTION



Symmetric beam and loading.  $R_A = R_E = \frac{1}{2}P$   
 $M_{max} = (\frac{1}{2}P)(2a) = Pa$

Draw M and  $\frac{M}{EI}$  diagrams.

$$A_1 = \frac{1}{2} \left( \frac{Pa}{2EI} \right) a = \frac{1}{4} \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2} \left( \frac{Pa}{4EI} \right) a = \frac{1}{8} \frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2} \left( \frac{Pa}{2EI} \right) a = \frac{1}{4} \frac{Pa^2}{EI}$$

Place reference tangent at C.  $\theta_C = 0$

(a) Slope at A.

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8} \frac{Pa^2}{EI}$$

(b) Deflection at C

$$|y_C| = t_{A/C} = A_1 \left( \frac{2}{3}a \right) + A_2 \left( \frac{4}{3}a \right) + A_3 \left( \frac{5}{3}a \right)$$

$$= \frac{1}{6} \frac{Pa^3}{EI} + \frac{1}{6} \frac{Pa^3}{EI} + \frac{5}{12} \frac{Pa^3}{EI}$$

$$= \frac{3}{4} \frac{Pa^3}{EI} \downarrow$$

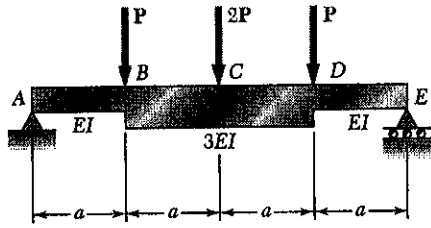




PROBLEM 9.118

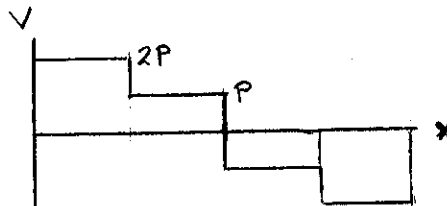
9.117 and 9.118 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C of the beam.

SOLUTION



Symmetric beam and loading.  $R_A = R_E = 2P$ .

Draw  $V$ ,  $M$ , and  $\frac{M}{EI}$  diagrams.

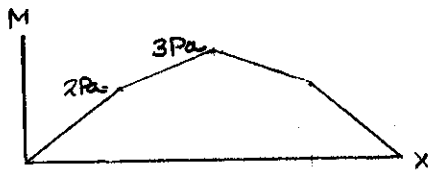


$$A_1 = \frac{1}{2} \left( \frac{2Pa}{EI} \right) a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2} \left( \frac{2Pa}{3EI} \right) a = \frac{1}{3} \frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2} \left( \frac{Pa}{EI} \right) a = \frac{1}{2} \frac{Pa^2}{EI}$$

Place reference tangent at C.  $\theta_C = 0$



(a) Slope at A

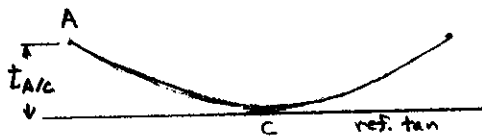
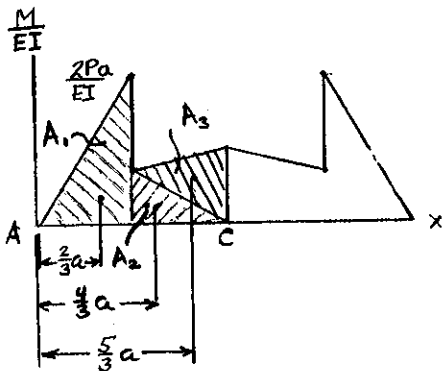
$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6} \frac{Pa^2}{EI}$$

(b) Deflection at C.

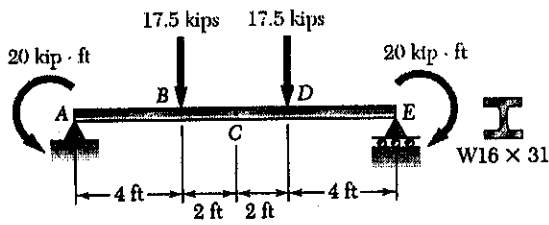
$$|y_C| = t_{A/C} = A_1 \left( \frac{2}{3}a \right) + A_2 \left( \frac{4}{3}a \right) + A_3 \left( \frac{5}{3}a \right)$$

$$= \frac{35}{18} \frac{Pa^3}{EI} \downarrow$$



**PROBLEM 9.119**

9.119 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the midpoint C of the beam. Use  $E = 29 \times 10^6$  psi.

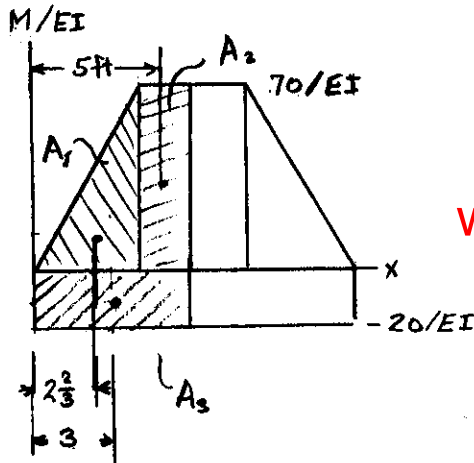


**SOLUTION**

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 375 \text{ in}^4$$

$$EI = (29 \times 10^3)(375) = 10.875 \times 10^6 \text{ kip}\cdot\text{in}^2 = 75521 \text{ kip}\cdot\text{ft}^2$$



Symmetric beam and loading.

$$R_A = R_E = 17.5 \text{ kips}$$

Bending moments:

$$M_A = 20 \text{ kip}\cdot\text{ft}$$

$$M_B = -20 + (17.5)(4) = -20 + 70 \text{ kip}\cdot\text{ft}$$

$$M_C = -20 + 70 \text{ kip}\cdot\text{ft}$$

Draw  $M/EI$  diagram by parts.

$$A_1 = \frac{1}{2}(70)(4) = 140/EI$$

$$A_2 = (70)(2) = 140/EI$$

$$A_3 = -(20)(6) = -120/EI$$

Place reference tangent at C.  $\theta_C = 0$

(a) Slope at A.  $\theta_A = \theta_C - \theta_{C/A}$

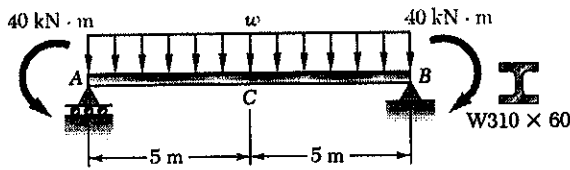
$$\theta_A = 0 - (A_1 + A_2 + A_3) = -160/EI = -\frac{160}{75521} = -2.119 \times 10^{-3} \text{ rad.}$$

(b) Deflection at C  $|y_C| = t_{A/C}$

$$|y_C| = \frac{1}{EI} \left\{ (140)\left(\frac{2}{3}\right) + (140)(5) - (120)(3) \right\} = \frac{713 \frac{1}{3}}{EI} = \frac{713 \frac{1}{3}}{75521} = 9.445 \times 10^{-3} \text{ ft} = 0.1133 \text{ in. } \downarrow$$

**PROBLEM 9.120**

9.120 For the beam and loading shown and knowing that  $w = 8 \text{ kN/m}$ , determine (a) the slope at end  $A$ , (b) the deflection at the midpoint  $C$  of the beam. Use  $E = 200 \text{ GPa}$ .



**SOLUTION**

$$E = 200 \times 10^9 \text{ Pa}$$

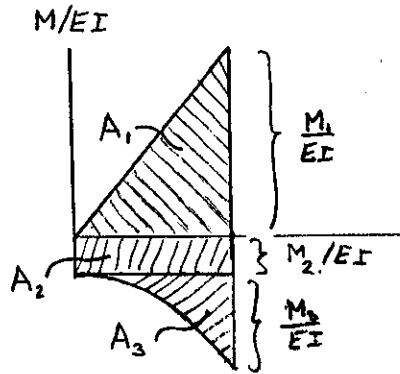
$$I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$= 25800 \text{ kN}\cdot\text{m}^2$$

Symmetrical beam and loading.

$$R_A = R_B = \frac{1}{2}(8)(10) = 40 \text{ kN}$$



[www.konkur.in](http://www.konkur.in)

$$M = 40x - 40 - \frac{1}{2}(8)x^2$$

At  $x = 5$

$$M = 200 - 40 - 100$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$\frac{M_1}{EI} = \frac{200}{25800} = 7.7519 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = \frac{40}{25800} = -1.5504 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_3}{EI} = \frac{100}{25800} = -3.8760 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(7.7519 \times 10^{-3})(5) = 19.380 \times 10^{-3}$$

$$\bar{x}_1 = \left(\frac{2}{3}\right)(5) = 3.3333 \text{ m}$$

$$A_2 = -(1.5504)(5) = -7.7520 \times 10^{-3}$$

$$\bar{x}_2 = \left(\frac{1}{2}\right)(5) = 2.5 \text{ m}$$

$$A_3 = -\frac{1}{3}(3.8760)(5) = -6.4600 \times 10^{-3}$$

$$\bar{x}_3 = \left(\frac{3}{4}\right)(5) = 3.75 \text{ m}$$

Place reference tangent at  $C$ .  $\theta_c = 0$

(a) Slope at  $A$ .  $\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$

$$\theta_A = -(19.380 \times 10^{-3} - 7.7520 \times 10^{-3} - 6.4600 \times 10^{-3}) = -5.17 \times 10^{-3} \text{ rad}$$

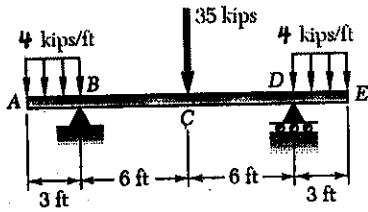
(b) Deflection at  $C$   $|y_c| = t_{A/C}$

$$= (19.380 \times 10^{-3})(3.3333) - (7.7520 \times 10^{-3})(2.5) - (6.4600 \times 10^{-3})(3.75)$$

$$= 21.0 \times 10^{-3} \text{ m} = 21.0 \text{ mm} \downarrow$$

PROBLEM 9.121

9.121 For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at point A. Use  $E = 29 \times 10^6$  psi.



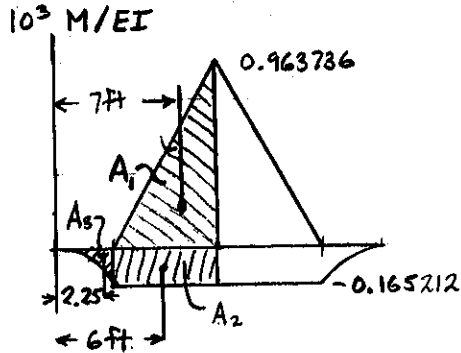
SOLUTION

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 541 \text{ in}^4$$

$$EI = (29 \times 10^3)(541) = 15.689 \times 10^6 \text{ ksi}$$

$$= 108951 \text{ kip} \cdot \text{ft}^2$$



Draw bending diagram by parts

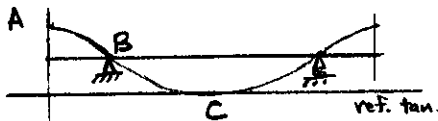
$$\frac{M_1}{EI} = \frac{(\frac{1}{2})(35)(6)}{108951} = 0.963736 \times 10^{-3} \text{ ft}^{-1}$$

$$A_1 = (\frac{1}{2})(0.963736 \times 10^{-3})(6) = 2.8912 \times 10^{-3}$$

$$\frac{M_2}{EI} = -\frac{(4)(3)(1.5)}{108951} = -0.165212 \times 10^{-3} \text{ ft}^{-1}$$

$$A_2 = -(0.165212 \times 10^{-3})(6) = -0.99127 \times 10^{-3}$$

$$A_3 = -\frac{1}{3}(0.165212 \times 10^{-3})(3) = -0.165212 \times 10^{-3}$$



Place reference at symmetry point C.

(a)  $\theta_c = \theta_A + \theta_{c/A} = 0$

$$\theta_A = -\theta_{c/A} = -A_1 - A_2 - A_3$$

$$= -2.8912 \times 10^{-3} + 0.99127 \times 10^{-3} + 0.165212 \times 10^{-3} = -1.735 \times 10^{-3} \text{ rad}$$

(b)  $t_{A/C} = (7)(2.8912 \times 10^{-3}) + (6)(-0.99127 \times 10^{-3}) + (2.25)(-0.165212)$

$$= 13.919 \times 10^{-3} \text{ ft}$$

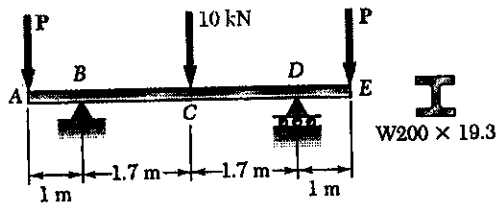
$$t_{B/C} = (4)(2.8912 \times 10^{-3}) + (3)(-0.99127 \times 10^{-3})$$

$$= 8.591 \times 10^{-3} \text{ ft}$$

$$y_A = t_{A/C} - t_{B/C} = 5.328 \times 10^{-3} \text{ ft} = 0.0639 \text{ in. } \uparrow$$

PROBLEM 9.122

9.122 Knowing that  $P = 8$  kN, determine (a) the slope at end A, (b) the deflection at midpoint C. Use  $E = 200$  GPa.



SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 16.6 \times 10^6 \text{ mm}^4 = 16.6 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(16.6 \times 10^{-6}) = 3.32 \times 10^6 \text{ N}\cdot\text{m}^2 = 3320 \text{ kN}\cdot\text{m}^2$$

Symmetric beam and loading

$$R_A = R_B = P + 5 = 8 + 5 = 13 \text{ kN.}$$

Bending moment

$$\text{Over AB} \quad M = -Px = -8x$$

$$\text{Over BC} \quad M = -8x + 13(x-1) = 5(x-1) - 8$$

Draw  $\frac{M}{EI}$  diagram by parts

$$A_1 = \frac{1}{2} \left( \frac{8.5}{EI} \right) (1.7) = \frac{7.225}{EI}$$

$$A_2 = -\frac{1}{2} \left( \frac{8}{EI} \right) (1) = -\frac{4}{EI}$$

$$A_3 = -\left( \frac{8}{EI} \right) (1.7) = -\frac{13.600}{EI}$$

Place reference tangent at C  $\theta_c = 0$

(a) Slope at A.  $\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2 + A_3)$

$$\theta_A = - \left( \frac{7.225}{EI} - \frac{4}{EI} - \frac{13.600}{EI} \right) = \frac{10.375}{EI} = \frac{10.375}{3320} = 3.125 \times 10^{-3} \text{ rad}$$

(b) Deflection at C  $y_c = -t_{B/C}$

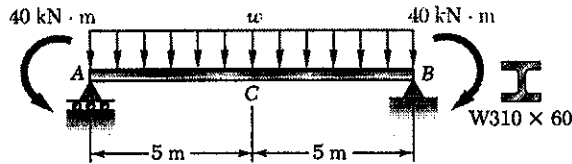
$$= - (A_1 \bar{x}_1 + A_3 \bar{x}_3)$$

$$= - \left[ \left( \frac{7.225}{EI} \right) \left( \frac{1}{3}(1.7) \right) - \left( \frac{13.600}{EI} \right) \left( \frac{1.7}{2} \right) \right] = \frac{3.3717}{EI} = \frac{3.3717}{3320}$$

$$= 1.016 \times 10^{-3} \text{ m} = 1.016 \text{ mm}$$

PROBLEM 9.123

9.123 For the beam and loading of Prob. 9.120, determine the value of  $w$  for which the deflection is zero at the midpoint  $C$  of the beam. Use  $E = 200$  GPa.



SOLUTION

Symmetric beam and loading.

$$R_A = R_B = 5w \quad (w \text{ in kN/m})$$

Bending moment in kN·m.

$$M = 5wx - 40 - \frac{1}{2}wx^2$$

At  $x = 5$  m

$$M = 25w - 40 - 12.5w$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$A_1 = \frac{1}{2} \left( \frac{25w}{EI} \right) (5) = \frac{62.5w}{EI}$$

$$A_2 = - \frac{(40)(5)}{EI} = - \frac{200}{EI}$$

$$A_3 = - \frac{1}{3} \left( \frac{12.5w}{EI} \right) (5) = - \frac{20.833w}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(5) = 3.3333 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2}(5) = 2.5 \text{ m}$$

$$\bar{x}_3 = \frac{3}{4}(5) = 3.75 \text{ m}$$

Place reference tangent at  $C$ .

Deflection at  $C$  is zero

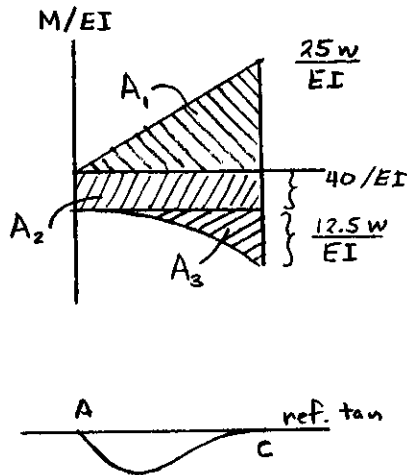
$$t_{AC} = y_A - y_C = 0$$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3 = 0$$

$$\left( \frac{62.5w}{EI} \right) (3.3333) - \left( \frac{200}{EI} \right) (2.5) - \left( \frac{20.833w}{EI} \right) (3.75) = 0$$

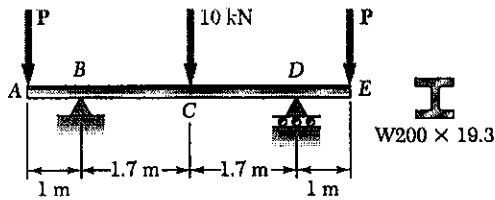
$$\frac{130.21w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$



PROBLEM 9.124

9.124 For the beam and loading of Prob. 9.122, determine the magnitude of the forces  $P$  for which the deflection is zero at end  $A$ . Use  $E = 200$  GPa.



SOLUTION

Symmetric beam and loading.

$$R_A = R_B = P + 5 \quad (P \text{ in kN})$$

Bending moment

Over AB  $M = -Px \quad \text{kN}\cdot\text{m}$

Over BC  $M = -Px + (P+5)(x-1)$   
 $= 5(x-1) - P(1)$

At  $x = 2.7 \text{ m}$

$$M = 8.5 - P(1)$$

Draw  $\frac{M}{EI}$  diagram by parts

$$A_1 = \frac{1}{2} \left( \frac{8.5}{EI} \right) (1.7) = \frac{7.225}{EI}$$

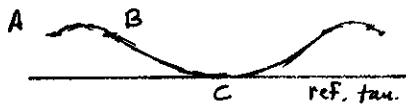
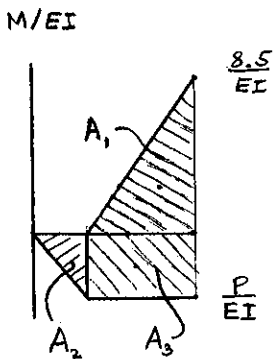
$$A_2 = -\frac{1}{2} \left( \frac{P}{EI} \right) (1) = -\frac{0.5P}{EI}$$

$$A_3 = -\left( \frac{P}{EI} \right) (1.7) = -\frac{1.7P}{EI}$$

Place reference tangent at C

$$y_A = y_B = 0$$

$$y_A - y_B = 0$$



$$t_{A/C} - t_{B/C} = 0$$

$$A_1 \left( 1 + \frac{2}{3} \cdot 1.7 \right) + A_3 \left( 1 + \frac{1}{2} \cdot 1.7 \right) + A_3 \left( \frac{2}{3} \right) - A_1 \left( \frac{2}{3} \cdot 1.7 \right) - A_3 \left( \frac{1}{2} \cdot 1.7 \right) = 0$$

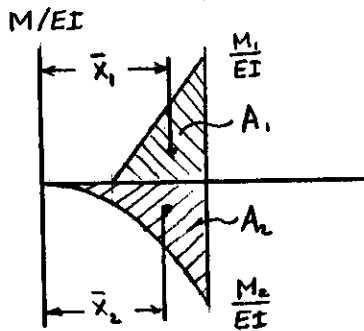
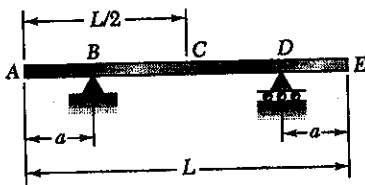
$$A_1 (1) + A_3 (1) + A_2 \left( \frac{2}{3} \right) = 0$$

$$\frac{7.225}{EI} - \frac{1.7P}{EI} - \frac{0.33333P}{EI} = 0$$

$$P = \frac{7.225}{2.0333} = 3.55 \text{ kN}$$

PROBLEM 9.125

\*9.125 A uniform rod  $AE$  is supported at two points  $B$  and  $D$ . Determine the distance  $a$  from the ends of the rod to the points of support if the downward deflections of points  $A$ ,  $C$ , and  $E$  are to be equal.



SOLUTION

Let  $w$  = weight per unit length of rod.

Symmetrical beam and loading.

$$R_B = R_D = \frac{1}{2}WL$$

Bending moment:

Over AB  $M = -\frac{1}{2}wx^2$

Over BCD  $M = -\frac{1}{2}wx^2 + \frac{1}{2}WL(x-a)$

Draw  $\frac{M}{EI}$  diagram by parts

$$\frac{M_1}{EI} = \frac{1}{2} \frac{WL(\frac{L}{2}-a)}{EI} = \frac{1}{4} \frac{WL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2} \frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8} \frac{WL^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} (\frac{L}{2}-a) = \frac{1}{16} \frac{WL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3} (\frac{M_2}{EI}) (\frac{L}{2}) = -\frac{1}{48} \frac{WL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3} (\frac{L}{2}-a) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}(\frac{L}{2}) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_C - y_C = t_{A/C} = 0$$

$$A_1 \bar{x}_1 + A_2 \bar{x}_2 = 0$$

$$\frac{1}{16} \frac{WL(L-2a)^2}{EI} \frac{1}{3}(L+a) - \frac{1}{48} \frac{WL^3}{EI} \frac{3}{8}L = 0$$

Let  $u = a/L$ . Divide by  $\frac{WL^4}{48EI}$

$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for  $u = 0.22315$

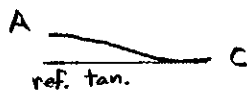
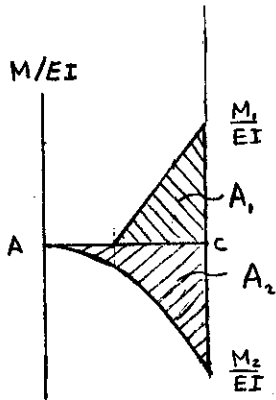
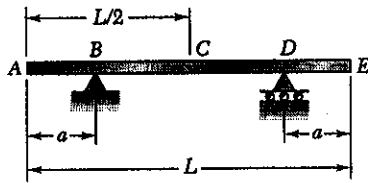
$$\frac{a}{L} = 0.223$$

$$a = 0.223 L$$



PROBLEM 9.126

9.126 A uniform rod  $AE$  is supported at two points  $B$  and  $D$ . Determine the distance  $a$  for which the slope at ends  $A$  and  $E$  is to be zero.



SOLUTION

Let  $w =$  weight per unit length of rod.  
Symmetrical beam and loading.

$$R_B = R_D = \frac{1}{2}WL$$

Bending moment

Over  $AB$   $M = -\frac{1}{2}wx^2$

Over  $BCD$   $M = -\frac{1}{2}wx^2 + \frac{1}{2}WL(x-a)$

Draw  $\frac{M}{EI}$  diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2} \frac{WL(\frac{L}{2}-a)}{EI} = \frac{1}{4} \frac{WL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2} \frac{W(\frac{L}{2})^2}{EI} = -\frac{1}{8} \frac{WL^2}{EI}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} (\frac{L}{2}-a) = \frac{1}{16} \frac{WL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3} (\frac{M_2}{EI}) \frac{L}{2} = -\frac{1}{48} \frac{WL^3}{EI}$$

Place reference tangent at  $C$ .  $\theta_c = 0$

$$\theta_A = \theta_c - \theta_{c/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16} \frac{WL(L-2a)^2}{EI} + \frac{1}{48} \frac{WL^3}{EI} = 0$$

Let  $u = \frac{a}{L}$  and divide by  $\frac{WL^3}{48EI}$

$$1 - 3(1-2u)^2 = 0$$

$$1 - 2u = \frac{\sqrt{3}}{3}$$

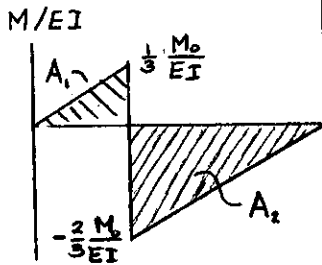
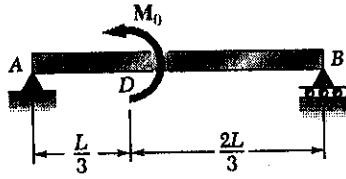
$$u = \frac{1}{2} (1 - \frac{\sqrt{3}}{3}) = 0.21132$$

$$\frac{a}{L} = 0.211$$

$$a = 0.211 L$$

PROBLEM 9.127

9.127 through 9.130 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



SOLUTION

Reactions:  $R_A = \frac{M_0}{L} \uparrow$ ,  $R_B = \frac{M_0}{L} \downarrow$

Draw  $\frac{M}{EI}$  diagram.

$$A_1 = \frac{1}{2} \left( \frac{1}{3} \frac{M_0}{EI} \right) \frac{L}{3} = \frac{1}{18} \frac{M_0 L}{EI}$$

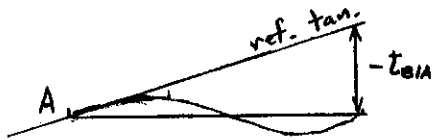
$$A_2 = -\frac{1}{2} \left( \frac{2}{3} \frac{M_0}{EI} \right) \frac{2L}{3} = -\frac{2}{9} \frac{M_0 L}{EI}$$

Place reference tangent at A

$$t_{B/A} = A_1 \left( \frac{L}{9} + \frac{2L}{3} \right) + A_2 \left( \frac{2}{3} - \frac{2L}{3} \right)$$

$$= \frac{7}{162} \frac{M_0 L^2}{EI} - \frac{8}{81} \frac{M_0 L^2}{EI} = -\frac{1}{18} \frac{M_0 L^2}{EI}$$

$$t_{D/A} = A_1 \frac{L}{9} = \frac{1}{162} \frac{M_0 L^2}{EI}$$



(a) Deflection at D

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$$

$$= \frac{1}{162} \frac{M_0 L^2}{EI} - \frac{1}{3} \left( -\frac{1}{18} \frac{M_0 L^2}{EI} \right)$$

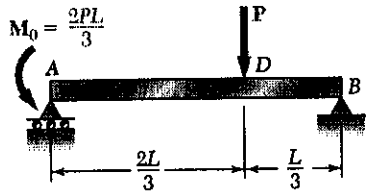
$$= \frac{2}{81} \frac{M_0 L^2}{EI} \uparrow$$

(b) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{1}{18} \frac{M_0 L}{EI}$$

PROBLEM 9.128

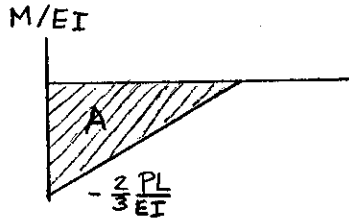
9.127 through 9.130 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



SOLUTION

$$\begin{aligned} \sum M_B = 0 & \quad \frac{2PL}{3} - R_A L + P \frac{L}{3} = 0 & R_A = P \\ + \sum M_A = 0 & \quad \frac{2PL}{3} - P \frac{2L}{3} + R_B L = 0 & R_B = 0 \end{aligned}$$

Draw  $\frac{M}{EI}$  diagram. Reference tangent at A.



$$A = -\frac{1}{2} \cdot \left( \frac{2PL}{3EI} \right) \left( \frac{2L}{3} \right) = -\frac{2}{9} \frac{PL^2}{EI}$$

$$t_{B/A} = \left( -\frac{2}{9} \frac{PL^2}{EI} \right) \left( \frac{2}{3} \frac{2L}{3} + \frac{L}{3} \right) = -\frac{14}{81} \frac{PL^3}{EI}$$

$$t_{D/A} = \left( -\frac{2}{9} \frac{PL^2}{EI} \right) \left( \frac{2}{3} \cdot \frac{2L}{3} \right) = -\frac{8}{81} \frac{PL^3}{EI}$$



(a) Deflection at D

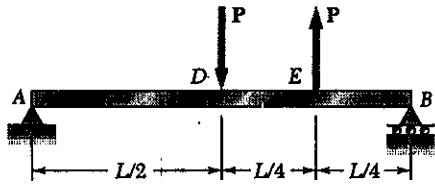
$$\begin{aligned} y_D &= t_{D/A} - \frac{x_D}{L} t_{B/A} \\ &= -\frac{8}{81} \frac{PL^3}{EI} + \frac{2}{3} \cdot \frac{14}{81} \frac{PL^3}{EI} = \frac{4}{243} \frac{PL^3}{EI} \uparrow \end{aligned}$$

(b) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = \frac{14}{81} \frac{PL^2}{EI}$$

PROBLEM 9.129

9.127 through 9.130 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.

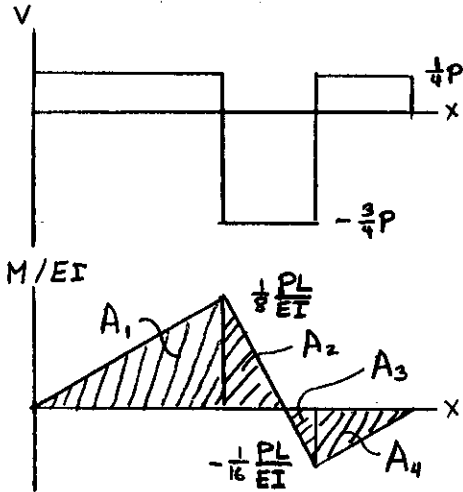


SOLUTION

$$\rightarrow \sum M_B = 0 \quad -R_A L + \frac{PL}{2} - \frac{PL}{4} = 0 \quad R_A = \frac{1}{4}P \uparrow$$

$$\rightarrow \sum M_A = 0 \quad -\frac{PL}{2} + P\frac{3L}{4} + R_B L = 0 \quad R_B = \frac{1}{4}P \downarrow$$

Draw V (shear) diagram and  $\frac{M}{EI}$  diagram.



$$A_1 = \frac{1}{2} \left( \frac{1}{8} \frac{PL}{EI} \right) \left( \frac{L}{2} \right) = \frac{1}{32} \frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2} \left( \frac{1}{8} \frac{PL}{EI} \right) \left( \frac{L}{4} \right) = \frac{1}{96} \frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2} \left( -\frac{1}{16} \frac{PL}{EI} \right) \left( \frac{L}{12} \right) = -\frac{1}{384} \frac{PL^2}{EI}$$

$$A_4 = \frac{1}{2} \left( -\frac{1}{16} \frac{PL}{EI} \right) \left( \frac{L}{4} \right) = -\frac{1}{128} \frac{PL^2}{EI}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= \left( \frac{1}{32} \frac{PL^2}{EI} \right) \left( \frac{2L}{3} \right) \\ &\quad + \left( \frac{1}{96} \frac{PL^2}{EI} \right) \left( \frac{L}{2} - \frac{1}{3} \cdot \frac{L}{6} \right) \\ &\quad + \left( -\frac{1}{384} \frac{PL^2}{EI} \right) \left( \frac{L}{4} + \frac{1}{3} \cdot \frac{L}{12} \right) \\ &\quad + \left( -\frac{1}{128} \frac{PL^2}{EI} \right) \left( \frac{2}{3} \cdot \frac{L}{4} \right) \\ &= \frac{1}{48} \frac{PL^3}{EI} + \frac{1}{216} \frac{PL^3}{EI} \\ &\quad - \frac{5}{6912} \frac{PL^3}{EI} - \frac{1}{768} \frac{PL^3}{EI} \\ &= \frac{3}{128} \frac{PL^3}{EI} \end{aligned}$$

$$t_{D/A} = \left( \frac{1}{32} \frac{PL^2}{EI} \right) \left( \frac{1}{3} \cdot \frac{L}{2} \right) = \frac{1}{192} \frac{PL^3}{EI}$$

(a) Deflection at D.

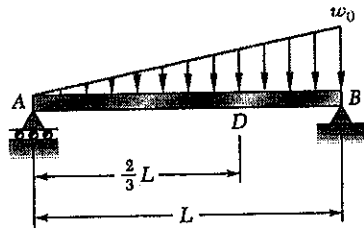
$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{1}{192} \frac{PL^3}{EI} - \frac{1}{2} \left( \frac{3}{128} \frac{PL^3}{EI} \right) = -\frac{5}{768} \frac{PL^3}{EI}$$

(b) Slope at A

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{3}{128} \frac{PL^2}{EI}$$

PROBLEM 9.130

9.127 through 9.130 For the prismatic beam and loading shown, determine (a) the deflection at point D, (b) the slope at end A.



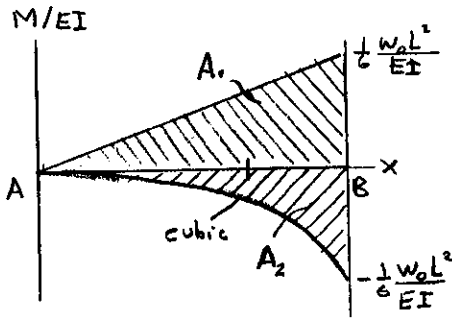
SOLUTION

$$\sum M_B = 0 \quad -R_A L + \left(\frac{1}{2} w_0 L\right) \left(\frac{2}{3} L\right) = 0 \quad R_A = \frac{1}{6} w_0 L$$

$$\begin{aligned} \text{Bending moment} \quad M &= R_A x - \frac{1}{6} \frac{w_0}{L} x^3 \\ &= \frac{1}{6} \frac{w_0}{L} (L^2 x - x^3) \end{aligned}$$

$$\text{At } x = L \quad M = \frac{1}{6} w_0 L^2 - \frac{1}{6} w_0 L^2$$

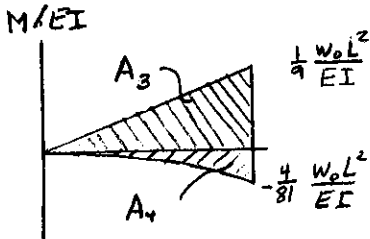
Draw  $\frac{M}{EI}$  diagram by parts.



$$A_1 = \frac{1}{2} \left(\frac{1}{6} \frac{w_0 L^2}{EI}\right) L = \frac{1}{12} \frac{w_0 L^3}{EI} \quad \bar{x}_1 = \frac{1}{3} L$$

$$A_2 = \frac{1}{4} \left(-\frac{1}{6} \frac{w_0 L^2}{EI}\right) L = -\frac{1}{24} \frac{w_0 L^3}{EI} \quad \bar{x}_2 = \frac{1}{5} L$$

Place reference tangent at A.



$$\begin{aligned} t_{B/A} &= A_1 \bar{x}_1 + A_2 \bar{x}_2 \\ &= \frac{1}{36} \frac{w_0 L^4}{EI} - \frac{1}{120} \frac{w_0 L^4}{EI} = \frac{7}{360} \frac{w_0 L^4}{EI} \end{aligned}$$

$$y_B = L \theta_A + t_{B/A}$$

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7}{360} \frac{w_0 L^3}{EI}$$

$$A_3 = \frac{1}{2} \left(\frac{1}{9} \frac{w_0 L^2}{EI}\right) \left(\frac{2}{3} L\right) = \frac{1}{27} \frac{w_0 L^3}{EI}$$

$$A_4 = \frac{1}{4} \left(-\frac{4}{81} \frac{w_0 L^2}{EI}\right) \left(\frac{2}{3} L\right) = -\frac{2}{243} \frac{w_0 L^3}{EI}$$

$$t_{D/A} = A_3 \bar{x}_3 + A_4 \bar{x}_4$$

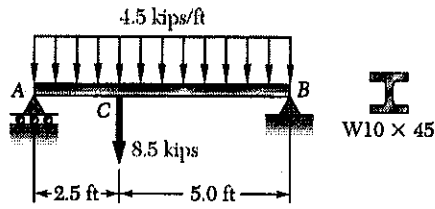
$$\begin{aligned} &= \left(\frac{1}{27} \frac{w_0 L^3}{EI}\right) \left(\frac{1}{3} \cdot \frac{2}{3} L\right) + \left(-\frac{2}{243} \frac{w_0 L^3}{EI}\right) \left(\frac{1}{5} \cdot \frac{2}{3} L\right) = \frac{2}{243} \frac{w_0 L^4}{EI} - \frac{4}{3645} \frac{w_0 L^4}{EI} \\ &= \frac{26}{3645} \frac{w_0 L^4}{EI} \end{aligned}$$

$$(a) \quad y_D = t_{D/A} + \frac{2}{3} L \theta_A = \frac{26}{3645} \frac{w_0 L^4}{EI} + \left(\frac{2}{3} L\right) \left(-\frac{7}{360} \frac{w_0 L^3}{EI}\right) = -\frac{17}{2916} \frac{w_0 L^4}{EI}$$

$$(b) \quad \theta_A = -\frac{7}{360} \frac{w_0 L^3}{EI}$$

PROBLEM 9.131

9.131 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point C. Use  $E = 29 \times 10^6$  psi.

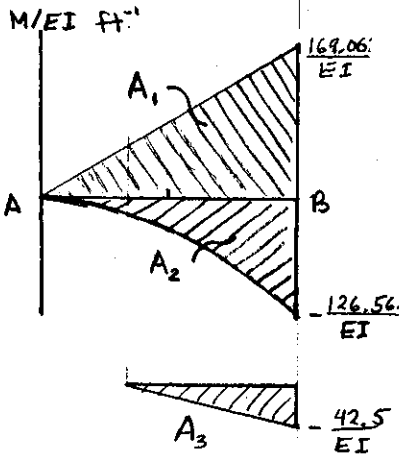


SOLUTION

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 248 \text{ in}^4$$

$$EI = (29 \times 10^3)(248) = 7.192 \times 10^6 \text{ kip}\cdot\text{in}^2 = 49944 \text{ kip}\cdot\text{ft}^2$$



$$\begin{aligned} +\sum M_B = 0 \quad & -7.5 R_A + (4.5)(7.5)(\frac{7.5}{2}) + (8.5)(5.0) = 0 \\ R_A = & 22.542 \text{ kips } \uparrow \end{aligned}$$

Bending moment

$$\text{Over AC} \quad M = 22.542x - 2.25x^2 \quad \text{kip}\cdot\text{ft}$$

$$\text{Over CB} \quad M = 22.542x - 2.25x^2 - 8.5(x - 2.5)$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$A_1 = \frac{1}{2}(7.5)(\frac{169.06}{EI}) = \frac{633.98}{EI}$$

$$A_2 = -\frac{1}{3}(7.5)(\frac{126.56}{EI}) = -\frac{316.40}{EI}$$

$$A_3 = -\frac{1}{2}(5)(\frac{42.5}{EI}) = -\frac{106.25}{EI}$$

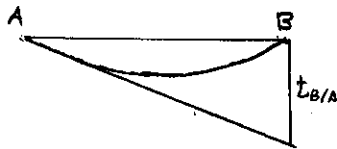
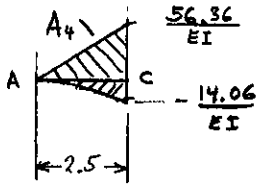
Place reference tangent at A.

$$t_{B/A} = A_1(\frac{7.5}{3}) + A_2(\frac{7.5}{4}) + A_3(\frac{5}{3}) = \frac{814.62}{EI}$$

$$A_4 = \frac{1}{2}(\frac{56.36}{EI})(2.5) = \frac{70.44}{EI}$$

$$A_5 = -\frac{1}{3}(\frac{14.06}{EI})(2.5) = -\frac{11.72}{EI}$$

$$t_{C/A} = A_4(\frac{2.5}{3}) + A_5(\frac{2.5}{4}) = \frac{51.375}{EI}$$



(a) Slope at A

$$\begin{aligned} \theta_A = -\frac{t_{B/A}}{L} &= -\frac{814.62}{7.5 EI} = -\frac{108.62}{EI} \\ &= -\frac{108.62}{49944} = -2.17 \times 10^{-3} \text{ rad} \end{aligned}$$

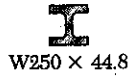
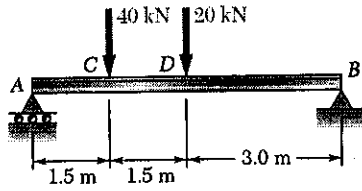
(b) Deflection at C

$$y_C = t_{C/A} - \frac{x_C}{L} t_{B/A}$$

$$\begin{aligned} y_C = \frac{51.375}{EI} - (\frac{2.5}{7.5}) \frac{814.62}{EI} &= -\frac{220.16}{EI} = -\frac{220.16}{49944} = -4.41 \times 10^{-3} \text{ ft} \\ &= 0.0529 \text{ in. } \downarrow \end{aligned}$$

PROBLEM 9.132

9.132 and 9.133 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D. Use  $E = 200 \text{ GPa}$ .



SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

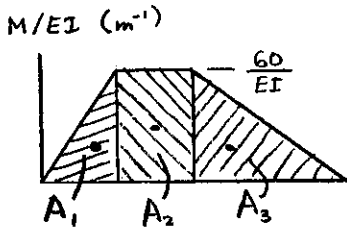
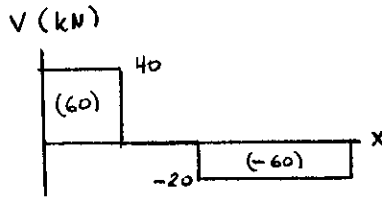
$$I = 71.1 \times 10^6 \text{ mm}^4 = 71.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(71.1 \times 10^{-6}) = 14.22 \times 10^6 \text{ N}\cdot\text{m}^2 = 14220 \text{ kN}\cdot\text{m}^2$$

$$\uparrow \Sigma M_B = 0 \quad -6R_A + (4.5)(40) + (3)(20) = 0$$

$$R_A = 40 \text{ kN.}$$

Draw shear and  $\frac{M}{EI}$  diagrams.



$$A_1 = \frac{1}{2} \left( \frac{60}{EI} \right) (1.5) = \frac{45}{EI}$$

$$A_2 = \left( \frac{60}{EI} \right) (1.5) = \frac{90}{EI}$$

$$A_3 = \frac{1}{2} \left( \frac{60}{EI} \right) (3) = \frac{90}{EI}$$

Place reference tangent at A.



$$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) + A_3(2.0) = \frac{742.5}{EI} \text{ m.}$$

$$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75) = \frac{157.5}{EI} \text{ m}$$

(a) Slope at A  $\theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14220} = -8.70 \times 10^{-3} \text{ rad.}$

(b) Deflection at D.

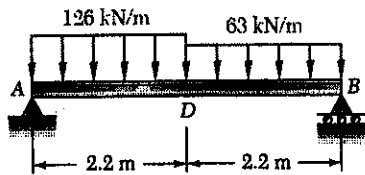
$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left( \frac{3}{6} \right) \left( \frac{742.5}{EI} \right) = -\frac{213.75}{EI}$$

$$= -\frac{213.75}{14220} = -15.03 \times 10^{-3} \text{ m}$$

$$= 15.03 \text{ mm} \downarrow$$

PROBLEM 9.133

9.132 and 9.133 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D. Use  $E = 200 \text{ GPa}$ .



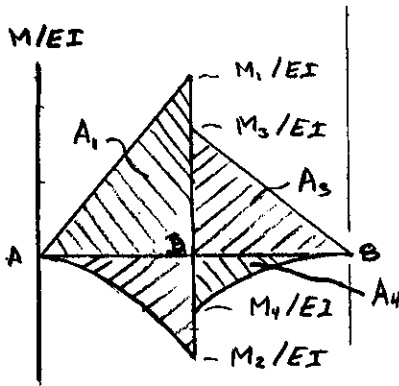
SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 556 \times 10^6 \text{ mm}^4 = 556 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(556 \times 10^{-6}) = 111.2 \times 10^6 \text{ N}\cdot\text{m}^2$$

$$= 111200 \text{ kN}\cdot\text{m}^2$$



$$+\circlearrowleft \sum M_B = 0$$

$$-4.4 R_A + (126)(2.2)(3.3) + (63)(2.2)(1.1) = 0$$

$$R_A = 242.55 \text{ kN } \uparrow$$

$$+\circlearrowleft \sum M_A = 0$$

$$-(126)(2.2)(1.1) - (63)(2.2)(3.3) + 4.4 R_B = 0$$

$$R_B = 173.25 \text{ kN } \uparrow$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$\frac{M_1}{EI} = \frac{(242.55)(2.2)}{111200} = 4.7987 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{1}{2} \frac{(126)(2.2)^2}{111200} = -2.7421 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_3}{EI} = \frac{(173.25)(2.2)}{111200} = 3.4276 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_4}{EI} = -\frac{1}{2} \frac{(63)(2.2)^2}{111200} = -1.3710 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2} \frac{M_1}{EI} (2.2) = 5.2785 \times 10^{-3}$$

$$A_2 = \frac{1}{3} \frac{M_2}{EI} (2.2) = -2.0109 \times 10^{-3}$$

$$A_3 = \frac{1}{2} \frac{M_3}{EI} (2.2) = 3.7704 \times 10^{-3}$$

$$A_4 = \frac{1}{3} \frac{M_4}{EI} (2.2) = -1.0054 \times 10^{-3}$$

Place reference tangent at A.

$$t_{B/A} = A_1(2.9333) + A_2(2.75) + A_3(1.4667) + A_4(1.65) = 13.824 \times 10^{-3} \text{ m}$$

$$t_{D/A} = A_1(0.7333) + A_2(0.55) = 2.7647 \times 10^{-3} \text{ m}$$

(a) Slope at A  $\theta_A = -\frac{t_{B/A}}{L} = -\frac{13.824 \times 10^{-3}}{4.4} = -3.14 \times 10^{-3} \text{ rad}$   $\blacktriangleleft$

(b) Deflection at D  $y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$

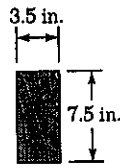
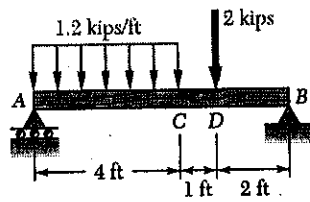
$$y_D = 2.7647 \times 10^{-3} - \left(\frac{2.2}{4.4}\right)(13.824 \times 10^{-3}) = -4.15 \times 10^{-3} \text{ m}$$

$$= 4.15 \text{ mm } \downarrow \blacktriangleleft$$



PROBLEM 9.134

9.134 For the timber beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D. Use  $E = 1.5 \times 10^6$  psi.



SOLUTION

$$I = \frac{1}{12}(3.5)(7.5)^3 = 123.047 \text{ in}^4$$

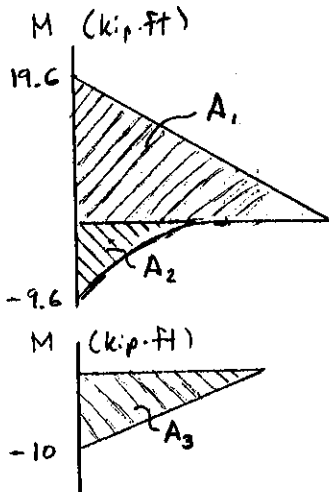
$$E = 1.5 \times 10^6 \text{ psi} = 1.5 \times 10^3 \text{ ksi}$$

$$EI = 184.57 \times 10 \text{ kip}\cdot\text{in}^2 = 1281.7 \text{ kip}\cdot\text{ft}^2$$

$$\odot \Sigma M_A = 0 \quad 7R_B - (2)(5) - (1.2)(4)(2) = 0$$

$$R_B = 2.8 \text{ kip.}$$

Draw bending moment diagram by parts.



$$M_1 = (2.8)(7) = 19.6 \text{ kip}\cdot\text{ft}$$

$$M_2 = -(1.2)(4)(2) = -9.6 \text{ kip}\cdot\text{ft}$$

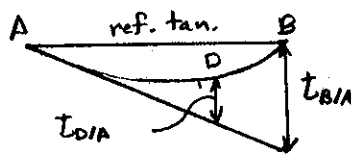
$$M_3 = -(2)(5) = -10 \text{ kip}\cdot\text{ft}$$

$$A_1 = \frac{1}{2}(7)(19.6) = 68.6 \text{ kip}\cdot\text{ft}^2$$

$$A_2 = \frac{1}{2}(4)(-9.6) = -12.8 \text{ kip}\cdot\text{ft}^2$$

$$A_3 = \frac{1}{2}(5)(-10) = -25.0 \text{ kip}\cdot\text{ft}^2$$

Draw reference tangent at A.

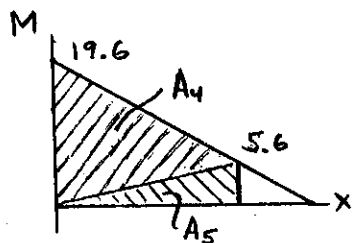


$$\theta_A = -\frac{t_{B/A}}{L}$$

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A}$$

$$EI t_{B/A} = A_1(7 - \frac{7}{2}) + A_2(7 - 1) + A_3(7 - \frac{5}{2}) = 110.0 \text{ kip}\cdot\text{ft}^2$$

$$(a) \quad \theta_A = -\frac{EI t_{B/A}}{EIL} = -\frac{110.0}{(1281.7)(7)} = -12.26 \times 10^{-3} \text{ rad}$$



$$A_4 = \frac{1}{2}(19.6)(5) = 49 \text{ kip}\cdot\text{ft}^2$$

$$A_5 = \frac{1}{2}(5.6)(5) = 14 \text{ kip}\cdot\text{ft}^2$$

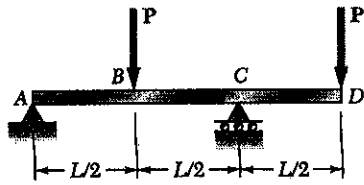
$$EI t_{D/A} = A_4(5 - \frac{5}{3}) + A_5(\frac{5}{3}) + A_2(5 - 1) + A_3(5 - \frac{5}{2}) = 52.133 \text{ kip}\cdot\text{ft}^3$$

$$EI y_D = 52.133 - \frac{5}{7}(110.0) = -26.438 \text{ kip}\cdot\text{ft}^3$$

$$y_D = -\frac{26.438}{1281.7} = -20.63 \times 10^{-3} \text{ ft} = 0.248 \text{ in. } \downarrow$$

PROBLEM 9.135

9.135 and 9.136 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D.

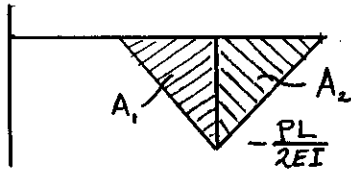


SOLUTION

$$\circlearrowleft \sum M_C = 0 \quad -R_A L + P \frac{L}{2} - P \frac{L}{2} = 0 \quad R_A = 0.$$

Draw  $\frac{M}{EI}$  diagram

M/EI

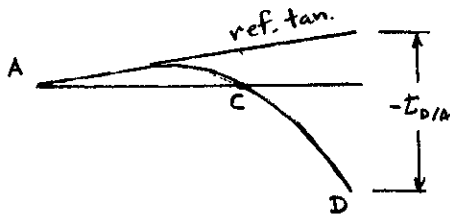


$$A_1 = -\frac{1}{2} \left( \frac{PL}{2EI} \right) \left( \frac{L}{2} \right) = -\frac{1}{8} \frac{PL^2}{EI}$$

$$A_2 = -\frac{1}{2} \left( \frac{PL}{2EI} \right) \left( \frac{L}{2} \right) = -\frac{1}{8} \frac{PL^2}{EI}$$

Place reference tangent at A

$$t_{C/A} = A_1 \cdot \left( \frac{1}{3} \cdot \frac{L}{2} \right) = -\frac{1}{48} \frac{PL^3}{EI}$$



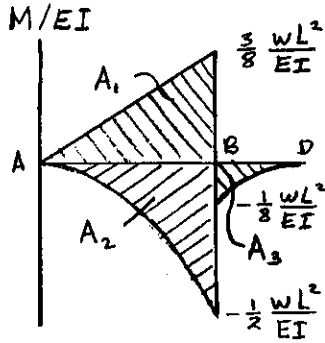
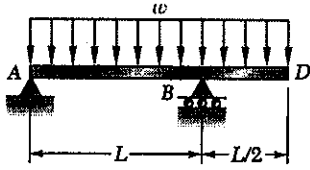
$$(a) \quad \theta_A = -\frac{t_{C/A}}{L} = \frac{1}{48} \frac{PL^2}{EI}$$

$$t_{D/A} = A_1 \left( \frac{L}{2} + \frac{L}{2} \right) + A_2 \left( \frac{2}{3} \cdot \frac{L}{2} \right) = -\frac{1}{8} \frac{PL^3}{EI}$$

$$y_D = t_{D/A} - \frac{x_D}{L} t_{C/A} = -\frac{1}{8} \frac{PL^3}{EI} - \left( \frac{3}{2} \right) \left( -\frac{1}{48} \frac{PL^3}{EI} \right) = -\frac{3}{32} \frac{PL^3}{EI}$$

PROBLEM 9.136

9.135 and 9.136 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D.



SOLUTION

$$\rightarrow \sum M_B = 0 \quad -R_A L + \left(\frac{3}{2} wL\right) \left(\frac{1}{4} L\right) = 0 \quad R_A = \frac{3}{8} wL$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$A_1 = \frac{1}{2} \left(\frac{3}{8} \frac{wL^2}{EI}\right) L = \frac{3}{16} \frac{wL^3}{EI}$$

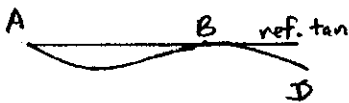
$$A_2 = -\frac{1}{3} \left(\frac{1}{2} \frac{wL^2}{EI}\right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

$$A_3 = -\frac{1}{3} \left(\frac{1}{8} \frac{wL^2}{EI}\right) \frac{L}{2} = -\frac{1}{48} \frac{wL^3}{EI}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= A_1 \frac{L}{3} + A_2 \frac{L}{4} \\ &= \frac{1}{16} \frac{wL^4}{EI} - \frac{1}{24} \frac{wL^4}{EI} = \frac{1}{48} \frac{wL^4}{EI} \end{aligned}$$

(a) Slope at A



$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{1}{48} \frac{wL^3}{EI}$$

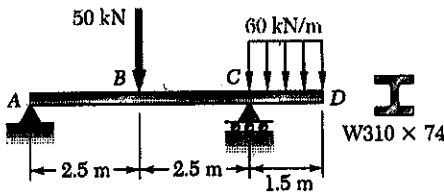
$$\begin{aligned} t_{D/A} &= A_1 \left(\frac{1}{3} + \frac{L}{2}\right) + A_2 \left(\frac{L}{4} + \frac{L}{2}\right) + A_3 \left(\frac{3}{4} \cdot \frac{L}{2}\right) \\ &= \frac{5}{32} \frac{wL^4}{EI} - \frac{1}{8} \frac{wL^4}{EI} - \frac{1}{128} \frac{wL^4}{EI} = \frac{3}{128} \frac{wL^4}{EI} \end{aligned}$$

(b) Deflection at D.

$$\begin{aligned} y_D &= t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{3}{128} \frac{wL^4}{EI} - \frac{3}{2} \cdot \frac{1}{48} \frac{wL^4}{EI} = -\frac{1}{128} \frac{wL^4}{EI} \\ y_D &= \frac{1}{128} \frac{wL^4}{EI} \downarrow \end{aligned}$$

PROBLEM 9.137

9.137 For the beam and loading shown, determine (a) the slope at point C, (b) the deflection at point D. Use  $E = 200 \text{ GPa}$ .

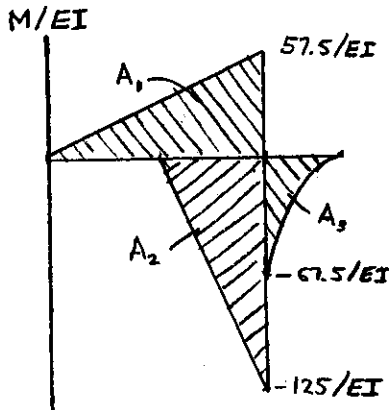


SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33.0 \times 10^6 \text{ N}\cdot\text{m}^2 = 33000 \text{ kN}\cdot\text{m}^2$$



$$\sum M_C = 0 \quad -5R_A + (50)(2.5) - (60)(1.5)(0.75) = 0$$

$$R_A = 11.5 \text{ kN}$$

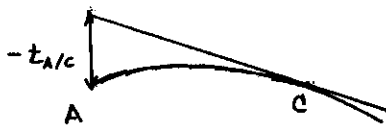
Draw  $\frac{M}{EI}$  diagram by parts.

$$A_1 = \frac{1}{2} \left( \frac{57.5}{EI} \right) (5) = \frac{143.75}{EI}$$

$$A_2 = -\frac{1}{2} \left( \frac{125}{EI} \right) (2.5) = -\frac{156.25}{EI}$$

$$A_3 = -\frac{1}{3} \left( \frac{67.5}{EI} \right) (1.5) = -\frac{33.75}{EI}$$

Place reference tangent at C



$$t_{A/C} = A_1 \left( \frac{2}{3} \cdot 5 \right) + A_2 \left( 2.5 + \frac{2}{3} \cdot 2.5 \right)$$

$$= -\frac{171.875}{EI} \text{ m}$$

(a) Slope at C

$$\theta_C = \frac{t_{A/C}}{L} = -\frac{171.875}{5EI} = -\frac{34.375}{EI}$$

$$= -\frac{34.375}{33000} = -1.042 \times 10^{-3} \text{ rad}$$

$$t_{D/C} = A_3 \left( \frac{2}{4} \cdot 1.5 \right) = -\frac{37.96875}{EI} \text{ m}$$

(b) Deflection at D

$$y_D = \theta_C \times D/C + t_{D/C}$$

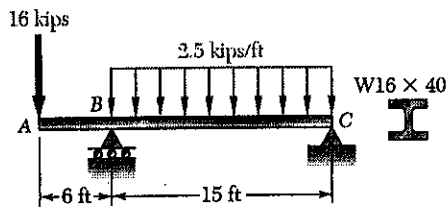
$$= -\left( \frac{34.375}{EI} \right) (1.5) - \frac{37.96875}{EI} = -\frac{89.53}{EI}$$

$$= -\frac{89.53}{33000} = -2.71 \times 10^{-3} \text{ m}$$

$$= 2.71 \text{ mm} \downarrow$$

PROBLEM 9.138

9.138 For the beam and loading shown, determine (a) the slope at point B, (b) the deflection at point A. Use  $E = 29 \times 10^6$  psi.



SOLUTION

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ kip}$$

$$I = 518 \text{ in}^4$$

$$EI = (29 \times 10^3)(518) = 15.022 \times 10^6 \text{ kip} \cdot \text{in}^2 = 104319 \text{ kip} \cdot \text{ft}^2$$

$$+\circlearrowleft \sum M_B = 0 \quad (16)(6) - (2.5)(15)(7.5) + 15 R_C = 0$$

$$R_C = 12.35 \text{ kN}$$

Draw  $\frac{M}{EI}$  diagram by parts

$$M_1 = (12.35)(15) = 185.25 \text{ kip} \cdot \text{ft}$$

$$M_2 = -\frac{1}{2}(2.5)(15)^2 = -281.25 \text{ kip} \cdot \text{ft}$$

$$M_3 = -(16)(6) = -96 \text{ kip} \cdot \text{ft}$$

$$A_1 = \frac{1}{2}(185.25)(15)/EI = 1389.375/EI$$

$$A_2 = -\frac{1}{3}(281.25)(15)/EI = -1406.25/EI$$

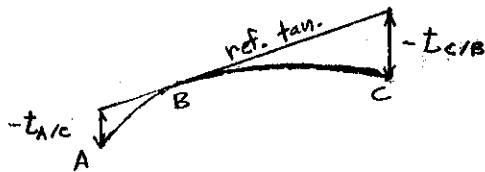
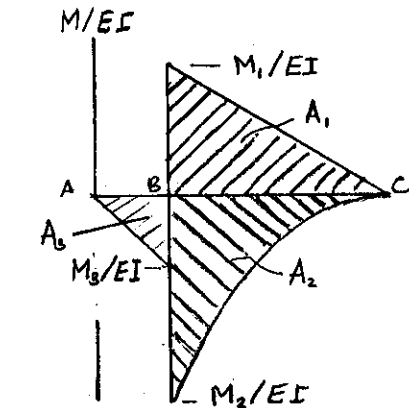
$$A_3 = -\frac{1}{2}(96)(6) = -288/EI$$

Place reference tangent at B.

$$t_{C/B} = A_1 \left(\frac{2}{3} \cdot 15\right) + A_2 \left(\frac{3}{4} \cdot 15\right)$$

$$= -1926.5625/EI$$

$$= -18.468 \times 10^{-3} \text{ ft}$$



(a) Slope at B  $\theta_B = -\frac{t_{C/B}}{L} = \frac{18.468 \times 10^{-3}}{15} = 1.231 \times 10^{-3} \text{ rad}$

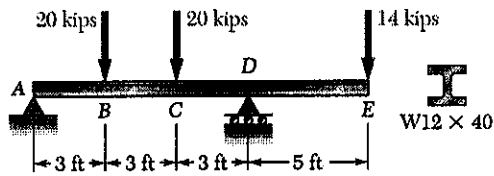
(b)  $t_{A/C} = A_3 \left(\frac{2}{3} \cdot 6\right) = -1152/EI$

$$y_A = t_{A/C} + \frac{x_{AB}}{L} t_{B/C} = -\frac{1152}{EI} - \frac{6}{15} \left(\frac{1926.5625}{EI}\right)$$

$$= -\frac{1922.6}{EI} = -18.43 \times 10^{-3} \text{ ft} = 0.221 \text{ in.} \downarrow$$

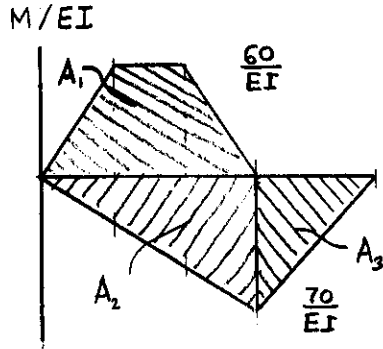
**PROBLEM 9.139**

9.139 For the beam and loading shown, determine (a) the slope at point D, (b) the deflection at point E. Use  $E = 29 \times 10^6$  psi.



**SOLUTION**

Draw bending moment diagram as the sum of two diagrams: one for the pair of 20 kip loads and one for the 14 kip load.



$$A_1 = [2 \cdot \frac{1}{2}(3)(60) + (3)(60)]/EI = 360/EI$$

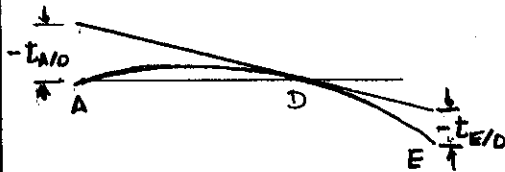
$$A_2 = \frac{1}{2}(9)(70)/EI = -315/EI$$

$$A_3 = \frac{1}{2}(5)(70) = -175/EI$$

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 310 \text{ in}^4$$

$$EI = (29 \times 10^3)(310) = 8.99 \times 10^6 \text{ kip} \cdot \text{in}^2 = 62430 \text{ kip} \cdot \text{ft}^2$$



Place reference tangent at D

$$t_{A/D} = A_1(4.5) + A_2(6) = -270/EI \text{ ft.}$$

(a) Slope at D  $\theta_D = \frac{t_{A/D}}{L} = -\frac{270}{9EI} = -\frac{30}{EI}$   
 $= -0.48054 \times 10^{-3} \text{ rad}$

$$t_{E/D} = A_3(\frac{2}{3} \cdot 5) = -583.333/EI = -9.3438 \times 10^{-3} \text{ ft}$$

(b) Deflection at E

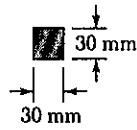
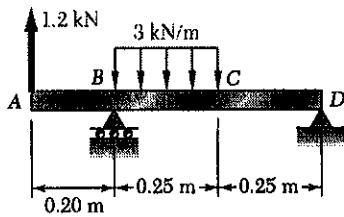
$$y_E = L_{DE} \theta_D + t_{E/D}$$

$$= -(5)(0.48054 \times 10^{-3}) - 9.3438 \times 10^{-3} = -11.75 \times 10^{-3} \text{ ft}$$

$$= 0.1410 \text{ in. } \downarrow$$

PROBLEM 9.140

9.140 Knowing the beam  $AD$  is made of a solid steel bar, determine the (a) slope at point  $B$ , (c) the deflection at point  $A$ . Use  $E = 200$  GPa.

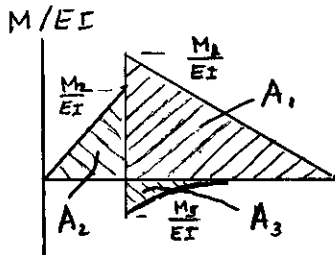


SOLUTION

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4 = 67.5 \times 10^{-9} \text{ m}^4$$

$$EI = (200 \times 10^9)(67.5 \times 10^{-9}) = 13500 \text{ N} \cdot \text{m}^2 = 13.5 \text{ kN} \cdot \text{m}^2$$



$$\sum M_B = 0 \quad -(0.2)(1.2) - (3)(0.25)(0.125) + 5R_D = 0$$

$$R_D = 0.6675 \text{ kN}$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375 \text{ kN} \cdot \text{m}$$

$$M_2 = (1.2)(0.2) = 0.240 \text{ kN} \cdot \text{m}$$

$$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$

$$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$

$$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at  $B$ .

$$t_{D/B} = A_1 \left( \frac{2}{3} \cdot 0.5 \right) + A_3 \left( \frac{3}{4} \cdot (0.25) + 0.25 \right) = 0.024395/EI$$

(a) Slope at  $B$       $\theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5 EI} = -\frac{0.048789}{EI}$

$$= -3.6140 \times 10^{-3} \text{ rad.}$$

$$t_{A/B} = A_2 \left( \frac{2}{3} (0.20) \right) = 0.0032/EI = 0.23704 \times 10^{-3} \text{ m}$$

(b) Deflection at  $A$

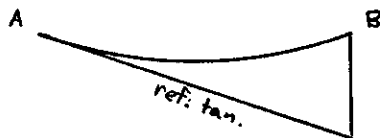
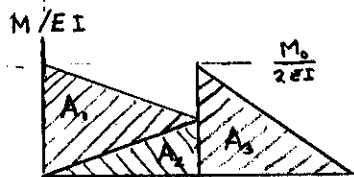
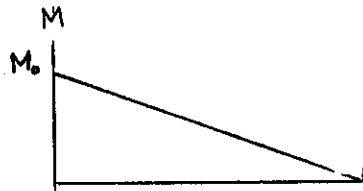
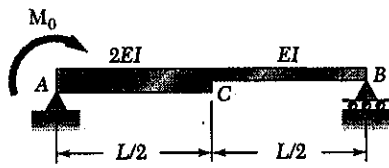
$$y_A = t_{A/B} - L_{AB} \theta_B$$

$$= 0.23704 \times 10^{-3} - (0.2)(-3.6140 \times 10^{-3}) = 0.960 \times 10^{-3} \text{ m}$$

$$= 0.960 \text{ mm} \uparrow$$

PROBLEM 9.141

9.141 and 9.142 For the beam and loading shown, determine (a) the slope at end A, (b) the slope at end B, (c) the deflection at the midpoint C.



SOLUTION

Draw bending moment and  $\frac{M}{EI}$  diagrams.

$$A_1 = \frac{1}{2} \left( \frac{M_0}{2EI} \right) \left( \frac{L}{2} \right) = \frac{1}{8} \frac{M_0 L}{EI}$$

$$A_2 = \frac{1}{2} \left( \frac{M_0}{4EI} \right) \left( \frac{L}{2} \right) = \frac{1}{16} \frac{M_0 L}{EI}$$

$$A_3 = \frac{1}{2} \left( \frac{M_0}{2EI} \right) \left( \frac{L}{2} \right) = \frac{1}{8} \frac{M_0 L}{EI}$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= A_1 \left( \frac{L}{2} + \frac{2}{3} \frac{L}{2} \right) + A_2 \left( \frac{L}{2} + \frac{1}{3} \frac{L}{2} \right) + A_3 \left( \frac{2}{3} \frac{L}{2} \right) \\ &= \left( \frac{1}{8} \frac{M_0 L}{EI} \right) \left( \frac{5}{3} L \right) + \left( \frac{1}{16} \frac{M_0 L}{EI} \right) \left( \frac{2}{3} L \right) + \left( \frac{1}{8} \frac{M_0 L}{EI} \right) \left( \frac{1}{3} L \right) \\ &= \frac{3}{16} \frac{M_0 L^2}{EI} \end{aligned}$$

(a) Slope at A

$$\theta_A = - \frac{t_{B/A}}{L} = - \frac{3}{16} \frac{M_0 L}{EI}$$

(b) Slope at B

$$\begin{aligned} \theta_B &= \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2 + A_3 \\ &= - \frac{3}{16} \frac{M_0 L}{EI} + \frac{1}{8} \frac{M_0 L}{EI} + \frac{1}{16} \frac{M_0 L}{EI} + \frac{1}{8} \frac{M_0 L}{EI} \\ &= \frac{1}{8} \frac{M_0 L}{EI} \end{aligned}$$

$$t_{C/A} = A_1 \left( \frac{2}{3} \frac{L}{2} \right) + A_2 \left( \frac{1}{3} \frac{L}{2} \right) = \left( \frac{1}{8} \frac{M_0 L}{EI} \right) \left( \frac{1}{3} L \right) + \left( \frac{1}{16} \frac{M_0 L}{EI} \right) \left( \frac{1}{2} L \right) = \frac{5}{96} \frac{M_0 L^2}{EI}$$

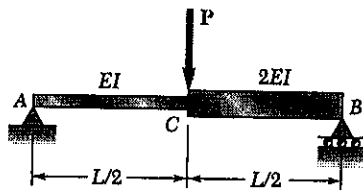
(c) Deflection at C

$$y_C = t_{C/A} + \frac{L}{2} \theta_A = \frac{5}{96} \frac{M_0 L^2}{EI} + \frac{3}{32} \frac{M_0 L^2}{EI} = - \frac{1}{24} \frac{M_0 L^2}{EI} = \frac{1}{24} \frac{M_0 L^2}{EI} \downarrow$$



PROBLEM 9.142

9.141 and 9.142 For the beam and loading shown, determine (a) the slope at end A, (b) the slope at end B, (c) the deflection at the midpoint C.



SOLUTION

Draw bending moment and  $\frac{M}{EI}$  diagrams.

$$A_1 = \frac{1}{2} \left( \frac{1}{4} \frac{PL}{EI} \right) \left( \frac{L}{2} \right) = \frac{1}{16} \frac{PL^2}{EI}$$

$$A_2 = \frac{1}{2} \left( \frac{1}{8} \frac{PL}{EI} \right) \left( \frac{L}{2} \right) = \frac{1}{32} \frac{PL^2}{EI}$$

Place reference tangent at A

$$t_{B/A} = A_1 \left( \frac{L}{2} + \frac{1}{3} \frac{L}{2} \right) + A_2 \left( \frac{2}{3} \frac{L}{2} \right)$$

$$= \left( \frac{1}{16} \frac{PL^2}{EI} \right) \left( \frac{2}{3} L \right) + \left( \frac{1}{32} \frac{PL^2}{EI} \right) \left( \frac{1}{3} L \right) = \frac{5}{96} \frac{PL^3}{EI}$$

(a) Slope at A

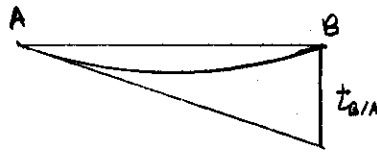
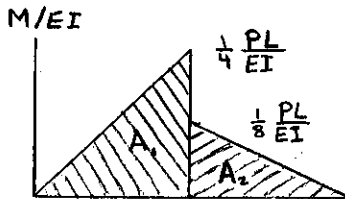
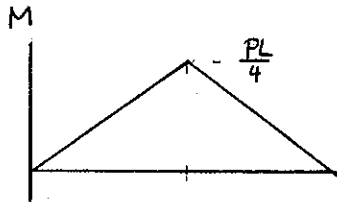
$$\theta_A = - \frac{t_{B/A}}{L} = - \frac{5}{96} \frac{PL^2}{EI}$$

(b) Slope at B

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2$$

$$= - \frac{5}{96} \frac{PL^2}{EI} + \frac{1}{16} \frac{PL^2}{EI} + \frac{1}{32} \frac{PL^2}{EI}$$

$$= \frac{1}{24} \frac{PL^2}{EI}$$



$$t_{C/A} = A_1 \left( \frac{1}{3} \frac{L}{2} \right) = \left( \frac{1}{16} \frac{PL^2}{EI} \right) \left( \frac{1}{8} L \right) = \frac{1}{96} \frac{PL^3}{EI}$$

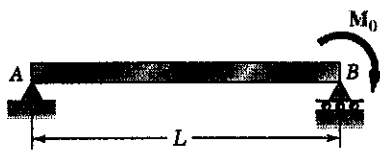
(c) Deflection at C

$$y_A = t_{C/A} - \frac{x_C}{L} t_{B/A} = \frac{1}{96} \frac{PL^3}{EI} - \frac{1}{2} \left( \frac{5}{96} \frac{PL^3}{EI} \right) = - \frac{1}{64} \frac{PL^3}{EI}$$

$$= \frac{1}{64} \frac{PL^3}{EI} \downarrow$$

PROBLEM 9.143

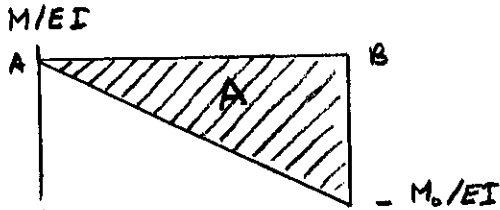
9.143 For the beam and loading shown, determine the magnitude and location of the maximum deflection.



SOLUTION

Draw  $\frac{M}{EI}$  diagram

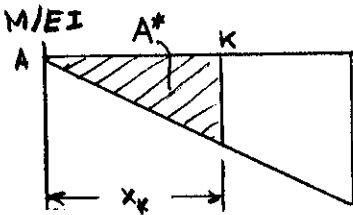
Place reference tangent at A.



$$A = \frac{1}{2} \left( -\frac{M_0}{EI} \right) L = -\frac{1}{2} \frac{M_0 L}{EI}$$

$$t_{B/A} = A \left( \frac{L}{3} \right) = -\frac{1}{6} \frac{M_0 L^2}{EI}$$

[www.konkur.in](http://www.konkur.in)  $\frac{t_{B/A}}{L} = \frac{1}{6} \frac{M_0 L}{EI}$



$$A^* = \frac{1}{2} \left( \frac{M_0 x_k}{EI L} \right) x_k = -\frac{1}{2} \frac{M_0 x_k^2}{EIL}$$

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A^* = 0$$

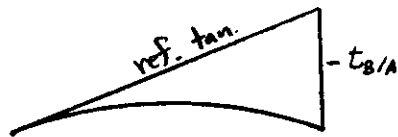
$$\frac{1}{6} \frac{M_0 L}{EI} - \frac{1}{2} \frac{M_0 x_k^2}{EIL} = 0$$

$$x_k = \frac{\sqrt{3}}{3} L$$

$$t_{K/A} = A^* \left( \frac{1}{3} x_k \right)$$

$$= -\frac{1}{2} \frac{M_0 x_k^2}{EIL} \left( \frac{1}{3} x_k \right)$$

$$= -\frac{1}{6} \frac{M_0 x_k^3}{EIL}$$



Maximum deflection

$$y_k = t_{K/A} - \frac{x_k}{L} t_{B/A} = -\frac{1}{6} \frac{M_0 x_k^3}{EIL} - \frac{x_k}{L} \left( -\frac{M_0 L^2}{6EI} \right)$$

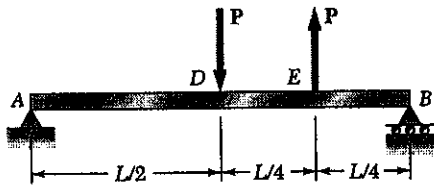
$$= \frac{M_0 x_k}{6EIL} (L^2 - x_k^2) = \frac{\sqrt{3}}{18} \frac{M_0}{EI} \left( L^2 - \frac{1}{3} L^2 \right) = \frac{\sqrt{3}}{27} \frac{M_0 L^2}{EI} \uparrow$$

PROBLEM 9.144

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.144 Beam and loading of Prob. 9.129

SOLUTION



Referring to the solution of Prob. 9.129

$$R_A = \frac{1}{4}P, \quad t_{B/A} = \frac{3}{128} \frac{PL^3}{EI}, \quad \theta_A = -\frac{3}{128} \frac{PL^2}{EI}$$

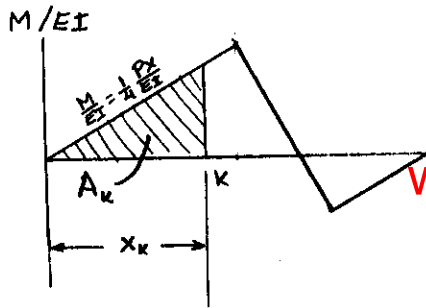
$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{3}{128} \frac{PL^2}{EI} + A_K$$

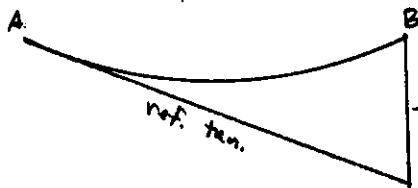
$$= -\frac{3}{128} \frac{PL^2}{EI} + \frac{1}{2} \left( \frac{1}{4} \frac{Px_K}{EI} \right) x_K$$

$$= \frac{P}{EI} \left( -\frac{3}{128} L^2 + \frac{1}{8} x_K^2 \right) = 0$$

$$x_K = \sqrt{\frac{3}{16}} L = \frac{1}{4} \sqrt{3} L = 0.433 L$$



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$$t_{K/A} = A_K \left( \frac{1}{3} x_K \right) = \frac{1}{2} \left( \frac{1}{4} \frac{Px_K^2}{EI} \right) \frac{x_K}{3}$$

$$= \frac{1}{24} \frac{Px_K^3}{EI} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI}$$

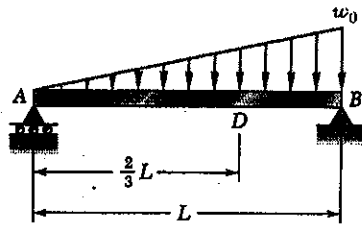
$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A} = \frac{\sqrt{3}}{512} \frac{PL^3}{EI} - \left( \frac{1}{4} \sqrt{3} \right) \frac{3}{128} \frac{PL^3}{EI} = -\frac{\sqrt{3}}{256} \frac{PL^3}{EI}$$

$$= 0.00677 \frac{PL^3}{EI} \downarrow$$

PROBLEM 9.145

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

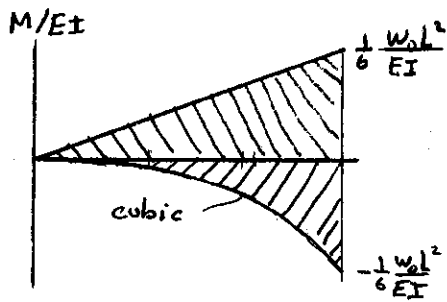
9.145 Beam and loading of Prob. 9.130



SOLUTION

$$+\circlearrowleft M_B = 0 \quad -R_A L + (\frac{1}{2} w_0 L)(\frac{1}{3} L) = 0 \quad R_A = \frac{1}{6} w_0 L$$

$$\begin{aligned} \text{Bending moment} \quad M &= R_A x - \frac{1}{6} \frac{w_0}{L} x^3 \\ &= \frac{1}{6} \frac{w_0}{L} (L^2 x - x^3) \end{aligned}$$



Draw  $\frac{M}{EI}$  diagram by parts

$$A_1 = \frac{1}{2} \left( \frac{1}{6} \frac{w_0 L^2}{EI} \right) L = \frac{1}{12} \frac{w_0 L^3}{EI} \quad \bar{x}_1 = \frac{1}{3} L$$

$$A_2 = \frac{1}{4} \left( -\frac{1}{6} \frac{w_0 L^2}{EI} \right) L = -\frac{1}{24} \frac{w_0 L^3}{EI} \quad \bar{x}_2 = \frac{1}{5} L$$

Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= A_1 \bar{x}_1 + A_2 \bar{x}_2 \\ &= \frac{1}{36} \frac{w_0 L^4}{EI} - \frac{1}{120} \frac{w_0 L^4}{EI} = \frac{7}{360} \frac{w_0 L^4}{EI} \end{aligned}$$

$$\text{Slope at A} \quad \theta_A = -\frac{t_{B/A}}{L} = -\frac{7}{360} \frac{w_0 L^3}{EI}$$

$$A_3 = A_1 \left( \frac{x_k}{L} \right)^2 = \frac{1}{12} \frac{w_0 L^3}{EI} u^2$$

$$A_4 = A_2 \left( \frac{x_k}{L} \right)^4 = -\frac{1}{24} \frac{w_0 L^3}{EI} u^4$$

$$\theta_{k/A} = A_3 + A_4 = \frac{w_0 L^3}{EI} \left( \frac{1}{12} u^2 - \frac{1}{24} u^4 \right) = -\theta_A = \frac{7}{360} \frac{w_0 L^3}{EI}$$

$$u^4 - 2u^2 + \frac{7}{15} = 0 \quad \text{Solving for } u \quad u = 0.51933$$

$$x_k = 0.51933 L$$

$$A_3 = \frac{1}{12} \frac{w_0 L^3}{EI} (0.51933)^2 = 0.0224753 \frac{w_0 L^3}{EI}, \quad \bar{x}_3 = \frac{1}{3} (0.51933) L$$

$$A_4 = -\frac{1}{24} \frac{w_0 L^3}{EI} (0.51933)^4 = -0.0030308 \frac{w_0 L^3}{EI}, \quad \bar{x}_4 = \frac{1}{5} (0.51933) L$$

$$t_{k/A} = A_3 \bar{x}_3 + A_4 \bar{x}_4 = 0.0035759 \frac{w_0 L^4}{EI}$$

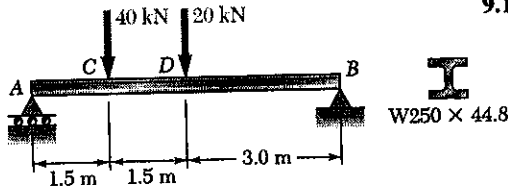
$$y_k = t_{k/A} - \frac{x_k}{L} t_{B/A} = 0.0035759 \frac{w_0 L^4}{EI} - (0.51933) \left( \frac{7}{360} \frac{w_0 L^4}{EI} \right)$$

$$= -0.00652 \frac{w_0 L^4}{EI} = 0.00652 \frac{w_0 L^4}{EI} \downarrow$$

PROBLEM 9.146

9.144 through 9.147 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

9.146 Beam and loading of Prob. 9.132



SOLUTION

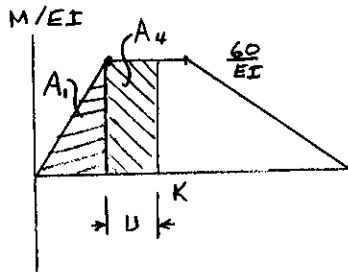
Referring to the solution to Prob. 9.129

$$EI = 14220 \text{ kN}\cdot\text{m}^2$$

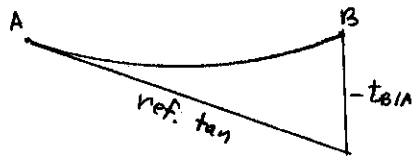
$$R_A = 40 \text{ kN}, \quad A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI} \text{ m}$$

$$\theta_A = -\frac{123.75}{EI}$$



Let K be the location of the maximum deflection. Assume that K lies between C and D.



$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{123.75}{EI} + A_1 + A_4$$

$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60U}{EI} = 0$$

$$U = \frac{123.75 - 45}{60} = 1.3125 \text{ m.}$$

$$x_K = 1.5 + U = 2.8125 \text{ m}$$

$$t_{K/A} = A_1(U + 0.5) + A_4\left(\frac{1}{2}U\right)$$

$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)\left(\frac{1}{2}\right)(1.3125)}{EI} = \frac{133.242}{EI}$$

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A}$$

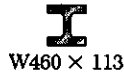
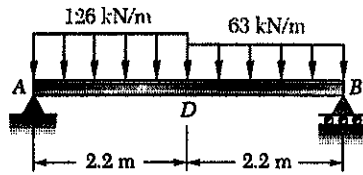
$$= \frac{133.242}{EI} - \frac{2.8125}{6} \left(\frac{742.5}{EI}\right) = -\frac{214.80}{EI} = -\frac{214.80}{14220}$$

$$= -15.11 \times 10^{-3} \text{ m} = 1.511 \text{ mm} \downarrow$$

**PROBLEM 9.147**

**9.144 through 9.147** For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

**9.147** Beam and loading of Prob. 9.133



**SOLUTION**

From the solution to Prob. 9.133

$$EI = 111200 \text{ KN}\cdot\text{m}^2, \quad R_A = 242.55 \text{ KN}$$

$$t_{B/A} = 13.824 \times 10^{-3} \text{ m}$$

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{13.824 \times 10^{-3}}{4.4} = -3.1418 \times 10^{-3} \text{ rad.}$$

Over portion AD of the beam

$$M = 242.55x - 63x^2 \text{ KN}\cdot\text{m}$$

$$\frac{M}{EI} = (2.1812x - 0.56655x^2) \times 10^{-3} \text{ m}^{-1}$$

$$\begin{aligned} \theta_{K/A} &= \int_0^{x_K} \frac{M}{EI} dx \\ &= (1.0906x_K^2 - 0.188849x_K^3) \times 10^{-3} \text{ rad} \end{aligned}$$

$$\theta_K = \theta_A + \theta_{K/A} = -3.1418 \times 10^{-3} + (1.0906x_K^2 - 0.188849x_K^3) \times 10^{-3} = 0$$

Solving for  $x_K$        $x_K = 2.13907 \text{ m}$        $x_K = 2.14 \text{ m} \blacktriangleleft$

$$A_5 = 1.0906 \times 10^{-3} x_K^2 = 4.99017 \times 10^{-3}, \quad \bar{x}_5 = \frac{1}{3} x_K = 0.71302 \text{ m}$$

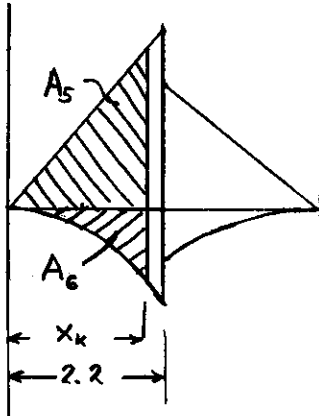
$$A_6 = -0.188849 \times 10^{-3} x_K^3 = -1.84837 \times 10^{-3}, \quad \bar{x}_6 = \frac{1}{4} x_K = 0.53477 \text{ m}$$

$$t_{K/A} = A_5 \bar{x}_5 + A_6 \bar{x}_6 = 2.5696 \times 10^{-3} \text{ m}$$

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A}$$

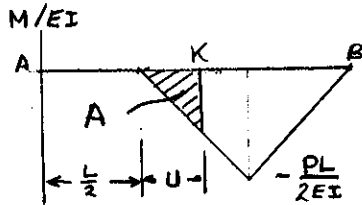
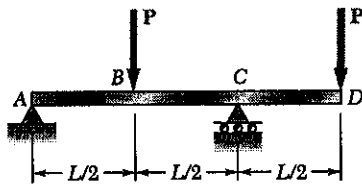
$$\begin{aligned} &= 2.5696 \times 10^{-3} - \frac{2.13907}{4.4} (13.824 \times 10^{-3}) = -4.15 \times 10^{-3} \text{ m} \\ &= 4.15 \text{ mm} \downarrow \blacktriangleleft \end{aligned}$$

M/EI



PROBLEM 9.148

9.148 For the beam and loading of Prob. 9.135, determine the magnitude and location of the largest upward deflection in span AC.



SOLUTION

From solution of Problem 9.135

$$R_A = 0, \quad \theta_A = \frac{1}{48} \frac{PL^2}{EI}$$

Draw  $M/EI$  diagram. Let  $K$  be location of maximum deflection.

$$\theta_K = \theta_A + \theta_{K/A} = \frac{1}{48} \frac{PL^2}{EI} + A = 0$$

$$\text{where } A = \frac{1}{2} \left( -\frac{PL}{2EI} \cdot \frac{2U}{L} \right) U = -\frac{1}{2} \frac{PLU^2}{EI}$$

$$\frac{1}{2} \frac{PLU^2}{EI} = \frac{1}{48} \frac{PL^2}{EI}$$

$$U^2 = \frac{1}{24} L^2 \quad U = 0.20412 L$$

$$x_K = \frac{1}{2} + U = 0.704 L$$

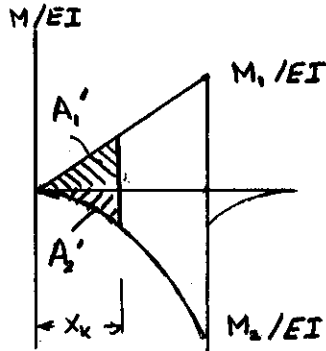
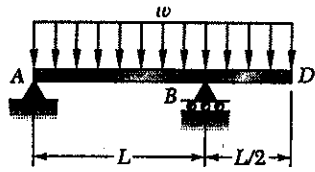
$$A = -\frac{1}{48} \frac{PL^2}{EI}$$

$$t_{A/K} = A \left( \frac{L}{2} + \frac{2}{3} U \right) = \left( -\frac{1}{48} \frac{PL^2}{EI} \right) (0.63608 L) = -0.01325 \frac{PL^3}{EI}$$

$$y_{max} = -t_{A/K} = 0.01325 \frac{PL^3}{EI}$$

PROBLEM 9.149

9.149 For the beam and loading of Prob. 9.136, determine the magnitude and location of the largest downward deflection in span AB.



SOLUTION

From solution of Prob. 9.146

$$\frac{M_1}{EI} = \frac{3}{8} \frac{WL^2}{EI}, \quad \frac{M_2}{EI} = -\frac{1}{2} \frac{WL^2}{EI}, \quad \theta_A = -\frac{1}{48} \frac{WL^3}{EI}$$

Let  $K$  be the location of maximum deflection.

$$U = \frac{x_K}{L}$$

From  $\frac{M}{EI}$  diagram

$$A_1' = \frac{1}{2} \left( \frac{M_1}{EI} U \right) (LU) = \frac{3}{16} \frac{WL^3}{EI} U^2$$

$$A_2' = \frac{1}{3} \left( \frac{M_2}{EI} U^3 \right) (LU) = -\frac{1}{6} \frac{WL^3}{EI} U^3$$

$$\begin{aligned} \theta_K &= \theta_A + \theta_{K/A} = \theta_A + A_1' + A_2' \\ &= -\frac{1}{48} \frac{WL^3}{EI} + \frac{3}{16} \frac{WL^3}{EI} U^2 - \frac{1}{6} \frac{WL^3}{EI} U^3 \\ &= -\left( \frac{1}{48} U^3 - \frac{3}{16} U^2 + \frac{1}{48} \right) \frac{WL^3}{EI} = 0 \end{aligned}$$

Multiplying by 48

$$8U^3 - 9U^2 + 1 = 0$$

Solving for  $U$

$$U = 0.421535$$

$$x_K = 0.4215 L$$

$$t_{A/K} = A_1' \left( \frac{2}{3} x_K \right) + A_2' \left( \frac{3}{4} x_K \right)$$

$$= \left( \frac{3}{16} \frac{WL^3}{EI} U^2 \right) \left( \frac{2}{3} LU \right) + \left( -\frac{1}{6} \frac{WL^3}{EI} U^3 \right) \left( \frac{3}{4} LU \right)$$

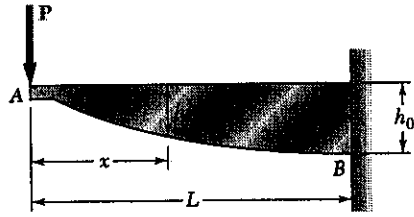
$$= \left( \frac{1}{8} U^3 - \frac{1}{8} U^4 \right) \frac{WL^4}{EI} = 0.00542 \frac{WL^4}{EI}$$

$$y_{max} = -t_{A/K} = -0.00542 \frac{WL^4}{EI} = 0.00542 \frac{WL^4}{EI} \downarrow$$



PROBLEM 9.150

\*9.150 The cantilever  $AB$  is a beam of constant strength. It has a rectangular cross section of uniform width  $b$  and variable depth  $h$ . Express the deflection at end  $A$  in terms of  $P$ ,  $L$  and the flexural rigidity  $EI_0$  at  $B$ . (Hint: Since the beam is of constant strength,  $Mc/I$  has a constant value along  $AB$ .)



SOLUTION

Bending moment  $M = -Px$

$$M_0 = -PL$$

$$M = M_0 \frac{x}{L}$$

For a constant strength beam

$$\frac{Mc}{I} = \frac{(M_0 x/L)(h/2)}{\frac{1}{12} b h^3} = \frac{6M_0 x/L}{h^2}$$

$$= \frac{6M_0}{h_0^2}$$

$$\left(\frac{h}{h_0}\right) = \left(\frac{x}{L}\right)^{\frac{1}{2}}$$

Moment of inertia  $I = \frac{1}{12} b h^3$ ,  $I_0 = \frac{1}{12} b h_0^3$

$$\frac{I}{I_0} = \left(\frac{h}{h_0}\right)^3 = \left(\frac{x}{L}\right)^{3/2}$$

Curvature  $\frac{M}{EI} = \frac{M_0(x/L)}{EI_0(x/L)^{3/2}} = \frac{M_0}{EI_0} \left(\frac{L}{x}\right)^{1/2}$

$$= -\frac{PL}{EI_0} \left(\frac{L}{x}\right)^{1/2}$$

Deflection at A

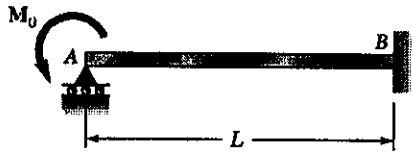
$$y_A = \Delta_{A/B} = \int_0^L x \frac{M}{EI} dx$$

$$= -\frac{PL^{3/2}}{EI_0} \int_0^L x^{1/2} dx = -\frac{PL^{3/2}}{EI_0} \left. \frac{x^{3/2}}{3/2} \right|_0^L$$

$$= -\frac{2}{3} \frac{PL^3}{EI_0} = \frac{2}{3} \frac{PL^3}{EI_0} \downarrow$$

PROBLEM 9.151

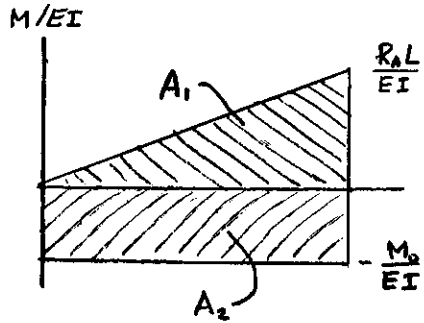
9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support A and treat  $R_A$  as redundant.

Draw  $M/EI$  diagram for loads  $M_0$  and  $R_A$ .

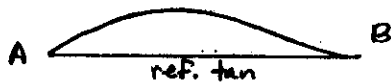


Place reference tangent at B

$$t_{B/A} = A_1 \left( \frac{2}{3}L \right) + A_2 \left( \frac{1}{2}L \right)$$

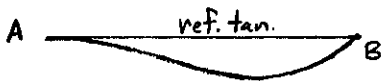
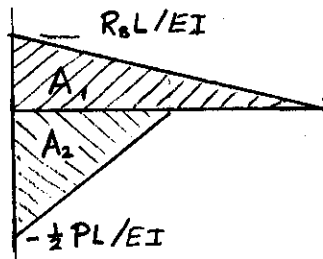
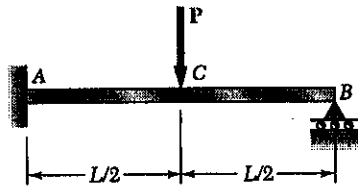
$$= \frac{1}{3} \frac{R_A L^2}{EI} - \frac{1}{2} \frac{M_0 L^2}{EI} = 0$$

$$R_A = \frac{3}{2} \frac{M_0}{L} \uparrow$$



PROBLEM 9.152

9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support B and treat  $R_B$  as redundant.

Draw  $M/EI$  diagram for loads  $P$  and  $R_B$ .

$$A_1 = \frac{1}{2} \left( \frac{R_B L}{EI} \right) (L) = \frac{1}{2} \frac{R_B L^2}{EI}$$

$$A_2 = \frac{1}{2} \left( \frac{1}{2} \frac{PL}{EI} \right) \left( \frac{L}{2} \right) = -\frac{1}{8} \frac{PL^2}{EI}$$

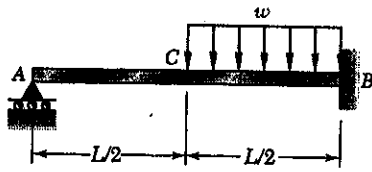
Place reference tangent at A.

$$\begin{aligned} t_{B/A} &= A_1 \left( \frac{2}{3} L \right) + A_2 \left( \frac{L}{2} + \frac{2}{3} \frac{L}{2} \right) \\ &= \frac{1}{3} \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} = 0 \end{aligned}$$

$$R_B = \frac{5}{16} P \uparrow$$

PROBLEM 9.153

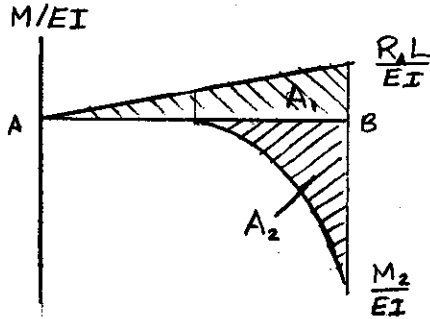
9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support A and treat  $R_A$  as redundant.

Draw  $M/EI$  diagram for loads  $R_A$  and  $w$ .



$$M_2 = -\frac{1}{2} w \left(\frac{L}{2}\right)^2 = -\frac{1}{8} w L^2$$

$$A_1 = \frac{1}{2} \left(\frac{R_A L}{EI}\right) L = \frac{1}{2} \frac{R_A L^2}{EI}$$

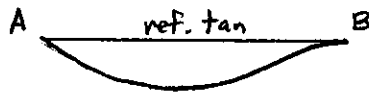
$$A_2 = \frac{1}{3} \left(-\frac{1}{8} \frac{w L^2}{EI}\right) \left(\frac{L}{2}\right) = -\frac{1}{48} \frac{w L^3}{EI}$$

Place reference tangent at B

$$t_{A/B} = A_1 \left(\frac{2}{3} L\right) + A_2 \left(\frac{L}{2} + \frac{3}{4} \frac{L}{2}\right)$$

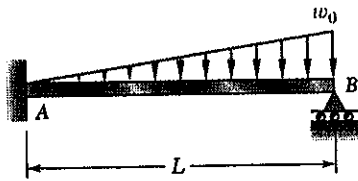
$$= \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

$$R_A = \frac{7}{128} w L \uparrow$$



PROBLEM 9.154

9.151 through 1.154 For the beam and loading shown, determine the reaction at the roller support.



SOLUTION

Remove support B and treat  $R_B$  as redundant.

Replace loading by equivalent shown at left.

Draw  $M/EI$  diagram for load  $w_0$  and  $R_B$ .

Use parts as shown.

$$A_1 = \frac{1}{2} \left( \frac{R_B L}{EI} \right) (L) = \frac{1}{2} \frac{R_B L^2}{EI}$$

$$M_2 = -\frac{1}{2} w_0 L^2$$

$$A_2 = \frac{1}{3} \left( -\frac{1}{2} \frac{w_0 L^2}{EI} \right) L = -\frac{1}{6} \frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6} \frac{w_0}{L} L^3 = \frac{1}{6} w_0 L^2$$

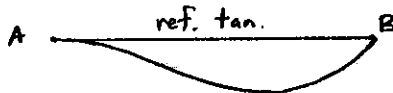
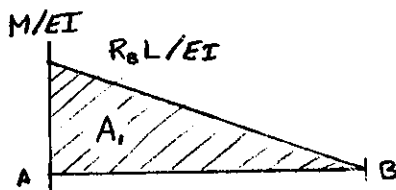
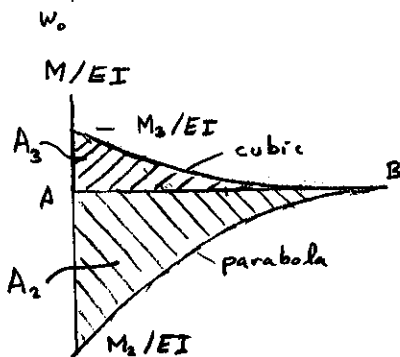
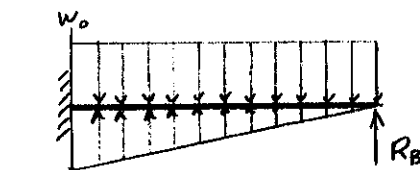
$$A_3 = \frac{1}{4} \left( \frac{1}{6} \frac{w_0 L^2}{EI} \right) L = \frac{1}{24} \frac{w_0 L^3}{EI}$$

Place reference tangent at A

$$t_{B/A} = A_1 \left( \frac{2}{3} L \right) + A_2 \left( \frac{3}{4} L \right) + A_3 \left( \frac{4}{5} L \right)$$

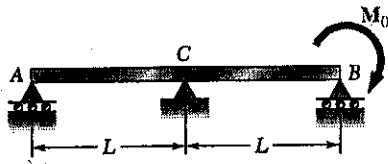
$$= \frac{1}{3} \frac{R_B L^3}{EI} - \frac{1}{8} \frac{w_0 L^4}{EI} + \frac{1}{30} \frac{w_0 L^4}{EI} = 0$$

$$R_B = \frac{11}{40} w_0 L = 0.275 w_0 L \uparrow$$



PROBLEM 9.155

9.155 and 9.156 For the beam and loading shown, determine the reaction at each support.

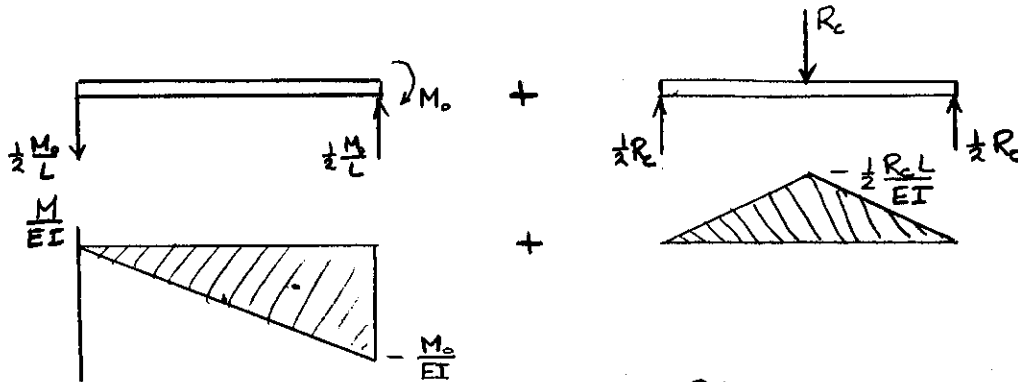


SOLUTION

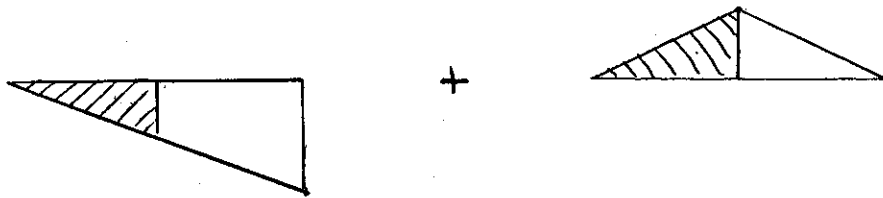
Remove support C and treat  $R_c$  as redundant.

Consider the loads  $M_0$  and  $R_c$  separately

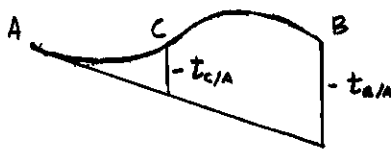
Place reference tangent at A.



$$t_{B/A} = -\frac{1}{2} \left( \frac{M_0}{EI} \right) (2L) \left( \frac{2L}{3} \right) + \frac{1}{2} \left( \frac{1}{2} \frac{R_c L}{EI} \right) (2L) (L) = -\frac{2}{3} \frac{M_0 L^2}{EI} + \frac{1}{2} \frac{R_c L^3}{EI}$$



$$t_{C/A} = -\frac{1}{2} \left( \frac{1}{2} \frac{M_0}{EI} \right) (L) \left( \frac{L}{3} \right) + \frac{1}{2} \left( \frac{1}{2} \frac{R_c L}{EI} \right) (L) \left( \frac{L}{3} \right) = -\frac{1}{12} \frac{M_0 L^2}{EI} + \frac{1}{12} \frac{R_c L^3}{EI}$$



$$y_c = t_{C/A} - \frac{x_{C/A}}{x_{B/A}} t_{B/A} = t_{C/A} - \frac{1}{2} t_{B/A} = 0$$

$$\left( -\frac{1}{12} + \frac{1}{3} \right) \frac{M_0 L^2}{EI} + \left( \frac{1}{12} - \frac{1}{4} \right) \frac{R_c L^3}{EI} = 0$$

$$R_c = \frac{3}{2} \frac{M_0}{L} \downarrow$$

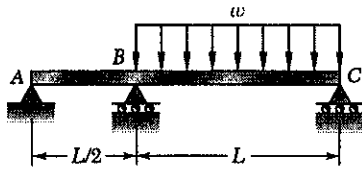
From Statics

$$R_A = \frac{1}{2} R_c - \frac{1}{2} \frac{M_0}{L} = \frac{1}{4} \frac{M_0}{L} \uparrow$$

$$R_B = \frac{1}{2} R_c + \frac{1}{2} \frac{M_0}{L} = \frac{5}{4} \frac{M_0}{L} \uparrow$$

PROBLEM 9.156

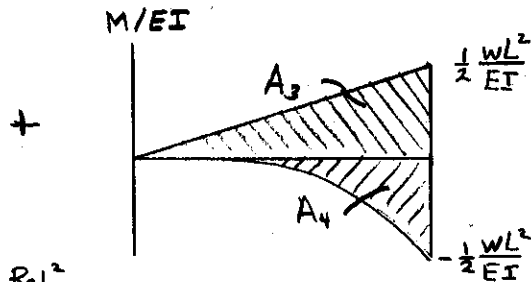
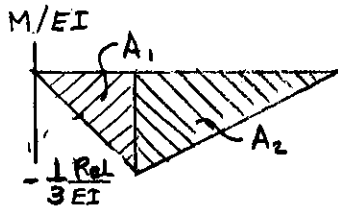
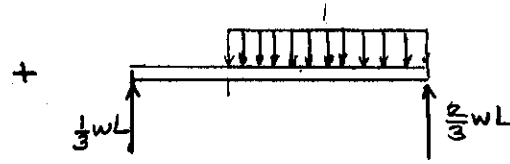
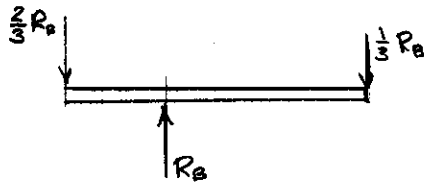
9.155 and 9.156 For the beam and loading shown, determine the reaction at each support.



SOLUTION

Remove support B and consider  $R_B$  as redundant.  
Consider loads  $R_B$  and  $w$  separately.

Place reference tangent at A.



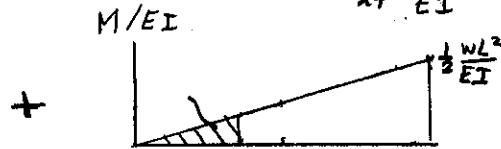
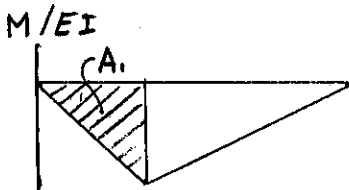
$$A_1 = \frac{1}{2} \cdot \left( -\frac{1}{3} \frac{R_B L}{EI} \right) \left( \frac{L}{2} \right) = -\frac{1}{12} \frac{R_B L^2}{EI}$$

$$A_2 = \frac{1}{2} \left( -\frac{1}{3} \frac{R_B L}{EI} \right) L = -\frac{1}{6} \frac{R_B L^2}{EI}$$

$$A_3 = \frac{1}{2} \left( \frac{1}{2} \frac{wL^2}{EI} \right) \frac{3L}{2} = \frac{3}{8} \frac{wL^3}{EI}$$

$$A_4 = \frac{1}{3} \left( -\frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

$$\begin{aligned} t_{C/A} &= A_1 \left( L + \frac{1}{3} \frac{L}{2} \right) + A_2 \left( \frac{2}{3} L \right) \\ &\quad + A_3 \left( \frac{1}{3} \cdot \frac{3L}{2} \right) + A_4 \left( \frac{1}{4} L \right) \\ &= -\frac{7}{72} \frac{R_B L^3}{EI} - \frac{1}{9} \frac{R_B L^3}{EI} \\ &\quad + \frac{3}{16} \frac{wL^4}{EI} - \frac{1}{24} \frac{wL^4}{EI} \\ &= -\frac{5}{24} \frac{R_B L^3}{EI} + \frac{7}{48} \frac{wL^3}{EI} \end{aligned}$$



$$\begin{aligned} A_5 &= \frac{1}{2} \left( \frac{1}{6} \frac{wL^2}{EI} \right) \frac{L}{2} \\ &= \frac{1}{24} \frac{wL^3}{EI} \end{aligned}$$

$$t_{B/A} = A_1 \left( \frac{1}{3} \frac{L}{2} \right) + A_5 \left( \frac{1}{3} \frac{L}{2} \right) = -\frac{1}{72} \frac{R_B L^3}{EI} + \frac{1}{144} \frac{wL^4}{EI}$$

$$y_B = t_{B/A} - \frac{L/2}{3L/2} t_{C/A} = \left( -\frac{1}{72} + \frac{5}{72} \right) \frac{R_B L^3}{EI} + \left( \frac{1}{144} - \frac{7}{144} \right) \frac{wL^4}{EI}$$

$$= \frac{1}{18} \frac{R_B L^3}{EI} - \frac{1}{24} \frac{wL^3}{EI} = 0$$

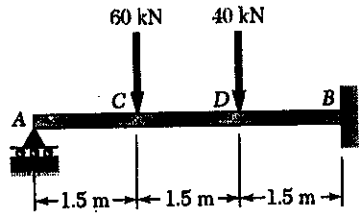
$$R_B = \frac{3}{4} wL \uparrow$$

$$R_A = \frac{1}{3} wL - \frac{2}{3} R_B = \frac{1}{3} wL - \frac{1}{2} wL = -\frac{1}{6} wL = \frac{1}{6} wL \downarrow$$

$$R_C = \frac{2}{3} wL - \frac{1}{3} R_B = \frac{2}{3} wL - \frac{1}{4} wL = \frac{5}{12} wL \uparrow$$

PROBLEM 9.157

9.157 and 9.158 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support at A and treat  $R_A$  as redundant.

Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2} \left( \frac{4.5 R_A}{EI} \right) (4.5) = \frac{10.125 R_A}{EI}$$

$$A_2 = -\frac{1}{2} \left( \frac{180}{EI} \right) (3.0) = -\frac{270}{EI}$$

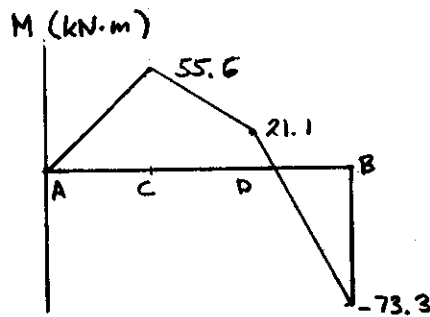
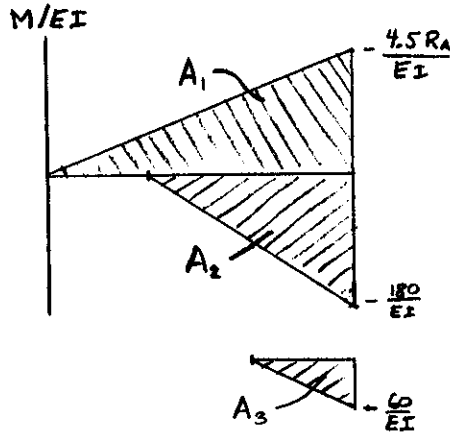
$$A_3 = -\frac{1}{2} \left( \frac{60}{EI} \right) (1.5) = -\frac{45}{EI}$$

Place reference tangent at B.  $\theta_B = 0$

$$t_{A/B} = A_1(3.0) + A_2(1.5 + 2.0) + A_3(3.0 + 1.0)$$

$$= \{ 30.375 R_A - 945 - 180 \} \frac{1}{EI} = 0$$

$$R_A = 37.037 \text{ kN } \uparrow$$



$$M_A = 0$$

$$M_C = (1.5)(37.037) = 55.6 \text{ kN}\cdot\text{m}$$

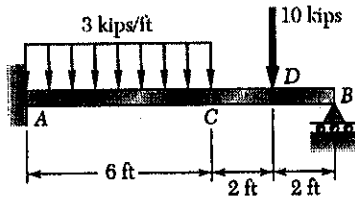
$$M_D = (3.0)(37.037) - (60)(1.5) = 21.1 \text{ kN}\cdot\text{m}$$

$$M_B = (4.5)(37.037) - (60)(3) - (40)(1.5) = -73.3 \text{ kN}\cdot\text{m}$$



PROBLEM 9.158

9.157 and 9.158 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support at B and treat  $R_B$  as redundant.

Draw bending moment diagram by parts

$$A_1 = \frac{1}{2} \left( \frac{10R_B}{EI} \right) (10) = \frac{50R_B}{EI}$$

$$A_2 = -\frac{1}{2} \left( \frac{30}{EI} \right) (8) = -\frac{320}{EI}$$

$$M_3 = -\frac{1}{2} (3)(6)^2 = -54 \text{ kN}\cdot\text{m}$$

$$A_3 = \frac{1}{3} \left( -\frac{54}{EI} \right) (6) = -\frac{108}{EI}$$

Place reference tangent at A.  $\theta_A = 0$

$$t_{B/A} = A_1 \left( \frac{2}{3} \cdot 10 \right) + A_2 \left( \frac{2}{3} \cdot 8 + 2 \right) + A_3 (10 - \frac{1}{4} \cdot 6)$$

$$= \frac{333.33R_B}{EI} - \frac{2346.7}{EI} - \frac{918}{EI} = 0$$

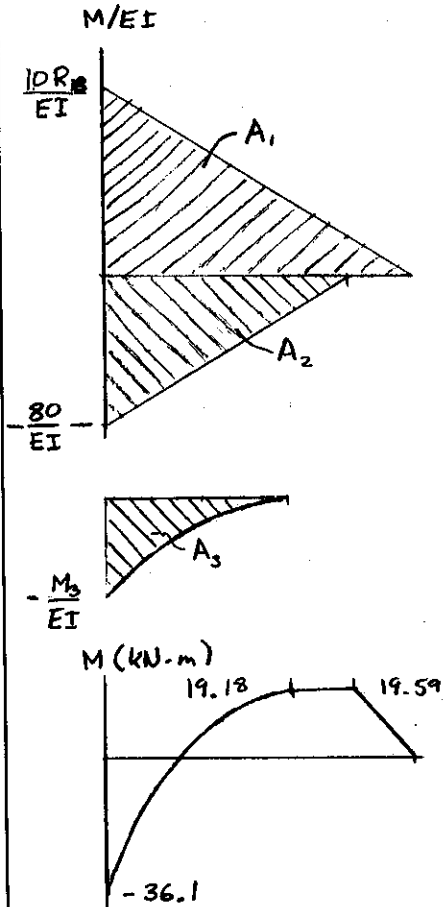
$$R_B = 9.79 \text{ kN } \uparrow$$

$$M_B = 0$$

$$M_D = (9.79)(2) = 19.59 \text{ kN}\cdot\text{m}$$

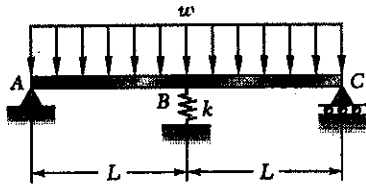
$$M_C = (9.79)(4) - (10)(2) = 19.18 \text{ kN}\cdot\text{m}$$

$$M_A = (9.79)(10) - (10)(8) - 54 = -36.1 \text{ kN}\cdot\text{m}$$

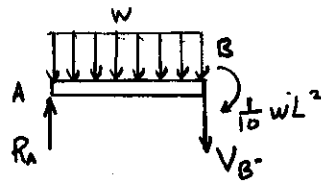


PROBLEM 9.159

9.159 For the beam and loading shown, determine the spring constant  $k$  for which the bending moment at  $B$  is  $M_B = -wL^2/10$ .



SOLUTION



Using free body AB

$$+\circlearrowleft M_B = 0$$

$$-R_A L + (wL)(\frac{L}{2}) - \frac{1}{10} wL^2 = 0$$

$$R_A = \frac{2}{5} wL \uparrow$$

Symmetric beam and loading  $R_C = R_A$

Using free body ABC  $+\uparrow \Sigma F_y = 0$

$$\frac{2}{5} wL + F + \frac{2}{5} wL - 2wL = 0$$

$$F = \frac{6}{5} wL$$

Draw  $\frac{M}{EI}$  diagram by parts

$$A_1 = \frac{1}{2} \left( \frac{2}{5} \frac{wL^2}{EI} \right) L = \frac{1}{5} \frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3} \left( \frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

Place reference tangent at B.  $\theta_B = 0$

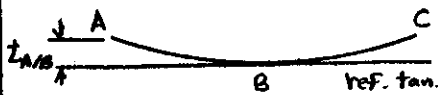
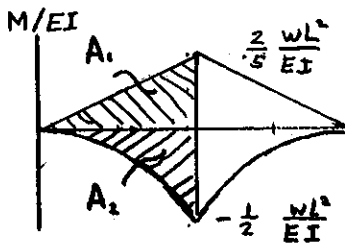
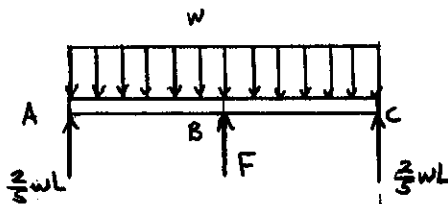
$$y_B = -t_{A/B}$$

$$= - \left( A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L \right)$$

$$= - \frac{1}{120} \frac{wL^4}{EI}$$

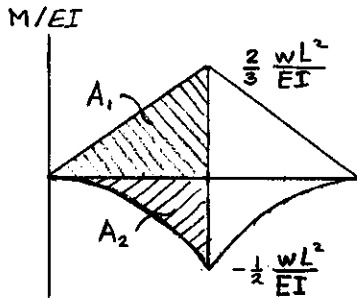
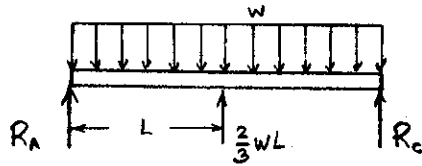
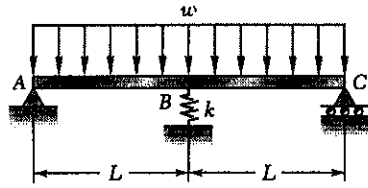
$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5} wL}{-\frac{1}{120} \frac{wL^4}{EI}} = 144 \frac{EI}{L^3}$$



PROBLEM 9.160

9.160 For the beam and loading shown, determine the spring constant  $k$  for which the force in the spring is equal to one-third of the total load on the beam.



SOLUTION

Symmetric beam and loading,  $R_C = R_A$

Spring force  $F = \frac{1}{3}(2wL) = \frac{2}{3}wL$

$$+\uparrow \Sigma F_y = 0 \quad R_A + F - 2wL + R_C = 0$$

$$R_A = R_C = \frac{2}{3}wL$$

Draw  $\frac{M}{EI}$  diagram by parts.

$$A_1 = \frac{1}{2} \left( \frac{2}{3} \frac{wL^2}{EI} \right) L = \frac{1}{3} \frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3} \left( \frac{1}{2} \frac{wL^2}{EI} \right) L = -\frac{1}{6} \frac{wL^3}{EI}$$

Place reference tangent at B.  $\theta_B = 0$

$$y_B = -t_{A/B}$$

$$= -(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L)$$

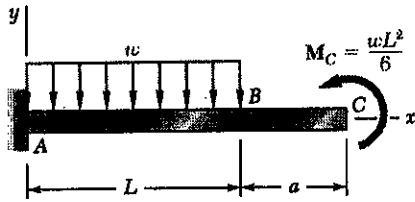
$$= -\frac{7}{72} \frac{wL^4}{EI}$$

$$F = -k y_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72} \frac{wL^4}{EI}} = \frac{48}{7} \frac{EI}{L^3}$$

PROBLEM 9.161

9.161 For the cantilever beam and loading shown, determine (a) the deflection at point B, (b) the slope at point B.

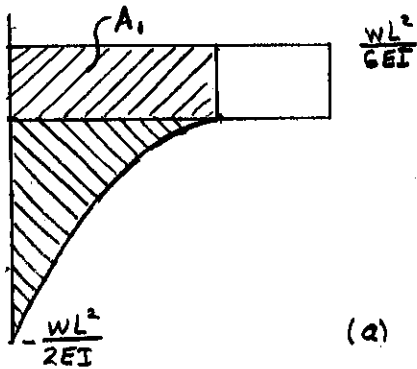


SOLUTION

Use moment area method.

Draw  $\frac{M}{EI}$  diagram by parts.

$M/EI$



Place reference tangent at A.  $\theta_A = 0$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_{B/A}$$

$$y_B = y_A + L\theta_A + t_{B/A} = t_{B/A}$$

$$A_1 = \left(\frac{WL^2}{6EI}\right)(L) = \frac{1}{6} \frac{WL^3}{EI}$$

$$A_2 = \frac{1}{3} \left(-\frac{WL^2}{2EI}\right)(L) = -\frac{1}{6} \frac{WL^3}{EI}$$

(a) Deflection at B

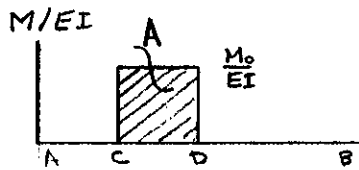
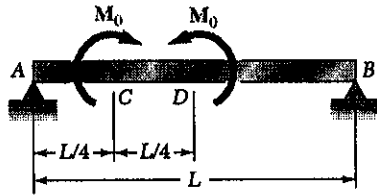
$$\begin{aligned} y_B &= t_{B/A} = A_1 \left(\frac{L}{2}\right) + A_2 \left(\frac{3}{4}L\right) \\ &= \frac{1}{12} \frac{WL^4}{EI} - \frac{1}{8} \frac{WL^4}{EI} = -\frac{1}{24} \frac{WL^4}{EI} \end{aligned}$$

(b) Slope at B

$$\begin{aligned} \theta_B &= \theta_{B/A} = A_1 + A_2 \\ &= \frac{1}{6} \frac{WL^3}{EI} - \frac{1}{6} \frac{WL^3}{EI} = 0 \end{aligned}$$

PROBLEM 9.162

9.162 For the beam and loading shown, determine (a) the slope at point A, (b) the deflection at point D.



SOLUTION

From Statics  $R_A = R_B = 0$ .

Draw  $\frac{M}{EI}$  diagram

$$A = \left(\frac{M_0}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{4} \frac{M_0 L}{EI}$$

Place reference tangent at A.

$$t_{B/A} = A\left(\frac{L}{2} + \frac{L}{8}\right) = \frac{5}{32} \frac{M_0 L^2}{EI}$$

$$t_{D/A} = A\left(\frac{L}{8}\right) = \frac{1}{32} \frac{M_0 L^2}{EI}$$

(a) Slope at A

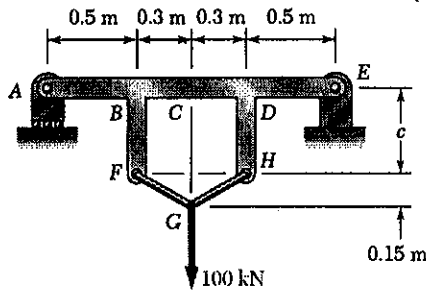
$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32} \frac{M_0 L}{EI}$$

(b) Deflection at D

$$\begin{aligned} y_D &= t_{D/A} - \frac{x_D}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A} \\ &= \frac{1}{32} \frac{M_0 L^2}{EI} - \frac{5}{64} \frac{M_0 L^2}{EI} = -\frac{3}{64} \frac{M_0 L^2}{EI} \end{aligned}$$

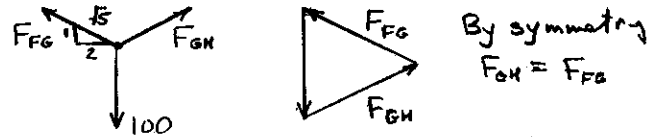
PROBLEM 9.163

9.163 The rigid bars  $BF$  and  $DH$  are welded to the rolled-steel beam  $AE$  as shown. Knowing that  $c = 0.4\text{m}$ , determine for the loading shown (a) the deflection at point  $B$ , (b) the deflection at the midpoint  $C$  of the beam. Use  $E = 200\text{ GPa}$ .



SOLUTION

Using joint  $G$  as a free body



$$2 F_{GHy} - 100 = 0 \quad F_{GHy} = 50 \text{ kN}$$

$$F_{GHx} = 2 F_{GHx} = 100 \text{ kN}$$

Forces in kN. Lengths in m

$$V = 50 - 50 \langle x - 0.5 \rangle^0 - 50 \langle x - 1.1 \rangle^0 \quad \text{kN}$$

$$M = 50x - 50 \langle x - 0.5 \rangle^1 - 50 \langle x - 1.1 \rangle^1 + 40 \langle x - 0.5 \rangle^0 - 40 \langle x - 1.1 \rangle^0 \quad \text{kN}\cdot\text{m}$$

$$EI \frac{dy}{dx} = 25x^2 - 25 \langle x - 0.5 \rangle^2 - 25 \langle x - 1.1 \rangle^2 - 40 \langle x - 0.5 \rangle^1 + 40 \langle x - 1.1 \rangle^1 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EI y = \frac{25}{3} x^3 - \frac{25}{3} \langle x - 0.5 \rangle^3 - \frac{25}{3} \langle x - 1.1 \rangle^3 - 20 \langle x - 0.5 \rangle^2 + 20 \langle x - 1.1 \rangle^2 + C_1 x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x = 0, y = 0] \quad C_2 = 0$$

$$[x = 1.6, y = 0]$$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \text{ kN}\cdot\text{m}^2$$

For  $EI y_B$ ,  $x = 0.5 \text{ m}$

$$EI y_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667 \text{ kN}\cdot\text{m}^2$$

For  $EI y_C$ ,  $x = 0.8 \text{ m}$

$$EI y_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0 = -0.8417 \text{ kN}\cdot\text{m}^2$$

For  $W 100 \times 19.3$  rolled steel section  $I = 4.77 \times 10^6 \text{ mm}^4 = 4.77 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(4.77 \times 10^{-6}) = 954 \times 10^3 \text{ N}\cdot\text{m}^2 = 954 \text{ kN}\cdot\text{m}^2$$

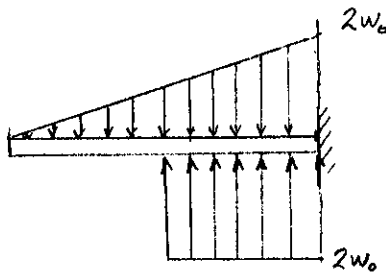
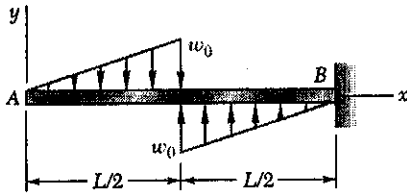
$$(a) y_B = \frac{0.1667}{954} = 0.175 \times 10^{-3} \text{ m} = 0.175 \text{ mm} \uparrow$$

$$(b) y_C = \frac{0.8417}{954} = 0.882 \times 10^{-3} \text{ m} = 0.882 \text{ mm} \uparrow$$

PROBLEM 9.164

9.164 For the beam and loading shown, determine the deflection at point A.

SOLUTION



Express loading in terms of singularity functions.

$$w = \frac{2w_0}{L}x - 2w_0 \langle x - \frac{L}{2} \rangle^0$$

$$\frac{dV}{dx} = -w = -\frac{2w_0}{L}x + 2w_0 \langle x - \frac{L}{2} \rangle^0$$

$$V = -\frac{w_0}{L}x^2 + 2w_0 \langle x - \frac{L}{2} \rangle^1 + C_1$$

$$[x=0, V=0]$$

$$0 + 0 + C_1 = 0 \quad C_1 = 0$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}x^2 + 2w_0 \langle x - \frac{L}{2} \rangle^1$$

$$M = -\frac{1}{3} \frac{w_0}{L} x^3 + w_0 \langle x - \frac{L}{2} \rangle^2 + C_2$$

$$[x=0, M=0]$$

$$0 + 0 + C_2 = 0 \quad C_2 = 0$$

$$EI \frac{d^2y}{dx^2} = M = -\frac{1}{3} \frac{w_0}{L} x^3 + w_0 \langle x - \frac{L}{2} \rangle^2$$

$$EI \frac{dy}{dx} = -\frac{1}{12} \frac{w_0}{L} x^4 + \frac{1}{3} w_0 \langle x - \frac{L}{2} \rangle^3 + C_3$$

$$[x=L, EI \frac{dy}{dx} = 0] \quad -\frac{1}{12} w_0 L^3 + \frac{1}{3} w_0 \left(\frac{L}{2}\right)^3 + C_3 = 0 \quad C_3 = \frac{1}{24} w_0 L^3$$

$$EI y = -\frac{1}{60} \frac{w_0}{L} x^5 + \frac{1}{12} w_0 \langle x - \frac{L}{2} \rangle^4 + C_3 x + C_4$$

$$[x=L, EI y = 0] \quad -\frac{1}{60} w_0 L^4 + \frac{1}{12} w_0 \left(\frac{L}{2}\right)^4 + \frac{1}{24} w_0 L^3 \cdot L + C_4 = 0$$

$$C_4 = -\frac{29}{960} w_0 L^4$$

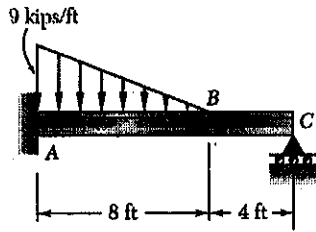
At point A,  $x=0$

$$EI y_A = 0 + 0 + 0 + C_4 = -\frac{29}{960} w_0 L^4$$

$$y_A = -\frac{29}{960} \frac{w_0 L^4}{EI}$$

**PROBLEM 9.165**

9.165 For the beam and loading shown, determine (a) the reaction at C, (b) the deflection at point B. Use  $E = 29 \times 10^6$  psi.



**SOLUTION**

$$E = 29 \times 10^6 \text{ psi} = 29 \times 10^3 \text{ ksi}$$

$$I = 310 \text{ in}^4$$

$$EI = (29 \times 10^3)(310) = 8.99 \times 10^6 \text{ kip}\cdot\text{in}^2 = 62430 \text{ kip}\cdot\text{ft}^2$$

Statically indeterminate beam. Remove support at C and treat  $R_c$  as redundant.

Draw  $\frac{M}{EI}$  diagram by parts.

For the uniformly varying load

$$k = \frac{9 \text{ kips/ft}}{8 \text{ ft}} = \frac{9}{8} \text{ kips/ft}^2$$

$$M_2 = -\frac{1}{2} k a^3 = -\frac{1}{2} \cdot \frac{9}{8} (8)^3 = -96 \text{ kip}\cdot\text{ft}$$

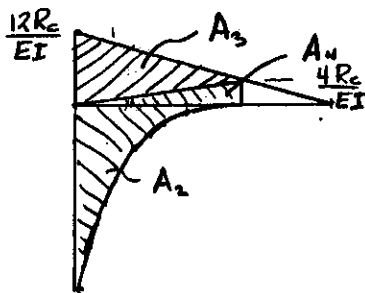
$$A_1 = \frac{1}{2} \left( \frac{12 R_c}{EI} \right) (12) = \frac{72 R_c}{EI}$$

$$A_2 = \frac{1}{4} \left( -\frac{96}{EI} \right) (8) = -\frac{192}{EI}$$

Place reference tangent at A.  $\theta_A = 0$   
 $y_A = 0$

$$y_C = y_A + \theta_A L + t_{C/A} = 0 + 0 + \left[ A_1 \left( \frac{2}{3} \cdot 12 \right) + A_2 \left( 12 - \frac{1}{3} \cdot 8 \right) \right]$$

$$= \frac{576 R_c}{EI} - \frac{1996.8}{EI} = 0 \quad R_c = 3.4667 \text{ kips } \uparrow$$



$$A_3 = \frac{1}{2} \left( \frac{12 R_c}{EI} \right) (8) = \frac{48 R_c}{EI}$$

$$A_4 = \frac{1}{2} \left( \frac{4 R_c}{EI} \right) (8) = \frac{16 R_c}{EI}$$

$$y_B = t_{B/A} = A_3 \left( \frac{2}{3} \cdot 8 \right) + A_4 \left( \frac{1}{3} \cdot 8 \right) + A_2 \left( \frac{4}{3} \cdot 8 \right)$$

$$= 298 \frac{2}{3} \frac{R_c}{EI} - \frac{6144}{5 EI} = -\frac{193.42}{EI}$$

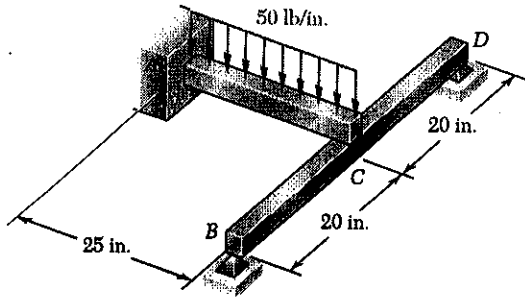
$$= -\frac{192.42}{62430} = -3.098 \times 10^{-3} \text{ ft}$$

$$= 0.0372 \text{ in. } \downarrow$$



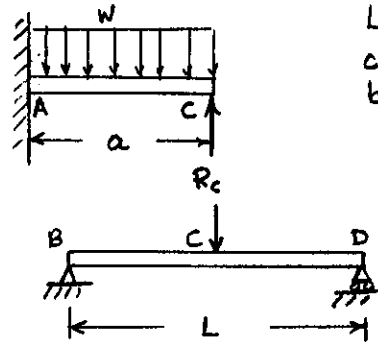
**PROBLEM 9.166**

9.166 For the loading shown, knowing that beams AC and BD have the same flexural rigidity, determine the reaction at B.



**SOLUTION**

Consider the two beams shown below.



Let  $R_c$  be the contact force between beams AC and BCD.

Applying Cases 1 and 2 of Appendix D to cantilever beam AC

$$y_c = \frac{R_c a^3}{3EI} - \frac{wa^4}{8EI}$$

Applying Case 4 of Appendix D to simply supported beam BCD.

$$y_c = -\frac{R_c L^3}{48EI}$$

Equating expressions for  $y_c$

$$\frac{R_c a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{R_c L^3}{48EI}$$

$$(16a^3 + L^3)R_c = 6wa^4$$

$$R_c = \frac{6wa}{16 + L^3/a^3}$$

Data:  $w = 50 \text{ lb/in}$ ,  $a = 25 \text{ in}$ ,  $L = 20 + 20 = 40 \text{ in}$ .

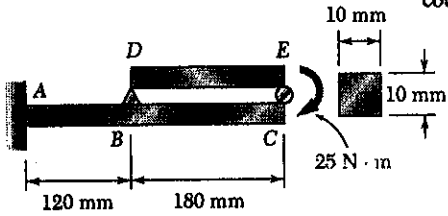
$$R_c = \frac{(6)(50)(25)}{16 + (40/25)^3} = 373.21 \text{ lb}$$

Using beam BCD as a free body

$$\sum M_D = 0 \quad -R_B L + R_c \frac{L}{2} = 0 \quad R_B = \frac{1}{2} R_c = 186.6 \text{ lb} \uparrow$$

**PROBLEM 9.167**

9.167 Beam  $DE$  rests on the cantilever beam  $AC$  as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end  $C$  if the 25-N·m couple is applied (a) to end  $E$  of beam  $DE$ , (b) to end  $C$  of beam  $AC$ . Use  $E = 200$  GPa.

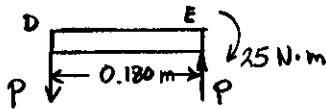


**SOLUTION**

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = 166.667 \text{ N} \cdot \text{m}^2$$

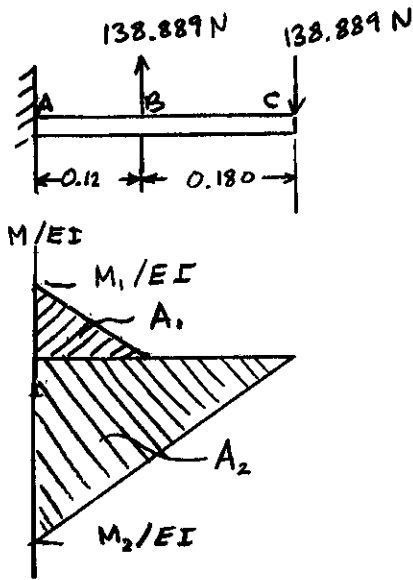


(a) Couple applied to beam  $DE$

Free body  $DE \quad \sum M = 0$

$$.180 P - 25 = 0 \quad P = 138.889 \text{ N}$$

For beam  $ABC$ , draw the  $\frac{M}{EI}$  diagram by parts.



$$\frac{M_1}{EI} = \frac{(138.889)(0.12)}{166.667} = 100 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(138.889)(0.30)}{166.667} = -250 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(100 \times 10^{-3})(0.12) = 6 \times 10^{-3}$$

$$A_2 = \frac{1}{2}(-250 \times 10^{-3})(0.30) = 37.5 \times 10^{-3}$$

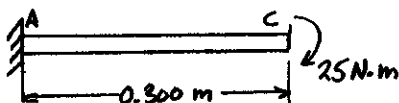
$$y_A = 0 \quad \theta_A = 0$$

Place reference tangent at  $A$

$$y_C = y_A + L\theta_A + t_{C/A}$$

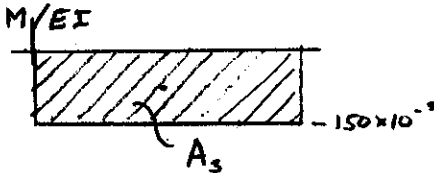
$$= 0 + 0 + A_1(0.180 + 0.080) + A_2(0.200)$$

$$= -5.94 \times 10^{-3} \text{ m} = 5.94 \text{ mm} \downarrow$$



(b) Couple applied to beam  $AC$

Draw  $\frac{M}{EI}$  diagram



$$\frac{M}{EI} = \frac{25}{166.667} = 150 \times 10^{-3} \text{ m}^{-1}$$

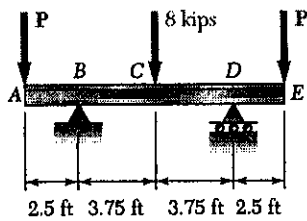
$$A_3 = (-150 \times 10^{-3})(0.30) = -45 \times 10^{-3}$$

$$y_C = t_{C/A} = A_3(0.15) = -6.75 \times 10^{-3} \text{ m}$$

$$= 6.75 \text{ mm} \downarrow$$

**PROBLEM 9.168**

9.168 For the beam and loading shown, determine the value of  $P$  for which the deflection is zero at end  $A$  of the beam. Use  $E = 29 \times 10^6$  psi.



**SOLUTION**

Symmetric beam and loading.  $\theta_c = 0$

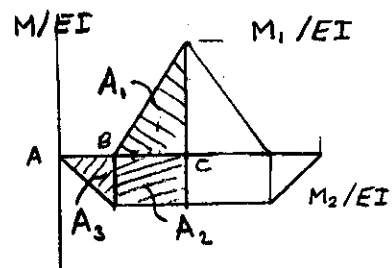
Place reference tangent at  $C$ .

Draw  $\frac{M}{EI}$  diagram by parts.

Assume  $EI$  in  $\text{kip} \cdot \text{ft}^2$

$$M_1 = (4)(3.75) = 15 \text{ kip} \cdot \text{ft}$$

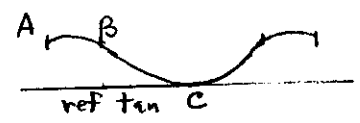
$$M_2 = -(P)(2.5) = -2.5P \text{ kip} \cdot \text{ft}$$



$$A_1 = \frac{1}{2} \left( \frac{15}{EI} \right) (3.75) = \frac{28.125}{EI}$$

$$A_2 = \frac{-2.5P}{EI} (3.75) = -\frac{9.375P}{EI}$$

$$A_3 = \frac{1}{2} \left( -\frac{2.5P}{EI} \right) (2.5) = -\frac{3.125P}{EI}$$



$$y_A = y_C + t_{A/C}$$

$$y_B = y_C + t_{B/C}$$

$$y_A - y_B = t_{A/C} - t_{B/C} = 0$$

$$A_1 (2.5 + 2.5) + A_2 (2.5 + 1.875) + A_3 \left( \frac{2}{3} \cdot 2.5 \right) - A_1 (2.5) - A_2 (1.875) = 0$$

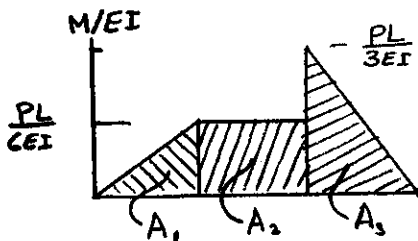
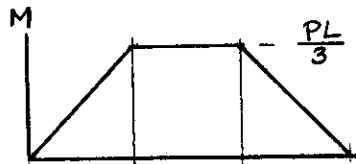
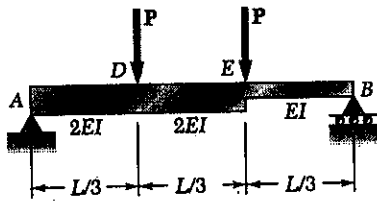
$$A_1 (2.5) + A_2 (2.5) + A_3 (1.6667) = 0$$

$$\frac{70.3125}{EI} - \frac{23.4375P}{EI} - \frac{5.208333P}{EI} = 0$$

$$P = 2.45 \text{ kips}$$

PROBLEM 9.169

9.169 For the beam and loading shown, determine the deflection (a) at point D, (b) at point E.



SOLUTION

Draw bending moment and  $\frac{M}{EI}$  diagrams.

$$A_1 = \frac{1}{2} \left( \frac{PL}{6EI} \right) \left( \frac{L}{3} \right) = \frac{1}{36} \frac{PL^2}{EI}$$

$$A_2 = \left( \frac{PL}{6EI} \right) \left( \frac{L}{3} \right) = \frac{1}{18} \frac{PL^2}{EI}$$

$$A_3 = \frac{1}{2} \left( \frac{PL}{3EI} \right) \left( \frac{L}{3} \right) = \frac{1}{18} \frac{PL^2}{EI}$$

Place reference tangent at A

$$\begin{aligned} t_{B/A} &= A_1 \left( \frac{2}{3}L + \frac{1}{3} \cdot \frac{1}{3}L \right) + A_2 \left( \frac{1}{2} \right) + A_3 \left( \frac{2}{3} \cdot \frac{L}{3} \right) \\ &= \frac{7}{324} \frac{PL^3}{EI} + \frac{1}{36} \frac{PL^3}{EI} + \frac{1}{81} \frac{PL^3}{EI} = \frac{5}{81} \frac{PL^3}{EI} \end{aligned}$$

$$\begin{aligned} t_{D/A} &= A_1 \left( \frac{1}{3} \cdot \frac{L}{3} \right) \\ &= \frac{1}{324} \frac{PL^3}{EI} \end{aligned}$$

$$\begin{aligned} t_{E/A} &= A_1 \left( \frac{1}{3} \cdot \frac{L}{3} + \frac{L}{3} \right) + A_2 \left( \frac{1}{2} \cdot \frac{L}{3} \right) \\ &= \frac{1}{81} \frac{PL^3}{EI} + \frac{1}{108} \frac{PL^3}{EI} = \frac{7}{324} \frac{PL^3}{EI} \end{aligned}$$

(a) Deflection at D

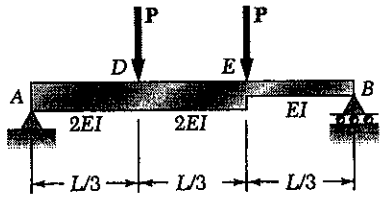
$$\begin{aligned} y_D &= t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{1}{324} \frac{PL^3}{EI} - \frac{1}{3} \cdot \frac{5}{81} \frac{PL^3}{EI} \\ &= -\frac{17}{972} \frac{PL^3}{EI} = -0.01749 \frac{PL^3}{EI} \end{aligned}$$

(b) Deflection at E

$$\begin{aligned} y_E &= t_{E/A} - \frac{x_E}{L} t_{B/A} = \frac{7}{324} \frac{PL^3}{EI} - \frac{2}{3} \cdot \frac{5}{81} \frac{PL^3}{EI} \\ &= -\frac{19}{972} \frac{PL^3}{EI} = -0.01955 \frac{PL^3}{EI} \end{aligned}$$

PROBLEM 9.170

9.170 For the beam and loading shown, determine the magnitude and location of the largest downward deflection.



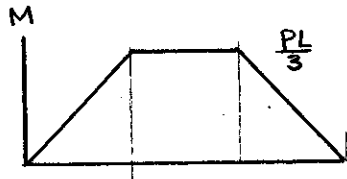
SOLUTION

Draw bending moment and  $\frac{M}{EI}$  diagrams.

$$A_1 = \frac{1}{2} \left( \frac{PL}{6EI} \right) \left( \frac{L}{3} \right) = \frac{1}{36} \frac{PL^2}{EI}$$

$$A_2 = \left( \frac{PL}{6EI} \right) \left( \frac{L}{3} \right) = \frac{1}{18} \frac{PL^2}{EI}$$

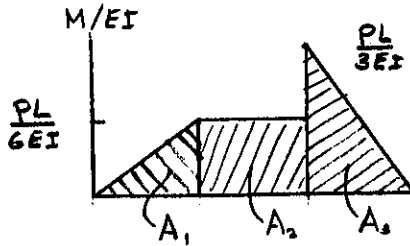
$$A_3 = \frac{1}{2} \left( \frac{PL}{3EI} \right) \left( \frac{L}{3} \right) = \frac{1}{18} \frac{PL^2}{EI}$$



Place reference tangent at A

$$t_{B/A} = A_1 \left( \frac{2}{3}L + \frac{1}{3} \cdot \frac{1}{3}L \right) + A_2 \left( \frac{L}{2} \right) + A_3 \left( \frac{2}{3} \cdot \frac{L}{3} \right)$$

$$= \frac{7}{324} \frac{PL^3}{EI} + \frac{1}{36} \frac{PL^3}{EI} + \frac{1}{81} \frac{PL^3}{EI} = \frac{5}{81} \frac{PL^3}{EI}$$



Slope at A

$$\theta_A = - \frac{t_{B/A}}{L} = - \frac{5}{81} \frac{PL^3}{EI}$$

Deflection is maximum at point K.

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1 + A_4 = 0$$

assuming that point K lies between D and E.

$$A_4 = \frac{1}{6} \left( \frac{PL}{EI} \right) U = \frac{1}{6} \frac{PLU}{EI}$$

$$- \frac{5}{81} \frac{PL^2}{EI} + \frac{1}{36} \frac{PL^2}{EI} + \frac{1}{6} \frac{PLU}{EI} = 0$$

$$U = 6 \left( \frac{5}{81} - \frac{1}{36} \right) L = \frac{11}{54} L$$

$$x_K = \frac{L}{3} + U = \left( \frac{1}{3} + \frac{11}{54} \right) L = \frac{29}{54} L$$

$$= 0.537 L$$

$$A_4 = \frac{11}{324} \frac{PL}{EI}$$

$$t_{K/A} = A_1 \left( \frac{1}{3} \cdot \frac{L}{3} + U \right) + A_4 \left( \frac{U}{2} \right) = \frac{17}{1944} \frac{PL^3}{EI} + \frac{121}{34992} \frac{PL^3}{EI} = \frac{427}{34992} \frac{PL^3}{EI}$$

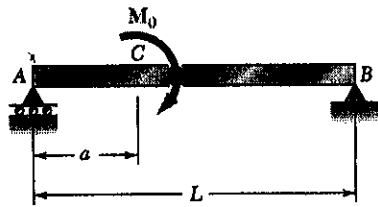
Maximum deflection

$$y_K = t_{K/A} - \frac{x_K}{L} t_{B/A} = \frac{427}{34992} \frac{PL^3}{EI} - \frac{29}{54} \cdot \frac{5}{81} \frac{PL^3}{EI} = - \frac{733}{34992} \frac{PL^3}{EI}$$

$$= -0.02095 \frac{PL^3}{EI}$$

PROBLEM 9.171

9.171 For the beam and loading shown, determine (a) the value of  $a$  for which the slope at end  $A$  is zero, (b) the corresponding deflection at point  $C$ .



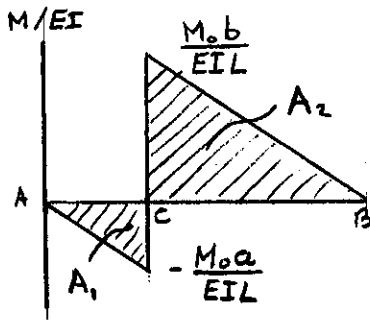
SOLUTION

Let  $b = L - a$

Place reference tangent at  $A$ .

$$y_e = y_A + L\theta_A + t_{B/A} = 0 + 0 + t_{B/A} = 0$$

Draw  $\frac{M}{EI}$  diagram.



$$A_1 = -\frac{1}{2} \frac{M_0 a}{EI} a = -\frac{1}{2} \frac{M_0 a^2}{EI}$$

$$A_2 = \frac{1}{2} \frac{M_0 b}{EI} b = \frac{1}{2} \frac{M_0 b^2}{EI}$$

$$t_{B/A} = A_1 \left( \frac{a}{3} + b \right) + A_2 \left( \frac{2}{3} b \right)$$

$$= -\frac{1}{6} \frac{M_0 a^3}{EI} + \frac{1}{2} \frac{M_0 b^2 b}{EI} + \frac{1}{3} \frac{M_0 b^3}{EI} = 0$$

Let  $u = \frac{a}{b}$

$$u^3 + 3u^2 - 2 = 0$$

Solving for  $u$ :  $u = 0.73205$

$$\frac{a}{b} = \frac{a}{L-a} = 0.73205$$

$$a = 0.73205(L-a)$$

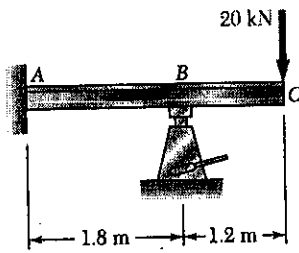
$$a = \frac{0.73205}{1.73205} L = 0.42265 L$$

$$A_1 = -\frac{1}{2} \frac{M_0 a^2}{EI} = -0.089316 \frac{M_0 L^2}{EI}$$

$$t_{C/A} = A_1 \left( \frac{1}{3} a \right) = -0.01258 \frac{M_0 L^2}{EI} = 0.01258 \frac{M_0 L^2}{EI} \downarrow$$

PROBLEM 9.172

9.172 A hydraulic jack may be used to raise point  $B$  of the cantilever beam  $ABC$ . Knowing that after the 20-kN load is applied, point  $C$  is to have the same elevation as point  $A$ , determine (a) how much  $B$  should be raised, (b) the reaction at  $B$  after point  $B$  has been raised and the 20-kN load has been applied. Use  $E = 200$  GPa.



SOLUTION

For W 130 x 23.8  $I_x = 8.80 \times 10^6 \text{ mm}^4$   
 $= 8.80 \times 10^{-6} \text{ m}^4$

$E = 200 \times 10^9 \text{ Pa}$

$EI = (200 \times 10^9)(8.80 \times 10^{-6}) = 1.760 \times 10^6 \text{ N}\cdot\text{m}^2$   
 $= 1760 \text{ kN}\cdot\text{m}^2$

Let  $R_B$  be the jack force in kN.

$A_1 = \frac{1}{2}(1.8 R_B)(1.8) = 1.62 R_B \text{ kN}\cdot\text{m}^2$

$A_2 = \frac{1}{2}(-60)(3) = -90 \text{ kN}\cdot\text{m}^2$

$EI t_{B/A} = (1.2 + 1.2) A_1 + (\frac{2}{3} - 3) A_2$   
 $= 3.888 R_B - 180 = 0 \text{ kN}\cdot\text{m}^2$

$R_B = \frac{180}{3.888} = 46.296 \text{ kN}$

$A_1 = 75 \text{ kN}\cdot\text{m}^2$

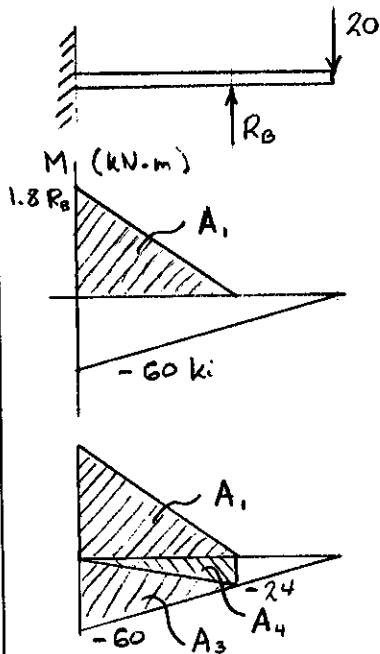
$A_3 = \frac{1}{2}(-60)(1.8) = -54 \text{ kN}\cdot\text{m}^2$

$A_4 = \frac{1}{2}(-24)(1.8) = -21.6 \text{ kN}\cdot\text{m}^2$

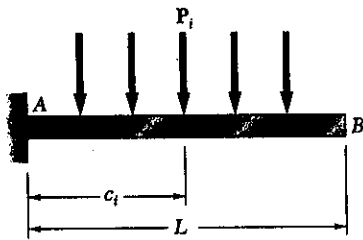
$EI t_{C/A} = 1.2 A_1 + 1.2 A_3 + 0.6 A_4$   
 $= 12.24 \text{ kN}\cdot\text{m}^2$

(a)  $y_B = t_{B/A} = \frac{EI t_{B/A}}{EI} = \frac{12.24}{1760} = 6.95 \times 10^{-3} \text{ m}$   
 $= 6.95 \text{ mm}$

(b)  $R_B = 46.3 \text{ kN}$



**PROBLEM 9.C1**



**9.C1** Several concentrated loads can be applied to the cantilever beam AB. Write a computer program to calculate the slope and deflection of beam AB from  $x = 0$  to  $x = L$ , using given increments  $\Delta x$ . Apply this program with increments  $\Delta x = 50$  mm to the beam and loading of Probs. 9.79 and 9.80.

**SOLUTION**

FOR EACH LOAD, ENTER

$$P_i, c_i$$

COMPUTE REACTION AT A

FOR  $i = 1$  TO NUMBER LOADS

$$R_A = R_A + P_i$$

$$M_A = M_A - P_i c_i$$

COMPUTE SLOPE AND DEFLECTION

USE METHOD OF INTEGRATION:  
STARTING WITH  $x=0$  AND UPDATING  
THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A:

$$\theta = (1/EI)(R_A x^2/2.0 + M_A x)$$

$$y = (1/EI)(R_A x^3/6.0 + M_A x^2/2.0)$$

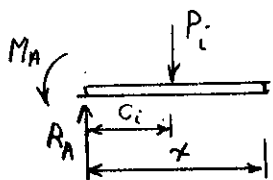
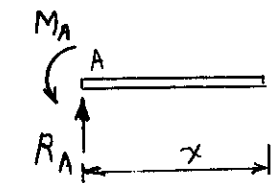
(2) DUE TO EACH LOAD WITH  $c_i < x$ :

$$\theta = -(1/EI)(P_i/2.0)(x - c_i)^2$$

$$y = -(1/EI)(P_i/6.0)(x - c_i)^3$$

AT  $x=0, y = \frac{dy}{dx} = 0$

∴ THE CONSTANTS OF  
INTEGRATION EQUAL ZERO



CONTINUED



PROBLEM 9.C1 CONTINUED

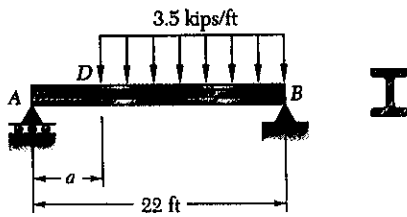
PROGRAM OUTPUT

Problem 9.79 and 9.80

At A: Force = 6.0 kN Couple = -6.0 kN·m

x m	Slope radians	Deflection m
.00	.000000	.000000
.05	-.000578	-.000015
.10	-.001126	-.000057
.15	-.001645	-.000127
.20	-.002134	-.000221
.25	-.002594	-.000340
.30	-.003024	-.000480
.35	-.003424	-.000642
.40	-.003794	-.000822
.45	-.004135	-.001021
.50	-.004447	-.001235
.55	-.004728	-.001465
.60	-.004980	-.001708
.65	-.005203	-.001962
.70	-.005395	-.002227
.75	-.005558	-.002501
.80	-.005699	-.002783
.85	-.005825	-.003071
.90	-.005936	-.003365
.95	-.006033	-.003664
1.00	-.006114	-.003968
1.05	-.006181	-.004275
1.10	-.006233	-.004586
1.15	-.006270	-.004898
1.20	-.006292	-.005213
1.25	-.006299	-.005527

**PROBLEM 9.C2**



**9.C2** The 22-ft beam  $AB$  consists of a  $W21 \times 62$  rolled-steel shape and supports a 3.5 kips/ft distributed load as shown. Write a computer program and use it to calculate for values of  $a$  from 0 to 22 ft, using 1-ft increments, (a) the slope and deflection at  $D$ , (b) the location and magnitude of the maximum deflection. Use  $E = 29 \times 10^6$  psi.

**SOLUTION**

ENTER LOAD  $w$ , LENGTH  $L$ ,  $a$   
COMPUTE REACTION AT A

$$R_A = w(L-a)^2 / (2.0L)$$

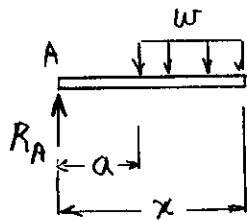
COMPUTE SLOPE AND DEFLECTION AT D

USING SINGULARITY FUNCTIONS:

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_AL^2$$

$$\theta = (1/EI)(R_A a^2 / 2.0 + C_1)$$

$$y = (1/EI)(R_A a^3 / 6.0 + C_1 a)$$



$$EI \frac{d^2 y}{dx^2} = R_A x - \frac{w}{2} \langle x-a \rangle^2$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{w}{6} \langle x-a \rangle^3 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{w}{24} \langle x-a \rangle^4 + C_1 x + C_2$$

FROM BOUNDARY CONDITIONS:

$$C_2 = 0$$

$$C_1 = -\frac{w}{24L}(L-a)^4 - \frac{1}{6}R_AL^2$$

COMPUTE LOCATION AND MAGNITUDE OF MAXIMUM DEFLECTION

MAXIMUM  $y$  AT  $\theta = 0$ :

$$0 = \frac{1}{2} R_A x^2 - \frac{w}{6} \langle x-a \rangle^3 + C_1$$

IF  $x_{max} \leq a$

$$\frac{1}{2} R_A x^2 + C_1 = 0$$

$$x_{max} = \sqrt{\frac{-2.0 C_1}{R_A}}$$

$$y_{max} = \frac{1}{6} R_A x_{max}^3 + C_1 x_{max}$$

ASSUME  $x < a$ :

$$x_{max} = (-2.0 C_1 / R_A)^{1/2}$$

IF  $x_{max} < a$ , THEN

$$y_{max} = (1/EI) \left( \frac{1}{6} R_A x_{max}^3 + C_1 x_{max} \right)$$

IF  $x_{max} > a$ , THEN

BEGIN WITH  $x = a$

$$\theta = (1/EI) \left( \frac{1}{2} R_A x - \frac{1}{6} \langle x-a \rangle^3 + C_1 \right)$$

INCREASE  $x$  BY SMALL AMOUNT UNTIL  $\theta$  IS APPROXIMATELY 0

$$y_{max} = (1/EI) \left( \frac{1}{6} R_A x^3 - \frac{w}{24} \langle x-a \rangle^4 + C_1 x \right)$$

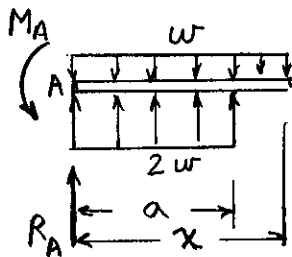
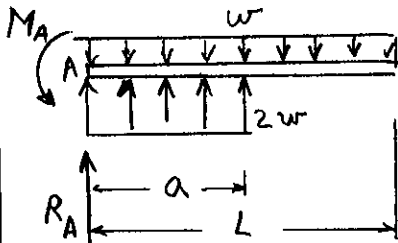
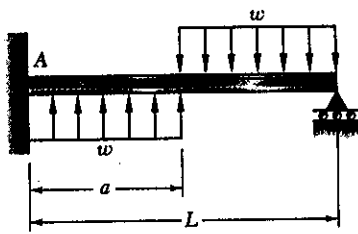
CONTINUED

PROBLEM 9.C2 CONTINUED

PROGRAM OUTPUT

a ft	theta D radians	yD in.	xm ft	ym in.
0.	-.00580	.000000	11.000	-.478290
1.	-.00569	-.068758	11.008	-.475922
2.	-.00539	-.133047	11.030	-.468860
3.	-.00494	-.189440	11.068	-.457231
4.	-.00439	-.235551	11.121	-.441245
5.	-.00378	-.269927	11.189	-.421192
6.	-.00314	-.291944	11.272	-.397443
7.	-.00250	-.301695	11.370	-.370441
8.	-.00188	-.299889	11.481	-.340699
9.	-.00131	-.287738	11.606	-.308795
10.	-.00080	-.266855	11.742	-.275364
11.	-.00036	-.239145	11.885	-.241090
12.	-.00001	-.206699	12.028	-.206700
13.	.00025	-.171684	12.159	-.172954
14.	.00043	-.136240	12.275	-.140603
15.	.00052	-.102374	12.376	-.110339
16.	.00054	-.071846	12.463	-.082792
17.	.00049	-.046069	12.537	-.058515
18.	.00039	-.026001	12.596	-.037987
19.	.00027	-.012036	12.643	-.021604
20.	.00014	-.003896	12.675	-.009677
21.	.00004	-.000530	12.695	-.002431
22.	.00000	.000000	12.702	.000000

**PROBLEM 9.C3**



AT  $x=0, y = \frac{dy}{dx} = 0$

∴ THE CONSTANTS OF INTEGRATION ARE ZERO

**9.C3** The cantilever beam  $AB$  carries the distributed loads shown. Write a computer program to calculate the slope and deflection of beam  $AB$  from  $x = 0$  to  $x = L$  using given increments  $\Delta x$ . Apply this program with increments  $\Delta x = 100$  mm, assuming that  $L = 2.4$  m,  $w = 36$  kN/m, and (a)  $a = 0.6$  m, (b)  $a = 1.2$  m, (c)  $a = 1.8$  m. Use  $E = 200$  GPa.

**SOLUTION**

ENTER  $w, a, L$

COMPUTE REACTION AT A

$$R_A = wL - 2.0wa$$

$$M_A = \frac{1}{2} wL^2 - \frac{1}{2} wa^2$$

COMPUTE SLOPE AND DEFLECTION

USE EQUATION OF ELASTIC CURVE

STARTING WITH  $x=0$  AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTIONS AT A

$$\theta = (1/EI) (\frac{1}{2} R_A x^2 + M_A x)$$

$$y = (1/EI) (\frac{1}{6} R_A x^3 + \frac{1}{2} M_A x^2)$$

(2) DUE TO LOAD  $w$

$$\theta = -(1/EI) (\frac{1}{6} w x^3)$$

$$y = -(1/EI) (\frac{1}{24} w x^4)$$

(3) DUE TO LOAD  $2w$

IF  $x \leq a$

$$\theta = (1/EI) (\frac{1}{3} w x^3)$$

$$y = (1/EI) (\frac{1}{12} w x^4)$$

IF  $x > a$

$$\theta = (1/EI) (\frac{1}{3} w x^3 - \frac{1}{3} w (x-a)^3)$$

$$y = (1/EI) (\frac{1}{12} w x^4 - \frac{1}{12} w (x-a)^4)$$

CONTINUED

**PROBLEM 9.C3 CONTINUED**

PROGRAM OUTPUT

Problem 9.C3 (a) a = 0.6 m

At A: Force = 43.2 kN Couple = -90.7 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000905	-.000046
.20	-.001762	-.000179
.30	-.002567	-.000396
.40	-.003318	-.000691
.50	-.004009	-.001058
.60	-.004638	-.001491
.70	-.005202	-.001983
.80	-.005703	-.002529
.90	-.006145	-.003122
1.00	-.006533	-.003756
1.10	-.006868	-.004427
1.20	-.007156	-.005128
1.30	-.007399	-.005856
1.40	-.007602	-.006607
1.50	-.007769	-.007376
1.60	-.007902	-.008160
1.70	-.008006	-.008955
1.80	-.008083	-.009760
1.90	-.008139	-.010571
2.00	-.008177	-.011387
2.10	-.008199	-.012206
2.20	-.008211	-.013027
2.30	-.008215	-.013848
2.40	-.008216	-.014669

Problem 9.C3 (b) a = 1.2 m

At A: Force = 0.0 kN Couple = -51.8 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	-.000529	-.000026
.20	-.001055	-.000106
.30	-.001574	-.000237
.40	-.002081	-.000420
.50	-.002574	-.000653
.60	-.003048	-.000934
.70	-.003500	-.001262
.80	-.003926	-.001633
.90	-.004323	-.002046
1.00	-.004687	-.002497
1.10	-.005014	-.002982
1.20	-.005301	-.003498
1.30	-.005544	-.004041
1.40	-.005747	-.004606
1.50	-.005913	-.005189
1.60	-.006047	-.005787
1.70	-.006150	-.006398
1.80	-.006228	-.007017
1.90	-.006284	-.007642
2.00	-.006321	-.008273
2.10	-.006344	-.008906
2.20	-.006356	-.009541
2.30	-.006360	-.010177
2.40	-.006361	-.010813

**CONTINUED**

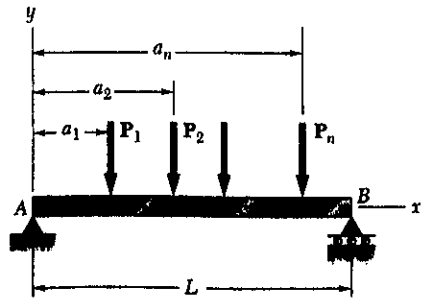
**PROBLEM 9.C3 PROGRAM OUTPUTS CONTINUED**

Problem 9.C3 (c) a = 1.8 m

At A: Force = -43.2 kN Couple = 13.0 kN·m

x m	slope radians	deflection m
.00	.000000	.000000
.10	.000111	.000006
.20	.000182	.000021
.30	.000215	.000041
.40	.000216	.000063
.50	.000187	.000083
.60	.000133	.000099
.70	.000056	.000109
.80	-.000039	.000110
.90	-.000149	.000101
1.00	-.000270	.000080
1.10	-.000398	.000046
1.20	-.000530	.000000
1.30	-.000662	-.000060
1.40	-.000790	-.000132
1.50	-.000911	-.000217
1.60	-.001021	-.000314
1.70	-.001116	-.000421
1.80	-.001193	-.000537
1.90	-.001248	-.000659
2.00	-.001286	-.000786
2.10	-.001309	-.000916
2.20	-.001320	-.001047
2.30	-.001325	-.001179
2.40	-.001325	-.001312

**PROBLEM 9.C4**



**9.C4** The simply supported beam  $AB$  is of constant flexural rigidity  $EI$  and carries several concentrated loads as shown. Using the *Method of Integration*, write a computer program to calculate the slope and deflection at points along the beam from  $x = 0$  to  $x = L$  using given increments  $\Delta x$ . Apply this program to the beam and loading of (a) Prob. 9.14 with  $\Delta x = 0.25$  m, (b) Prob. 9.15 with  $\Delta x = 0.05$  m, (c) Prob. 9.132 with  $\Delta x = 0.25$  m.

**SOLUTION**

FOR EACH LOAD, ENTER  $P_i, a_i$

COMPUTE REACTION AT A

FOR  $i = 1$  TO NUMBER LOADS:

$$M_A = M_A + P_i a_i$$

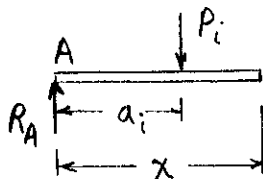
$$\text{LOAD} = \text{LOAD} + P_i$$

THEN:

$$R_B = M_A / L$$

$$R_A = \text{LOAD} - R_B$$

FOR LOAD  $P_i$ :



FOR  $x < a_i$

$$EI \frac{d^2 y}{dx^2} = R_A x$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 + C_1$$

$$EI y = \frac{1}{6} R_A x^3 + C_1 x + C_2$$

FOR  $x > a_i$

$$EI \frac{d^2 y}{dx^2} = R_A x - P_i (x - a_i)$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_A x^2 - \frac{1}{2} P_i (x - a_i)^2 + C_3$$

$$EI y = \frac{1}{6} R_A x^3 - \frac{1}{6} (x - a_i)^3 + C_3 x + C_4$$

FROM BOUNDARY CONDITIONS

$$C_2 = C_4 = 0$$

$$C_1 = C_3 = \frac{P_i}{6L} (L - a_i)^3 - \frac{1}{6} R_A L^2$$

NOTE:  $R_A$  FOR LOAD  $P_i$

COMPUTE SLOPE AND DEFLECTION

STARTING WITH  $x = 0$  AND UPDATING THROUGH INCREMENTS, SUPERPOSE:

(1) DUE TO REACTION AT A

$$\theta = (1/EI) (\frac{1}{2} R_A x^2)$$

$$y = (1/EI) (\frac{1}{6} R_A x^3)$$

(2) DUE TO LOADS - CONSTANT PART

$$\text{CONST}_1 = -\frac{1}{6} R_A L^2$$

FOR 1 TO NUMBER LOADS

$$\text{CONST}_2 = \frac{1}{6L} P_i (L - a_i)^3 + \text{CONST}_2$$

THEN, TOTAL CONTRIBUTION FOR CONSTANT

$$\text{CONST} = (1/EI) (\text{CONST}_1 + \text{CONST}_2)$$

(3) DUE TO LOADS - REMAINING PART

IF  $x \leq a_i$

$$\theta = (1/EI) (\frac{1}{2.0} R_A x^2)$$

$$y = (1/EI) (\frac{1}{6.0} R_A x^3)$$

IF  $x > a_i$

$$\theta = (1/EI) (\frac{1}{2.0} R_A x^2 - \frac{1}{2.0} P_i (x - a_i)^2)$$

$$y = (1/EI) (\frac{1}{6.0} R_A x^3 - \frac{1}{6.0} P_i (x - a_i)^3)$$

**CONTINUED**

**PROBLEM 9.C4 CONTINUED**

PROGRAM OUTPUT

Problem 9.14

x m	theta rad*10**3	y mm
.000	-6.058	.000
.250	-5.831	-1.496
.500	-5.150	-2.878
.750	-4.014	-4.033
1.000	-2.423	-4.847
1.250	-.719	-5.235
1.500	.757	-5.225
1.750	2.007	-4.875
2.000	3.029	-4.241
2.250	3.824	-3.379
2.500	4.392	-2.348
2.750	4.733	-1.202
3.000	4.847	.000

Problem 9.15

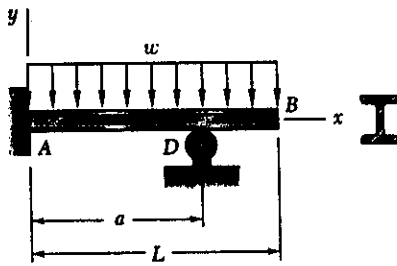
x m	theta rad*10**3	y mm
.000	-2.490	.000
.050	-2.485	-.124
.100	-2.471	-.248
.150	-2.448	-.371
.200	-2.416	-.493
.250	-2.375	-.613
.300	-2.325	-.730
.350	-2.265	-.845
.400	-2.197	-.957
.450	-2.119	-1.065
.500	-2.032	-1.168
.550	-1.936	-1.268
.600	-1.831	-1.362
.650	-1.716	-1.451
.700	-1.593	-1.533
.750	-1.460	-1.610
.800	-1.318	-1.679
.850	-1.172	-1.741
.900	-1.025	-1.796
.950	-.879	-1.844
1.000	-.732	-1.884
1.050	-.586	-1.917
1.100	-.439	-1.943
1.150	-.293	-1.961
1.200	-.146	-1.972
1.250	.000	-1.976
1.300	.146	-1.972
1.350	.293	-1.961
1.400	.439	-1.943
1.450	.586	-1.917
1.500	.732	-1.884
1.550	.879	-1.844
1.600	1.025	-1.796
1.650	1.172	-1.741
1.700	1.318	-1.679
1.750	1.460	-1.610
1.800	1.593	-1.533
1.850	1.716	-1.451
1.900	1.831	-1.362
1.950	1.936	-1.268
2.000	2.032	-1.168
2.050	2.119	-1.065
2.100	2.197	-.957
2.150	2.265	-.845
2.200	2.325	-.730
2.250	2.375	-.613
2.300	2.416	-.493
2.350	2.448	-.371
2.400	2.471	-.248
2.450	2.485	-.124
2.500	2.490	.000

Problem 9.132

x m	theta rad*10**3	y mm
.000	-8.703	.000
.250	-8.615	-2.168
.500	-8.351	-4.293
.750	-7.911	-6.329
1.000	-7.296	-8.234
1.250	-6.505	-9.962
1.500	-5.538	-11.472
1.750	-4.483	-12.724
2.000	-3.428	-13.713
2.250	-2.373	-14.438
2.500	-1.319	-14.900
2.750	-.264	-15.098
3.000	.791	-15.032
3.250	1.802	-14.706
3.500	2.725	-14.138
3.750	3.560	-13.350
4.000	4.307	-12.365
4.250	4.967	-11.204
4.500	5.538	-9.889
4.750	6.021	-8.442
5.000	6.417	-6.886
5.250	6.725	-5.241
5.500	6.944	-3.531
5.750	7.076	-1.776
6.000	7.120	.000



**PROBLEM 9.C5**



9.C5 The supports of beam AB consist of a fixed support at end A and a roller located at point D. Write a computer program to calculate the slope and deflection at the free end of the beam for values of a from 0 to L using given increments  $\Delta a$ . Apply this program to calculate the slope and deflection at point B for each of the following cases:

	L	$\Delta a$	w	E	Shape
(a)	12 ft	0.5 ft	1.6 kips/ft	$29 \times 10^6$ psi	W16 $\times$ 57
(b)	3 m	0.2 m	18 kN/m	200 GPa	W460 $\times$ 113

**SOLUTION**

BEAM IS INDETERMINATE

USE APPENDIX D AND SUPERPOSITION

DETERMINE REACTION AT D

DUE TO DISTRIBUTED LOAD

$$(y_D)_w = -\frac{w}{24EI} (a^4 - 4La^3 + 6L^2a^2)$$

DUE TO REDUNDANT LOAD:

$$(y_D)_R = \frac{R_D L^3}{3EI}$$

REDUNDANT REACTION:

$$\text{SINCE } (y_D)_w + (y_D)_R = 0:$$

$$R_D = \frac{3EI}{L^3} (y_D)_w$$

COMPUTE SLOPE AND DEFLECTION AT B

SUPERPOSE:

DUE TO DISTRIBUTED LOAD:

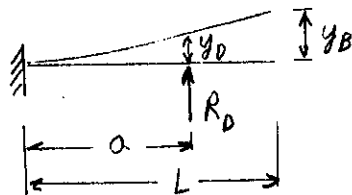
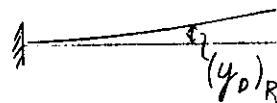
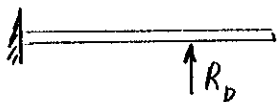
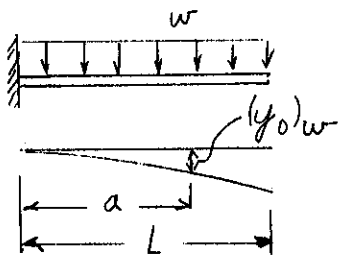
$$\theta_B = -\frac{wL^3}{6EI}$$

$$y_B = -\frac{wL^4}{8EI}$$

DUE TO  $R_D$ :

$$\theta_B = \frac{Pa^2}{2EI}$$

$$y_B = \frac{Pa^3}{3EI} + (L-a) \frac{Pa^2}{2EI}$$



$$\theta_B = \theta_D$$

$$y_B = y_D + (L-a)\theta_D$$

**CONTINUED**

## PROBLEM 9.C5 CONTINUED

PROGRAM OUTPUT

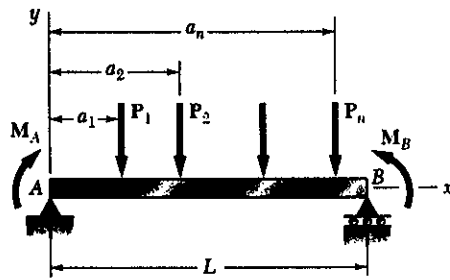
Problem 9.C5 (a)

a ft	theta B rad*10 <sup>-3</sup>	y at B in.
.0	-3.019	-.3260
.5	-2.743	-.2869
1.0	-2.483	-.2511
1.5	-2.238	-.2183
2.0	-2.007	-.1885
2.5	-1.790	-.1614
3.0	-1.586	-.1369
3.5	-1.395	-.1149
4.0	-1.216	-.0953
4.5	-1.049	-.0778
5.0	-.893	-.0624
5.5	-.748	-.0490
6.0	-.613	-.0374
6.5	-.488	-.0274
7.0	-.373	-.0191
7.5	-.266	-.0122
8.0	-.168	-.0067
8.5	-.077	-.0025
9.0	.006	.0006
9.5	.082	.0027
10.0	.152	.0037
10.5	.216	.0039
11.0	.274	.0033
11.5	.328	.0020
12.0	.377	.0000

Problem 9.C5 (b)

a m	theta B rad*10 <sup>-3</sup>	y at B mm
.0	-.728	-1.6389
.2	-.624	-1.3324
.4	-.529	-1.0663
.6	-.442	-.8374
.8	-.364	-.6426
1.0	-.293	-.4789
1.2	-.230	-.3435
1.4	-.174	-.2338
1.6	-.124	-.1472
1.8	-.079	-.0813
2.0	-.040	-.0337
2.2	-.006	-.0024
2.4	.023	.0149
2.6	.049	.0198
2.8	.072	.0143

**PROBLEM 9.C6**



9.C6 For the beam and loading shown, use the *Moment-Area Method* to write a computer program to calculate the slope and deflection at points along the beam from  $x = 0$  to  $x = L$  using given increments  $\Delta x$ . Apply this program to calculate the slope and deflection at each concentrated load for the beam of (a) Prob. 9.76 with  $\Delta x = 0.5$  m, (b) Prob. 9.116 with  $\Delta x = 3$  in., (c) Prob. 9.119 with  $\Delta x = 0.5$  ft.

**SOLUTION**

ENTER  $M_A$  AND  $M_B$   
 FOR EACH LOAD ENTER  $P_i$  AND  $a_i$   
DETERMINE REACTION AT A

DUE TO MOMENTS AT ENDS:

$$(R_A)_1 = -(M_A - M_B) / L$$

DUE TO LOADS  $P_i$ :

FOR  $i = 1$  TO NUMBER OF LOADS

$$R_B = R_B + P_i a_i / L$$

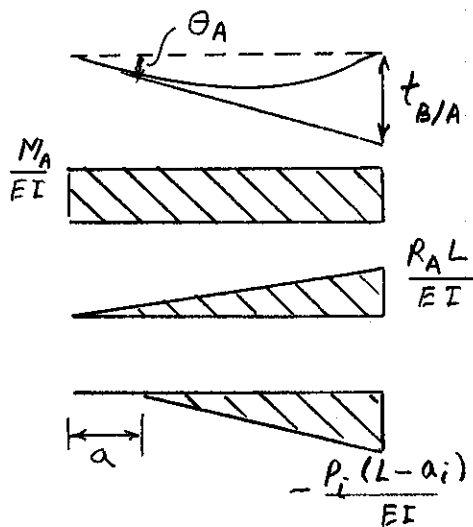
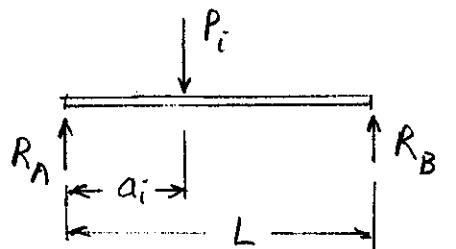
$$\text{LOAD} = \text{LOAD} + P_i$$

$$(R_A)_2 = \text{LOAD} - R_B$$

$$R_A = (R_A)_1 + (R_A)_2$$

DETERMINE SLOPE AT A

USE SECOND MOMENT-AREA THEOREM TO GET TANGENTIAL DEVIATION AT B



DUE TO  $M_A$ :

$$t_{B/A} = M_A L^2 / (2.0 EI)$$

DUE TO  $R_A$ :

$$t_{B/A} = R_A L^3 / (6.0 EI)$$

DUE TO LOADS  $P_i$ :

FOR  $i = 1$  TO NUMBER OF LOADS

$$t_{B/A} = -P_i (L - a_i)^3 / (6.0 EI)$$

SUM  $t_{B/A}$ :

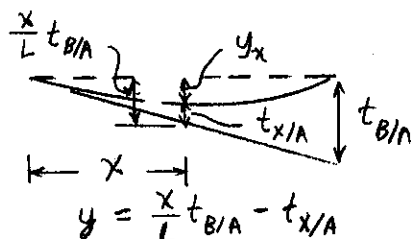
$$\theta_A = -t_{B/A} / L$$

DETERMINE SLOPE AND DEFLECTIONS

FOR  $x = 0$  TO  $L$ , SUPERPOSE:

DUE TO  $M_A$  AND  $R_A$ :

$$\theta_x = \theta_A + (M_A x + R_A x^2 / 2.0) / EI$$



CONTINUED

PROBLEM 9.C6 CONTINUED

$$y_x = \frac{x}{L} t_{B/A} - M_A x^2 / (2.0 EI) - R_A x^3 / (6.0 EI)$$

DUE TO LOADS  $P_i$  :

DO FOR ALL LOADS WITH  $a_i < x$

$$\theta_x = P_i (x - a_i)^2 / (2.0 EI)$$

$$y_x = P_i (x - a_i)^3 / (6.0 EI)$$

PROGRAM OUTPUT

Problem 9.76

x m	theta rad*1000	y at x mm
.000	-.600962	.000000
.500	-1.602564	.574252
1.000	-2.043269	1.509081
1.500	-1.923077	2.524039
2.000	-1.241987	3.338675
2.500	.000000	3.672543
3.000	1.241987	3.338676
3.500	1.923077	2.524039
4.000	2.043269	1.509082
4.500	1.602564	.574253
5.000	.600962	.000000

Problem 9.119

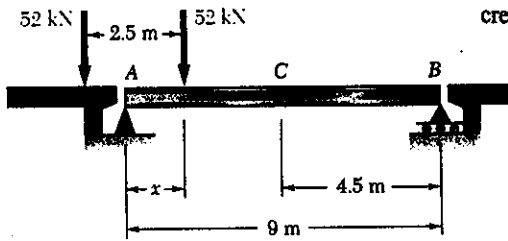
x ft	theta rad*1000	y at x in.
.000	-2.118621	.000000
.500	-2.222069	.013051
1.000	-2.267586	.026549
1.500	-2.255172	.040146
2.000	-2.184828	.053495
2.500	-2.056552	.066248
3.000	-1.870345	.078058
3.500	-1.626207	.088577
4.000	-1.324138	.097457
4.500	-.993103	.104408
5.000	-.662069	.109374
5.500	-.331034	.112353
6.000	.000000	.113346
6.500	.331034	.112353
7.000	.662069	.109374
7.500	.993103	.104408
8.000	1.324138	.097457
8.500	1.626207	.088577
9.000	1.870345	.078058
9.500	2.056552	.066248
10.000	2.184828	.053495
10.500	2.255172	.040146
11.000	2.267586	.026549
11.500	2.222069	.013051
12.000	2.118621	.000000

Problem 9.116

x ft	theta rad*1000	y at x in.
.000	-8.937931	.000000
.250	-8.813793	.026690
.500	-8.441380	.052634
.750	-7.820690	.077090
1.000	-6.951724	.099310
1.250	-5.834483	.118552
1.500	-4.468966	.134069
1.750	-2.979310	.145241
2.000	-1.489655	.151945
2.250	.000000	.154179
2.500	1.489655	.151945
2.750	2.979310	.145241
3.000	4.468966	.134069
3.250	5.834483	.118552
3.500	6.951724	.099310
3.750	7.820690	.077090
4.000	8.441380	.052634
4.250	8.813793	.026690
4.500	8.937931	.000000

PROBLEM 9.C7

9.C7 Two 52-kN loads are maintained 2.5 m apart as they are moved slowly across beam AB. Write a computer program to calculate the deflection at the midpoint C of the beam for values of x from 0 to 9 m, using 0.5-m increments. Use  $E = 200 \text{ GPa}$ .



SOLUTION

ENTER LOAD P, BEAM LENGTH L AND SPACE BETWEEN LOADS D

WILL SOLVE WITH MOMENT-AREA METHOD

DETERMINE DEFLECTION AT C

FOR  $x = 0$  TO L

IF  $0 \leq x \leq D$ :

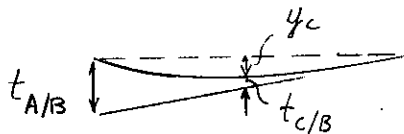
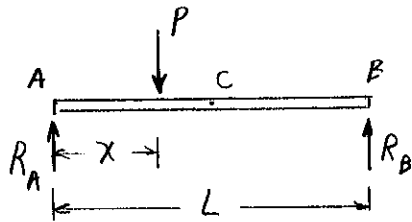
HAVE ONE LOAD TO LEFT OF C

$$R_B = Px/L$$

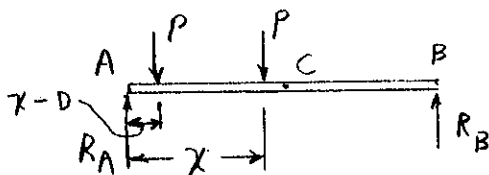
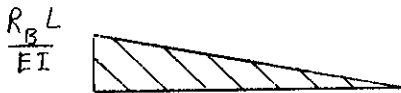
$$t_{A/B} = (R_B L^3 - Px^3)/(6.0EI)$$

$$t_{C/B} = R_B L^3/(48.0EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$



IF  $D < x \leq L/2$

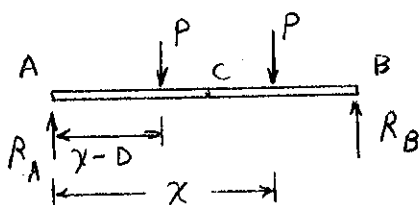
HAVE TWO LOADS TO LEFT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

$$t_{C/B} = R_B L^3/(48.0EI)$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

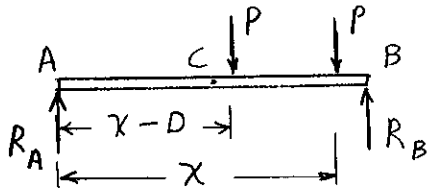


IF  $L/2 < x \leq (L/2 + D)$

HAVE ONE LOAD TO LEFT OF C AND ONE TO RIGHT OF C OR AT C

CONTINUED

PROBLEM 9.C7 CONTINUED



$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

$$t_{C/B} = (R_B L^3/48.0 - P(x - \frac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

IF  $(L/2 + D) < x < L$

HAVE BOTH LOADS TO RIGHT OF C

$$R_B = Px/L + P(x-D)/L$$

$$t_{A/B} = (R_B L^3 - Px^3 - P(x-D)^3)/(6.0EI)$$

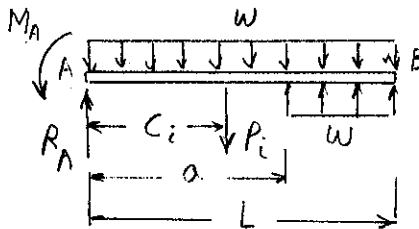
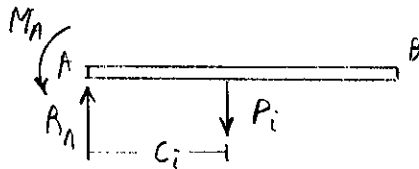
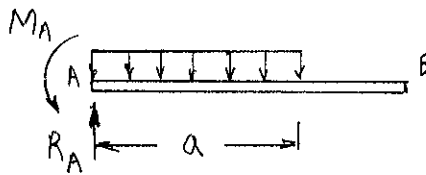
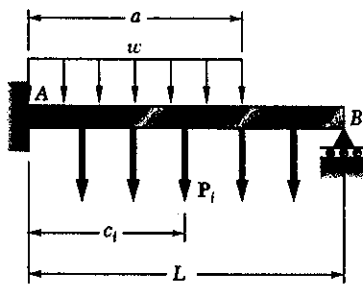
$$t_{C/B} = (R_B L^3/48.0 - P(x - \frac{L}{2})^3/6.0 - P(x-D - \frac{L}{2})^3/6.0)/EI$$

$$y_C = \frac{1}{2} t_{A/B} - t_{C/B}$$

PROGRAM OUTPUT

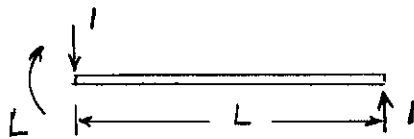
x m	RB kN	ThetaB rad	YC mm
.0	.000	.00000	.00000
.5	2.889	.00315	1.17881
1.0	5.778	.00624	2.32839
1.5	8.667	.00921	3.41951
2.0	11.556	.01200	4.42296
2.5	14.444	.01456	5.30950
3.0	20.222	.01998	7.22872
3.5	26.000	.02499	8.94335
4.0	31.778	.02947	10.39493
4.5	37.556	.03331	11.52503
5.0	43.333	.03639	12.28492
5.5	49.111	.03859	12.66487
6.0	54.889	.03980	12.66487
6.5	60.667	.03989	12.28492
7.0	66.444	.03876	11.52503
7.5	72.222	.03629	10.39493
8.0	78.000	.03235	8.94335
8.5	83.778	.02684	7.22872
9.0	89.556	.01963	5.30950

PROBLEM 9.C8



AT  $x=0, y = \frac{dy}{dx} = 0$

∴ THE CONSTANTS OF INTEGRATION ARE ZERO



$$EI \frac{d^2y}{dx^2} = -x + L$$

$$EI \frac{dy}{dx} = -\frac{1}{2}x^2 + Lx + C_1$$

$$EI y = -\frac{1}{6}x^3 + \frac{1}{2}Lx^2 + C_1x + C_2$$

BOUNDARY CONDITIONS GIVE  $C_1 = C_2 = 0$

9.C8 A uniformly distributed load  $w$  and several concentrated loads  $P_i$  may be applied to the cantilever beam  $AB$ . Write a computer program to determine the reaction at the roller support and apply this program to the beam and loading of (a) Prob. 9.57a, (b) Prob. 9.58a.

SOLUTION

THE BEAM IS INDETERMINATE

USE EQUATION OF ELASTIC CURVE

ENTER  $w$  AND FOR EACH LOAD  $P_i$  AND  $c_i$  COMPUTE DISPLACEMENT AT B DUE TO LOADS

REACTION AT A:

DUE TO  $w$

$$R_A = wa$$

$$M_A = \frac{1}{2}wa^2$$

FOR  $i = 1$  TO NUMBER LOADS  $P_i$

$$R_A = R_A - P_i$$

$$M_A = M_A - P_i c_i$$

FOR DISPLACEMENT AT B, SUPERPOSE:

DUE TO REACTION AT A

$$EI y_B = \frac{1}{6}R_A L^3 + \frac{1}{2}M_A L^2$$

DUE TO DISTRIBUTED LOADS

$$EI y_B = \frac{1}{24}(-wL^4 + w(L-a)^4)$$

DUE TO  $P_i$

FOR  $i = 1$  TO NUMBER LOADS

$$EI y_B = \frac{1}{6}P_i (L - c_i)^3$$

COMPUTE DISPLACEMENT AT B DUE TO UNIT  $R_B$

$$EI(y_B)_{UNIT} = \frac{1}{3}L^3$$

COMPUTE REACTION AT B

$$\text{FROM } EI y_B + R_B EI (y_B)_{UNIT} = 0$$

$$R_B = -y_B / (y_B)_{UNIT}$$

CONTINUED

**PROBLEM 9.C8 CONTINUED**

PROGRAM OUTPUT

Problem 9.57 (a)

Reaction at Roller Support = 41.2500 kN

Problem 9.58 (a)

Reaction at Roller Support = 11.5356 kN

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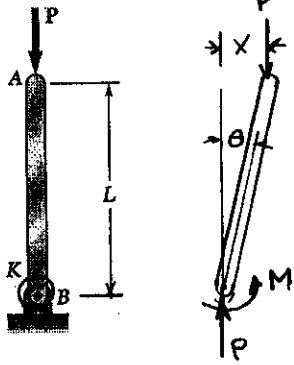


# CHAPTER 10

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**PROBLEM 10.1**

10.1 Knowing that the torsional spring at  $B$  is of constant  $K$  and that the bar  $AB$  is rigid, determine the critical load  $P_{cr}$ .



**SOLUTION**

Let  $\theta$  be the angle change of bar  $AB$ .

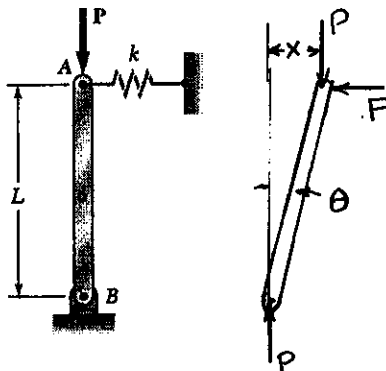
$$M = K\theta, \quad x = L \sin \theta \approx L\theta$$

$$\sum M_B = 0 \quad M - Px = 0 \quad K\theta - PL\theta = 0$$

$$(K - PL)\theta = 0 \quad P_{cr} = K/L$$

**PROBLEM 10.2**

10.2 Knowing that the spring at  $A$  is of constant  $k$  and that the bar  $AB$  is rigid, determine the critical load  $P_{cr}$ .



**SOLUTION**

Let  $\theta$  be the angle change of bar  $AB$ .

$$F = kx = kL \sin \theta$$

$$\sum M_B = 0 \quad Fx \cos \theta - Px = 0$$

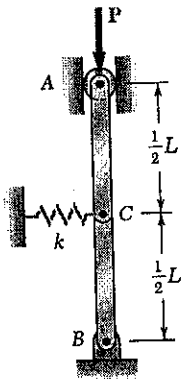
$$kL^2 \sin \theta \cos \theta - PL \sin \theta = 0$$

Using  $\sin \theta \approx \theta$  and  $\cos \theta \approx 1$  
$$kL^2 \theta - PL \theta = 0$$

$$(kL^2 - PL)\theta = 0 \quad P_{cr} = kL$$

**PROBLEM 10.3**

10.3 Two rigid bars  $AC$  and  $BC$  are connected as shown to a spring of constant  $k$ . Knowing that the spring can act in either tension or compression, determine the critical load  $P_{cr}$  for the system.



**SOLUTION**

Let  $x$  be the lateral deflection of point  $C$

$$x = \frac{1}{2}L \sin \theta \quad F_c = kx = \frac{1}{2}kL \sin \theta$$

$$\text{Joint C: } +\uparrow F_y = 0 \quad F_{AC} \cos \theta - F_{CB} \cos \theta = 0$$

$$F_{AC} = F_{CB}$$

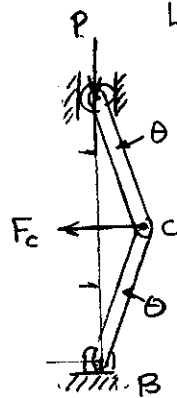
$$+\rightarrow \Sigma F_x = 0 \quad F_{AB} \sin \theta + F_{CB} \sin \theta - F_c = 0$$

$$-2F_{AB} \sin \theta - \frac{1}{2}kL \sin \theta = 0$$

$$-(F_{AB} + \frac{1}{4}kL) \sin \theta = 0 \quad F_{AB} = -\frac{1}{4}kL$$

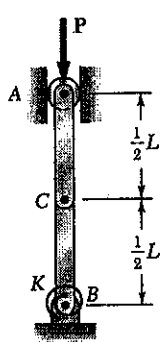
$$\text{Joint A: } \Sigma F_y = 0 \quad -P + F_{AB} \cos \theta = 0 \quad P = -F_{AB} \cos \theta = \frac{1}{4}kL \cos \theta$$

$$\text{with } \theta \rightarrow 0 \quad P_{cr} = \frac{1}{4}kL$$



**PROBLEM 10.4**

10.4 Two rigid bars  $AC$  and  $BC$  are connected by a pin at  $C$  as shown. Knowing that the torsional spring at  $B$  is of constant  $K$ , determine the critical load  $P_{cr}$  for the system.



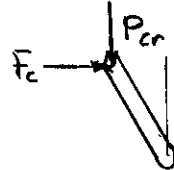
**SOLUTION**

Let  $\theta$  be the angle change of each bar.

$$M_B = K\theta$$

$$+\circlearrowleft M_B = 0 \quad K\theta - F_A L = 0 \quad F_A = \frac{K\theta}{L}$$

$$\text{Bar AC } \circlearrowleft \Sigma M_C = 0$$

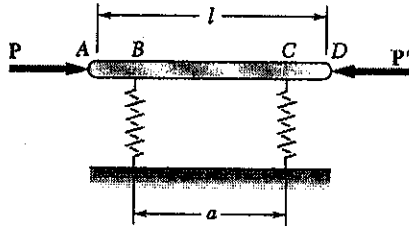


$$P_{cr} \frac{1}{2}L \theta - \frac{1}{2}L F_A = 0$$

$$P_{cr} = \frac{F_A}{\theta} = \frac{K}{L}$$

**PROBLEM 10.5**

10.5 The rigid bar  $AD$  is attached to two springs of constant  $k$  and is in equilibrium in the position shown. Knowing that the equal and opposite loads  $P$  and  $P'$  remain horizontal, determine the magnitude  $P_{cr}$  of the critical load for the system.



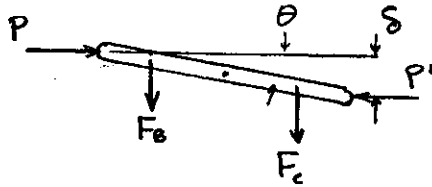
**SOLUTION**

Let  $y_B$  and  $y_C$  be the deflections of points B and C, positive upward.

Then  $F_B = -ky_B$      $F_C = -ky_C$

$\uparrow \Sigma F_y = 0$      $F_B + F_C = 0$      $F_C = -F_B$

$y_C = -y_B$      $F_B$  and  $F_C$  form a couple  $\odot$



Let  $\theta$  be the angle change:  $y_B = -y_C = \frac{1}{2}a \sin \theta$ ,     $\delta = l \sin \theta$

$P$  and  $P'$  form a couple  $\odot$  of amount  $PS$

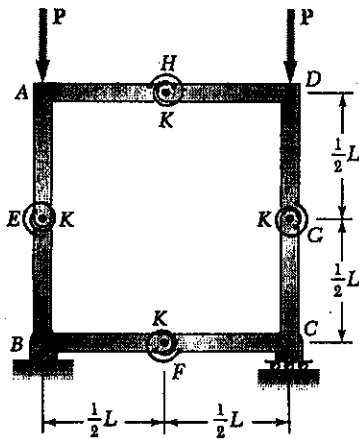
$\odot \Sigma M = 0$ ;  $k(\frac{1}{2}a \sin \theta)a \cos \theta - Pl \sin \theta = 0$      $P = \frac{ka^2}{2l} \cos \theta$

Let  $\theta \rightarrow 0$

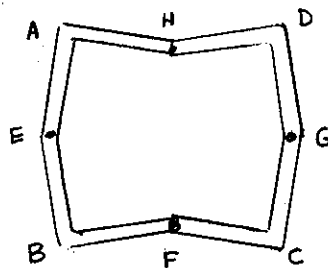
$P_{cr} = \frac{ka^2}{2l}$

**PROBLEM 10.6**

10.6 A frame consists of four L-shaped members connected by four torsional springs, each of constant  $K$ . Knowing that equal loads  $P$  are applied at points  $A$  and  $D$  as shown, determine the critical value  $P_{cr}$  of the loads applied to the frame.



**SOLUTION**



Let  $\theta$  be the rotation of each L-shaped member.

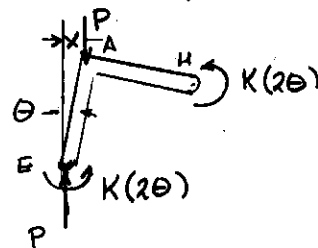
Angle change across each torsional spring is  $2\theta$

$x = \frac{1}{2}L \sin \theta \approx \frac{1}{2}L\theta$

$\Sigma M_E = 0$

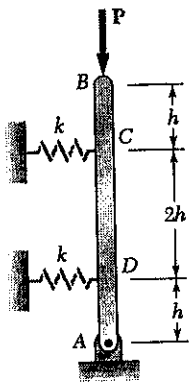
$K(2\theta) + K(2\theta) - Px = 0$

$P_{cr} = \frac{4K\theta}{x} = \frac{8K}{L}$

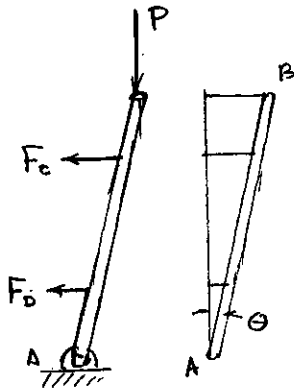


**PROBLEM 10.7**

10.7 The rigid rod  $AB$  is attached to a hinge at  $A$  and to two springs, each of constant  $k = 2.0$  kip/in., that can act in either tension or compression. Knowing that  $h = 2.0$  ft, determine the critical load.



**SOLUTION**



Let  $\theta$  be the small rotation angle

$$x_D \approx h\theta, \quad x_C \approx 3h\theta, \quad x_B \approx 4h\theta$$

$$F_C = kx_C \approx 3kh\theta$$

$$F_D = kx_D \approx kh\theta$$

$$\circlearrowleft \sum M_A = 0 \quad hF_D + 3hF_C - Px_B = 0$$

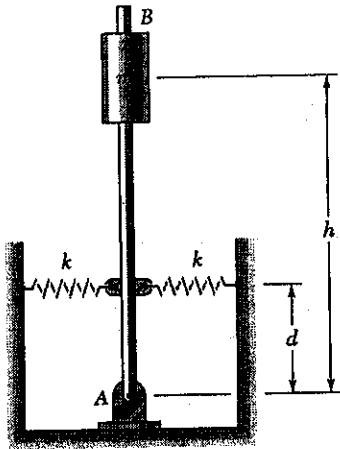
$$kh^2\theta + 9kh^2\theta - 4hP = 0, \quad P = \frac{5}{2}kh$$

Data:  $k = 2.0$  kip/in.,  $h = 2$  ft = 24 in

$$P = \frac{5}{2}(2.0)(24) = 120 \text{ kips.}$$

**PROBLEM 10.8**

10.8 If  $m = 125$  kg,  $h = 700$ , and the constant of each spring is  $k = 2.8$  kN/m, determine the range of values of the distance  $d$  for which the equilibrium of the rigid rod  $AB$  is stable in the position shown. Each spring can act in either tension or compression.



**SOLUTION**

$$h = 700 \text{ mm.} = 700 \times 10^{-3} \text{ m}$$

Let  $\theta$  be the small rotation of  $AB$

$$x = d\theta \quad F = kx = kd\theta$$

$$\circlearrowleft \sum M_A = 0 \quad 2Fd - mgh\theta = 0$$

$$2kd^2\theta - mgh = 0$$

$$d_{cr}^2 = \frac{mgh}{2k}$$

$$d_{cr} = \sqrt{\frac{mgh}{k}} = \sqrt{\frac{(125)(9.81)(700 \times 10^{-3})}{2(2.8 \times 10^3)}}$$

$$= 0.392 \text{ m} = 392 \text{ mm}$$

$d > 392 \text{ mm}$  for stability

**PROBLEM 10.9**

**10.9** Determine the critical load of a round wooden dowel that is 48-in. long and has a diameter of (a) 0.375 in., (b) 0.5 in. Use  $E = 1.6 \times 10^6$  psi.

**SOLUTION**

$$(a) \quad c = \frac{1}{2}d = 0.1875 \text{ in} \quad I = \frac{\pi}{4}c^4 = 970.7 \times 10^{-12} \text{ in}^4$$

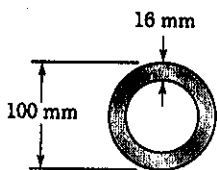
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (1.6 \times 10^6) (970.7 \times 10^{-12})}{(48)^2} = 6.65 \text{ lb.} \quad \blacktriangleleft$$

$$(b) \quad c = \frac{1}{2}d = 0.25 \text{ in.} \quad I = \frac{\pi}{4}c^4 = 3.068 \times 10^{-3} \text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (1.6 \times 10^6) (3.068 \times 10^{-3})}{(48)^2} = 21.0 \text{ lb.} \quad \blacktriangleleft$$

**PROBLEM 10.10**

**10.10** Determine the critical load of a steel tube that is 5.0 m long and has a 100-mm outer diameter and a 16 mm wall thickness. Use  $E = 200$  GPa.



**SOLUTION**

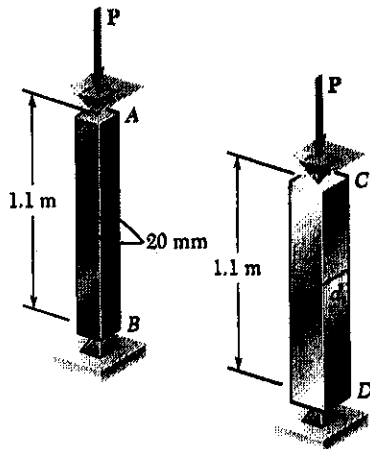
$$c_o = \frac{1}{2}d_o = 50 \text{ mm} \quad c_i = c_o - t = 50 - 16 = 34 \text{ mm}$$

$$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 3.859 \times 10^8 \text{ mm}^4 = 3.859 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (3.859 \times 10^{-6})}{(5.0)^2} = 305 \times 10^3 \text{ N} = 305 \text{ kN} \quad \blacktriangleleft$$

**PROBLEM 10.11**

**10.11** Determine (a) the critical load for the brass strut, (b) the dimension  $d$  for which the aluminum strut will have the same critical load, (c) the weight of the aluminum strut as a percent of the weight of the brass strut.



Brass  
 $E = 120 \text{ GPa}$   
 $\rho = 8740 \text{ kg/m}^3$

Aluminum  
 $E = 70 \text{ GPa}$   
 $\rho = 2710 \text{ kg/m}^3$

**SOLUTION**

(a) Brass strut  $I = \frac{1}{12}(20)(20)^3 = 13.333 \times 10^3 \text{ mm}^4$   
 $= 13.333 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 E_b I_b}{L^2} = \frac{\pi^2 (120 \times 10^9)(13.333 \times 10^{-9})}{(1.1)^2}$$

$$= 13.06 \times 10^3 \text{ N} = 13.06 \text{ kN}$$

(b) Aluminum strut

$$P_{cr} = \frac{\pi^2 E_a I_a}{L^2} = \frac{\pi^2 E_a (d^4/12)}{L^2}$$

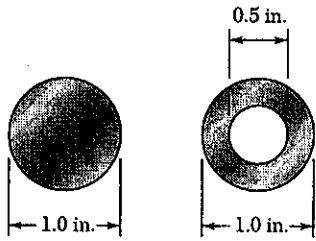
$$d^4 = \frac{12 P_{cr} L^2}{\pi^2 E_a} = \frac{(12)(13.06 \times 10^3)(1.1)^2}{\pi^2 (70 \times 10^9)} = 274.3 \times 10^{-9} \text{ m}^4$$

$$d = 22.9 \times 10^{-3} \text{ m} = 22.9 \text{ mm}$$

(c)  $\frac{m_a}{m_b} = \frac{\gamma_a L d^2}{\gamma_b L d_b^2} = \left(\frac{\gamma_a}{\gamma_b}\right) \left(\frac{d}{d_b}\right)^2 = \left(\frac{2710}{8740}\right) \left(\frac{22.9}{20}\right)^2 = 0.406 = 40.6\%$

PROBLEM 10.12

10.12 A compression member of 20 in. effective length consists of a solid 1.0-in.-diameter aluminum rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (a) the percent reduction in the critical load, (b) the value of the critical load for the hollow rod. Use  $E = 10.6 \times 10^6$  psi.



SOLUTION

Solid  $A_s = \frac{\pi}{4} d_o^2$   $I_s = \frac{\pi}{64} \left(\frac{d_o}{2}\right)^4 = \frac{\pi}{64} d_o^4$

Hollow:  $A_H = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{3}{4} A_s = \frac{3}{4} \frac{\pi}{4} d_o^2$

$d_i^2 = \frac{1}{4} d_o^2$   $d_i = \frac{1}{2} d_o = 0.5$  in.

Solid rod:  $I_s = \frac{\pi}{64} (1.0)^4 = 0.049087$  in<sup>4</sup>

$P_{cr} = \frac{\pi^2 E I_s}{L^2} = \frac{\pi^2 (10.6 \times 10^6) (0.049087)}{(20)^2} = 12.839 \times 10^3$  lb.

Hollow rod:  $I_H = \frac{\pi}{64} (d_o^4 - d_i^4) = \frac{\pi}{64} \left[ (1)^4 - \left(\frac{1}{2}\right)^4 \right] = 0.046019$  in<sup>4</sup>

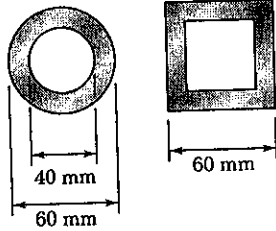
(b)  $P_{cr} = \frac{\pi^2 E I_H}{L^2} = \frac{\pi^2 (10.6 \times 10^6) (0.046019)}{(20)^2} = 12.036 \times 10^3$  lb. = 12.04 kips

(a)  $\frac{P_s - P_H}{P_s} = \frac{12.839 \times 10^3 - 12.036 \times 10^3}{12.839 \times 10^3} = 0.0625 = 6.25\%$



**PROBLEM 10.13**

10.13 Two brass rods used as compression members, each of 3-m effective length, have the cross sections shown. (a) Determine the wall thickness of the hollow square rod for which the rods have the same cross-sectional area. (b) Using  $E = 105 \text{ GPa}$ , determine the critical load of each rod.



**SOLUTION**

(a) Same area  $\frac{\pi}{4}(d_o^2 - d_i^2) = b_o^2 - b_i^2$

$$b_i^2 = b_o^2 - \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$= 60^2 - \frac{\pi}{4}(60^2 - 40^2) = 2.0292 \text{ mm}^2$$

$$b_i = 45.047 \text{ mm} \quad t = \frac{1}{2}(b_o - b_i) = 7.48 \text{ mm} \quad \blacktriangleleft$$

(b) Circular:  $I = \frac{\pi}{64}(d_o^4 - d_i^4) = 510.51 \times 10^3 \text{ mm}^4 = 510.51 \times 10^{-9} \text{ m}^4$

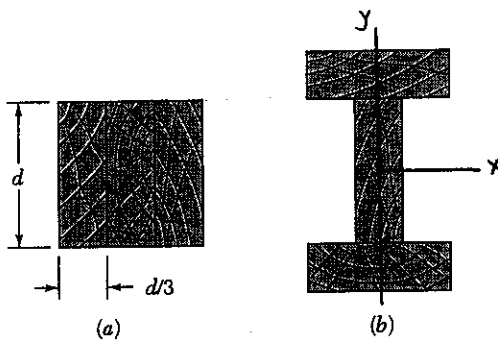
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (105 \times 10^9)(510.51 \times 10^{-9})}{(3.0)^2} = 58.8 \times 10^3 \text{ N} = 58.8 \text{ kN} \quad \blacktriangleleft$$

Square:  $I = \frac{1}{12}(b_o^4 - b_i^4) = 736.85 \times 10^3 \text{ mm}^4 = 736.85 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (105 \times 10^9)(736.85 \times 10^{-9})}{(3.0)^2} = 84.8 \times 10^3 \text{ N} = 84.8 \text{ kN} \quad \blacktriangleleft$$

**PROBLEM 10.14**

10.14 A column of effective length  $L$  can be made by gluing together identical planks in each of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b.



**SOLUTION**

Arrangement (a)

$$I_a = \frac{1}{12} d^4$$

$$P_{cr,a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

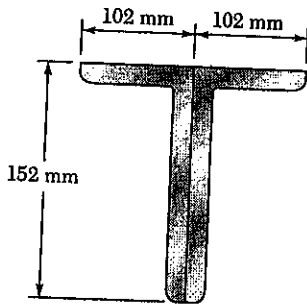
Arrangement (b)  $I_{min} = I_y = \frac{1}{12} \left(\frac{d}{3}\right)^3 + \frac{1}{12} \left(d\left(\frac{d}{3}\right)\right)^3 + \frac{1}{12} \left(\frac{d}{3}\right) \left(d\right)^3 = \frac{19}{324} d^4$

$$P_{cr,b} = \frac{\pi^2 EI}{L_e^2} = \frac{19 \pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19} = 1.421 \quad \blacktriangleleft$$

**PROBLEM 10.15**

10.15 A compression member of 7-m effective length is made by welding together two L152 × 102 × 12.7 angles as shown. Using  $\sigma = 200$  GPa, determine the allowable centric load for the member if a factor of safety of 2.2 is required.



**SOLUTION**

Angle L 152 × 102 × 12.7

$$A = 3060 \text{ mm}^2$$

$$I_x = 7.20 \times 10^6 \text{ mm}^4$$

$$I_y = 2.64 \times 10^6 \text{ mm}^4$$

$$y = 50.3 \text{ mm}$$

$$x = 25.3 \text{ mm}$$

Two angles:  $I_x = (2)(7.20 \times 10^6) = 14.00 \times 10^6 \text{ mm}^4$

$$I_y = 2 [ 2.64 \times 10^6 + (3060)(25.3)^2 ] = 9.197 \times 10^6 \text{ mm}^4$$

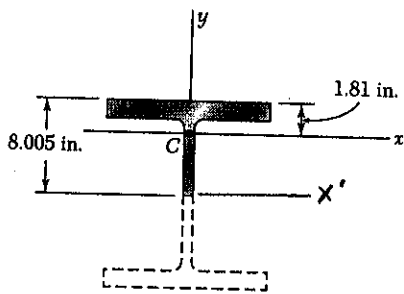
$$I_{\min} = I_y = 9.197 \times 10^6 \text{ mm}^4 = 9.197 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (9.197 \times 10^{-6})}{(7.0)^2} = 370.5 \times 10^3 \text{ N} = 370.5 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{370.5}{2.2} = 168.4 \text{ kN}$$

**PROBLEM 10.16**

10.16 A column of 26-ft effective length is made from half a W16 × 40 rolled-steel shape. Knowing that the centroid of the cross section is located as shown, determine the factor of safety if the allowable centric load is 20 kips. Use  $E = 29 \times 10^6$  psi.



**SOLUTION**

Full W 16 × 40

$$A = 11.8 \text{ in}^2$$

$$I_x = 518 \text{ in}^4, \quad I_y = 28.9 \text{ in}^4$$

Half W 16 × 40

$$A = (\frac{1}{2})(11.8) = 5.90 \text{ in}^2$$

$$I_x = \frac{1}{2}(518) - (5.90)(8.005 - 1.81)^2 = 32.57 \text{ in}^4$$

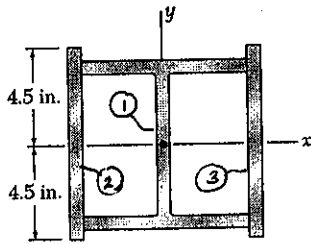
$$I_y = \frac{1}{2}(28.9) = 14.45 \text{ in}^4 = I_{\min}$$

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_e^2} = \frac{\pi^2 (29 \times 10^6) (14.45)}{(26 \times 12)^2} = 42.5 \times 10^3 \text{ lb} = 42.5 \text{ kips}$$

$$P_{all} = \frac{P_{cr}}{F.S.} \quad F.S. = \frac{P_{cr}}{P_{all}} = \frac{42.5}{20} = 2.125$$

PROBLEM 10.17

10.17 A column of 22-ft effective length is to be made by welding two  $9 \times 0.5$  in. plates to a  $W8 \times 35$  as shown. Determine the allowable centric load if a factor of safety of 2.3 is required. Use  $E = 29 \times 10^6$  psi.



SOLUTION

①  $W8 \times 35$      $I_x = 127 \text{ in}^4$      $I_y = 42.6 \text{ in}^4$   
 $b_f = 8.02 \text{ in}$

② and ③    For each plate     $A = (0.5)(9.0) = 4.5 \text{ in}^2$

$$I_x = \frac{1}{12}(0.5)(9)^3 = 30.375 \text{ in}^4$$

$$I_y = \frac{1}{12}(9)(0.5)^3 + (4.5)\left(\frac{8.02}{2} + \frac{0.5}{2}\right)^2 = 81.758 \text{ in}^4$$

Total:     $I_x = 127 + (2)(30.375) = 187.75 \text{ in}^4 = I_{\min}$

$I_y = 42.6 + (2)(81.758) = 206.12 \text{ in}^4$

$L = 22 \text{ ft} = 264 \text{ in.}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^6) (187.75)}{264^2} = 771.0 \times 10^3 \text{ lb} = 771 \text{ kips}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{771}{2.3} = 335 \text{ kips}$$

PROBLEM 10.18

10.18 A column of 3-m effective length is to be made by welding together two C130 × 13 rolled-steel channels. Using  $E = 200 \text{ GPa}$ , determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.



(a)



(b)

SOLUTION

For channel C 130 × 13

$$I_x = 3.70 \times 10^6 \text{ mm}^4$$

$$A = 1710 \text{ mm}^2$$

$$b_x = 48 \text{ mm}$$

$$I_y = 0.264 \times 10^6 \text{ mm}^4$$

$$\bar{x} = 12.2 \text{ mm}$$

Arrangement (a)

$$I_x = (2)(3.70 \times 10^6) = 7.40 \times 10^6 \text{ mm}^4$$

$$I_y = 2[0.264 \times 10^6 + (1710)(12.2)^2] = 1.0370 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 1.0370 \times 10^6 \text{ mm}^4 = 1.0370 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(1.0370 \times 10^{-6})}{(3.0)^2} = 227 \times 10^3 \text{ N} = 227 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{227}{2.4} = 94.8 \text{ kN}$$

Arrangement (b)

$$I_x = (2)(3.70 \times 10^6) \text{ mm}^4$$

$$I_y = 2[0.264 \times 10^6 + (1710)(48 - 12.2)^2] = 4.911 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 4.911 \times 10^6 \text{ mm}^4 = 4.911 \times 10^{-6} \text{ m}^4$$

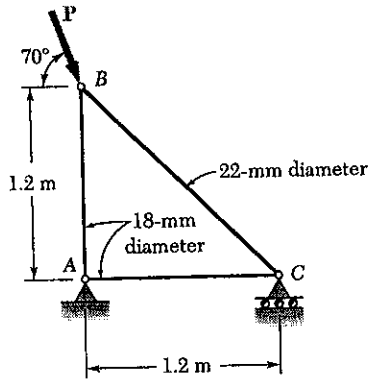
$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(4.911 \times 10^{-6})}{(3.0)^2} = 1077 \times 10^3 \text{ N} = 1077 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1077}{2.4} = 449 \text{ kN}$$

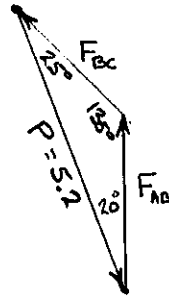
**PROBLEM 10.19**

10.19 Knowing that  $P = 5.2$  kN, determine the factor of safety for the structure shown. Use  $E = 200$  GPa and consider only buckling in the plane of the structure.

**SOLUTION**



Joint B: From force triangle



$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN (comp)}$$

$$F_{BC} = 2.5152 \text{ kN (comp)}$$

Member AB:  $I_{AB} = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{18}{2}\right)^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$

$$F_{AB,cr} = \frac{\pi^2 EI_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9) (5.153 \times 10^{-9})}{(1.2)^2} = 7.0636 \times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

Member BC:  $I_{BC} = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

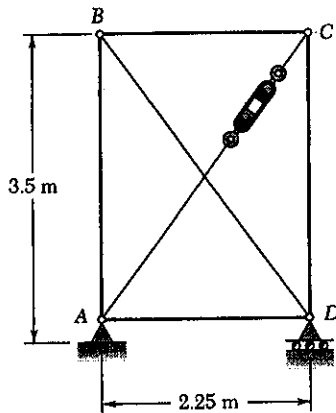
$$F_{BC,cr} = \frac{\pi^2 EI_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{2.88} = 7.8813 \times 10^3 \text{ N} = 7.8813 \text{ kN}$$

$$F.S. = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs.

$$F.S. = 2.27$$

PROBLEM 10.20



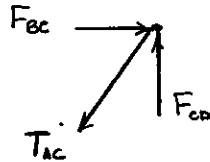
10.20 Members  $AB$  and  $CD$  are 30-mm-diameter steel rods, and members  $BC$  and  $AD$  are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member  $AC$  is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in  $AC$ . Use  $E = 200$  GPa and consider only buckling in the plane of the structure.

SOLUTION

$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$

Joint C  $\pm \sum F_x = 0 \quad F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$

$$T_{AC} = 1.84926 F_{BC}$$



$$+ \sum F_y = 0 \quad F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888 F_{CD}$$

Members  $BC$  and  $AD$ :  $I_{BC} = \frac{\pi}{4} \left(\frac{d_{BC}}{2}\right)^4 = \frac{\pi}{4} \left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

$$L_{BC} = 2.25 \text{ m} \quad F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$$

$$F_{BC,al} = \frac{F_{BC,cr}}{F.S.} = 1.6304 \times 10^3 \text{ N} \quad T_{AC,al} = 3.02 \times 10^3 \text{ N}$$

Members  $AB$  and  $CD$ :  $I_{CD} = \frac{\pi}{4} \left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{4} \left(\frac{30}{2}\right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$

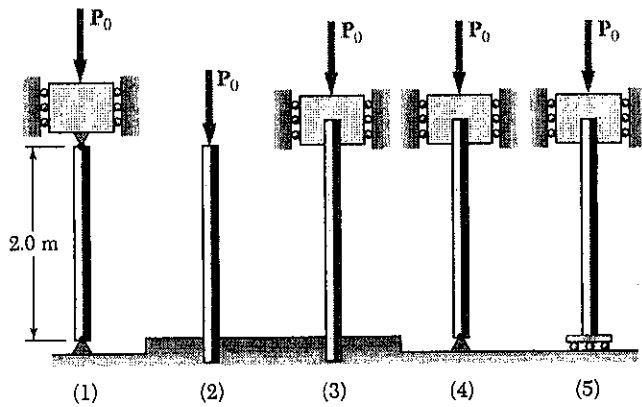
$$L_{CD} = 3.5 \text{ m} \quad F_{CD,cr} = \frac{\pi^2 E I_{CD}}{L_{CD}^2} = \frac{\pi^2 (200 \times 10^9) (39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$$

$$F_{CD,al} = \frac{F_{CD,cr}}{F.S.} = 2.3298 \times 10^3 \text{ N} \quad T_{AC,al} = 2.77 \times 10^3 \text{ N}$$

Smaller value for  $T_{AC,al}$  governs  $T_{AC,al} = 2.77 \times 10^3 \text{ N} = 2.77 \text{ kN}$   $\blacktriangleleft$

PROBLEM 10.21

10.21 Each of the five struts consists of an aluminum tube that has a 32-mm outer diameter and a 4-mm wall thickness. Using  $E = 70 \text{ GPa}$  and a factor of safety of 2.3, determine the allowable load  $P_0$  for each support condition shown.



SOLUTION

$$c_o = \frac{1}{2} d_o = \frac{1}{2}(32) = 16 \text{ mm}$$

$$c_i = c_o - t = 16 - 4 = 12 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = 35.1858 \times 10^3 \text{ mm}^4$$

$$= 35.1858 \times 10^{-9} \text{ m}^4$$

$$\pi^2 EI = \pi^2 (70 \times 10^9) (35.1858 \times 10^{-9})$$

$$= 24309 \text{ N}\cdot\text{m}^2$$

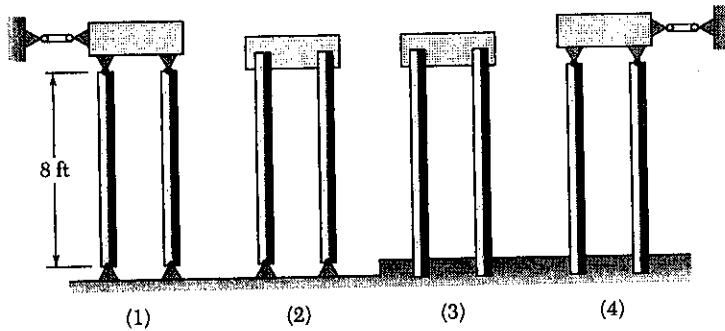
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309}{L_e^2}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569}{L_e^2}$$

- |   |  |   |
|---|--|---|
| (1) $L_e = (1)(2.0) = 2.0 \text{ m},$           | $P_{all} = 2642 \text{ N} = 2.64 \text{ kN}$   | ▲ |
| (2) $L_e = (2)(2.0) = 4.0 \text{ m},$           | $P_{all} = 661 \text{ N} = 0.661 \text{ kN}$   | ▲ |
| (3) $L_e = (\frac{1}{2})(2.0) = 1.0 \text{ m},$ | $P_{all} = 10569 \text{ N} = 10.57 \text{ kN}$ | ▲ |
| (4) $L_e = (0.7)(2.0) = 1.4 \text{ m},$         | $P_{all} = 5392 \text{ N} = 5.39 \text{ kN}$   | ▲ |
| (5) $L_e = (1.0)(2.0) = 2.0 \text{ m},$         | $P_{all} = 2642 \text{ N} = 2.64 \text{ kN}$   | ▲ |

PROBLEM 10.22

10.22 Two columns are used to support a block weighing 3.25 kips in each of the four ways shown. (a) Knowing that the column of Fig. (1) is made of steel with a 1.25-in.-diameter, determine the factor of safety with respect to buckling for the loading shown. (b) Determine the diameter of each of the other columns for which the factor of safety is the same as the factor of safety obtained in part a. Use  $E = 29 \times 10^6$  psi.



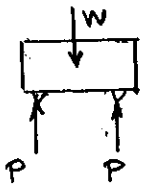
SOLUTION

$$(a) I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{1.25}{2}\right)^4 = 0.119842 \text{ in}^4$$

$$L = 8 \text{ ft} = 96 \text{ in}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 (29 \times 10^6) (0.119842)}{(96)^2} = 3722 \text{ lb} = 3.722 \text{ kip. for one column.}$$



$$P = \frac{1}{2} W = \frac{3.25}{2} = 1.625 \text{ kip.}$$

$$F.S. = \frac{P_{cr}}{P} = \frac{3.722}{1.625} = 2.29$$

$$P_{cr(1)} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr(a)} = \frac{\pi^2 E I_a}{(L_{e,a})^2}$$

$$\frac{P_{cr(a)}}{P_{cr(1)}} = 1 \quad \frac{I_a}{I(1)} \cdot \frac{L^2}{L_{e,a}^2} = 1 \quad \left(\frac{d_a}{d(1)}\right)^4 \left(\frac{L_e}{L}\right)^2 = 1$$

$$d_a = d(1) \sqrt[4]{\frac{L_{e,a}}{L}}$$

$$(2) L_{e(2)}/L = 2.0$$

$$d_{(2)} = 1.25 \sqrt[4]{2.0} = 1.768 \text{ in.}$$

$$(3) L_{e(3)}/L = 1.0$$

$$d_{(3)} = 1.25 \text{ in.}$$

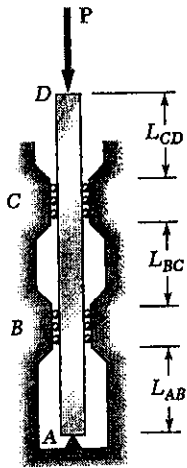
$$(4) L_{e(4)}/L = 0.7$$

$$d_{(4)} = 1.25 \sqrt[4]{0.7} = 1.046 \text{ in.}$$



**PROBLEM 10.23**

10.23 A 25-mm-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that  $L_{AB} = 1.0$  m,  $L_{BC} = 1.25$  m, and  $L_{CD} = 0.5$  m, determine the allowable load *P* using a factor of safety with respect to buckling of 2.8. Consider only buckling in the plane of the figure and use  $E = 75$  GPa.



**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (25)(25)^3 = 32.552 \times 10^3 \text{ mm}^3 = 32.552 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.) L_e^2} = \frac{\pi^2 EI}{(F.S.) (L_{e,max})^2}$$

Portion AB:  $L_e = 0.7 L_{AB} = (0.7)(1.0) = 0.7$  m

Portion BC:  $L_e = 0.5 L_{BC} = (0.5)(1.25) = 0.625$  m

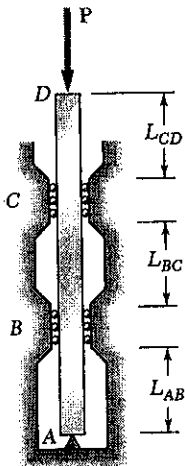
Portion CD:  $L_e = 2 L_{CD} = (2.0)(0.5) = 1.0$  m

$$L_{e,max} = 1.0 \text{ m}$$

$$P_{all} = \frac{\pi^2 (75 \times 10^9) (32.552 \times 10^{-9})}{(2.8)(1.0)^2} = 8.61 \times 10^3 \text{ N} = 8.61 \text{ kN}$$

**PROBLEM 10.24**

10.24 A 32-mm-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that  $L_{AB} = 1.4$  m, determine (a) the largest values of  $L_{BC}$  and  $L_{CD}$  that may be used if the allowable load *P* is to be as large as possible, (b) the magnitude of the corresponding allowable load if the factor of safety is 2.8. Consider only buckling in the plane of the figure and use  $E = 72$  GPa.



**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (32)(32)^3 = 87.381 \times 10^3 \text{ mm}^3 = 87.381 \times 10^{-9} \text{ m}^4$$

Equivalent lengths: AB  $L_e = 0.7 L_{AB} = 0.98$  m

BC  $L_e = 0.5 L_{BC}$

CD  $L_e = 2 L_{CD}$

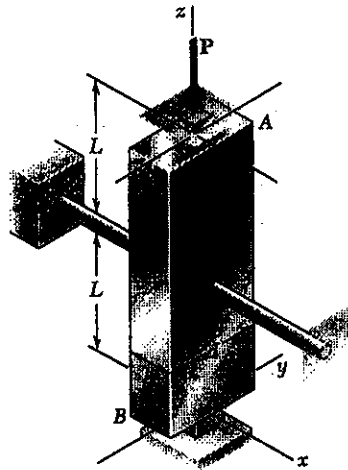
Equating  $L_{BC} = \frac{0.7}{0.5} L_{AB} = (1.4)(1.4) = 1.96$  m

$L_{CD} = \frac{0.7}{2} L_{AB} = (0.35)(1.4) = 0.49$  m

(b)  $P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.) L_e^2} = \frac{\pi^2 (72 \times 10^9) (87.381 \times 10^{-9})}{(2.8)(0.98)^2} = 23.1 \times 10^3 \text{ N} = 23.1 \text{ kN}$

PROBLEM 10.25

10.25 Column  $ABC$  has a uniform rectangular cross section and is braced in the  $xz$  plane at its midpoint  $C$ . (a) Determine the ratio  $b/d$  for which the factor of safety is the same with respect to buckling in the  $xz$  and  $yz$  planes. (b) Using the ratio found in part a, design the cross section of the column so that the factor of safety will be 2.7 when  $P = 1.2$  kips,  $L = 24$  in., and  $E = 10.6 \times 10^6$  psi.



SOLUTION

Buckling in  $xz$ -plane:  $L_e = L = 24$  in.



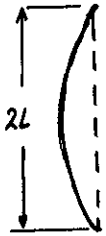
$$I = \frac{1}{12} db^3$$

$$P = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{2.8 L_e^2} = \frac{\pi^2 E db^3}{12(F.S.) L_e^2}$$

$$db^3 = \frac{12 P (F.S.) L_e^2}{\pi^2 E} = \frac{(12)(1.2 \times 10^3)(2.7)(24)^2}{\pi^2 (10.6 \times 10^6)}$$

$$= 0.21406 \text{ in}^4$$

Buckling in  $yz$ -plane:  $L_e = 2L = (2)(24) = 48$  in  $I = \frac{1}{12} bd^3$



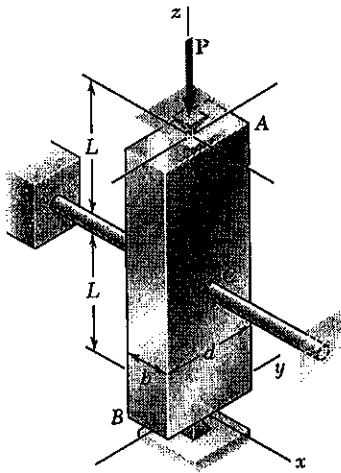
$$P = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{2.8 L_e^2} = \frac{\pi^2 E bd^3}{12(F.S.) L_e^2}$$

$$bd^3 = \frac{12 P (F.S.) L_e^2}{\pi^2 E} = \frac{(12)(1.2 \times 10^3)(2.7)(48)^2}{\pi^2 (10.6 \times 10^6)} = 0.85625 \text{ in}^4$$

(a)  $\frac{db^3}{bd^3} = \frac{b^2}{d^2} = \frac{0.21406}{0.85625} = \frac{1}{4} \quad \frac{b}{d} = \frac{1}{2}$

$db^3 = d(\frac{1}{8}d^3) = \frac{1}{8}d^4 = 0.21406 \text{ in}^4, \quad d = 1.144 \text{ in.}$   
 $b = \frac{1}{2}d = 0.572 \text{ in.}$

PROBLEM 10.26



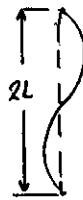
10.26 The aluminum column  $ABC$  has a uniform rectangular cross section with  $b = \frac{1}{2}$  in. and  $d = \frac{7}{8}$  in. The column is braced in the  $xz$  plane at its midpoint  $C$  and carries a centric load  $P$  of magnitude 1.1 kips. Knowing that a factor of safety of 2.5 is required, determine the largest allowable length  $L$ . Use  $E = 10.6 \times 10^6$  psi.

SOLUTION

$$P_{cr} = (\text{F.S.})P = (2.5)(1.1 \times 10^3) = 2.75 \times 10^3 \text{ lb.}$$

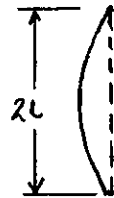
$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$$

Buckling in  $xz$ -plane:  $I = \frac{1}{12} db^3 = \frac{1}{12} \left(\frac{7}{8}\right) \left(\frac{1}{2}\right)^3 = 9.1146 \times 10^{-3} \text{ in}^4$



$$L = L_e = \pi \sqrt{\frac{EI}{P_{cr}}} = \pi \sqrt{\frac{(10.6 \times 10^6)(9.1146 \times 10^{-3})}{2.75 \times 10^3}} = 18.62 \text{ in.}$$

Buckling in  $yz$ -plane.

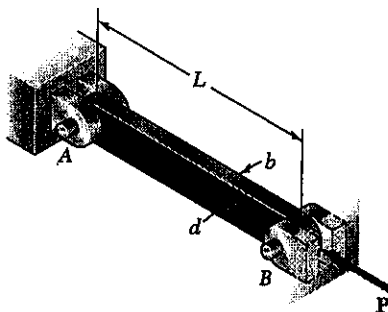


$$I = \frac{1}{12} bd^3 = \frac{1}{12} \left(\frac{1}{2}\right) \left(\frac{7}{8}\right)^3 = 27.913 \times 10^{-5} \text{ in}^4 \quad L_e = 2L$$

$$L = \frac{1}{2} L_e = \frac{\pi}{2} \sqrt{\frac{EI}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(10.6 \times 10^6)(27.913 \times 10^{-5})}{2.75 \times 10^3}} = 16.29 \text{ in.}$$

Smaller length governs  $L = 16.29 \text{ in.}$

PROBLEM 10.27



10.27 The uniform brass bar  $AB$  has a rectangular cross section and is supported by pins and brackets as shown. Each end of the bar can rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. (a) Determine the ratio  $b/d$  for which the factor of safety is the same about the horizontal and vertical axes. (b) Determine the factor of safety if  $P = 1.8$  kips,  $L = 7$  ft,  $d = 1.5$  in., and  $E = 15 \times 10^6$  psi.

SOLUTION

Buckling in horizontal plane:  $L_e = \frac{1}{2}L$ ,  $I = \frac{1}{12} db^3$

$$P_{cr1} = \frac{\pi^2 EI}{L_e^2} = \frac{4\pi^2 Edb^3}{12L^2} \quad (1)$$

Buckling in vertical plane:  $L_e = L$ ,  $I = \frac{1}{12} bd^3$

$$P_{cr2} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 Ebd^3}{12L^2} \quad (2)$$

(a) Equating  $P_{cr1} = P_{cr2}$   $\frac{4\pi^2 Edb^3}{12L^2} = \frac{\pi^2 Ebd^3}{12L^2}$   $4b^2 = d^2$   $b = \frac{1}{2}d$

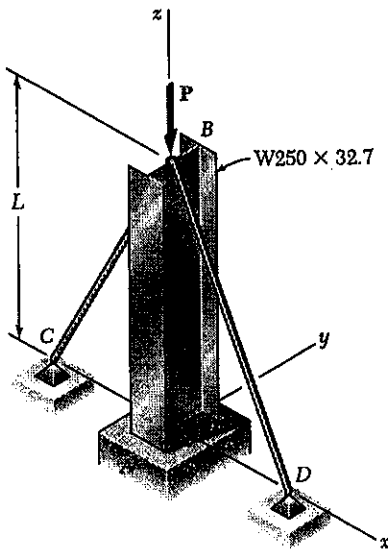
(b)  $b = \frac{1}{2}d = 0.75 \text{ in.}$   $L = 7 \text{ ft} = 84 \text{ in.}$

Using (2)  $P_{cr} = \frac{\pi^2 (15 \times 10^6)(0.75)(1.5)^3}{(12)(84)^2} = 4.426 \times 10^3 \text{ lb} = 4.426 \text{ kips.}$

$$\text{F.S.} = \frac{P_{cr}}{P} = \frac{4.426}{1.8} = 2.46$$

**PROBLEM 10.28**

**10.28** Column  $AB$  carries a centric load  $P$  of magnitude 72 kN. Cables  $BC$  and  $BD$  are taut and prevent motion of point  $B$  in the  $xz$  plane. Using Euler's formula and a factor of safety of 2.3, and neglecting the tension in the cables, determine the maximum allowable length  $L$ . Use  $E = 200$  GPa.



**SOLUTION**

W 250 x 32.7       $I_x = 48.9 \times 10^6 \text{ mm}^4 = 48.9 \times 10^{-6} \text{ m}^4$   
 $I_y = 4.73 \times 10^6 \text{ mm}^4 = 4.73 \times 10^{-6} \text{ m}^4$

$P = 72 \times 10^3 \text{ N}$        $P_{cr, \min} = (\text{F.S.})(P) = 165.3 \times 10^3 \text{ N}$

Buckling in  $xz$ -plane:  $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 EI_y}{(0.7L)^2}$$

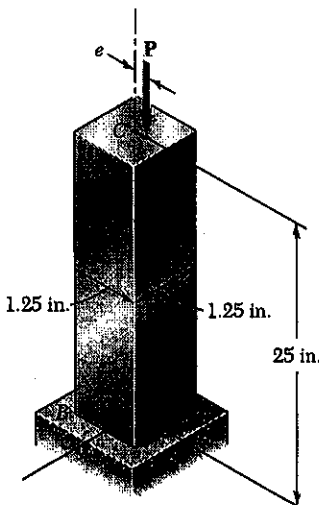
$$L = \frac{\pi \sqrt{EI_y}}{0.7 \sqrt{P_{cr}}} = \frac{\pi \sqrt{(200 \times 10^9)(4.73 \times 10^{-6})}}{0.7 \sqrt{165.3 \times 10^3}} = 10.74 \text{ m}$$

Buckling in  $yz$ -plane:  $L_e = 2L$        $P_{cr} = \frac{\pi^2 EI_x}{(2L)^2}$

$$L = \frac{\pi \sqrt{EI_x}}{2 \sqrt{P_{cr}}} = \frac{\pi \sqrt{(200 \times 10^9)(48.9 \times 10^{-6})}}{2 \sqrt{165.3 \times 10^3}} = 12.08 \text{ m}$$

**PROBLEM 10.29**

**10.29** An axial load  $P$  is applied to the 1.25-in.-square aluminum bar  $ABC$  as shown. When  $P = 3.8$  kips, the horizontal deflection at end  $C$  is 0.16 in. Using  $E = 10.1 \times 10^6$  psi, determine (a) the eccentricity  $e$  of the load, (b) the maximum stress in the rod.



**SOLUTION**

$I = \frac{1}{12} (1.25)^4 = 0.20345 \text{ in}^4$        $A = 1.25^2 = 1.5625 \text{ in}^2$

$L_e = 2L = 50 \text{ in}$        $L_e = 2L = 50 \text{ in}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (10.1 \times 10^6)(0.20345)}{(50)^2} = 8.1122 \times 10^3 \text{ lb}$$

$$\frac{P}{P_{cr}} = \frac{3.8 \times 10^3}{8.1122 \times 10^3} = 0.46842$$

(a)  $y_{\max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = e \left[ \sec\left(\frac{\pi}{2} \sqrt{0.46842}\right) - 1 \right]$   
 $= e \left[ \sec(1.07508) - 1 \right] = 1.1023 e$

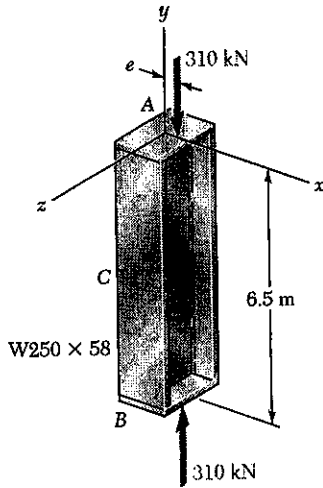
$$e = \frac{y_{\max}}{1.1023} = \frac{0.16}{1.1023} = 0.1451 \text{ in.}$$

(b)  $M_{\max} = P(e + y_{\max}) = (3.8 \times 10^3)(0.1451 + 0.16) = 1.15957 \text{ lb}\cdot\text{in}$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{3.8 \times 10^3}{1.5625} + \frac{(1.15957)(0.625)}{0.20345} = 5.99 \times 10^3 \text{ psi} = 5.99 \text{ ksi}$$

**PROBLEM 10.30**

10.30 The line of action of the 310-kN axial load is parallel to the geometric axis of the column  $AB$  and intersects the  $x$  axis at  $x = e$ . Using  $E = 200$  GPa, determine (a) the eccentricity  $e$  when the deflection of the midpoint  $C$  of the column is 9 mm, (b) the corresponding maximum stress in the column.



**SOLUTION**

For  $W250 \times 58$

$$A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.8 \times 10^6 \text{ mm}^4 = 18.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$$

$$L = 6.5 \text{ m} \quad L_e = 6.5 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.8 \times 10^{-6})}{(6.5)^2} = 878.3 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{310 \times 10^3}{878.3 \times 10^3} = 0.35294$$

$$y_{max} = e \left[ \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = 0.67990 e$$

(a)  $e = \frac{y_{max}}{0.67990} = \frac{9 \times 10^{-3}}{0.67990} = 13.24 \times 10^{-3} \text{ m} = 13.24 \text{ mm}$   $\blacktriangleleft$

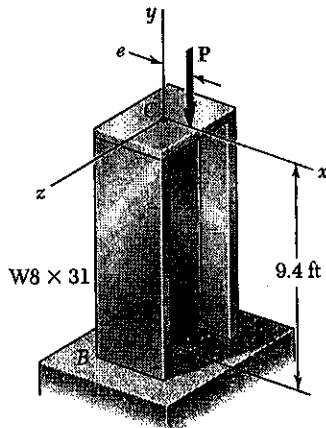
(b)  $M_{max} = P(e + y_{max}) = (310 \times 10^3)(9 + 13.24)(10^{-3}) = 6893.5 \text{ N}\cdot\text{m}$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{310 \times 10^3}{7420 \times 10^{-6}} + \frac{6893.5}{185 \times 10^{-6}}$$

$$= 41.78 \times 10^6 + 37.26 \times 10^6 = 79.04 \times 10^6 \text{ Pa} = 79.0 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 10.31**

10.31 The axial load  $P$  is applied at a point located on the  $x$  axis at a distance  $e$  from the geometric axis of the rolled-steel column  $BC$ . When  $P = 82$  kips, the horizontal deflection of the top of the column is 0.20 in. Using  $E = 29 \times 10^6$  psi, determine (a) the eccentricity  $e$  of the load, (b) the maximum stress in the column.



**SOLUTION**

$$W8 \times 31: A = 9.13 \text{ in}^2, I_y = 37.1 \text{ in}^4, S_y = 9.27 \text{ in}^3$$

$$L = 9.4 \text{ ft} = 112.8 \text{ in} \quad L_e = 2L = 225.6 \text{ in}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^6) (37.1)}{(225.6)^2} = 208.63 \times 10^3$$

$$\frac{P}{P_{cr}} = \frac{82 \times 10^3}{208.63 \times 10^3} = 0.39304$$

(a)  $y_{max} = e \left[ \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = 0.80816 e$

$$e = \frac{y_{max}}{0.80816} = \frac{0.20}{0.80816} = 0.247 \text{ in.} \quad \blacktriangleleft$$

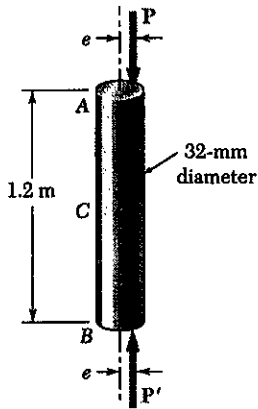
(b)  $M_{max} = P(e + y_{max}) = (82 \times 10^3)(0.247 + 0.20) = 36.693 \times 10^3 \text{ lb}\cdot\text{in}$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{82 \times 10^3}{9.13} + \frac{36.693 \times 10^3}{9.27} = 12.94 \times 10^3 \text{ psi}$$

$$= 12.94 \text{ ksi} \quad \blacktriangleleft$$

**PROBLEM 10.32**

10.32 An axial load  $P$  is applied to the 32-mm-diameter steel rod  $AB$  as shown. For  $P = 37 \text{ kN}$  and  $e = 1.2 \text{ mm}$ , determine (a) the deflection at the midpoint  $C$  of the rod, (b) the maximum stress in the rod. Use  $E = 200 \text{ GPa}$ .



**SOLUTION**

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{32}{2}\right)^4 = 51.47 \times 10^3 \text{ mm}^4 = 51.47 \times 10^{-9} \text{ m}^4$$

$$L_e = L = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (51.47 \times 10^{-9})}{(1.2)^2} = 70.556 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{37 \times 10^3}{70.556 \times 10^3} = 0.52440$$

$$(a) \ y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = 1.3817 e = (1.3817)(1.2) = 1.658 \text{ mm} \quad \blacktriangleleft$$

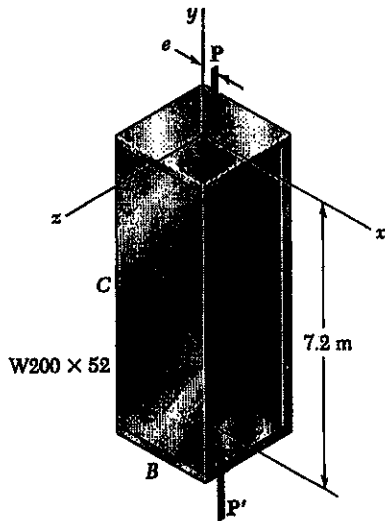
$$(b) \ M_{max} = P(e + y_{max}) = (37 \times 10^3)(1.2 + 1.658)(10^{-3}) = 105.75 \text{ N}\cdot\text{m}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2, \quad c = 16 \times 10^{-3} \text{ m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{37 \times 10^3}{804.25 \times 10^{-6}} + \frac{(105.75)(16 \times 10^{-3})}{51.47 \times 10^{-9}} = 78.9 \times 10^6 \text{ Pa} = 78.9 \text{ MPa} \quad \blacktriangleleft$$

**PROBLEM 10.33**

10.33 The line of action of the axial load  $P$  of magnitude 270 kN is parallel to the geometric axis of the column  $AB$  and intersects the  $x$  axis at  $e = 14 \text{ mm}$ . Using  $E = 200 \text{ GPa}$ , determine (a) the deflection of the midpoint  $C$  of the column, (b) the maximum stress in the column.



**SOLUTION**

$$W 200 \times 52 \quad A = 6660 \text{ mm}^2 = 6660 \times 10^{-6} \text{ m}^2$$

$$I_y = 17.8 \times 10^6 \text{ mm}^4 = 17.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 175 \times 10^3 \text{ mm}^3 = 175 \times 10^{-6} \text{ m}^3$$

$$L = 7.2 \text{ m} \quad L_e = 7.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (17.8 \times 10^{-6})}{(7.2)^2} = 677.77 \times 10^3 \text{ N}$$

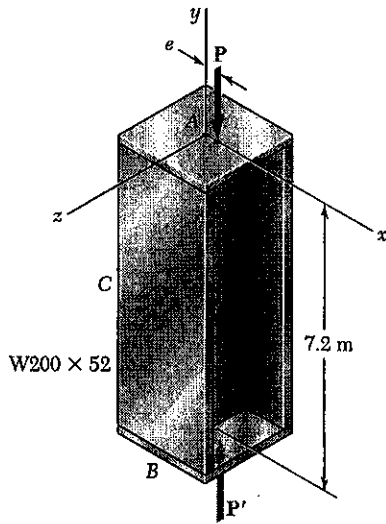
$$\frac{P}{P_{cr}} = \frac{270 \times 10^3}{677.77 \times 10^3} = 0.39836$$

$$(a) \ y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = 0.82648 e = (0.82648)(14) = 11.57 \text{ mm} \quad \blacktriangleleft$$

$$(b) \ M_{max} = P(e + y_{max}) = (270 \times 10^3)(14 + 11.57)(10^{-3}) = 6904 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{270 \times 10^3}{6660 \times 10^{-6}} + \frac{6904}{175 \times 10^{-6}} = 80.0 \times 10^6 \text{ Pa} = 80.0 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 10.34



10.33 The line of action of the axial load  $P$  of magnitude 270 kN is parallel to the geometric axis of the column  $AB$  and intersects the  $x$  axis at  $e = 14$  mm. Using  $E = 200$  GPa, determine (a) the deflection of the midpoint  $C$  of the column, (b) the maximum stress in the column.

10.34 Solve Prob. 10.33 if the load  $P$  is applied parallel to the geometric axis of the column  $AB$  so that it intersects the  $x$  axis at  $e = 21$  mm.

SOLUTION

$$W 200 \times 52 \quad A = 6660 \text{ mm}^2 = 6660 \times 10^{-6} \text{ m}^2$$

$$I_y = 17.8 \times 10^6 \text{ mm}^4 = 17.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 175 \times 10^3 \text{ mm}^3 = 175 \times 10^{-6} \text{ m}^3$$

$$L = 7.2 \text{ m} \quad L_e = 7.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (17.8 \times 10^{-6})}{(7.2)^2}$$

$$= 677.77 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{270 \times 10^3}{677.77 \times 10^3} = 0.39836$$

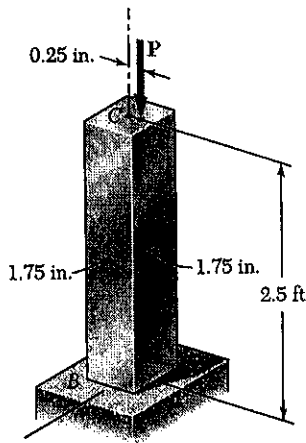
$$(a) \quad y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = 0.82648 e = (0.82648)(21) = 17.36 \text{ mm}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (270 \times 10^3)(21 + 17.36)(10^{-3}) = 10356 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{270 \times 10^3}{6660 \times 10^{-6}} + \frac{10356}{175 \times 10^{-6}} = 99.7 \times 10^6 \text{ Pa} = 99.7 \text{ MPa}$$

**PROBLEM 10.35**

**10.35** An axial load  $P$  is applied at a point  $D$  that is 0.25 in. from the geometric axis of the square aluminum bar  $BC$ . Determine (a) the load  $P$  for which the horizontal deflection of end  $C$  is 0.50 in., (b) the corresponding maximum stress in the column. Use  $E = 10.1 \times 10^3$  ksi.



**SOLUTION**

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (1.75)(1.75)^3 = 0.78157 \text{ in}^4$$

$$A = (1.75)^2 = 3.0625 \text{ in}^2 \quad c = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$L = 2.5 \text{ ft} = 30 \text{ in} \quad L_e = 2L = 60 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (10.1 \times 10^3)(0.78157)}{(60)^2} = 21.641 \text{ kips.}$$

$$y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right]$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}, \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$(a) \quad \frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{e}{e + y_{max}} \right]^2 = \left[ \frac{2}{\pi} \arccos \frac{0.25}{0.25 + 0.50} \right]^2$$

$$= 0.61411$$

$$P = 0.61411 P_{cr} = 13.29 \text{ kips} \leftarrow$$

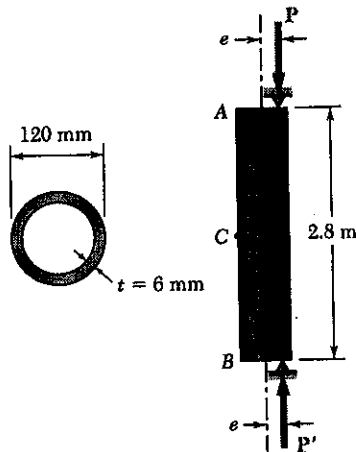
$$(b) \quad M_{max} = P(e + y_{max}) = (13.29)(0.25 + 0.50) = 9.9675 \text{ kip-in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{13.29}{3.0625} + \frac{(9.9675)(0.875)}{0.78157} = 15.50 \text{ ksi} \leftarrow$$



PROBLEM 10.36

10.36 A brass pipe having the cross section shown has an axial load  $P$  applied 5 mm from its geometric axis. Using  $E = 120$  GPa, determine (a) the load  $P$  for which the horizontal deflection at the midpoint  $C$  is 5 mm, (b) the corresponding maximum stress in the column.



SOLUTION

$$c_o = \frac{1}{2} d_o = 60 \text{ mm} \quad c_i = c_o - t = 54 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = 3.5005 \times 10^6 \text{ mm}^4 = 3.5005 \times 10^{-6} \text{ m}^4$$

$$L = 2.8 \text{ m} \quad L_e = 2.8 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9) (3.5005 \times 10^{-6})}{(2.8)^2}$$

$$= 528.8 \times 10^3 \text{ N} = 528.8 \text{ kN}$$

$$(a) \quad y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e} \quad \frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2$$

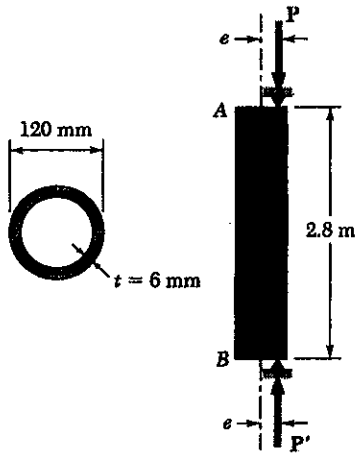
$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{5}{5+5} \right]^2 = 0.44444 \quad P = 0.44444 P_{cr} = 235 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = P(e + y_{max}) = (235 \times 10^3)(5 + 5)(10^{-3}) = 2350 \text{ N}\cdot\text{m}$$

$$A = \pi (c_o^2 - c_i^2) = \pi (60^2 - 54^2) = 2.1488 \times 10^3 \text{ mm}^2 = 2.1488 \times 10^{-3} \text{ m}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{235 \times 10^3}{2.1488 \times 10^{-3}} + \frac{(2350)(60 \times 10^{-3})}{3.5005 \times 10^{-6}} = 149.6 \times 10^6 \text{ Pa} = 149.6 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 10.37



10.36 A brass pipe having the cross section shown has an axial load  $P$  applied 5 mm from its geometric axis. Using  $E = 120$  GPa, determine (a) the load  $P$  for which the horizontal deflection at the midpoint  $C$  is 5 mm, (b) the corresponding maximum stress in the column.

10.37 Solve Prob. 10.36, assuming that the axial load  $P$  is applied 10 mm from the geometric axis of the column.

SOLUTION

$$C_o = \frac{1}{2} d_o = 60 \text{ mm} \quad C_i = C_o - t = 54 \text{ mm}$$

$$I = \frac{\pi}{4} (C_o^4 - C_i^4) = 3.5005 \times 10^6 \text{ mm}^4 = 3.5005 \times 10^{-6} \text{ m}^4$$

$$L = 2.8 \text{ m} \quad L_e = 2.8 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9) (3.5005 \times 10^{-6})}{(2.8)^2}$$

$$= 528.8 \times 10^3 \text{ N} = 528.8 \text{ kN}$$

$$(a) \quad y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e} \quad \frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2$$

$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{10}{5 + 10} \right]^2 = 0.28670 \quad P = 0.28670 P_{cr} = 151.6 \text{ kN}$$

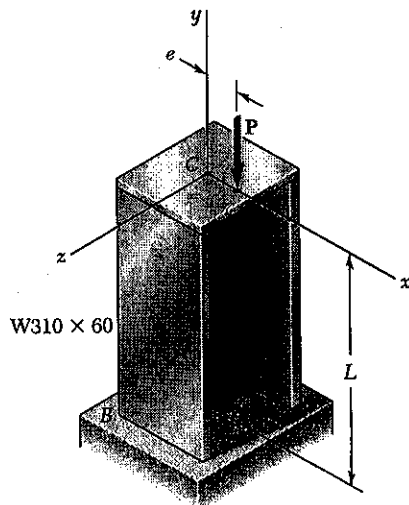
$$(b) \quad M_{max} = P(e + y_{max}) = (151.6 \times 10^3)(10 + 5)(10^{-3}) = 2274 \text{ N}\cdot\text{m}$$

$$A = \pi (C_o^2 - C_i^2) = \pi (60^2 - 54^2) = 2.1488 \times 10^3 \text{ mm}^2 = 2.1488 \times 10^{-3} \text{ m}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{151.6 \times 10^3}{2.1488 \times 10^{-3}} + \frac{(2274)(60 \times 10^{-3})}{3.5005 \times 10^{-6}} = 109.5 \times 10^6 \text{ Pa} = 109.5 \text{ MPa}$$

PROBLEM 10.38

10.38 An axial load  $P$  is applied at a point located on the  $x$  axis at a distance  $e = 12$  mm from the geometric axis of the  $W310 \times 60$  rolled-steel column  $BC$ . Assuming that  $L = 3.5$  m and using  $E = 200$  GPa, determine (a) the load  $P$  for which the horizontal deflection at end  $C$  is 15 mm, (b) the corresponding maximum stress in the column.



SOLUTION

$W310 \times 60$

$$A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$$

$$L = 3.5 \text{ m} \quad l_e = 2L = 7.0 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 (200 \times 10^9) (18.3 \times 10^{-6})}{(7.0)^2}$$

$$= 737.2 \times 10^3 \text{ N} = 737.2 \text{ kN}$$

$$y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[ \frac{2}{\pi} \arccos \frac{12}{15 + 12} \right]^2 = 0.49957$$

(a)  $P = 0.49957 P_{cr} = 368.28 \text{ kN}$

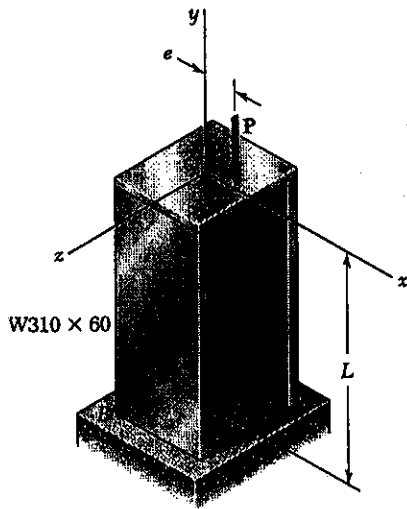
$$M_{max} = P(e + y_{max}) = (368.28 \times 10^3)(12 + 15)(10^{-3}) = 9944 \text{ N}\cdot\text{m}$$

(b)  $\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28 \times 10^3}{7590 \times 10^{-6}} + \frac{9944}{180 \times 10^{-6}} = 103.8 \times 10^6 \text{ Pa}$   
 $= 103.8 \text{ MPa}$

**PROBLEM 10.39**

**10.38** An axial load  $P$  is applied at a point located on the  $x$  axis at a distance  $e = 12$  mm from the geometric axis of the  $W310 \times 60$  rolled-steel column  $BC$ . Assuming that  $L = 3.5$  m and using  $E = 200$  GPa, determine (a) the load  $P$  for which the horizontal deflection at end  $C$  is 15 mm, (b) the corresponding maximum stress in the column.

**10.39** Solve Prob. 10.38, assuming that  $L$  is 4.5 m.



**SOLUTION**

W 310 x 60      $A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$   
 $I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$   
 $S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$

$L = 4.5 \text{ m}$       $L_e = 2L = 9.0 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.3 \times 10^{-6})}{(9.0)^2}$$

$$= 445.96 \times 10^3 \text{ N} = 445.96 \text{ kN}$$

$$y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

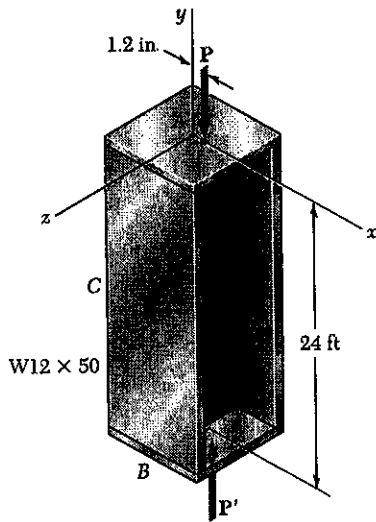
$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[ \frac{2}{\pi} \arccos \frac{12}{15 + 12} \right]^2 = 0.49957$$

(a)  $P = 0.49957 P_{cr} = 222.79 \text{ kN}$

$$M_{max} = P(e + y_{max}) = (222.79 \times 10^3)(12 + 15)(10^{-3}) = 6015 \text{ N}\cdot\text{m}$$

(b)  $\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{222.79 \times 10^3}{7590 \times 10^{-6}} + \frac{6015}{180 \times 10^{-6}} = 62.8 \times 10^6 \text{ Pa}$   
 $= 62.8 \text{ MPa}$

**PROBLEM 10.40**



10.40 The line of action of an axial load  $P$  is parallel to the geometric axis of the column  $AB$  and intersects the  $x$  axis at  $x = 1.2$  in. Using  $E = 29 \times 10^6$  psi., determine (a) the load  $P$  for which the horizontal deflection of the midpoint  $C$  of the column is 0.8 in., (b) the corresponding maximum stress in the column.

**SOLUTION**

$$W12 \times 50 \quad A = 14.7 \text{ in}^2, \quad I_y = 56.3 \text{ in}^4, \quad S_y = 13.9 \text{ in}^3$$

$$L = 24 \text{ ft} = 288 \text{ in} \quad L_e = 288 \text{ in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(56.3)}{(288)^2} = 194.28 \times 10^3 \text{ lb.}$$

$$= 194.28 \text{ kips}$$

$$y_{max} = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

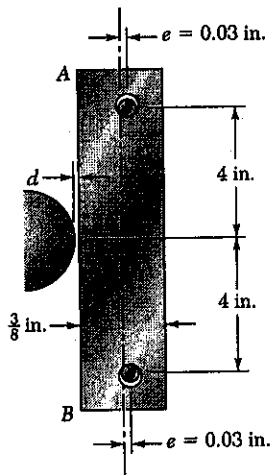
$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e} \quad \frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2$$

$$(a) \quad \frac{P}{P_{cr}} = \left[ \frac{2}{\pi} \arccos \frac{1.2}{0.8 + 1.2} \right]^2 = 0.34849 \quad P = 0.34849 P_{cr} = 67.7 \text{ kips} \quad \blacktriangleleft$$

$$M_{max} = P(e + y_{max}) = (67.7)(1.2 + 0.8) = 135.4 \text{ kip}\cdot\text{in}$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{67.7}{14.7} + \frac{135.4}{13.9} = 14.3 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 10.41



10.41 The steel bar  $AB$  has a  $\frac{3}{8} \times \frac{3}{8}$ -in. square cross section and is held by pins that are a fixed distance apart and are located at a distance  $e = 0.03$  in. from the geometric axis of the bar. Knowing that at temperature  $T_0$  the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point  $C$  if  $d = 0.01$  in. Use  $E = 29 \times 10^6$  psi. and the coefficient of thermal expansion  $\alpha = 6.5 \times 10^{-6}/^\circ\text{F}$ .

SOLUTION

$$A = \left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = 0.140625 \text{ in}^2$$

$$I = \frac{1}{12}\left(\frac{3}{8}\right)^4 = 1.64795 \times 10^{-5} \text{ in}^4$$

$$EI = (29 \times 10^6)(1.64795 \times 10^{-5}) = 47791 \text{ lb}\cdot\text{in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (47791)}{(8)^2} = 7370 \text{ lb.}$$

Calculate  $P$  using the secant formula

$$y_{max} = d = e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 1 + \frac{d}{e}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = \cos^{-1}\left(1 + \frac{d}{e}\right)^{-1} = \cos^{-1}\left(1 + \frac{0.01}{0.03}\right)^{-1} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (0.72273) \right]^2 = 0.21170 \quad P = 0.21170 P_{cr} = 1560.2 \text{ lb.}$$

Thermal analysis.

(1) Simple approximation by ignoring eccentricity.

$$\text{Total elongation} = \alpha L(\Delta T) - \frac{PL}{EA} = 0$$

$$\Delta T = \frac{PL}{EA} \frac{1}{\alpha L} = \frac{P}{EA\alpha} = \frac{1560.2}{(29 \times 10^6)(0.140625)(6.5 \times 10^{-6})} = 58.9^\circ\text{F} \blacktriangleleft$$

(2) Analysis with inclusion of eccentricity.

$$\text{Total elongation of centroidal axis} = \alpha L(\Delta T) - \frac{PL}{EA} = 2e \left. \frac{dy}{dx} \right|_{x=0}$$

To calculate  $\frac{dy}{dx}$ , differentiate eq. (10.26)

$$\frac{dy}{dx} = e \left( p \tan \frac{pL}{2} \cos px - p \sin px \right)$$

$$\text{At } x = 0 \quad \left. \frac{dy}{dx} \right|_{x=0} = ep \tan \frac{pL}{2} = e \sqrt{\frac{P}{EI}} \tan \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}$$

$$\text{The elongation of the centroidal axis is } 2e^2 \sqrt{\frac{P}{EI}} \tan\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

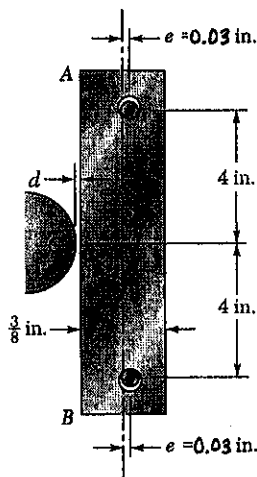
$$= (2)(0.03)^2 \sqrt{\frac{1560.2}{47791}} \tan(0.72273) = 286.8 \times 10^{-6} \text{ in.}$$

$$\alpha L(\Delta T) = \frac{PL}{EA} + 2e \left. \frac{dy}{dx} \right|_{x=0}$$

$$\Delta T = \frac{P}{EA\alpha} + \frac{286.8 \times 10^{-6}}{\alpha L} = 58.9 + \frac{286.8 \times 10^{-6}}{(6.5 \times 10^{-6})(8)} = 58.9 + 5.5^\circ$$

$$= 64.4^\circ\text{F} \blacktriangleleft$$

PROBLEM 10.42



10.41 The steel bar  $AB$  has a  $\frac{3}{8} \times \frac{3}{8}$ -in. square cross section and is held by pins that are a fixed distance apart and are located at a distance  $e = 0.03$  in. from the geometric axis of the bar. Knowing that at temperature  $T_0$  the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point  $C$  if  $d = 0.01$  in. Use  $E = 29 \times 10^6$  psi. and the coefficient of thermal expansion  $\alpha = 6.5 \times 10^{-6}/^\circ\text{F}$ .

10.42 For the bar of Prob. 10.41, determine the required distance  $d$  for which the bar will just make contact with point  $C$  when the temperature increases by  $120^\circ\text{F}$ .

SOLUTION

$$A = \left(\frac{3}{8}\right)\left(\frac{3}{8}\right) = 0.140625 \text{ in}^2$$

$$I = \frac{1}{12}\left(\frac{3}{8}\right)^4 = 1.64795 \times 10^{-3} \text{ in}^4$$

$$EI = (29 \times 10^6)(1.64795 \times 10^{-3}) = 47791 \text{ lb}\cdot\text{in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (47791)}{(8)^2} = 7370 \text{ lb.}$$

Calculate  $P$  from thermal analysis. To obtain an approximate value, neglect the effect of eccentricity in the thermal analysis.

$$\text{Total elongation} = \alpha L(\Delta T) - \frac{PL}{EA} = 0$$

$$P = EA\alpha(\Delta T) = (29 \times 10^6)(0.140625)(6.5 \times 10^{-6})(120) = 3181 \text{ lb.}$$

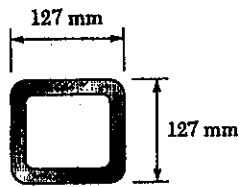
Calculate the deflection using the secant formula

$$\begin{aligned} d = y_{max} &= e \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = (0.03) \left[ \sec\left(\frac{\pi}{2} \sqrt{\frac{3181}{7370}}\right) - 1 \right] \\ &= (0.03) \left[ \sec(1.03197) - 1 \right] = (0.03)(0.94883) = 0.0285 \text{ in.} \end{aligned}$$

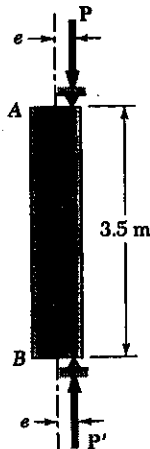
For an improved thermal analysis including eccentricity, see solution of Prob. 10.41.

PROBLEM 10.43

10.43 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used  $\sigma_y = 250$  MPa and  $E = 200$  GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load  $P$  when the eccentricity  $e$  is (a) 15 mm, (b) 7.5 mm. (Hint: Since the factor of safety must be applied to the load  $P$ , not to the stress, use Fig. 10.24 to determine  $P_T$ ).



$A = 3400 \text{ mm}^2$   
 $I = 7.93 \times 10^{-6} \text{ m}^4$   
 $r = 48.3 \text{ mm}$



SOLUTION

$A = 3400 \times 10^{-6} \text{ m}^2$        $r = 48.3 \times 10^{-3} \text{ m}$

$L_e = 3.5 \text{ m}$        $\frac{L_e}{r} = \frac{3.5}{48.3 \times 10^{-3}} = 72.46$

$c = \frac{127}{2} = 63.5 \text{ mm}$

(a)  $e = 15 \text{ mm}$        $\frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.24 with  $L_e/r = 72.46$  and  $ec/r^2 = 0.40829$

$P/A = 144.75 \text{ MPa} = 144.75 \times 10^6 \text{ Pa}$

$P = (144.75 \times 10^6)(3400 \times 10^{-6}) = 492 \times 10^3 \text{ N}$

Using factor of safety       $P_{all} = \frac{492 \times 10^3}{2.6} = 189 \times 10^3 \text{ N} = 189 \text{ kN}$        $\blacktriangleleft$

(b)  $e = 7.5 \text{ mm}$        $\frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

Using Fig. 10.24 with  $L_e/r = 72.46$  and  $ec/r^2 = 0.20415$

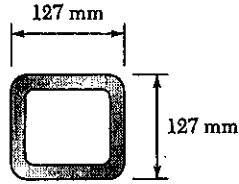
$P/A = 175.2 \text{ MPa} = 175.2 \times 10^6 \text{ Pa}$

$P = (175.2 \times 10^6)(3400 \times 10^{-6}) = 596 \times 10^3 \text{ N}$

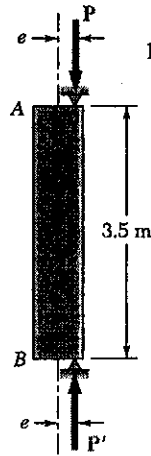
Using factor of safety       $P_{all} = \frac{596 \times 10^3}{2.6} = 229 \times 10^3 \text{ N} = 229 \text{ kN}$        $\blacktriangleleft$



**PROBLEM 10.44**



$A = 3400 \text{ mm}^2$   
 $I = 7.93 \times 10^{-6} \text{ m}^4$   
 $r = 48.3 \text{ mm}$



**10.43** A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used  $\sigma_r = 250 \text{ MPa}$  and  $E = 200 \text{ GPa}$ . Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load  $P$  when the eccentricity  $e$  is (a) 15 mm, (b) 7.5 mm. (Hint: Since the factor of safety must be applied to the load  $P$ , not to the stress, use Fig. 10.24 to determine  $P_r$ ).

**10.44** Solve Prob. 10.43, assuming that the length of the steel tube is increased to 5 m.

**SOLUTION**

$A = 3400 \times 10^{-6} \text{ m}^2$        $r = 48.3 \times 10^{-3} \text{ m}$

$L_e = 5 \text{ m}$        $\frac{L_e}{r} = \frac{5}{48.3 \times 10^{-3}} = 103.52$

$C = \frac{127}{2} = 63.5 \text{ mm}$

(a)  $e = 15 \text{ mm}$        $\frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.24 with  $\frac{L_e}{r} = 103.52$

and  $\frac{ec}{r^2} = 0.40829$  gives  $\frac{P}{A} = 112.75 \text{ MPa} = 112.75 \times 10^6 \text{ Pa}$

$P = (112.75 \times 10^6)(3400 \times 10^{-6}) = 383 \times 10^3 \text{ N}$

Using factor of safety  $P_{all} = \frac{383 \times 10^3}{2.6} = 147 \times 10^3 \text{ N} = 147 \text{ kN}$  ▶

(b)  $e = 7.5 \text{ mm}$        $\frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

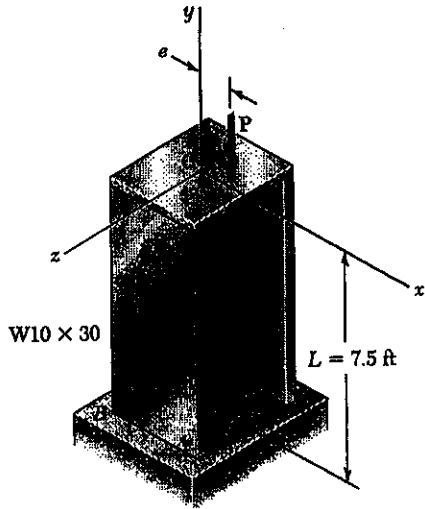
Using Fig. 10.24 gives  $\frac{P}{A} = 133.2 \text{ MPa} = 133.2 \times 10^6 \text{ Pa}$

$P = (133.2 \times 10^6)(3400 \times 10^{-6}) = 453 \times 10^3 \text{ N}$

Using factor of safety  $P_{all} = \frac{453 \times 10^3}{2.6} = 174 \times 10^3 \text{ N} = 174 \text{ kN}$  ▶

**PROBLEM 10.45**

10.45 An axial load  $P$  is applied to the  $W10 \times 30$  rolled-steel column  $BC$  that is free at its top  $C$  and fixed at its base  $B$ . Knowing that the eccentricity of the load is  $e = 0.5$  in. and that for the grade of steel used  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi., determine (a) the magnitude of  $P$  of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part a to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43.)



**SOLUTION**

$$W10 \times 30 \quad A = 8.84 \text{ in}^2 \quad r_y = 1.37 \text{ in.}$$

$$c = \frac{b_f}{2} = \frac{5.810}{2} = 2.905 \text{ in.} \quad I_y = 16.7 \text{ in}^4$$

$$L = 7.5 \text{ ft} = 90 \text{ in.} \quad L_e = 2L = 180 \text{ in.}$$

$$\frac{L_e}{r} = \frac{180}{1.37} = 131.39 \quad e = 0.5 \text{ in}$$

$$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$$

$$\text{Using Fig 10.24} \quad \frac{P}{A} = 10.47 \text{ ksi}$$

$$P = (10.47)(8.84) = 92.6 \text{ kips}$$

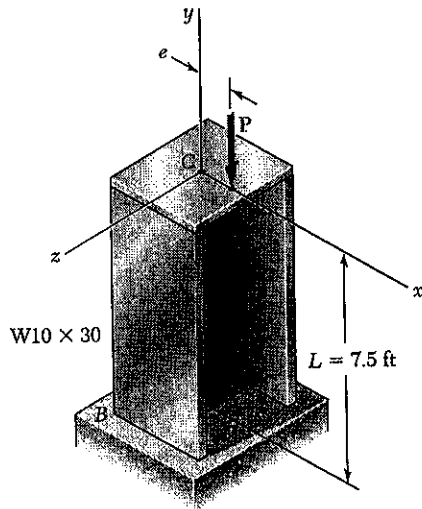
(a) Using factor of safety  $P_{all} = \frac{92.6}{2.4} = 38.6 \text{ kips}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000)(16.7)}{(180)^2} = 147.5 \text{ kips}$$

Using factor of safety  $P_{all} = \frac{147.5}{2.4} = 61.5 \text{ kips}$

(b)  $\text{ratio} = \frac{38.6}{61.5} = 0.628$

PROBLEM 10.46



10.45 An axial load  $P$  is applied to the  $W10 \times 30$  rolled-steel column  $BC$  that is free at its top  $C$  and fixed at its base  $B$ . Knowing that the eccentricity of the load is  $e = 0.5$  in. and that for the grade of steel used  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi., determine (a) the magnitude of  $P$  of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part a to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43.)

10.46 Solve Prob. 10.45, assuming that the length of the column is reduced to 5.0 ft.

SOLUTION

$W10 \times 30 \quad A = 8.84 \text{ in}^2 \quad I_y = 16.7 \text{ in}^4$   
 $r_y = 1.37 \text{ in} \quad c = \frac{bf}{2} = \frac{5.810}{2} = 2.905 \text{ in}$

$L = 5.0 \text{ ft} = 60 \text{ in} \quad L_e = 2L = 120 \text{ in.}$

$\frac{L_e}{r} = \frac{120}{1.37} = 87.6$

$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$

Using Fig 10.24  $\frac{P}{A} = 14.90 \text{ ksi} \quad P = (14.90)(8.84) = 131.7 \text{ kips}$

(a) Using factor of safety  $P_{all} = \frac{131.7}{2.4} = 54.9 \text{ kips}$   $\blacktriangleleft$

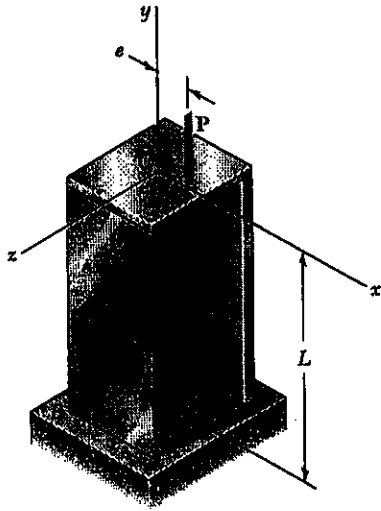
$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000)(16.7)}{(120)^2} = 332 \text{ kips}$

Using factor of safety  $P_{all} = \frac{332}{2.4} = 138.3 \text{ kips}$

(b) ratio =  $\frac{54.9}{138.3} = 0.397$   $\blacktriangleleft$

PROBLEM 10.47

10.47 A 55-kip axial load  $P$  is applied to a  $W8 \times 24$  rolled-steel column  $BC$  that is free at its top  $C$  and fixed at its base  $B$ . Knowing that the eccentricity of the load is  $e = 0.25$  in., determine the largest permissible length  $L$  if the allowable stress in the column is 14 ksi. Use  $E = 29 \times 10^6$  psi.



SOLUTION

Data:  $P = 55$  kips,  $e = 0.25$  in

$E = 29 \times 10^6$  psi = 29000 ksi

$W8 \times 24$ :  $A = 7.08$  in<sup>2</sup>  $b_f = 6.495$  in

$c = \frac{b_f}{2} = 3.25$  in,  $I_y = 18.3$  in<sup>4</sup>,  $r_y = 1.61$  in.

$\sigma_{max} = 14$  ksi

$$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left( \frac{A\sigma_{max}}{P} - 1 \right) = \frac{(1.61)^2}{(0.25)(3.25)} \left[ \frac{(7.08)(14)}{55} - 1 \right] = 2.5592$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.39075 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 1.16935$$

$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (1.16935) \right]^2 = 0.55418$$

$$P_{cr} = \frac{P}{0.55418} = \frac{\pi^2 EI}{L_e^2}$$

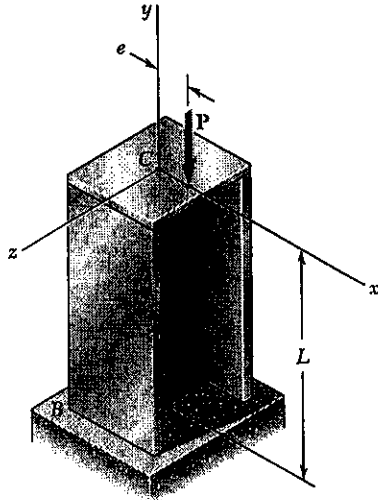
$$L_e^2 = \frac{0.55418 \pi^2 EI}{P} = \frac{0.55418 \pi^2 (29000)(18.3)}{55} = 52.78 \times 10^3 \text{ in}^2$$

$$L_e = 229.7 \text{ in} = 2L$$

$$L = 114.8 \text{ in} = 9.57 \text{ ft.} \quad \blacktriangleleft$$

**PROBLEM 10.48**

10.48 A 26-kip axial load  $P$  is applied to a  $W6 \times 12$  rolled-steel column  $BC$  that is free at its top  $C$  and fixed at its base  $B$ . Knowing that the eccentricity of the load is  $e = 0.25$  in., determine the largest permissible length  $L$  if the allowable stress in the column is 14 ksi. Use  $E = 29 \times 10^6$  psi.



**SOLUTION**

Data:  $P = 26$  kips,  $e = 0.25$  in.  
 $E = 29 \times 10^6$  psi = 29000 ksi

$W6 \times 12$ :  $A = 3.55$  in<sup>2</sup>,  $b_f = 4.000$  in  
 $c = \frac{b_f}{2} = 2.000$  in,  $I_y = 2.99$  in<sup>4</sup>,  $r_y = 0.918$  in.

$\sigma_{max} = 14$  ksi

$$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left( \frac{A\sigma_{max}}{P} - 1 \right) = \frac{(0.918)^2}{(0.25)(2.000)} \left[ \frac{(3.55)(14)}{26} - 1 \right] = 1.53635$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.65089 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.86204$$

$$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (0.86204) \right]^2 = 0.30117$$

$$P_{cr} = \frac{P}{0.30117} = \frac{\pi^2 EI}{L_e^2}$$

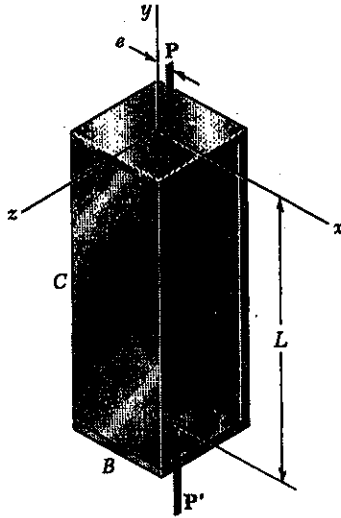
$$L_e^2 = \frac{0.30117 \pi^2 EI}{P} = \frac{0.30117 \pi^2 (29000)(2.99)}{26} = 9.913 \times 10^3 \text{ in}^2$$

$$L_e = 99.56 \text{ in} = 2L$$

$$L = 49.78 \text{ in} = 4.15 \text{ ft} \quad \blacktriangleleft$$

PROBLEM 10.49

10.49 Axial loads of magnitude  $P = 84 \text{ kN}$  are applied parallel to the geometric axis of a  $W200 \times 22.5$  rolled-steel column  $AB$  and intersect the  $x$  axis at a distance  $e$  from its geometric axis. Knowing that allowable stress  $\sigma_{all} = 75 \text{ MPa}$  and  $E = 200 \text{ GPa}$ , determine the largest permissible length  $L$  when (a)  $e = 5 \text{ mm}$ , (b)  $e = 12 \text{ mm}$ .



SOLUTION

Data:  $P = 84 \times 10^3 \text{ N}$      $E = 200 \times 10^9 \text{ Pa}$

$W 200 \times 22.5$      $A = 2860 \text{ mm}^2 = 2860 \times 10^{-6} \text{ m}^2$

$b_f = 102 \text{ mm}$      $c = \frac{b_f}{2} = 51 \text{ mm}$      $r_y = 22.3 \text{ mm}$

$I_y = 1.42 \times 10^6 \text{ mm}^4 = 1.42 \times 10^{-6} \text{ m}^4$

$\sigma_{all} = \sigma_{max} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2} \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

$$\sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{r^2}{ec} \left( \frac{A\sigma_{max}}{P} - 1 \right)$$

(a)  $e = 5 \text{ mm}$      $\sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{(22.3)^2}{(5)(51)} \left[ \frac{(2860 \times 10^{-6})(75 \times 10^6)}{84 \times 10^3} - 1 \right] = 3.0297$

$\cos \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 0.33006$      $\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 1.2344$

$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (1.2344) \right]^2 = 0.61757$

$P_{cr} = \frac{P}{0.61757} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.61757 \pi^2 EI}{P} = \frac{0.61757 \pi^2 (200 \times 10^9)(1.42 \times 10^{-6})}{84 \times 10^3} = 20.61 \text{ m}^2$

$L_e = 4.54 \text{ m}$      $L = L_e = 4.54 \text{ m}$      $\blacktriangleleft$

(b)  $e = 12 \text{ mm}$      $\sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{(22.3)^2}{(12)(51)} \left[ \frac{(2860 \times 10^{-6})(75 \times 10^6)}{84 \times 10^3} - 1 \right] = 1.26238$

$\cos \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 0.79216$      $\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.6564635$

$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (0.65646) \right]^2 = 0.17466$

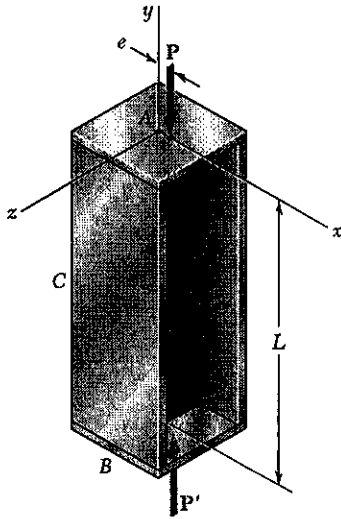
$P_{cr} = \frac{P}{0.17466} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.17466 \pi^2 EI}{P} = \frac{0.17466 \pi^2 (200 \times 10^9)(1.42 \times 10^{-6})}{84 \times 10^3} = 5.828 \text{ m}^2$

$L_e = 2.41 \text{ m}$      $L = L_e = 2.41 \text{ m}$      $\blacktriangleleft$

PROBLEM 10.50

10.50 Axial loads of magnitude  $P = 580 \text{ kN}$  are applied parallel to the geometric axis of a  $W250 \times 80$  rolled-steel column  $AB$  and intersect the  $x$  axis at a distance  $e$  from its geometric axis. Knowing that allowable stress  $\sigma_{all} = 75 \text{ MPa}$  and  $E = 200 \text{ GPa}$ , determine the largest permissible length  $L$  when (a)  $e = 5 \text{ mm}$ , (b)  $e = 10 \text{ mm}$ .



SOLUTION

Data:  $P = 580 \times 10^3 \text{ N}$        $E = 200 \times 10^9 \text{ Pa}$

$W 250 \times 80$        $A = 10200 \text{ mm}^2 = 10200 \times 10^{-6} \text{ m}^2$

$b_f = 255 \text{ mm}$        $c = \frac{b_f}{2} = 127.5 \text{ mm}$        $r_y = 65.0 \text{ mm}$

$I_y = 43.1 \times 10^6 \text{ mm}^4 = 43.1 \times 10^{-6} \text{ m}^4$

$\sigma_{all} = \sigma_{max} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{r_y^2}{ec} \left( \frac{A\sigma_{max}}{P} - 1 \right)$$

(a)  $e = 5 \text{ mm}$        $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{(65.0)^2}{(5)(127.5)} \left[ \frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1 \right] = 2.1139$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.47305$        $\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 1.07804$

$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (1.07804) \right]^2 = 0.47101$

$P_{cr} = \frac{P}{0.47101} = \frac{\pi^2 EI_y}{L_e^2}$

$L_e^2 = \frac{0.47101 \pi^2 EI_y}{P} = \frac{0.47101 \pi^2 (200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 69.09 \text{ m}^2$

$L_e = 8.31 \text{ m}$        $L = L_e = 8.31 \text{ m}$

(b)  $e = 10 \text{ mm}$        $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{(65)^2}{(10)(127.5)} \left[ \frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1 \right] = 1.05696$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.94611$        $\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.32980$

$\frac{P}{P_{cr}} = \left[ \frac{2}{\pi} (0.32980) \right]^2 = 0.044083$

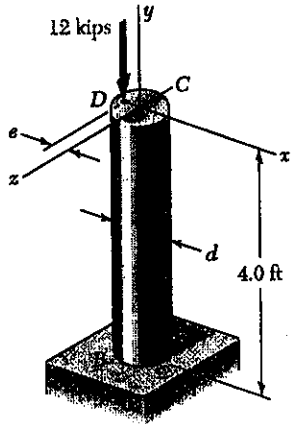
$P_{cr} = \frac{P}{0.044083} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.044083 \pi^2 EI}{P} = \frac{0.044083 \pi^2 (200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 6.466 \text{ m}^2$

$L_e = 2.54 \text{ m}$        $L = L_e = 2.54 \text{ m}$

PROBLEM 10.51

10.51 A 12-kip axial load is applied with an eccentricity  $e = 0.375$  in. to the circular steel rod  $BC$  that is free at its top  $C$  and fixed at its base  $B$ . Knowing that the stock of rods available for use have diameters in increments of  $\frac{1}{8}$  in. from 1.5 in. to 3.0 in., determine the lightest rod that may be used if  $\sigma_{all} = 15$  ksi. Use  $E = 29 \times 10^6$  psi.



SOLUTION

$$E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi} \quad d = \text{diameter (in.)}$$

$$A = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64} \quad c = \frac{1}{2} d \quad e = 0.375 \text{ in}$$

$$L = 4.0 \text{ ft} = 48 \text{ in} \quad L_e = 2L = 96 \text{ in}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000) \pi d^4}{(64)(96)^2} = 1.52449 d^4 \text{ kips}$$

$$r^2 = \frac{I}{A} = \frac{\pi d^4}{64} \cdot \frac{4}{\pi d^2} = \frac{d^2}{16} \quad P = 12 \text{ kips}$$

$$\frac{ec}{r^2} = \frac{(0.375)(\frac{1}{2}d)}{\frac{1}{16}d^2} = \frac{3}{d}$$

$$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

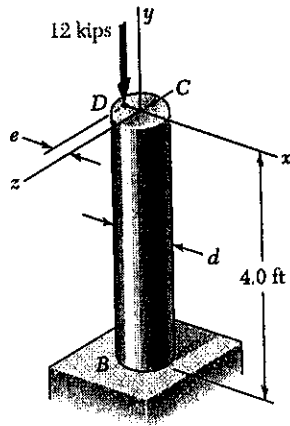
Use  $d = 2.125$  in.  $\blacktriangleleft$

$$\sigma_{max} = 11.90 \text{ ksi} < 15 \text{ ksi}$$

$d$ (in)	$A$ (in <sup>2</sup> )	$P_{cr}$ (kips)	$\frac{ec}{r^2}$	$\sigma_{max}$ (ksi)
2.25	3.976	39.07	1.3333	9.26
2.0	3.1416	24.39	1.5	16.49
2.125	3.546	31.09	1.4118	11.90



**PROBLEM 10.52**



10.51 A 12-kip axial load is applied with an eccentricity  $e = 0.375$  in. to the circular steel rod  $BC$  that is free at its top  $C$  and fixed at its base  $B$ . Knowing that the stock of rods available for use have diameters in increments of  $\frac{1}{8}$  in. from 1.5 in. to 3.0 in., determine the lightest rod that may be used if  $\sigma_{all} = 15$  ksi. Use  $E = 29 \times 10^6$  psi.

10.52 Solve Prob. 10.51, assuming that the 12-kip axial load will be applied to the rod with an eccentricity  $e = \frac{1}{2} d$ .

**SOLUTION**

$E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$        $d = \text{diameter (in)}$

$A = \frac{\pi}{4} d^2$        $I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{64} d^4$        $c = \frac{1}{2} d$        $e = \frac{1}{2} d$

$L = 4 \text{ ft} = 48 \text{ in}$        $L_e = 2L = 96 \text{ in}$

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000) (\frac{\pi d^4}{64})}{(96)^2} = 1.52449 d^4$

$r^2 = \frac{I}{A} = \frac{\frac{\pi d^4}{64}}{\frac{\pi d^2}{4}} = \frac{4}{16} d^2$        $P = 12 \text{ kips}$

$\frac{ec}{r^2} = \frac{(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{4}{16}d^2} = 4.0$

$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right] = \frac{P}{A} \left[ 1 + 4.0 \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$

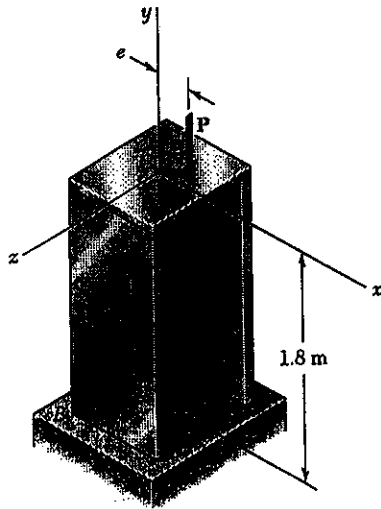
$d$ (in)	$A$ (in <sup>2</sup> )	$P_{cr}$ (kips)	$\sigma_{max}$ (ksi)
2.25	3.976	39.07	21.75
3.0	7.068	123.48	9.39
2.5	4.909	59.55	15.28
2.625	5.412	72.38	13.27

Use  $d = 2.625$  in.

$\sigma_{max} = 13.27 \text{ ksi} < 15 \text{ ksi}$

PROBLEM 10.53

10.53 An axial load of magnitude  $P = 220 \text{ kN}$  is applied at a point located on the  $x$  axis at a distance  $e = 6 \text{ mm}$  from the geometric axis of the wide-flange column  $BC$ . Knowing that  $E = 200 \text{ GPa}$ , chose the lightest W200 shape that may be used if  $\sigma_{all} = 120 \text{ MPa}$ .



SOLUTION

$$P = 220 \times 10^3 \text{ N} \quad L = 1.8 \text{ m} \quad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{3.6^2} = 152.3 \times 10^9 I_y \text{ N}$$

$$e = 6 \text{ mm} \quad c = \frac{b_f}{2} \quad \frac{ec}{r^2} = \frac{e b_f}{2 r_y^2} =$$

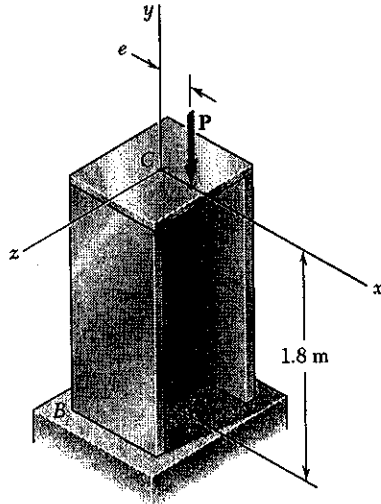
$$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

Shape	$A (10^6 \text{ m}^2)$	$b_f (\text{mm})$	$I_y (10^6 \text{ m}^4)$	$r_y (\text{mm})$	$P_{cr} (\text{kN})$	$\frac{ec}{r^2}$	$\sigma_{max} (\text{MPa})$
W200 x 41.7	5310	166	9.01	41.2	1372	0.2934	56.5
W200 x 26.6	3390	133	3.30	31.2	502.6	0.4099	117.4
W200 x 22.5	2860	102	1.42	22.3	*216.3		

\* < P

Use W200 x 26.6  $\Rightarrow \sigma_{max} = 117.4 \text{ MPa}$

PROBLEM 10.54



10.53 An axial load of magnitude  $P = 220$  kN is applied at a point located on the  $x$  axis at a distance  $e = 6$  mm from the geometric axis of the wide-flange column  $BC$ . Knowing that  $E = 200$  GPa, choose the lightest W200 shape that may be used if  $\sigma_{all} = 120$  MPa.

10.54 Solve Prob. 10.53, assuming that the magnitude of the axial load is  $P = 345$  kN.

SOLUTION

$P = 345 \times 10^3$  N       $L = 1.8$  m       $L_e = 2L = 3.6$  m

$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{(3.6)^2} = 152.3 \times 10^9 I_y$  N

$e = 6$  mm       $c = \frac{b_f}{2}$        $\frac{ec}{r^2} = \frac{e b_f}{2r^2}$

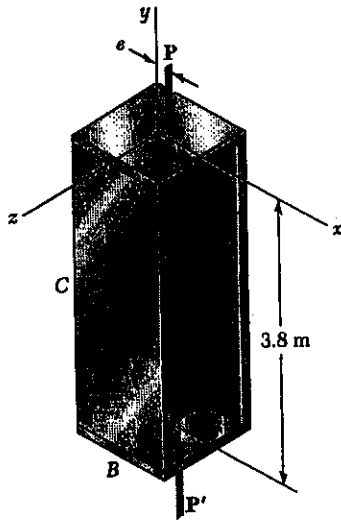
$\sigma_{max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$

Shape	$A (10^6 \text{ m}^2)$	$b_f$ (mm)	$I_y (10^6 \text{ m}^4)$	$r_y$ (mm)	$P_{cr}$ (kN)	$\frac{ec}{r^2}$	$\sigma_{max}$ (MPa)
W 200 x 41.7	5310	166	9.01	41.2	1372	0.2934	92.0
W 200 x 26.6	3390	133	3.30	31.2	502.6	0.4099	258
W 200 x 35.9	4580	165	7.64	40.8	1164	0.2974	109.5 ←
W 200 x 31.3	4000	134	4.10	32.0	624.4	0.3926	172.6

Use W 200 x 35.9 ←       $\sigma_{max} = 109.5$  MPa

**PROBLEM 10.55**

10.55 Axial loads of magnitude  $P = 175$  kN are applied to a point located on the  $x$  axis at a distance  $e = 12$  mm from the geometric axis of the  $W250 \times 44.8$  rolled-steel column  $AB$ . Knowing that  $\sigma_y = 250$  MPa and  $E = 200$  GPa, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load  $P$ , not to the stresses, use Fig. 10.24 to determine  $P_r$ .)



**SOLUTION**

For  $W 250 \times 44.8$       $A = 5720 \text{ mm}^2$  ,      $r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm}$       $L_e/r = 108.26$

$C = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm}$       $e = 12 \text{ mm}$

$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$

Using Fig 10.24 with  $L_e/r = 108.26$  and  $\frac{ec}{r^2} = 0.72077$

$P_r/A = 90.37 \text{ MPa} = 90.37 \times 10^6 \text{ N/m}^2$

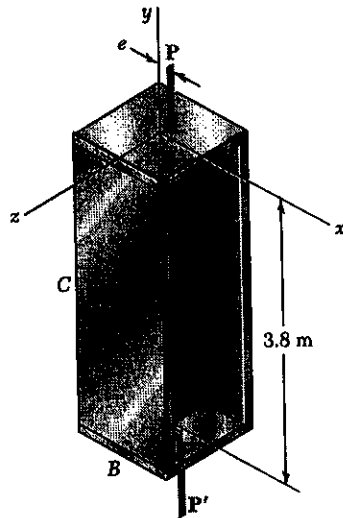
$P_r = A P_r/A = (5720 \times 10^{-6})(90.37 \times 10^6) = 517 \times 10^3 \text{ N} = 517 \text{ kN}$

F.S. =  $\frac{P_r}{P} = \frac{517}{175} = 2.95$

**PROBLEM 10.56**

10.55 Axial loads of magnitude  $P = 175$  kN are applied to a point located on the  $x$  axis at a distance  $e = 12$  mm from the geometric axis of the  $W250 \times 44.8$  rolled-steel column  $AB$ . Knowing that  $\sigma_y = 250$  MPa and  $E = 200$  GPa, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load  $P$ , not to the stresses, use Fig. 10.24 to determine  $P_r$ .)

10.56 Solve Prob. 10.55, assuming that  $e = 16$  mm and  $P = 155$  kN.



**SOLUTION**

For  $W 250 \times 44.8$       $A = 5720 \text{ mm}^2$  ,      $r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm}$       $L_e/r = 108.26$

$C = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm}$       $e = 16 \text{ mm}$

$\frac{ec}{r^2} = \frac{(16)(74)}{(35.1)^2} = 0.96103$

Using Fig 10.24 with  $L_e/r = 108.26$  and  $\frac{ec}{r^2} = 0.96103$

$P_r/A = 81.17 \text{ MPa} = 81.17 \text{ N/m}^2$

$P_r = A(P_r/A) = (5720 \times 10^{-6})(81.17 \times 10^6) = 464 \times 10^3 \text{ N} = 464 \text{ kN}$

F.S. =  $\frac{P_r}{P} = \frac{464}{155} = 3.00$

**PROBLEM 10.57**

**10.57** Using allowable stress design, determine the allowable centric load for a column of 6.5-m effective length that is made from the following rolled-steel shape: (a) W250 × 49.1, (b) W250 × 80. Use  $\sigma_y = 250$  MPa and  $E = 200$  GPa.

**SOLUTION**

Steel:  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.664$

(a) W250 × 49.1  $A = 6250 \times 10^{-6} \text{ m}^2$   $r_{\min} = 49.2 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{6.5}{49.2 \times 10^{-3}} = 132.11 > C_c$

$\sigma_{\text{all}} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(132.11)^2} = 58.9 \times 10^6 \text{ Pa}$

$P_{\text{all}} = A \sigma_{\text{all}} = (6250 \times 10^{-6})(58.9 \times 10^6) = 368 \times 10^3 \text{ N} = 368 \text{ kN}$  ▶

(b) W250 × 80  $A = 10200 \times 10^{-6} \text{ m}^2$   $r_{\min} = 65.0 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{6.5}{65.0 \times 10^{-3}} = 100 < C_c$   $\frac{L/r}{C_c} = 0.79577$

F.S. =  $\frac{5}{3} + \frac{3}{8}(0.79577) - \frac{1}{8}(0.79577)^3 = 1.90209$

$\sigma_{\text{all}} = \frac{\sigma_y}{\text{F.S.}} \left[ 1 - \frac{1}{2} \left( \frac{L/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.90209} \left[ 1 - \frac{1}{2} (0.79577)^2 \right] = 89.82 \times 10^6 \text{ Pa}$

$P_{\text{all}} = A \sigma_{\text{all}} = (10200 \times 10^{-6})(89.82 \times 10^6) = 916 \times 10^3 \text{ N} = 916 \text{ kN}$  ▶

**PROBLEM 10.58**

**10.58** A W8 × 31 rolled-steel shape is used to form a column of 21-ft effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (a)  $\sigma_y = 36$  ksi, (b)  $\sigma_y = 50$  ksi. Use  $E = 29 \times 10^6$  psi.

**SOLUTION**

Steel:  $E = 29000 \text{ ksi}$   $W8 \times 31$   $A = 9.13 \text{ in}^2$   $r_{\min} = 2.02 \text{ in}$

$L_e = 21 \text{ ft} = 252 \text{ in}$   $L_e/r = 124.75$

(a)  $\sigma_y = 36 \text{ ksi}$   $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$

$L_e/r < C_c$   $\frac{L/r}{C_c} = 0.98932$

F.S. =  $\frac{5}{3} + \frac{3}{8}(0.98932) - \frac{1}{8}(0.98932)^3 = 1.91662$

$\sigma_{\text{all}} = \frac{\sigma_y}{\text{F.S.}} \left[ 1 - \frac{1}{2} \left( \frac{L/r}{C_c} \right)^2 \right] = \frac{36}{1.91662} \left[ 1 - \frac{1}{2} (0.98932)^2 \right] = 9.59 \text{ ksi}$

$P_{\text{all}} = \sigma_{\text{all}} A = (9.59)(9.13) = 87.6 \text{ kips}$  ▶

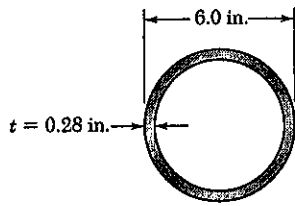
(b)  $\sigma_y = 50 \text{ ksi}$   $C_c = \sqrt{\frac{2\pi^2 (29000)}{50}} = 107.00$

$L_e/r > C_c$   $\sigma_{\text{all}} = \frac{\pi^2 E}{\frac{32}{12} (L/r)^2} = 9.59 \text{ ksi}$

$P_{\text{all}} = \sigma_{\text{all}} A = (9.59)(9.13) = 87.6 \text{ kips}$  ▶

**PROBLEM 10.59**

10.59 A steel pipe having the cross section shown is used as a column. Using allowable stress design, determine the allowable centric load if the effective length of the column is (a) 18 ft, (b) 26 ft. Use  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi.



**SOLUTION**

$$C_o = \frac{d_o}{2} = 3.0 \text{ in.} \quad C_i = C_o - t = 2.72 \text{ in.}$$

$$A = \pi (C_o^2 - C_i^2) = 5.0316 \text{ in}^2 \quad r = \sqrt{\frac{I}{A}} = 2.0247 \text{ in}$$

$$I = \frac{\pi}{4} (C_o^4 - C_i^4) = 20.627 \text{ in}^4$$

Steel:  $E = 29000$  ksi  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$

(a)  $l_e = 18 \text{ ft} = 216 \text{ in.}$   $l_e/r = 106.68 < C_c$   $\frac{l_e/r}{C_c} = 0.84601$

F.S. =  $\frac{5}{3} + \frac{3}{8}(0.84601) - \frac{1}{8}(0.84601)^3 = 1.9082$

$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[ 1 - \frac{1}{2} \left( \frac{l_e/r}{C_c} \right)^2 \right] = \frac{36}{1.9082} \left[ 1 - \frac{1}{2} (0.84601)^2 \right] = 12.11 \text{ ksi}$

$P_{all} = \sigma_{all} A = (12.11)(5.0316) = 61.0 \text{ kips}$

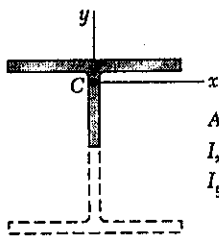
(b)  $l_e = 26 \text{ ft} = 312 \text{ in.}$   $l_e/r = 154.097 > C_c$

$\sigma_{all} = \frac{\pi^2 E}{1.92 (l_e/r)^2} = \frac{\pi^2 (29000)}{(1.92)(154.097)^2} = 6.28 \text{ ksi}$

$P_{all} = \sigma_{all} A = (6.28)(5.0316) = 31.6 \text{ kips}$

**PROBLEM 10.60**

10.60 A column is made from half of a W360 x 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (a) 4.0 m, (b) 6.5 m. Use  $\sigma_y = 345$  MPa and  $E = 200$  GPa.



$A = 13.8 \times 10^3 \text{ mm}^2$   
 $I_x = 26.0 \times 10^6 \text{ mm}^4$   
 $I_y = 142.0 \times 10^6 \text{ mm}^4$

**SOLUTION**

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.8 \times 10^3}} = 43.406 \text{ mm}$   
 $= 43.406 \times 10^{-3} \text{ m}$

$A = 13.8 \times 10^{-3} \text{ m}^2$

Steel  $C_c = \frac{2\pi^2 E}{\sigma_y} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.97$

(a)  $l_e = 4.0 \text{ m}$   $\frac{l_e}{r} = 92.153 < C_c$   $\frac{l_e/r}{C_c} = 0.86149$

F.S. =  $\frac{5}{3} + \frac{3}{8}(0.86149) - \frac{1}{8}(0.86149)^3 = 1.9098$

$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[ 1 - \frac{1}{2} \left( \frac{l_e/r}{C_c} \right)^2 \right] = \frac{345 \times 10^6}{1.9098} \left[ 1 - \frac{1}{2} (0.86149)^2 \right] = 113.61 \times 10^6 \text{ Pa}$

$P_{all} = \sigma_{all} A = (113.61 \times 10^6)(13.8 \times 10^{-3}) = 1568 \times 10^3 \text{ N} = 1568 \text{ kN}$

(b)  $l_e = 6.5 \text{ m}$   $\frac{l_e}{r} = 149.75 > C_c$

$\sigma_{all} = \frac{\pi^2 E}{1.92 (l_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(1.92)(149.75)^2} = 45.845 \times 10^6 \text{ Pa}$

$P_{all} = \sigma_{all} A = (45.845 \times 10^6)(13.8 \times 10^{-3}) = 633 \times 10^3 \text{ N} = 633 \text{ kN}$

## PROBLEM 10.61

10.61 A 3.5-m effective length column is made of sawn lumber with a 114 × 140-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is  $\sigma_c = 7.6$  MPa and  $E = 10$  GPa, determine the maximum allowable centric load for the column.

## SOLUTION

Sawn lumber:  $c = 0.8$ ,  $\sigma_c = 7.6$  MPa  $K_{CE} = 0.3$   $E = 10000$  MPa

$$A = (114)(140) = 15960 \text{ mm}^2 = 15960 \times 10^{-6} \text{ m}^2$$

$$d = 114 \text{ mm} = 114 \times 10^{-3} \text{ m}$$

$$L/d = 3.5 / 114 \times 10^{-3} = 30.70$$

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.3)(10000)}{(30.70)^2} = 3.1827 \text{ MPa} \quad \frac{\sigma_{CE}}{\sigma_c} = 0.41878$$

$$U = \frac{1 + \sigma_{CE}/\sigma_c}{2c} = \frac{1.41878}{(2)(0.8)} = 0.88673 \quad v = \frac{\sigma_{CE}/\sigma_c}{c} = 0.523475$$

$$C_p = U - \sqrt{U^2 - v} = 0.37408$$

$$\sigma_{all} = \sigma_c C_p = (7.6)(0.37408) = 2.84 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (2.84 \times 10^6)(15960 \times 10^{-6}) = 45.4 \times 10^3 \text{ N} = 45.4 \text{ kN}$$

## PROBLEM 10.62

10.62 A sawn lumber column with a 7.5 × 5.5-in. cross section has a 18-ft effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is  $\sigma_c = 1220$  psi and that  $E = 1.3 \times 10^6$  psi, determine the maximum allowable centric load for the column.

## SOLUTION

Sawn lumber:  $c = 0.8$ ,  $\sigma_c = 1220$  psi  $E = 1.3 \times 10^6$  psi  $K_{CE} = 0.3$

$$A = (7.5)(5.5) = 41.25 \text{ in}^2 \quad d = 5.5 \text{ in.} \quad L = 18 \text{ ft} = 216 \text{ in}$$

$$L/d = 216 / 5.5 = 39.273$$

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.3)(1.3 \times 10^6)}{(39.273)^2} = 252.86 \text{ ps} \quad \frac{\sigma_{CE}}{\sigma_c} = 0.20726$$

$$U = \frac{1 + \sigma_{CE}/\sigma_c}{2c} = \frac{1.20726}{(2)(0.8)} = 0.754537 \quad v = \frac{\sigma_{CE}/\sigma_c}{c} = 0.259075$$

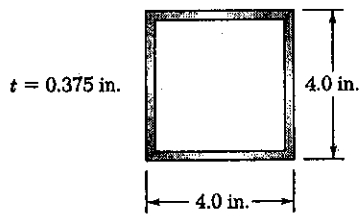
$$C_p = U - \sqrt{U^2 - v} = 0.197535$$

$$\sigma_{all} = \sigma_c C_p = (1220)(0.197535) = 241.0 \text{ psi}$$

$$P_{all} = \sigma_{all} A = (241.0)(41.25) = 9.94 \times 10^3 \text{ lb.} = 9.94 \text{ kips}$$

**PROBLEM 10.63**

10.63 A compression member has the cross section shown and an effective length of 5 ft. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.



**SOLUTION**

$$b_o = 4.0 \quad b_i = b_o - 2t = 3.25 \text{ in.}$$

$$A = (4.0)^2 - (3.25)^2 = 5.4375 \text{ in}^2$$

$$I = \frac{1}{12} [(4.0)^4 - (3.25)^4] = 12.036 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{12.036}{5.4375}} = 1.488 \text{ in.} \quad L_e = 5 \text{ ft} = 60 \text{ in.}$$

$$\frac{L_e}{r} = \frac{60}{1.488} = 40.33 < 55 \quad \text{for 2014-T6 aluminum alloy}$$

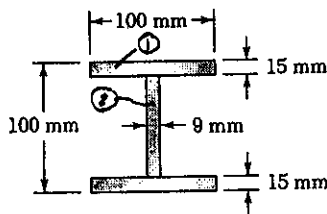
$$\sigma_{all} = 30.7 - 0.23(L/r) = 30.7 - (0.23)(40.33) = 21.42 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (21.42)(5.4375) = 116.5 \text{ kips}$$

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**PROBLEM 10.64**

10.64 A compression member has the cross section shown and an effective length of 1.55 m. Knowing that the aluminum alloy used is 6061-T6, determine the allowable centric load.



**SOLUTION**

$$I_{x1} = \frac{1}{12} (100)(15)^3 + (100)(15)(42.5)^2 = 2.7375 \times 10^6 \text{ mm}^4$$

$$I_{x2} = \frac{1}{12} (9)(70)^3 = 257.25 \times 10^3 \text{ mm}^4$$

$$I_x = 2I_{x1} + I_{x2} = 5.73225 \times 10^6 \text{ mm}^4$$

$$I_y = 2 \left[ \frac{1}{12} (15)(100)^3 \right] + \frac{1}{12} (70)(9)^3 = 2.50425 \times 10^6 \text{ mm}^4$$

$$A = 2(100)(15) + (9)(70) = 3630 \text{ mm}^2 = 3630 \times 10^{-6} \text{ m}^2$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.50425 \times 10^6}{3630}} = 26.265 \text{ mm} = 26.265 \times 10^{-3} \text{ m}$$

$$L_e = 1.55 \text{ m} \quad L_e/r = 59.01 < 66 \quad (6061-T6 \text{ aluminum})$$

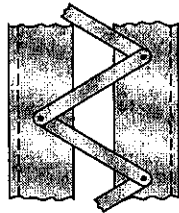
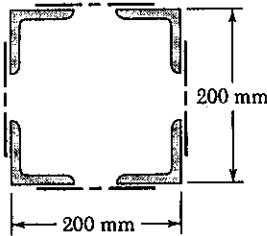
$$\sigma_{all} = 139 - 0.868(L/r) = 139 - (0.868)(59.01) = 87.78 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (87.78 \times 10^6)(3630 \times 10^{-6}) = 319 \times 10^3 \text{ N} = 319 \text{ kN}$$



**PROBLEM 10.65**

**10.65** A column of 6.4-m effective length is obtained by connecting four  $89 \times 89 \times 9.5$ -mm steel angles with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use  $\sigma_y = 345$  MPa and  $E = 200$  GPa.



**SOLUTION**

Steel:  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.97$

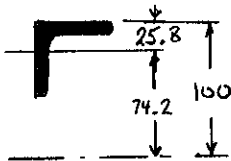
89x89x9.5 mm angle

$A_L = 1600 \text{ mm}^2$

$x = 25.8 \text{ mm}$

$I_x = 1.19 \times 10^6 \text{ mm}^2$

$d = 100 - x = 74.2 \text{ mm}$



$I = 4(Ad^2 + I_x) = 4[(1600)(74.2)^2 + 1.19 \times 10^6] = 89.976 \times 10^6 \text{ mm}^4$

$A = 4A_L = 6400 \text{ mm}^2 = 6400 \times 10^{-6} \text{ m}^2$

$r = \sqrt{\frac{I}{A}} = 79.053 \text{ mm} = 79.053 \times 10^{-3} \text{ m}$

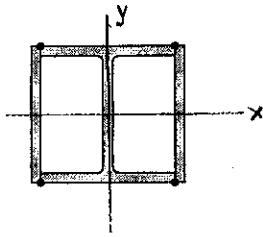
$\frac{L_e}{r} = \frac{6.4}{79.053 \times 10^{-3}} = 80.958 < C_c \quad \frac{L_e/r}{C_c} = 0.75683$

F.S. =  $\frac{5}{3} + \frac{2}{3}(0.75683) - \frac{1}{8}(0.75683)^2 = 1.8963$

$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{345 \times 10^6}{1.8963} \left[ 1 - \frac{1}{2} (0.75683)^2 \right] = 129.83 \times 10^6 \text{ Pa}$

$P_{all} = \sigma_{all} A = (129.83 \times 10^6)(6400 \times 10^{-6}) = 831 \times 10^3 \text{ N} = 831 \text{ kN} \quad \blacktriangleleft$

**PROBLEM 10.68**



10.68 A column of 23-ft effective length is obtained by welding two  $\frac{3}{8}$ -in. steel plates to a W10 × 33 rolled-steel shape as shown. Using allowable stress design, determine the allowable centric load for the column. Use  $\sigma_r = 50$  ksi and  $E = 29 \times 10^6$  psi.

**SOLUTION**

For W10 × 33  $A = 9.71 \text{ in}^2$ ,  $d = 9.73 \text{ in}$ ,  $b_f = 7.960 \text{ in}$   
 $I_x = 170 \text{ in}^4$ ,  $I_y = 36.6 \text{ in}^4$

For column:  $A = 9.71 + (2)(\frac{3}{8})(9.73) = 17.0075 \text{ in}^2$   
 $I_x = 170 + (2)(\frac{1}{12})(\frac{3}{8})(9.73)^3 = 227.57 \text{ in}^4$   
 $I_y = 36.6 + (2)[(\frac{3}{8})(9.73)(\frac{7.960}{2} + \frac{3}{16})^2 + \frac{1}{12}(9.73)(\frac{3}{8})^3]$   
 $= 36.6 + (2)[63.37 + 0.043] = 163.43 \text{ in}^4$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{163.43}{17.0075}} = 3.100 \text{ in}$

Steel:  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2(29000)}{50}} = 107.00$

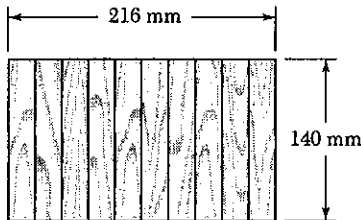
$L_c = 23 \text{ ft} = 276 \text{ in}$ ,  $\frac{L_c}{r} = \frac{276}{3.100} = 89.03 < C_c$ ,  $\frac{L_c/r}{C_c} = 0.83208$

F.S. =  $\frac{5}{3} + \frac{3}{8}(0.83208) - \frac{1}{8}(0.83208)^3 = 1.9067$

$\sigma_{all} = \frac{\sigma_r}{F.S.} [1 - \frac{1}{2}(\frac{L_c/r}{C_c})^2] = \frac{50}{1.9067} [1 - \frac{1}{2}(0.83208)^2] = 17.145 \text{ ksi}$

$P_{all} = \sigma_{all} A = (17.145)(17.0075) = 292 \text{ kips}$

**PROBLEM 10.69**



10.69 A rectangular column with a 4.4-m effective length is made of glued laminated wood. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is  $\sigma_c = 8.3$  MPa and that  $E = 10$  GPa, determine the maximum allowable centric load for the column.

**SOLUTION**

Glued laminated column  $C = 0.9$ ,  $K_{CE} = 0.418$

$\sigma_c = 8.3 \text{ MPa}$ ,  $E = 10000 \text{ MPa}$

$A = (216)(140) = 30240 \text{ mm}^2 = 30240 \times 10^{-6} \text{ m}^2$

$d = 140 \text{ mm} = 0.140 \text{ m}$ ,  $L = 4.4 \text{ m}$ ,  $L/d = 31.429$

$\sigma_{cE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.418)(10000)}{(31.429)^2} = 4.2318 \text{ MPa}$ ,  $\sigma_{cE}/\sigma_c = 0.50986$

$U = \frac{1 + \sigma_{cE}/\sigma_c}{2C} = \frac{1.50986}{(2)(0.9)} = 0.838811$ ,  $V = \frac{\sigma_{cE}/\sigma_c}{C} = 0.566111$

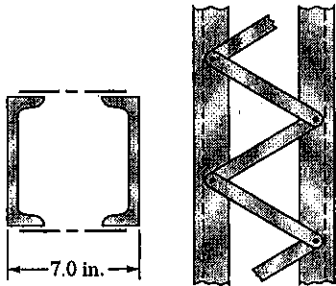
$C_p = U - \sqrt{U^2 - V} = 0.46801$

$\sigma_{all} = \sigma_c C_p = (8.3)(0.46801) = 3.8845 \text{ MPa}$

$P_{all} = \sigma_{all} A = (3.8845 \times 10^6)(30240 \times 10^{-6}) = 117.5 \times 10^3 \text{ N} = 117.5 \text{ kN}$

**PROBLEM 10.66**

10.66 A column of 21-ft effective length is obtained by connecting two C10 × 20 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi.



**SOLUTION**



C10 × 20     $A = 5.88 \text{ in}^2$      $x = 0.606 \text{ in}$   
 $I_x = 78.9 \text{ in}^4$      $I_y = 2.81 \text{ in}^4$   
 $d = 3.5 - x = 2.894 \text{ in}$   
 For the column:  $A = (2)(5.88) = 11.76 \text{ in}^2$   
 $I_x = (2)(78.9) = 157.8 \text{ in}^4$   
 $I_y = 2[2.81 + (5.88)(2.894)^2] = 104.11 \text{ in}^4$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{104.11}{11.76}} = 2.975 \text{ in}$      $L_e = 21 \text{ ft} = 252 \text{ in}$

$\frac{L_e}{r} = 84.69$      $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

$\frac{L_e}{r} < C_c$      $\frac{L_e/r}{C_c} = 0.67165$

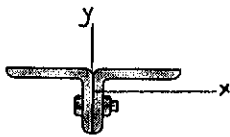
F.S. =  $\frac{5}{3} + \frac{3}{8}(0.67165) - \frac{1}{8}(0.67165)^3 = 1.8807$

$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.8807} \left[ 1 - \frac{1}{2} (0.67165)^2 \right] = 14.82 \text{ ksi}$

$P_{all} = \sigma_{all} A = (14.82)(11.76) = 174.3 \text{ kips}$

**PROBLEM 10.67**

10.67 A compression member of 2.3-m effective length is obtained by bolting together two 127 × 76 × 12.7-mm steel angles as shown. Using allowable stress design, determine the allowable centric load for the column. Use  $\sigma_y = 250$  MPa and  $E = 200$  GPa.



**SOLUTION**

Steel:  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(200 \times 10^9)}{250 \times 10^6}} = 125.66$

L 127 × 76 × 12.7 mm    Table gives  $A = 2420 \text{ mm}^2$ ,  $I_x = 3.93 \times 10^6 \text{ mm}^4$

$y = 44.4 \text{ mm}$ ,  $I_y = 1.06 \times 10^6 \text{ mm}^4$ ,  $x = 19.0 \text{ mm}$ ,  $r_y =$

For column     $I_x = 2(I_y)_{angle} = (2)(1.06 \times 10^6) = 2.12 \times 10^6 \text{ mm}^4$   
 $I_y > I_x \therefore I_{min} = I_x = 2.12 \times 10^6 \text{ mm}^4 = 2.12 \times 10^{-6} \text{ m}^4$   
 $A = 2A_1 = 4840 \text{ mm}^2 = 4840 \times 10^{-6} \text{ m}^2$   
 $r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.12 \times 10^6}{4840 \times 10^{-6}}} = 20.93 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{2.3}{20.93 \times 10^{-3}} = 109.90 < C_c$      $\frac{L_e/r}{C_c} = 0.87455$

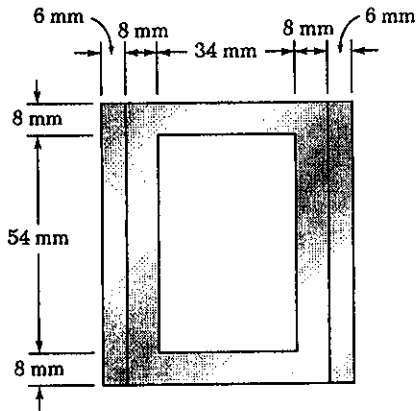
F.S. =  $\frac{5}{3} + \frac{3}{8}(0.87455) - \frac{1}{8}(0.87455)^3 = 1.9110$

$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.9110} \left[ 1 - \frac{1}{2} (0.87455)^2 \right] = 80.79 \times 10^6 \text{ Pa}$

$P_{all} = \sigma_{all} A = (80.79 \times 10^6)(4840 \times 10^{-6}) = 391 \times 10^3 \text{ N} = 391 \text{ kN}$

PROBLEM 10.70

10.70 An aluminum structural tube is reinforced by riveting two plates to it as shown for use as a column of 1.7-m effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.



SOLUTION

$$b_o = 6 + 8 + 34 + 8 + 6 = 62 \text{ mm}$$

$$b_i = 34 \text{ mm}$$

$$h_o = 8 + 54 + 8 = 70 \text{ mm}$$

$$h_i = 54 \text{ mm}$$

$$A = b_o h_o - b_i h_i = (62)(70) - (34)(54) \\ = 2.504 \times 10^3 \text{ mm}^2 = 2.504 \times 10^{-3} \text{ m}^2$$

$$I_x = \frac{1}{12} [b_o h_o^3 - b_i h_i^3] = \frac{1}{12} [(62)(70)^3 - (34)(54)^3] \\ = 1.32602 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12} [h_o b_o^3 - h_i b_i^3] = \frac{1}{12} [(70)(62)^3 - (54)(34)^3] = 1.21337 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.21337 \times 10^6}{2.504 \times 10^3}} = 22.013 \text{ mm} = 22.013 \times 10^{-3} \text{ m} \quad L = 1.7 \text{ m}$$

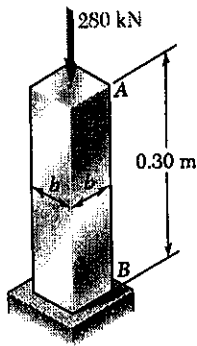
$$\frac{L}{r} = \frac{1.7}{22.013 \times 10^{-3}} = 77.23 > 55 \text{ for aluminum alloy 2014-T6}$$

$$\sigma_{\text{all}} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{77.23^2} = 62.37 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (62.37 \times 10^6)(2.504 \times 10^{-3}) = 156.2 \times 10^3 \text{ N} = 156.2 \text{ kN} \quad \blacktriangleleft$$

**PROBLEM 10.71**

10.71 A 280-kN centric load is applied to the column shown, that is free at its top *A* and fixed at its base *B*. Using aluminum alloy 2014-T6, select the smallest square cross section that can be used.



**SOLUTION**

$$L_e = 2L = (2)(0.30) = 0.60 \text{ m}$$

$$A = b^2 \quad I = \frac{1}{12} b^4 \quad r = \sqrt{\frac{I}{A}} = \frac{b}{\sqrt{12}}$$

$$\frac{L}{r} = \frac{0.60 \sqrt{12}}{b} = \frac{2.0785}{b}$$

2014-T6 aluminum alloy

Assume  $\frac{L}{r} < 55$        $\sigma_{all} = 212 - 1.585(L/r) = 212 - (1.585)(2.0785/b)$   
 $= (212 - \frac{3.294}{b}) \text{ MPa} = [212 - \frac{3.294}{b}] (10^6) \text{ Pa}$

$$P_{all} = \sigma_{all} A = [212 b^2 - 3.294 b] (10^6) = 280 \times 10^3$$

$$212 b^2 - 3.294 b - 280 \times 10^{-3} = 0$$

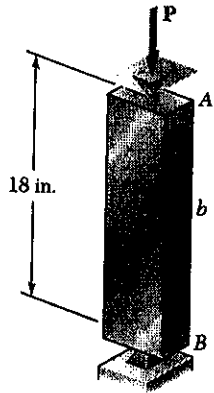
$$b = \frac{3.294 + \sqrt{(3.294)^2 + (4)(212)(280 \times 10^{-3})}}{(2)(212)} = 44.9 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{2.0785}{b} = \frac{2.0785}{44.9 \times 10^{-3}} = 46.26 < 55$$

Answer:  $b = 44.9 \times 10^{-3} \text{ m} = 44.9 \text{ mm}$

PROBLEM 10.72

10.72 A 16-kip centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension  $b$  that can be used.



SOLUTION

$$L_e = L = 18 \text{ in} \quad A = 2b^2 \quad I_{\min} = \frac{1}{12}(2b)(b)^3 = \frac{1}{6}b^4$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \frac{b}{\sqrt{12}} \quad \frac{L}{r} = \frac{18\sqrt{12}}{b} = \frac{62.354}{b}$$

6061-T aluminum alloy. Assume  $\frac{L}{r} < 66$

$$\sigma_{\text{all}} = 20.2 - 0.126(L/r) = 20.2 - (0.126) \frac{62.354}{b}$$

$$= 20.2 - \frac{7.8566}{b} \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = \left(20.2 - \frac{7.8566}{b}\right)(2b^2) = 40.4 b^2 - 15.713 b \text{ kip}$$

$$40.4 b^2 - 11.111 b = 16 \quad b = \frac{15.713 + \sqrt{(15.713)^2 + (4)(40.4)(16)}}{(2)(40.4)} = 0.853 \text{ in}$$

$$\frac{L}{r} = \frac{62.354}{b} = \frac{62.354}{0.853} = 73.09 > 66 \quad \text{Assumption not verified.}$$

Assume  $\frac{L}{r} > 66$   $\sigma_{\text{all}} = \frac{51000}{(L/r)^2} = \frac{51000 b^2}{(62.354)^2} = 13.117 b^2 \text{ ksi}$

$$P_{\text{all}} = \sigma_{\text{all}} A = (13.117 b^2)(2b^2) = 26.234 b^4 = 16 \text{ kips}$$

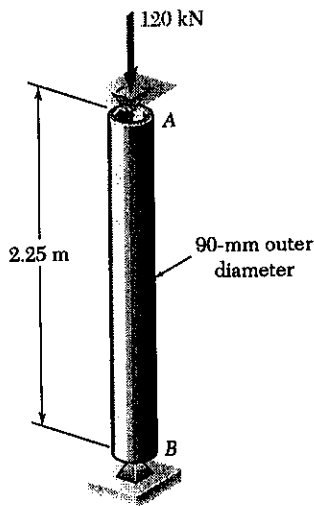
$$b = \sqrt[4]{\frac{16}{26.234}} = 0.884 \text{ in.}$$

$$\frac{L}{r} = \frac{62.354}{0.884} = 70.56 > 66 \quad \text{Assumption verified.}$$

$$b = 0.884 \text{ in.}$$

PROBLEM 10.73

10.73 An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thickness in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.



SOLUTION

$$L = 2250 \text{ mm}, P = 120 \times 10^3 \text{ N} \quad r_o = 45 \text{ mm}$$

$$r_i = r_o - t \quad A = \pi(r_o^2 - r_i^2) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$r = \sqrt{I/A}$$

For 2014-T6 aluminum alloy

$$\sigma_{all} = 212 - 1.585(L/r) \text{ MPa} \quad \text{if } L/r < 55$$

$$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} \text{ MPa} \quad \text{if } L/r > 55$$

$$P_{all} = \sigma_{all} A$$

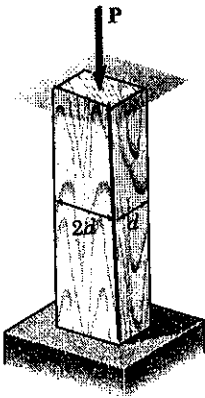
Calculate  $P_{all}$  for each thickness.

$t$ mm	$r_i$ mm	$A$ $\text{mm}^2$	$I$ $10^6 \text{ mm}^4$	$r$ mm	$L/r$	$\sigma_{all}$ MPa	$P_{all}$ kN
6	39	1583	1.404	29.78	75.56	65.16	103.1
→ 9	36	2290	1.901	28.82	78.08	61.01	139.7 ←
12	33	2941	2.289	27.90	80.65	57.20	168.2
15	30	3534	2.584	27.04	83.20	53.74	189.9

Since  $P_{all}$  must be greater than 120 kN, use  $t = 9 \text{ mm}$

PROBLEM 10.74

10.74 A 18-kip centric load is applied to a rectangular sawn lumber column of 22-ft effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is  $\sigma_c = 1050$  psi and knowing that  $E = 10 \times 10^6$  psi, determine the smallest cross section that can be used for the column if  $b = 2d$ .



SOLUTION

Sawn lumber  $c = 0.8$   $K_{CE} = 0.3$

$\sigma_c = 1050$  psi  $E = 10 \times 10^6$  psi

$A = 2d^2$   $L = 22 \text{ ft} = 264$   $L/d = \frac{264}{d}$

Assumed  $C_p = 0.5$

$\sigma_{all} = \sigma_c C_p = (1050)(0.5) = 525$  psi

$P_{all} = \sigma_{all} A = 2 \sigma_{all} d^2$

$d = \sqrt{\frac{P_{all}}{2 \sigma_{all}}} = \sqrt{\frac{18000}{2 \cdot 525}} = \frac{94.868}{\sqrt{525}} = 4.14$  in.

$L/d = 63.76$   $\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.3)(10 \times 10^6)}{(63.76)^2} = \frac{3 \times 10^6}{(L/d)^2} = 737.9$  psi

$\sigma_{CE} / \sigma_c = 0.7028$

Checked  $C_p = \frac{1 + \sigma_{CE} / \sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2c}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{c}} = 0.5601$

Results of similar trials are summarized in the table below.

Assumed $C_p$	$\sigma_{all}$ (psi)	$d$ (in)	$L/d$	$\sigma_{CE}$ (psi)	$\sigma_{CE} / \sigma_c$	Checked $C_p$	$\Delta C_p$
0.5	525	4.14	63.76	737.9	0.7028	0.5601	0.0601
0.56	588	3.91	67.48	658.8	0.6275	0.5169	-0.0431
0.535	561.75	4.00	66.00	688.7	0.6559	0.5337	-0.0013
0.5343	561.0	4.005	65.92	690.4	0.6575	0.5346	$\approx 0$

Answer  $d = 4.01$  in.



## PROBLEM 10.77

10.77 A column of 5.6-m effective length must carry a centric load of 2750 kN. Knowing that  $\sigma_y = 250$  MPa and  $E = 200$  GPa, use allowable stress design to select the wide-flange shape of 360-mm nominal depth that should be used.

## SOLUTION

$$P < \frac{\sigma_y A}{F.S.}$$

$$A > \frac{(F.S.) P}{\sigma_y} = \frac{(5/3)(2750 \times 10^3)}{250 \times 10^6} = 18.33 \times 10^{-3} \text{ m}^2 = 18330 \text{ mm}^2$$

$$P < \frac{\pi^2 EI}{1.92 L^2}$$

$$I > \frac{1.92 PL^2}{\pi^2 E} = \frac{(1.92)(2750 \times 10^3)(5.6)^2}{\pi^2 (200 \times 10^9)} = 83.9 \times 10^{-6} \text{ m}^4 = 83.9 \times 10^6 \text{ mm}^4$$

Try W 360 × 216       $A = 27600 \text{ mm}^2 = 27600 \times 10^{-6} \text{ m}^2$       o.k.  
 $I_{\min} = 283 \times 10^6 \text{ mm}^4$       o.k.  
 $r_y = 101 \text{ mm} = 101 \times 10^{-3} \text{ m}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.66$$

$$\frac{L_e}{r} = \frac{5.6}{101 \times 10^{-3}} = 55.45 < C_c \quad \frac{L_e/r}{C_c} = 0.44123$$

$$F.S. = \frac{5}{3} + \frac{3}{8} (0.44123) - \frac{1}{8} (0.44123)^3 = 1.8214$$

$$\sigma_{\text{all}} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.8214} \left[ 1 - \frac{1}{2} (0.44123)^2 \right] = 123.9 \times 10^6 \text{ Pa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (123.9 \times 10^6)(27600 \times 10^{-6}) = 3420 \times 10^3 \text{ N} = 3420 \text{ kN}$$

$$3420 \text{ kN} > 2750 \text{ kN} \quad \text{Use W 360} \times 216$$

## PROBLEM 10.78

10.78 A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that  $\sigma_y = 345$  MPa and  $E = 200$  GPa, use allowable stress design to select the wide-flange shape of 200-mm nominal depth that should be used.

## SOLUTION

$$P < \frac{\sigma_y A}{\text{F.S.}}$$

$$A > \frac{(\text{F.S.}) P}{\sigma_y} = \frac{(5/3)(525 \times 10^3)}{345 \times 10^6} = 2.54 \times 10^{-3} \text{ m}^2 = 2540 \text{ mm}^2$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2}$$

$$I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(525 \times 10^3)(4.6)^2}{\pi^2 (200 \times 10^9)} = 10.89 \times 10^{-6} \text{ m}^4 = 10.89 \times 10^6 \text{ mm}^4$$

Try W 200 × 46.1     $A = 5860 \text{ mm}^2$ ,     $I_{\min} = 15.3 \times 10^6 \text{ mm}^4$ ,     $r = 51.1 \times 10^{-3} \text{ m}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{345 \times 10^6}} = 106.97$$

$$\frac{L_e}{r} = \frac{4.6}{51.1 \times 10^{-3}} = 90.02 < C_c \quad \frac{L_e/r}{C_c} = 0.84154$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.84154) - \frac{1}{8}(0.84154)^3 = 1.9077$$

$$\sigma_{\text{all}} = \frac{\sigma_y}{\text{F.S.}} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{345 \times 10^6}{1.9077} \left[ 1 - \frac{1}{2} (0.84154)^2 \right] = 116.8 \times 10^6 \text{ Pa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (116.8 \times 10^6)(5860 \times 10^{-6}) = 684 \text{ kN} > 525 \text{ kN}$$

Use W 200 × 46.1

**PROBLEM 10.79**

10.79 A column of 22.5-ft effective length must carry a centric load of 288 kips. Using allowable stress design, select the wide-flange shape of 14-in. nominal depth that should be used. Use  $\sigma_r = 50$  ksi and  $E = 29 \times 10^6$  psi.

**SOLUTION**

$$P < \frac{\sigma_y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(288)}{50} = 9.6 \text{ in}^2$$

$$L_e = 22.5 \text{ ft} = 270 \text{ in} \quad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \quad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(288)(270)^2}{\pi^2 (29000)} = 140.8 \text{ in}^4$$

Try W 14 x 82  $A = 24.1 \text{ in}^2$ ,  $I_{\min} = 148 \text{ in}^4$ ,  $r = 2.48 \text{ in}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (29000)}{50}} = 107.00$$

$$\frac{L_e}{r} = \frac{270}{2.48} = 108.87 > 107.00$$

$$\sigma_{\text{all}} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = \frac{\pi^2 (29000)}{(1.92)(108.87)^2} = 12.58 \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (12.58)(24.1) = 303 \text{ kips} > 288 \text{ kips}$$

Use W 14 x 82

**PROBLEM 10.80**

10.80 A column of 17-ft effective length must carry a centric load of 235 kips. Using allowable stress design, select the wide-flange shape of 10-in. nominal depth that should be used. Use  $\sigma_r = 36$  ksi and  $E = 29 \times 10^6$  psi.

**SOLUTION**

$$P < \frac{\sigma_y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(235)}{36} = 10.88 \text{ in}^2$$

$$L_e = 17 \text{ ft} = 204 \text{ in} \quad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \quad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(235)(204)^2}{\pi^2 (29000)} = 65.6 \text{ in}^4$$

Try W 10 x 54  $A = 15.8 \text{ in}^2$   $I_y = 103 \text{ in}^4$   $r = 2.56 \text{ in}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$\frac{L_e}{r} = \frac{204}{2.56} = 79.69 < C_c \quad \frac{L_e/r}{C_c} = \frac{79.69}{126.10} = 0.63194$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.63194) - \frac{1}{8}(0.63194)^3 = 1.8721$$

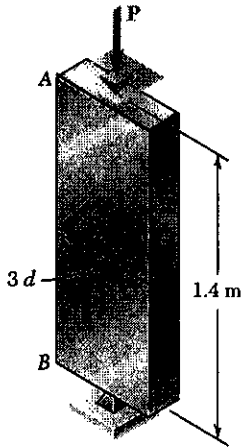
$$\sigma_{\text{all}} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.8721} \left[ 1 - \frac{1}{2} (0.63194)^2 \right] = 15.39 \text{ ksi}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (15.39)(15.8) = 243 \text{ kips} > 235 \text{ kips}$$

Use W 10 x 54

PROBLEM 10.81

10.81 A centric load  $P$  must be supported by the steel bar  $AB$ . Using allowable stress design, determine the smallest dimension  $d$  of the cross section that can be used when (a)  $P = 108$  kN, (b)  $P = 166$  kN. Use  $\sigma_r = 250$  MPa and  $E = 200$  GPa.



SOLUTION

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250}} = 125.66$$

$$L_e = L = 1.4 \text{ m}$$

$$A = (3d)(d) = 3d^2$$

$$I = \frac{1}{12}(3d)(d)^3 = \frac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675 d$$

(a)  $P = 108 \times 10^3 \text{ N}$  Assume  $\frac{L_e}{r} > C_c$

$$P_{all} = \frac{\pi^2 EI}{1.92 L_e^2} \quad I = \frac{(1.92)(P_{all} L_e^2)}{\pi^2 E} = \frac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(108 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 823.59 \times 10^{-9} \text{ m}^4$$

$$d = 30.125 \times 10^{-3} \text{ m} \quad r = 8.696 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{8.696 \times 10^{-3}} = 160.99 > 125.66 \checkmark \quad d = 30.1 \text{ mm}$$

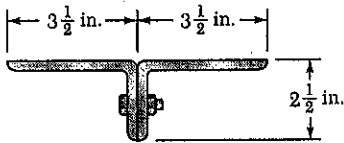
(b)  $P = 166 \times 10^3 \text{ N}$  Assume  $\frac{L_e}{r} > C_c$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(166 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 1.26588 \times 10^{-6} \text{ m}^4$$

$$d = 33.543 \times 10^{-3} \text{ m} \quad r = 9.68295 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{9.68295 \times 10^{-3}} = 144.58 > 125.66 \checkmark \quad d = 33.5 \text{ mm}$$

PROBLEM 10.82



10.82 Two  $3\frac{1}{2} \times 2\frac{1}{2}$  -in. angles are bolted together as shown for use as a column of 8-ft effective length to carry a centric load of 41 kips. Knowing that the angles available have thicknesses of  $\frac{1}{4}$  in.,  $\frac{3}{8}$  in., and  $\frac{1}{2}$  in., use allowable stress design to determine the lightest angles that can be used. Use  $\sigma_r = 36$  ksi and  $E = 29 \times 10^6$  psi.

SOLUTION

Steel:  $E = 29000$  ksi       $L_e = 8 \text{ ft} = 96 \text{ in.}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

Try L  $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$  in.       $A = (2)(2.11) = 4.22 \text{ in.}^2$   
 $I_x = (2)(1.09) = 2.18 \text{ in.}^2 < I_y$   
 $r = \sqrt{\frac{I_x}{A}} = 0.719 \text{ in.}$

$$\frac{L_e}{r} = \frac{96}{0.719} = 133.52 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2 (29000)}{1.92 (133.52)^2} = 8.36 \text{ ksi.}$$

$$P_{all} = \sigma_{all} A = (8.36)(4.22) = 35.3 \text{ kips} < 41 \text{ kips} \quad \text{Do not use.}$$

Try L  $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$  in.       $A = (2)(2.75) = 5.50 \text{ in.}^2$   
 $r = 0.704 \text{ in.}$

$$\frac{L_e}{r} = \frac{96}{0.704} = 136.36 > C_c$$

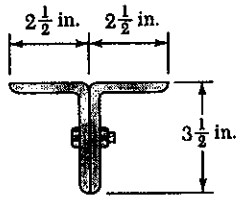
$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2 (29000)}{(1.92)(136.36)^2} = 8.02 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (8.02)(5.50) = 44.1 \text{ kips} > 41 \text{ kips}$$

Use L  $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$  in.

PROBLEM 10.83

10.83 Two  $3\frac{1}{2} \times 2\frac{1}{2}$  -in. angles are bolted together as shown for use as a column of 6-ft effective length to carry a centric load of 54 kips. Knowing that the angles available have thicknesses of  $\frac{1}{4}$  in.,  $\frac{3}{8}$  in., and  $\frac{1}{2}$  in., use allowable stress design to determine the lightest angles that can be used. Use  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi.



SOLUTION

Steel:  $E = 29000$  ksi  $Le = 6 \text{ ft} = 72$  in

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

Try L  $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$  in.

$$A = (2)(2.11) = 4.22 \text{ in}^2$$

$$I_x = (2)(2.56) = 5.12 \text{ in}^4$$

$$I_y = 2[1.09 + (2.11)(0.660)^2] = 4.018 \text{ in}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{4.018}{4.22}} = 0.9758$$

$$\frac{Le}{r} = \frac{72}{0.9758} = 73.78 < C_c$$

$$\frac{Le/r}{C_c} = \frac{73.78}{126.10} = 0.58509$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.58509) - \frac{1}{8}(0.58509)^3 = 1.8610$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L/r}{C_c} \right)^2 \right] = \frac{36}{1.8610} \left[ 1 - \frac{1}{2} (0.58509)^2 \right] = 16.03 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (16.03)(4.22) = 67.7 \text{ kips} > 54 \text{ kips (allowed)}$$

Try L  $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$  in.

$$A = (2)(1.44) = 2.88 \text{ in}^2$$

$$I_x = (2)(1.80) = 3.60 \text{ in}^4$$

$$I_y = (2)[0.777 + (1.44)(0.614)^2] = 2.6397 \text{ in}^4 = I_{min}$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.6397}{2.88}} = 0.97538 \text{ in.}$$

$$\frac{Le}{r} = \frac{72}{0.97538} = 75.205 < C_c$$

$$\frac{L/r}{C_c} = \frac{75.205}{126.10} = 0.59633$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.59633) - \frac{1}{8}(0.59633)^3 = 1.8638$$

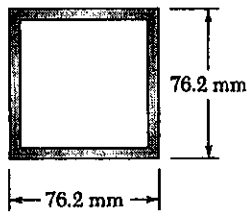
$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L/r}{C_c} \right)^2 \right] = \frac{36}{1.8638} \left[ 1 - \frac{1}{2} (0.59633)^2 \right] = 15.88 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.88)(2.88) = 45.7 \text{ kips} < 54 \text{ kips Do not use}$$

Use L  $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$  in. ▶

PROBLEM 10.84

10.84 A square structural tube having the cross section shown is used as a column of 3.1-m effective length to carry a centric load of 129 kN. Knowing that the tubes available for use are made with wall thicknesses of 3.2 mm, 4.8 mm, 6.4 mm, and 7.9 mm, use allowable stress design to determine the lightest tube that can be used. Use  $\sigma_r = 250$  MPa and  $E = 200$  GPa.



SOLUTION

$$b_o = 76.2 \text{ mm} \quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4)$$

Steel:  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.66$

Try  $t = 4.8 \text{ mm}$

$$b_i = 76.2 - 9.6 = 66.6 \text{ mm}$$

$$A = (76.2)^2 - (66.6)^2 = 1.37088 \times 10^3 \text{ mm}^2 = 1.37088 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12} [(76.2)^4 - (66.6)^4] = 1.17005 \times 10^6 \text{ mm}^4 = 1.17005 \times 10^{-6} \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = 29.21 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{3.1}{29.21 \times 10^{-3}} = 106.11 < C_c \quad \frac{L_e/r}{C_c} = 0.84443$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.84443) - \frac{1}{8}(0.84443)^3 = 1.9081$$

$$\sigma_{all} = \frac{\sigma_r}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.9081} \left[ 1 - \frac{1}{2} (0.84443)^2 \right] = 84.3 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (84.3 \times 10^6)(1.37088 \times 10^{-3}) = 115.6 \times 10^3 \text{ N}$$

$$= 115.6 \text{ kN} < 129 \text{ kN} \quad \text{Do not use.}$$

Try  $t = 6.4 \text{ mm}$

$$b_i = 76.2 - 12.8 = 63.4 \text{ mm}$$

$$A = (76.2)^2 - (63.4)^2 = 1.78688 \times 10^3 \text{ mm}^2 = 1.78688 \times 10^{-3} \text{ m}^2$$

$$I = \frac{1}{12} [(76.2)^4 - (63.4)^4] = 1.46316 \times 10^6 \text{ mm}^4 = 1.46316 \times 10^{-6} \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = 28.615 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{3.1}{28.615 \times 10^{-3}} = 108.33 < C_c \quad \frac{L_e/r}{C_c} = 0.86212$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.86212) - \frac{1}{8}(0.86212)^3 = 1.9099$$

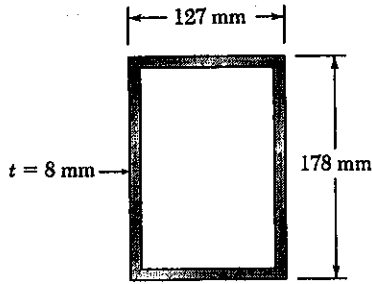
$$\sigma_{all} = \frac{\sigma_r}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.9099} \left[ 1 - \frac{1}{2} (0.86212)^2 \right] = 82.25 \times 10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all} A = (82.25 \times 10^6)(1.78688 \times 10^{-3}) = 147.0 \times 10^3 \text{ N}$$

$$= 147.0 \text{ kN} > 129 \text{ kN}$$

Use  $t = 6.4 \text{ mm}$

PROBLEM 10.85



10.85 A rectangular tube having the cross section shown is used as a column of 4.5-m effective length. Knowing that  $\sigma_T = 250$  MPa and  $E = 200$  GPa, use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 140 kN. Use a dead load factor  $\gamma_D = 1.2$ , a live load factor  $\gamma_L = 1.6$  and the resistance factor  $\phi = 0.85$ .

SOLUTION

$$h_o = 178 \text{ mm} \quad b_o = 127 \text{ mm} \quad h_i = h_o - 2t = 111 \text{ mm}$$

$$b_i = b_o - 2t = 111 \text{ mm}$$

$$A = b_o h_o - b_i h_i = (178)(127) - (111)(111)$$

$$= 4624 \text{ mm}^2 = 4624 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} [b_o h_o^3 - b_i h_i^3] = \frac{1}{12} [(178)(127)^3 - (111)(111)^3] = 11.9213 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9213 \times 10^6}{4624}} = 50.775 \text{ mm} = 50.775 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{4.5}{50.775 \times 10^{-3}} = 88.63$$

$$\lambda_c = \frac{L}{r} \sqrt{\frac{\sigma_T}{E}} = \frac{88.63}{\pi} \sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 0.9974 < 1.5$$

$$\lambda_c^2 = 0.9948$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_T = (4624 \times 10^{-6}) (0.658)^{0.9948} (250 \times 10^6) = 762.3 \times 10^3 \text{ N}$$

$$= 762.3 \text{ kN}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$(1.2)(140) + 1.6 P_L = (0.85)(762.3)$$

$$P_L = 300 \text{ kN}$$



## PROBLEM 10.86

10.86 A column with a 19.5-ft effective length supports a centric load, with ratio of dead to live load equal to 1.35. The dead load factor is  $\gamma_D = 1.2$ , the live load factor  $\gamma_L = 1.6$ , and the resistance factor  $\phi = 0.85$ . Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shape: (a) W10  $\times$  39, (b) W 14  $\times$  68. Use  $E = 29 \times 10^6$  psi and  $\sigma_y = 50$  ksi.

## SOLUTION

$$L_e = 19.5 \text{ ft} = 234 \text{ in.}$$

$$(a) \text{ W10} \times 39 \quad A = 11.5 \text{ in}^2 \quad r_y = 1.98 \text{ in} \quad L_e/r_y = 118.18$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{118.18}{\pi} \sqrt{\frac{50}{29000}} = 1.5620 > 1.5$$

$$P_u = A \left( \frac{0.877}{\lambda_c^2} \right) \sigma_y = \frac{(11.5)(0.877)(50)}{(1.5620)^2} = 206.67 \text{ kips}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$(1.2)(1.35 P_L) + 1.6 P_L = (0.85)(206.67)$$

$$P_D = 73.7 \text{ kips}$$

$$P_L = 54.6 \text{ kips}$$

$$(b) \text{ W 14} \times 68 \quad A = 20.0 \text{ in}^2 \quad r_y = 2.46 \text{ in} \quad L_e/r_y = 95.12$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{95.12}{\pi} \sqrt{\frac{50}{29000}} = 1.2572 < 1.5$$

$$\lambda_c^2 = 1.5806$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_y = (20.0)(0.658)^{1.5806} (50) = 516 \text{ kips}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

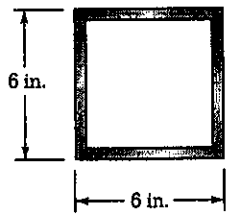
$$(1.2)(1.35 P_L) + 1.6 P_L = (0.85)(516)$$

$$P_D = 183.9 \text{ kips}$$

$$P_L = 136.2 \text{ kips}$$

PROBLEM 10.87

10.87 The structural tube having the cross section shown is used as a column of 15-ft effective length to carry a centric dead load of 51 kips and a centric live load of 58 kips. Knowing that the tubes available for use are made with wall thicknesses in increments of  $\frac{1}{16}$  in. from  $\frac{3}{16}$  in. to  $\frac{5}{8}$  in., use load and resistance factor design to determine the lightest tube that can be used. Use  $\sigma_r = 36$  ksi and  $E = 29 \times 10^6$  psi. The dead load factor  $\gamma_D = 1.2$ , the live load factor  $\gamma_L = 1.6$  and the resistance factor  $\phi = 0.85$ .



SOLUTION

$$L_e = 15 \text{ ft} = 180 \text{ in}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$\text{Required } P_u = \frac{\gamma_D P_D + \gamma_L P_L}{\phi} = \frac{(1.2)(51) + (1.6)(58)}{0.85} = 181.2 \text{ kips}$$

$$\text{Try } t = \frac{1}{4} \text{ in.} = 0.25 \text{ in.} \quad b_o = 6.0 \text{ in.} \quad b_i = b_o - 2t = 5.5 \text{ in.}$$

$$A = b_o^2 - b_i^2 = (6)^2 - (5.5)^2 = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}[(6)^4 - (5.5)^4] = 31.74 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.74}{5.75}} = 2.3496 \text{ in.} \quad \frac{L_e}{r} = \frac{180}{2.3496} = 76.61$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_r}{E}} = \frac{76.61}{\pi} \sqrt{\frac{36}{29000}} = 0.85916 < 1.5 \quad \lambda_c^2 = 0.73815$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_r = (5.75)(0.658)^{0.73815} (36) = 152.0 \text{ kips} < 181.2 \text{ kips}$$

Thickness is too small.

Since  $P_u$  is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{req}}{0.25} \approx \frac{P_u(req)}{152} = \frac{181.18}{152} \quad t_{req} \approx 0.296 \text{ in.}$$

$$\text{Try } t = \frac{5}{16} \text{ in.} = 0.3125 \text{ in.,} \quad b_i = 5.375$$

$$A = 7.1094 \text{ in}^2, \quad I = 38.44 \text{ in}^4, \quad r = 2.3254 \text{ in.} \quad \frac{L_e}{r} = 77.41$$

$$\lambda_c = \frac{77.41}{\pi} \sqrt{\frac{36}{29000}} = 0.86811 < 1.5 \quad \lambda_c^2 = 0.75361$$

$$P_u = (7.1094)(0.658)^{0.75361} (36) = 186.7 \text{ kips} > 181.2 \text{ kips}$$

$$\text{Use } t = \frac{5}{16} \text{ in.}$$

## PROBLEM 10.88

\*10.88 A column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that  $\sigma_r = 250$  MPa and  $E = 200$  GPa, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor  $\gamma_D = 1.2$ , the live load factor  $\gamma_L = 1.6$  and the resistance factor  $\phi = 0.85$ .

## SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$\text{Required } P_u = \frac{\gamma_D P_D + \gamma_L P_L}{\phi} = \frac{(1.2)(310) + (1.6)(375)}{0.85} = 1143 \text{ kN}$$

Preliminary calculations

$$P_u < \sigma_r A \quad \therefore A > \frac{P_u}{\sigma_r} = \frac{1143 \times 10^3}{250 \times 10^6} = 4.572 \times 10^{-3} \text{ m}^2 = 4572 \text{ mm}^2$$

$$P_u < \frac{\pi^2 EI}{L^2} \quad \therefore I > \frac{P_u L^2}{\pi^2 E} = \frac{(1143 \times 10^3)(5.5)^2}{\pi^2 (200 \times 10^9)} = 17.52 \times 10^{-6} \text{ m}^4 = 17.52 \times 10^6 \text{ mm}^4$$

$$\text{Try W 310} \times 60 \quad A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4, \quad r_y = 49.1 \text{ mm} = 49.1 \times 10^{-3} \text{ m}$$

$$\lambda_c = \frac{L_e}{\pi r} \sqrt{\frac{\sigma_r}{E}} = \frac{5.5}{\pi (49.1 \times 10^{-3})} \sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2606 < 1.5$$

$$\lambda_c^2 = 1.5892$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_r = (7590 \times 10^{-6}) (0.658)^{1.5892} (250 \times 10^6)$$

$$= 975 \times 10^3 \text{ N} = 975 \text{ kN} < 1143 \text{ kN}$$

Too light. Do not use.

$$\text{Try W 310} \times 74 \quad A = 9480 \text{ mm}^2 = 9480 \times 10^{-6} \text{ m}^2$$

$$r_y = 49.7 \text{ mm} = 49.7 \times 10^{-3} \text{ m}$$

$$\lambda_c = \frac{5.5}{\pi (49.7 \times 10^{-3})} \sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2454 \quad \lambda_c^2 = 1.5510$$

$$P_u = (9480 \times 10^{-6}) (0.658)^{1.5510} (250 \times 10^6) = 1238 \times 10^3 \text{ N}$$

$$= 1238 \text{ kN} > 1143 \text{ kN}$$

Use W 310  $\times$  74

## PROBLEM 10.89

10.89 A sawn lumber column with a 125-mm-square cross section and a 3.6-m effective length is made of a grade of wood that has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 9.2$  MPa and a modulus of elasticity  $E = 12$  GPa. Using the allowable-stress method, determine the maximum load  $P$  that can be safely supported with an eccentricity of 50 mm.

## SOLUTION

$$d = 125 \text{ mm} = 0.125 \text{ m} \quad A = d^2 = 15.625 \times 10^{-3} \text{ m}^2 \quad \frac{L}{d} = \frac{3.6}{0.125} = 28.8$$

$$\sigma_c = 9.2 \text{ MPa}, \quad E = 12000 \text{ MPa}, \quad \text{sawn lumber: } c = 0.8, \quad K_E = 0.300$$

$$\sigma_{cE} = \frac{K_E E}{(L/d)^2} = \frac{(0.300)(12000)}{(28.8)^2} = 4.34 \text{ MPa} \quad \sigma_{cE}/\sigma_c = 0.47177$$

$$C_P = \frac{1 + (\sigma_{cE}/\sigma_c)}{2c} - \sqrt{\left(\frac{1 + \sigma_{cE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}} = 0.41347$$

$$\sigma_{all} = \sigma_c C_c = (9.2)(0.41347) = 3.804 \text{ MPa} \quad e = 50 \text{ mm} = 0.050 \text{ m}$$

$$I = \frac{1}{12} d^4 = \frac{1}{12} (0.125)^4 = 20.345 \times 10^{-6} \text{ m}^4 \quad c = \frac{1}{2} d = 0.0625 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I} \leq \sigma_{all} \quad \left(\frac{1}{A} + \frac{ec}{I}\right)P < \sigma_{all}$$

$$P < \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I}} = \frac{3.804 \times 10^6}{\frac{1}{15.625 \times 10^{-3}} + \frac{(0.050)(0.0625)}{20.345 \times 10^{-6}}} = 17.48 \times 10^3 \text{ N}$$

$$P < 17.48 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 10.90

10.89 A sawn lumber column with a 125-mm-square cross section and a 3.6-m effective length is made of a grade of wood that has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 9.2$  MPa and a modulus of elasticity  $E = 12$  GPa. Using the allowable-stress method, determine the maximum load  $P$  that can be safely supported with an eccentricity of 50 mm.

SOLUTION

10.90 Solve Prob. 10.89 using the interaction method and an allowable stress in bending of 12.8 MPa.

$$d = 125 \text{ mm} = 0.125 \text{ m} \quad A = d^2 = 15.625 \times 10^{-3} \text{ m}^2 \quad \frac{L}{d} = \frac{3.6}{0.125} = 28.8$$

$$\sigma_c = 9.2 \text{ MPa} \quad E = 12000 \text{ MPa} \quad \text{sawn lumber: } c = 0.8, K_{CE} = 0.300$$

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.300)(12000)}{(28.8)^2} = 4.34 \text{ MPa} \quad \sigma_{CE}/\sigma_c = 0.47177$$

$$C_P = \frac{1 + (\sigma_{CE}/\sigma_c)}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_c}{c}} = 0.41347$$

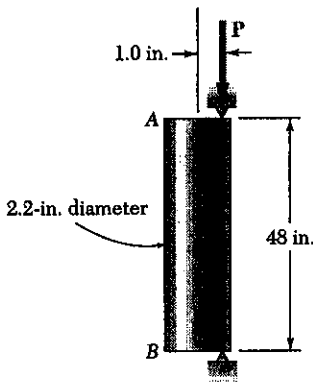
$$\sigma_{all,c} = \sigma_c C_P = (9.2)(0.41347) = 3.804 \text{ MPa} \quad e = 50 \text{ mm} = 0.050 \text{ m}$$

$$I = \frac{1}{12} d^4 = \frac{1}{12} (0.125)^4 = 20.345 \times 10^{-6} \text{ m}^4 \quad c = \frac{1}{2} d = 0.0625 \text{ m}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} < 1$$

$$P < \frac{1}{\frac{1}{A \sigma_{all,c}} + \frac{e c}{I \sigma_{all,b}}} = \frac{1}{\frac{1}{(15.625 \times 10^{-3})(3.804 \times 10^6)} + \frac{(0.050)(0.0625)}{(20.345 \times 10^{-6})(12.8 \times 10^6)}} = 34.7 \times 10^3 \text{ N} = 34.7 \text{ kN}$$

**PROBLEM 10.91**



10.91 An eccentric load is applied at a point 1 in. from the geometric axis of a 2.2-in.-diameter rod made of a steel for which  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi. Using the allowable-stress method, determine the allowable load P.

**SOLUTION**

$$c = \frac{1}{2}d = 1.1 \text{ in.} \quad A = \pi c^2 = 3.8013 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 1.1499 \text{ in}^4 \quad r = \sqrt{\frac{I}{A}} = 0.550 \text{ in}$$

$$L_e = 48 \text{ in} \quad L_e/r = 48/0.550 = 87.2724$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10 > L_e/r$$

$$\frac{L_e/r}{C_c} = 0.6921$$

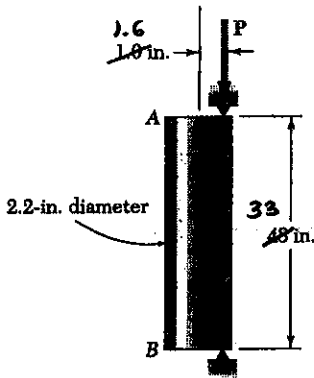
$$F.S. = \frac{5}{3} + \frac{3}{8}(0.6921)^2 - \frac{1}{8}(0.6921)^3 = 1.8848$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.8848} \left[ 1 - \frac{1}{2} (0.6921)^2 \right] = 14.526 \text{ ksi}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \sigma_{all} \quad \left( \frac{1}{A} + \frac{ec}{I} \right) P_{all} = \sigma_{all} \quad P_{all} = \sigma_{all} \left[ \frac{1}{A} + \frac{ec}{I} \right]^{-1}$$

$$P_{all} = (14.526) \left[ \frac{1}{3.8013} + \frac{(1.0)(1.1)}{1.1499} \right]^{-1} = 11.91 \text{ kips}$$

**PROBLEM 10.92**



10.91 An eccentric load is applied at a point 1 in. from the geometric axis of a 2.2-in.-diameter rod made of a steel for which  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi. Using the allowable-stress method, determine the allowable load P.

10.92 Solve Prob. 10.91, assuming that the load is applied at a point 1.6 in. from the geometric axis and that the effective length is 33 in.

**SOLUTION**

$$c = \frac{1}{2}d = 1.1 \text{ in.} \quad A = \pi c^2 = 3.8013 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 1.1499 \text{ in}^4 \quad r = \sqrt{\frac{I}{A}} = 0.550 \text{ in}$$

$$L_e = 33 \text{ in} \quad L_e/r = 33/0.550 = 60$$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$$\frac{L_e/r}{C_c} = 0.4758 \quad F.S. = \frac{5}{3} + \frac{3}{8}(0.4758)^2 - \frac{1}{8}(0.4758)^3 = 1.8316$$

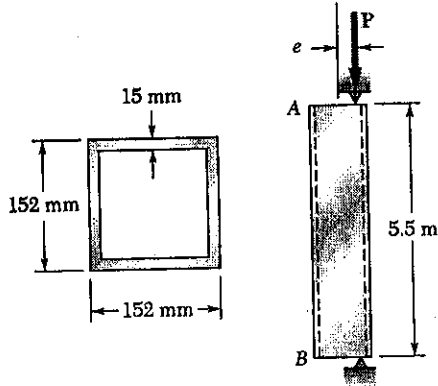
$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.8316} \left[ 1 - \frac{1}{2} (0.4758)^2 \right] = 17.430 \text{ ksi}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \sigma_{all} \quad \left( \frac{1}{A} + \frac{ec}{I} \right) P_{all} = \sigma_{all} \quad P_{all} = \sigma_{all} \left[ \frac{1}{A} + \frac{ec}{I} \right]^{-1}$$

$$P_{all} = (17.430) \left[ \frac{1}{3.8013} + \frac{(1.6)(1.1)}{1.1499} \right]^{-1} = 9.72 \text{ kips}$$

**PROBLEM 10.93**

10.93 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load  $P$ , knowing that when the eccentricity is (a)  $e = 0$ , (b)  $e = 40$  mm.



**SOLUTION**

$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$$

$$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(97.76)^2} = 38.92 \text{ MPa for centric loading}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{Pec}{I \sigma_{all,b}} = 1$$

(a)  $e = 0 \quad P = A \sigma_{all,c} = (8220 \times 10^{-6})(38.92 \times 10^6) = 320 \times 10^3 \text{ N} = 320 \text{ kN}$   $\blacktriangleleft$

(b)  $e = 40 \times 10^{-3} \text{ m} \quad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$

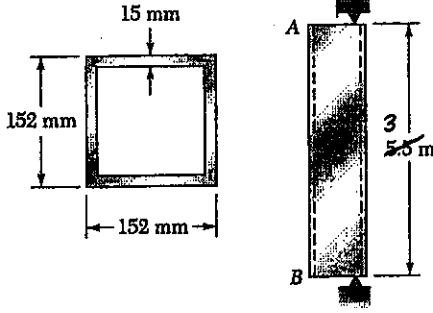
$$\frac{P}{(8220 \times 10^{-6})(38.92 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 3.6568 \times 10^6 P = 1$$

$$P = 273 \times 10^3 \text{ N} = 273 \text{ kN} \quad \blacktriangleleft$$

**PROBLEM 10.94**

**10.93** A column of 5.5-m effective length is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load  $P$ , knowing that when the eccentricity is (a)  $e = 0$ , (b)  $e = 40$  mm.

**10.94** Solve Prob. 10.93, assuming that the effective length of a column is 3.0 m.



**SOLUTION**

$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8200 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < \text{www.konkur.in}$$

$$\sigma_{all,c} = 212 - 1.585 (L/r) = 212 - (1.585)(53.32) = 127.5 \text{ MPa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

(a)  $e = 0 \quad P = A\sigma_{all} = (8220 \times 10^{-6})(127.5 \times 10^6) = 1048 \times 10^3 \text{ N} = 1048 \text{ kN}$   $\blacktriangleleft$

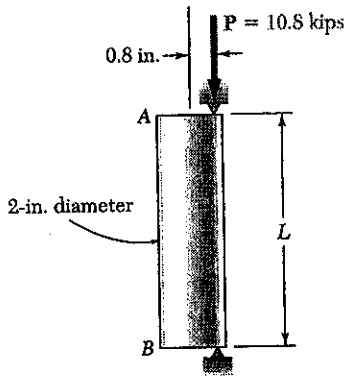
(b)  $e = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \quad c = (\frac{1}{2})(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$

$$\frac{P}{(8220 \times 10^{-6})(127.5 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 1.4852 \times 10^{-6} P = 1$$

$$P = 673 \times 10^3 \text{ N} = 673 \text{ kN} \quad \blacktriangleleft$$



**PROBLEM 10.95**



10.95. An eccentric load  $P = 10.8$  kips is applied at a point 0.8 in. from the geometric axis of a 2-in.-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 21 ksi, determine the largest allowable effective length  $L$  that can be used.

**SOLUTION**

$$c = \frac{1}{2}d = 1.0 \text{ in} \quad A = \pi c^2 = 3.1416 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 0.7854 \text{ in}^4 \quad r = \sqrt{\frac{I}{A}} = 0.5 \text{ in.}$$

$$e = 0.8 \text{ in} \quad \sigma_{all,b} = 21 \text{ ksi}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1 \quad \frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

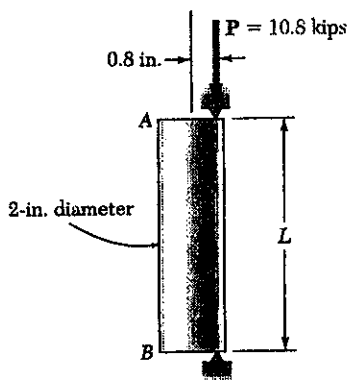
$$\frac{1}{\sigma_{all,c}} = \frac{A}{P} \left( 1 - \frac{Pec}{I\sigma_{all,b}} \right) = \frac{3.1416}{10.8} \left[ 1 - \frac{(10.8)(0.8)(1.0)}{(0.7854)(21)} \right] = 0.1385 \text{ ksi}^{-1}$$

$$\sigma_{all,c} = 7.22 \text{ ksi} \quad \text{Assume } L/r > 66$$

$$\sigma_{all,c} = \frac{51000}{(L/r)^2} \quad \frac{L}{r} = \sqrt{\frac{51000}{\sigma_{all,c}}} = 84.05 > 66$$

$$L = 84.05 r = (84.05)(0.5) = 42.0 \text{ in.}$$

**PROBLEM 10.96**



10.95 An eccentric load  $P = 10.8$  kips is applied at a point 0.8 in. from the geometric axis of a 2-in.-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 21 ksi, determine the largest allowable effective length  $L$  that can be used.

10.96 Solve Prob. 10.95, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 26 ksi.

**SOLUTION**

$$c = \frac{1}{2}d = 1.0 \text{ in.} \quad A = \pi c^2 = 3.1416 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 0.7854 \text{ in}^4 \quad r = \sqrt{\frac{I}{A}} = 0.5 \text{ in.}$$

$$e = 0.8 \text{ in.} \quad \sigma_{all,b} = 26 \text{ ksi}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1 \quad \frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P} \left( 1 - \frac{Pec}{I\sigma_{all,b}} \right) = \frac{3.1416}{10.8} \left[ 1 - \frac{(10.8)(0.8)(1.0)}{(0.7854)(26)} \right] = 0.1678 \text{ ksi}^{-1}$$

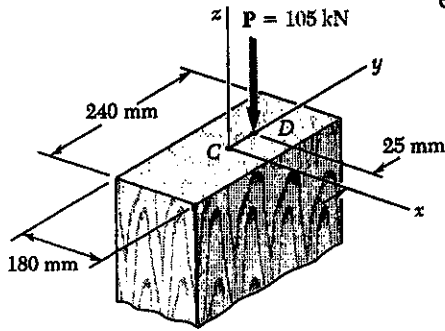
$$\sigma_{all,c} = 5.96 \text{ ksi} \quad \text{Assume } L/r > 55$$

$$\sigma_{all,c} = \frac{54000}{(L/r)^2} \quad \frac{L}{r} = \sqrt{\frac{54000}{\sigma_{all,c}}} = \sqrt{\frac{54000}{5.96}} = 95.19 > 55$$

$$L = 95.19 r = (95.19)(0.5) = 47.6 \text{ in.}$$

**PROBLEM 10.97**

10.97 A rectangular column is made of sawn lumber that has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 8.3$  MPa and a modulus of elasticity  $E = 11.1$  GPa. Using the allowable-stress method, determine the largest allowable effective length  $L$  that can be used.



**SOLUTION**

$d = 180 \text{ mm} = 0.180 \text{ m}$      $b = 240 \text{ mm} = 0.240 \text{ m}$

$A = bd = 43.2 \times 10^{-3} \text{ m}^2$      $E = 11100 \text{ MPa}$

$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$

$e = 25 \text{ mm} = 0.025 \text{ m}$      $c = \frac{b}{2} = 0.120 \text{ m}$

$\frac{P}{A} + \frac{Pec}{I_x} \leq \sigma_{all}$      $\sigma_{all} \geq \frac{105 \times 10^3}{43.2 \times 10^{-3}} + \frac{(105 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa} = 3.9496 \text{ MPa}$

$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.47586 = y$     Let  $x = \sigma_{ce}/\sigma_c$

$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$     where  $c = 0.8$  for sawn lumber

$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$

$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$

$\left(\frac{1+x}{2c}\right)^2 - \frac{1+x}{c}y + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$

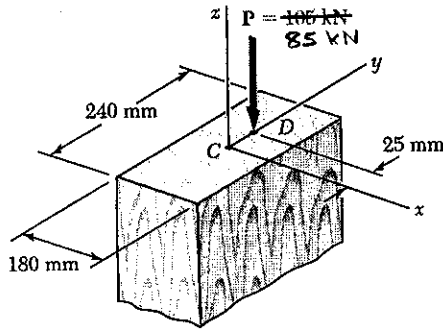
$x = y \frac{1-cy}{1-y} = (0.47586) \frac{1 - (0.8)(0.47586)}{1 - 0.47586} = 0.56227$

$\sigma_{ce} = \sigma_c (0.56227) = (8.3)(0.56227) = 4.6668 \text{ MPa}$

$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2}$      $L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}}$     where  $K_{ce} = 0.300$

$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = 0.180 \sqrt{\frac{(0.300)(11100)}{4.6668}} = 4.81 \text{ m}$

PROBLEM 10.98



10.97 A rectangular column is made of sawn lumber that has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 8.3$  MPa and a modulus of elasticity  $E = 11.1$  GPa. Using the allowable-stress method, determine the largest allowable effective length  $L$  that can be used.

10.98 Solve Prob. 10.97, assuming that  $P = 85$  kN.

SOLUTION

$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I} \leq \sigma_{all} \quad \sigma_{all} \geq \frac{85 \times 10^3}{43.2 \times 10^{-3}} + \frac{(85 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa}$$

$$= \text{MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.1973}{8.3} = 0.38522 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c}\right)^2 - y\left(\frac{1+x}{c}\right) + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$x = y \frac{(1-cy)}{1-y} = (0.38522) \frac{1 - (0.8)(0.38522)}{1 - 0.38522} = 0.43350$$

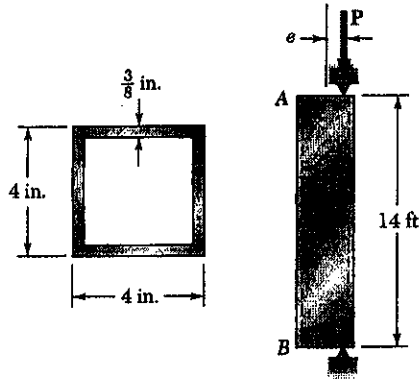
$$\sigma_{ce} = \sigma_c (0.43350) = (8.3)(0.43350) = 3.598 \text{ MPa}$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = (0.180) \sqrt{\frac{(0.300)(11100)}{3.598}} = 5.48 \text{ m}$$

PROBLEM 10.99

10.99 A column of 14-ft effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity  $e$  if (a)  $P = 55$  kips, (b)  $P = 35$  kips. Use  $\sigma_y = 36$  ksi and  $E = 29 \times 10^6$  psi.



SOLUTION

Steel:  $\sigma_y = 36$  ksi  $E = 29000$  ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$$b_o = 4.0 \text{ in} \quad b_i = b_o - 2t = 3.25 \text{ in} \quad c = 2.0 \text{ in}$$

$$A = b_o^2 - b_i^2 = 5.4375 \text{ in}^2 \quad I = \frac{1}{12}(b_o^4 - b_i^4) = 12.036 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.4878 \text{ in} \quad L_e = 14 \text{ ft} = 168 \text{ in}$$

$$L_e/r = 112.92 < C_c \quad \frac{L_e/r}{C_c} = 0.89547$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.89547) - \frac{1}{8}(0.89547)^3 = 1.9127$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.9127} \left[ 1 - \frac{1}{2} (0.89547)^2 \right] = 11.275 \text{ ksi}$$

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I} = \sigma_{all} \quad \frac{P_{all} e c}{I} = \sigma_{all} - \frac{P_{all}}{A} \quad e = \frac{I}{c P_{all}} \left( \sigma_{all} - \frac{P_{all}}{A} \right)$$

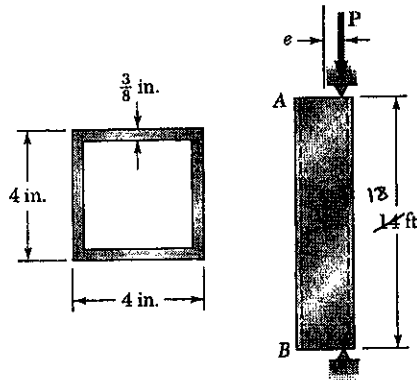
(a)  $P_{all} = 55$  kips

$$e = \frac{12.036}{(2.0)(55)} \left[ 11.275 - \frac{55}{5.4375} \right] = 0.127 \text{ in}$$

(b)  $P_{all} = 35$  kips

$$e = \frac{12.036}{(2.0)(35)} \left[ 11.275 - \frac{35}{5.4375} \right] = 0.832 \text{ in}$$

PROBLEM 10.100



10.99 A column of 14-ft effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity  $e$  if (a)  $P = 55$  kips, (b)  $P = 35$  kips. Use  $\sigma_r = 36$  ksi and  $E = 29 \times 10^6$  psi.

10.100 Solve Prob. 11.99, assuming that the effective length of the column is increased to 18 ft and that (a)  $P = 28$  kips, (b)  $P = 18$  kips.

SOLUTION

Steel:  $\sigma_r = 36$  ksi       $E = 29000$  ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$$

$b_o = 4.0$  in     $b_i = b_o - 2t = 3.25$  in     $c = 2.0$  in.

$A = b_o^2 - b_i^2 = 5.4375$  in<sup>2</sup>     $I = \frac{1}{12}(b_o^4 - b_i^4) = 12.036$  in<sup>4</sup>

$r = \sqrt{\frac{I}{A}} = 1.4878$        $L_e = 18$  ft = 216 in       $L_e/r = 145.18 > C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2 (29000)}{(1.92)(145.18)^2} = 7.0726$$
 ksi

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I} = \sigma_{all} \qquad \frac{P_{all} e c}{I} = \sigma_{all} - \frac{P_{all}}{A} \qquad e = \frac{I}{c P_{all}} \left( \sigma_{all} - \frac{P_{all}}{A} \right)$$

(a)  $P_{all} = 28$  kips

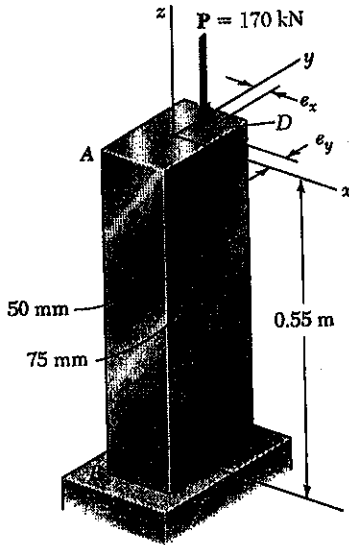
$$e = \frac{12.036}{(2.0)(28)} \left[ 7.0726 - \frac{28}{5.4375} \right] = 0.413$$
 in.

(b)  $P_{all} = 18$  kips

$$e = \frac{12.036}{(2.0)(18)} \left[ 7.0726 - \frac{18}{5.4375} \right] = 1.258$$
 in.

**PROBLEM 10.101**

10.101 The compression member  $AB$  is made of a steel for which  $\sigma_y = 250$  MPa and  $E = 200$  GPa. It is free at its top  $A$  and fixed at its base  $B$ . Using the allowable-stress method, determine the largest allowable eccentricity  $e_x$ , knowing that (a)  $e_y = 0$ , (b)  $e_y = 8$  mm.



**SOLUTION**

Steel:  $\sigma_y = 250$  MPa  $E = 200000$  MPa  
 $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 \cdot 200000}{250}} = 125.66$  MPa

$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$   
 $I_y = \frac{1}{12} (75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$   
 $r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m} = r_{\min}$   
 $I_x = \frac{1}{12} (50 \times 10^{-3})(75 \times 10^{-3})^3 = 1.7578 \times 10^{-6} \text{ m}^4$   
 $r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$

$L_e = 2L = (2)(0.55) = 1.10 \text{ m}$   $L_e/r_{\min} = 1.10/14.434 \times 10^{-3} = 76.21 < C_c$

$\frac{L_e/r_{\min}}{C_c} = \frac{76.21}{125.66} = 0.6065$   $F.S. = \frac{5}{3} + \frac{3}{8}(0.6065) - \frac{1}{8}(0.6065)^3 = 1.8662$

$\sigma_{\text{all}} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r_{\min}}{C_c} \right)^2 \right] = \frac{250}{1.8662} \left[ 1 - \frac{1}{2} (0.6065)^2 \right] = 109.32 \text{ MPa}$

$\frac{P}{A} + \frac{Pe_x}{S_y} + \frac{Pe_y}{S_x} = \sigma_{\text{all}}$   $\frac{Pe_x}{S_y} = \sigma_{\text{all}} - \frac{P}{A} - \frac{Pe_y}{S_x}$

$e_x = \frac{S_y}{P} \left[ \sigma_{\text{all}} - \frac{P}{A} - \frac{Pe_y}{S_x} \right] = S_y \left[ \frac{\sigma_{\text{all}}}{P} - \frac{1}{A} - \frac{e_y}{S_x} \right]$

$S_y = \frac{I_y}{x} = \frac{781.25 \times 10^{-9}}{25 \times 10^{-3}} = 31.25 \times 10^{-6} \text{ m}^3$

$S_x = \frac{I_x}{y} = \frac{1.7578 \times 10^{-6}}{37.5 \times 10^{-3}} = 46.875 \times 10^{-6} \text{ m}^3$

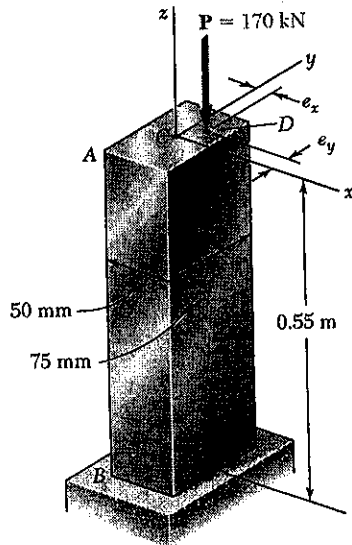
$P = 170 \times 10^3 \text{ N}$

(a)  $e_y = 0$   $e_x = 31.25 \times 10^{-6} \left[ \frac{109.32 \times 10^6}{170 \times 10^3} - \frac{1}{3750 \times 10^{-6}} - 0 \right]$   
 $= 11.76 \times 10^{-3} \text{ m} = 11.76 \text{ mm}$

(b)  $e_y = 8 \times 10^{-3} \text{ m}$   $e_x = 31.25 \times 10^{-6} \left[ \frac{109.32 \times 10^6}{170 \times 10^3} - \frac{1}{3750 \times 10^{-6}} - \frac{8 \times 10^{-3}}{46.875 \times 10^{-6}} \right]$   
 $= 6.43 \times 10^{-3} \text{ m} = 6.43 \text{ mm}$

**PROBLEM 10.102**

10.102 The compression member  $AB$  is made of a steel for which  $\sigma_r = 250$  MPa and  $E = 200$  GPa. It is free at its top  $A$  and fixed at its base  $B$ . Using the interaction method with an allowable bending stress equal to 120 MPa and knowing that the eccentricities  $e_x$  and  $e_y$  are equal, determine the largest allowable common value.



**SOLUTION**

Steel:  $\sigma_r = 250$  MPa  $E = 200000$  MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

$$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$$

$$I_y = \frac{1}{12}(75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m}$$

$$I_x = \frac{1}{12}(50 \times 10^{-3})(75 \times 10^{-3})^3 = 1.7578 \times 10^{-6} \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.55) = 1.10 \text{ m} \quad L_e / r_{\min} = 1.10 / 14.434 \times 10^{-3} = 76.21 < C_c$$

$$\frac{L_e / r_{\min}}{C_c} = \frac{76.21}{125.66} = 0.6065 \quad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.6065) - \frac{1}{8}(0.6065)^3 = 1.8662$$

$$\sigma_{\text{all(centric)}} = \frac{\sigma_r}{\text{F.S.}} \left[ 1 - \frac{1}{2} \left( \frac{L_e / r_{\min}}{C_c} \right)^2 \right] = \frac{250}{1.8662} \left[ 1 - \frac{1}{2} (0.6065)^2 \right] = 109.32 \text{ MPa}$$

$$\sigma_{\text{all(bending)}} = 120 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all(centric)}}} + \frac{P e_x y}{I_x \sigma_{\text{all(bending)}}} + \frac{P e_y x}{I_y \sigma_{\text{all(bending)}}} = 1 \quad \text{with } e_x = e_y$$

$$\frac{P}{\sigma_{\text{all(bending)}}} \left( \frac{y}{I_x} + \frac{x}{I_y} \right) e = 1 - \frac{P}{A \sigma_{\text{all(centric)}}}$$

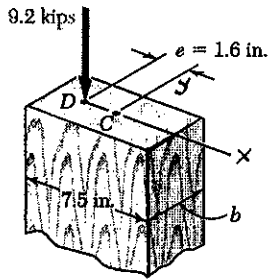
$$\frac{170 \times 10^3}{120 \times 10^6} \left( \frac{37.5 \times 10^{-3}}{1.7578 \times 10^{-6}} + \frac{25 \times 10^{-3}}{781.25 \times 10^{-9}} \right) e = 1 - \frac{170 \times 10^3}{(3750 \times 10^{-6})(109.32 \times 10^6)}$$

$$75.556 e = 1 - 0.41468$$

$$e = 7.75 \times 10^{-3} \text{ m} = 7.75 \text{ mm}$$

**PROBLEM 10.103**

**10.103** A sawn lumber column of rectangular cross section has a 7.2-ft effective length and supports a 9.2 kip load as shown. The sizes available for use have  $b$  equal to 3.5 in., 5.5 in., 7.5 in. and 9.5 in. The grade of wood has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 1180$  psi and  $E = 1.2 \times 10^6$  psi. Use the allowable-stress method to determine the lightest section that can be used.



**SOLUTION**

Sawn lumber:  $\sigma_c = 1180$  psi       $E = 1.2 \times 10^6$  psi  
 $C = 0.8$        $K_{CE} = 0.300$   
 $L_e = 7.2$  ft = 86.4 in

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all} \quad P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{e c}{I}}$$

$e = 1.6$  in       $c = \frac{1}{2}(7.5) = 3.75$  in.       $A = 7.5 b$

$I_x = \frac{1}{12} b (7.5)^3 = 35.156 b$

$\frac{1}{A} + \frac{e c}{I_x} = \frac{1}{7.5 b} + \frac{(1.6)(3.75)}{35.156 b} = 3.2895 b \quad P_{all} = 3.2895 b \sigma_{all}$

$d = 7.5$  in. or  $b$ , whichever is smaller.

$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{K_{CE} E d^2}{L^2} = \frac{(0.300)(1.2 \times 10^6) d^2}{(86.4)^2} = 48.225 d^2$  (psi)

$\sigma_{CE} / \sigma_c = (48.225 d^2) / 1180 = 0.04087 d^2$

$C_p = \frac{1 + \sigma_{CE} / \sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2C}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{C}}$

$\sigma_{all} = \sigma_c C_p = 1180 C_p$

$P_{all} = (3.2895) b (1180 C_p) = 3882 b C_p$  (lb.)

Calculate  $P_{all}$  for all four values of  $b$ . See table below.

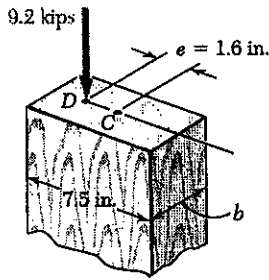
$b$ (in.)	$d$ (in.)	$\sigma_{CE} / \sigma_c$	$C_p$	$P_{all}$ (lb.)
3.5	3.5	0.5007	0.4341	5900
5.5	5.5	1.2363	0.7588	16200
7.5	7.5	2.299	0.8882	25900
9.5	7.5	2.299	0.8882	32800

←  $P = 9200$  lb.

Use  $b = 5.5$  in. →



**PROBLEM 10.104**



**10.103** A sawn lumber column of rectangular cross section has a 7.2-ft effective length and supports a 9.2 kip load as shown. The sizes available for use have  $b$  equal to 3.5 in., 5.5 in., 7.5 in. and 9.5 in. The grade of wood has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 1180$  psi and  $E = 1.2 \times 10^6$  psi. Use the allowable-stress method to determine the lightest section that can be used.

**10.104** Solve Prob. 10.103, assuming that  $e = 3.2$  in.

**SOLUTION**

Sawn lumber:  $\sigma_c = 1180$  psi       $E = 1.2 \times 10^6$  psi  
 $c = 0.8$        $K_{CE} = 0.300$   
 $L_e = 7.2$  ft = 86.4 in.

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all} \quad P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{e c}{I_x}}$$

$e = 3.2$  in       $c = \frac{1}{2}(7.5) = 3.75$  in       $A = 7.5 b$

$I_x = \frac{1}{12} b (7.5)^3 = 35.156 b$

$$\frac{1}{\frac{1}{A} + \frac{e c}{I_x}} = \frac{1}{\frac{1}{7.5 b} + \frac{(3.2)(3.75)}{35.156 b}} = 2.1067 b \quad P_{all} = 2.1067 b \sigma_{all}$$

$d = 7.5$  in. or  $b$ , whichever is smaller

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{K_{CE} E d^2}{L^2} = \frac{(0.300)(1.2 \times 10^6) d^2}{(86.4)^2} = 48.225 d^2 \text{ (psi)}$$

$\sigma_{CE} / \sigma_c = 48.225 d^2 / 1180 = 0.04087 d^2$

$$C_p = \frac{1 + \sigma_{CE} / \sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2c}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{c}}$$

$\sigma_{all} = \sigma_c C_p = 1180 C_p$

$P_{all} = (2.1067) b (1180 C_p) = 2486 b C_p$

Calculate  $P_{all}$  for all four values of  $b$ . See table below.

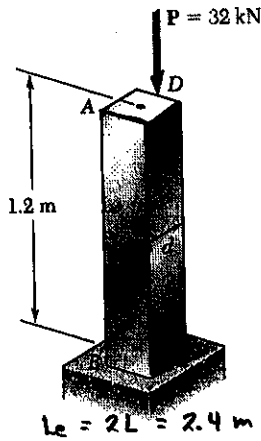
$b$ (in.)	$d$ (in.)	$\sigma_{CE} / \sigma_c$	$C_p$	$P_{all}$ (lb)
3.5	3.5	0.5007	0.4341	3780
5.5	5.5	1.2363	0.7588	10370
7.5	7.5	2.299	0.8882	16560
9.5	7.5	2.299	0.8882	20100

←  $P = 9200$  lb.

Use  $b = 5.5$  in.

**PROBLEM 10.105**

10.105 A 32-kN vertical load  $P$  is applied at the midpoint of one edge of the square cross section of the aluminum compression member  $AB$  that is free at its top  $A$  and fixed at its base  $B$ . Knowing that the alloy used is 6061-T6, use the allowable-stress method to determine the smallest allowable dimension  $d$ .



**SOLUTION**

$$A = d^2 \quad I = \frac{1}{12} d^4 \quad r = \sqrt{\frac{I}{A}} = \frac{1}{\sqrt{12}} d \quad c = \frac{1}{2} d \quad e = \frac{1}{2} d$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{P}{d^2} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4} = \frac{4P}{d^2} = \sigma_{all}$$

Assume  $L/r > 66$        $\sigma_{all} = \frac{B}{(L/r)^2}$        $B = 351 \times 10^9 \text{ Pa}$

$$\frac{Br^2}{L^2} = \frac{Bd^2}{12L^2} = \frac{4P}{d^2} \quad d^4 = \frac{48PL^2}{B}$$

$$d = \sqrt[4]{\frac{48PL^2}{B}} = \sqrt[4]{\frac{(48)(32 \times 10^3)(2.4)^2}{351 \times 10^9}} = 70.9 \times 10^{-3} \text{ m}$$

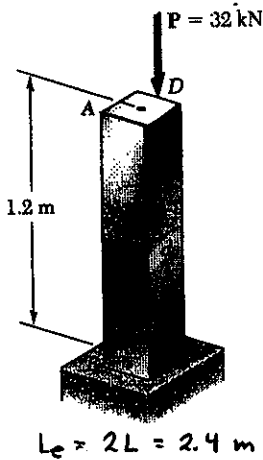
$$r = \frac{70.9 \times 10^{-3}}{\sqrt{12}} = 20.45 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = \frac{2.4}{20.45 \times 10^{-3}} = 117.3 > 66$$

answer  $d = 70.9 \text{ mm}$   $\blacktriangleleft$

**PROBLEM 10.106**

10.105 A 32-kN vertical load  $P$  is applied at the midpoint of one edge of the square cross section of the aluminum compression member  $AB$  that is free at its top  $A$  and fixed at its base  $B$ . Knowing that the alloy used is 6061-T6, use the allowable-stress method to determine the smallest allowable dimension  $d$ .

10.106 Solve Prob. 10.105, assuming that the vertical load  $P$  is applied at a corner of the square cross section of the compression member  $AB$ .



**SOLUTION**

$$A = d^2, \quad I = \frac{1}{12} d^4, \quad r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} \quad x = y = \frac{1}{2} d$$

$$e_x = e_y = \frac{1}{2} d$$

$$\frac{P}{A} + \frac{Pe_x x}{I_y} + \frac{Pe_y y}{I_x} = \frac{P}{d^2} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4}$$

$$= \frac{7P}{d^2} = \sigma_{all}$$

Assume  $L/r > 66$        $\sigma_{all} = \frac{B}{(L_e/r)^2}$        $B = 351 \times 10^9 \text{ Pa}$

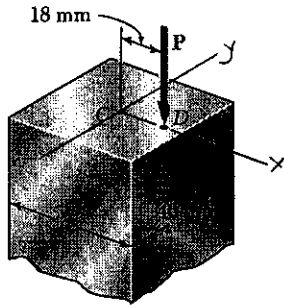
$$\frac{Br^2}{L_e^2} = \frac{Bd^2}{12L_e^2} = \frac{7P}{d^2} \quad d^4 = \frac{84PL_e^2}{B}$$

$$d = \sqrt[4]{\frac{84PL_e^2}{B}} = \sqrt[4]{\frac{(84)(32 \times 10^3)(2.4)^2}{351 \times 10^9}} = 81.5 \times 10^{-3} \text{ m} \quad d = 81.5 \text{ mm} \quad \blacktriangleleft$$

$$r = \frac{d}{\sqrt{12}} = 23.5 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = \frac{2.4}{23.5 \times 10^{-3}} = 102.0 > 66$$

PROBLEM 10.107

10.107 A compression member made of steel has a 720-mm effective length and must support the 198-kN load  $P$  as shown. For the material used  $\sigma_y = 250$  MPa and  $E = 200$  GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension  $d$  of the cross section that can be used.



SOLUTION

Using dimensions in meters

$$A = 40 \times 10^{-3} d \quad L_e = 720 \text{ mm} = 0.720 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{d}{2}, \quad |y| = 20 \text{ mm} = 0.020 \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

Steel:  $\sigma_y = 250$  MPa  $E = 200000$  MPa  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$

$$C_c = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

Assume  $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$ . Then  $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{40 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m}, \quad \frac{L_e}{r} = 62.35 < C_c$$

$$\frac{L_e/r}{C_c} = 0.49621 \quad F.S. = \frac{5}{3} + \frac{3}{8}(0.49621) - \frac{1}{8}(0.49621)^3 = 1.83747$$

$$\sigma_{all, centric} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.83747} \left[ 1 - \frac{1}{2} (0.49621)^2 \right] = 119.31 \text{ MPa}$$

$$\sigma_{all, bending} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, centric}} + \frac{P e_x x}{I_y \sigma_{all, bending}} = 1$$

$$\frac{198 \times 10^3}{(40 \times 10^{-3} d)(119.31 \times 10^6)} + \frac{(198 \times 10^3)(18 \times 10^{-3})(\frac{1}{2} d)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

$$\frac{41.489 \times 10^{-3}}{d} + \frac{3.5640 \times 10^{-3}}{d^2} = 1$$

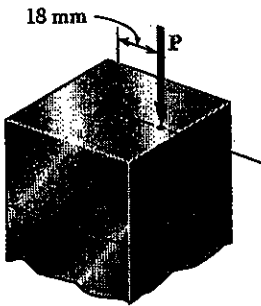
$$d^2 - 41.489 \times 10^{-3} d - 3.5640 \times 10^{-3} = 0$$

$$d = \frac{1}{2} \left\{ 41.489 \times 10^{-3} + \sqrt{(41.489 \times 10^{-3})^2 + 4(3.5640 \times 10^{-3})} \right\}$$

$$= 83.9 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 83.9 \text{ mm}$$

PROBLEM 10.108



10.107 A compression member made of steel has a 720-mm effective length and must support the 198-kN load  $P$  as shown. For the material used  $\sigma_y = 250$  MPa and  $E = 200$  GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension  $d$  of the cross section that can be used.

10.108 Solve Prob. 10.107, assuming that the effective length is 1.62 m and that the magnitude  $P$  of the eccentric load is 128 kN.

SOLUTION

Using dimensions in meters

$$A = 40 \times 10^{-3} d \quad L_e = 1.62 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{1}{2} d, \quad |y| = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

Steel:  $\sigma_y = 250$  MPa  $E = 200000$  MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

Assume  $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$  Then  $I_{\min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{3.3333 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 140.29 > C_c$$

$$\sigma_{\text{all, centric}} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = \frac{\pi^2 (200000)}{(1.92)(140.29)^2} = 52.236 \text{ MPa} \quad \sigma_{\text{all, bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e_x x}{I_y \sigma_{\text{all, bending}}} = 1$$

$$\frac{128 \times 10^3}{(40 \times 10^{-3} d)(52.236 \times 10^6)} + \frac{(128 \times 10^3)(18 \times 10^{-3})(\frac{1}{2} d)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

$$\frac{61.260 \times 10^{-3}}{d} + \frac{2.304 \times 10^{-3}}{d^2} = 1$$

$$d^2 - 61.260 \times 10^{-3} d - 2.304 \times 10^{-3} = 0$$

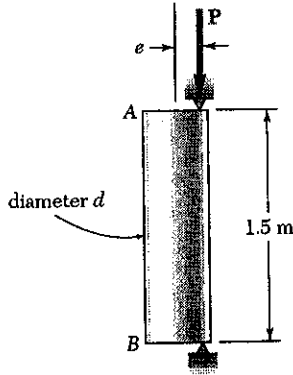
$$d = \frac{1}{2} \left\{ 61.260 \times 10^{-3} + \sqrt{(61.260 \times 10^{-3})^2 + (4)(2.304 \times 10^{-3})} \right\}$$

$$= 87.6 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 87.6 \text{ mm}$$

PROBLEM 10.109

10.109 The eccentric load  $P$  has a magnitude of 85 kN and is applied at a point located at a distance  $e = 30$  mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Use the interaction method with a 140-MPa allowable stress in bending to determine the smallest diameter  $d$  that can be used.



SOLUTION

Assume  $L/r > 66$        $\sigma_{all} = \frac{B}{(L/r)^2}$        $B = 351 \times 10^9 \text{ Pa}$

$c = \frac{d}{2}$        $A = \pi c^2 = \frac{\pi}{4} d^2$        $I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$

$r = \sqrt{\frac{I}{A}} = \frac{1}{4} d$

$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I \sigma_{all, bending}} = 1$

$\frac{PL^2}{ABr^2} + \frac{Ped}{2I \sigma_{all, bending}} = 1$

$\frac{64 PL^2}{\pi B d^4} + \frac{32 Ped}{\pi \sigma_{all, bending}} = 1$

$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi(351 \times 10^9)d^4} + \frac{(32)(85 \times 10^3)(30 \times 10^{-3})}{\pi(140 \times 10^6)d^3} = 1$       let  $x = \frac{1}{d}$

$11.1 \times 10^{-6} x^4 + 185.53 \times 10^{-6} x^3 = 1$       Solving  $x = 14.2725 \text{ m}^{-1}$

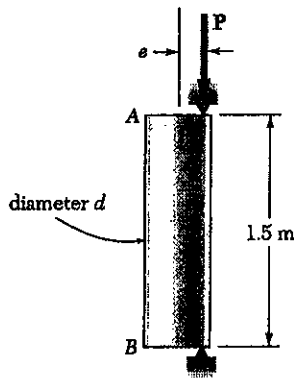
$d = \frac{1}{x} = 70.0 \times 10^{-3} \text{ m}$

$d = 70.0 \text{ mm}$        $\blacktriangleleft$

$r = \frac{d}{4} = 17.50 \times 10^{-3} \text{ m}$

$\frac{L}{r} = \frac{1.5}{17.5 \times 10^{-3}} = 85.7 > 66$

PROBLEM 10.110



10.109 The eccentric load  $P$  has a magnitude of 85 kN and is applied at a point located at a distance  $e = 30$  mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Use the interaction method with a 140-MPa allowable stress in bending to determine the smallest diameter  $d$  that can be used.

10.110 Solve Prob. 10.109, using the allowable-stress method and assuming that the aluminum alloy used is 2014-T6.

SOLUTION

Assume  $L/r > 55$        $\sigma_{all} = \frac{B}{(L/r)^2}$        $B = 372 \times 10^9 \text{ Pa}$

$c = \frac{d}{2}$        $A = \pi c^2 = \frac{\pi}{4} d^2$        $I = \frac{\pi}{4} e^4 = \frac{\pi}{64} d^4$

$r = \sqrt{\frac{I}{A}} = \frac{1}{4} d$

$\frac{P}{A} + \frac{Pec}{I} = \sigma_{all} = \frac{Br^2}{L^2}$

$\frac{PL^2}{ABr^2} + \frac{PL^2 e \frac{1}{2}d}{I B r^2} = 1$        $\frac{64 PL^2}{\pi d^4 B} + \frac{32 PL^2}{\pi d^3 B} = 1$       Let  $x = \frac{1}{d}$

$\frac{64 PL^2}{\pi B} x^4 + \frac{(16)(64) PL^2 e}{2\pi B} x^5 = 1$

$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi (372 \times 10^9)} x^4 + \frac{(16)(64)(85 \times 10^3)(1.5)^2 (30 \times 10^{-3})}{2\pi (372 \times 10^9)} = 1$

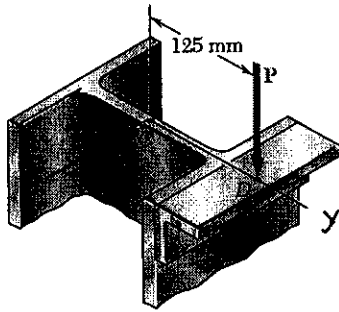
$10.473 \times 10^{-6} x^4 + 2.5136 \times 10^{-6} x^5 = 1$        $x = 12.441 \text{ m}^{-1}$

$d = \frac{1}{x} = 80.4 \times 10^{-3} \text{ m}$        $d = 80.4 \text{ mm}$       ←

$r = \frac{d}{4} = 20.1 \times 10^{-3} \text{ m}$        $\frac{L}{r} = \frac{1.5}{20.1 \times 10^{-3}} = 74.5 > 55$

PROBLEM 10.111

10.111 A steel compression member of 5.8-m effective length is to support a 296-kN eccentric load  $P$ . Using the interaction method, select the wide-flange shape of 200-mm nominal depth that should be used. Use  $E = 200$  GPa,  $\sigma_y = 250$  MPa and  $\sigma_{all} = 150$  MPa in bending.



SOLUTION

Steel:  $E = 200000$  MPa  $\sigma_y = 250$  MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = 125.66$$

$L_e = 5.8$  m  $\sigma_{all, bending} = 150$  MPa

For 200 mm nominal depth wide flange section

$$r_x \approx 88 \text{ mm} = 88 \times 10^{-3} \text{ m}, \quad y \approx \frac{210}{2} = 105 \text{ mm} = 105 \times 10^{-3} \text{ m}$$

$$r_y \approx 48 \text{ mm} = 48 \times 10^{-3} \text{ m} \quad \frac{L_e}{r_y} \approx \frac{5.8}{48 \times 10^{-3}} = 121 \quad \frac{h/r_x}{C_c} \approx 0.96$$

$$F.S. \approx \frac{5}{3} + \frac{3}{8}(0.96) - \frac{1}{8}(0.96)^3 = 1.916$$

$$\sigma_{all} \approx \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r_y}{C_c} \right)^2 \right] = \frac{250}{1.916} \left[ 1 - \frac{1}{2} (0.96)^2 \right] = 70 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, centric}} + \frac{P e_y y}{I_x \sigma_{all, bending}} = \frac{1}{A} \left[ \frac{P}{\sigma_{all, centric}} + \frac{P e_y y}{r_x^2 \sigma_{all, bending}} \right] = 1$$

$$A = \frac{P}{\sigma_{all, centric}} + \frac{P e_y y}{r_x^2 \sigma_{all, bending}}$$

$$= \frac{296 \times 10^3}{70 \times 10^6} + \frac{(296 \times 10^3)(125 \times 10^{-3})(105 \times 10^{-3})}{(88 \times 10^{-3})^2 (150 \times 10^6)} = 7.573 \times 10^{-3} \text{ m}^2 = 7573 \text{ mm}^2$$

Try W200 x 59  $A = 7560 \times 10^{-6} \text{ m}^2$ ,  $y = 105 \times 10^{-3} \text{ m}$ ,  $I_x = 61.1 \times 10^{-6} \text{ m}^4$

$r_y = 51.9 \times 10^{-3} \text{ m}$ ,  $L_e/r_y = 111.75 < C_c$

$\frac{L_e/r_y}{C_c} = 0.8893$   $F.S. = 1.9122$   $\sigma_{all, centric} = 79.04$  MPa

$$\frac{P}{A \sigma_{all, centric}} + \frac{P e_y y}{I_x \sigma_{all, bending}}$$

$$= \frac{296 \times 10^3}{(7560 \times 10^{-6})(79.04 \times 10^6)} + \frac{(296 \times 10^3)(125 \times 10^{-3})(105 \times 10^{-3})}{(61.1 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.4954 + 0.4239 = 0.9193 < 1 \quad (\text{allowed})$$

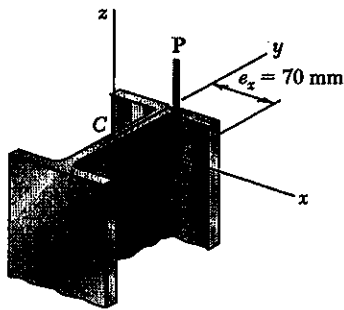
Trying W200 x 52 leads to

$$\frac{P}{A \sigma_{all, centric}} + \frac{P e_y y}{I_x \sigma_{all, bending}} = 1.047 > 1 \quad (\text{not allowed})$$

Use W200 x 59

PROBLEM 10.112

10.112 A steel column of 7.2-m effective length is to support an 83-kN eccentric load  $P$  at a point  $D$  located on the  $x$  axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use  $E = 200$  GPa,  $\sigma_y = 250$  MPa.



SOLUTION

Steel:  $E = 200000$  MPa  $\sigma_y = 250$  MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$$

$$L_e = 7.2 \text{ m}$$

Try W 250  $\times$  49.1  $A = 6250 \times 10^{-6}$

$$b_f = 202 \times 10^{-3} \text{ m}, \quad c = 101 \times 10^{-3} \text{ m}, \quad I_y = 15.1 \times 10^{-6} \text{ m}^4, \quad r_y = 49.2 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_y} = \frac{7.2}{49.2 \times 10^{-3}} = 146.34 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r_y)^2} = \frac{\pi^2 (200000)}{(1.92)(146.34)^2} = 48.01 \text{ MPa}$$

$$\begin{aligned} \frac{P}{A} + \frac{P e_x c}{I_y} &= \frac{83 \times 10^3}{6250 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101 \times 10^{-3})}{15.1 \times 10^{-6}} \\ &= 13.28 \times 10^6 + 38.86 \times 10^6 = 52.14 \text{ MPa} > 48.01 \text{ MPa} \\ &\quad \text{(not allowed)} \end{aligned}$$

Required area  $A \approx \left(\frac{52.14}{48.01}\right)(6250 \text{ mm}^2) = 6788 \text{ mm}^2$

Try W 250  $\times$  58

$$\frac{L_e}{r_y} = \frac{7.2}{50.3 \times 10^{-3}} = 143.14 \quad \sigma_{all} = \frac{\pi^2 (200000)}{(1.92)(143.14)^2} = 50.18 \text{ MPa}$$

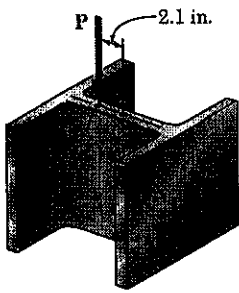
$$\begin{aligned} \frac{P}{A} + \frac{P e_x c}{I_y} &= \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101.5 \times 10^{-3})}{18.8 \times 10^{-6}} \\ &= 11.19 \times 10^6 + 31.37 \times 10^6 = 42.56 \text{ MPa} < 50.18 \text{ MPa} \end{aligned}$$

Use W 250  $\times$  58



PROBLEM 10.113

10.113 A steel column of 21-ft effective length must carry a load of 82 kips with an eccentricity of 2.1 in. as shown. Using the interaction method, select the wide-flange shape of 12-in. nominal depth that should be used. Use  $E = 29 \times 10^6$  psi,  $\sigma_y = 36$  ksi, and  $\sigma_{all} = 22$  ksi in bending.



SOLUTION

Steel:  $E = 29000$  ksi  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (29000)}{36}} = 126.10$

$L_e = 21$  ft = 252 in.

Try W 12x35  $r_y = 1.54$  in  $\frac{L_e}{r_y} = 163.64 > C_c$

$\sigma_{all, centric} = \frac{\pi^2 E}{(1.92)(L/r)^2} = \frac{\pi^2 (29000)}{(1.92)(163.64)^2} = 5.57$  ksi

$$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I_x \sigma_{all, bending}} = \frac{82}{(10.3)(5.57)} + \frac{(82)(2.1)(\frac{1}{2} \cdot 12.50)}{(285)(22)}$$

$$= 1.429 + 0.172 = 1.601 \quad (\text{not allowed})$$

Approximate required  $A = (1.596)(10.3) = 16.4$  in<sup>2</sup>

Try W 12x50  $r_y = 1.96$  in  $\frac{L_e}{r_y} = 128.57 > C_c$

$\sigma_{all, centric} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2 (29000)}{(1.92)(128.57)^2} = 9.02$  ksi

$$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I_x \sigma_{all, bending}} = \frac{82}{(14.7)(9.02)} + \frac{(82)(2.1)(\frac{1}{2} \cdot 12.19)}{(394)(22)}$$

$$= 0.618 + 0.121 = 0.739 \quad (\text{allowed})$$

Try W 12x40  $r_y = 1.93$  in  $\frac{L_e}{r_y} = \frac{252}{1.93} = 130.57 > C_c$

$\sigma_{all, centric} = \frac{\pi^2 E}{(1.92)(L/r)^2} = \frac{\pi^2 (29000)}{(1.92)(130.57)^2} = 8.74$  ksi

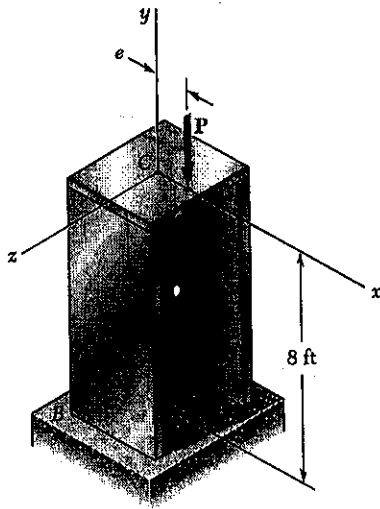
$$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I_x \sigma_{all, bending}} = \frac{82}{(11.8)(8.74)} + \frac{(82)(2.1)(\frac{1}{2} \cdot 11.94)}{(310)(22)}$$

$$= 0.795 + 0.151 = 0.946 \quad (\text{allowed}) \leftarrow$$

Use W 12x40

PROBLEM 10.114

10.114 A 43-kip axial load  $P$  is applied to the rolled-steel column  $BC$  at a point on the  $x$  axis at a distance  $e = 2.5$  in. from the geometric axis of the column. Using the allowable-stress method, select the wide-flange shape of 8-in. nominal depth that should be used. Use  $E = 29 \times 10^6$  psi. and  $\sigma_y = 36$  ksi.



SOLUTION

Steel:  $E = 29000$  ksi  $\sigma_y = 36$  ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$$L = 8 \text{ ft} = 96 \text{ in.} \quad L_e = 2L = 192 \text{ in}$$

Try W 8x31:  $r_y = 2.02$  in,  $\frac{L_e}{r_y} = 95.05 < C_c$

$$\frac{L_e/r_y}{C_c} = 0.754$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.754) - \frac{1}{8}(0.754)^2 = 1.896$$

$$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r_y}{C_c} \right)^2 \right] = \frac{36}{1.896} \left[ 1 - \frac{1}{2} (0.754)^2 \right] = 13.59 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{9.13} + \frac{(43)(2.5)(\frac{1}{2} \cdot 7.995)}{37.1} = 4.71 + 11.58 = 16.29 \text{ ksi} > 13.59 \text{ ksi} \text{ (not allowed)}$$

Approximate required area  $\left( \frac{16.29}{13.59} \right) (9.13) = 10.9 \text{ in}^2$

Try W 8x35  $r_y = 2.03$   $\frac{L_e}{r_y} = 94.58 < C_c$   $\frac{L_e/r_y}{C_c} = 0.750$

$$F.S. = 1.895 \quad \sigma_{all} = 13.65 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{10.3} + \frac{(43)(2.5)(\frac{1}{2} \cdot 8.020)}{42.6} = 14.29 \text{ ksi} > 13.65 \text{ ksi} \text{ (not allowed)}$$

Try W 8x40  $r_y = 2.04$   $\frac{L_e}{r_y} = 94.12 < C_c$   $\frac{L_e/r_y}{C_c} = 0.746$

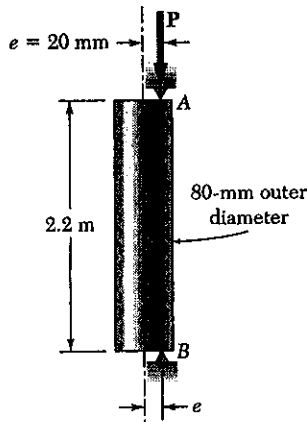
$$F.S. = 1.895 \quad \sigma_{all} = 13.71 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{11.7} + \frac{(43)(2.5)(\frac{1}{2} \cdot 8.07)}{49.1} = 12.51 \text{ ksi} < 13.71 \text{ ksi} \text{ (allowed)}$$

Use W 8x40 ▶

**PROBLEM 10.115**

10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load  $P$  with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume  $E = 200$  GPa,  $\sigma_y = 250$  MPa.



**SOLUTION**

$$r_o = \frac{1}{2} d_o = 40 \text{ mm}, \quad r_i = r_o - t$$

$$A = \pi (r_o^2 - r_i^2), \quad I = \frac{\pi}{4} (r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

$t$ mm	$r_i$ mm	$A$ $\text{mm}^2$	$I$ $10^6 \text{ mm}^4$	$r$ mm
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$l_e = 2.2 \text{ m}$

$P = 93 \times 10^3 \text{ N}$

Steel:  $E = 200000 \text{ MPa}$        $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.66$

Try  $t = 9 \text{ mm}$        $\frac{l_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c$        $\frac{l_e/r}{C_c} = 0.6917$

F.S. =  $\frac{5}{3} + \frac{2}{8} (0.6917) - \frac{1}{8} (0.6917)^2 = 1.885$

$\sigma_{all} = \frac{\sigma_y}{\text{F.S.}} \left[ 1 - \left( \frac{l_e/r}{C_c} \right)^2 \right] = \frac{250}{1.885} \left[ 1 - \frac{1}{2} (0.6917)^2 \right] = 100.9 \text{ MPa}$

$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}} = 104.2 \text{ MPa} > 100.9 \text{ MPa}$   
(not allowed)

Approximate required area  $\left( \frac{104.2}{100.9} \right) (2007 \times 10^{-6}) = 2073 \times 10^{-6} \text{ m}^2 = 2073 \text{ mm}^2$

For  $t = 12 \text{ mm}$        $\frac{l_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c$        $\frac{l_e/r}{C_c} = 0.7172$

F.S. = 1.890       $\sigma_{all} = 98.3 \text{ MPa}$

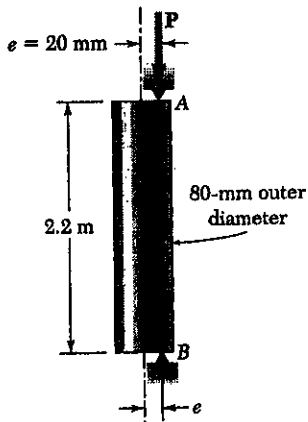
$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}} = 85.0 \text{ MPa} < 98.3 \text{ MPa}$

Use  $t = 12 \text{ mm}$        $\blacktriangleleft$

**PROBLEM 10.116**

**10.115** A steel tube of 80-mm outer diameter is to carry a 93-kN load  $P$  with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume  $E = 200$  GPa,  $\sigma_r = 250$  MPa.

**10.116** Solve Prob. 10.115, using the interaction method with  $P = 165$  kN,  $e = 15$  mm, and an allowable stress in bending of 150 MPa.



**SOLUTION**

$$r_o = \frac{1}{2} d_o = 40 \text{ mm} \quad r_i = r_o - t$$

$$A = \pi(r_o^2 - r_i^2) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

t mm	$r_i$ mm	A mm <sup>2</sup>	I 10 <sup>6</sup> mm <sup>4</sup>	r mm
3	37	726	0.539	27.24
6	34	1075	0.795	27.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2 \text{ m}$

$P = 165 \times 10^3 \text{ N}$

$\sigma_{all, bending} = 150 \text{ MPa}$

Steel:  $E = 200000 \text{ MPa}$   $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$

Try  $t = 9 \text{ mm}$   $\frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c$   $\frac{L_e/r}{C_c} = 0.6917$

F.S. =  $\frac{5}{3} + \frac{2}{3}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$

$\sigma_{all, centric} = \frac{\sigma_r}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.885} \left[ 1 - \frac{1}{2} (0.6917)^2 \right] = 100.9 \text{ MPa}$

$$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I \sigma_{all, bending}} = \frac{165 \times 10^3}{(2007 \times 10^{-6})(100.9 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.815 + 0.514 = 1.329 > 1 \quad (\text{not allowed})$$

Approximate required area  $A = (1.329)(2007) = 2667 \text{ mm}^2$

For  $t = 12 \text{ mm}$   $\frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c$   $\frac{L_e/r}{C_c} = 0.7172$

F.S. = 1.890  $\sigma_{all, centric} = 98.3 \text{ MPa}$

$$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I \sigma_{all, bending}} = \frac{165 \times 10^3}{(2564 \times 10^{-6})(98.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.655 + 0.432 = 1.087 > 1 \quad (\text{not allowed})$$

Try  $t = 15 \text{ mm}$   $\frac{L_e}{r} = \frac{2.2}{23.59 \times 10^{-3}} = 93.26 < C_c$   $(L_e/r)/C_c = 0.7422$

F.S. = 1.894  $\sigma_{all, centric} = 95.64 \text{ MPa}$

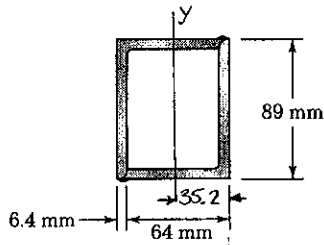
$$\frac{P}{A \sigma_{all, centric}} + \frac{Pec}{I \sigma_{all, bending}} = \frac{165 \times 10^3}{(3063 \times 10^{-6})(95.64 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.563 + 0.387 = 0.950 < 1 \quad (\text{allowed})$$

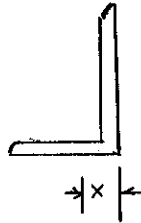
Use  $t = 15 \text{ mm}$

**PROBLEM 10.117**

**10.117** A column of 3.5-m effective length is made by welding together two  $89 \times 64$  mm angles as shown. Using  $E = 200$  GPa, determine the allowable centric load if a factor of safety of 2.8 is required.



**SOLUTION**



One angle  $x = 15.8$  mm

$$I_y = \bar{I}_y + A (35.2 - 15.8)^2$$

$$= 0.333 \times 10^6 + (938)(19.4)^2$$

$$= 0.686 \times 10^6 \text{ mm}^4$$

Two angles  $I_y = (2)(0.686 \times 10^6) = 1.372 \times 10^6 \text{ mm}^4 = 1.372 \times 10^{-6} \text{ m}^4$

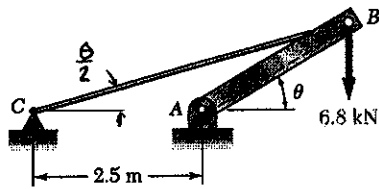
$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 E I_y}{(F.S.)^2 L_e^2} = \frac{\pi^2 (200 \times 10^9) (1.372 \times 10^{-6})}{(2.8)^2 (3.5)^2} = 79.0 \times 10^3 \text{ N}$$

$$= 79.0 \text{ kN}$$

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**PROBLEM 10.118**

10.118 Member  $AB$  consists of a single  $C130 \times 10.4$  steel channel of length 2.5 m. Knowing that the pins at  $A$  and  $B$  pass through the centroid of the cross section of the channel, determine the factor of safety for the load shown with respect to buckling in the plane of the figure when  $\theta = 30^\circ$ . Use Euler's formula with  $E = 200$  GPa.



**SOLUTION**

Since  $AB = 2.5$  m, triangle  $ABC$  is isosceles.

$$+\uparrow \Sigma F_y = 0$$

$$F_{AB} \sin 30^\circ - F_{AC} \sin 15^\circ - 6.8 = 0$$

$$F_{AB} \left( \sin 30^\circ - \frac{\sin 15^\circ \cos 30^\circ}{\cos 15^\circ} \right) = 0.26795 F_{AB} = 6.8$$

$$F_{AB} = 25.378 \text{ kN}$$

$$\Sigma F_x = 0$$

$$- F_{AC} \cos 15^\circ + F_{AB} \cos 30^\circ = 0$$

$$F_{AC} = \frac{F_{AB} \cos 30^\circ}{\cos 15^\circ}$$

$$C 130 \times 10.4 \quad I_{min} = 0.229 \times 10^6 \text{ mm}^4 = 0.229 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{min}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9) (0.229 \times 10^{-6})}{(2.5)^2} = 72.324 \times 10^3 \text{ N} = 72.324 \text{ kN}$$

$$F.S. = \frac{P_{cr}}{F_{AB}} = \frac{72.324}{25.378} = 2.85$$

**PROBLEM 10.119**

10.119 Supports  $A$  and  $B$  of the pin-ended column shown are at a fixed distance  $L$  from each other. Knowing that at a temperature  $T_0$  the force in the column is zero and that buckling occurs when the temperature is  $T_1 = T_0 + \Delta T$ , express  $\Delta T$  in terms of  $b$ ,  $L$ , and the coefficient of thermal expansion  $\alpha$ .



**SOLUTION**

Let  $P$  be the compressive force in the column.

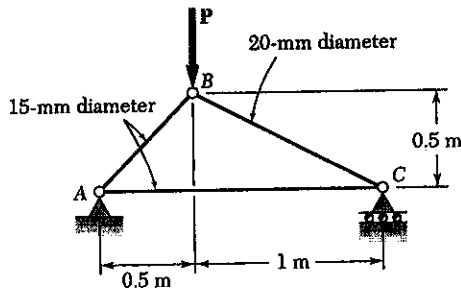
$$L\alpha(\Delta T) - \frac{PL}{EA} = 0 \quad P = EA\alpha(\Delta T)$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = P = EA\alpha(\Delta T)$$

$$\Delta T = \frac{\pi^2 EI}{L^2 EA\alpha} = \frac{\pi^2 E b^4 / 12}{L^2 E b^2} = \frac{\pi^2 b^2}{12 L^2 \alpha}$$

**PROBLEM 10.120**

10.120 Knowing that a factor of safety of 2.6 is required, determine the largest load  $P$  that can be applied to the structure shown. Use  $E = 200$  GPa and consider only buckling in the plane of the structure.



**SOLUTION**

BC:  $L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180 \text{ m}$

$$I = \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64} (20)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (7.854 \times 10^{-9})}{(1.1180)^2} = 12.403 \times 10^3 \text{ N} = 12.403 \text{ kN}$$

$$F_{BC,all} = \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770 \text{ kN}$$

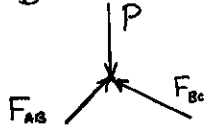
AB:  $L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711 \text{ m}$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (15)^4 = 2.485 \times 10^3 \text{ mm}^4 = 2.485 \times 10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3 \text{ N} = 9.8106 \text{ kN}$$

$$F_{AB,all} = \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773 \text{ kN}$$

Joint B



$$\sum F_x = 0 \quad \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$$

$$F_{BC} = 0.79057 F_{AB}$$

$$\sum F_y = 0 \quad \frac{0.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} + P = 0$$

$$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$$

$$P = 1.06066 F_{AB}$$

$$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC}$$

Allowable value for  $P$ .

$$P < 1.06066 F_{AB,all} = (1.06066)(3.773) = 4.00 \text{ kN}$$

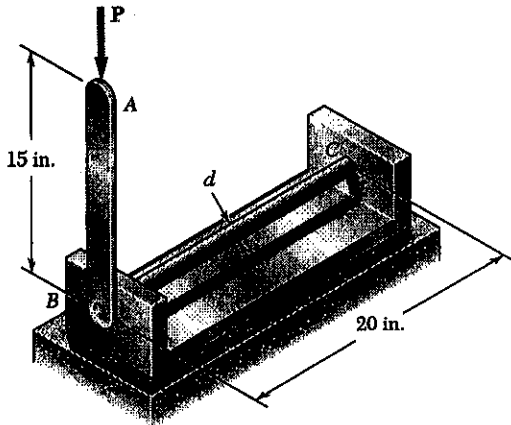
$$P < 1.3416 F_{BC,all} = (1.3416)(4.770) = 6.40 \text{ kN}$$

$$P_{all} = 4.00 \text{ kN}$$

PROBLEM 10.121

10.121 The steel rod  $BC$  is attached to the rigid bar  $AB$  and to the fixed support at  $C$ . Knowing that  $G = 11.2 \times 10^6$  psi, determine the diameter of rod  $BC$  for which the critical load  $P_{cr}$  of the system is 80 lb.

SOLUTION



Look at torsion spring  $BC$

$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ}{L} \phi = K\phi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

$$L = 20 \text{ in}$$

$$K = \frac{(11.2 \times 10^6) \pi d^4}{(20)(32)} = 54978 d^4$$



$$\sum M_B = 0$$

$$T - Pl \sin \phi = 0$$

$$K\phi - Pl \sin \phi = 0$$

$$P = \frac{K\phi}{l \sin \phi}$$

$$P_{cr} = \frac{K}{l}$$

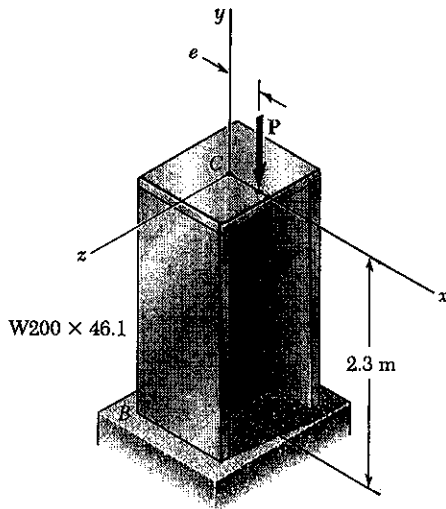
$$K = 54978 d^4 = P_{cr} l = (80)(15) = 1200$$

$$d = \sqrt[4]{\frac{1200}{54978}} = 0.384 \text{ in.}$$



PROBLEM 10.122

10.122 An axial load  $P$  of magnitude 560 kN is applied at a point on the  $x$  axis at a distance  $e = 8$  mm from the geometric axis of the W 200  $\times$  46.1 rolled-steel column BC. Using  $E = 200$  GPa, determine (a) the horizontal deflection of end C, (b) the maximum stress in the column.



SOLUTION

$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \quad e = 8 \times 10^{-3} \text{ m}$$

$$W 200 \times 46.1 \quad A = 5860 \text{ mm}^2 = 5860 \times 10^{-6} \text{ m}^2$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(15.3 \times 10^{-6})}{(4.6)^2}$$

$$= 1.42727 \times 10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560 \times 10^3}{1.42727 \times 10^6} = 0.39236$$

$$y_m = e \left[ \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} - 1 \right] = (8 \times 10^{-3}) \left[ \sec \left( \frac{\pi}{2} \sqrt{0.39236} \right) - 1 \right]$$

$$= (8 \times 10^{-3}) \left[ \sec(0.98393) - 1 \right] = (8 \times 10^{-3}) \left[ 1.8058 - 1 \right]$$

$$= 6.447 \times 10^{-3} \text{ m} \quad = 6.45 \text{ mm} \quad \blacktriangleleft$$

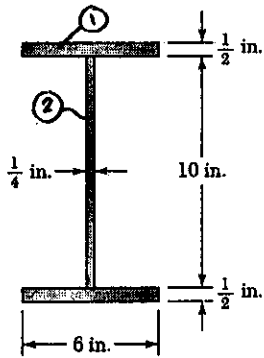
$$M_{max} = P(y_m + e) = (560 \times 10^3)(8 \times 10^{-3} + 6.447 \times 10^{-3}) = 8.090 \times 10^3 \text{ N}\cdot\text{m}$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560 \times 10^3}{5860 \times 10^{-6}} + \frac{8.090 \times 10^3}{151 \times 10^{-6}} = 149.1 \times 10^6 \text{ Pa} = 149.1 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 10.123

10.123 A column with the cross section shown has a 13.5-ft effective length. Knowing that  $\sigma_r = 36$  ksi, and  $E = 29 \times 10^6$  psi., use the AISC allowable stress design formulas to determine the largest centric load that can be applied to the column.



SOLUTION

$$A = 2A_1 + A_2 = (2)\left(\frac{1}{2}\right)(6) + (10)\left(\frac{1}{4}\right) = 8.5 \text{ in}^2$$

$$I_y = 2I_1 + I_2 = (2)\left(\frac{1}{12}\right)\left(\frac{1}{2}\right)(6)^3 + \left(\frac{1}{12}\right)(10)\left(\frac{1}{4}\right)^3 = 18.013 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{18.013}{8.5}} = 1.4557 \text{ in.}$$

$$L_e = 13.5 \text{ ft} = 162 \text{ in} \quad \frac{L_e}{r} = 111.29 < C_c$$

Steel:  $E = 29000 \text{ ksi}$ ,  $\sigma_r = 36 \text{ ksi}$   $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_r}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

$$\frac{L_e/r}{C_c} = 0.8826 \quad F.S. = \frac{5}{3} + \frac{3}{8}(0.8826) - \frac{1}{8}(0.8826)^2 = 1.912$$

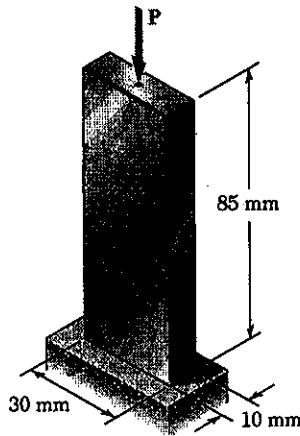
$$\sigma_{all} = \frac{\sigma_r}{F.S.} \left[ 1 - \frac{1}{2} \left( \frac{L_e/r}{C_c} \right)^2 \right] = \frac{36}{1.912} \left[ 1 - \frac{1}{2} (0.8826)^2 \right] = 11.49 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (11.49)(8.5) = 97.7 \text{ kips}$$

## PROBLEM 10.125

10.125 Bar  $AB$  is free at its end  $A$  and fixed at its base  $B$ . Determine the allowable centric load  $P$  if the aluminum alloy is (a) 6061-T6, (b) 2014-T6.

## SOLUTION



$$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$$

$$I_{\min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$$

$$r_{\min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$$

$$L_c = 2L = (2)(85) = 170 \text{ mm} \quad \frac{L_c}{r_{\min}} = 58.88$$

(a) 6061-T6  $L/r < 66$

$$\begin{aligned} \sigma_{\text{all}} &= 139 - 0.868(L/r) = 139 - (0.868)(58.88) \\ &= 87.9 \text{ MPa} \end{aligned}$$

$$\begin{aligned} P_{\text{all}} &= \sigma_{\text{all}} A = (87.9 \times 10^6)(300 \times 10^{-6}) = 26.4 \times 10^3 \text{ N} \\ &= 26.4 \text{ kN} \end{aligned} \quad \blacktriangleleft$$

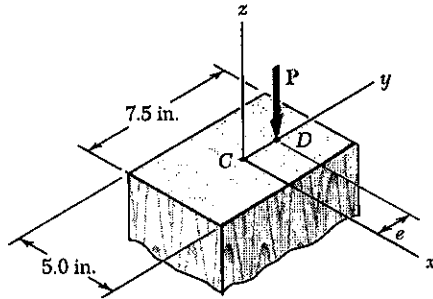
(b) 2014-T6  $L/r > 55$

$$\sigma_{\text{all}} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(58.88)^2} = 107.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (107.3 \times 10^6)(300 \times 10^{-6}) = 32.2 \times 10^3 \text{ N} = 32.2 \text{ kN} \quad \blacktriangleleft$$

PROBLEM 10.126

10.126 A sawn lumber column of 5.0 × 7.5-in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain  $\sigma_c = 1180$  psi and a modulus of elasticity  $E = 1.2 \times 10^6$  psi. Using the allowable-stress method, determine the largest eccentric load  $P$  that can be applied when (a)  $e = 0.5$  in., (b)  $e = 1.0$  in.



SOLUTION

Sawn lumber:  $\sigma_c = 1180$  psi     $E = 1.2 \times 10^6$  psi  
 $C = 0.8$      $K_{CE} = 0.300$

$L_e = 8.5 \text{ ft} = 102 \text{ in.}$

$b = 7.5 \text{ in.}, d = 5.0 \text{ in.}, c = \frac{b}{2} = 3.75 \text{ in.}$

$A = bd = (7.5)(5.0) = 37.5 \text{ in}^2$      $I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in}^4$

$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{K_{CE} E d^2}{L^2} = \frac{(0.300)(1.2 \times 10^6)(5.0)^2}{(102)^2} = 865 \text{ psi}$

$\sigma_{CE} / \sigma_c = 865 / 1180 = 0.7331$

$C_P = \frac{1 + \sigma_{CE} / \sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2C}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{C}} = 0.5763$

$\sigma_{all} = \sigma_c C_P = (1180)(0.5763) = 680 \text{ psi}$

$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all}$

$P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{e c}{I_x}}$

(a)  $e = 0.5 \text{ in.}$

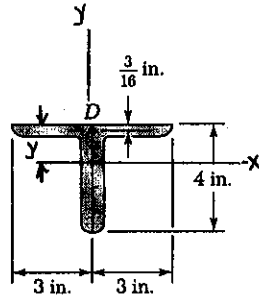
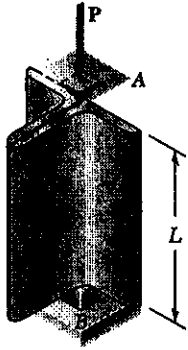
$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(0.5)(3.75)}{175.78}} = 18210 \text{ lb.} = 18.21 \text{ kips}$      $\blacktriangleleft$

(b)  $e = 1.0 \text{ in.}$

$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(1.0)(3.75)}{175.78}} = 14170 \text{ lb.} = 14.17 \text{ kips}$      $\blacktriangleleft$

PROBLEM 10.127

10.127 Two  $4 \times 3 \times \frac{3}{8}$  -in. steel angles are welded together to form the column  $AB$ . An axial load  $P$  of magnitude 14 kips is applied at point  $D$ . Using the allowable-stress method, determine the largest allowable length  $L$ . Assume  $E = 29 \times 10^6$  psi and  $\sigma_y = 36$  ksi.



SOLUTION

One angle  $L$   $4 \times 3 \times \frac{3}{8}$   $A = 2.48 \text{ in}^2$   
 $I_x = 3.96 \text{ in}^4$ ,  $S_x = 1.46 \text{ in}^3$ ,  $r_x = 1.26 \text{ in}$ ,  $y = 1.28 \text{ in}$ .  
 $I_y = 1.92 \text{ in}^4$ ,  $r_y = 0.879 \text{ in}$ ,  $x = 0.782 \text{ in}$

Two angles  $A = (2)(2.48) = 4.96 \text{ in}^2$

$I_x = (2)(3.96) = 7.92 \text{ in}^4$ ,  $S_x = (2)(1.46) = 2.92 \text{ in}^3$ ,  $r_x = 1.26$ ,  $y = 1.28 \text{ in}$ .

$I_y = 2 [I_{y0} + Ax^2] = (2)[1.92 + (2.48)(0.782)^2] = 6.873 \text{ in}^4 = I_{\min}$

$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = 1.177 \text{ in}$ ,  $e = y - \frac{3}{16} = 1.28 - \frac{3}{16} = 1.0925 \text{ in}$

$P = 14 \text{ kips}$ ,  $\sigma_{\text{all}} = \frac{P}{A} + \frac{Pe y}{I_x} = \frac{14}{4.96} + \frac{(14)(1.0925)(1.28)}{7.92} = 5.294 \text{ ksi}$

$E = 29000 \text{ ksi}$ ,  $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.1$

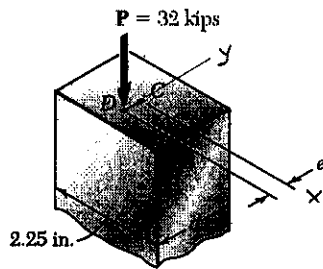
Assume  $\frac{L_e}{r} > C_c$ ,  $\sigma_{\text{all}} = \frac{\pi^2 E}{1.92 (L/r_{\min})^2}$ ,  $\left(\frac{L}{r_{\min}}\right)^2 = \frac{\pi^2 E}{1.92 \sigma_{\text{all}}}$

$\frac{L}{r_{\min}} = \sqrt{\frac{\pi^2 E}{1.92 \sigma_{\text{all}}}} = \sqrt{\frac{\pi^2(29000)}{(1.92)(5.294)}} = 167.8 > C_c$

$L = 167.8 r_{\min} = (167.8)(1.177) = 197.5 \text{ in.} = 16.46 \text{ ft}$

PROBLEM 10.128

10.128 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension  $d$  of the cross section that can be used when  $e = 0.4$  in.



SOLUTION

$$A = 2.25 d \quad c = \frac{1}{2} d \quad e = 0.4 \text{ in.} \quad L_e = 36 \text{ in}$$

$$\sigma_{all,b} = 24 \text{ ksi} \quad P = 32 \text{ kips}$$

$$I_x = \frac{1}{12} (2.25) d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume  $r_x = r_{min}$ , i.e.  $d < 2.25$

$$L_e/r_{min} = \sqrt{12} L_e/d$$

Assume  $L_e/r_{min} > 55$ .

$$\sigma_{all,c} = \frac{54000}{(L_e/r_x)^2} = \frac{54000 d^2}{12 L_e^2} = \frac{54000}{(12)(36)^2} d^2 = 3.47222 d^2$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} = \frac{32}{(2.25 d)(3.47222 d^2)} + \frac{(12)(32)(0.4)(\frac{1}{2} d)}{(2.25 d^3)(24)} = 1$$

$$\frac{4.096}{d^3} + \frac{1.42222}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 4.096 x^3 + 1.42222 x^2 = 1$$

Solving for  $x$ ,  $x = 0.528118$ ,  $d = \frac{1}{x} = 1.894 \text{ in.} < 2.25 \text{ in.}$

$$L/r_x = (\sqrt{12})(36)/1.894 = 65.8 > 66 \quad d = 1.894 \text{ in.} \blacktriangleleft$$

**PROBLEM 10.C1**

**10.C1** A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load  $P$ . For the grade of steel used  $E = 200$  GPa and  $\sigma_y = 245$  MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load  $P_{all}$  for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

**SOLUTION**

ENTER RADIUS RAD, EFFECTIVE LENGTH  $L_e$   
AND FACTOR OF SAFETY FS

COMPUTE RADIUS OF GYRATION

$$A = \pi \text{ RAD}^2$$

$$I = \frac{1}{4} \pi \text{ RAD}^4$$

$$r = \sqrt{\frac{I}{A}}$$

DETERMINE ALLOWABLE CENTRIC LOAD

CRITICAL STRESS :

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

LET  $\sigma$  EQUAL SMALLER OF  $\sigma_{cr}$  AND  $\sigma_y$

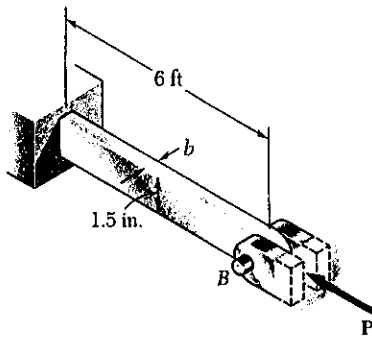
$$P_{all} = \frac{\sigma A}{FS}$$

PROGRAM OUTPUT

Radius of rod m	Critical stress MPa	Allowable load kN
.006	71.1	2.87
.008	126.3	9.07
.010	197.4	22.15
-----		
.012	284.2	39.58
.014	386.9	53.88
.016	505.3	70.37
.018	639.6	89.06
.020	789.6	109.96
.022	955.4	133.05
.024	1137.0	158.34

Below the dashed line we have:  
critical stress > yield strength

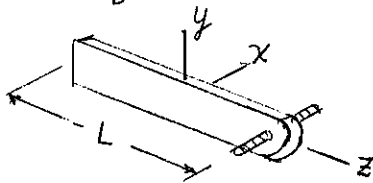
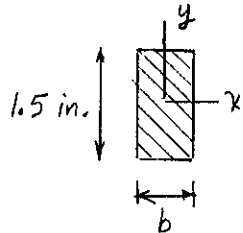
**PROBLEM 10.C2**



**10.C2** An aluminum bar is fixed at end A and supported at end B so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end B is prevented by the brackets. Knowing that  $E = 10.1 \times 10^6$  psi, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load P for values of b from 0.75 in. to 1.5 in., using 0.125-in. increments.

**SOLUTION**

ENTER E, LENGTH L AND FACTOR OF SAFETY FS FOR  $b = 0.75$  TO  $1.5$  WITH  $0.125$  INCREMENTS



COMPUTE RADII OF GYRATION

$$A = 1.5 b$$

$$I_x = \frac{1}{12} b (1.5)^3 \quad r_x = \sqrt{\frac{I_x}{A}}$$

$$I_y = \frac{1}{8} b^3 \quad r_y = \sqrt{\frac{I_y}{A}}$$

COMPUTE CRITICAL STRESSES

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

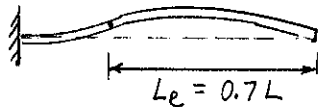
$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

LET  $\sigma_{cr}$  EQUAL SMALLER STRESS

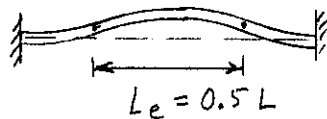
COMPUTE ALLOWABLE CENTRIC LOAD

$$P_{all} = \frac{\sigma_{cr} A}{FS}$$

BUCKLING IN yz PLANE



BUCKLING IN xz PLANE

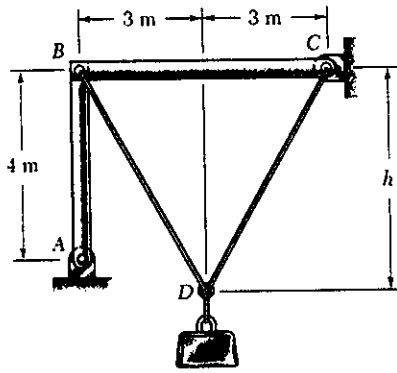


PROGRAM OUTPUT

b	Critical stress x axis	Critical stress y axis	Allowable load
in.	ksi	ksi	kips
.750	7.358	3.6	1.62
.875	7.358	4.9	2.58
1.000	7.358	6.4	3.85
1.125	7.358	8.1	4.97
1.250	7.358	10.0	5.52
1.375	7.358	12.1	6.07
1.500	7.358	14.4	6.62



**PROBLEM 10.C3**



**10.C3** The pin-ended members  $AB$  and  $BC$  consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass  $m$  of the largest block that can be supported by the cable arrangement shown for values of  $h$  from 4 m to 8 m, using 0.25-m increments. Use  $E = 70$  GPa and consider only buckling in the plane of the structure.

**SOLUTION**

COMPUTE MOMENT OF INERTIA

$$I = \frac{\pi}{4} (0.06^4 - 0.05^4)$$

FOR  $h = 4$  TO  $8$  USING  $0.25$  INCREMENTS

COMPUTE ALLOWABLE LOADS FOR MEMBERS

$$(F_{AB})_{cr} = \frac{\pi^2 EI}{3.5(4)^2}; (F_{BC})_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

DETERMINE ALLOWABLE  $W$

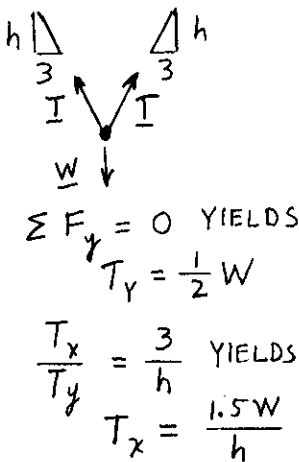
$$(W_{all})_1 = 2(F_{AB})_{cr}; (W_{all})_2 = \frac{h}{1.5}(F_{BC})_{cr}$$

$W_{all}$  EQUALS SMALLER VALUE

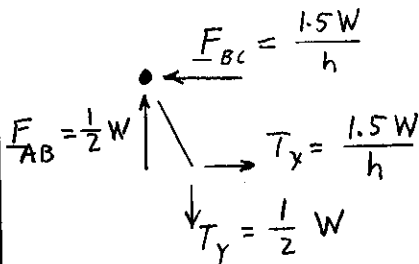
COMPUTE MASS  $m$

$$m = \frac{W_{all}}{9.81}$$

JOINT D:



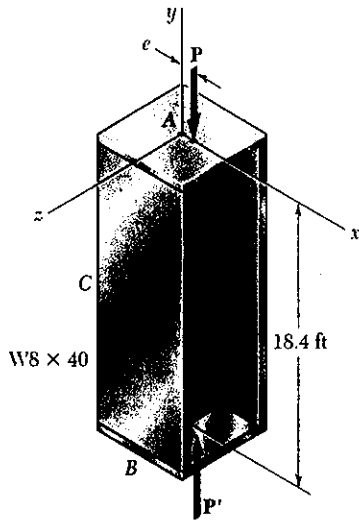
JOINT B:



PROGRAM OUTPUT

h	Weight critical stress AB	Weight critical stress BC	mass
m	kN	kN	kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10309.52
5.50	455.11	370.8	10800.45
5.75	455.11	387.7	11291.38
6.00	455.11	404.5	11782.31
6.25	455.11	421.4	12273.24
6.50	455.11	438.3	12764.17
6.75	455.11	455.1	13255.10
7.00	455.11	472.0	13255.10
7.25	455.11	488.8	13255.10
7.50	455.11	505.7	13255.10
7.75	455.11	522.5	13255.10
8.00	455.11	539.4	13255.10

**PROBLEM 10.C4**



**10.C4** An axial load  $P$  is applied at a point located on the  $x$  axis at a distance  $e = 0.5$  in. from the geometric axis of the  $W8 \times 40$  rolled-steel column  $AB$ . Using  $E = 29 \times 10^6$  psi, write a computer program and use it to calculate for values of  $P$  from 25 to 75 kips, using 5-kip increments, (a) the horizontal deflection at the midpoint  $C$ , (b) the maximum stress in the column.

**SOLUTION**

ENTER LENGTH  $L$ , ECCENTRICITY  $e$

ENTER PROPERTIES  $A, I_y, r_y, b_f$

COMPUTE CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 E I_y}{L^2}$$

FOR  $P = 25$  TO  $75$  IN INCREMENTS OF  $5$

COMPUTE HORIZONTAL DEFLECTION AT  $C$

$$y_c = e \left( \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1.0 \right)$$

COMPUTE MAXIMUM STRESS

$$\sigma_{max} = \frac{P}{A} \left( 1 + \frac{e b_f}{2 r_y^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

PROGRAM OUTPUT

Load kip	maximum deflection in.	maximum stress kips
25.0	.059	3.29
30.0	.072	3.99
35.0	.086	4.69
40.0	.100	5.41
45.0	.115	6.14
50.0	.130	6.88
55.0	.146	7.65
60.0	.163	8.43
65.0	.181	9.22
70.0	.199	10.04
75.0	.219	10.88

## PROBLEM 10.C5

**10.C5** A column of effective length  $L$  is made from a rolled-steel shape and carries a centric axial load  $P$ . The yield strength for the grade of steel used is denoted by  $\sigma_Y$ , the modulus of elasticity by  $E$ , the cross-sectional area of the selected shape by  $A$ , and its smallest radius of gyration by  $r$ . Using the AISC design formulas for allowable stress design, write a computer program that can be used with either SI or U.S. customary units to determine the allowable load  $P$ . Use this program to solve (a) Prob. 10.57, (b) Prob. 10.58, (c) Prob. 10.60.

## SOLUTION

ENTER  $L, E, \sigma_Y$ ENTER PROPERTIES  $A, r_y$ DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$$

$$\text{IF } L/r_y \geq C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r_y)^2}$$

$$\text{IF } L/r_y < C_c$$

$$FS = \frac{5}{3} + \frac{3}{8} \left( \frac{L/r_y}{C_c} \right) - \frac{1}{8} \left( \frac{L/r_y}{C_c} \right)^3$$

$$\sigma_{all} = \frac{\sigma_Y}{FS} \left( 1 - \frac{(L/r_y)^2}{2 C_c^2} \right)$$

CALCULATE ALLOWABLE LOAD:

$$P_{all} = \sigma_{all} A$$

CONTINUED

PROBLEM 10.C5 CONTINUED

PROGRAM OUTPUT

Problem 10.57 (a)

Effective Length = 6.50 m  
A = 6250.0 mm\*\*2  
ry = 49.2 mm  
Yield strength = 250.0 MPa  
E = 200 GPa

-----  
Allowable centroid load: P = 368.139 kN

Problem 10.57 (b)

Effective Length = 6.50 m  
A = 10200.0 mm\*\*2  
ry = 65.0 mm  
Yield strength = 250.0 MPa  
E = 200 GPa

-----  
Allowable centroid load: P = 916.148 kN

Problem 10.58 (a)

Effective Length = 21.00 ft  
A = 9.130 in\*\*2  
ry = 2.020 in.  
Yield strength = 36.0 ksi  
E = 29000 ksi

-----  
Allowable centroid load: P = 87.566 kips

Problem 10.58 (b)

Effective Length = 21.00 ft  
A = 9.130 in\*\*2  
ry = 2.020 in.  
Yield strength = 50.0 ksi  
E = 29000 ksi

-----  
Allowable centroid load: P = 87.452 kips

Problem 10.60 (a)

Effective Length = 4.00 m  
A = 13800.0 mm\*\*2  
ry = 43.4 mm  
Yield strength = 345.0 MPa  
E = 200 GPa

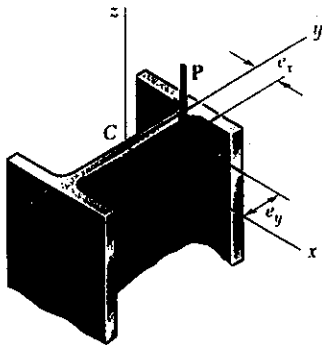
-----  
Allowable centroid load: P = 1567.879 kN

Problem 10.60 (b)

Effective Length = 6.50 m  
A = 13800.0 mm\*\*2  
ry = 43.4 mm  
Yield strength = 345.0 MPa  
E = 200 GPa

-----  
Allowable centroid load: P = 632.667 kN

**PROBLEM 10.C6**



**10.C6** A column of effective length  $L$  is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by  $\sigma_Y$ , the allowable stress in bending by  $\sigma_{all}$ , the modulus of elasticity by  $E$ , the cross-sectional area of the selected shape by  $A$ , and its smallest radius of gyration by  $r$ . Write a computer program that can be used with either SI or U.S. customary units to determine the allowable load  $P$ , using either the allowable-stress method or the interaction method. Use this program to check the given answer for (a) Prob. 10.111, (b) Prob. 10.112, (c) Prob. 10.113.

**SOLUTION**

ENTER  $L, E, \sigma_Y, (\sigma_{all})_{bending}, e_x, e_y$

ENTER PROPERTIES  $A, S_x, S_y, r_y$

DETERMINE ALLOWABLE STRESS

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$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$$

IF  $L/r_y \geq C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2}$$

IF  $L/r_y < C_c$

$$FS = \frac{5}{3} + \frac{3}{8} \left(\frac{L/r_y}{C_c}\right) - \frac{1}{8} \left(\frac{L/r_y}{C_c}\right)^3$$

$$\sigma_{all} = \frac{\sigma_Y}{FS} \left(1 - \frac{(L/r_y)^2}{2C_c^2}\right)$$

FOR ALLOWABLE-STRESS METHOD

$$COEF = \frac{1}{A} + \frac{e_x}{S_x} + \frac{e_y}{S_y}$$

$$P_{all} = \frac{\sigma_{all}}{COEF}$$

FOR INTERACTION METHOD

$$COEF = \frac{1}{A\sigma_{all}} + \frac{(e_x/S_x) + (e_y/S_y)}{(\sigma_{all})_{bending}}$$

$$P_{all} = \frac{1.0}{COEF}$$

**CONTINUED**

PROBLEM 10.C6 CONTINUED

PROGRAM OUTPUT

Problem 10.111

Effective Length = 5.80 m  
A = 7560.0 mm\*\*2  
ry = 51.900 mm  
Sx = 582000.0 mm\*\*3  
Yield strength = 250.0 MPa  
E = 200 GPa

-----  
Using Interaction Method  
Allowable load: P = 322.022 kN  
-----

Problem 10.112

Effective Length = 7.20 m  
A = 7420.0 mm\*\*2  
ry = 50.300 mm  
Sy = 185000.0 mm\*\*3  
Yield strength = 250.0 MPa  
E = 200 GPa

-----  
Using Allowable-Stress Method  
Allowable load: P = 97.781 kN  
-----

Problem 10.113

Effective Length = 21.00 ft  
A = 11.800 in\*\*2  
ry = 1.930 in.  
Sx = 51.90 in\*\*3  
Yield strength = 36.0 ksi  
E = 29 x 10^3 ksi

-----  
Using Interaction Method  
Allowable load: P = 86.722 kips  
-----

# CHAPTER 11

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**PROBLEM 11.1**

**11.1** Determine the modulus of resilience for each of the following grades of structural steel:

**SOLUTION**

(a) ASTM	A709 Grade 50:	$\sigma_r = 50$ ksi
(b) ASTM	A913 Grade 65:	$\sigma_r = 65$ ksi
(c) ASTM	A709 Grade 100:	$\sigma_r = 100$ ksi

Structural steel  $E = 29 \times 10^6$  psi for all three steels given.

(a)  $\sigma_r = 50$  ksi  $= 50 \times 10^3$  psi

$$U_r = \frac{\sigma_r^2}{2E} = \frac{(50 \times 10^3)^2}{(2)(29 \times 10^6)} = 43.1 \text{ in}\cdot\text{lb}/\text{in}^3$$

(b)  $\sigma_r = 65$  ksi  $= 65 \times 10^3$  psi

$$U_r = \frac{\sigma_r^2}{2E} = \frac{(65 \times 10^3)^2}{(2)(29 \times 10^6)} = 72.8 \text{ in}\cdot\text{lb}/\text{in}^3$$

(c)  $\sigma_r = 100$  ksi  $= 100 \times 10^3$  psi

$$U_r = \frac{\sigma_r^2}{2E} = \frac{(100 \times 10^3)^2}{(2)(29 \times 10^6)} = 172.4 \text{ in}\cdot\text{lb}/\text{in}^3$$

**PROBLEM 11.2**

**11.2** Determine the modulus of resilience for each of the following aluminum alloys:

**SOLUTION**

(a) 1100-H14:	$E = 70$ GPa,	$\sigma_r = 55$ MPa
(b) 2014-T6:	$E = 72$ GPa	$\sigma_r = 220$ MPa
(c) 6061-Y6:	$E = 69$ GPa	$\sigma_r = 140$ MPa

Aluminum alloys

(a)  $E = 70 \times 10^9$  Pa,  $\sigma_r = 55 \times 10^6$  Pa

$$U_r = \frac{\sigma_r^2}{2E} = \frac{(55 \times 10^6)^2}{(2)(70 \times 10^9)} = 21.6 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^2 = 21.6 \text{ kJ}/\text{m}^3$$

(b)  $E = 72 \times 10^9$  Pa,  $\sigma_r = 220 \times 10^6$  Pa

$$U_r = \frac{\sigma_r^2}{2E} = \frac{(220 \times 10^6)^2}{(2)(72 \times 10^9)} = 336 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^2 = 336 \text{ kJ}/\text{m}^3$$

(c)  $E = 69 \times 10^9$  Pa,  $\sigma_r = 140 \times 10^6$  Pa

$$U_r = \frac{\sigma_r^2}{2E} = \frac{(140 \times 10^6)^2}{(2)(69 \times 10^9)} = 142.0 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^2 = 142.0 \text{ kJ}/\text{m}^3$$



## PROBLEM 11.3

11.3 Determine the modulus of resilience for each of the following metals:

- (a) Stainless steel AISI 302 (annealed):  $E = 190 \text{ GPa}$ ,  $\sigma_Y = 260 \text{ MPa}$   
 (b) Stainless steel AISI 302 (cold-rolled):  $E = 190 \text{ GPa}$ ,  $\sigma_Y = 520 \text{ MPa}$   
 (c) Malleable cast iron:  $E = 165 \text{ GPa}$ ,  $\sigma_Y = 230 \text{ MPa}$

## SOLUTION

$$(a) \quad E = 190 \times 10^9 \text{ Pa}, \quad \sigma_Y = 260 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(260 \times 10^6)^2}{(2)(190 \times 10^9)} = 177.9 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 177.9 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(b) \quad E = 190 \times 10^9 \text{ Pa}, \quad \sigma_Y = 520 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(520 \times 10^6)^2}{(2)(190 \times 10^9)} = 712 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 712 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(c) \quad E = 165 \times 10^9 \text{ Pa}, \quad \sigma_Y = 230 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(230 \times 10^6)^2}{(2)(165 \times 10^9)} = 160.3 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 160.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

## PROBLEM 11.4

11.4 Determine the modulus of resilience for each of the following alloys:

- (a) Titanium:  $E = 16.5 \times 10^6 \text{ psi}$ ;  $\sigma_Y = 120 \text{ ksi}$   
 (b) Magnesium:  $E = 6.5 \times 10^6 \text{ psi}$ ;  $\sigma_Y = 29 \text{ ksi}$   
 (c) Cupronickel (annealed):  $E = 20 \times 10^6 \text{ psi}$ ;  $\sigma_Y = 16 \text{ ksi}$

## SOLUTION

$$(a) \quad E = 16.5 \times 10^6 \text{ psi}, \quad \sigma_Y = 120 \times 10^3 \text{ psi}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(120 \times 10^3)^2}{(2)(16.5 \times 10^6)} = 436 \text{ in}\cdot\text{lb}/\text{in}^3 \quad \blacktriangleleft$$

$$(b) \quad E = 6.5 \times 10^6 \text{ psi}, \quad \sigma_Y = 29 \times 10^3 \text{ psi}$$

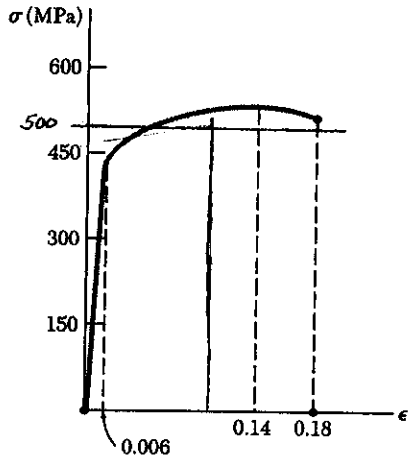
$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(29 \times 10^3)^2}{(2)(6.5 \times 10^6)} = 64.7 \text{ in}\cdot\text{lb}/\text{in}^3 \quad \blacktriangleleft$$

$$(c) \quad E = 20 \times 10^6 \text{ psi}, \quad \sigma_Y = 16 \times 10^3 \text{ psi}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(16 \times 10^3)^2}{(2)(20 \times 10^6)} = 6.40 \text{ in}\cdot\text{lb}/\text{in}^3 \quad \blacktriangleleft$$

**PROBLEM 11.5**

11.5 The stress-strain diagram shown has been drawn from data obtained during a tensile test of an aluminum alloy. Using  $E = 72 \text{ GPa}$ , (a) determine the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.



**SOLUTION**

(a)  $\sigma_Y = E \epsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2} E \epsilon_Y^2 = \frac{1}{2} (72 \times 10^9) (0.006)^2$$

$$= 1296 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 1296 \text{ kJ}/\text{m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

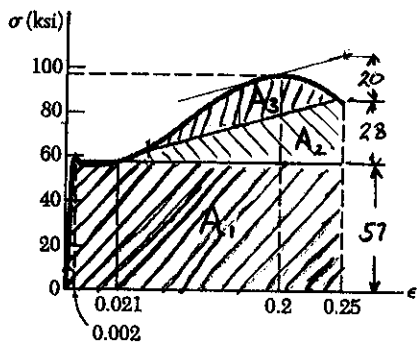
The average ordinate of the stress-strain curve is  $500 \text{ MPa} = 500 \times 10^6 \text{ N}/\text{m}^2$

The area under the curve is  $A = (500 \times 10^6) (0.18) = 90 \times 10^6 \text{ N}/\text{m}^2$

modulus of toughness =  $90 \times 10^6 \text{ J}/\text{m}^3 = 90 \text{ MJ}/\text{m}^3$

**PROBLEM 11.6**

11.6 The stress-strain diagram shown has been drawn from data obtained during the tensile test of a specimen of structural steel. Using  $E = 29 \times 10^6 \text{ psi}$ , (a) determine the modulus of resilience of the steel, (b) determine the modulus of toughness of the steel.



**SOLUTION**

(a)  $\sigma_Y = E \epsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2} E \epsilon_Y^2 = \frac{1}{2} (29 \times 10^6) (0.002)^2$$

$$= 58.0 \text{ in}\cdot\text{lb}/\text{in}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

$$A_1 = (57)(0.25 - 0.002) = 14.14 \text{ kips}/\text{in}^2 = 14.14 \text{ in}\cdot\text{kip}/\text{in}^3$$

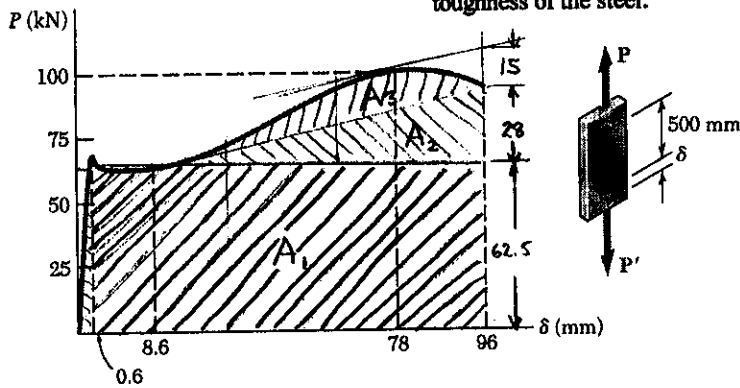
$$A_2 = \frac{1}{2}(28)(0.25 - 0.021) = 3.21 \text{ kips}/\text{in}^2 = 3.21 \text{ in}\cdot\text{kip}/\text{in}^3$$

$$A_3 = \frac{2}{3}(20)(0.25 - 0.075) = 2.33 \text{ kips}/\text{in}^2 = 2.33 \text{ in}\cdot\text{kip}/\text{in}^3$$

$$\text{modulus of toughness} = U_Y + A_1 + A_2 + A_3 \approx 20 \text{ in}\cdot\text{kip}/\text{in}^3$$

**PROBLEM 11.7**

11.7 The load-deformation diagram shown has been drawn from data obtained during the tensile test of a specimen of structural steel. Knowing that the cross-sectional area of the specimen is 250 mm<sup>2</sup> and that the deformation was measured using a 500-mm gage length, determine (a) the modulus of resilience of the steel, (b) the modulus of toughness of the steel.



**SOLUTION**

Assuming that yielding occurs at  $P = 62.5 \text{ kN}$  and  $\delta = 0.6 \text{ mm}$

$$U_Y = \frac{1}{2} (62.5 \times 10^3) (0.6 \times 10^{-3})$$

$$= 18.75 \text{ N}\cdot\text{m}$$

$$= 18.75 \text{ J}$$

Volume of stressed material  $V = AL = (250)(500) = 125 \times 10^3 \text{ mm}^3$

$$= 125 \times 10^{-6} \text{ m}^3$$

$$U_Y = \frac{U_Y}{V} = \frac{18.75}{125 \times 10^{-6}} = 150 \times 10^3 = 150 \text{ kJ/m}^3$$

$$A_1 = (62.5 \times 10^3) (96 \times 10^{-3}) = 6 \times 10^3 \text{ N}\cdot\text{m} = 6 \times 10^3 \text{ J}$$

$$A_2 = \frac{1}{2} (28 \times 10^3) (96 - 8.6) \times 10^{-3} = 1.22 \times 10^3 \text{ N}\cdot\text{m} = 1.22 \times 10^3 \text{ J}$$

$$A_3 = \frac{2}{3} (15 \times 10^3) (61 \times 10^{-3}) = 0.61 \times 10^3 \text{ N}\cdot\text{m} = 0.61 \times 10^3 \text{ J}$$

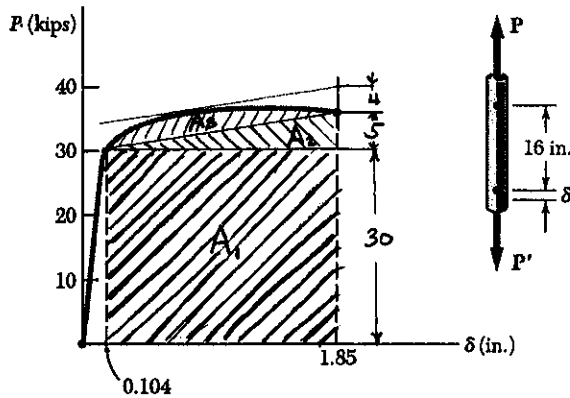
Total energy  $U = U_Y + A_1 + A_2 + A_3 = 7.85 \times 10^3 \text{ J}$

modulus of toughness =  $\frac{U}{V} = \frac{7.85 \times 10^3}{125 \times 10^{-6}} = 63 \times 10^6 \text{ J/m}^3$

$$= 63 \text{ MJ/m}^3$$

**PROBLEM 11.8**

11.8 The load-deformation diagram shown has been drawn from data obtained during the tensile test of a 0.75-in.-diameter rod of an aluminum alloy. Knowing that the deformation was measured using a 16-in. gage length, determine (a) the modulus of resilience of the alloy, (b) modulus of toughness of the alloy.



**SOLUTION**

Volume of stressed material involved in the measurement.

$$V = \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4} (0.75)^2 (16) = 7.0686 \text{ in}^3$$

(a) Modulus of resilience.

$$P_Y = 30 \text{ kips}, \quad \delta_Y = 0.104 \text{ in.}$$

$$U_Y = \frac{1}{2} P_Y \delta_Y = \frac{1}{2} (30)(0.104) = 1.56 \text{ in. kip} = 1560 \text{ in. lb.}$$

modulus of resilience 
$$u_Y = \frac{U_Y}{V} = \frac{1560}{7.0686} = 221 \text{ in. lb/in}^3$$

(b) modulus of toughness

$$A_1 = (30)(1.85 - 0.104) = 52.38 \text{ kip}\cdot\text{in} = 52380 \text{ in}\cdot\text{lb/in}^3$$

$$A_2 = \frac{1}{2}(5)(1.85 - 0.104) = 4.365 \text{ kip}\cdot\text{in} = 4365 \text{ in}\cdot\text{lb/in}^3$$

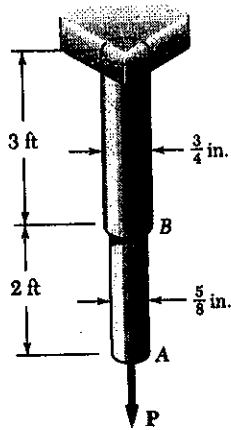
$$A_3 = \frac{2}{3}(4)(1.85 - 0.104) = 4.656 \text{ kip}\cdot\text{in} = 4656 \text{ in}\cdot\text{lb/in}^3$$

$$U = U_Y + A_1 + A_2 + A_3 = 62961 \text{ in}\cdot\text{lb/in}^3$$

modulus of toughness 
$$\frac{U}{V} = \frac{62961}{7.0686} = 8900 \text{ in}\cdot\text{lb/in}^3$$

**PROBLEM 11.9**

11.9 Using  $E=29 \times 10^6$  psi, determine (a) the strain energy of the steel rod  $ABC$  when  $P=8$  kips, (b) the corresponding strain energy density in portions  $AB$  and  $BC$  of the rod.



**SOLUTION**

$P = 8 \text{ kips}, E = 29 \times 10^3 \text{ ksi}$

$A = \frac{\pi}{4} d^2, V = AL, \sigma = \frac{P}{A}, u = \frac{\sigma^2}{2E}$

$U = uV$

Portion	d in.	L in.	A in <sup>2</sup>	V in <sup>3</sup>	$\sigma$ Ksi	u in·kip/in <sup>3</sup>	U in·kip
AB	0.625	24	0.3608	7.363	26.08	$11.72 \times 10^{-3}$	$86.32 \times 10^{-3}$
BC	0.75	36	0.4418	15.904	18.11	$5.65 \times 10^{-3}$	$89.92 \times 10^{-3}$
$\Sigma$							$176.24 \times 10^{-3}$

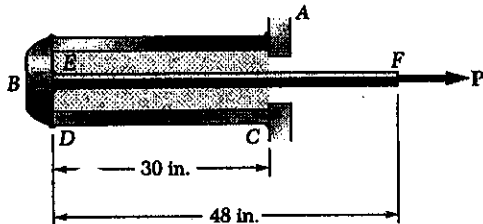
(a)  $U = 176.2 \times 10^{-3} \text{ in·kip} = 176.2 \text{ in·lb}$  ◀

(b) In AB  $u = 11.72 \times 10^{-3} \text{ in·kip/in}^3 = 11.72 \text{ in·lb/in}^3$  ◀

In BC  $u = 5.65 \times 10^{-3} \text{ in·kip/in}^3 = 5.65 \text{ in·lb/in}^3$  ◀

**PROBLEM 11.10**

11.10 A 30-in. length of aluminum pipe of cross-sectional area 1.85 in<sup>2</sup> is welded to a fixed support  $A$  and to a rigid cap  $B$ . The steel rod  $EF$ , of 0.75-in. diameter, is welded to cap  $B$ . Knowing that the modulus of elasticity is  $29 \times 10^6$  psi for steel and  $10.6 \times 10^6$  for aluminum, determine (a) the total strain energy of the system when  $P=10$  kips, (b) the corresponding strain-energy density in the pipe  $CD$  and in the rod  $EF$ .



**SOLUTION**

For  $EF: A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$

CD:  $U_{CD} = \frac{P^2 L}{2EA} = \frac{(10 \times 10^3)^2 (30)}{(2)(10.6 \times 10^6)(1.85)} = 76.49 \text{ in·lb}$

EF:  $U_{EF} = \frac{P^2 L}{2EA} = \frac{(10 \times 10^3)^2 (48)}{(2)(29 \times 10^6)(0.4418)} = 187.33 \text{ in·lb}$

Total:  $U = U_{CD} + U_{EF} = 264 \text{ in·lb}$  ◀

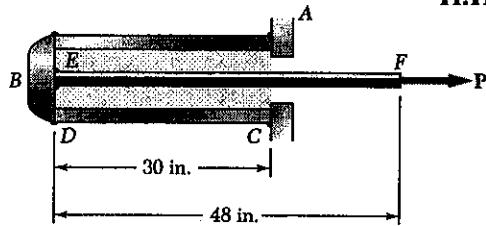
CD:  $\sigma = -\frac{10000}{1.85} = -5405 \text{ psi}, u = \frac{\sigma^2}{2E} = \frac{(-5405)^2}{(2)(10.6 \times 10^6)} = 1.378 \text{ in·lb/in}^3$  ◀

EF:  $\sigma = \frac{10000}{0.4418} = 22635 \text{ psi}, u = \frac{\sigma^2}{2E} = \frac{22635^2}{(2)(29 \times 10^6)} = 8.83 \text{ in·lb/in}^3$  ◀

PROBLEM 11.11

11.10 A 30-in. length of aluminum pipe of cross-sectional area  $1.85 \text{ in}^2$  is welded to a fixed support  $A$  and to a rigid cap  $B$ . The steel rod  $EF$ , of 0.75-in. diameter, is welded to cap  $B$ . Knowing that the modulus of elasticity is  $29 \times 10^6 \text{ psi}$  for steel and  $10.6 \times 10^6$  for aluminum, determine (a) the total strain energy of the system when  $P = 10 \text{ kips}$ , (b) the corresponding strain-energy density in the pipe  $CD$  and in the rod  $EF$ .

11.11 Solve Prob. 11.10, when  $P = 8 \text{ kips}$ .



SOLUTION

For  $EF$ :  $A = \frac{\pi}{4} d^2 = 0.4418 \text{ in}^2$

$$CD: U_{CD} = \frac{P^2 L}{2EA} = \frac{(-8000)^2 (30)}{(2)(10.6 \times 10^6)(1.85)} = 48.95 \text{ in}\cdot\text{lb}$$

$$EF: U_{EF} = \frac{P^2 L}{2EA} = \frac{(8000)^2 (48)}{(2)(29 \times 10^6)(0.4418)} = 119.89 \text{ in}\cdot\text{lb}$$

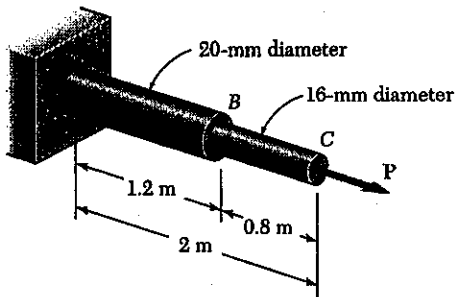
$$\text{Total } U = U_{CD} + U_{EF} = 168.8 \text{ in}\cdot\text{lb} \quad \blacktriangleleft$$

$$CD: \sigma = -\frac{8000}{1.85} = -4324 \text{ psi}, \quad u = \frac{\sigma^2}{2E} = \frac{(-4324)^2}{(2)(10.6 \times 10^6)} = 0.882 \text{ in}\cdot\text{lb}/\text{in}^3 \quad \blacktriangleleft$$

$$EF: \sigma = \frac{8000}{0.4418} = 18108 \text{ psi}, \quad u = \frac{\sigma^2}{2E} = \frac{(18108)^2}{(2)(29 \times 10^6)} = 5.65 \text{ in}\cdot\text{lb}/\text{in}^3 \quad \blacktriangleleft$$

**PROBLEM 11.12**

11.12 Using  $E = 200$  GPa, determine (a) the strain energy of the steel rod  $ABC$  when  $P = 25$  kN, (b) the corresponding strain-energy density of portions  $AB$  and  $BC$  of the rod.



**SOLUTION**

$$A_{AB} = \frac{\pi}{4} (20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = 25 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 L}{2EA} = \frac{(25 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} + \frac{(25 \times 10^3)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})}$$

$$(a) \quad U = 5.968 + 6.213 = 12.18 \text{ N}\cdot\text{m} = 12.18 \text{ J}$$

$$(b) \quad \sigma_{AB} = \frac{P}{A_{AB}} = \frac{25 \times 10^3}{314.16 \times 10^{-6}} = 79.58 \times 10^6 \text{ Pa}$$

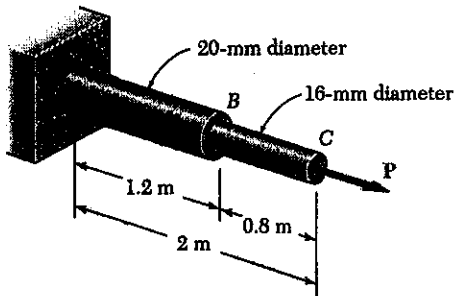
$$U_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58 \times 10^6)^2}{(2)(200 \times 10^9)} = 15.83 \times 10^3 = 15.83 \text{ kJ/m}^3$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{25 \times 10^3}{201.06 \times 10^{-6}} = 124.28 \times 10^6 \text{ Pa}$$

$$U_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28 \times 10^6)^2}{(2)(200 \times 10^9)} = 38.6 \times 10^3 = 38.6 \text{ kJ/m}^3$$

**PROBLEM 11.13**

11.13 The steel rod  $ABC$  is made of a steel for which the yield strength is  $\sigma_y = 250$  MPa and the modulus of elasticity is  $E = 200$  GPa. Determine, for the loading shown, the maximum strain energy that can be acquired by the rod without causing any permanent deformation.



**SOLUTION**

$$A_{AB} = \frac{\pi}{4} (20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

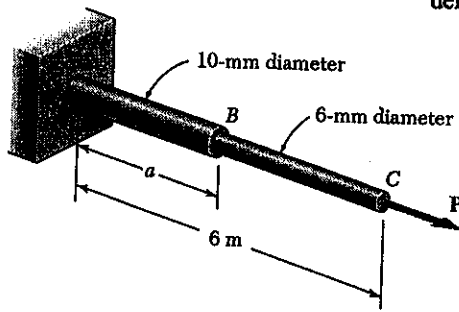
$$A_{BC} = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = \sigma_y A_{min} = (250 \times 10^6)(201.06 \times 10^{-6}) = 50.265 \times 10^3 \text{ Pa}$$

$$U = \sum \frac{P^2 L}{2EA} = \frac{(50265)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} + \frac{(50265)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})} = 24.13 + 25.13 = 49.3 \text{ J}$$

**PROBLEM 11.14**

11.14 The steel rods  $AB$  and  $BC$  are made of a steel for which the yield strength is  $\sigma_Y = 300$  MPa and the modulus of elasticity is  $E = 200$  GPa. Determine the maximum strain energy that can be acquired by the assembly without causing any permanent deformation when the length  $a$  of rod  $AB$  is (a) 2 m, (b) 4 m.



**SOLUTION**

$$A_{AB} = \frac{\pi}{4}(10)^2 = 78.54 \text{ mm}^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(6)^2 = 28.274 \text{ mm}^2 = 28.274 \times 10^{-6} \text{ m}^2$$

$$P = \sigma_Y A_{\min} = (300 \times 10^6)(28.274 \times 10^{-6}) = 8.4822 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 L}{2EA}$$

(a)  $a = 2 \text{ m}$       $L - a = 6 - 2 = 4 \text{ m}$

$$U = \frac{(8.4822 \times 10^3)^2 (2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(8.4822 \times 10^3)^2 (4)}{(2)(200 \times 10^9)(28.274 \times 10^{-6})}$$

$$= 4.5803 + 25.4466 = 30.0 \text{ N}\cdot\text{m} = 30.0 \text{ J}$$

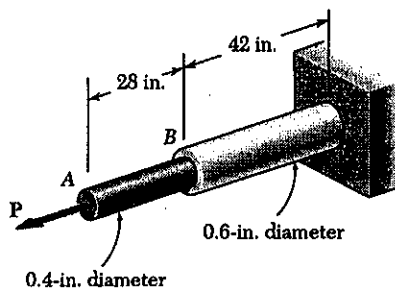
(b)  $a = 4 \text{ m}$       $L - a = 6 - 4 = 2 \text{ m}$

$$U = \frac{(8.4822 \times 10^3)^2 (4)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(8.4822 \times 10^3)^2 (2)}{(2)(200 \times 10^9)(28.274 \times 10^{-6})}$$

$$= 9.1606 + 12.7233 = 21.9 \text{ N}\cdot\text{m} = 21.9 \text{ J}$$

**PROBLEM 11.15**

11.15 Rod  $AB$  is made of a steel for which the yield strength is  $\sigma_Y = 65$  ksi and the modulus of elasticity is  $E = 29 \times 10^6$  psi; rod  $BC$  is made of an aluminum alloy for which  $\sigma_Y = 40$  ksi and  $E = 10.6 \times 10^6$  psi. Determine the maximum strain energy that can be acquired by the composite rod  $ABC$  without causing permanent deformation.



**SOLUTION**

$$A_{AB} = \frac{\pi}{4}(0.4)^2 = 0.12566 \text{ in}^2 \quad E = 29000 \text{ ksi}$$

$$A_{BC} = \frac{\pi}{4}(0.6)^2 = 0.28274 \text{ in}^2 \quad E = 10600 \text{ ksi}$$

$$P_{\text{all}} = \sigma_Y A \text{ for each part}$$

$$AB: P_{\text{all}} = (65)(0.12566) = 8.1679 \text{ kips}$$

$$BC: P_{\text{all}} = (40)(0.28274) = 11.3096 \text{ kips}$$

Use smaller value      $P = 8.1679 \text{ kips}$

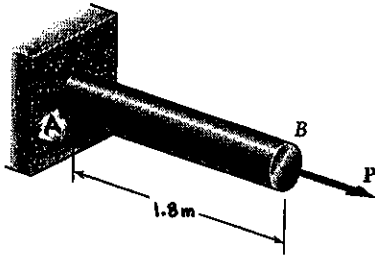
$$U = \sum \frac{P^2 L}{2EA} = \frac{(8.1679)^2 (28)}{(2)(29000)(0.12566)} + \frac{(8.1679)^2 (42)}{(2)(10600)(0.28274)}$$

$$= 256.3 \times 10^{-3} + 467.5 \times 10^{-3} = 724 \times 10^{-3} \text{ in}\cdot\text{kip} = 724 \text{ in}\cdot\text{lb}$$



**PROBLEM 11.16**

11.16 Rod *AB* is made of a steel for which the yield strength is  $\sigma_y = 300$  MPa and the modulus of elasticity is  $E = 200$  GPa. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load *P* is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.



**SOLUTION**

For factor of safety of six on the energy

$$U_Y = (6)(10) = 60 \text{ J}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(300 \times 10^6)^2}{(2)(200 \times 10^9)} = 225 \times 10^3 \text{ J/m}^3$$

$$A = \frac{U_Y}{LU_Y} = \frac{60}{(1.8)(225 \times 10^3)} = 148.148 \times 10^{-6} \text{ m}^2$$

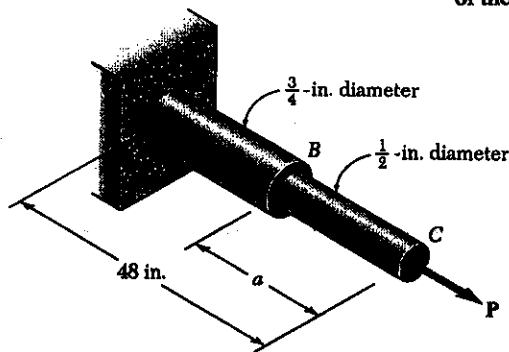
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148 \times 10^{-6})}{\pi}} = 13.73 \times 10^{-3} \text{ m} = 13.73 \text{ mm}$$

$$U_Y = ALU_Y'$$

$$A = \frac{\pi}{4} d^2$$

**PROBLEM 11.17**

11.17 The rod *ABC* is made of a steel for which the yield strength is  $\sigma_y = 65$  ksi and the modulus of elasticity is  $E = 29 \times 10^6$  psi. Knowing that a strain energy of 90 in·lb must be acquired by the rod as the axial load *P* is applied, determine the factor of safety of the rod with respect to permanent deformation when  $a = 18$  in.



**SOLUTION**

$$A_{AB} = \frac{\pi}{4} \left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$P_Y = \sigma_Y A_{\min} = (65000)(0.19635) = 12763 \text{ lb.}$$

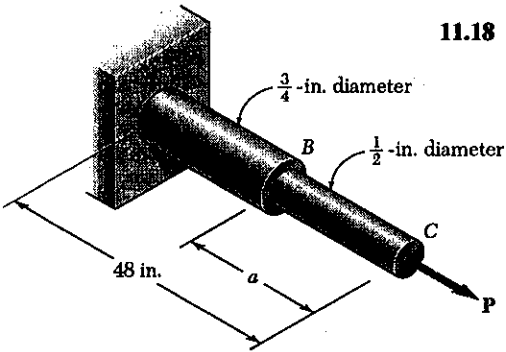
$$U_Y = \sum \frac{P_Y^2 L}{2EI}$$

$$U_Y = \frac{(12763)^2 (48 - 18)}{(2)(29 \times 10^6)(0.4418)} + \frac{(12763)^2 (18)}{(2)(29 \times 10^6)(0.19635)} = 448 \text{ in}\cdot\text{lb.}$$

$$F.S. = \frac{U_Y}{U_{\text{design}}} = \frac{448}{90} = 4.98$$

**PROBLEM 11.18**

11.18 The rod  $ABC$  is made of a steel for which the yield strength is  $\sigma_y = 65$  ksi and the modulus of elasticity is  $E = 29 \times 10^6$  psi. Knowing that a strain energy of 90 in·lb must be acquired by the rod as the axial load  $P$  is applied, determine the factor of safety of the rod with respect to permanent deformation when  $a = 18$  in.



11.18 Solve Prob. 11.17, assuming that  $a = 30$  in.

**SOLUTION**

$$A_{AB} = \frac{\pi}{4} \left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} \left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$$

$$P_Y = \sigma_Y A_{\min} = (65000)(0.19635) = 12763 \text{ lb.}$$

$$U_Y = \sum \frac{P_Y^2 L}{2EA} = \frac{(12763)^2 (48-30)}{(2)(29 \times 10^6)(0.4418)} + \frac{(12763)^2 (30)}{(2)(29 \times 10^6)(0.19635)} = 543.5 \text{ in}\cdot\text{lb}$$

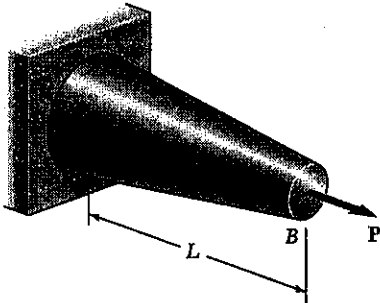
$$\text{F.S.} = \frac{U_Y}{U_{\text{design}}} = \frac{543.5}{90} = 6.04$$

**PROBLEM 11.19**

11.19 Show by integration that the strain energy of the tapered rod  $AB$  is

$$U = \frac{1}{4} \frac{P^2 L}{EA_{\min}}$$

where  $A_{\min}$  is the cross-sectional area at end  $B$ .



**SOLUTION**

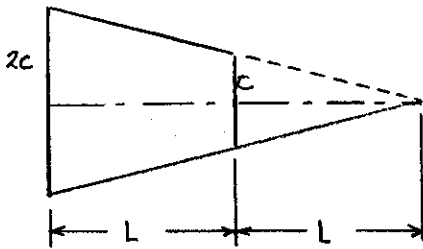
$$\text{radius } r = \frac{cx}{L} \quad A_{\min} = \pi c^2$$

$$A = \pi r^2 = \frac{\pi c^2}{L^2} x^2$$

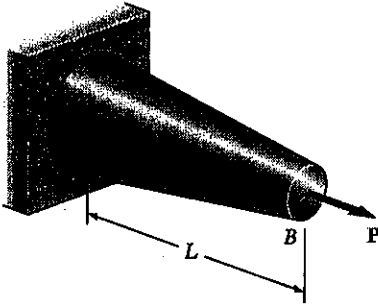
$$U = \int_L^{2L} \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int_L^{2L} \frac{L^2}{\pi c^2 x^2} dx$$

$$= \frac{P^2 L^2}{2E \pi c^2} \left(-\frac{1}{x}\right) \Big|_L^{2L}$$

$$= \frac{P^2 L^2}{2EA_{\min}} \left(-\frac{1}{2L} + \frac{1}{L}\right) = \frac{P^2 L^2}{4EA_{\min}}$$



**PROBLEM 11.20**

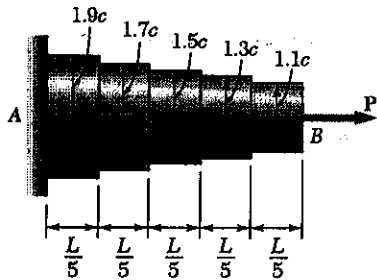


11.19 Show by integration that the strain energy of the tapered rod  $AB$  is

$$U = \frac{1}{4} \frac{P^2 L}{EA_{\min}}$$

where  $A_{\min}$  is the cross-sectional area at end  $B$ .

11.20 Solve Prob. 11.19, using the stepped rod shown as an approximation of the tapered rod. What is the percentage error in the answer obtained?



**SOLUTION**

$$A_i = \pi v_i^2 \quad A_{\min} = \pi c^2$$

$$U = \sum \frac{P^2 l_i}{2EA_i} = \frac{P^2 (L/5)}{2E} \sum \frac{1}{A_i}$$

$$= \frac{P^2 L}{10 \pi E} \sum \frac{1}{v_i^2}$$

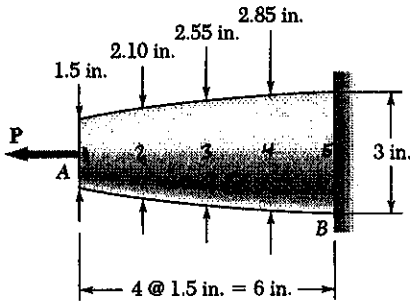
$$= \frac{P^2 L}{10 \pi E} \left\{ \frac{1}{(1.9c)^2} + \frac{1}{(1.7c)^2} + \frac{1}{(1.5c)^2} + \frac{1}{(1.3c)^2} + \frac{1}{(1.1c)^2} \right\}$$

$$= \frac{P^2 L}{10 E (\pi c^2)} \{ 2.4856 \} = 0.24856 \frac{P^2 L}{EA_{\min}}$$

$$\% \text{ error} = \frac{0.24856 - 0.25}{0.25} \times 100\% = -0.575\%$$

PROBLEM 11.21

11.21 Using  $E = 10.6 \times 10^6$  psi, determine by approximate means the maximum strain energy that can be acquired by the aluminum rod shown if the allowable normal stress is  $\sigma_{all} = 22$  ksi.



SOLUTION

$$A_{min} = \frac{\pi}{4} (1.5)^2 = 1.7671 \text{ in}^2$$

$$\sigma_{all} = 22000 \text{ psi}$$

$$P_{all} = \sigma_{all} A_{min} = 38877 \text{ lb.}$$

$$U = \int \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int \frac{dx}{\frac{\pi}{4} d^2} = \frac{2P^2}{\pi E} \int \frac{dx}{d^2}$$

Use Simpson's rule to compute the integral

$$h = 0.15 \text{ in.}$$

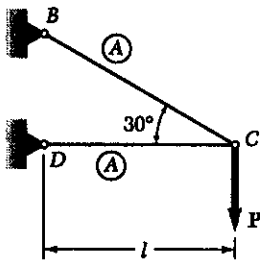
Section	d(in)	1/d <sup>2</sup> (in <sup>-2</sup> )	multiplier	m(1/d <sup>2</sup> ) (in <sup>-2</sup> )
1	1.50	0.4444	1	0.4444
2	2.10	0.22675	4	0.9070
3	2.55	0.15379	2	0.3076
4	2.85	0.12311	4	0.4924
5	3.00	0.11111	1	0.1111
$\Sigma$				2.2625

$$\int \frac{dx}{d^2} = \frac{h}{3} \Sigma m\left(\frac{1}{d^2}\right) = \frac{1.5}{3} (2.2625) = 1.13125 \text{ in}^{-1}$$

$$U = \frac{(2)(38877)^2 (1.13125)}{\pi (10.6 \times 10^6)} = 102.7 \text{ in. lb.}$$

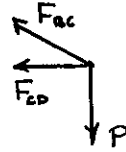
**PROBLEM 11.22**

**11.22** In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load  $P$  is applied.



**SOLUTION**

Joint C



$$+\uparrow \sum F_y = 0$$

$$\frac{1}{2} F_{bc} - P = 0 \quad F_{bc} = 2P$$

$$+\rightarrow \sum F_x = 0$$

$$-F_{cd} - \frac{\sqrt{3}}{2} F_{bc} = 0 \quad F_{cd} = -\sqrt{3}P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

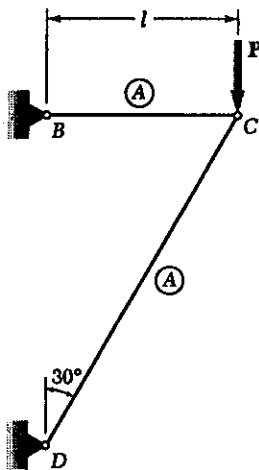
Member	F	L	A	$F^2 L/A$
BC	$2P$	$\frac{2}{\sqrt{3}}l$	A	$\frac{8}{\sqrt{3}}P^2 l/A$
CD	$-\sqrt{3}P$	$l$	A	$3P^2 l/A$
$\Sigma$				$7.62P^2 l/A$

$$U = \frac{1}{2E} \left( 7.62 \frac{P^2 l}{A} \right)$$

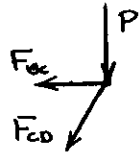
$$= 3.81 \frac{P^2 l}{EA}$$

**PROBLEM 11.23**

**11.23** In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load  $P$  is applied.



**SOLUTION**



Joint C

$$+\uparrow \sum F_y = 0$$

$$-\frac{\sqrt{3}}{2} F_{cd} - P = 0$$

$$F_{cd} = -\frac{2}{\sqrt{3}}P$$

$$+\rightarrow \sum F_x = 0$$

$$-F_{bc} - \frac{1}{2} F_{cd} = 0$$

$$F_{bc} = \frac{1}{\sqrt{3}}P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

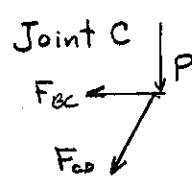
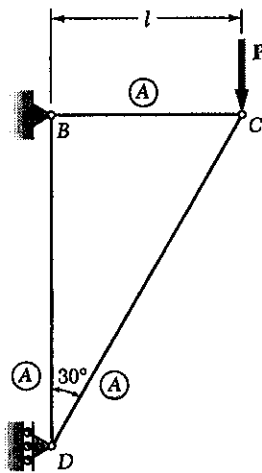
Member	F	L	A	$F^2 L/A$
BC	$\frac{1}{\sqrt{3}}P$	$l$	A	$\frac{1}{3}P^2 l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2 l/A$
$\Sigma$				$3P^2 l/A$

$$U = \frac{1}{2E} \left( 3 \frac{P^2 l}{A} \right)$$

$$= 1.5 \frac{P^2 l}{EA}$$

PROBLEM 11.24

11.24 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load  $P$  is applied.

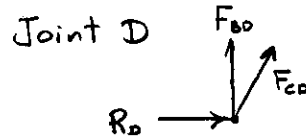


$$+\uparrow \sum F_y = 0 \quad -\frac{\sqrt{3}}{2} F_{cd} - P = 0$$

$$F_{cd} = -\frac{2}{\sqrt{3}} P$$

$$+\rightarrow \sum F_x = 0 \quad -F_{bc} - \frac{1}{2} F_{cd} = 0$$

$$F_{bc} = \frac{1}{\sqrt{3}} P$$



$$+\uparrow \sum F_y = 0$$

$$F_{bd} + \frac{\sqrt{3}}{2} F_{cd} = 0$$

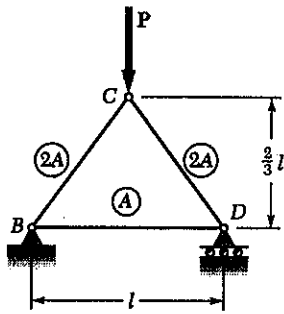
$$F_{bd} = P$$

Member	F	L	A	$F^2 L / A$
BC	$\frac{1}{\sqrt{3}} P$	$l$	A	$\frac{1}{3} P^2 l / A$
CD	$-\frac{2}{\sqrt{3}} P$	$2l$	A	$\frac{8}{3} P^2 l / A$
BD	$P$	$\sqrt{3} l$	A	$\sqrt{3} P^2 l / A$
$\Sigma$				$4.732 P^2 l / A$

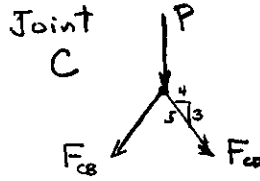
$$\begin{aligned}
 U &= \sum \frac{1}{2} \frac{F^2 L}{EA} \\
 &= \frac{1}{2E} \sum \frac{F^2 L}{A} \\
 &= \frac{1}{2E} \left( 4.732 \frac{P^2 l}{A} \right) \\
 &= 2.37 \frac{P^2 l}{EA}
 \end{aligned}$$

PROBLEM 11.25

11.25 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load  $P$  is applied



SOLUTION



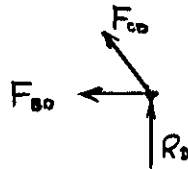
$$+\rightarrow \sum F_x = 0 \quad \frac{3}{5} F_{CD} - \frac{3}{5} F_{CB} = 0$$

$$F_{CB} = F_{CD}$$

$$+\uparrow \sum F_y = 0 \quad -P - 2 \cdot \frac{4}{5} F_{CD} = 0$$

$$F_{CB} = F_{CD} = -\frac{5}{8} P$$

Joint D



$$+\rightarrow \sum F_x = 0$$

$$-F_{BD} - \frac{3}{5} F_{CD} = 0$$

$$F_{BD} = -\frac{3}{5} \cdot \frac{5}{8} P = \frac{3}{8} P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

$$= \frac{1}{2E} \left( \frac{179}{384} \frac{P^2 l}{A} \right)$$

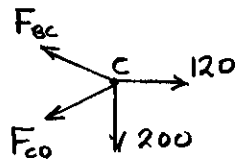
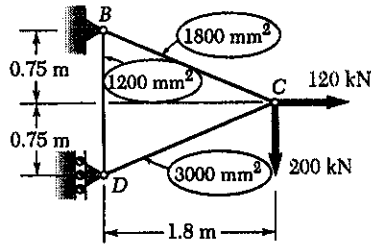
$$= \frac{179}{768} \frac{P^2 l}{EA}$$

$$= 0.233 \frac{P^2 l}{EA}$$

Member	F	L	A	$F^2 L / A$
CB	$-\frac{5}{8} P$	$\frac{5}{5} l$	2A	$\frac{125}{768} P^2 l / A$
CD	$-\frac{5}{8} P$	$\frac{5}{5} l$	2A	$\frac{125}{768} P^2 l / A$
BD	$\frac{3}{8} P$	l	A	$\frac{9}{64} P^2 l / A$
$\Sigma$				$\frac{179}{384} P^2 l / A$

PROBLEM 11.26

11.26 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area indicated. Using  $E = 72 \text{ GPa}$ , determine the strain energy of the truss for the loading shown.



SOLUTION

$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

Joint C

$$+\rightarrow \sum F_x = 0 \quad -\frac{1.8}{1.95} F_{BC} - \frac{1.8}{1.95} F_{CD} + 120 = 0 \quad (1)$$

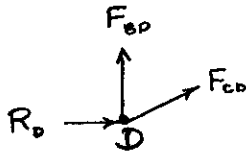
$$+\uparrow \sum F_y = 0 \quad \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0 \quad (2)$$

Solving (1) and (2) simultaneously,

$$F_{BC} = 325 \text{ kN} \quad F_{CD} = -195 \text{ kN}$$

Joint D

$$+\uparrow \sum F_y = 0 \quad F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \quad F_{BD} = 75 \text{ kN}$$



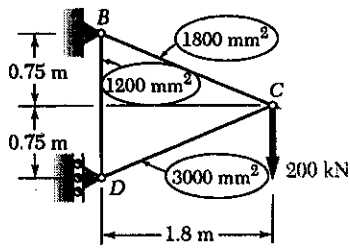
$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A ( $10^{-6} \text{ m}^2$ )	$F^2 L / A$ ( $\text{N}^2 / \text{m}$ )
BC	325	1.95	1800	$114.43 \times 10^{12}$
BD	75	1.5	1200	$7.03 \times 10^{12}$
CD	-195	1.95	3000	$24.72 \times 10^{12}$
$\Sigma$				$146.18 \times 10^{12}$

$$U = \frac{146.18 \times 10^{12}}{(2)(72 \times 10^9)} = 1.015 \times 10^3 \text{ N-m} = 1015 \text{ J}$$



**PROBLEM 11.27**



11.27 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area indicated. Using  $E = 72 \text{ GPa}$ , determine the strain energy of the truss for the loading shown.

11.27 Solve Prob. 11.26, assuming that the 120-kN load is removed.

**SOLUTION**

$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

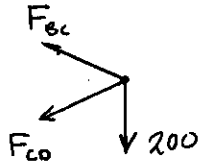
Joint C

$$+\rightarrow \Sigma F_x = 0 \quad -\frac{1.8}{1.95} F_{BC} - \frac{1.8}{1.95} F_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0 \quad \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0$$

Solving (1) and (2) simultaneously,

$$F_{BC} = 260 \text{ kN} \quad F_{CD} = -260 \text{ kN}$$



Joint D

$$+\uparrow \Sigma F_y = 0 \quad F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \quad F_{BD} = 100 \text{ kN}$$



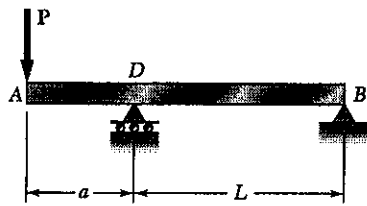
$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A ( $10^6 \text{ m}^2$ )	$F^2 L / A$ ( $\text{N}^2/\text{m}$ )
BC	260	1.95	1800	$73.23 \times 10^{12}$
BD	100	1.5	1200	$12.50 \times 10^{12}$
CD	-260	1.95	3000	$43.94 \times 10^{12}$
$\Sigma$				$129.67 \times 10^{12}$

$$U = \frac{129.67 \times 10^{12}}{(2)(72 \times 10^9)} = 900 \text{ N}\cdot\text{m} = 900 \text{ J}$$

PROBLEM 11.28

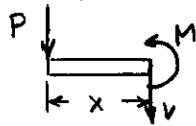
11.28 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam  $AB$  for the loading shown.



SOLUTION

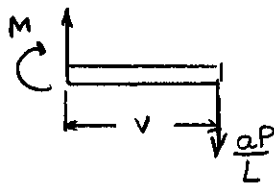
$$\begin{aligned} \sum M_D = 0 \\ aP + LR_B = 0 \quad R_B = -\frac{aP}{L} = \frac{aP}{L} \downarrow \end{aligned}$$

Over portion AD :  $M = -Px$



$$\begin{aligned} U_{AD} &= \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2}{2EI} \left[ \frac{x^3}{3} \right]_0^a \\ &= \frac{P^2 a^3}{6EI} \end{aligned}$$

Over portion DB :  $M = -\frac{aP}{L}v$

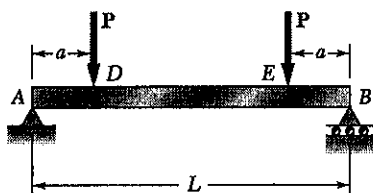


$$\begin{aligned} U_{DB} &= \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \frac{a^2 P^2}{L^2} v^2 dv \\ &= \frac{P^2 a^2}{2EIL^2} \int_0^L v^2 dv = \frac{P^2 a^2}{2EIL^2} \left[ \frac{v^3}{3} \right]_0^L = \frac{P^2 a^2 L}{6EI} \end{aligned}$$

Total  $U = U_{AD} + U_{DB} = \frac{P^2 a^2}{6EI} (a+L)$  ▶

PROBLEM 11.29

11.29 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam  $AB$  for the loading shown.



SOLUTION

Symmetric beam and loading  $R_A = R_B$   
 $\uparrow \sum F_y = 0 \quad R_A + R_B - 2P = 0 \quad R_A = R_B = P$

Over portion AD :  $M = R_A x = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2}{2EI} \left[ \frac{x^3}{3} \right]_0^a = \frac{P^2 a^3}{6EI}$$

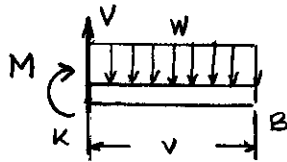
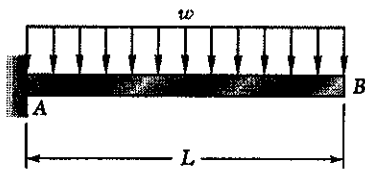
Over portion DE :  $M = Pa \quad U_{DE} = \frac{P^2 a^2 (L-2a)}{2EI}$

Over portion EB : By symmetry  $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

Total  $U = U_{AD} + U_{DE} + U_{EB} = \frac{P^2 a^2}{6EI} (3L - 4a)$  ▶

**PROBLEM 11.30**

11.30 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam  $AB$  for the loading shown.



**SOLUTION**

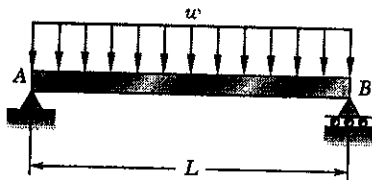
$$\sum M_k = 0 \quad -M - (wv)\left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2} wv^2$$

$$\begin{aligned} U &= \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} wv^2\right)^2 dv \\ &= \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \left[\frac{v^5}{5}\right]_0^L \\ &= \frac{w^2 L^5}{40EI} \end{aligned}$$

**PROBLEM 11.31**

11.31 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam  $AB$  for the loading shown.



**SOLUTION**

$$\sum M_B = 0 \quad -R_A L + (wL)\left(\frac{L}{2}\right) = 0 \quad R_A = \frac{wL}{2}$$

Bending moment  $M = R_A x - \frac{1}{2} wx^2$   
 $= \frac{w}{2} (Lx - x^2)$

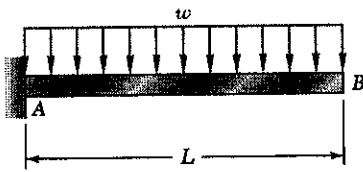
$$\begin{aligned} U &= \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (Lx - x^2)^2 dx \\ &= \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[ \frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^L \\ &= \frac{w^2 L^5}{8EI} \left[ \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = \frac{w^2 L^5}{240EI} \end{aligned}$$

PROBLEM 11.32

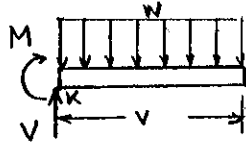
11.32 Assuming that the prismatic beam  $AB$  has a rectangular cross section, show that for the given loading the maximum value of the strain energy-density in the beam is

$$u_{\max} = 15 \frac{U}{V}$$

where  $U$  is the strain energy of the beam and  $V$  is its volume.



SOLUTION



$$\begin{aligned} \sum M_x = 0 \quad -M - (wv) \frac{v}{2} &= 0 \\ M &= -\frac{1}{2} wv^2 \end{aligned}$$

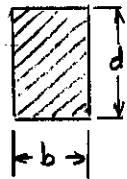
$$U = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} wv^2\right)^2 dv = \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \frac{x^5}{5} \Big|_0^L = \frac{w^2 L^5}{40EI}$$

$$M_{\max} = \frac{1}{2} wL^2 \quad \sigma_{\max} = \frac{M_{\max} c}{I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{M_{\max}^2 c^2}{2EI^2} = \frac{\frac{1}{4} w^2 L^4 c^2}{2EI^2} = \frac{w^2 L^4 c^2}{8EI^2}$$

$$\frac{U}{U_{\max}} = \frac{LI}{5c^2} = \frac{L \left(\frac{1}{12} bd^3\right)}{5 \left(\frac{d}{2}\right)^2} = \frac{1}{15} Lbd = \frac{1}{15} V$$

$$U_{\max} = 15 \frac{U}{V} \quad \blacktriangleleft$$

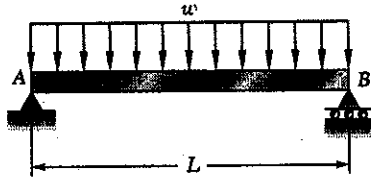


PROBLEM 11.33

11.33 Assuming that the prismatic beam  $AB$  has a rectangular cross section, show that for the given loading the maximum value of the strain energy-density in the beam is

$$u_{\max} = \frac{45 U}{8 V}$$

where  $U$  is the strain energy of the beam and  $V$  is its volume.



SOLUTION

$$+\circlearrowleft M_B = 0 \quad -R_A L + (wL)\frac{L}{2} = 0 \quad R_A = \frac{1}{2}wL$$

$$M = R_A x - \frac{1}{2}wLx = \frac{1}{2}w(Lx - x^2)$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (L^2x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[ \frac{L^2x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^L$$

$$= \frac{w^2 L^5}{8EI} \left[ \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = \frac{w^2 L^5}{240EI}$$

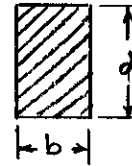
$$M_{\max} = \frac{1}{2}w \left[ L \cdot \frac{L}{2} - \left(\frac{L}{2}\right)^2 \right] = \frac{1}{8}wL^2$$

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{wL^2 c}{8I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{w^2 L^4 c^2}{128EI^2}$$

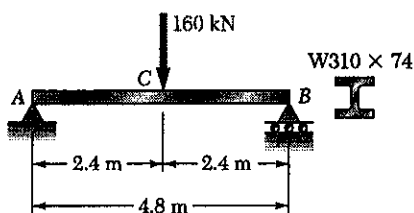
$$\frac{U}{U_{\max}} = \frac{8LI}{15c^2} = \frac{8L(\frac{1}{2}bd^3)}{15(\frac{d}{2})^2} = \frac{8}{45}Lbd = \frac{8}{45}V$$

$$U_{\max} = \frac{45}{8} \frac{U}{V}$$



PROBLEM 11.34

11.34 Using  $E = 200$  GPa, determine the strain energy due to bending for the steel beam and loading shown.



SOLUTION

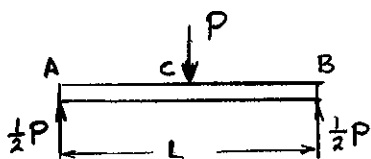
Over portion AC  $M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx = \frac{P^2}{8EI} \int_0^{\frac{L}{2}} x^2 dx$$

$$= \frac{P^2}{8EI} \left. \frac{x^3}{3} \right|_0^{\frac{L}{2}} = \frac{P^2 L^3}{192EI}$$

By symmetry  $U_{CB} = U_{AC} = \frac{P^2 L^3}{192EI}$

Total:  $U = U_{AC} + U_{CB} = \frac{P^2 L^3}{96EI}$

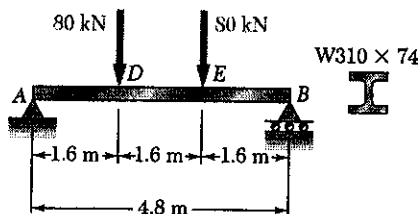


Data:  $P = 160 \times 10^3$  N,  $L = 4.8$  m,  $E = 200 \times 10^9$  Pa  
 $I = 165 \times 10^6$  mm<sup>4</sup> =  $165 \times 10^{-6}$  m<sup>4</sup>

$$U = \frac{(160 \times 10^3)^2 (4.8)^3}{(96)(200 \times 10^9)(165 \times 10^{-6})} = 894 \text{ N}\cdot\text{m} = 894 \text{ J}$$

PROBLEM 11.35

11.35 Using  $E = 200$  GPa, determine the strain energy due to bending for the steel beam and loading shown.



SOLUTION

Over portion AD:  $M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a (Px)^2 dx$$

$$= \frac{P^2}{2EI} \left. \frac{x^3}{3} \right|_0^a = \frac{P^2 a^3}{6EI}$$

Over portion DE:  $M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2 a^3}{2EI}$$

By symmetry  $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

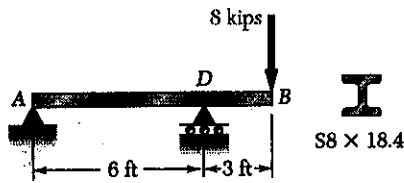
$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6} \frac{P^2 a^3}{EI}$$

Data:  $P = 80 \times 10^3$  N,  $a = 1.6$  m,  $E = 200 \times 10^9$  Pa  
 $I = 165 \times 10^6$  mm<sup>4</sup> =  $165 \times 10^{-6}$  m<sup>4</sup>

$$U = \frac{5}{6} \frac{(80 \times 10^3)^2 (1.6)^3}{(200 \times 10^9)(165 \times 10^{-6})} = 662 \text{ N}\cdot\text{m} = 662 \text{ J}$$

PROBLEM 11.36

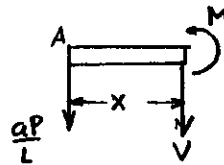
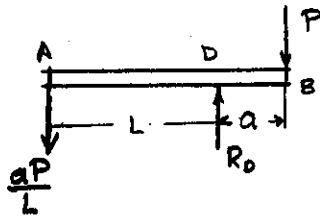
11.36 Using  $E = 29 \times 10^6$  psi, determine the strain energy due to bending for the steel beam and loading shown.



SOLUTION

$$\sum M_B = 0 \quad -R_A L - aP = 0 \quad R_A = \frac{aP}{L} \downarrow$$

Over portion AD  $M = -\frac{aP}{L}x$



$$U_{AD} = \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^L \left(\frac{aP}{L}x\right)^2 dx$$

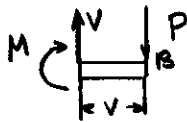
$$= \frac{P^2 a^3}{2EIL^2} \int_0^L x^2 dx$$

$$= \frac{P^2 a^3 L}{6EI}$$

[www.konkur.in](http://www.konkur.in)

Over portion DB

$$M = -Px$$



$$U_{DB} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2 a^3}{6EI}$$

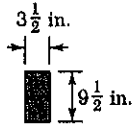
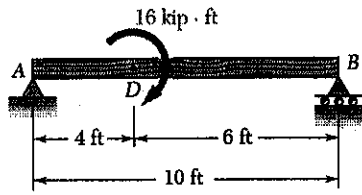
$$\text{Total: } U = U_{AD} + U_{DB} = \frac{P^2 a^3}{6EI} (a + L)$$

Data:  $P = 8000$  lb.,  $L = 6$  ft. = 72 in,  $a = 3$  ft = 36 in,  $E = 29 \times 10^6$  psi  
 $I = 57.6$  in<sup>4</sup>

$$U = \frac{(8000)^2 (36)^3 (72 + 36)}{(6)(29 \times 10^6)(57.6)} = 894 \text{ in. lb.}$$

PROBLEM 11.37

11.37 Using  $E = 1.8 \times 10^6$  psi, determine the strain energy due to bending for the timber beam and loading shown.



SOLUTION

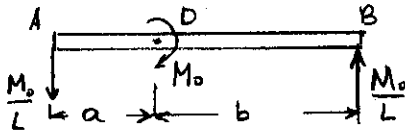
$$\sum M_A = 0 \quad -M_0 + R_B L = 0 \quad R_B = \frac{M_0}{L} \uparrow$$

$$R_A = \frac{M_0}{L} \downarrow$$

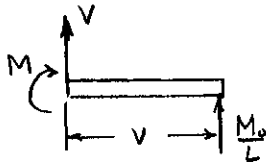
Over portion AD  $M = \frac{M_0 x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EIL^2} \int_0^a x^2 dx$$

$$= \frac{M_0^2 a^3}{6EIL^2}$$



Over portion DB  $M = \frac{M_0}{L} v$



$$U_{DB} = \int_0^b \frac{M^2}{2EI} dx = \frac{M_0^2}{2EIL^2} \int_0^b x^2 dx = \frac{M_0^2 b^3}{6EIL^2}$$

Total  $U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EIL^2}$

Data:  $M_0 = 16 \text{ kip}\cdot\text{ft}$ ,  $a = 4 \text{ ft}$ ,  $b = 6 \text{ ft}$ ,  $L = 10 \text{ ft}$

$E = 1.8 \times 10^3 \text{ ksi}$

$I = \frac{1}{12} (3\frac{1}{2})(9\frac{1}{2})^3 = 250.07 \text{ in}^4$

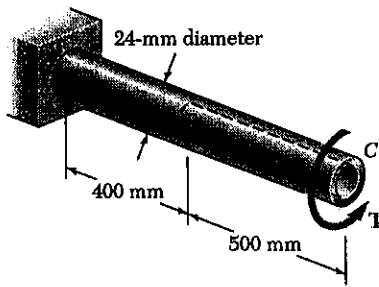
$EI = (1.8 \times 10^3)(250.07) = 450.13 \times 10^3 \text{ kip}\cdot\text{in}^2 = 3126 \text{ kip}\cdot\text{ft}^2$

$$U = \frac{(16)^2 (4^3 + 6^3)}{(6)(3126)(10)^2} = 38.2 \times 10^{-3} \text{ kip}\cdot\text{ft} = 38.2 \text{ ft}\cdot\text{lb}$$

$$= 458 \text{ in}\cdot\text{lb}$$



**PROBLEM 11.38**



11.38 Rod AC is made of aluminum ( $G = 73 \text{ GPa}$ ) and is subjected to a torque  $T$  applied at end C. Knowing that portion BC of the rod is hollow and has an inside diameter of 16 mm, determine the strain energy of the rod for a maximum shearing stress of 120 MPa.

**SOLUTION**

$$C_o = \frac{d_o}{2} = 12 \text{ mm}, \quad C_i = \frac{d_i}{2} = 8 \text{ mm}$$

$$J_{AB} = \frac{\pi}{2} C_o^4 = \frac{\pi}{2} (12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} (C_o^4 - C_i^4) = \frac{\pi}{2} (12^4 - 8^4) = 26.138 \times 10^3 \text{ mm}^4 = 26.138 \times 10^{-9} \text{ m}^4$$

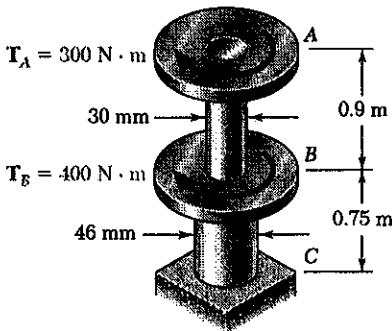
$$\tau_{\text{all}} = \frac{T C}{J_{\text{min}}} \quad T = \frac{J_{\text{min}} \tau_{\text{all}}}{C} = \frac{(26.138 \times 10^{-9})(120 \times 10^6)}{12 \times 10^{-3}} = 261.38 \text{ N}\cdot\text{m}$$

$$U_{AB} = \frac{T^2 L_{AB}}{2G J_{AB}} = \frac{(261.38)^2 (400 \times 10^{-3})}{(2)(73 \times 10^9)(32.572 \times 10^{-9})} = 5.747 \text{ J}$$

$$U_{BC} = \frac{T^2 L_{BC}}{2G J_{BC}} = \frac{(261.38)^2 (500 \times 10^{-3})}{(2)(73 \times 10^9)(26.138 \times 10^{-9})} = 8.951 \text{ J}$$

$$\text{Total } U = U_{AB} + U_{BC} = 14.70 \text{ J}$$

**PROBLEM 11.39**



**SOLUTION**

Over portion AB

$$T_{AB} = T_A = 300 \text{ N}\cdot\text{m}$$

$$J_{AB} = \frac{\pi}{2} C^4 = \frac{\pi}{2} \left(\frac{30}{2}\right)^4 = 79.52 \times 10^3 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = 0.9 \text{ m}$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2G J_{AB}} = \frac{(300)^2 (0.9)}{(2)(73 \times 10^9)(79.52 \times 10^{-9})} = 6.977 \text{ J}$$

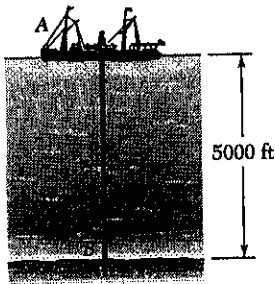
Over portion BC:  $T_{BC} = T_A + T_B = 300 + 400 = 700 \text{ N}\cdot\text{m}$ ,  $L_{BC} = 0.75 \text{ m}$

$$J_{BC} = \frac{\pi}{2} \left(\frac{46}{2}\right)^4 = 439.57 \times 10^3 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2G J_{BC}} = \frac{(700)^2 (0.75)}{(2)(73 \times 10^9)(439.57 \times 10^{-9})} = 5.726 \text{ J}$$

$$\text{Total } U = U_{AB} + U_{BC} = 6.977 + 5.726 = 12.70 \text{ J}$$

PROBLEM 11.40



11.40 The ship at *A* has just started to drill for oil on the ocean floor at a depth of 5000 ft. The steel drill pipe has an outside diameter of 8 in. and a uniform wall thickness of 0.5 in. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at *B* starts to operate and using  $G = 11.2 \times 10^6$  psi, determine the maximum strain energy acquired by the drill pipe.

SOLUTION

$$\phi = (2)(2\pi) = 4\pi \text{ rad}$$

$$L = 5000 \text{ ft} = 60 \times 10^3 \text{ in.}$$

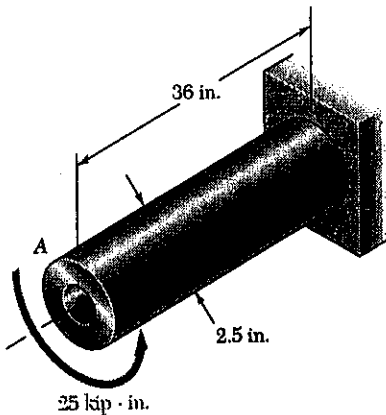
$$C_o = \frac{d_o}{2} = 4 \text{ in.} \quad C_i = C_o - t = 3.5 \text{ in.}$$

$$J = \frac{\pi}{2}(C_o^4 - C_i^4) = 166.406 \text{ in}^4$$

$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ\phi}{L} \quad U = \frac{T^2 L}{2GJ} = \left(\frac{GJ\phi}{L}\right)^2 \frac{L}{2GJ} = \frac{GJ\phi^2}{2L}$$

$$U = \frac{(11.2 \times 10^6)(166.406)(4\pi)^2}{(2)(60 \times 10^3)} = 2.45 \times 10^6 \text{ in}\cdot\text{lb.}$$

PROBLEM 11.41



11.41. The design specifications for the steel shaft *AB* require that the shaft acquire a strain energy of 300 in-lb as the 25-kip-in. torque is applied. Using  $G = 11.2 \times 10^6$  psi, determine (a) the largest inside diameter of the shaft that can be used, (b) the corresponding maximum shearing stress in the shaft.

SOLUTION

$$U = 300 \text{ in}\cdot\text{lb}$$

$$T = 25 \text{ kip}\cdot\text{in} = 25 \times 10^3 \text{ lb}\cdot\text{in}$$

$$L = 36 \text{ in.}$$

$$U = \frac{T^2 L}{2GJ}$$

$$J = \frac{T^2 L}{2GU} = \frac{(25 \times 10^3)^2 (36)}{(2)(11.2 \times 10^6)(300)} = 3.3482 \text{ in}^4$$

$$\text{But } J = \frac{\pi}{2} \left[ \left(\frac{d_o}{2}\right)^4 - \left(\frac{d_i}{2}\right)^4 \right] = \frac{\pi}{32} (d_o^4 - d_i^4)$$

$$d_i^4 = d_o^4 - \frac{32}{\pi} J = 2.5^4 - \frac{32}{\pi} (3.3482) = 4.95787 \text{ in}^4$$

$$d_i = 1.492 \text{ in.}$$

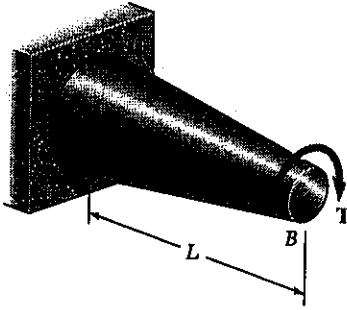
$$\tau = \frac{T C_o}{J} = \frac{(25 \times 10^3)(1.25)}{3.3482} = 9.33 \times 10^3 \text{ psi} = 9.33 \text{ ksi}$$

PROBLEM 11.42

11.42 Show by integration that the strain energy in the tapered rod  $AB$  is

$$U = \frac{7}{48} \frac{T^2 L}{GJ_{\min}}$$

where  $J_{\min}$  is the polar moment of inertia of the rod at end  $B$ .



SOLUTION

$$r = \frac{Cx}{L}$$

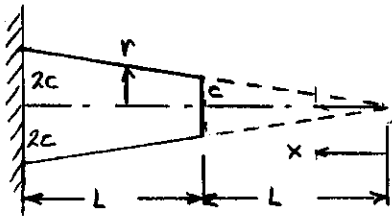
$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2} \frac{C^4}{L^4} x^4, \quad J_{\min} = \frac{\pi}{2} C^4$$

$$U = \int_L^{2L} \frac{T^2 dx}{2GJ} = \int_L^{2L} \frac{T^2 dx}{2G \left( \frac{\pi}{2} \frac{C^4}{L^4} x^4 \right)}$$

$$= \frac{T^2 L^4}{2G J_{\min}} \int_L^{2L} \frac{dx}{x^4}$$

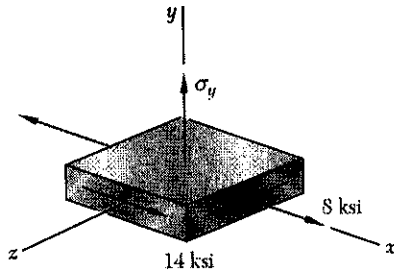
$$= \frac{T^2 L^4}{2G J_{\min}} \left( -\frac{1}{3x^3} \right) \Big|_L^{2L}$$

$$U = \frac{T^2 L^2}{2G J_{\min}} \left( -\frac{1}{3(2L)^3} + \frac{1}{3L^3} \right) = \frac{7}{48} \frac{T^2 L}{G J_{\min}}$$

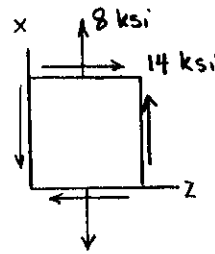


**PROBLEM 11.43**

11.43 The state of stress shown occurs in a machine component made of a grade of steel for which  $\sigma_Y = 65$  ksi. Using the maximum-distortion-energy criterion, determine the range of values of  $\sigma_y$  for which the factor of safety associated with the yield strength is equal to or larger than 2.2.



**SOLUTION**



$$\sigma_{ave} = \frac{1}{2} (0 + 8) = 4 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{8 - 0}{2} = 4 \text{ ksi}$$

$$\tau_{xz} = 14 \text{ ksi}$$

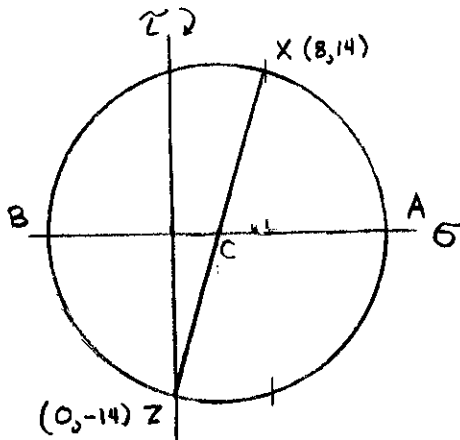
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 18.56 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -10.56 \text{ ksi}$$

$$\sigma_c = \sigma_y$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

$$(18.56 + 10.56)^2 + (-10.56 - \sigma_y)^2 + (\sigma_y - 18.56)^2 = 2\left(\frac{65}{2.2}\right)^2$$

$$847.97 + (111.51 + 21.12 \sigma_y + \sigma_y^2) + (\sigma_y^2 - 37.12 \sigma_y + 344.47) = 1745.87$$

$$2\sigma_y^2 - 16\sigma_y - 441.92 = 0$$

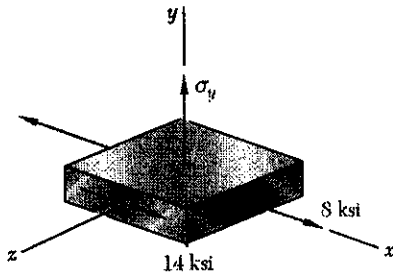
$$\sigma_y = \frac{16 \pm \sqrt{16^2 + (4)(2)(441.92)}}{(2)(2)} = 4 \pm 15.39$$

$$\sigma_y = 19.39 \text{ ksi}, -11.39 \text{ ksi}$$

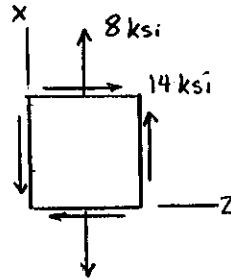
$$-11.39 \text{ ksi} \leq \sigma_y \leq 19.39 \text{ ksi}$$

**PROBLEM 11.44**

11.44 The state of stress shown occurs in a machine component made of a grade of steel for which  $\sigma_y = 65$  ksi. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (a)  $\sigma_y = +16$  ksi, (b)  $\sigma_y = -16$  ksi,



**SOLUTION**



$$\sigma_{ave} = \frac{1}{2}(0 + 8) = 4 \text{ ksi}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{8 - 0}{2} = 4 \text{ ksi}$$

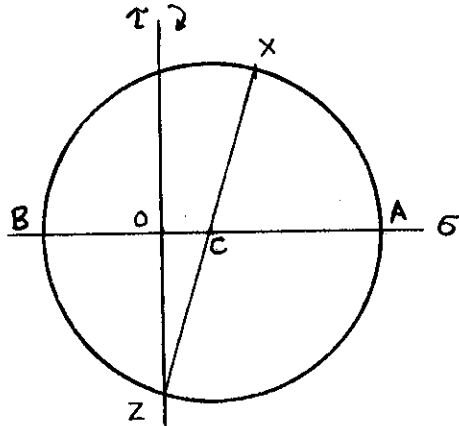
$$\tau_{xz} = 14 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{4^2 + 14^2} = 14.56 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 18.56$$

$$\sigma_b = \sigma_{ave} - R = -10.56$$

$$\sigma_c = \sigma_y$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

(a)  $\sigma_c = \sigma_y = 16 \text{ ksi}$

$$(18.56 + 10.56)^2 + (-10.56 - 16)^2 + (16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$

$$847.97 + 705.43 + 6.55 = \frac{8450}{(F.S.)^2} \quad F.S. = 2.33 \quad \blacktriangleleft$$

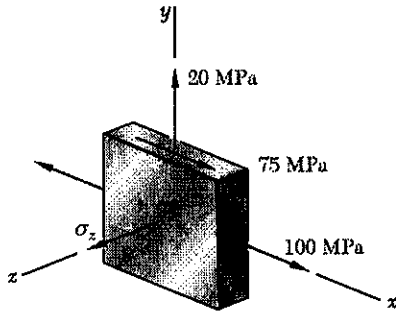
(b)  $\sigma_c = \sigma_y = -16 \text{ ksi}$

$$(18.56 + 10.56)^2 + (-10.56 + 16)^2 + (-16 - 18.56)^2 = 2\left(\frac{65}{F.S.}\right)^2$$

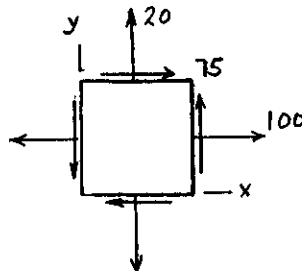
$$847.97 + 29.59 + 1194.39 = \frac{8450}{(F.S.)^2} \quad F.S. = 2.02 \quad \blacktriangleleft$$

PROBLEM 11.45

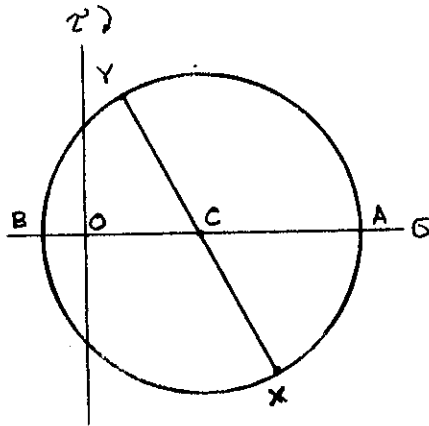
11.45 The state of stress shown occurs in a machine component made of a brass for which  $\sigma_Y = 160$  MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a)  $\sigma_z = +45$  MPa, (b)  $\sigma_z = -45$  MPa.



SOLUTION



$$\begin{aligned} \sigma_{ave} &= \frac{1}{2}(100 + 20) = 60 \text{ MPa} \\ \frac{\sigma_x - \sigma_y}{2} &= \frac{100 - 20}{2} = 40 \text{ MPa} \\ \tau_{xy} &= 75 \text{ MPa} \\ R &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \sqrt{40^2 + 75^2} = 85 \text{ MPa} \end{aligned}$$



$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 \stackrel{?}{\leq} 2\sigma_Y^2$$

(a)  $\sigma_c = \sigma_z = +45$  MPa

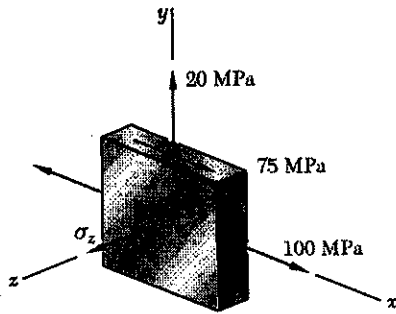
$$\begin{aligned} (145 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 &\stackrel{?}{<} 2(160)^2 = 51200 \\ 28900 + 4900 + 10000 &= 43800 < 51200 \quad (\text{No yield}) \end{aligned}$$

(b)  $\sigma_c = \sigma_z = -45$  MPa

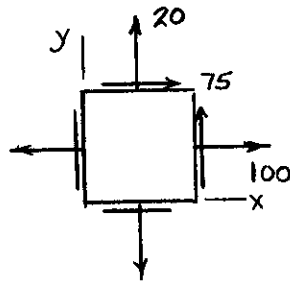
$$\begin{aligned} (145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 &\stackrel{?}{<} 51200 \\ 28900 + 400 + 36100 &= 65400 > 51200 \quad (\text{Yield occurs}) \end{aligned}$$

PROBLEM 11.46

11.46 The state of stress shown occurs in a machine component made of a brass for which  $\sigma_y = 160$  MPa. Using the maximum-distortion-energy criterion, determine the range of values of  $\sigma_z$  for which yield does not occur.



SOLUTION



$$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

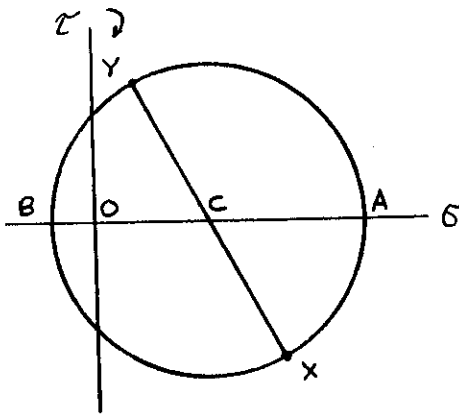
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\sigma_y^2$$

$$(145 + 25)^2 + (-25 - \sigma_z)^2 + (\sigma_z - 145)^2 = (2)(160)^2$$

$$28900 + (625 + 50\sigma_z + \sigma_z^2) + (\sigma_z^2 - 290\sigma_z + 21025) = 51200$$

$$2\sigma_z^2 - 240\sigma_z - 650 = 0$$

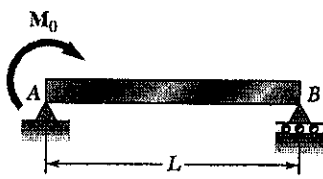
$$\sigma_z = \frac{240 \pm \sqrt{240^2 + (4)(2)(650)}}{(2)(2)} = 60 \pm 62.65$$

$$\sigma_z = 122.65 \text{ MPa}, -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \sigma_z < 122.65 \text{ MPa}$$

PROBLEM 11.47

11.47 Determine the strain energy of the prismatic beam AB, taking into account the effect of both normal and shearing stresses.

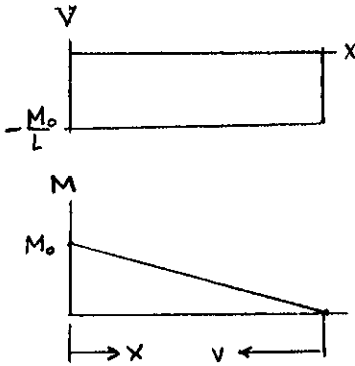


SOLUTION

Reactions  $R_A = \frac{M_0}{L} \downarrow$ ,  $R_B = \frac{M_0}{L} \uparrow$

Shear:  $V = -\frac{M_0}{L}$

Bending moment:  $M = \frac{M_0}{L} x$



For bending

$$U_1 = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^L x^2 dx = \frac{M_0^2 L^3}{6EI L^2} = \frac{M_0^2 L}{6EI}$$

For shear

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} \left(1 - \frac{y^2}{c^2}\right) \quad c = \frac{1}{2}d$$

$$u = \frac{\tau_{xy}^2}{2G} = \frac{9V^2}{8GA^2} \left(1 - \frac{y^2}{c^2}\right)^2 = \frac{9M_0^2}{8G(bd)^2 L^2} \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)$$

$$U_2 = \int u dV = \int_0^L \int_{-c}^c u b dy dx = \frac{9M_0^2 b}{8G b^2 d^2 L^2} \int_0^L \int_{-c}^c \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right) dy dx$$

$$= \frac{9M_0^2}{8G b d^2 L^2} \int_0^L \left(y - \frac{2}{3} \frac{y^3}{c^2} + \frac{1}{5} \frac{y^5}{c^4}\right) \Big|_{-c}^c dx = \frac{9M_0^2}{8G b d^2 L^2} \int_0^L \left(2c - \frac{4}{3}c + \frac{2}{5}c\right) dx$$

$$\frac{9M_0^2}{8G b d^2 L^2} \left(\frac{16}{15}c\right) L = \frac{6}{5} \frac{M_0^2 c}{G b d^2 L} = \frac{3}{5} \frac{M_0^2}{G b d L}$$

Total  $U = U_1 + U_2 = \frac{M_0^2 L}{6EI} + \frac{3}{5} \frac{M_0^2}{G b d L}$

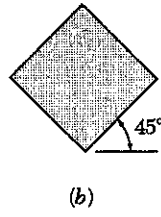
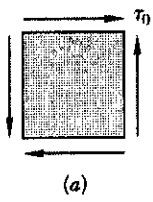
With  $I = \frac{1}{12} b d^3$

$$U = \frac{2M_0^2 L}{E b d^3} + \frac{3}{5} \frac{M_0^2}{G b d L} = \frac{2M_0^2 L}{E b d^3} \left\{ 1 + \frac{3}{10} \frac{E}{G} \frac{d^2}{L^2} \right\}$$



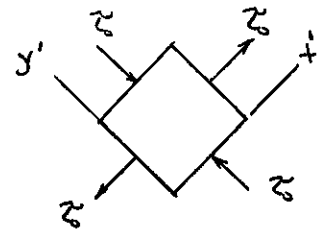
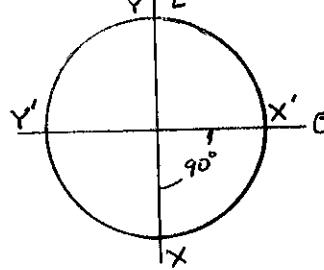
**PROBLEM 11.48**

11.48 For the state of stress shown in Fig. a, determine the stresses in an element oriented as shown in Fig. b. Compare the strain energy density in the given state first by using Fig. a and then by using Fig. b. Equating the two results obtained, show that



$$G = \frac{E}{2(1+\nu)}$$

Using Mohr's circle



**SOLUTION**

(a)  $\sigma_x = 0, \sigma_y = 0, \tau_{xy} = \tau_0$

$$U = \frac{1}{2E} (\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{1}{2G} \tau_{xy}^2 = \frac{\tau_0^2}{2G}$$

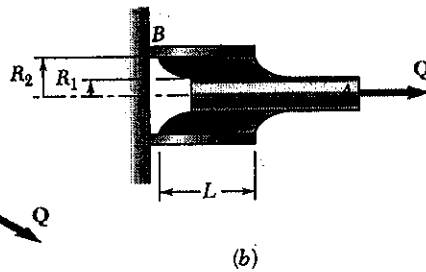
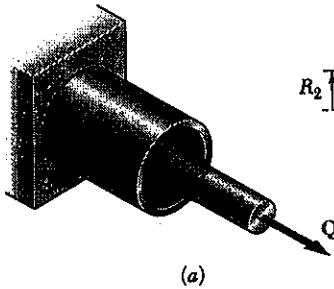
(b)  $\sigma_{x'} = \tau_0, \sigma_{y'} = -\tau_0, \tau_{x'y'} = 0$

$$U = \frac{1}{2E} (\sigma_{x'}^2 + \sigma_{y'}^2 - 2\nu\sigma_{x'}\sigma_{y'}) + \frac{1}{2G} \tau_{x'y'}^2 = \frac{(2+2\nu)\tau_0^2}{2E}$$

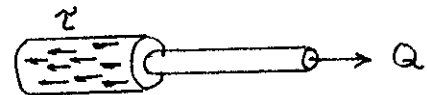
Equate  $\frac{\tau_0^2}{2G} = \frac{(2+2\nu)\tau_0^2}{2E}$   $G = \frac{E}{2(1+\nu)}$

**PROBLEM 11.49**

11.49 A vibration isolation support is made by bonding a rod A, of radius  $R_1$ , and a tube B, of inner radius  $R_2$  to a hollow rubber cylinder. Denoting by  $G$  the modulus of rigidity of the rubber, determine the strain energy of the hollow rubber cylinder for the loading shown.



**SOLUTION**



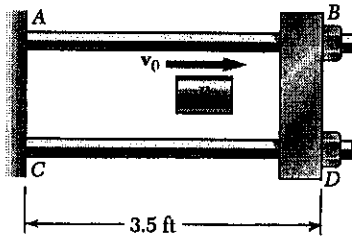
$$\begin{aligned} + \rightarrow \sum F_x = 0 \quad -\tau(2\pi r L) + Q = 0 \\ \tau = \frac{Q}{2\pi r L} \end{aligned}$$

$$U = \frac{\tau^2}{2G} = \frac{Q^2}{8\pi^2 r^2 L^2 G}$$

$$U = \int U dV = \frac{Q^2}{8\pi^2 G L^2} \int \frac{dV}{r^2} = \frac{Q^2}{8\pi^2 G L^2} \int_0^L \int_{R_1}^{R_2} \frac{2\pi r dr}{r^2} dx$$

$$= \frac{Q^2}{4\pi G L^2} \int_0^L \int_{R_1}^{R_2} \frac{dr}{r} dx = \frac{Q^2}{4\pi G L^2} \int_0^L (\ln r) \Big|_{R_1}^{R_2} dx = \frac{Q^2}{4\pi G L} \ln \frac{R_2}{R_1}$$

PROBLEM 11.50



11.50 The cylindrical block  $E$  has a speed  $v_0 = 16$  ft/s when it strikes squarely the yoke  $BD$  that is attached to the  $\frac{7}{8}$ -in.-diameter rods  $AB$  and  $CD$ . Knowing that the rods are made of a steel for which  $\sigma_y = 50$  ksi and  $E = 29 \times 10^6$  psi, determine the weight of the block  $E$  for which the factor of safety is five with respect to permanent deformation of the rods.

SOLUTION

At the onset of yielding the force in each rod is

$$F = \sigma_y A$$

Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_y^2 A^2 L}{2EA} = \frac{\sigma_y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_y^2 AL}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_y^2 AL}{E}$$

$$U_m = \left(\frac{1}{2} m v_0^2\right) (\text{F.S.}) = \left(\frac{1}{2} \frac{W}{g} v_0^2\right) (\text{F.S.})$$

Solving for  $W$ : 
$$W = \frac{2g U_m}{v_0^2 (\text{F.S.})} = \frac{2g \sigma_y^2 AL}{v_0^2 (\text{F.S.}) E}$$

Data:  $g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2$ ,  $\sigma_y = 50 \times 10^3 \text{ psi}$ ,

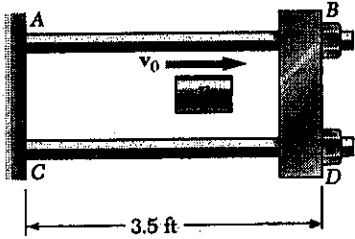
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2 \quad E = 29 \times 10^6 \text{ psi}$$

$$L = 3.5 \text{ ft} = 42 \text{ in} \quad \text{F.S.} = 5$$

$$v_0 = 16 \text{ ft/sec} = 192 \text{ in/sec}$$

$$W = \frac{(2)(386)(50 \times 10^3)^2 (0.60132)(42)}{(192)^2 (5)(29 \times 10^6)} = 9.12 \text{ lb.}$$

PROBLEM 11.51



11.51 The 18-lb cylindrical block  $E$  has a horizontal velocity  $v_0$  when it strikes squarely the yoke  $BD$  that is attached to the  $\frac{7}{8}$ -in.-diameter rods  $AB$  and  $CD$ . Knowing that the rods are made of a steel for which  $\sigma_y = 50$  ksi and  $E = 29 \times 10^6$  psi, determine the maximum allowable speed  $v_0$  if the rods are not to be permanently deformed.

SOLUTION

At the onset of yielding the force in each rod is

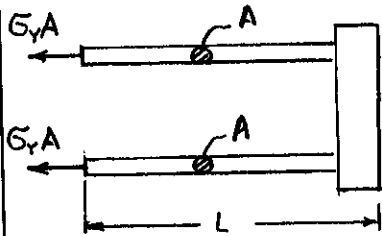
$$F = \sigma_y A$$

Corresponding strain energy

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_y^2 A^2 L}{2EA} = \frac{\sigma_y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_y^2 AL}{2E}$$

$$\text{Total } U_m = U_{AB} + U_{CD} = \frac{\sigma_y^2 AL}{E}$$



$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2} \frac{W}{g} v_0^2$$

$$\text{Solving for } v_0^2 \quad v_0^2 = \frac{2gU_m}{W} = \frac{2g\sigma_y^2 AL}{EW}$$

$$v_0 = \sqrt{\frac{2g\sigma_y^2 AL}{EW}}$$

Data:  $g = 32.17 \text{ ft/sec}^2 = 386 \text{ in/sec}^2$ ,  $\sigma_y = 50 \times 10^3 \text{ psi}$

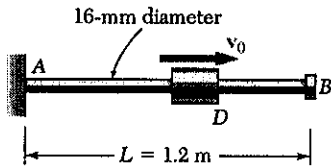
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$$
,  $E = 29 \times 10^6 \text{ psi}$

$$L = 3.5 \text{ ft} = 42 \text{ in.}$$
,  $W = 18 \text{ lb.}$

$$v_0 = \sqrt{\frac{(2)(386)(50 \times 10^3)^2(0.60132)(42)}{(29 \times 10^6)(18)}} = 305.6 \text{ in/sec}$$

$$= 25.5 \text{ ft./sec} \quad \blacktriangleleft$$

**PROBLEM 11.52**



11.52 The uniform rod  $AB$  is made of a brass for which  $\sigma_y = 125 \text{ MPa}$  and  $E = 105 \text{ GPa}$ . Collar  $D$  moves along the rod and has a speed  $v_0 = 3 \text{ m/s}$  as it strikes a small plate attached to end  $B$  of the rod. Using a factor of safety of four, determine the largest allowable mass of the collar if the rod is not to be permanently deformed.

**SOLUTION**

At onset of yielding  $P_m = \sigma_y A$

$\sigma_y = 125 \times 10^6 \text{ Pa}$

$A = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$

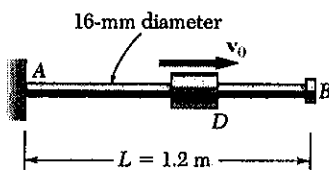
$P_m = 25133 \text{ N}$

Corresponding strain energy  $U_m = \frac{P_m^2 L}{2EA} = \frac{(25133)^2 (1.2)}{(2)(105 \times 10^9)(201.06 \times 10^{-6})}$   
 $= 17.953 \text{ J}$

Kinetic energy times safety factor  $= \frac{1}{2} m v_0^2 (\text{F.S.}) = 2 m v_0^2$

$2 m v_0^2 = U_m, \quad m = \frac{U_m}{2 v_0^2} = \frac{17.953}{(2)(3)^2} = 0.997 \text{ kg}$

**PROBLEM 11.53**



11.52 The uniform rod  $AB$  is made of a brass for which  $\sigma_y = 125 \text{ MPa}$  and  $E = 105 \text{ GPa}$ . Collar  $D$  moves along the rod and has a speed  $v_0 = 3 \text{ m/s}$  as it strikes a small plate attached to end  $B$  of the rod. Using a factor of safety of four, determine the largest allowable mass of the collar if the rod is not to be permanently deformed

11.53 Solve Prob. 11.52, assuming that the length of the brass rod is increased from 1.2 m to 2.4 m.

**SOLUTION**

At onset of yielding  $P_m = \sigma_y A$        $\sigma_y = 125 \times 10^6 \text{ Pa}$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (16) = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$

$P_m = 25133 \text{ N}$

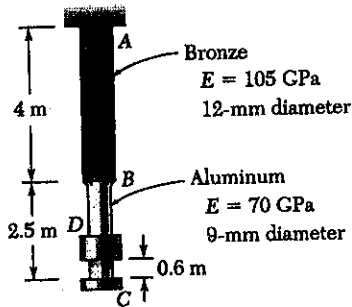
Corresponding strain energy  $U_m = \frac{P_m^2 L}{2EA} = \frac{(25133)^2 (2.4)}{(2)(105 \times 10^9)(201.06 \times 10^{-6})}$   
 $= 35.906 \text{ J}$

Kinetic energy times safety factor  $= \frac{1}{2} m v_0^2 (4) = 2 m v_0^2$

$2 m v_0^2 = U_m, \quad m = \frac{U_m}{2 v_0^2} = \frac{35.906}{(2)(3)^2} = 1.995 \text{ kg}$

PROBLEM 11.54

11.54 Collar  $D$  is released from rest in the position shown and is stopped by a small plate attached at end  $C$  of the vertical rod  $ABC$ . Determine the mass of the collar for which the maximum normal stress in portion  $BC$  is 125 MPa.



SOLUTION

Portion  $BC$ :  $\sigma_m = 125 \times 10^6 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E_{BC} A_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E_{AB} A_{AB}} = \frac{(7952)^2 (4)}{(2)(105 \times 10^9)(113.907 \times 10^{-6})} = 10.574 \text{ J}$$

$$U_m = U_{BC} + U_{AB} = 28.324 \text{ J}$$

Corresponding elongation  $\Delta_m$   $\frac{1}{2} P_m \Delta_m = U_m$

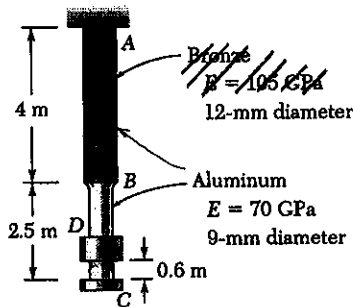
$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12 \times 10^{-3} \text{ m}$$

Falling distance  $h = 0.6 + 7.12 \times 10^{-3} = 0.60712 \text{ m}$ .

Work of weight =  $U_m$   $Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{28.324}{(9.81)(0.60712)} = 4.76 \text{ kg}$$

PROBLEM 11.55



11.54 Collar  $D$  is released from rest in the position shown and is stopped by a small plate attached at end  $C$  of the vertical rod  $ABC$ . Determine the mass of the collar for which the maximum normal stress in portion  $BC$  is 125 MPa.

11.55 Solve Prob. 11.54, assuming that both portions of rod  $ABC$  are made of aluminum.

SOLUTION

Portion  $BC$ :  $\sigma_m = 125 \times 10^6 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy

$$U_{BC} = \frac{P_m^2 L_{BC}}{2E A_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2E A_{AB}} = \frac{(7952)^2 (4)}{(2)(70 \times 10^9)(113.907 \times 10^{-6})} = 15.861 \text{ J}$$

$$\text{Total } U_m = U_{BC} + U_{AB} = 33.611 \text{ J}$$

Corresponding elongation  $\Delta_m$   $\frac{1}{2} P_m \Delta_m = U_m$

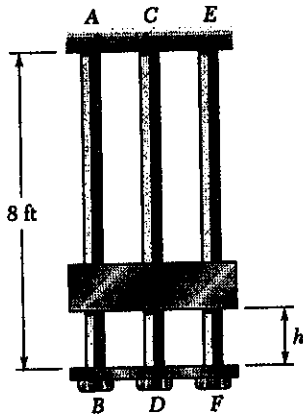
$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(33.611)}{7952} = 8.45 \times 10^{-3} \text{ m}$$

Falling distance  $h = 0.6 + \Delta_m = 0.60845 \text{ m}$

Work of weight =  $U_m$   $Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{33.611}{(9.81)(0.60845)} = 5.63 \text{ kg}$$

PROBLEM 11.56



11.56 The 100-lb collar  $G$  is released from rest in the position shown and is stopped by plate  $BDF$  that is attached to the  $\frac{7}{8}$ -in.-diameter steel rod  $CD$  and to the  $\frac{5}{8}$ -in.-diameter steel rods  $AB$  and  $EF$ . Knowing that for the grade of steel used  $\sigma_{all} = 24$  ksi and  $E = 29 \times 10^6$  psi, determine the largest allowable distance  $h$ .

SOLUTION

Let  $\Delta_m$  be the elongation

$$\Delta_m = \frac{\sigma_{AB} L}{E} = \frac{\sigma_{CD} L}{E} = \frac{\sigma_{EF} L}{E}$$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 24 \times 10^3 \text{ psi}$$

$$L = 8 \text{ ft} = 96 \text{ in}$$

$$\Delta_m = \frac{(24 \times 10^3)(96)}{29 \times 10^6} = 79.448 \times 10^{-3} \text{ in.}$$

For each rod.  $U = \frac{F_m^2 L}{2EA} = \frac{(EA \Delta_m / L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$

Rod  $CD$ :  $A_{CD} = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$

$$U_{CD} = \frac{(29 \times 10^6)(0.60132)(79.448 \times 10^{-3})^2}{(2)(96)} = 573.28 \text{ in}\cdot\text{lb.}$$

Rods  $AB$  and  $EF$ :  $A_{AB} = A_{EF} = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$

$$U_{AB} = U_{EF} = \frac{(29 \times 10^6)(0.30680)(79.448 \times 10^{-3})^2}{(2)(96)} = 292.49 \text{ in}\cdot\text{lb.}$$

Total  $U_m = U_{AB} + U_{CD} + U_{EF} = 1158.27 \text{ in}\cdot\text{lb.}$

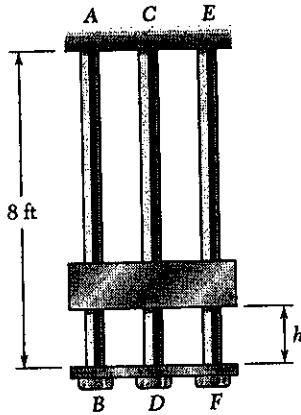
Falling distance is  $h + \Delta_m$ ,  $W = 100 \text{ lb}$

$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{1158.27}{100} = 11.583 \text{ in.}$$

$$h = 11.583 - 79.448 \times 10^{-3} = 11.50 \text{ in.}$$

PROBLEM 11.57



11.56 The 100-lb collar  $G$  is released from rest in the position shown and is stopped by plate  $BDF$  that is attached to the  $\frac{7}{8}$ -in.-diameter steel rod  $CD$  and to the  $\frac{5}{8}$ -in.-diameter steel rods  $AB$  and  $EF$ . Knowing that for the grade of steel used  $\sigma_{all} = 24$  ksi and  $E = 29 \times 10^6$  psi, determine the largest allowable distance  $h$ .

11.57 Solve Prob. 11.56, assuming that the  $\frac{7}{8}$ -in.-diameter steel rod  $CD$  is replaced by a  $\frac{7}{8}$ -in.-diameter rod made of a grade of aluminum for which  $\sigma_{all} = 20$  ksi and  $E = 10.6 \times 10^6$  psi.

SOLUTION

Let  $\Delta_m$  be the elongation.  $L = 8 \text{ ft} = 96 \text{ in}$

$$\Delta_m = \frac{\sigma_{AB} L}{E_{AB}} = \frac{\sigma_{CD} L}{E_{CD}} = \frac{\sigma_{EF} L}{E_{EF}}$$

If  $\sigma_{AB} = 24 \times 10^3$  psi,  $\Delta_m = \frac{(24 \times 10^3)(96)}{29 \times 10^6} = 79.448 \times 10^{-3} \text{ in.}$

If  $\sigma_{CD} = 20 \times 10^3$  psi,  $\Delta_m = \frac{(20 \times 10^3)(96)}{10.6 \times 10^6} = 181.13 \times 10^{-3} \text{ in.}$

Smaller value governs  $\Delta_m = 79.448 \times 10^{-3} \text{ in.}$

For each rod  $U = \frac{F^2 L}{2EA} = \frac{(EA \Delta_m / L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$

Rod  $CD$ :  $A_{CD} = \frac{\pi}{4} \left(\frac{7}{8}\right)^2 = 0.60132 \text{ in}^2$ ,  $E_{CD} = 10.6 \times 10^6$  psi

$$U_{CD} = \frac{(10.6 \times 10^6)(0.60132)(79.448 \times 10^{-3})^2}{(2)(96)} = 209.54 \text{ in}\cdot\text{lb}$$

Rods  $AB$  and  $EF$ :  $A_{AB} = A_{EF} = \frac{\pi}{4} \left(\frac{5}{8}\right)^2 = 0.30680 \text{ in}^2$

$$U_{AB} = U_{EF} = \frac{(29 \times 10^6)(0.30680)(79.448 \times 10^{-3})^2}{(2)(96)} = 292.49 \text{ in}\cdot\text{lb}$$

Total  $U_m = U_{AB} + U_{CD} + U_{EF} = 794.52 \text{ in}\cdot\text{lb.}$

Falling distance is  $h + \Delta_m$   $W = 100 \text{ lb.}$

$$W(h + \Delta_m) = U_m$$

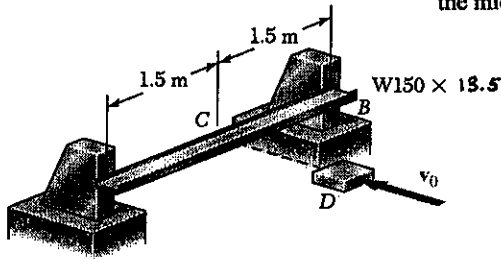
$$h + \Delta_m = \frac{U_m}{W} = \frac{794.52}{100} = 7.9452 \text{ in}$$

$$h = 7.9452 - 79.448 \times 10^{-3} = 7.87 \text{ in.}$$



PROBLEM 11.58

11.58 The steel beam  $AB$  is struck squarely at its midpoint  $C$  by a 45-kg block moving horizontally with a speed  $v_0 = 2$  m/s. Using  $E = 200$  GPa, determine (a) the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint  $C$  of the beam.



SOLUTION

From Appendix C, for  $W 150 \times 13.5$

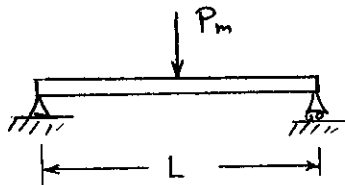
$$I_x = 6.87 \times 10^6 \text{ mm}^4 = 6.87 \times 10^{-6} \text{ m}^4$$

$$S_x = 91.6 \times 10^3 \text{ mm}^3 = 91.6 \times 10^{-6} \text{ m}^3$$

$$\text{Kinetic energy } T = \frac{1}{2} m v_0^2 = \frac{1}{2} (45)(2)^2 = 90 \text{ J}$$

From Appendix D, Case 4

$$|y_m| = \frac{P L^3}{48 E I}, \quad M_{max} = \frac{P L}{4}$$



$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96 E I} = T$$

$$(a) \quad P_m = \sqrt{\frac{96 E I T}{L^3}} = \sqrt{\frac{(96)(200 \times 10^9)(6.87 \times 10^{-6})(90)}{(3.0)^3}} = 20.968 \times 10^3 \text{ N} = 21.0 \text{ kN}$$

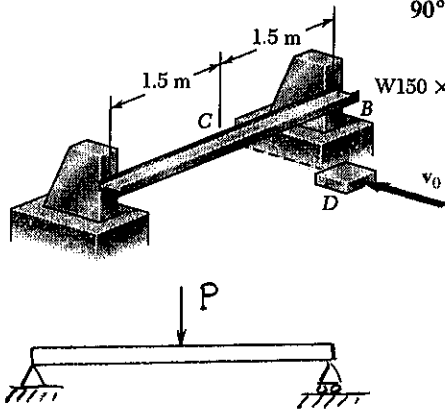
$$(b) \quad \sigma_m = \frac{M_{max}}{S} = \frac{P_m L}{4 S} = \frac{(20.968 \times 10^3)(3.0)}{(4)(91.6 \times 10^{-6})} = 171.7 \times 10^6 \text{ Pa} = 171.7 \text{ MPa}$$

$$(c) \quad |y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{20.968 \times 10^3} = 8.58 \times 10^{-3} \text{ m} = 8.58 \text{ mm}$$

**PROBLEM 11.59**

**11.58** The steel beam  $AB$  is struck squarely at its midpoint  $C$  by a 45-kg block moving horizontally with a speed  $v_0 = 2$  m/s. Using  $E = 200$  GPa, determine (a) the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint  $C$  of the beam.

**11.59** Solve Prob. 11.58, assuming that the  $W150 \times 13.5$  rolled-steel beam is rotated by  $90^\circ$  about its longitudinal axis so that its web is vertical.



**SOLUTION**

From Appendix C, for  $W 150 \times 13.5$

$$I_y = 0.918 \times 10^6 \text{ mm}^4 = 0.918 \times 10^{-6} \text{ m}^4$$

$$S_y = 18.4 \times 10^3 \text{ mm}^3 = 18.4 \times 10^{-6} \text{ m}^3$$

$$\text{Kinetic energy } T = \frac{1}{2} m v_0^2 = \frac{1}{2} (45)(2)^2 = 90 \text{ J}$$

From Appendix D, Case 4

$$|y_m| = \frac{P L^3}{48 E I} \quad M_{\max} = \frac{P L}{4}$$

$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96 E I} = T$$

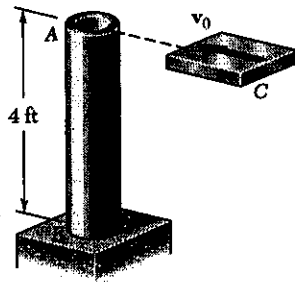
$$(a) P_m = \sqrt{\frac{96 E I T}{L^3}} = \sqrt{\frac{(96)(200 \times 10^9)(0.918 \times 10^{-6})(90)}{(3.0)^3}} = 7.665 \times 10^3 \text{ N} = 7.67 \text{ kN}$$

$$(b) \sigma_m = \frac{M_{\max}}{S} = \frac{P_m L}{4 S} = \frac{(7.665 \times 10^3)(3.0)}{(4)(18.4 \times 10^{-6})} = 312 \times 10^6 \text{ Pa} = 312 \text{ MPa}$$

$$(c) |y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{7.665 \times 10^3} = 23.5 \times 10^{-3} \text{ m} = 23.5 \text{ mm}$$

**PROBLEM 11.60**

11.60 The post  $AB$  consists of a steel pipe of 3.5-in outer diameter and 0.3-in. wall thickness. A 15-lb block  $C$  moving horizontally with a velocity  $v_0$  hits the post squarely at  $A$ . Using  $E = 29 \times 10^6$  psi, determine the largest speed  $v_0$  for which the maximum normal stress in the pipe does not exceed 24 ksi.



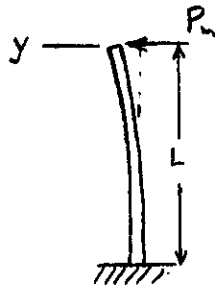
**SOLUTION**

$$c_o = \frac{1}{2} d_o = \frac{1}{2} (3.5) = 1.75 \text{ in.}, \quad c_i = c_o - t = 1.75 - 0.3 = 1.45 \text{ in.}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = 3.8943 \text{ in}^4 \quad \sigma_m = 24000 \text{ psi}$$

$$\sigma_m = \frac{M_m c}{I}, \quad M_m = \frac{I \sigma_m}{c} = \frac{(3.8943)(24000)}{1.75} = 53407 \text{ lb-in}$$

$$P_m = \frac{M_m}{L} = \frac{53407}{48} = 1112.66 \text{ lb.}$$



By Appendix D, Case 1

$$y_m = \frac{P_m L^3}{3EI} = \frac{(1112.66)(48)^3}{(3)(29 \times 10^6)(3.8943)} = 0.36319 \text{ in}$$

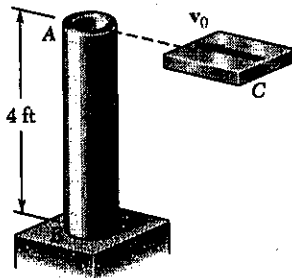
$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (1112.66)(0.36319) = 202.05 \text{ in-lb.}$$

$$\frac{1}{2} \frac{W}{g} v_0^2 = U_m \quad v_0^2 = \frac{2gU_m}{W} = \frac{(2)(386)(202.05)}{15} = 10399 \text{ in}^2/\text{sec}^2$$

$$v_0 = 102.0 \text{ in/sec} = 8.50 \text{ ft/sec}$$

**PROBLEM 11.61**

11.61 The post  $AB$  consists of a steel pipe of 3.5-in outer diameter and 0.3-in. wall thickness. A 15-lb block  $C$  moving horizontally with a velocity  $v_0$  hits the post squarely at  $A$ . Using  $E = 29 \times 10^6$  psi, determine the largest speed  $v_0$  for which the maximum normal stress in the pipe does not exceed 24 ksi.



11.61 Solve Prob 11.60, assuming that the post  $AB$  consists of a solid steel rod of 3.5-in outer diameter.

**SOLUTION**

$$c = \frac{1}{2} d = 1.75 \text{ in.} \quad I = \frac{\pi}{4} c^4 = 7.3662 \text{ in}^4$$

$$\sigma_m = 24000 \text{ psi} \quad L = 4 \text{ ft} = 48 \text{ in.}$$

$$\sigma_m = \frac{M_m c}{I}, \quad M_m = \frac{I \sigma_m}{c} = \frac{(7.3662)(24000)}{1.75} = 101022 \text{ lb-in}$$

$$P_m = \frac{M_m}{L} = 2104.6 \text{ lb.}$$



By Appendix D, Case 1

$$y_m = \frac{P_m L^3}{3EI} = \frac{(2104.6)(48)^3}{(3)(29 \times 10^6)(7.3662)} = 0.36319 \text{ in}$$

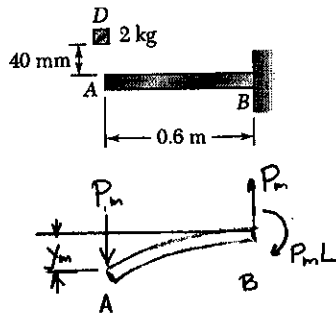
$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (2104.6)(0.36319) = 382.19 \text{ in-lb.}$$

$$\frac{1}{2} \frac{W}{g} v_0^2 = U_m, \quad v_0^2 = \frac{2gU_m}{W} = \frac{(2)(386)(382.19)}{15} = 19670 \text{ in}^2/\text{sec}^2$$

$$v_0 = 140.25 \text{ in/sec} = 11.69 \text{ ft/sec}$$

PROBLEM 11.62

11.62 The 2-kg block  $D$  is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that  $E = 200$  GPa, determine (a) the maximum deflection of end  $A$ , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



SOLUTION

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad L_{AB} = 0.6 \text{ m}$$

Appendix D, Case 1

$$y_m = \frac{P_m L_{AB}^3}{3EI} \quad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3} y_m = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.9361 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (8.9361 \times 10^3) y_m^2 = 4.4681 \times 10^3 y_m^2$$

$$\begin{aligned} \text{Work of dropped weight} \quad mg(h + y_m) &= (2)(9.81)(0.040 + y_m) \\ &= 0.7848 + 19.62 y_m \end{aligned}$$

Equating work and energy

$$0.7848 + 19.62 y_m = 4.4681 \times 10^3 y_m^2$$

$$y_m^2 - 4.3911 \times 10^{-3} y_m - 1.75.645 \times 10^{-6} = 0$$

$$\begin{aligned} \text{(a)} \quad y_m &= \frac{1}{2} \left\{ 4.3911 \times 10^{-3} + \sqrt{(4.3911 \times 10^{-3})^2 + (4)(1.75.645 \times 10^{-6})} \right\} \\ &= 15.629 \times 10^{-3} \text{ m} = 15.63 \text{ mm} \end{aligned}$$

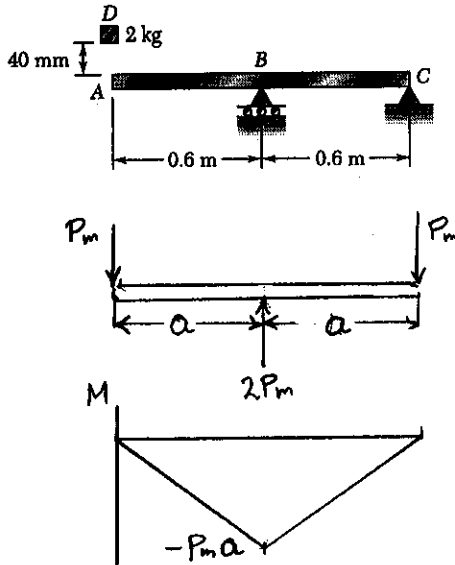
$$P_m = (8.9361 \times 10^3)(15.629 \times 10^{-3}) = 139.66 \text{ N}$$

$$\text{(b)} \quad M_m = -P_m L_{AB} = -(139.66)(0.6) = -83.8 \text{ N}\cdot\text{m}$$

$$\text{(c)} \quad \sigma_m = \frac{M_m c}{I} = \frac{(83.8)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 208 \times 10^6 \text{ Pa} = 208 \text{ MPa}$$

PROBLEM 11.63

11.63 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that  $E = 200 \text{ GPa}$ , determine (a) the maximum deflection of end A, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



SOLUTION

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad a = 0.6 \text{ m}$$

Over AB  $M = -P_m x$   $M_m = -P_m a$

$$U_{AB} = \int_0^a \frac{P_m^2 x^2}{2EI} dx = \frac{P_m^2 a^3}{6EI}$$

$$= \frac{(0.6)^3}{(6)(200 \times 10^9)(3.2170 \times 10^{-9})} P_m^2$$

$$= 55.953 \times 10^{-6} P_m^2$$

By symmetry of bending moment diagram

$$U_{BC} = U_{AB} = 55.953 \times 10^{-6} P_m^2$$

$$U_m = U_{AB} + U_{BC} = 111.906 \times 10^{-6} P_m^2$$

$$\frac{1}{2} P_m y_m = U_m = 111.906 \times 10^{-6} P_m^2 \quad P_m = 4.4681 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 2.2340 \times 10^3 y_m^2$$

Work of dropped weight  $mg(h + y_m) = (2)(9.81)(0.040 + y_m)$

$$= 0.7848 + 19.62 y_m$$

Equating work and energy

$$0.7848 + 19.62 y_m = 2.2340 \times 10^3 y_m^2$$

$$y_m^2 - 8.7825 \times 10^{-3} y_m - 351.298 \times 10^{-6} = 0$$

$$(a) \quad y_m = \frac{1}{2} \left\{ 8.7825 \times 10^{-3} + \sqrt{(8.7825 \times 10^{-3})^2 + (4)(351.298 \times 10^{-6})} \right\}$$

$$= 23.636 \times 10^{-3} \text{ m} = 23.6 \text{ mm}$$

$$P_m = (4.4681 \times 10^3)(23.636 \times 10^{-3}) = 105.61 \text{ N}$$

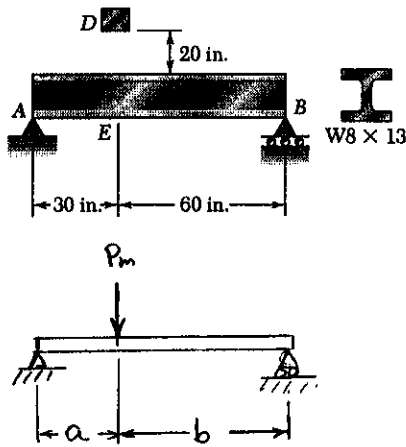
$$(b) \quad M_m = -(105.61)(0.6) = -64.4 \text{ N}\cdot\text{m}$$

$$(c) \quad \sigma_m = \frac{|M_m|c}{I} = \frac{(64.4)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 157.6 \times 10^6 \text{ Pa}$$

$$= 157.6 \text{ MPa}$$

PROBLEM 11.64

11.64 The 50-lb block  $D$  is dropped from a height of 20 in. onto the steel beam  $AB$ . Knowing that  $E = 29 \times 10^6$  psi, determine (a) the maximum deflection at point  $E$ , (b) the maximum normal stress in the beam.



SOLUTION

$$I_x = 39.6 \text{ in}^4, \quad S_x = 9.91 \text{ in}^3$$

Appendix D, Case 5

$$y_E = \frac{P_m a^2 b^2}{3EI} = \frac{(30)^2 (60)^2 P_m}{(3)(29 \times 10^6)(39.6)(90)}$$

$$= 10.4493 \times 10^{-6} P_m$$

$$P_m = 95700 y_E$$

$$W(h + y_E) = 50(20 + y_E) = 1000 + 50 y_E$$

Work of falling weight

$$W(h + y_E) = 50(20 + y_E) = 1000 + 50 y_E$$

Equating work and energy:

$$1000 + 50 y_E = 47850 y_E^2$$

$$y_E^2 - 1.04493 \times 10^{-3} - 20.899 \times 10^{-3} = 0$$

$$(a) \quad y_E = \frac{1}{2} \left\{ 1.04493 \times 10^{-3} + \sqrt{(1.04493 \times 10^{-3})^2 + (4)(20.899 \times 10^{-3})} \right\}$$

$$= 0.1451 \text{ in}$$

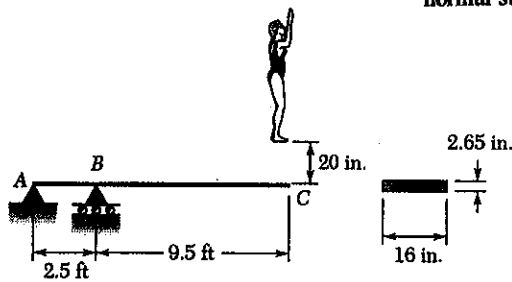
$$P_m = (95700)(0.1451) = 13885 \text{ lb.}$$

$$M_m = \frac{P_m ab}{L} = \frac{(13885)(30)(60)}{90} = 277.7 \times 10^3 \text{ lb}\cdot\text{in.}$$

$$(b) \quad \sigma_m = \frac{M_m}{S_x} = \frac{277.7 \times 10^3}{9.91} = 28.0 \times 10^3 \text{ psi} = 28.0 \text{ ksi}$$

PROBLEM 11.65

11.65 A 160-lb diver jumps from a height of 20 in. onto end C of a diving board having the uniform cross section shown. Assuming that the diver's legs remain rigid and using  $E = 1.8 \times 10^6$  psi, determine (a) the maximum deflection at point C, (b) the maximum normal stress in the board, (c) the equivalent static load.

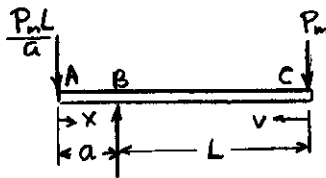


SOLUTION

$$I = \frac{1}{12}(16)(2.65)^3 = 24.813 \text{ in}^4$$

$$L = 9.5 \text{ ft.} = 114 \text{ in.}, \quad a = 2.5 \text{ ft} = 30 \text{ in.}$$

$$c = \frac{1}{2}(2.65) = 1.325 \text{ in.}$$



Over portion AB  $M = -\frac{P_m L}{a} x$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \frac{P_m^2 L^2}{2EI a^2} \int_0^a x^2 dx = \frac{P_m^2 L^2 a}{6EI}$$

Over portion BC  $M = -P_m v$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dv = \frac{P_m^2}{2EI} \int_0^L v^2 dv = \frac{P_m^2 L^3}{6EI}$$

Total  $U = U_{AB} + U_{BC} = \frac{P_m^2 L^2 (a+L)}{6EI}$

$$\frac{1}{2} P_m y_m = U_m \quad y_m = \frac{2U_m}{P_m} = \frac{P_m L^2 (a+L)}{3EI}$$

$$P_m = \frac{3EI}{L^2 (a+L)} y_m = \frac{(3)(1.8 \times 10^6)(24.813)}{(114)^2 (114 + 30)} y_m = 71.598 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 35.799 y_m^2$$

Work of weight =  $W(h + y_m) = (160)(20 + y_m) = 3200 + 160 y_m$

Equating  $3200 + 160 y_m = 35.799 y_m^2$

$$y_m^2 - 4.4694 y_m - 89.388 = 0$$

(a)  $y_m = \frac{1}{2} \left\{ 4.4694 + \sqrt{4.4694^2 + (4)(89.388)} \right\} = 11.95 \text{ in}$

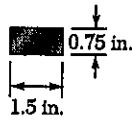
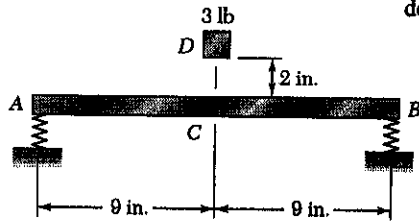
(c)  $P_m = (71.598)(11.95) = 856 \text{ lb}$

$$M_m = -(856)(114) = 97535 \text{ lb}\cdot\text{in}$$

(b)  $\sigma_m = \frac{M_m c}{I} = \frac{(97535)(1.325)}{24.813} = 5210 \text{ psi}$

PROBLEM 11.66

11.66 The 3-lb block  $D$  is released from rest in the position shown and strikes a steel bar  $AB$  having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using  $E = 29 \times 10^6$  psi, determine the maximum deflection at the midpoint of the bar.



SOLUTION

$$k = 20 \text{ kips/in} = 20 \times 10^3 \text{ lb/in}$$

$$R_A = R_B = \frac{1}{2} P_m$$

For spring A,  $U_A = \frac{1}{2} R_A y_A = \frac{1}{2} \frac{R_A^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

For spring B,  $U_B = \frac{1}{2} R_B y_B = \frac{1}{2} \frac{R_B^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

Portion AC of beam ACB  $M = \frac{1}{2} P_m x$

$$U_{AC} = \int_0^{L_{AC}} \frac{M^2}{2EI} dx = \frac{P_m^2}{8EI} \int_0^{L_{AC}} x^2 dx = \frac{P_m^2 L_{AC}^3}{24EI}$$

Portion CB of beam

By symmetry  $U_{CB} = U_{AC} = \frac{P_m^2 L_{AC}^3}{24EI}$

$$\text{Total } U = U_A + U_B + U_{AC} + U_{CB} = \frac{P_m^2}{4k} + \frac{P_m^2 L_{AC}^3}{12EI}$$

$$I = \frac{1}{12} b d^3 = \frac{1}{12} (1.5)(0.75)^3 = 52.734 \times 10^{-3} \text{ in}^4$$

$$U = \left\{ \frac{1}{(4)(20 \times 10^3)} + \frac{(9)^3}{(12)(29 \times 10^6)(52.734 \times 10^{-3})} \right\} P_m^2 = 52.224 \times 10^{-6} P_m^2 = \frac{1}{2} P_m y_m$$

$$y_m = \frac{2U}{P_m} = 104.448 \times 10^{-6} P_m \quad P_m = 9.5741 \times 10^3 y_m$$

$$U = (52.224 \times 10^{-6})(9.5741 \times 10^3)^2 y_m^2 = 4.7871 \times 10^3 y_m^2$$

Work of falling weight  $W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$

Equating  $6 + 3y_m = 4.7871 \times 10^3 y_m^2$

$$y_m^2 - 626.69 \times 10^{-6} y_m - 1.25338 \times 10^{-3} = 0$$

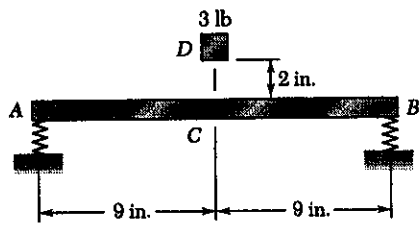
$$y_m = \frac{1}{2} \left\{ 626.69 \times 10^{-6} + \sqrt{(626.69 \times 10^{-6})^2 + (4)(1.25338 \times 10^{-3})} \right\}$$

$$= 35.7 \times 10^{-3} \text{ in.} = 0.0357 \text{ in.}$$

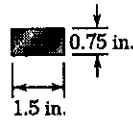


**PROBLEM 11.67**

11.67 The 3-lb block *D* is released from rest in the position shown and strikes a steel bar *AB* having the uniform cross section shown. The bar is supported at each end by springs of constant 20 kips/in. Using  $E = 20 \times 10^6$  psi, determine the maximum deflection at the midpoint of the bar.



11.67 Solve Prob 11.66, assuming that the constant of each spring is 40 kips/in.



**SOLUTION**

$$k = 40 \text{ kips/in} = 40 \times 10^3 \text{ lb/in.}$$

$$R_A = R_B = \frac{1}{2} P_m$$

For spring A,  $U_A = \frac{1}{2} R_A y_A = \frac{1}{2} \frac{R_A^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

For spring B,  $U_B = \frac{1}{2} R_B y_B = \frac{1}{2} \frac{R_B^2}{k} = \frac{1}{8} \frac{P_m^2}{k}$

Portion AC of beam ACB  $M = \frac{1}{2} P_m x$

$$U_{Ac} = \int_0^{L_{Ac}} \frac{M^2}{2EI} dx = \frac{P_m^2}{8EI} \int_0^{L_{Ac}} x^2 dx = \frac{P_m^2 L_{Ac}^3}{24 EI}$$

Portion CB of beam

By symmetry  $U_{Cb} = U_{Ac} = \frac{P_m^2 L_{Ac}^3}{24 EI}$

$$\text{Total } U = U_A + U_B + U_{Ac} + U_{Cb} = \frac{P_m^2}{4k} + \frac{P_m^2 L_{Ac}^3}{12 EI}$$

$$I = \frac{1}{12} b d^3 = \frac{1}{12} (1.5)(0.75)^3 = 52.734 \times 10^{-3} \text{ in}^4$$

$$U = \left\{ \frac{1}{(4)(40 \times 10^3)} + \frac{(9)^3}{(12)(29 \times 10^6)(52.734 \times 10^{-3})} \right\} P_m^2 = 45.974 \times 10^{-6} P_m^2 = \frac{1}{2} P_m y_m$$

$$y_m = \frac{2U}{P_m} = 91.949 \times 10^{-6} P_m \quad P_m = 10.8756 \times 10^3 y_m$$

$$U = (45.974 \times 10^{-6})(10.8756 \times 10^3)^2 y_m^2 = 5.4378 \times 10^3 y_m^2$$

Work of falling weight  $W(h + y_m) = (3)(2 + y_m) = 6 + 3y_m$

Equating  $6 + 3y_m = 5.4378 \times 10^3 y_m^2$

$$y_m^2 - 551.70 \times 10^{-6} y_m - 1.1034 \times 10^{-3} = 0$$

$$y_m = \frac{1}{2} \left\{ 551.70 \times 10^{-6} + \sqrt{(551.70 \times 10^{-6})^2 + (4)(1.1034 \times 10^{-3})} \right\}$$

$$= 33.5 \times 10^{-3} \text{ in.} = 0.0335 \text{ in.}$$

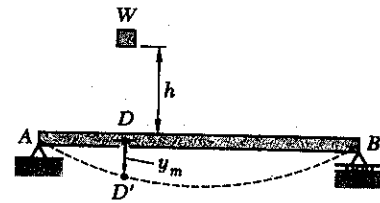
PROBLEM 11.68

11.68 A block of weight  $W$  is placed in contact with a beam at some given point  $D$  and released. Show that the resulting maximum deflection at point  $D$  is twice as large as the deflection due to a static weight  $W$  applied at  $D$ .

SOLUTION

Consider dropping the weight from a height  $h$  above the beam. The work done by the weight is

$$\text{Work} = W(h + y_m)$$



Strain energy  $U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$

where  $k$  is the spring constant of the beam for loading at point  $D$ .

Equating work and energy  $W(h + y_m) = \frac{1}{2} k y_m^2$

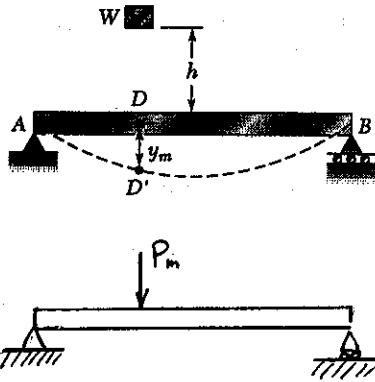
Setting  $h = 0$ ,  $W y_m = \frac{1}{2} k y_m^2$ ,  $y_m = \frac{2W}{k}$

The static deflection at point  $D$  due to weight applied at  $D$  is

$$S_{st} = \frac{W}{k}$$

Thus  $y_m = 2 S_{st}$

PROBLEM 11.69



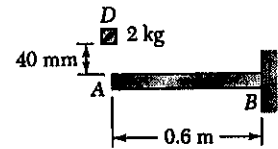
11.69 A block of weight  $W$  is dropped from a height  $h$  onto the horizontal beam  $AB$  and hits it at point  $D$ . (a) Show that the maximum deflection  $y_m$  at point  $D$  can be expressed as

$$y_m = y_{st} \left( 1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

where  $y_m$  represents the deflection at  $D$  caused by a static load  $W$  applied at that point and where the quantity in parentheses is referred to as the *impact factor*. (b) Compute the impact factor for the beam and impact factor of Prob. 11.62.

11.62 The 2-kg block  $D$  is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that  $E = 200$  GPa, determine (a) the maximum deflection of end  $A$ , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.

SOLUTION



Work of falling weight

$$Work = W(h + y_m)$$

$$Strain\ energy \quad U = \frac{1}{2} P y_m = \frac{1}{2} k y_m^2$$

where  $k$  is the spring constant for a load applied at point  $D$ .

Equating work and energy

$$W(h + y_m) = \frac{1}{2} k y_m^2$$

$$y_m^2 - \frac{2W}{k} y_m - \frac{2Wh}{k} = 0$$

$$y_m^2 - 2y_{st} y_m - 2y_{st} h = 0$$

$$\text{where } y_{st} = \frac{W}{k}$$

$$y_m = \frac{2y_{st} + \sqrt{4y_{st}^2 + 8y_{st}h}}{2} = y_{st} \left( 1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

For Prob. 11.62

$$W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa} \quad I = \frac{\pi}{4} \left( \frac{16}{2} \right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m}$$

$$h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

Using Appendix D Case 1

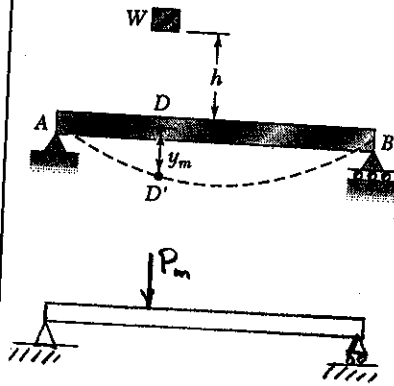
$$y_{st} = \frac{WL^3}{3EI}$$

$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-3} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-3}} = 36.44$$

$$\text{impact factor} = 1 + \sqrt{1 + 36.44} = 7.12$$

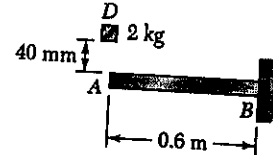
PROBLEM 11.70



11.70 A block of weight  $W$  is dropped from a height  $h$  onto the horizontal beam  $AB$  and hits it at point  $D$ . (a) Denoting by  $y_m$  the exact value of the maximum deflection at  $D$  and by  $y'_m$  the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is  $(y_m - y'_m)/y_m$  never exceeds  $y_m/2h$ . (b) Check the result obtained in part a by solving part a of Prob. 11.62 without taking  $y'_m$  into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

11.62 The 2-kg block  $D$  is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that  $E = 200$  GPa, determine (a) the maximum deflection of end  $A$ , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.

SOLUTION



$$U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2 \quad \text{where } k \text{ is the spring constant for a load at point } D.$$

Work of falling weight:   
 exact:  $Work = W(h + y_m)$    
 approximate:  $Work \approx Wh$

Equating work and energy:   
 $\frac{1}{2} k y_m^2 = W(h + y_m)$  (1) exact   
 $\frac{1}{2} k y'_m{}^2 = Wh$  (2) approximate

where  $y'_m$  is the approximate value for  $y_m$

Subtracting  $\frac{1}{2} k (y_m^2 - y'_m{}^2) = W y_m$

$$y_m^2 - y'_m{}^2 = (y_m - y'_m)(y_m + y'_m) = \frac{2W}{k} y_m$$

Relative error  $\frac{y_m - y'_m}{y_m} = \frac{2W}{k(y_m + y'_m)}$

But  $\frac{2W}{k} = \frac{y'_m{}^2}{h}$  from equation (2)

(a) Relative error =  $\frac{y_m - y'_m}{y_m} = \frac{y'_m{}^2}{h(y_m + y'_m)} < \frac{y'_m}{2h}$

(b) From the solution to Prob. 11.62  $y_m = 15.63$  mm

Approximate solution:  $W = mg = (2)(9.81) = 19.62$  N

$E = 200 \times 10^9$  Pa  $I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.217 \times 10^3$  mm<sup>4</sup> =  $3.217 \times 10^{-9}$  m<sup>4</sup>

$L = 0.6$  m,  $h = 40$  mm =  $40 \times 10^{-3}$  m

$k = \frac{3EI}{L^3} = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.936 \times 10^3$  N/m

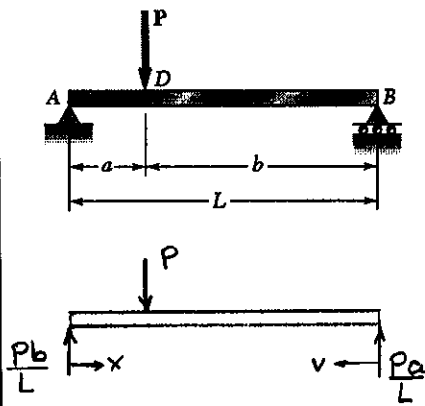
$y'_m{}^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40 \times 10^{-3})}{8.936 \times 10^3} = 175.65 \times 10^{-6}$  m<sup>2</sup>

$y'_m = 13.25 \times 10^{-3}$  m = 13.25 mm

relative error =  $\frac{15.63 - 13.25}{15.63} = 0.152 \blacktriangleleft \frac{y'_m}{2h} = 0.166 \blacktriangleleft$

**PROBLEM 11.71**

11.71 Using the method of work-energy, determine the deflection at point  $D$  caused by the load  $P$



**SOLUTION**

Reactions:  $R_A = \frac{Pb}{L}$  ,  $R_B = \frac{Pa}{L}$

Over AD  $M = R_A x = \frac{Pbx}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2 b^2}{2EI L^2} \int_0^a x^2 dx$$

$$= \frac{P^2 b^2 a^3}{6EI L^2}$$

Over DB  $M = R_B v = \frac{Pav}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^b v^2 dv$$

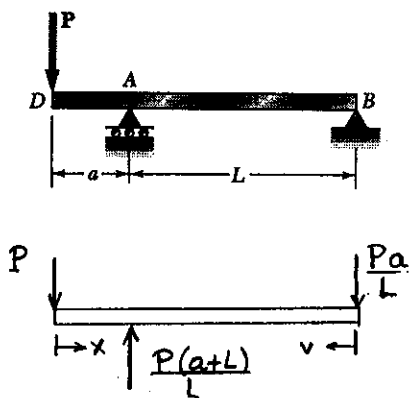
$$= \frac{P^2 a^2 b^3}{6EI L^2}$$

Total  $U = U_{AD} + U_{DB} = \frac{P^2 a^2 b^2 (a+b)}{6EI L^2} = \frac{P^2 a^2 b^2}{6EI L}$

$\frac{1}{2} P S_D = U$        $S_D = \frac{2U}{P} = \frac{Pa^2 b^2}{3EI L} \downarrow$

**PROBLEM 11.72**

11.72 Using the method of work-energy, determine the deflection at point  $D$  caused by the load  $P$



**SOLUTION**

$\sum M_A = 0$        $Pa + R_B L = 0$        $R_B = -\frac{Pa}{L}$

Over portion DA  $M = -Px$

$$U_{DA} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over portion AB  $M = -\frac{Pav}{L}$

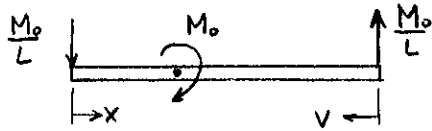
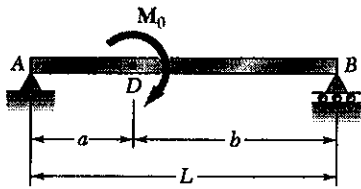
$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^L v^2 dv = \frac{Pa^2 L}{6EI}$$

Total  $U = U_{DA} + U_{AB} = \frac{P^2 a^2 (a+L)}{6EI}$

$\frac{1}{2} P S_D = U$        $S_D = \frac{2U}{P} = \frac{Pa^2 (a+L)}{3EI} \downarrow$

PROBLEM 11.73

11.73 Using the method of work-energy, determine the slope at point  $D$  caused by the couple  $M_0$ .



SOLUTION

Reactions  $R_A = \frac{M_0}{L} \downarrow$       $R_B = \frac{M_0}{L} \uparrow$

Over portion AD      $M = -\frac{M_0 x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EIL^2} \int_0^a x^2 dx$$

$$= \frac{M_0^2 a^3}{6EIL^2}$$

Over portion DB      $M = \frac{M_0 v}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EIL^2} \int_0^b v^2 dv$$

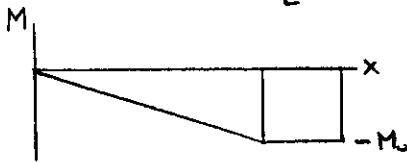
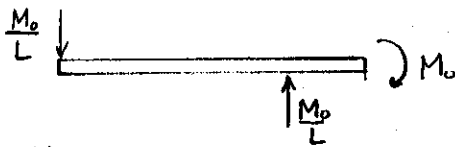
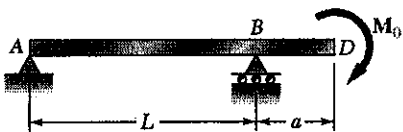
$$= \frac{M_0^2 b^3}{6EIL^2}$$

Total  $U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EIL^2}$

$\frac{1}{2} M_0 \theta_D = U$       $\theta_D = \frac{2U}{M_0} = \frac{M_0 (a^3 + b^3)}{3EIL^2}$

PROBLEM 11.74

11.74 Using the method of work-energy, determine the slope at point  $D$  caused by the couple  $M_0$ .



SOLUTION

Reactions  $R_A = \frac{M_0}{L} \downarrow$       $R_B = \frac{M_0}{L} \uparrow$

Over portion AB      $M = -\frac{M_0 x}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EIL^2} \int_0^L x^2 dx$$

$$= \frac{M_0^2 L}{6EI}$$

Over portion BD      $M = -M_0$

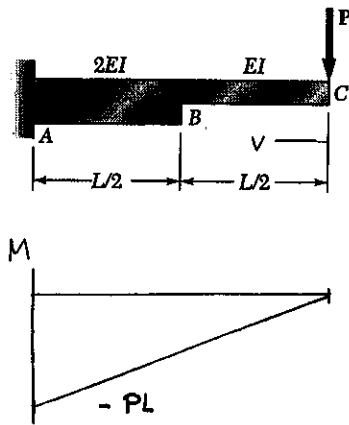
$$U_{BD} = \frac{M_0^2 a}{2EI}$$

Total  $U = U_{AB} + U_{BD} = \frac{M_0^2 (L + 3a)}{6EI}$

$\frac{1}{2} M_0 \theta_D = U$       $\theta_D = \frac{2U}{M_0} = \frac{M_0 (L + 3a)}{3EI}$

**PROBLEM 11.75**

11.75 Using the method of work and energy, determine the deflection at point C caused by the load P.



**SOLUTION**

Bending moment  $M = -Pv$

Over AB

$$U_{AB} = \int_{\frac{L}{2}}^L \frac{M^2}{4EI} dv = \frac{P^2}{4EI} \int_{\frac{L}{2}}^L v^2 dv$$

$$= \frac{P^2}{12EI} \left[ L^3 - \left(\frac{L}{2}\right)^3 \right] = \frac{7}{96} \frac{P^2 L^3}{EI}$$

Over BC  $U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^2 dv$

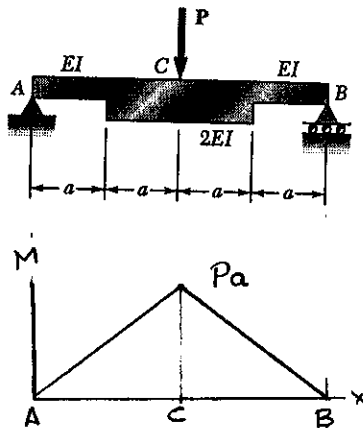
$$= \frac{1}{48} \frac{P^2 L^3}{EI}$$

Total  $U = U_{AB} + U_{BC} = \frac{3}{32} \frac{P^2 L^3}{EI}$

$\frac{1}{2} P \delta_c = 0 \quad \delta_c = \frac{2U}{P} = \frac{3}{16} \frac{PL^3}{EI} \downarrow$

**PROBLEM 11.76**

11.76 Using the method of work and energy, determine the deflection at point C caused by the load P.



**SOLUTION**

Symmetric beam and loading  $R_A = R_B = \frac{1}{2} P$

From A to C  $M = R_A x = \frac{1}{2} Px$

$$U_{AC} = \int_0^a \frac{M^2}{2EI} dx + \int_a^{2a} \frac{M^2}{4EI} dx$$

$$= \frac{P^2}{8EI} \int_0^a x^2 dx + \frac{P^2}{16EI} \int_a^{2a} x^2 dx$$

$$\frac{P^2 a^3}{24EI} + \frac{P^2}{48EI} [(2a)^3 - a^3] = \frac{3}{16} \frac{P^2 a^3}{EI}$$

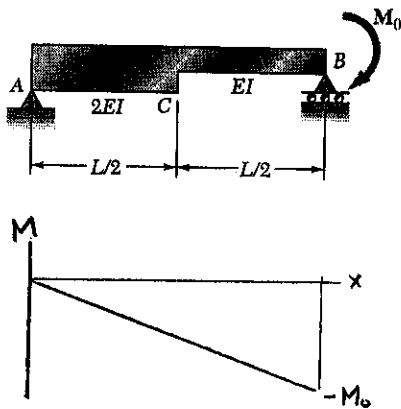
By symmetry  $U_{CB} = U_{AB} = \frac{3}{16} \frac{P^2 a^3}{EI}$

Total  $U = U_{AB} + U_{BC} = \frac{3}{8} \frac{P^2 a^3}{EI}$

$\frac{1}{2} P \delta_c = U \quad \delta_c = \frac{2U}{P} = \frac{3}{4} \frac{Pa^3}{EI}$

PROBLEM 11.77

11.77 Using the method of work and energy, determine the slope at point B caused by the couple  $M_0$ .



SOLUTION

$$\sum M_0 = 0 \quad -R_A L - M_0 = 0 \quad R_A = -\frac{M_0}{L}$$

$$M = R_A x = -\frac{M_0}{L} x$$

Over portion AC  $U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2(2EI)} dx$

$$U_{AC} = \frac{M_0^2}{4EIL^2} \int_0^{\frac{L}{2}} x^2 dx = \frac{1}{96} \frac{M_0^2 L}{EI}$$

Over portion CB  $U_{CB} = \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dx$

$$U_{CB} = \frac{M_0^2}{2EIL^2} \int_{\frac{L}{2}}^L x^2 dx = \frac{M_0^2}{6EIL^2} \left[ L^3 - \left(\frac{L}{2}\right)^3 \right]$$

$$= \frac{7}{48} \frac{M_0^2 L}{EI}$$

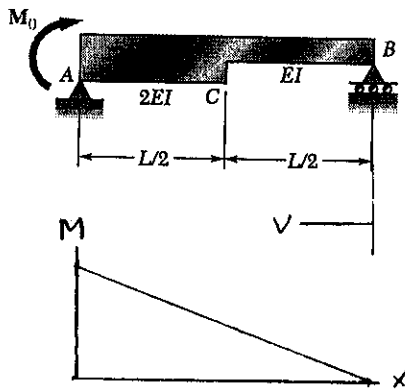
Total  $U = U_{AC} + U_{CB} = \frac{5}{32} \frac{M_0^2 L}{EI}$

$$\frac{1}{2} M_0 \theta_B = U$$

$$\theta_B = \frac{2U}{M_0} = \frac{5}{16} \frac{M_0 L}{EI}$$

PROBLEM 11.78

11.78 Using the method of work-energy, determine the slope at point A caused by the couple  $M_0$ .



SOLUTION

$$R_B = \frac{M_0}{L}$$

$$M = R_B v = \frac{M_0}{L} v$$

Over AC  $U_{AC} = \int_{\frac{L}{2}}^L \frac{M^2}{2(2EI)} dv$

$$U_{AC} = \frac{M_0^2}{4EIL^2} \int_{\frac{L}{2}}^L v^2 dv = \frac{M_0^2}{12EIL^2} \left[ L^3 - \left(\frac{L}{2}\right)^3 \right]$$

$$= \frac{7}{96} \frac{M_0^2 L}{EI}$$

Over CB  $U_{CB} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$

$$U_{CB} = \frac{M_0^2}{2EIL^2} \int_0^{\frac{L}{2}} v^2 dv = \frac{1}{48} \frac{M_0^2 L}{EI}$$

Total  $U = U_{AC} + U_{CB} = \frac{3}{32} \frac{M_0^2 L}{EI}$

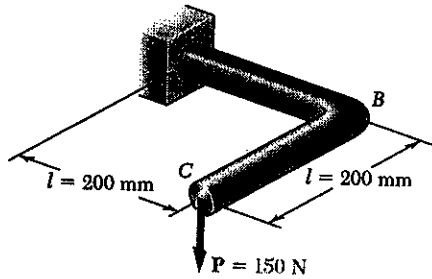
$$\frac{1}{2} M_0 \theta_A = U$$

$$\theta_A = \frac{2U}{M_0} = \frac{3}{16} \frac{M_0 L}{EI}$$



PROBLEM 11.79

11.79 The 12-mm-diameter steel rod ABC has been bent into the shape shown. Knowing that  $E = 200 \text{ GPa}$  and  $G = 77.2 \text{ GPa}$ , determine the deflection of end C caused by the 150-N force.



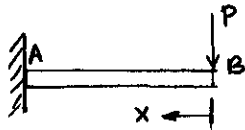
SOLUTION

$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} (12)^4 = 2.0358 \times 10^3 \text{ mm}^4 = 2.0358 \times 10^{-9} \text{ m}^4$$

$$I = \frac{1}{2} J = 1.0179 \times 10^{-9} \text{ m}^4$$

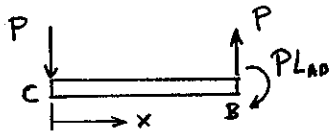
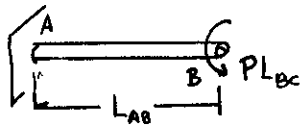
Portion AB: bending  $M = -Px$

$$\begin{aligned} U_{AB,b} &= \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{AB}} x^2 dx \\ &= \frac{P^2 L_{AB}^3}{6EI} = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} \\ &= 0.14736 \text{ J} \end{aligned}$$



torsion  $T = PL_{BC}$

$$\begin{aligned} U_{AB,t} &= \frac{T^2 L_{AB}}{2GJ} = \frac{P^2 L_{BC}^2 L_{AB}}{2GJ} \\ &= \frac{(150)^2 (200 \times 10^{-3})^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(2.0358 \times 10^{-9})} \\ &= 0.57265 \text{ J} \end{aligned}$$



Portion BC:  $M = -Px$

$$\begin{aligned} U_{BC} &= \int_0^{L_{BC}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{BC}} x^2 dx = \frac{P^2 L_{BC}^3}{6EI} \\ &= \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} = 0.14736 \text{ J} \end{aligned}$$

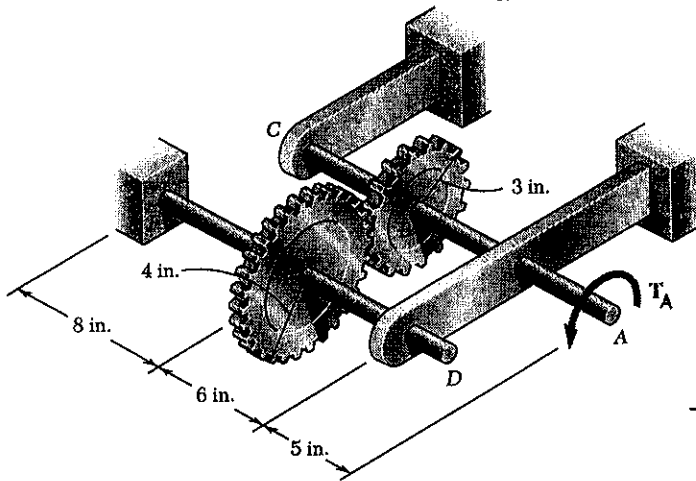
Total:  $U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737 \text{ J}$

Work-energy  $\frac{1}{2} P \delta = U$   $\delta = \frac{2U}{P} = \frac{(2)(0.86737)}{150}$

$= 11.57 \times 10^{-3} \text{ m} = 11.57 \text{ mm} \downarrow$

PROBLEM 11.80

11.80 Two steel shafts, each of 0.75-in. diameter, are connected by the gears shown. Knowing that  $G = 11.2 \times 10^6$  psi and that shaft  $DF$  is fixed at  $F$ , determine the angle through which end  $A$  rotates when a 750-lb-in. torque is applied at  $A$ . (Ignore the strain energy due to the bending of the shafts)



SOLUTION

Work - energy equation

$$\frac{1}{2} T_A \phi_A = U$$

$$\phi_A = \frac{2U}{T_A}$$

Portion AB of shaft ABC:

$$T_{AB} = T_A = 750 \text{ lb}\cdot\text{in}$$

$$L_{AB} = 5 + 6 = 11 \text{ in}$$

$$J_{AB} = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} \left(\frac{0.75}{2}\right)^4 = 31.063 \times 10^{-3} \text{ in}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2 G J_{AB}} = \frac{(750)^2 (11)}{(2)(11.2 \times 10^6)(31.063 \times 10^{-3})} = 8.892 \text{ in}\cdot\text{lb}$$

Portion BC of shaft ABC:  $U_{BC} = 0$

Gear B  $F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{750}{3} = 250 \text{ lb}$

Gear E  $T_E = r_E F_{BE} = (4)(250) = 1000 \text{ lb}\cdot\text{in}$

Portion DE of shaft DEF.  $U_{DE} = 0$

Portion EF of shaft DEF:  $T_{EF} = T_E = 1000 \text{ lb}\cdot\text{in}$

$$L_{EF} = 8 \text{ in} \quad J_{EF} = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = 31.063 \times 10^{-3} \text{ in}^4$$

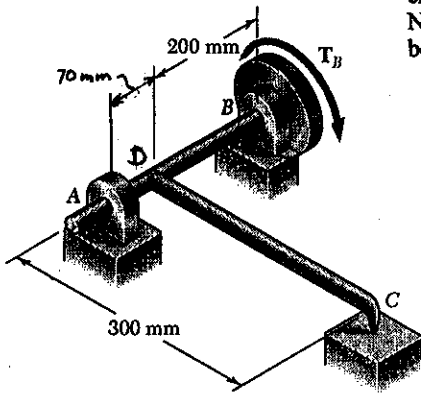
$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2 G J_{EF}} = \frac{(1000)^2 (8)}{(2)(11.2 \times 10^6)(31.063 \times 10^{-3})} = 11.497 \text{ lb}\cdot\text{in}$$

Total:  $U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 20.389 \text{ in}\cdot\text{lb}$

$$\phi_A = \frac{2U}{T_A} = \frac{(2)(20.389)}{750} = 54.4 \times 10^{-3} \text{ rad} = 3.12^\circ$$

PROBLEM 11.81

11.81 The 20-mm-diameter steel rod CD is welded to the 20-mm-diameter steel shaft AB as shown. End C of rod CD is touching the rigid surface shown when a couple  $T_B$  is applied to a disk attached to shaft AB. Knowing that the bearings are self-aligning and exert no couples on the shaft, determine the angle of rotation of the disk when  $T_B = 400$  N·m. Use  $E = 200$  GPa and  $G = 77.2$  GPa. (Consider the strain energy due to both bending and twisting in shaft AB and to bending in arm CD.)



SOLUTION

$$\ominus \sum M_{AB} = 0 \quad Y_{CD} F_C = T_B \quad F_C = \frac{T_B}{Y_{CD}}$$

$$F_C = \frac{400}{300 \times 10^{-3}} = 1333.3 \text{ N}, \quad F_D = 1333.3 \text{ N}$$

Bending of rod CD:

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{20}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$M = F_C x$$

$$U = \int_0^{L_{CD}} \frac{(F_C x)^2}{2EI} dx = \frac{F_C^2 L_{CD}^3}{6EI}$$

$$= \frac{(1333.3)^2 (300 \times 10^{-3})^3}{(6)(200 \times 10^9)(7.854 \times 10^{-9})} = 5.093 \text{ J}$$

Bending of shaft ADB

$$\ominus \sum M_B = 0 \quad -F_A L_{AB} + F_D b = 0 \quad F_A = \frac{F_D b}{L_{AB}}$$

$$\oplus \sum M_A = 0 \quad +F_A L_{AB} - F_D a = 0 \quad F_A = \frac{F_D a}{L_{AB}}$$

$$U = \frac{1}{2EI} \left\{ \int_0^a \left(\frac{F_D b}{L_{AB}}\right)^2 dx + \int_0^b \left(\frac{F_D a}{L_{AB}}\right)^2 dx \right\} = \frac{F_D^2 a^2 b^2}{6EI L_{AB}}$$

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = (270 \times 10^{-3}) \text{ m}$$

$$U = \frac{(1333.3)^2 (70 \times 10^{-3})^2 (200 \times 10^{-3})^2}{(6)(200 \times 10^9)(7.854 \times 10^{-9})(270 \times 10^{-3})} = 0.137 \text{ J}$$

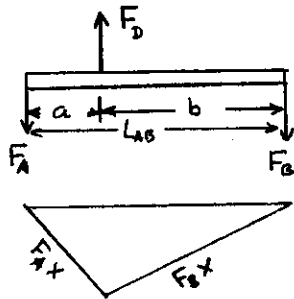
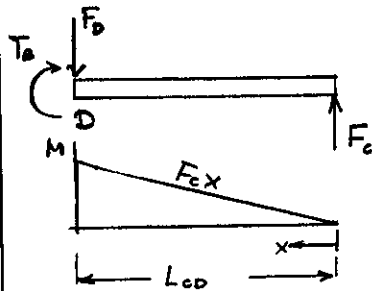
Torsion: Only portion DB carries torque.  $J = 2J = 15.708 \times 10^{-9} \text{ m}^4$

$$U = \frac{T_B^2 L_{DB}}{2GJ} = \frac{(400)^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 13.194 \text{ J}$$

Total:  $U = 5.093 + 0.137 + 13.194 = 18.424 \text{ J}$

$$\frac{1}{2} T_B \phi_B = U$$

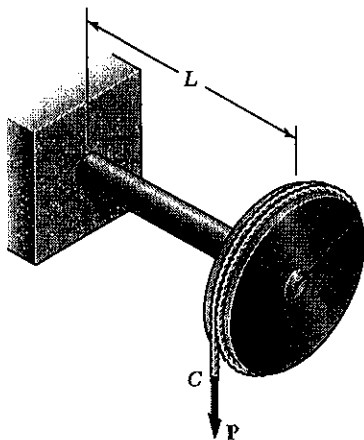
$$\phi_B = \frac{2U}{T_B} = \frac{2(18.424)}{400} = 92.1 \times 10^{-3} \text{ rad}$$



PROBLEM 11.82

11.82 A disk of radius  $a$  has been welded to end  $B$  of the solid steel shaft  $AB$ . A cable is then wrapped around the disk and a vertical force  $P$  is applied to end  $C$  of the cable. Knowing that the radius of the shaft is  $r$  and neglecting the deformations of the disk and of the cable, show that the deflection of point  $C$  caused by the application of  $P$  is

$$\delta_C = \frac{PL^3}{3EI} \left( 1 + 15 \frac{Ea^2}{GL^2} \right)$$



SOLUTION

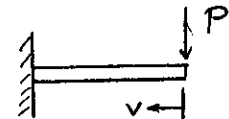
Torsion:  $T = Pa$

$$U_t = \frac{T^2 L}{2GJ} = \frac{P^2 a^2 L}{2GJ}$$

Bending:  $M = Pv$

$$U_b = \int_0^L \frac{M^2 dv}{2EI} = \int_0^L \frac{P^2 v^2 dv}{2EI}$$

$$= \frac{P^2 L^3}{6EI}$$



Total  $U = \frac{P^2 a^2 L}{2GJ} + \frac{P^2 L^3}{6EI} = \frac{1}{2} P \delta_C$

$$\delta_C = \frac{Pa^2 L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI} \left( 1 + \frac{3EIa^2}{GJL^2} \right)$$

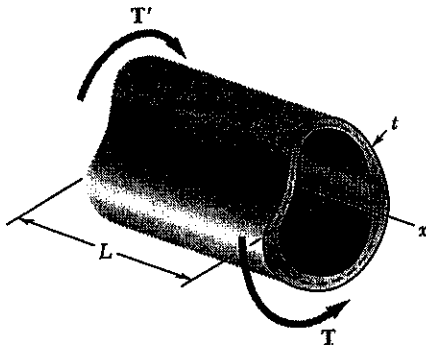
Since  $J = 2I$   $\delta_C = \frac{PL^3}{3EI} \left( 1 + \frac{3}{2} \frac{Ea^2}{GL^2} \right)$

PROBLEM 11.83

11.83 The thin-walled hollow cylindrical member  $AB$  has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.53) of Sec. 3.13, and the expression for the strain-energy density given in Eq. (11.19) of Sec. 11.4, show that the angle of twist of member  $AB$  is

$$\phi = \frac{TL}{4A^2G} \oint \frac{ds}{t}$$

where  $ds$  is an element of the centerline of wall cross section and  $A$  is the area enclosed by that centerline.



SOLUTION

From equation (3.53)  $\gamma = \frac{T}{2tA}$

Strain energy density

$$U = \frac{\gamma^2}{2G} = \frac{T^2}{8Gt^2A^2}$$

$$U = \int_0^L \oint U t ds dx$$

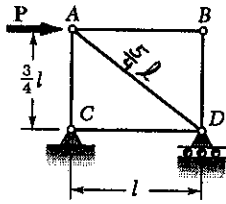
$$= \int_0^L \frac{T^2}{8GA^2} \oint \frac{ds}{t} dx = \frac{T^2L}{8GA^2} \oint \frac{ds}{t}$$

Work of torque =  $\frac{1}{2}T\phi = \frac{T^2L}{8GA^2} \oint \frac{ds}{t}$

$$\phi = \frac{TL}{4GA^2} \oint \frac{ds}{t}$$

**PROBLEM 11.84**

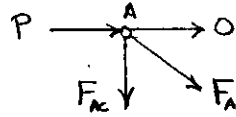
11.84 Each member of the truss shown has a uniform cross-sectional area  $A$ . Using the method of work and energy, determine the horizontal deflection of the point of application of the load  $P$ .



**SOLUTION**

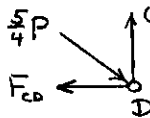
Members AB and BD are zero force members.

Joint A



$$\begin{aligned}
 +\rightarrow \Sigma F_x &= 0 \\
 \frac{4}{5} F_{AD} + P &= 0 & F_{AD} &= -\frac{5}{4} P \\
 +\uparrow \Sigma F_y &= 0 \\
 -F_{AC} - \frac{3}{5} F_{AD} &= 0 & F_{AC} &= \frac{3}{4} P
 \end{aligned}$$

Joint D



$$\begin{aligned}
 +\rightarrow \Sigma F_x &= 0 \\
 \frac{4}{5} \cdot \frac{5}{4} P - F_{CD} &= 0 \\
 F_{CD} &= P
 \end{aligned}$$

Member	F	L	F <sup>2</sup> L
AB	0	l	0
BD	0	3/4 l	0
AD	-5/4 P	5/4 l	125/64 P <sup>2</sup> l
CD	P	l	P <sup>2</sup> l
AC	3/4 P	3/4 l	27/64 P <sup>2</sup> l
$\Sigma$			27/8 P <sup>2</sup> l

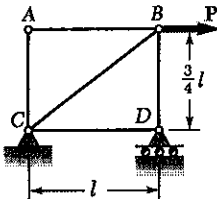
$$\begin{aligned}
 U &= \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L \\
 &= \frac{27}{16} \frac{P^2 l}{EA}
 \end{aligned}$$

Work of  $P = \frac{1}{2} P \Delta = U$

$$\Delta = \frac{2U}{P} = \frac{27}{8} \frac{Pl}{EA} = 3.375 \frac{Pl}{EA}$$

**PROBLEM 11.85**

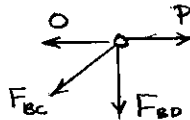
11.85 Each member of the truss shown has a uniform cross-sectional area  $A$ . Using the method of work and energy, determine the horizontal deflection of the point of application of the load  $P$ .



**SOLUTION**

Members AB, AC, and CD are zero force members.

Joint B



$$\begin{aligned}
 +\rightarrow \Sigma F_x &= 0 \\
 P - \frac{4}{5} F_{BC} &= 0 & F_{BC} &= \frac{5}{4} P \\
 +\uparrow \Sigma F_y &= 0 \\
 -F_{BD} - \frac{3}{5} F_{BC} &= 0 & F_{BD} &= -\frac{3}{4} P
 \end{aligned}$$

$$\begin{aligned}
 U &= \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L \\
 &= \frac{19}{16} \frac{P^2 l}{EA}
 \end{aligned}$$

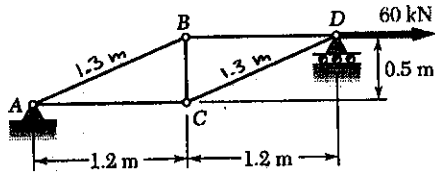
Member	F	L	F <sup>2</sup> L
AB	0	l	0
AC	0	3/4 l	0
CD	0	l	0
BC	5/4 P	5/4 l	125/64 P <sup>2</sup> l
BD	-3/4 P	3/4 l	27/64 P <sup>2</sup> l
$\Sigma$			19/8 P <sup>2</sup> l

Work of  $P = \frac{1}{2} P \Delta = U$

$$\Delta = \frac{2U}{P} = \frac{19}{8} \frac{Pl}{EA} = 2.375 \frac{Pl}{EA}$$

PROBLEM 11.86

11.86 Each member of the truss shown is made of steel; the cross-sectional area of member BC is 800 mm<sup>2</sup> and for all other members the cross-sectional area is 400 mm<sup>2</sup>. Using E = 200 GPa, determine the deflection of point D caused by the 60-kN load shown.



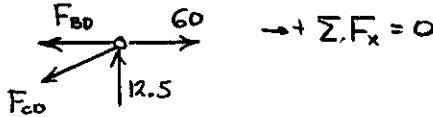
SOLUTION

Entire truss  $\circlearrowleft \sum M_A = 0$

$$2.4 R_D - (0.5)(60) = 0 \quad R_D = 12.5 \text{ kN}$$

Joint D  $\uparrow \sum F_y = 0$

$$12.5 - \frac{0.5}{1.3} F_{CD} = 0 \quad F_{CD} = 32.5 \text{ kN}$$

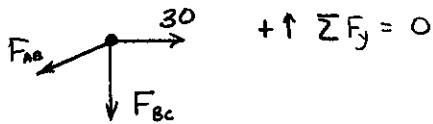


$\rightarrow \sum F_x = 0$

$$60 - F_{BD} - \frac{1.2}{1.3} F_{CD} = 0 \quad F_{BD} = 30 \text{ kN}$$

Joint B  $\rightarrow \sum F_x = 0$

$$30 - \frac{1.2}{1.3} F_{AB} = 0 \quad F_{AB} = 32.5 \text{ kN}$$

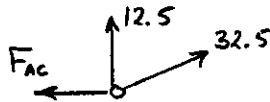


$\uparrow \sum F_y = 0$

$$-\frac{0.5}{1.3} F_{AB} + F_{BC} = 0 \quad F_{BC} = 12.5 \text{ kN}$$

Joint C  $\rightarrow \sum F_x = 0$

$$-F_{AC} + \frac{1.2}{1.3}(32.5) = 0 \quad F_{AC} = 30 \text{ kN}$$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10 <sup>6</sup> m <sup>2</sup> )	F <sup>2</sup> L/A (N <sup>2</sup> /m)
CD	32.5	1.3	400	3.4328 × 10 <sup>12</sup>
BD	30	1.2	400	2.7 × 10 <sup>12</sup>
AB	32.5	1.3	400	3.4328 × 10 <sup>12</sup>
BC	12.5	0.5	800	0.0977 × 10 <sup>12</sup>
AC	30	1.2	400	2.7 × 10 <sup>12</sup>
				12.3633 × 10 <sup>12</sup>

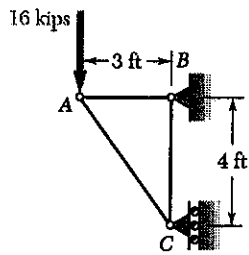
$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^9)} = 30.908 \text{ J}$$

$\frac{1}{2} P \Delta = U$

$$\Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^3} = 1.030 \times 10^{-3} \text{ m} = 1.030 \text{ mm} \rightarrow$$

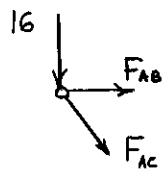
**PROBLEM 11.87**

11.87 Each member of the truss shown is made of steel and has a uniform cross-sectional area of  $3 \text{ in}^2$ . Using  $E = 29 \times 10^6 \text{ psi}$ , determine the vertical deflection of the point of application of joint  $A$  caused by the 16-kip load.



**SOLUTION**

Joint A



$$+\uparrow \Sigma F_y = 0$$

$$-16 - \frac{4}{3} F_{AC} = 0$$

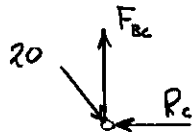
$$F_{AC} = -20 \text{ kips}$$

$$+\rightarrow \Sigma F_x = 0$$

$$\frac{3}{5} F_{AC} + F_{AB} = 0$$

$$F_{AB} = 12 \text{ kips}$$

Joint C



$$+\uparrow \Sigma F_y = 0$$

$$F_B - \frac{4}{3}(20) = 0$$

$$F_{BC} = 16 \text{ kips}$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \Sigma F^2 L$$

$$E = 29 \times 10^3 \text{ ksi}$$

$$A = 3 \text{ in}^2$$

Member	F (kips)	L (in)	F <sup>2</sup> L (kip <sup>2</sup> ·in)
AB	12	36	5184
AC	-20	60	24000
BC	16	48	12288
			41472

$$U = \frac{41472}{(2)(29 \times 10^3)(3)} = 0.23834 \text{ kip} \cdot \text{in.}$$

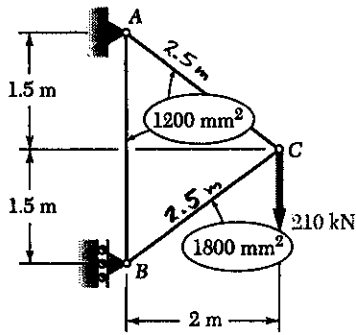
$$\frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{(2)(0.23834)}{16} = 0.0298 \text{ in.} \downarrow$$



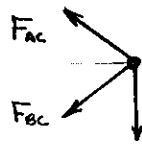
**PROBLEM 11.88**

11.88 Members of the truss shown are made of steel and have the cross-sectional areas shown. Using  $E = 200 \text{ GPa}$ , determine the vertical deflection of joint C caused by the application of the 210-kN load.



**SOLUTION**

Joint C



$$+\rightarrow \sum F_x = 0$$

$$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} = 0$$

$$+\uparrow \sum F_y = 0$$

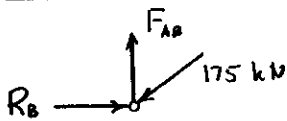
$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - 210 = 0$$

Solving simultaneously

$$F_{AC} = 175 \text{ kN}$$

$$F_{BC} = -175 \text{ kN}$$

Joint B



$$+\uparrow \sum F_y = 0$$

$$F_{AB} - \left(\frac{3}{5}\right)(175) = 0$$

$$F_{AB} = 105 \text{ kN}$$

$$U_m = \sum \frac{F^2 L}{2EA}$$

Member	F (kN)	L (m)	A ( $10^{-6} \text{ m}^2$ )	$F^2 L / A$ ( $\text{N}^2/\text{m}$ )
AB	105	3.0	1200	$27.5625 \times 10^{12}$
AC	175	2.5	1200	$63.8021 \times 10^{12}$
BC	-175	2.5	1800	$42.5347 \times 10^{12}$
				$133.8993 \times 10^{12}$

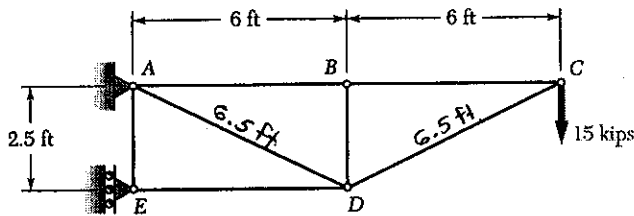
$$U_m = \frac{1}{2E} \sum \frac{F^2 L}{A} = \frac{133.8993 \times 10^{12}}{(2)(200 \times 10^9)} = 334.75 \text{ J}$$

$$\frac{1}{2} P_n \Delta_n = U_m$$

$$\Delta_n = \frac{2U_m}{P_n} = \frac{(2)(334.75)}{210 \times 10^3} = 3.19 \times 10^{-3} \text{ m} = 3.19 \text{ mm}$$

PROBLEM 11.89

11.89. Each member of the truss shown is made of steel and has a uniform cross-sectional area of 5 in<sup>2</sup>. Using  $E = 29 \times 10^6$  psi, determine the vertical deflection of the point of application of joint C caused by the 15-kip load.



SOLUTION

Members BD and AE are zero force members.

For entire truss  $\sum M_A = 0$

$$2.5 R_D - (12)(15) = 0$$

$$R_D = 72 \text{ kips}$$

For equilibrium of joint E

$$F_{ED} = -R_D = -72 \text{ kips}$$

Joint C

$$+\uparrow \sum F_y = 0$$

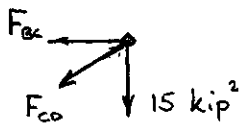
$$-\frac{2.5}{6.5} F_{CD} - 15 = 0$$

$$F_{CD} = -39 \text{ kips}$$

$$+\rightarrow \sum F_x = 0$$

$$-\frac{6}{6.5} F_{CD} - F_{BC} = 0$$

$$F_{BC} = 36 \text{ kips}$$



Joint D

$$+\rightarrow \sum F_x = 0$$

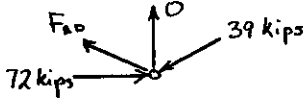
$$72 - \frac{6}{6.5} (F_{AD} + 39) = 0$$

$$F_{AD} = 39 \text{ kips}$$

Joint B  $\sum F_x = 0$

$$-F_{AB} + F_{BC} = 0$$

$$F_{AB} = 36 \text{ kips}$$



Strain energy  $U_m = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$

Member	F (kips)	L (in)	F <sup>2</sup> L (kip <sup>2</sup> ·in)
AB	36	72	93312
BC	36	72	93312
CD	-39	78	118638
DE	-72	72	373248
BD	0	30	0
AE	0	30	0
AD	39	78	118638
$\Sigma$			797148

Data:  $E = 29 \times 10^3$  ksi  
 $A = 5 \text{ in}^2$

$$U_m = \frac{797148}{(2)(29 \times 10^3)(5)}$$

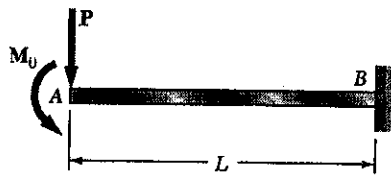
$$= 2.7488 \text{ kip}\cdot\text{in}$$

$$\frac{1}{2} P_m \Delta_m = U$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(2.7488)}{15} = 0.366 \text{ in. } \downarrow$$

PROBLEM 11.90

11.90 Using the information provided in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load  $P$  is applied first, (b) if the couple  $M_0$  is applied first.



SOLUTION

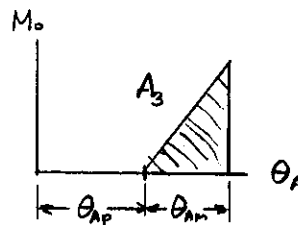
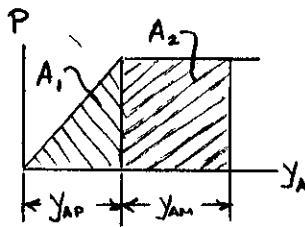
From Appendix D, Case 1

$$y_{AP} = \frac{PL^3}{3EI} \quad \theta_{AP} = \frac{PL^2}{2EI}$$

From Appendix D, Case 3

$$y_{AM} = \frac{M_0L^2}{2EI} \quad \theta_{AM} = \frac{M_0L}{EI}$$

(a) First  $P$ , then  $M_0$ .

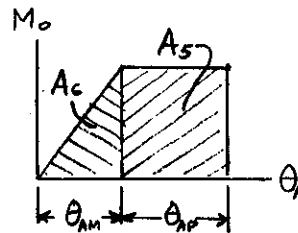
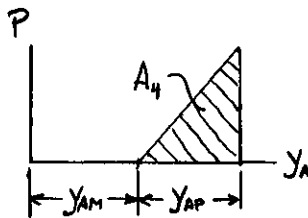


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{AP} + P y_{AM} + \frac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2 L^3}{6EI} + \frac{P M_0 L^2}{2EI} + \frac{M_0^2 L}{2EI}$$

(b) First  $M_0$ , then  $P$



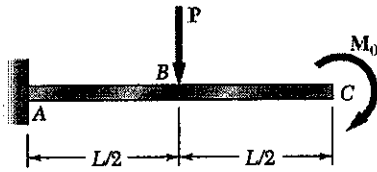
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2} P y_{AP} + M_0 \theta_{AP} + \frac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2 L^3}{6EI} + \frac{M_0 P L^2}{2EI} + \frac{M_0^2 L}{2EI}$$

**PROBLEM 11.91**

11.91 Using the information provided in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load  $P$  is applied first, (b) if the couple  $M_0$  is applied first.



**SOLUTION**

Appendix D Cases 1 and 3

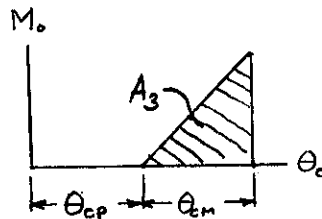
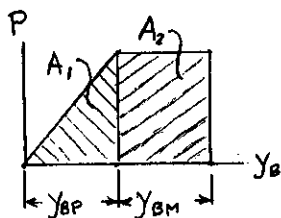
$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0L^2}{8EI}$$

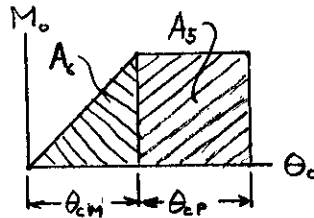
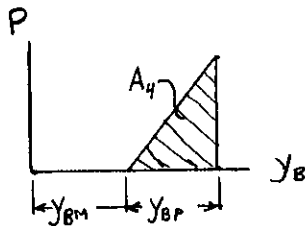
$$\theta_{BM} = \frac{M_0L}{EI}$$

(a) First  $P$ , then  $M_0$



$$\begin{aligned} U &= A_1 + A_2 + A_3 \\ &= \frac{1}{2} P y_{BP} + P y_{BM} + \frac{1}{2} M_0 \theta_{CM} \\ &= \frac{P^2 L^3}{48EI} + \frac{P M_0 L^2}{8EI} + \frac{M_0^2 L}{2EI} \end{aligned}$$

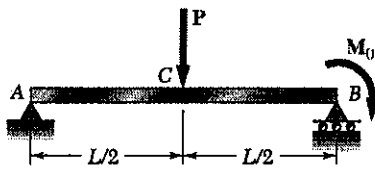
(b) First  $M_0$ , then  $P$



$$\begin{aligned} U &= A_4 + A_5 + A_6 \\ &= \frac{1}{2} P y_{BP} + M_0 \theta_{CP} + \frac{1}{2} M_0 \theta_{CM} \\ &= \frac{P^2 L^3}{48EI} + \frac{M_0 P L^2}{8EI} + \frac{M_0^2 L}{2EI} \end{aligned}$$

**PROBLEM 11.92**

11.92 Using the information provided in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load  $P$  is applied first, (b) if the couple  $M_0$  is applied first.



**SOLUTION**

From Appendix D, Case 4

$$\downarrow y_c = \frac{PL^3}{48EI} \quad \curvearrowleft \theta_B = -\frac{PL^2}{16EI}$$

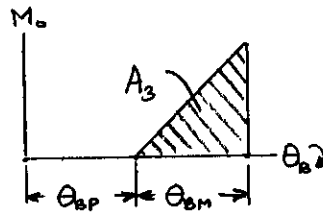
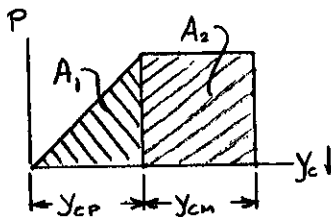
From Appendix D, Case 7

$$\downarrow y_c = \frac{M_0}{6EIL} \left( (L/2)^3 - L^2(L/2) \right) = -\frac{M_0 L^2}{16EI}$$

$$\curvearrowright \theta_B = \frac{M_0 L}{3EI}$$

(a) First  $P$ , then  $M_0$

[www.konkur.in](http://www.konkur.in)

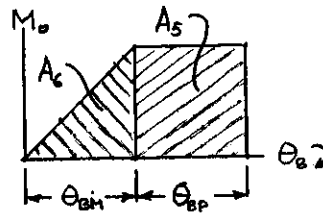
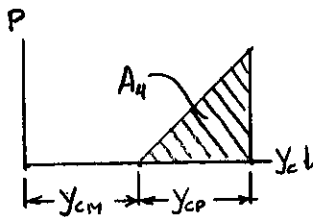


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{cp} + P y_{cn} + \frac{1}{2} M_0 \theta_{BM}$$

$$= \frac{P^2 L^3}{96EI} - \frac{P M_0 L^2}{16EI} + \frac{M_0^2 L}{6EI}$$

(b) First  $M_0$ , then  $P$



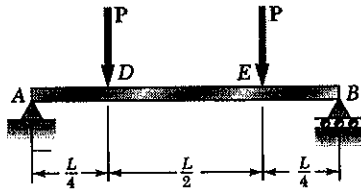
$$U = A_4 + A_5 + A_c$$

$$= \frac{1}{2} P y_{cp} + M_0 \theta_{BP} + \frac{1}{2} M_0 \theta_{BM}$$

$$= \frac{P^2 L^3}{96EI} - \frac{M_0 P L^2}{16EI} + \frac{M_0^2 L}{6EI}$$

PROBLEM 11.93

11.93 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



SOLUTION

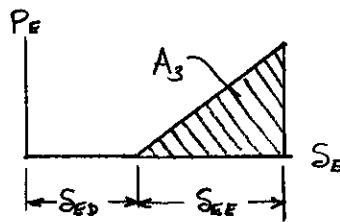
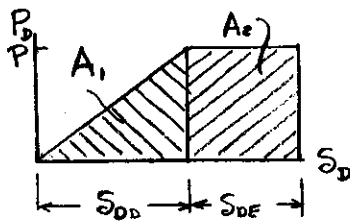
(a) Label the forces  $P_D$  and  $P_E$ .

Using Appendix D Case 5

$$S_{EE} = \frac{P_E a^2 b^2}{3EI L} = \frac{P_E \left(\frac{3L}{4}\right)^2 \left(\frac{L}{4}\right)^2}{3EI L} = \frac{3}{256} \frac{P_E L^3}{EI}$$

$$S_{DE} = \frac{P_E b}{6EI L} \left[ (L^2 - b^2)x - x^3 \right] = \frac{P_E \left(\frac{L}{4}\right)}{6EI L} \left[ \left(L^2 - \left(\frac{L}{4}\right)^2\right)\left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^3 \right] = \frac{7}{768} \frac{P_E L^3}{EI}$$

Likewise  $S_{DD} = \frac{3}{256} \frac{P_D L^3}{EI}$  and  $S_{ED} = \frac{7}{768} \frac{P_D L^3}{EI}$



Let  $P_D$  be applied first.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_D S_{DD} + P_D S_{DE} + \frac{1}{2} P_E S_{EE} = \frac{3}{512} \frac{P_D^2 L^3}{EI} + \frac{7}{768} \frac{P_D P_E L^3}{EI} + \frac{3}{512} \frac{P_E^2 L^3}{EI}$$

With  $P_D = P_E = P$  
$$U = \frac{1}{48} \frac{P^2 L^3}{EI}$$

(b) Reactions  $R_A = R_B = P$

Over portion AD  $0 < x < \frac{L}{4}$   $M = Px$

$$U_{AD} = \int_0^{\frac{L}{4}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{\frac{L}{4}} x^2 dx = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{4}\right)^3 = \frac{1}{384} \frac{P^2 L^3}{EI}$$

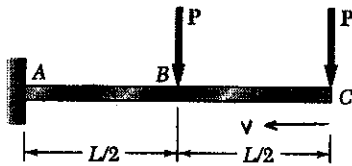
Over portion DE  $M = \frac{PL}{4}$  
$$U_{DE} = \frac{M^2 \left(\frac{L}{2}\right)}{2EI} + \frac{P^2 L^2}{2EI} \cdot \frac{1}{16} \cdot \frac{L}{2} = \frac{P^2 L^3}{64 EI}$$

Over portion EB: By symmetry 
$$U_{EB} = U_{AD} = \frac{1}{384} \frac{P^2 L^3}{EI}$$

Total 
$$U = U_{AD} + U_{DE} + U_{EB} = \left(\frac{1}{384} + \frac{1}{64} + \frac{1}{384}\right) \frac{P^2 L^3}{EI} = \frac{1}{48} \frac{P^2 L^3}{EI}$$

PROBLEM 11.94

11.94 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



SOLUTION

(a) Label the forces  $P_B$  and  $P_C$

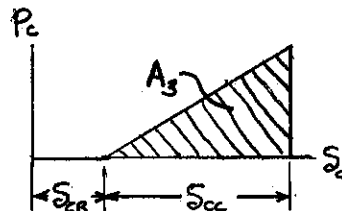
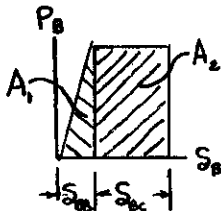
Using Appendix D Case 1

$$S_{BB} = \frac{P_B(L/2)^3}{3EI} = \frac{1}{24} \frac{P_B L^3}{EI}$$

$$S_{CB} = S_{BB} + \frac{1}{2} \theta_B = \frac{1}{24} \frac{P_B L^3}{EI} + \frac{1}{2} \frac{P_B(L/2)^2}{2EI} = \left(\frac{1}{24} + \frac{1}{16}\right) \frac{P_B L^3}{EI} = \frac{5}{48} \frac{P_B L^3}{EI}$$

$$S_{CC} = \frac{1}{3} \frac{P_C L^3}{EI}$$

$$S_{BC} = \frac{P_C}{6EI} (3Lx^2 - x^3) = \frac{P_C}{6EI} \left(3L\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3\right) = \frac{5}{48} \frac{P_C L^3}{EI}$$



Apply  $P_B$  first, then  $P_C$

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_B S_{BB} + P_B S_{BC} + \frac{1}{2} P_C S_{CC} = \frac{1}{48} \frac{P_B L^3}{EI} + \frac{5}{48} \frac{P_B P_C L^3}{EI} + \frac{1}{6} \frac{P_C^2 L^3}{EI}$$

$$\text{With } P_B = P_C = P \quad U = \left(\frac{1}{48} + \frac{5}{48} + \frac{1}{6}\right) \frac{P^2 L^3}{EI} = \frac{7}{24} \frac{P^2 L^3}{EI}$$

$$\text{Over AB} \quad M = Pv + P\left(v - \frac{L}{2}\right) = P\left(2v - \frac{L}{2}\right)$$

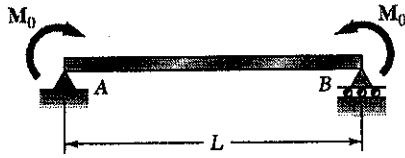
$$\begin{aligned} U_{AB} &= \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_{\frac{L}{2}}^L \left(4v^2 - 2Lv + \frac{1}{4}L^2\right) dv \\ &= \frac{P^2}{2EI} \left\{ \frac{4}{3} \left[L^3 - \left(\frac{L}{2}\right)^3\right] - 2L \cdot \frac{1}{2} \left[L^2 - \left(\frac{L}{2}\right)^2\right] + \frac{1}{4} L^2 \left[L - \frac{L}{2}\right] \right\} \\ &= \frac{P^2}{2EI} \left\{ \frac{7}{6} L^3 - \frac{3}{4} L^3 + \frac{1}{8} L^3 \right\} = \frac{13}{48} \frac{P^2 L^3}{EI} \end{aligned}$$

$$\begin{aligned} \text{Over BC} \quad M &= Pv \quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^2 dv = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{2}\right)^3 \\ &= \frac{P^2 L^3}{48 EI} \end{aligned}$$

$$\text{Total} \quad U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48}\right) \frac{P^2 L^3}{EI} = \frac{7}{24} \frac{P^2 L^3}{EI}$$

**PROBLEM 11.95**

11.95 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.

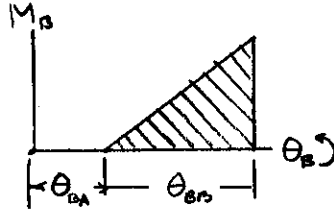
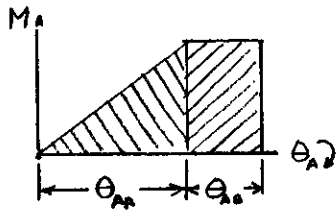


**SOLUTION**

(a) Label the couples  $M_A$  and  $M_B$

Using Appendix D, Case 7

$$C \theta_{AA} = \frac{M_A L}{3EI} \quad C \theta_{BA} = \frac{M_A L}{6EI} \quad C \theta_{BB} = \frac{M_B L}{3EI} \quad C \theta_{AB} = \frac{M_B L}{6EI}$$



Apply  $M_A$  first, then  $M_B$ .

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} M_A \theta_{AA} + M_A \theta_{AB} + \frac{1}{2} M_B \theta_{BB} = \frac{1}{6} \frac{M_A^2 L}{EI} + \frac{1}{6} \frac{M_A M_B L}{EI} + \frac{1}{6} \frac{M_B^2 L}{EI}$$

With  $M_A = M_B = M_0$        $U = \frac{1}{2} \frac{M_0^2 L}{EI}$

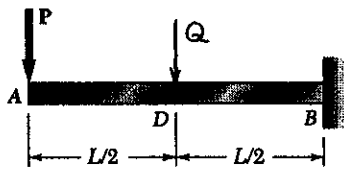
(b) Bending moment       $M = M_0$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2 L}{2EI}$$



**PROBLEM 11.96**

11.96 For the prismatic beam shown, determine the deflection at point D.



**SOLUTION**

Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD  $0 < x < \frac{L}{2}$   $M = -Px$ ,  $\frac{\partial M}{\partial Q} = 0$

Over portion DB  $\frac{L}{2} < x < L$   $M = -Px - Q(x - \frac{L}{2})$ ,  $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set  $Q = 0$

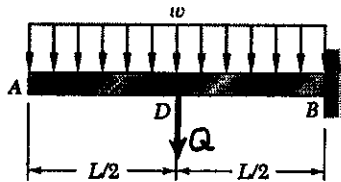
$$S_D = \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px) \left[ -(x - \frac{L}{2}) \right] dx$$

$$= \frac{P}{EI} \int_{\frac{L}{2}}^L (x^2 - \frac{L}{2}x) dx = \frac{P}{EI} \left\{ \frac{1}{3}L^3 - \frac{1}{3}\left(\frac{L}{2}\right)^3 - \left(\frac{L}{2}\right)\frac{1}{2}L^2 + \frac{L}{2}\frac{1}{2}\left(\frac{L}{2}\right)^2 \right\}$$

$$= \left( \frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16} \right) \frac{PL^3}{EI} = \frac{5}{48} \frac{PL^3}{EI} \blacktriangleleft$$

**PROBLEM 11.97**

11.97 For the prismatic beam shown, determine the deflection at point D.



**SOLUTION**

Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD  $0 < x < \frac{L}{2}$   $M = -\frac{1}{2}wx^2$   $\frac{\partial M}{\partial Q} = 0$

Over portion DB  $\frac{L}{2} < x < L$   $M = -\frac{1}{2}wx^2 - Q(x - \frac{L}{2})$   $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set  $Q = 0$

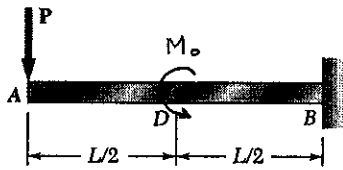
$$S_D = \frac{1}{EI} \int_0^{\frac{L}{2}} \left(-\frac{1}{2}wx^2\right)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L \left(-\frac{1}{2}wx^2\right) \left[ -(x - \frac{L}{2}) \right] dx$$

$$= \frac{w}{2EI} \int_{\frac{L}{2}}^L (x^3 - \frac{L}{2}x^2) dx = \frac{w}{2EI} \left\{ \frac{1}{4}L^4 - \frac{1}{4}\left(\frac{L}{2}\right)^4 - \left(\frac{L}{2}\right)\frac{1}{3}L^3 + \left(\frac{L}{2}\right)\frac{1}{3}\left(\frac{L}{2}\right)^3 \right\}$$

$$= \frac{1}{2} \left( \frac{1}{4} - \frac{1}{64} - \frac{1}{6} + \frac{1}{48} \right) \frac{wL^4}{EI} = \frac{17}{384} \frac{wL^4}{EI} = 0.04427 \frac{wL^4}{EI} \blacktriangleleft$$

**PROBLEM 11.98**

**11.98** For the prismatic beam shown, determine the slope at point  $D$ .



**SOLUTION**

Add couple  $M_0$  at point  $D$ .

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx$$

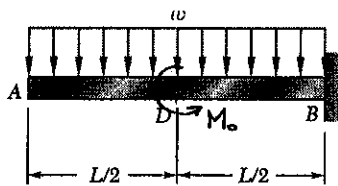
Over portion AD  $0 < x < \frac{L}{2}$   $M = -Px$   $\frac{\partial M}{\partial M_0} = 0$

Over portion DB  $\frac{L}{2} < x < L$   $M = -Px - M_0$   $\frac{\partial M}{\partial M_0} = -1$

Set  $M_0 = 0$ .  $\theta_D = \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px)(-1) dx + \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx$   
 $= \frac{P}{EI} \int_{\frac{L}{2}}^L x dx = \frac{P}{EI} \left[ \frac{1}{2} L^2 - \frac{1}{2} \left(\frac{L}{2}\right)^2 \right]$   
 $= \left(\frac{1}{2} - \frac{1}{8}\right) \frac{PL^2}{EI} = \frac{3}{8} \frac{PL^2}{EI} \triangleleft$

**PROBLEM 11.99**

**11.99** For the prismatic beam shown, determine the slope at point  $D$ .



**SOLUTION**

Add couple  $M_0$  at point  $D$ .

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx$$

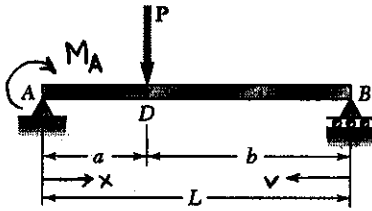
Over portion AD  $0 < x < \frac{L}{2}$   $M = -\frac{1}{2}wx^2$   $\frac{\partial M}{\partial M_0} = 0$

Over portion DB  $\frac{L}{2} < x < L$   $M = -\frac{1}{2}wx^2 - M_0$   $\frac{\partial M}{\partial M_0} = -1$

Set  $M_0 = 0$ .  $\theta_D = \frac{1}{EI} \int_{\frac{L}{2}}^L \left(-\frac{1}{2}wx^2\right)(-1) dx + \frac{1}{EI} \int_0^{\frac{L}{2}} \left(-\frac{1}{2}wx^2\right)(0) dx$   
 $= \frac{w}{2EI} \int_{\frac{L}{2}}^L x^2 dx = \frac{w}{2EI} \left[ \frac{1}{3} L^3 - \frac{1}{3} \left(\frac{L}{2}\right)^3 \right]$   
 $= \frac{1}{6} \left(1 - \frac{1}{8}\right) \frac{wL^3}{EI} = \frac{7}{48} \frac{wL^3}{EI} \triangleleft$

PROBLEM 11.100

11.100 and 11.101 For the prismatic beam shown, determine the slope at point A.



SOLUTION

Add couple  $M_A$  at point A.

Reactions:  $R_A = \frac{Pb}{L} - \frac{M_A}{L}$ ,  $R_B = \frac{Pa}{L} + \frac{M_A}{L}$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a M^2 dx + \frac{1}{2EI} \int_0^b M^2 dv$$

$$\delta \theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI} \int_0^a M \frac{\partial M}{\partial M_A} dx + \frac{1}{EI} \int_0^b M \frac{\partial M}{\partial M_A} dv$$

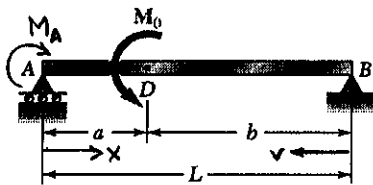
Over portion AD ( $0 < x < a$ )  $M = M_A + R_A x = M_A (1 - \frac{x}{L}) + \frac{Pbx}{L}$ ,  $\frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$

Over portion DB ( $0 < v < b$ )  $M = R_B v = \frac{Pav}{L} + \frac{M_A v}{L}$ ,  $\frac{\partial M}{\partial M_A} = \frac{v}{L}$

$$\begin{aligned} \text{Set } M_A = 0 \quad \theta_A &= \frac{1}{EI} \int_0^a \left(\frac{Pbx}{L}\right) \left(1 - \frac{x}{L}\right) dx + \frac{1}{EI} \int_0^b \left(\frac{Pav}{L}\right) \left(\frac{v}{L}\right) dv \\ &= \frac{P}{EIL^2} \left(\frac{1}{2} bLa^2 - \frac{1}{3} ba^3 + \frac{1}{3} ab^3\right) \\ &= \frac{Pab}{6EIL^2} (3La - 2a^2 + 2b^2) \quad \blacktriangleleft \end{aligned}$$

PROBLEM 11.101

11.100 and 11.101 For the prismatic beam shown, determine the slope at point A.



SOLUTION

Add couple  $M_A$  at point a

Reactions: Positive if upward

$$R_A = \frac{M_0 - M_A}{L}, \quad R_B = \frac{M_A - M_0}{L}$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a M^2 dx + \frac{1}{2EI} \int_0^b M^2 dv$$

$$\delta \theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI} \int_0^a M \frac{\partial M}{\partial M_A} dx + \frac{1}{EI} \int_0^b M \frac{\partial M}{\partial M_A} dv$$

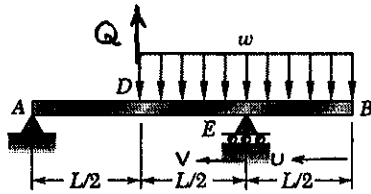
Over portion AD ( $0 < x < a$ )  $M = M_A + R_A x = M_A (1 - \frac{x}{L}) + \frac{M_0 x}{L}$ ,  $\frac{\partial M}{\partial M_A} = 1 - \frac{x}{L}$

Over portion DB ( $0 < v < b$ )  $M = R_B v = \frac{(M_A - M_0)v}{L}$ ,  $\frac{\partial M}{\partial M_A} = \frac{v}{L}$

$$\begin{aligned} \text{Set } M_A = 0 \quad \theta_A &= \frac{1}{EI} \int_0^a \left(\frac{M_0 x}{L}\right) \left(1 - \frac{x}{L}\right) dx + \frac{1}{EI} \int_0^b \left(\frac{M_0 v}{L}\right) \left(-\frac{v}{L}\right) dv \\ &= \frac{M_0}{EIL^2} \left(\frac{1}{2} La^2 - \frac{1}{3} a^3 - \frac{1}{3} b^3\right) \\ &= \frac{M_0}{6EIL^2} (3La^2 - 2a^3 - 2b^3) \quad \blacktriangleleft \end{aligned}$$

PROBLEM 11.102

11.102 For the prismatic beam shown, determine the deflection at point D.



SOLUTION

Add force Q at point D.

Reactions:  $R_A = -\frac{1}{2}Q$ ,  $R_B = wL - \frac{1}{2}Q$

$U = U_{AD} + U_{DE} + U_{EB}$ ;  $S_D = \frac{\partial U}{\partial Q}$

Over portion AD: with  $Q = 0$   $M = 0$   $\frac{\partial U_{AD}}{\partial Q} = 0$

Over portion DE:  $M = R_B v - \frac{1}{2} w (v + \frac{L}{2})^2 = wL v - \frac{1}{2} w (v + \frac{L}{2})^2 - \frac{1}{2} Q v$

$\frac{\partial M}{\partial Q} = -\frac{1}{2} v$   $U_{DE} = \frac{11}{2EI} \int_0^{\frac{L}{2}} M^2 dv$  Set  $Q = 0$

$\frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial Q} dv = \frac{1}{EI} \int_0^{\frac{L}{2}} [wL v - \frac{1}{2} w (v + \frac{L}{2})^2] (-\frac{1}{2} v) dv$

$= \frac{w}{2EI} \int_0^{\frac{L}{2}} [-Lv^2 + \frac{1}{2} (v^3 + Lv^2 + \frac{1}{4} L^2 v)] dv$

$= \frac{w}{2EI} [-L \cdot \frac{1}{3} (\frac{L}{2})^3 + \frac{1}{2} (\frac{1}{4} (\frac{L}{2})^4 + L \cdot \frac{1}{3} (\frac{L}{2})^3 + \frac{1}{4} L^2 \frac{1}{2} (\frac{L}{2})^2)]$

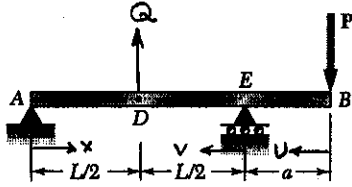
$= \frac{1}{2} (-\frac{1}{24} + \frac{1}{128} + \frac{1}{48} + \frac{1}{64}) \frac{wL^4}{EI} = \frac{1}{768} \frac{wL^4}{EI}$

Over portion EB:  $M = -\frac{1}{2} w u^2$   $\frac{\partial M}{\partial Q} = 0$   $\frac{\partial U_{EB}}{\partial Q} = 0$

$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = 0 + \frac{1}{768} \frac{wL^4}{EI} + 0 = \frac{1}{768} \frac{wL^4}{EI}$

PROBLEM 11.103

11.103 For the prismatic beam shown, determine the deflection at point D.



SOLUTION

Add force Q at point D.

Reactions  $R_A = -\frac{Pa}{L} - \frac{1}{2}Q$ ,  $R_E = \frac{P(a+L)}{L} - \frac{1}{2}Q$

$U = U_{AD} + U_{DE} + U_{EB}$  ;  $S_D = \frac{\partial U}{\partial Q}$

Over portion AD:  $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx$ ,  $M = R_A x = -\frac{Pa}{L}x - \frac{1}{2}Qx$ ,  $\frac{\partial M}{\partial Q} = -\frac{1}{2}x$

Set  $Q = 0$ .  $\frac{\partial U_{AD}}{\partial Q} = \frac{1}{EI} \int_0^{L/2} (-\frac{Pa}{L}x)(-\frac{1}{2}x) dx = \frac{Pa}{2EIL} \int_0^{L/2} x^2 dx$   
 $= \frac{Pa}{2EIL} \frac{1}{3}(\frac{L}{2})^3 = \frac{PaL^2}{48EI}$

Over portion DE:  $U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$   $\frac{\partial M}{\partial Q} = -\frac{1}{2}v$

$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{1}{2}Qv - P(a+v) = \frac{Pa}{L}v - Pa - \frac{1}{2}Qv$

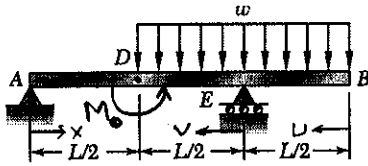
Set  $Q = 0$ .  $\frac{\partial U_{DE}}{\partial Q} = \frac{1}{EI} \int_0^{L/2} (\frac{Pa}{L}v - Pa)(-\frac{1}{2}v) dv = \frac{Pa}{2EIL} \int_0^{L/2} (-v^2 + Lv) dv$   
 $= \frac{Pa}{2EIL} [-\frac{1}{3}(\frac{L}{2})^3 + (L)\frac{1}{2}(\frac{L}{2})^2] = \frac{Pa}{2EIL} [-\frac{L^3}{24} + \frac{L^3}{8}]$   
 $= \frac{1}{24} \frac{PaL^2}{EI}$

Over portion EB  $M = -Pu$   $\frac{\partial M}{\partial Q} = 0$   $\frac{\partial U_{EB}}{\partial Q} = 0$

$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{PaL^2}{48EI} + \frac{PaL^2}{24EI} + 0 = \frac{1}{16} \frac{PaL^2}{EI}$

PROBLEM 11.104

11.104 For the prismatic beam shown, determine the slope at point D.



SOLUTION

Add couple  $M_0$  at point D.

Reactions:  $R_A = \frac{M_0}{L}$ ,  $R_E = wL - \frac{M_0}{L}$

$U = U_{AD} + U_{DE} + U_{EB}$ ,  $\theta_D = \frac{\partial U}{\partial M_0}$

Over portion AD:  $M = \frac{M_0}{L}x = 0$  with  $M_0 = 0$   $\frac{\partial U_{AD}}{\partial M_0} = 0$

Over portion DE:  $M = R_E v - \frac{1}{2}w(v + \frac{L}{2})^2 = wLv - \frac{1}{2}w(v + \frac{L}{2})^2 - \frac{M_0}{L}v$

$\frac{\partial M}{\partial M_0} = -\frac{1}{L}v$ ,  $U_{DE} = \frac{1}{2EI} \int_0^{\frac{L}{2}} M^2 dv$  Set  $M_0 = 0$

$\frac{\partial U_{DE}}{\partial M_0} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial M_0} dv = \frac{1}{EI} \int_0^{\frac{L}{2}} [wLv - \frac{1}{2}w(v + \frac{L}{2})^2] (-\frac{1}{L}v) dv$

$= \frac{w}{EIL} \int_0^{\frac{L}{2}} [-Lv^2 + \frac{1}{2}(v^3 + Lv^2 + \frac{1}{4}L^2v)] dv$

$= \frac{w}{EIL} [-L \cdot \frac{1}{3}(\frac{L}{2})^3 + \frac{1}{2}(\frac{1}{4}(\frac{L}{2})^4 + L \cdot \frac{1}{3}(\frac{L}{2})^3 + \frac{1}{4}L^2 \cdot \frac{1}{2}(\frac{L}{2})^2)]$

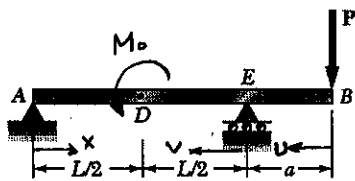
$= (-\frac{1}{24} + \frac{1}{128} + \frac{1}{48} + \frac{1}{64}) \frac{wL^3}{EI} = \frac{1}{384} \frac{wL^3}{EI}$

Over portion EB:  $M = -\frac{1}{2}wv^2$   $\frac{\partial M}{\partial M_0} = 0$   $\frac{\partial U_{EB}}{\partial M_0} = 0$

$S_D = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0} = 0 + \frac{1}{384} \frac{wL^3}{EI} + 0 = \frac{1}{384} \frac{wL^3}{EI}$

PROBLEM 11.105

11.105 For the prismatic beam shown, determine the slope at point D.



SOLUTION

Add couple  $M_o$  at point D.

Reactions:  $R_A = -\frac{Pa}{L} + \frac{M_o}{L}$ ,  $R_B = \frac{P(a+L)}{L} - \frac{M_o}{L}$

$U = U_{AD} + U_{DE} + U_{EB}$        $\theta_D = \frac{\partial U}{\partial M_o}$

Over portion AD:  $U_{AD} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx$ ,  $M = R_A x = -\frac{Pa}{L}x + \frac{M_o}{L}x$ ,  $\frac{\partial M}{\partial M_o} = \frac{1}{L}x$

Set  $M_o = 0$ ,  $\frac{\partial U_{AD}}{\partial M_o} = \frac{1}{EI} \int_0^{\frac{L}{2}} \left(-\frac{Pa}{L}x\right)\left(\frac{1}{L}x\right) dx = -\frac{Pa}{EIL^2} \int_0^{\frac{L}{2}} x^2 dx$   
 $= -\frac{Pa}{EIL^2} \cdot \frac{1}{3}\left(\frac{L}{2}\right)^3 = -\frac{PaL}{24EI}$

Over portion DE:  $U_{DE} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$        $\frac{\partial M}{\partial M_o} = -\frac{1}{L}v$

$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{M_o}{L}v - P(a+v) = \frac{Pa}{L}v - Pa - \frac{M_o}{L}v$

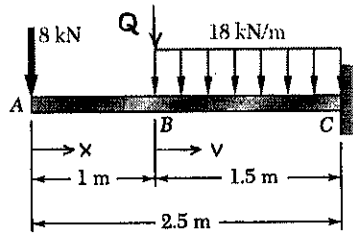
Set  $M_o = 0$ ,  $\frac{\partial U_{DE}}{\partial M_o} = \frac{1}{EI} \int_0^{\frac{L}{2}} \left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{L}v\right) dv = -\frac{Pa}{EIL^2} \int_0^{\frac{L}{2}} (v^2 - Lv) dv$   
 $= -\frac{Pa}{EIL^2} \left[ \frac{1}{3}\left(\frac{L}{2}\right)^3 - L \cdot \frac{1}{2}\left(\frac{L}{2}\right)^2 \right] = -\frac{Pa}{EIL^2} \left[ \frac{1}{24}L^3 - \frac{1}{8}L^3 \right]$   
 $= \frac{1}{12} \frac{PaL}{EI}$

Over portion EB:  $M = -Pv$        $\frac{\partial M}{\partial M_o} = 0$        $\frac{\partial U_{EB}}{\partial M_o} = 0$

Total  $\theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = -\frac{1}{24} \frac{PaL}{EI} + \frac{1}{12} \frac{PaL}{EI} + 0 = \frac{1}{24} \frac{PaL}{EI}$

PROBLEM 11.106

11.106 For the beam and loading shown, determine the deflection at point B. Use  $E = 200 \text{ GPa}$ .



SOLUTION

Add force  $Q$  at point B.

Units: Forces in kN, lengths in m.

Over AB  $M = -8x$   $\frac{\partial M}{\partial Q} = 0$

Over BC  $M = -8(v+1) - \frac{1}{2}(18)v^2 - Qv$   $\frac{\partial M}{\partial Q} = -v$

$E = 200 \times 10^9 \text{ Pa}$ ,  $I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$

$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6 \text{ N}\cdot\text{m}^2 = 5780 \text{ kN}\cdot\text{m}^2$

$U = \int_0^1 \frac{M^2}{2EI} dx + \int_0^{1.5} \frac{M^2}{2EI} dv$

$\delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^1 M \frac{\partial M}{\partial Q} dx + \int_0^{1.5} M \frac{\partial M}{\partial Q} dv \right\}$

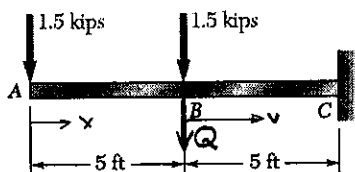
$= \frac{1}{EI} \left\{ 0 + \int_0^{1.5} [-8(v+1) - \frac{1}{2}(18)v^2](-v) dv \right\} = \frac{1}{EI} \int_0^{1.5} (9v^3 + 8v^2 + 8v) dv$

$= \frac{1}{EI} \left\{ \frac{9}{4}(1.5)^4 + \frac{8}{3}(1.5)^3 + \frac{8}{2}(1.5)^2 \right\} = \frac{29.391}{EI} = \frac{29.391}{5780}$

$= 5.08 \times 10^{-3} \text{ m} = 5.08 \text{ mm} \downarrow$

PROBLEM 11.107

11.107 For the beam and loading shown, determine the deflection at point B. Use  $E = 29 \times 10^3 \text{ ksi}$ .



SOLUTION

Add force  $Q$  at point B

Units: forces in kips, lengths in ft.

$E = 29 \times 10^3 \text{ ksi}$   $I = 39.6 \text{ in}^4$

$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6 \text{ kip}\cdot\text{in}^2 = 7975 \text{ kip}\cdot\text{ft}^2$

$U = \int_0^5 \frac{M^2}{2EI} dx + \int_0^5 \frac{M^2}{2EI} dv$

$\delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^5 M \frac{\partial M}{\partial Q} dx + \int_0^5 M \frac{\partial M}{\partial Q} dv \right\}$

Over AB:  $M = -1.5x$ ,  $\frac{\partial M}{\partial Q} = 0$   $\int_0^5 M \frac{\partial M}{\partial Q} dx = 0$

Over BC:  $M = -1.5(v+5) - 1.5v - Qv = -3v - 7.5 - Qv$ ,  $\frac{\partial M}{\partial Q} = -v$

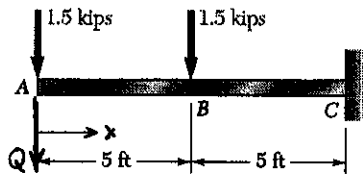
$\int_0^5 M \frac{\partial M}{\partial Q} dv = \int_0^5 (3v^2 + 7.5v) dv = (3)(\frac{1}{3})(5)^3 + (7.5)(\frac{1}{2})(5)^2 = 218.75$

$\delta_B = \frac{1}{EI} \{ 0 + 218.75 \} = \frac{218.75}{7975} = 27.43 \times 10^{-3} \text{ ft} = 0.329 \text{ in.} \downarrow$



PROBLEM 11.108

11.108 For the beam and loading shown, determine the deflection at point A. Use  $E = 29 \times 10^3$  ksi.



SOLUTION



Add force  $Q$  at point A.

Units: forces in kips, lengths in ft.

$E = 29 \times 10^3$  ksi,  $I = 39.6$  in<sup>4</sup>

$EI = (29 \times 10^3)(39.6) = 1.148 \times 10^6$  kip-in<sup>2</sup> =  $7975$  kip-ft<sup>2</sup>

$U = \int_0^{10} \frac{M^2}{2EI} dx$        $S_A = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{10} M \frac{\partial M}{\partial Q} dx$

Over portion AB  $0 < x < 5$ ,  $M = -1.5x - Qx$        $\frac{\partial M}{\partial Q} = -x$

$\int_0^5 M \frac{\partial M}{\partial Q} dx = \int_0^5 (1.5x)(x) dx = 1.5 \int_0^5 x^2 dx = (1.5)(\frac{1}{3})(5)^3 = 62.5$

Over portion BC  $5 < x < 10$   $M = -1.5x - 1.5(x-5) - Qx$

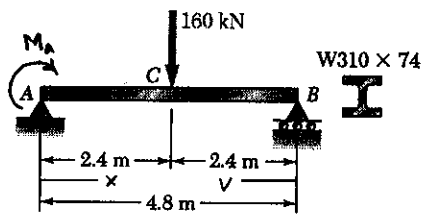
$M = -3x + 7.5 - Qx$        $\frac{\partial M}{\partial Q} = -x$

$\int_5^{10} M \frac{\partial M}{\partial Q} dx = \int_5^{10} (3x^2 - 7.5x) dx = (3)(\frac{1}{3})(10^3 - 5^3) - (7.5)(\frac{1}{2})(10^2 - 5^2)$   
 $= 593.75$

$S_A = \frac{1}{EI} \{ 62.5 + 593.75 \} = \frac{656.25}{7975} = 82.29 \times 10^3$  ft =  $0.987$  in. ↓

PROBLEM 11.109

11.109 For the beam and loading shown, determine the slope at end A. Use  $E = 200$  GPa



SOLUTION

Add couple  $M_A$  at point A.

Units: forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}, \quad I = 165 \times 10^4 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33 \times 10^6 \text{ N}\cdot\text{m}^2 = 33000 \text{ kN}\cdot\text{m}^2$$

Reactions:  $R_A = 80 - \frac{M_A}{4.8}$        $R_B = 80 + \frac{M_A}{4.8}$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI} dx + \int_0^{2.4} \frac{M^2}{2EI} dv \quad \delta \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

Over AB:  $M = M_A + R_A x = M_A + 80x - \frac{M_A}{4.8} x$        $\frac{\partial M}{\partial M_A} = (1 - \frac{x}{4.8})$

Set  $M_A = 0$        $\frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80x)(1 - \frac{x}{4.8}) dx = \frac{1}{EI} \int_0^{2.4} (80x - 16.6667x^2) dx$   
 $= \frac{1}{EI} \left\{ (80)(\frac{1}{2})(2.4)^2 - (16.6667)(\frac{1}{3})(2.4)^3 \right\} = \frac{153.6}{EI}$

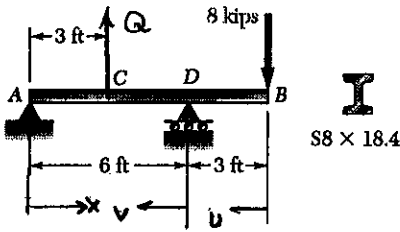
Over BC:  $M = R_B v = 80v + \frac{M_A}{4.8} v$ ,       $\frac{\partial M}{\partial M_A} = \frac{1}{4.8} v$

Set  $M_A = 0$        $\frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80v)(\frac{1}{4.8} v) dv = \frac{16.6667}{EI} \int_0^{2.4} v^2 dv$   
 $= \frac{(16.6667)(2.4)^3}{3EI} = \frac{76.8}{EI}$

$$\delta \theta_A = \frac{1}{EI} \{ 153.6 + 76.8 \} = \frac{230.4}{33000} = 6.98 \times 10^{-3} \text{ rad. } \curvearrowright$$

PROBLEM 11.110

11.110 For the beam and loading shown, determine the deflection at point C. Use  $E = 29 \times 10^3$  ksi.



SOLUTION

Units: Forces in kip, lengths in ft.

$E = 29 \times 10^3$  ksi     $I = 57.6$  in<sup>4</sup>

$EI = (29 \times 10^3)(57.6) = 1.6704 \times 10^6$  kip·in<sup>2</sup> = 11600 kip·ft<sup>2</sup>

Add dummy force  $Q$  at point C. Reactions  $R_A = 4 + \frac{1}{2}Q \downarrow$ ,  $R_D = 12 - \frac{1}{2}Q \uparrow$

$U = U_{AC} + U_{CD} + U_{DB}$        $S_c = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$

Over AC  $0 < x < 3$        $M = -(4 + \frac{1}{2}Q)x$        $\frac{\partial M}{\partial Q} = -\frac{1}{2}x$       Set  $Q = 0$ .

$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI} \int_0^3 (4x)(\frac{1}{2}x) dx = \frac{2}{EI} \int_0^3 x^2 dx = \frac{(2)(3)^3}{3EI} = \frac{18}{EI}$

Over CD  $0 < v < 3$        $M = R_D v - 8(v+3) = 12v - \frac{1}{2}Qv - 8v - 24 = 4v - 24 - \frac{1}{2}Qv$

$\frac{\partial M}{\partial Q} = -\frac{1}{2}v$       Set  $Q = 0$

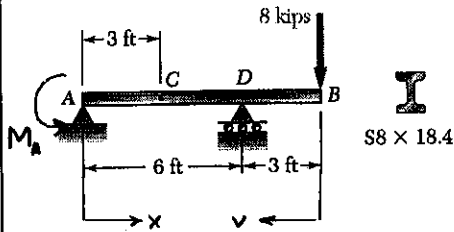
$\frac{\partial U_{CD}}{\partial Q} = \frac{1}{EI} \int_0^3 (24 - 4v)(\frac{1}{2}v) dv = \frac{1}{EI} \int_0^3 (12v - 2v^2) dv = \frac{1}{EI} \left\{ (12) \frac{(3)^2}{2} - (2) \frac{(3)^3}{3} \right\}$   
 $= \frac{36}{EI}$

Over DB  $0 < v < 3$        $M = -8v$        $\frac{\partial M}{\partial Q} = 0$        $\frac{\partial U_{DB}}{\partial Q} = 0$

$S_c = \frac{18}{EI} + \frac{36}{EI} + 0 = \frac{54}{11600} = 4.655 \times 10^{-3}$  ft = 0.0559 in.  $\uparrow$

PROBLEM 11.111

11.111 For the beam and loading shown, determine the slope at end A. Use  $E = 29 \times 10^3$  ksi.



SOLUTION

Units: Forces in kips, lengths in ft.

$E = 29 \times 10^3$  ksi,  $I = 57.6$  in<sup>4</sup>

$EI = (29 \times 10^3)(57.6) = 1.6704 \times 10^6$  kip·in<sup>2</sup> = 11660 kip·ft<sup>2</sup>

Add dummy couple  $M_A$  at end A. Reactions:  $R_A = -4 + \frac{M_A}{6}$ ,  $R_B = 12 - \frac{M_A}{6}$

$U = U_{AD} + U_{DB}$        $\delta \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}$

Over AD  $0 < x < 6$        $M = -M_A + R_A x = -M_A - 4x + \frac{M_A}{6} x$

$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{6}\right)$       Set  $M_A = 0$ .

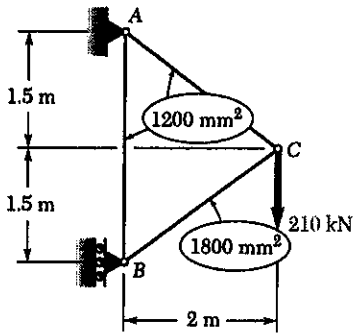
$\frac{\partial U_{AD}}{\partial M_A} = \frac{1}{EI} \int_0^6 (4x)\left(1 - \frac{x}{6}\right) dx = \frac{1}{EI} \int_0^6 \left(4x - \frac{2}{3}x^2\right) dx = \frac{1}{EI} \left\{ (4) \frac{6^2}{2} - \frac{2}{3} \frac{6^3}{3} \right\}$   
 $= \frac{24}{EI}$

Over DB  $0 < v < 3$        $M = -8v$        $\frac{\partial M}{\partial M_A} = 0$        $\frac{\partial U_{DB}}{\partial M_A} = 0$

$\delta \theta_A = \frac{24}{EI} + 0 = \frac{24}{11660} = 2.07 \times 10^{-5}$  rad

PROBLEM 11.112

11.112 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using  $E = 200 \text{ GPa}$ , determine the vertical deflection of joint C.

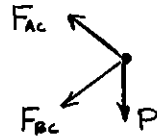


SOLUTION

Call the vertical load  $P$ . The vertical deflection of joint C is  $S_p$

$$S_p = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

Joint C:  $\rightarrow +\sum F_x = 0 \quad -\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$

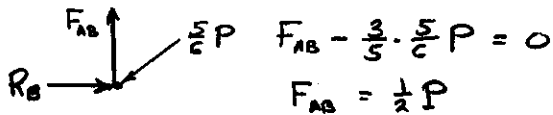


$\uparrow +\sum F_y \quad \frac{3}{5}F_{AC} + \frac{3}{5}F_{BC} - P = 0$

Solving simultaneously

$F_{AC} = \frac{5}{6}P \quad F_{BC} = -\frac{5}{6}P$

Joint B  $\uparrow +\sum F_y = 0$

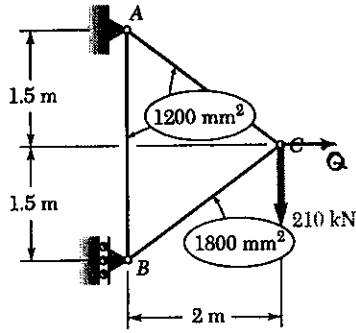


Member	F	$\partial F / \partial P$	L (m)	A ( $10^6 \text{ m}^2$ )	$F(\partial F / \partial P)L/A$
AB	$\frac{1}{2}P$	$\frac{1}{2}$	3	1200	625 P
AC	$\frac{5}{6}P$	$\frac{5}{6}$	2.5	1200	1446.76 P
BC	$-\frac{5}{6}P$	$-\frac{5}{6}$	2.5	1800	964.51 P
$\Sigma$					3036.27 P

$$S_p = \frac{1}{E} (3036.27 P) = \frac{(3036.27)(210 \times 10^3)}{200 \times 10^9} = 3.19 \times 10^{-3} \text{ m} = 3.19 \text{ mm} \downarrow$$

PROBLEM 11.113

11.113 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using  $E = 200$  GPa, determine the horizontal deflection of joint C.



SOLUTION

Call the vertical force P. Add a dummy horizontal force Q (positive  $\rightarrow$ ) at joint C. The horizontal deflection of joint C is

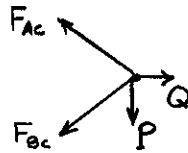
$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EI} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Joint C  $\rightarrow \sum F_x = 0$

$$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} + Q = 0$$

$\uparrow \sum F_y = 0$

$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - P = 0$$



Solving simultaneously

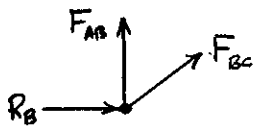
$$F_{AC} = \frac{5}{6} P + \frac{5}{8} Q$$

$$F_{BC} = -\frac{5}{6} P + \frac{5}{8} Q$$

Joint B  $\uparrow \sum F_y = 0$

$$F_{AB} + \frac{3}{5} F_{BC} = 0$$

$$F_{AB} = -\frac{3}{5} F_{BC} = \frac{1}{2} P - \frac{5}{8} Q$$



Member	F	$\partial F / \partial Q$	L (m)	A ( $10^4$ m <sup>2</sup> )	$F(\partial F / \partial Q) L / A$
AB	$\frac{1}{2} P - \frac{5}{8} Q$	$-\frac{5}{8}$	3	1200	$-468.75 P$
AC	$\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1200	$1085.07 P$
BC	$-\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1800	$-723.38 P$
					$-107.06 P$

$$\delta_Q = \frac{1}{E} (-107.06 P) = -\frac{(107.06)(210 \times 10^3)}{200 \times 10^9} = -0.1124 \times 10^{-3} \text{ m}$$

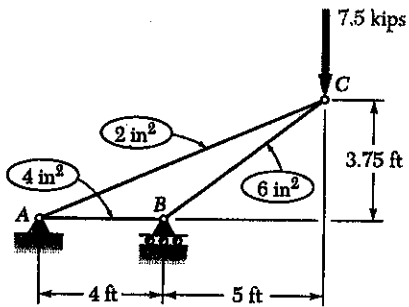
$$= 0.1124 \text{ mm} \leftarrow$$

PROBLEM 11.114

11.114 and 11.115 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using  $E = 29 \times 10^6$  psi, determine the deflection indicated.

11.114 Vertical deflection of joint C.

SOLUTION



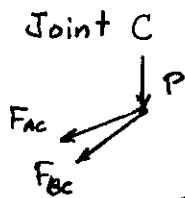
Call the vertical load  $P$ . The vertical deflection of joint C is  $\delta_P$

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

Geometry  $AC = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in}$

$BC = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in}$

$4 \text{ ft} = 48 \text{ in}, 5 \text{ ft} = 60 \text{ in}, 3.75 \text{ ft} = 45 \text{ in}$

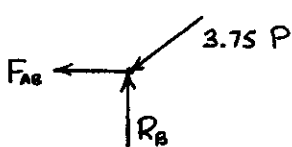


$$+\rightarrow \sum F_x = 0 \quad -\frac{108}{117} F_{AC} - \frac{60}{75} F_{BC} = 0$$

$$+\uparrow \sum F_y = 0 \quad -\frac{45}{117} F_{AC} - \frac{45}{75} F_{BC} - P = 0$$

Solving simultaneously  $F_{AC} = 3.25 P, F_{BC} = -3.75 P$

Joint B



$$+\rightarrow \sum F_{AB} = 0 \quad -F_{AB} - \frac{60}{75} F_{AC} = 0$$

$$F_{AB} = -3.00 P$$

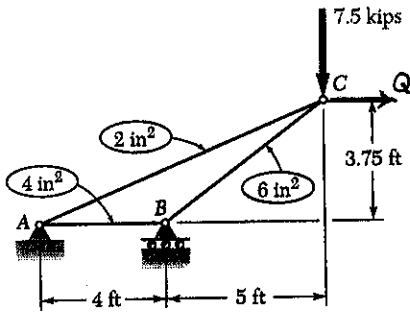
Member	F	$\partial F / \partial P$	L (in)	A (in <sup>2</sup> )	$F(\partial F / \partial P)L / A$
AB	$-3.00 P$	$-3.00$	48	4	$108.00 P$
AC	$3.25 P$	$3.25$	117	2	$617.91 P$
BC	$-3.75 P$	$-3.75$	75	6	$175.78 P$
$\Sigma$					$901.69 P$

$$\delta_P = \frac{901.69 P}{E} = \frac{(901.69)(7.5 \times 10^3)}{29 \times 10^6} = 0.233 \text{ in. } \downarrow$$

PROBLEM 11.115

11.114 and 11.115 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using  $E = 29 \times 10^6$  psi, determine the deflection indicated.

11.115 Horizontal deflection of joint C.



SOLUTION

Call the vertical load  $P$ . Add horizontal dummy load  $Q$  at joint C. The horizontal deflection of joint C is

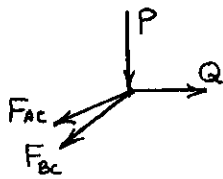
$$S_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Geometry  $AC = \sqrt{9^2 + 3.75^2} = 9.75 \text{ ft} = 117 \text{ in}$

$BC = \sqrt{5^2 + 3.75^2} = 6.25 \text{ ft} = 75 \text{ in}$

$4 \text{ ft} = 48 \text{ in}, 5 \text{ ft} = 60 \text{ in}, 3.75 \text{ ft} = 45 \text{ in}$

Joint C



$$+\rightarrow \sum F_x = 0 \quad -\frac{108}{117} F_{AC} - \frac{60}{75} F_{BC} + Q = 0$$

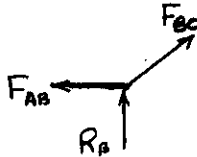
$$+\uparrow \sum F_y = 0 \quad -\frac{45}{117} F_{AC} - \frac{45}{75} F_{BC} - P = 0$$

Solving simultaneously

$$F_{AC} = 3.25 P + 2.4375 Q$$

$$F_{BC} = -3.75 P - 1.5625 Q$$

Joint B



$$+\rightarrow \sum F_x = 0 \quad \frac{4}{5} F_{AC} - F_{AB} = 0$$

$$F_{AB} = \frac{4}{5} F_{BC} = -3.00 P - 1.25 Q$$

Member	F	$\partial F / \partial Q$	L (in.)	A (in <sup>2</sup> )	$F(\partial F / \partial Q)L/A$
AB	$-3.00 P - 1.25 Q$	-1.25	48	4	$45.00 P$
AC	$3.25 P + 2.4375 Q$	2.4375	117	2	$463.43 P$
BC	$-3.75 P - 1.5625 Q$	-1.5625	75	6	$73.24 P$
$\Sigma$					$581.67 P$

$$S_Q = \frac{581.67 P}{E} = \frac{(581.67)(7.5 \times 10^3)}{29 \times 10^6} = 0.1504 \text{ in.} \rightarrow$$

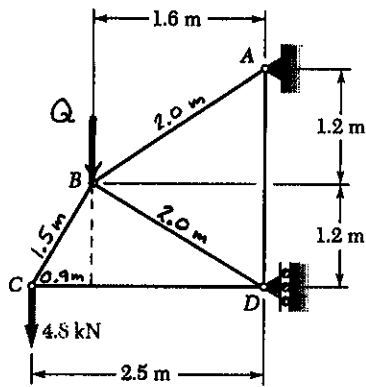


PROBLEM 11.116

11.116 and 11.117 Each member of the truss shown is made of steel and has a cross-sectional area of 500 mm<sup>2</sup>. Using  $E = 200$  GPa, determine the deflection indicated.

11.116 Vertical deflection of joint B.

SOLUTION



Find the length of each member as shown.

Add dummy vertical force  $Q$  at joint B.

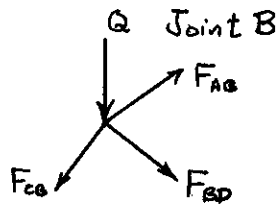
$$S_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial Q} L$$

Joint C  $+\uparrow \Sigma F_y = 0$   $\frac{4}{5} F_{CB} - 4.8 = 0$

$$F_{CB} = 6.0 \text{ kN}$$

$\rightarrow \Sigma F_x = 0$   $\frac{3}{5} F_{CB} + F_{CD} = 0$

$$F_{CD} = -3.6 \text{ kN}$$



$$+\rightarrow \Sigma F_x = 0 \quad \frac{4}{5} F_{AB} + \frac{4}{5} F_{BD} - 3.6 = 0$$

$$+\uparrow \Sigma F_y = 0 \quad \frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} - 4.8 - Q = 0$$

Solving simultaneously  $F_{AB} = 6.25 + 0.8333 Q$  kN

$$F_{BD} = -1.75 - 0.8333 Q$$
 kN

Joint D



$$+\uparrow \Sigma F_y = 0 \quad \frac{3}{5} F_{BD} + F_{AD} = 0$$

$$F_{AD} = -\frac{3}{5} F_{BD} = 1.05 + 0.5 Q$$

Member	$F$ ( $10^3$ N)	$\partial F / \partial Q$	$L$ (m)	with $Q = 0$ $F (\partial F / \partial Q) L$ ( $10^3$ N·m)
AB	$6.25 + 0.8333 Q$	0.8333	2.0	10.4167
AD	$1.05 + 0.5 Q$	0.5	2.4	1.26
BD	$-1.75 - 0.8333 Q$	-0.8333	2.0	2.9167
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
$\Sigma$				14.593

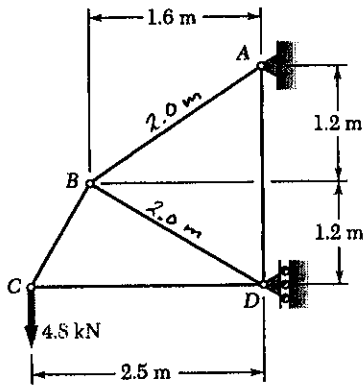
$$S_B = \frac{1}{EA} \sum F (\partial F / \partial Q) L = \frac{14.593 \times 10^3}{(200 \times 10^9)(500 \times 10^{-6})} = 145.9 \times 10^{-6} \text{ m} = 0.1459 \text{ mm} \downarrow$$

PROBLEM 11.117

11.116 and 11.117 Each member of the truss shown is made of steel and has a cross-sectional area of 500 mm<sup>2</sup>. Using  $E = 200$  GPa, determine the deflection indicated.

11.117 Horizontal deflection of joint B.

SOLUTION



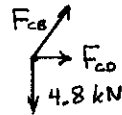
Find the length of each member as shown.

Add dummy horizontal force  $Q$  at joint B.

$$S_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial Q} L$$

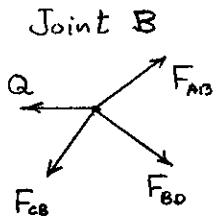
Joint C  $\uparrow \Sigma F_y = 0$   $\frac{4}{5} F_{CB} - 4.8 = 0$

$$F_{CB} = 6.0 \text{ kN}$$



$\rightarrow \Sigma F_x = 0$   $\frac{3}{5} F_{CB} + F_{CD} = 0$

$$F_{CD} = -3.6 \text{ kN}$$



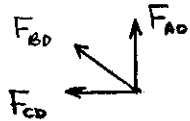
$\rightarrow \Sigma F_x = 0$   $\frac{4}{5} F_{AB} + \frac{4}{5} F_{BD} - 3.6 - Q = 0$

$\uparrow \Sigma F_y = 0$   $\frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} - 4.8 = 0$

Solving simultaneously  $F_{AB} = 6.25 + 0.625Q$  kN

$$F_{BD} = -1.75 + 0.625Q \text{ kN}$$

Joint D



$\uparrow \Sigma F_y = 0$   $\frac{3}{5} F_{BD} + F_{AD} = 0$

$$F_{AD} = -\frac{3}{5} F_{BD} = 1.05 - 0.375Q$$

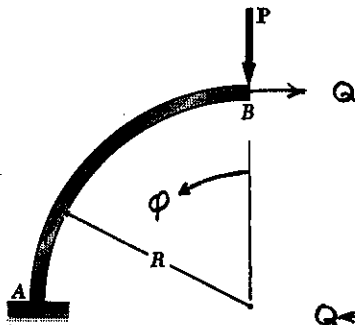
Member	F 10 <sup>3</sup> N	$\partial F / \partial Q$	L (m)	F( $\partial F / \partial Q$ )L (10 <sup>3</sup> N·m)
AB	6.25 + 0.625Q	0.625	2.0	7.8125
AD	1.05 + 0.375Q	-0.375	2.4	-0.9450
BD	-1.75 + 0.625Q	0.625	2.0	-2.1875
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
$\Sigma$				4.680

$$S_B = \frac{1}{EA} \sum F (\partial F / \partial Q) L = \frac{4.680 \times 10^3}{(200 \times 10^9)(500 \times 10^{-6})} = 46.8 \times 10^{-6} \text{ m}$$

$$= 0.0468 \text{ mm} \leftarrow$$

PROBLEM 11.118

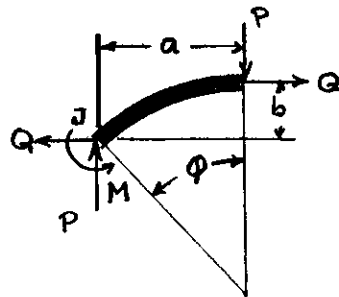
\*11.118 For the uniform rod and loading shown and using Castigliano's theorem, determine (a) the horizontal deflection of point B, (b) the vertical deflection of point B.



SOLUTION

Add dummy load Q at point B.

Use polar coordinate  $\phi$



$$U = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} R d\phi$$

Bending moment

$$+\circlearrowleft \sum M_I = 0 \quad M - Pa - Qb = 0$$

$$M = Pa + Qb \\ = PR \sin\phi + QR(1 - \cos\phi)$$

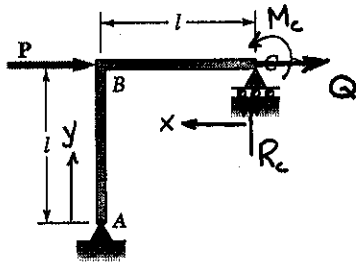
$$\frac{\partial M}{\partial P} = R \sin\phi \qquad \frac{\partial M}{\partial Q} = R(1 - \cos\phi) \qquad \text{Set } Q = 0$$

$$\begin{aligned} (a) \quad S_a &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial Q} R d\phi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin\phi R(1 - \cos\phi) R d\phi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} (\sin\phi - \sin\phi \cos\phi) d\phi = \frac{PR^3}{EI} (-\cos\phi - \frac{1}{2} \sin^2\phi) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{PR^3}{EI} (-\cos \frac{\pi}{2} + \cos 0 - \frac{1}{2} \sin^2 \frac{\pi}{2} + \frac{1}{2} \sin^2 0) \\ &= \frac{PR^3}{EI} (0 + 1 - \frac{1}{2} + 0) = \frac{1}{2} \frac{PR^3}{EI} \end{aligned}$$

$$\begin{aligned} (b) \quad S_p &= \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial P} R d\phi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin\phi R \sin\phi R d\phi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \sin^2\phi d\phi = \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 2\phi) d\phi \\ &= \frac{PR^3}{EI} (\frac{1}{2}\phi - \frac{1}{2} \sin 2\phi) \Big|_0^{\frac{\pi}{2}} = \frac{PR^3}{EI} (\frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot 0 - \frac{1}{2} \sin \pi + \frac{1}{2} \cdot \sin 0) \\ &= \frac{PR^3}{EI} (\frac{\pi}{4} - 0 - 0 + 0) = \frac{\pi}{4} \frac{PR^3}{EI} \end{aligned}$$

PROBLEM 11.119

11.119 Two rods  $AB$  and  $BC$  of the same flexural rigidity  $EI$  are welded together at  $B$ . For the loading shown, determine (a) the deflection of point  $C$ , (b) the slope of member  $BC$  at point  $C$ .



SOLUTION

Add dummy force  $Q$  and dummy couple  $M_c$  at  $C$ .

$$\circlearrowleft \sum M_A = 0 \quad R_c l + M_c + (P+Q)l = 0$$

$$R_c = P + Q + \frac{M_c}{l}$$

$$\rightarrow \sum F_x = 0 \quad P + Q + R_{Ax} = 0 \quad R_{Ax} = P + Q \leftarrow$$

Member  $AB$ :  $M = R_{Ax}y = (P+Q)y, \quad \frac{\partial M}{\partial Q} = y, \quad \frac{\partial M}{\partial M_c} = 0$

$$U_{AB} = \int_0^l \frac{M^2}{2EI} dy \quad \text{Set } Q = 0 \text{ and } M_c = 0$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{1}{3} \frac{Pl^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = 0$$

Member  $BC$ :  $M = M_c + R_c x = M_c + (P + Q + \frac{M_c}{l})x$

$$\frac{\partial M}{\partial Q} = x, \quad \frac{\partial M}{\partial M_c} = 1 + \frac{x}{l}$$

$$U_{BC} = \int_0^l \frac{M^2}{2EI} dx \quad \text{Set } Q = 0 \text{ and } M_c = 0$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^l (Px)x dx = \frac{1}{3} \frac{Pl^3}{EI}$$

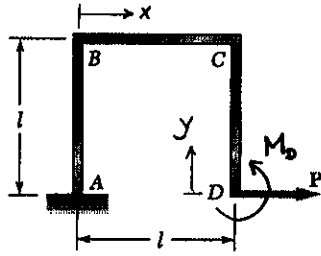
$$\begin{aligned} \frac{\partial U}{\partial M_c} &= \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = \frac{1}{EI} \int_0^l (Px) \left(1 + \frac{x}{l}\right) dx = \frac{P}{EI} \int_0^l \left(x + \frac{x^2}{l}\right) dx \\ &= \frac{P}{EI} \left(\frac{1}{2}l^2 + \frac{1}{3}l^2\right) = \frac{1}{6} \frac{Pl^3}{EI} \end{aligned}$$

(a) Deflection at  $C$   $\delta_c = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{2}{3} \frac{Pl^3}{EI} \rightarrow$

(b) Slope at  $C$   $\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c} = \frac{1}{6} \frac{Pl^3}{EI} \curvearrowright$

PROBLEM 11.120

11.120 A uniform rod of flexural rigidity  $EI$  is bent and loaded as shown. Determine (a) the horizontal deflection of point  $D$ , (b) the slope at point  $D$ .



SOLUTION

Add dummy couple  $M_D$  at point  $D$ .

Reactions at  $A$ :  $R_{Ay} = 0$ ,  $R_{Ax} = P \leftarrow$ ,  $M_A = M_D \curvearrowright$

Member AB:  $M = M_A + R_{Ay} = M_D + P_y$        $\frac{\partial M}{\partial P} = y$ ,  $\frac{\partial M}{\partial M_D} = 1$

$U_{AB} = \int_0^l \frac{M^2}{2EI} dy$       Set  $M_D = 0$

$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI} \int_0^l (P_y) y dy = \frac{Pl^3}{3EI}$

$\frac{\partial U_{AB}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI} \int_0^l (P_y)(1) dy = \frac{Pl^2}{2EI}$

Member BC:  $M = M_A + R_{Ax} l = M_D + Pl$        $\frac{\partial M}{\partial P} = l$ ,  $\frac{\partial M}{\partial M_D} = 1$

$U_{BC} = \int_0^l \frac{M^2}{2EI} dx$       Set  $M_D = 0$

$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^l (Pl)(l) dx = \frac{Pl^3}{EI}$

$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dx = \frac{1}{EI} \int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$

Member CD:  $M = M_D + P_y$        $\frac{\partial M}{\partial P} = y$        $\frac{\partial M}{\partial M_D} = 1$

$U_{CD} = \int_0^l \frac{M^2}{2EI} dy$       Set  $M_D = 0$

$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI} \int_0^l (P_y)(y) dy = \frac{Pl^3}{3EI}$

$\frac{\partial U_{CD}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI} \int_0^l (P_y)(1) dy = \frac{Pl^2}{2EI}$

(a) horizontal deflection of point  $D$ .

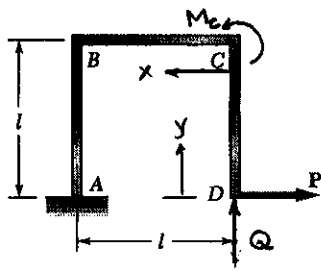
$S_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right) \frac{Pl^3}{EI} = \frac{5}{3} \frac{Pl^3}{EI} \rightarrow$

(b) slope at point  $D$

$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_D} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right) \frac{Pl^2}{EI} = 2 \frac{Pl^2}{EI} \curvearrowright$

PROBLEM 11.121

11.121 A uniform rod of flexural rigidity  $EI$  is bent and loaded as shown. Determine (a) the vertical deflection of point D, (b) the slope of BC at point C.



SOLUTION

Add dummy force  $Q$  at point D and dummy couple  $M_c$  at point C.

Reactions at A:  $R_{Ax} = P \leftarrow$ ,  $R_{Ay} = Q \downarrow$   
 $M_A = Ql + M_c \curvearrowright$

Member AB:  $M = M_A + R_{Ay}y = Ql + M_c + Py$ ,  $\frac{\partial M}{\partial Q} = l$ ,  $\frac{\partial M}{\partial M_c} = 1$   
 $U_{AB} = \int_0^l \frac{M^2}{2EI} dy$  Set  $Q=0$  and  $M_c=0$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dy = \frac{1}{EI} \int_0^l (Py)(l) dy = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

Member BC  $M = M_c + Pl + Qx$   $\frac{\partial M}{\partial Q} = x$   $\frac{\partial M}{\partial M_c} = 1$

$U_{BC} = \frac{1}{EI} \int_0^l \frac{M^2}{2EI} dx$  Set  $Q=0$  and  $M_c=0$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^l (Pl)(x) dx = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{BC}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = \frac{1}{EI} \int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$$

Member CD  $M = Py$   $\frac{\partial M}{\partial Q} = 0$   $\frac{\partial M}{\partial M_c} = 0$

$$\frac{\partial U_{CD}}{\partial Q} = 0 \quad \frac{\partial U_{CD}}{\partial M_c} = 0$$

(a) vertical deflection of point D

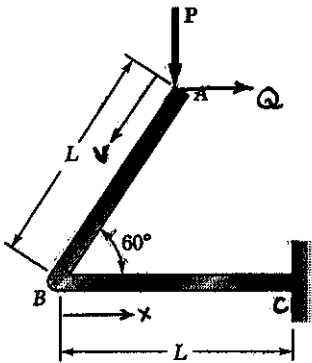
$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} = \left(\frac{1}{2} + \frac{1}{2} + 0\right) \frac{Pl^3}{EI} = \frac{Pl^3}{EI} \uparrow$$

(b) slope of BC at C

$$\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c} + \frac{\partial U_{CD}}{\partial M_c} = \left(\frac{1}{2} + 1 + 0\right) \frac{Pl^2}{EI} = \frac{3}{2} \frac{Pl^2}{EI} \triangleleft$$

PROBLEM 11.122

11.122 A uniform rod of flexural rigidity  $EI$  is bent and loaded as shown. Determine (a) the vertical deflection of point A, (b) the horizontal deflection of point A.



SOLUTION

Add dummy horizontal force  $Q$  at point A.

Over AB  $M = \frac{1}{2} Pv + \frac{\sqrt{3}}{2} Qv$

$$\frac{\partial M}{\partial P} = \frac{1}{2}v \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dv = \frac{1}{EI} \int_0^L \left(\frac{1}{2}Pv\right)\left(\frac{1}{2}v\right) dv$$

$$= \frac{1}{12} \frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dv = \frac{1}{EI} \int_0^L \left(\frac{1}{2}Pv\right)\frac{\sqrt{3}}{2} dv$$

$$= \frac{\sqrt{3}}{12} \frac{PL^3}{EI}$$

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Over BC  $M = -P\left(x - \frac{L}{2}\right) + \frac{\sqrt{3}}{2}QL$ ,  $\frac{\partial M}{\partial P} = -\left(x - \frac{L}{2}\right)$ ,  $\frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}L$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^L P\left(x - \frac{L}{2}\right)^2 dx = \frac{P}{3EI} \left(x - \frac{L}{2}\right)^3 \Big|_0^L = \frac{1}{12} \frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L P\left(x - \frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)L dx = -\frac{\sqrt{3}P}{4EI} \left(x - \frac{L}{2}\right)^2 \Big|_0^L = 0$$

(a) vertical deflection of point A

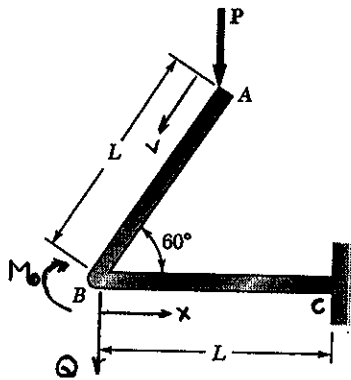
$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} = \frac{1}{6} \frac{PL^3}{EI} \downarrow$$

(b) horizontal deflection of point A

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12} \frac{PL^3}{EI} = 0.1443 \frac{PL^3}{EI} \rightarrow$$

PROBLEM 11.123

11.123 A uniform rod of flexural rigidity  $EI$  is bent and loaded as shown. Determine (a) the vertical deflection of point B, (b) the slope of BC at point B.



SOLUTION

Add dummy vertical  $Q$  and dummy couple  $M_0$  at B.

Over AB  $M = \frac{1}{2}Pv$ ,  $\frac{\partial M}{\partial Q} = 0$ ,  $\frac{\partial M}{\partial M_0} = 0$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv$$

$$\frac{\partial U_{AB}}{\partial Q} = 0$$

$$\frac{\partial U_{AB}}{\partial M_0} = 0$$

Over BC  $M = -P(x - \frac{L}{2}) - Qx + M_0$ ,  $\frac{\partial M}{\partial Q} = -x$ ,  $\frac{\partial M}{\partial M_0} = 1$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx$$

Set  $Q = 0$  and  $M_0 = 0$

$$\begin{aligned} \frac{\partial U_{BC}}{\partial Q} &= \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L P(x - \frac{L}{2}) x dx = \frac{P}{EI} \left[ \frac{L^3}{3} - (\frac{L}{2}) \frac{L^2}{2} \right] \\ &= \frac{1}{12} \frac{PL^3}{EI} \end{aligned}$$

$$\frac{\partial U_{BC}}{\partial M_0} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L P(x - \frac{L}{2}) dx = 0$$

(a) vertical deflection of point B

$$\delta_B = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{1}{12} \frac{PL^3}{EI} \downarrow$$

(b) slope of BC at point B

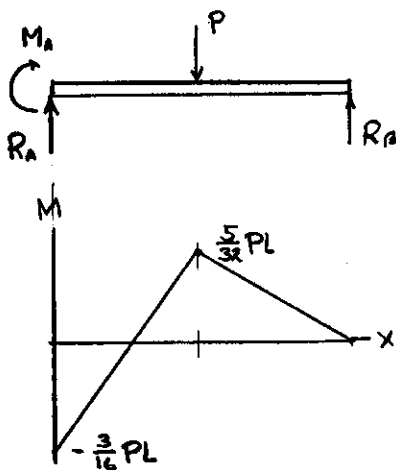
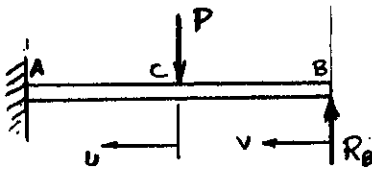
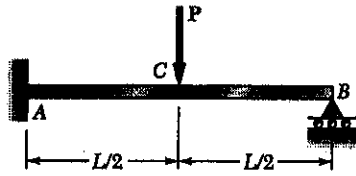
$$\theta_B = \frac{\partial U_{AB}}{\partial M_0} + \frac{\partial U_{BC}}{\partial M_0} = 0$$



PROBLEM 11.124

11.124 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Remove support B and add reaction  $R_B$  as a load.

$$U = U_{AC} + U_{CB} = \int_0^{L/2} \frac{M^2}{2EI} du + \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AB}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$

Over AC:  $M = R_B(u + \frac{L}{2}) - Pu$ ,  $\frac{\partial M}{\partial R_B} = (u + \frac{L}{2})$

$$\begin{aligned} \frac{\partial U_{AB}}{\partial R_B} &= \frac{1}{EI} \int_0^{L/2} [R_B(u + \frac{L}{2}) - Pu](u + \frac{L}{2}) du \\ &= \frac{R_B}{EI} \int_0^{L/2} (u + \frac{L}{2})^2 du - \frac{P}{EI} \int_0^{L/2} u(u + \frac{L}{2}) du \\ &= \frac{R_B}{3EI} [L^3 - (\frac{L}{2})^3] - \frac{P}{EI} [\frac{1}{3}(\frac{L}{2})^3 + \frac{L}{2} \cdot \frac{1}{2}(\frac{L}{2})^2] \\ &= \frac{7}{24} \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} \end{aligned}$$

Over CB:  $M = R_B v$ ,  $\frac{\partial M}{\partial R_B} = v$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI} \int_0^{L/2} (R_B v) v dv = \frac{R_B}{3EI} (\frac{L}{2})^3 = \frac{1}{24} \frac{R_B L^3}{EI}$$

$$y_B = (\frac{7}{24} + \frac{1}{24}) \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} = 0$$

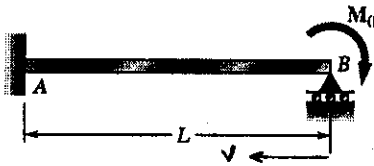
$$R_B = \frac{5}{16} P \quad \uparrow$$

$$M_C = R_B \frac{L}{2} = \frac{5}{32} PL$$

$$M_A = R_B L - P \frac{L}{2} = (\frac{5}{16} - \frac{1}{2}) PL = -\frac{3}{16} PL$$

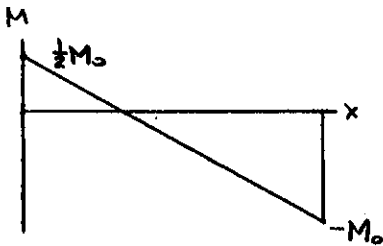
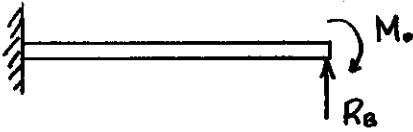
PROBLEM 11.125

11.125 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support \$B\$ and add reaction \$R\_B\$ as a load.



$$U = \int_0^L \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_B} dv = 0$$

$$M = R_B v - M_0 \quad \frac{\partial M}{\partial R_B} = v$$

$$y_B = \frac{1}{EI} \int_0^L (R_B v - M_0) v dv$$

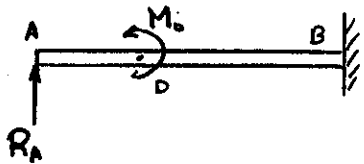
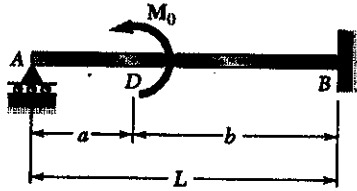
$$= \frac{R_B}{EI} \int_0^L v^2 dv - \frac{M_0}{EI} \int_0^L v dv$$

$$= \frac{R_B L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \quad R_B = \frac{3}{2} \frac{M_0}{L} \uparrow$$

$$M_A = R_B - M_0 = \frac{3}{2} M_0 - M_0 = \frac{1}{2} M_0 \quad \blacktriangleright$$

PROBLEM 11.126

11.126 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support A and add reaction  $R_A$  as a load.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx = 0$$

Portion AD  $0 < x < a$   $M = R_A x$   $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI} \int_0^a (R_A x)(x) dx = \frac{R_A a^3}{3EI}$$

Portion DB ( $a < x < L$ )  $M = R_A x - M_0$   $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \int_a^L (R_A x - M_0)(x) dx = \frac{1}{EI} \left\{ \frac{1}{3} R_A (L^3 - a^3) - \frac{1}{2} M_0 (L^2 - a^2) \right\}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \left\{ R_A \left( \frac{1}{3} a^3 + \frac{1}{3} L^3 - \frac{1}{3} a^3 \right) - \frac{1}{2} M_0 (L^2 - a^2) \right\} = 0$$

$$R_A = \frac{3}{2} \frac{M_0 (L^2 - a^2)}{L^3} = \frac{3}{2} \frac{M_0 b (L+a)}{L^3} \quad \uparrow$$

$$M_A = 0$$

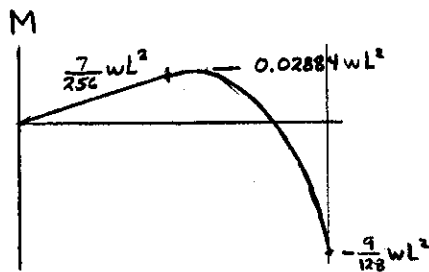
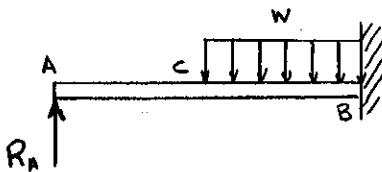
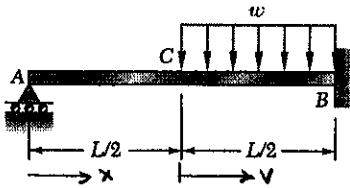
$$M_{D-} = R_A a = \frac{3}{2} \frac{M_0 a b (L+a)}{L^3}$$

$$M_{D+} = M_{D-} + M_0 = \frac{3}{2} \frac{M_0 a b (L+a)}{L^3} - M_0$$

$$M_B = R_A L - M_0 = \frac{3}{2} \frac{M_0 b (L+a)}{L^2} - M_0$$

PROBLEM 11.127

11.127 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support A and add reaction  $R_A$  as a load.

$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx + \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dv = 0$$

Portion AC:  $0 < x < \frac{L}{2}$      $M = R_A x$      $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A L^3}{24 EI}$$

Portion CB  $0 < v < \frac{L}{2}$

$$M = R_A \left(v + \frac{L}{2}\right) - \frac{1}{2} w v^2 \quad \frac{\partial M}{\partial R_A} = \left(v + \frac{L}{2}\right)$$

$$\frac{\partial U_{CB}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} \left[ R_A \left(v + \frac{L}{2}\right) - \frac{1}{2} w v^2 \right] \left(v + \frac{L}{2}\right) dv$$

$$= \frac{1}{EI} \left\{ R_A \int_0^{\frac{L}{2}} \left(v + \frac{L}{2}\right)^2 dv - \frac{1}{2} w \int_0^{\frac{L}{2}} \left(v^3 + \frac{L}{2} v^2\right) dv \right\}$$

$$= \frac{R_A}{EI} \left[ \frac{1}{3} L^3 - \frac{1}{3} \left(\frac{L}{2}\right)^3 \right] - \frac{w}{2EI} \left[ \frac{1}{4} \left(\frac{L}{2}\right)^4 + \frac{L}{2} \frac{1}{3} \left(\frac{L}{2}\right)^3 \right]$$

$$= \left(\frac{1}{3} - \frac{1}{24}\right) \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI}$$

$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

$$R_A = \frac{7}{128} w L \quad \uparrow$$

Over AC  $M = \frac{7}{128} w L x$

$$M_c = \frac{7}{256} w L^2 = 0.02734 w L^2 \quad \blacktriangleleft$$

Over CB  $M = \frac{7}{128} w L \left(v + \frac{L}{2}\right) - \frac{1}{2} w v^2$

$$M_B = \frac{7}{128} w L^2 - \frac{1}{2} w \left(\frac{L}{2}\right)^2 = -\frac{9}{128} w L^2 = -0.07031 w L^2 \quad \blacktriangleleft$$

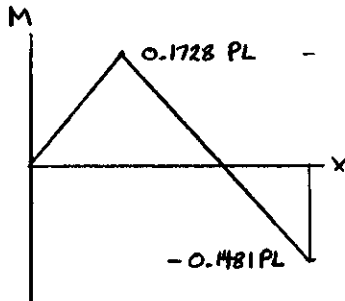
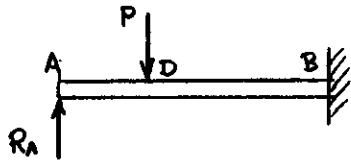
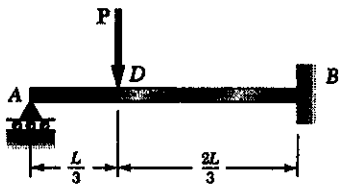
$$\frac{dM}{dv} = \frac{7}{128} w L - w v_m = 0 \quad v_m = \frac{7}{128} L$$

$$M_m = \frac{7}{128} w L \left(\frac{7}{128} L + \frac{L}{2}\right) - \frac{1}{2} w \left(\frac{7}{128} L\right)^2$$

$$= \frac{945}{32768} w L^2 = 0.02884 w L^2 \quad \blacktriangleleft$$

PROBLEM 11.128

11.128 Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.



SOLUTION

Remove support A and add reaction  $R_A$  as a load.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx$$

Portion AD  $0 < x < \frac{L}{3}$   $M = R_A x$   $\frac{\partial M}{\partial R_A} = x$

$$\begin{aligned} \frac{\partial U_{AD}}{\partial R_A} &= \frac{1}{EI} \int_0^{\frac{L}{3}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{3}} (R_A x)(x) dx \\ &= \frac{R_A}{3EI} \left(\frac{L}{3}\right)^3 = \frac{1}{81} \frac{R_A L^3}{EI} \end{aligned}$$

Portion DB  $\frac{L}{3} < x < L$   $M = R_A x - P(x - \frac{L}{3})$

$$\frac{\partial M}{\partial R_A} = x$$

$$\begin{aligned} \frac{\partial U_{DB}}{\partial R_A} &= \frac{1}{EI} \int_{\frac{L}{3}}^L M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_{\frac{L}{3}}^L [R_A x - P(x - \frac{L}{3})] x dx \\ &= \frac{R_A}{EI} \int_{\frac{L}{3}}^L x^2 dx - \frac{P}{EI} \int_{\frac{L}{3}}^L (x^2 - \frac{L}{3} x) dx \\ &= \frac{R_A}{3EI} [L^3 - (\frac{L}{3})^3] - \frac{P}{EI} \left[ \frac{1}{3} (L^3 - (\frac{L}{3})^3) - \frac{L}{6} (L^2 - (\frac{L}{3})^2) \right] \\ &= \left(\frac{1}{3} - \frac{1}{81}\right) \frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54}\right) \frac{PL^3}{EI} \end{aligned}$$

$$\delta_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81}\right) \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = 0$$

$$R_A = \frac{14}{27} P$$

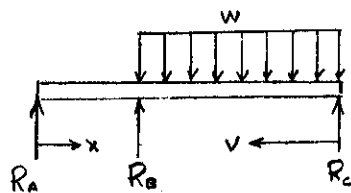
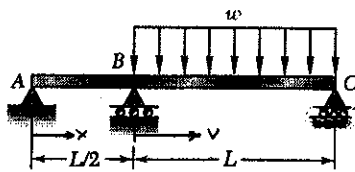
Bending moments

$$M_D = R_A \left(\frac{L}{3}\right) = \frac{14}{81} PL = 0.1728 PL$$

$$M_B = R_A L - P\left(\frac{2L}{3}\right) = -\frac{4}{27} PL = -0.1481 PL$$

PROBLEM 11.129

11.129 For the uniform beam and loading shown, determine the reaction at each support.



SOLUTION

Remove support A and add reaction  $R_A$  as a load.

$$\sum M_B = 0 \quad -R_A \frac{L}{2} - \frac{1}{2} w L^2 + R_C L = 0$$

$$R_C = \frac{1}{2} R_A + \frac{1}{2} w L$$

$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^L \frac{M^2}{2EI} dv$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

Portion AB:  $M = R_A x, \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A}{3EI} \left(\frac{L}{2}\right)^3 = \frac{1}{24} \frac{R_A L^3}{EI}$$

Portion BC:  $M = R_C v - \frac{1}{2} w v^2 = \frac{1}{2} R_A v + \frac{1}{2} w L v - \frac{1}{2} w L v^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2} v$$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI} \int_0^L \left[ \frac{1}{2} R_A v + \frac{1}{2} w (L v - v^2) \right] \left( \frac{1}{2} v \right) dv = \frac{1}{4EI} \int_0^L [R_A v^2 + w (L v^2 - v^3)] dv$$

$$= \frac{1}{4EI} \left[ R_A \frac{L^3}{3} + w \left( \frac{L^4}{3} - \frac{L^4}{4} \right) \right] = \frac{R_A L^3}{12EI} + \frac{w L^4}{48EI}$$

$$S_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left( \frac{1}{24} + \frac{1}{12} \right) \frac{R_A L^3}{EI} + \frac{w L^4}{48EI} = 0$$

$$R_A = -\frac{1}{6} w L = \frac{1}{6} w L \downarrow$$

$$R_C = \frac{1}{2} \left( -\frac{1}{6} w L \right) + \frac{1}{2} w L = \frac{5}{12} w L \uparrow$$

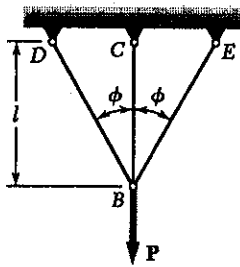
$$+\uparrow \sum F_y = 0 \quad R_A + R_B + R_C - w L = 0$$

$$-\frac{1}{6} w L + R_B + \frac{5}{12} w L - w L = 0$$

$$R_B = \frac{3}{4} w L$$

PROBLEM 11.130

11.130 Three members of the same material and same cross-sectional area are used to support the load  $P$ . Determine the force in member  $BC$ .



SOLUTION

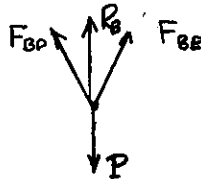
Detach member  $BC$  at support  $C$ .

Add reaction  $R_c$  as a load

$$U = \sum \frac{F^2 L}{2EA} \quad y_c = \frac{\partial U}{\partial R_c} = \sum \frac{FL}{EA} \frac{\partial F}{\partial R_c} = 0$$

Joint  $C$  ,  $F_{bc} = R_c$

Joint  $B$



$$\rightarrow \sum F_x = 0 \quad F_{BE} \sin \phi - F_{BD} \sin \phi = 0 \quad F_{BE} = F_{BD}$$

$$\uparrow \sum F_y = 0 \quad F_{BD} \cos \phi + F_{BE} \cos \phi + R_B - P$$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2 \cos \phi}$$

Member	F	$\partial F / \partial R_B$	L	$(FL/EA) \partial F / \partial R_B$
BD	$(P - R_B) / 2 \cos \phi$	$-1 / 2 \cos \phi$	$l / \cos \phi$	$(R_B - P) l / 4EA \cos^3 \phi$
BE	$(P - R_B) / 2 \cos \phi$	$-1 / 2 \cos \phi$	$l / \cos \phi$	$(R_B - P) l / 4EA \cos^3 \phi$
BC	$R_B$	1	$l$	$R_B l / EA$

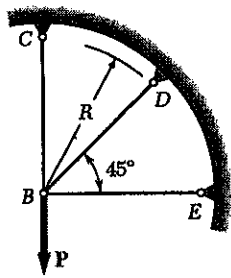
$$y_B = -Pl / 2EA \cos^3 \phi + R_B l / 2EA \cos^3 \phi + R_B l / EA = 0$$

$$R_B = \frac{P}{1 + 2 \cos^3 \phi}$$

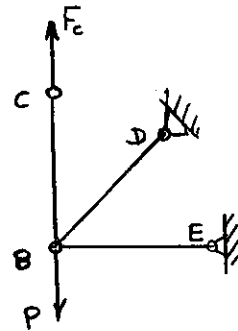
$$F_{bc} = R_B = \frac{P}{1 + 2 \cos^3 \phi}$$

PROBLEM 11.131

11.131 Three members of the same material and same cross-sectional area are used to support the load  $P$ . Determine the force in member  $BC$ .

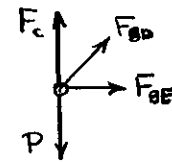


SOLUTION



Detach member  $BC$  from its support at point  $C$ . Add reaction  $F_c$  as a load.

Joint B.



$$\uparrow \Sigma F_y = 0$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_c - P = 0$$

$$F_{BD} = \sqrt{2} P - \sqrt{2} F_c$$

$$\rightarrow \Sigma F_x = 0$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_{BE} = 0$$

$$F_{BE} = -P + F_c$$

$$S_c = \frac{R}{EA} (-3P + 4F_c) = 0$$

$$F_c = \frac{3}{4} P$$

$$F_{BC} = F_c = \frac{3}{4} P$$

$$U = \frac{\Sigma F^2 R}{2EA} = \frac{R}{2EA} \Sigma F^2$$

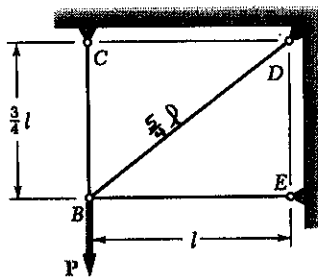
$$S_c = \frac{\partial U}{\partial F_c} = \frac{R}{EA} \Sigma F \frac{\partial F}{\partial F_c} = 0$$

Member	F	$\partial F / \partial F_c$	$F(\partial F / \partial F_c)$
BC	$F_c$	1	$F_c$
BD	$\sqrt{2} P - \sqrt{2} F_c$	$-\sqrt{2}$	$-2P + 2F_c$
BE	$-P + F_c$	1	$-P + F_c$
$\Sigma$			$-3P + 4F_c$



PROBLEM 11.132

11.132 Three members of the same material and same cross-sectional area are used to support the load  $P$ . Determine the force in member  $BC$ .



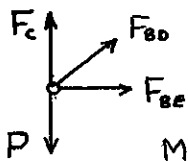
SOLUTION

Detach member  $BC$  from support  $C$ . Add reaction  $F_c$  as a load.

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$S_c = \frac{\partial U}{\partial F_c} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_c} L$$

Joint B



$$+\uparrow \sum F_y = 0 \quad F_c - P + \frac{3}{5} F_{BD} = 0 \quad F_{BD} = \frac{5}{3} P - \frac{5}{3} F_c$$

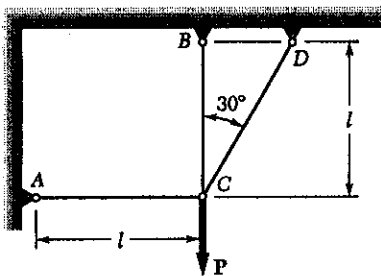
$$\pm \sum F_x = 0 \quad F_{BE} + \frac{4}{5} F_{BD} = 0 \quad F_{BE} = -\frac{4}{3} P + \frac{4}{3} F_c$$

Member	F	$\partial F / \partial F_c$	L	$F(\partial F / \partial F_c) L$
BC	$F_c$	1	$\frac{3}{4} l$	$\frac{3}{4} F_c l$
BD	$\frac{5}{3} P - \frac{5}{3} F_c$	$-\frac{5}{3}$	$\frac{5}{4} l$	$-\frac{125}{36} P l + \frac{125}{36} F_c l$
BE	$-\frac{4}{3} P + \frac{4}{3} F_c$	$\frac{4}{3}$	$l$	$-\frac{16}{9} P l + \frac{16}{9} F_c l$
$\Sigma$				$-\frac{31}{4} P l + 6 F_c l$

$$S_c = \frac{1}{EA} \left( -\frac{31}{4} P l + 6 F_c l \right) = 0 \quad F_c = \frac{7}{8} P \quad F_{BC} = F_c = \frac{7}{8} P$$

PROBLEM 11.133

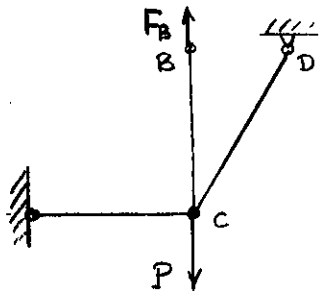
11.133 Three members of the same material and same cross-sectional area are used to support the load P. Determine the force in member BC.



SOLUTION

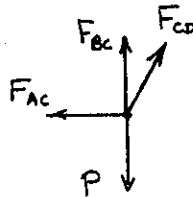
Cut member BC at end B and replace member force  $F_{bc}$  by load  $F_B$  acting on member BC at B.

$$S_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_B} L$$



Joint C  $\uparrow \Sigma F_y = 0 \quad \frac{\sqrt{3}}{2} F_{cd} + F_{bc} - P = 0$

$$F_{cd} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$$



$\pm \Sigma F_x = 0 \quad F_{ac} - \frac{1}{2} F_{cd} = 0$

$$F_{ac} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$$

Member	F	$\partial F / \partial F_B$	L	$F(\partial F / \partial F_B)L$
AC	$F_B$	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{\sqrt{3}} P l + \frac{1}{\sqrt{3}} F_B l$
CD	$\frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} l$	$-\frac{8}{\sqrt{3}} P l + \frac{8}{\sqrt{3}} F_B l$
$\Sigma$				$-\left(\frac{1}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) P l + \left(\frac{4}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) F_B l$

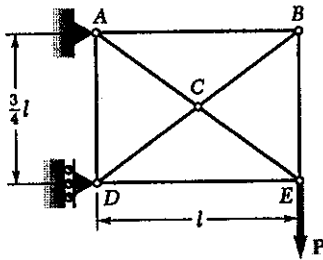
$$S_B = -\left(\frac{1}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) \frac{P l}{EA} + \left(\frac{4}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) \frac{F_B l}{EA}$$

$$F_B = \frac{\frac{1}{\sqrt{3}} + \frac{8}{\sqrt{3}}}{\frac{4}{\sqrt{3}} + \frac{8}{\sqrt{3}}} P = \frac{8 + \sqrt{3}}{8 + 4\sqrt{3}} P = 0.652 P$$

$$F_{bc} = F_B = 0.652 P$$

PROBLEM 11.134

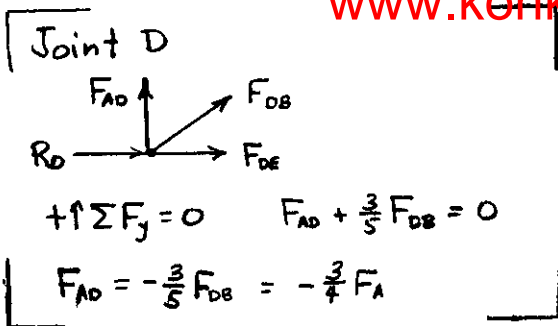
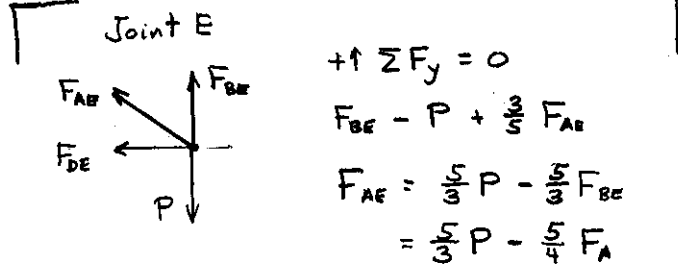
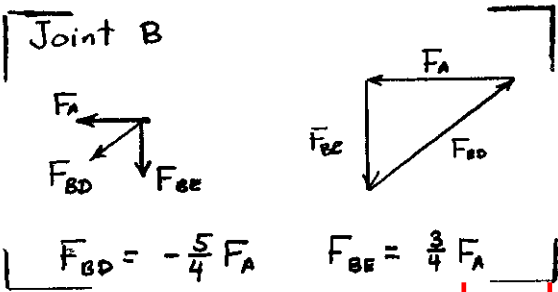
11.134 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



SOLUTION

Cut member AB at end A and replace member force  $F_{AB}$  by load  $F_A \leftarrow$  acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$



$$+\uparrow \Sigma F_y = 0$$

$$F_{BE} - P + \frac{3}{5} F_{AE} = 0$$

$$F_{AE} = \frac{5}{3} P - \frac{5}{3} F_{BE}$$

$$= \frac{5}{3} P - \frac{5}{4} F_A$$

$$+\rightarrow \Sigma F_x = 0 \quad -\frac{4}{5} F_{AE} - F_{DE} = 0$$

$$F_{DE} = -\frac{4}{5} F_{AE} = -\frac{4}{3} P + F_A$$

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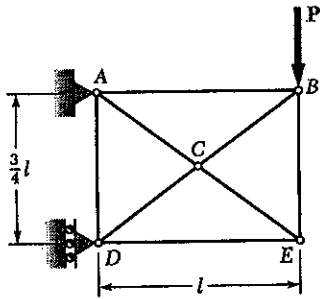
Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A) L$
AB	$F_A$	1	$l$	$F_A l$
AD	$-\frac{3}{4} F_A$	$-\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$\frac{125}{64} F_A l$
BE	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	$l$	$-\frac{4}{3} P l + F_A l$
$\Sigma$				$-\frac{63}{16} P l + \frac{27}{4} F_A l$

$$S_A = \frac{1}{EA} \left( -\frac{63}{16} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{7}{12} P$$

$$F_{AB} = F_A = \frac{7}{12} P = 0.583 P$$

PROBLEM 11.135

11.135 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



SOLUTION

Cut member AB at end A and replace member force  $F_{AB}$  by load  $F_A \leftarrow$  acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B  $\rightarrow \Sigma F_x = 0$      $-F_A - \frac{4}{5} F_{BD} = 0$      $F_{BD} = -\frac{5}{4} F_A$

$\uparrow \Sigma F_y = 0$      $-P - F_{BE} - \frac{3}{5} F_{BD} = 0$      $F_{BE} = -P + \frac{3}{4} F_A$

Joint E  $\uparrow \Sigma F_y = 0$      $F_{BE} + \frac{3}{5} F_{AE} = 0$      $F_{AE} = \frac{5}{3} P - \frac{5}{4} F_A$

$\rightarrow \Sigma F_x = 0$      $-\frac{4}{5} F_{AE} = F_{DE} = 0$      $F_{DE} = -\frac{4}{3} P + F_A$

Joint D  $\uparrow \Sigma F_y = 0$      $F_{AD} + \frac{3}{5} F_{BD} = 0$

$$F_{AD} = \frac{3}{4} F_A$$

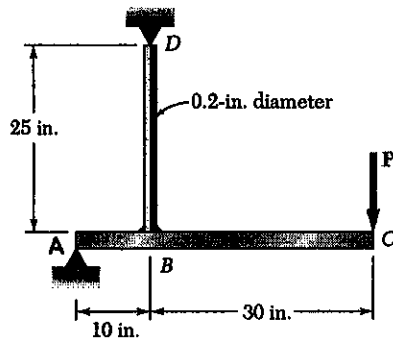
Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A)L$
AB	$F_A$	1	$l$	$F_A l$
AD	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$\frac{125}{64} F_A l$
BE	$-P + \frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{9}{16} P l + \frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	$l$	$-\frac{4}{3} P l + F_A l$
$\Sigma$				$-\frac{9}{2} P l + \frac{27}{4} F_A l$

$$S_A = \frac{1}{EA} \left( -\frac{9}{2} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{2}{3} P$$

$$F_{AB} = F_A = \frac{2}{3} P = 0.667 P$$

PROBLEM 11.136

11.136 The steel bar  $ABC$  has a square cross section of side 0.75 in. and is subjected to a 50-lb load  $P$ . Using  $E = 29 \times 10^6$ , determine the deflection of point  $C$ .



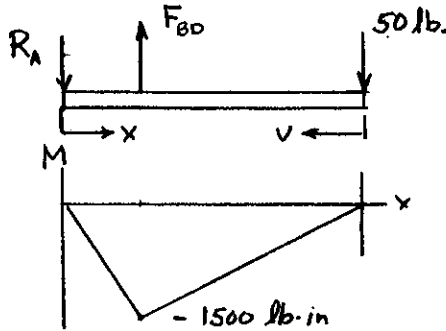
SOLUTION

Assume member  $BD$  is a two-force member.

$$\sum M_A = 0 \quad 10 F_{BD} - (40)(50) = 0 \quad F_{BD} = 200 \text{ lb.}$$

$$A_{BD} = \frac{\pi}{4} (0.2)^2 = 31.416 \times 10^{-3} \text{ in}^2$$

$$U_{BD} = \frac{F_{BD}^2 L_{BD}}{2EA} = \frac{(200)^2 (25)}{(2)(29 \times 10^6)(31.416 \times 10^{-3})} = 0.5488 \text{ in}\cdot\text{lb.}$$



Member  $ABC$

$$I = \frac{1}{12} (0.75)(0.75)^3 = 26.367 \times 10^{-3} \text{ in}^4$$

Portion  $AB \quad M = -1500 \frac{x}{10} = -150x$

$$U_{AB} = \int_0^{10} \frac{M^2}{2EI} dx = \frac{150^2}{2EI} \int_0^{10} x^2 dx = \frac{(150)^2 (10^3)}{(2)(29 \times 10^6)(26.367 \times 10^{-3})(3)} = 4.904 \text{ in}\cdot\text{lb.}$$

Portion  $BC: M = -50v \quad U_{BC} = \int_0^{30} \frac{M^2}{2EI} dv = \frac{50^2}{2EI} \int_0^{30} v^2 dv = \frac{(50)^2 (30)^3}{(2)(29 \times 10^6)(26.367 \times 10^{-3})(3)} = 14.713 \text{ in}\cdot\text{lb.}$

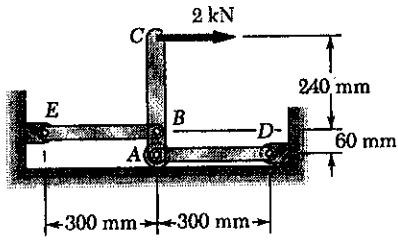
Total  $U = U_{BD} + U_{AB} + U_{BC} = 20.166 \text{ in}\cdot\text{lb.}$

$$\frac{1}{2} P \delta_c = U$$

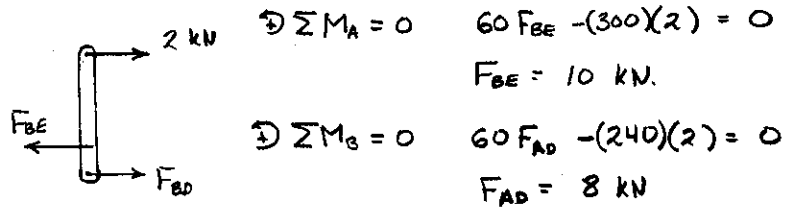
$$\delta_c = \frac{2U}{P} = \frac{(2)(20.166)}{50} = 0.807 \text{ in.} \downarrow$$

PROBLEM 11.137

11.137 The steel bars  $BE$  and  $AD$  have each a  $5 \times 15$ -mm cross section. Assuming that lever  $ABC$  is rigid and using  $E = 200$  GPa, determine the deflection of point  $C$ .



SOLUTION



$$\begin{aligned} \sum M_A = 0 & \quad 60 F_{BE} - (300)(2) = 0 \\ & \quad F_{BE} = 10 \text{ kN} \\ \sum M_B = 0 & \quad 60 F_{AD} - (240)(2) = 0 \\ & \quad F_{AD} = 8 \text{ kN} \end{aligned}$$

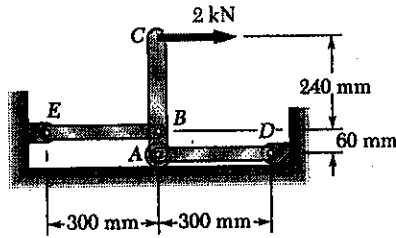
For bars  $BE$  and  $AD$   $A = 5 \times 15 = 75 \text{ mm}^2 = 75 \times 10^{-6} \text{ m}^2$

$$\begin{aligned} U &= \frac{F_{BE}^2 L_{BE}}{2EA} + \frac{F_{AD}^2 L_{AD}}{2EA} = \frac{(10 \times 10^3)^2 (300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} + \frac{(8 \times 10^3)^2 (300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} \\ &= 1.0000 + 0.6400 = 1.6400 \text{ N}\cdot\text{m} \end{aligned}$$

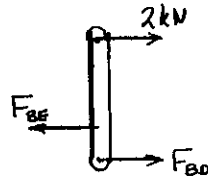
$$\frac{1}{2} P \delta_c = U \quad \delta_c = \frac{2U}{P} = \frac{(2)(1.6400)}{2 \times 10^3} = 1.64 \times 10^{-3} \text{ m} = 1.64 \text{ mm} \rightarrow$$

PROBLEM 11.138

11.138 The steel bars BE and AD have each a 5 × 15-mm cross section and the steel lever ABC has a square cross section of side 25 mm. Using E = 200 GPa, determine the deflection of point C.



SOLUTION



$$\rightarrow \sum M_A = 0 \quad 60F_{BE} - (300)(2) = 0$$

$$F_{BE} = 10 \text{ kN}$$

$$\rightarrow \sum M_B = 0 \quad 60F_{BD} - (240)(2) = 0$$

$$F_{BD} = 8 \text{ kN}$$

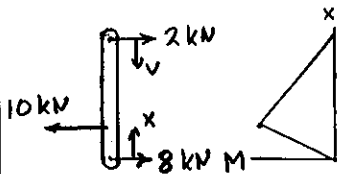
For bars BE and AD

$$A = 5 \times 15 = 75 \text{ mm}^2 = 75 \times 10^{-6} \text{ m}^2$$

$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2EA} = \frac{(10)^2 (300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} = 1.0000 \text{ J}$$

$$U_{AD} = \frac{F_{AD}^2 L_{AD}}{2EA} = \frac{(8)^2 (300 \times 10^{-3})}{(2)(200 \times 10^9)(75 \times 10^{-6})} = 0.6400 \text{ J}$$

Beam ABC:  $I = \frac{1}{12}(25)(25)^3 = 32.552 \times 10^3 \text{ mm}^4 = 32.552 \times 10^{-9} \text{ m}^4$



Bending moment at B

$$M_B = (2 \times 10^3)(240 \times 10^{-3}) = 480 \text{ N}\cdot\text{m}$$

Portion AB:  $M = \frac{480}{L_{AB}} x$

$$U_{AB} = \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{(480)^2}{2EI L_{AB}^2} \int_0^{L_{AB}} x^2 dx$$

$$= \frac{(480)^2 L_{AB}^3}{6EI L_{AB}^2} = \frac{(480)^2 L_{AB}}{6EI}$$

$$= \frac{(480)^2 (60 \times 10^{-3})}{(6)(200 \times 10^9)(32.552 \times 10^{-9})} = 0.3539 \text{ J}$$

Portion BC:  $M = \frac{480}{L_{BC}} v$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI} dv = \frac{(480)^2}{2EI L_{BC}^2} \int_0^{L_{BC}} v^2 dv = \frac{(480)^2 L_{BC}^3}{6EI L_{BC}^2} = \frac{(480)^2 L_{BC}}{6EI}$$

$$= \frac{(480)^2 (240 \times 10^{-3})}{(6)(200 \times 10^9)(32.552 \times 10^{-9})} = 1.4156 \text{ J}$$

Total  $U = U_{BE} + U_{AD} + U_{AB} + U_{BC}$

$$= 1.0000 + 0.6400 + 0.3539 + 1.4156 = 3.4095 \text{ J}$$

$$\frac{1}{2} P \delta_c = U$$

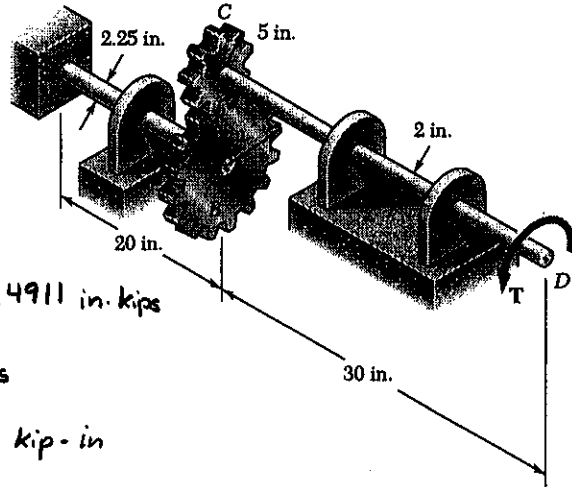
$$\delta_c = \frac{2U}{P} = \frac{2(3.4095)}{2 \times 10^3} = 3.41 \times 10^{-3} \text{ m}$$

$$= 3.41 \text{ mm} \rightarrow$$

PROBLEM 11.139

11.139 Two solid steel shafts are connected by the gears shown. Using  $G = 11.2 \times 10^6$  psi, determine the strain energy in each shaft when a 24 kip-in. torque is applied at  $D$ . (Ignore the strain energy due to bending of the shafts.)

SOLUTION



Shaft CD:  $T_{CD} = T_C = 24 \text{ kip-in}$

$$J_{CD} = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} \left(\frac{2.0}{2}\right)^4 = 1.5708 \text{ in}^4$$

$L_{CD} = 30 \text{ in.}, G = 11.2 \times 10^6 \text{ psi} = 11.2 \times 10^3 \text{ ksi}$

$$U_{CD} = \frac{T_{CD}^2 L_{CD}}{2GJ_{CD}} = \frac{(24)^2 (30)}{(2)(11.2 \times 10^3)(1.5708)} = 0.4911 \text{ in-kips}$$

Gear C  $F_{CB} = \frac{T_C}{r_C} = \frac{T_{CD}}{r_C} = \frac{24}{5} = 4.8 \text{ kips}$

Gear B  $T_B = r_B F_{CB} = (8)(4.8) = 38.4 \text{ kip-in}$

Shaft AB  $T_{AB} = T_B = 38.4 \text{ kip-in}$   $L_{AB} = 20 \text{ in}$

$$J_{AB} = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} \left(\frac{2.25}{2}\right)^4 = 2.5161 \text{ in}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(38.4)^2 (20)}{(2)(11.2 \times 10^3)(2.5161)} = 0.5233 \text{ in-kips}$$

Total  $U = U_{AB} + U_{CD} = 0.5233 + 0.4911 = 1.0144 \text{ in-kips.}$

PROBLEM 11.140

11.140 Two solid steel shafts are connected by the gears shown. Using  $G = 11.2 \times 10^6$  psi, determine the angle through which end  $D$  rotates when  $T = 24 \text{ kip-in}$ .

(Ignore the strain energy of bending of the shafts.)

SOLUTION

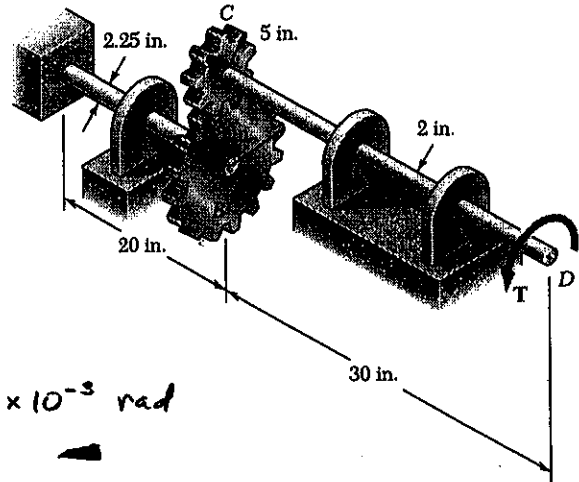
From Prob. 11.139

$$U = 1.0144 \text{ in-lb.}$$

$$\frac{1}{2} T_D \phi_D = U$$

$$\phi_D = \frac{2U}{T_D} = \frac{(2)(1.0144)}{24} = 84.5 \times 10^{-3} \text{ rad}$$

$$= 4.84^\circ$$





PROBLEM 11.141

11.141 (a) Determine the modulus of resilience of a grade of structural steel for which  $\sigma_y = 300$  MPa and  $E = 200$  GPa. (b) Determine the required yield strength of an aluminum alloy for which  $E = 72$  GPa if the modulus of resilience of the alloy is to be the same as that of the structural steel.

SOLUTION

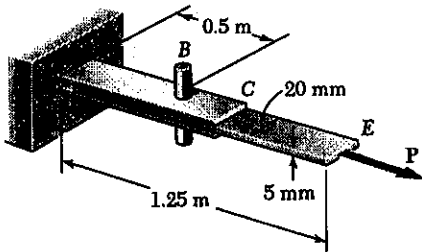
(a)  $E = 200 \times 10^9$  Pa,  $\sigma_y = 300 \times 10^6$  Pa

$$U_y = \frac{\sigma_y^2}{2E} = \frac{(300 \times 10^6)^2}{2(200 \times 10^9)} = 225 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 225 \text{ kJ}/\text{m}^3$$

(b)  $\sigma_{ya} = \sqrt{2E_a U_{ya}} = \sqrt{2(72 \times 10^9)(225 \times 10^3)} = 180 \times 10^6 \text{ Pa} = 180 \text{ MPa}$

PROBLEM 11.142

11.142 A single 6-mm-diameter steel pin B is used to connect the steel strip DE to two aluminum strips, each of 20-mm width and 5-mm thickness. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that for the pin at B the allowable shearing stress is  $\tau_{all} = 85$  MPa, determine, for the loading shown, the maximum strain energy that can be acquired by the assembled strips.



SOLUTION

$$A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (6)^2 = 28.274 \text{ mm}^2 = 28.274 \times 10^{-6} \text{ m}^2$$

$$\tau_{all} = 85 \times 10^6 \text{ Pa}$$

$$\text{Double shear } P = 2A\tau = (2)(28.274 \times 10^{-6})(85 \times 10^6) = 4.8066 \times 10^3 \text{ N}$$

For strips AB, DB, BE  $A = (20)(5) = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$

$$F_{AB} = F_{DB} = \frac{1}{2}P = 2.4033 \times 10^3 \text{ N}$$

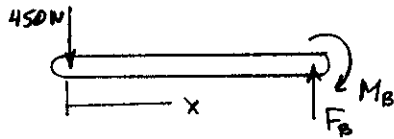
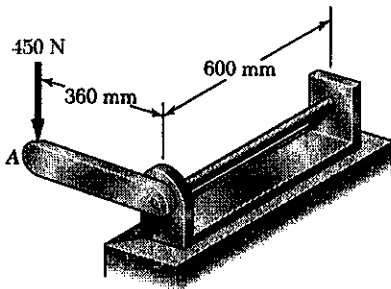
$$U_{AB} = U_{DB} = \frac{F_{AB}^2 L_{AB}}{2E_a A_{AB}} = \frac{(2.4033)^2 (0.5)}{2(70 \times 10^9)(100 \times 10^{-6})} = 206.3 \times 10^{-3} \text{ J}$$

$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2E_s A_{BE}} = \frac{(4.8066 \times 10^3)^2 (1.25 - 0.5)}{2(200 \times 10^9)(100 \times 10^{-6})} = 433.2 \times 10^{-3} \text{ J}$$

Total:  $U = U_{AB} + U_{DB} + U_{BE} = 846 \times 10^{-3} \text{ J} = 0.846 \text{ J}$

PROBLEM 11.143

11.143 The 18-mm-diameter steel rod  $BC$  is attached to the lever  $AB$  and to the fixed support  $C$ . The uniform steel lever  $AB$  is 9 mm wide and 24 mm deep. Using  $E = 200$  GPa,  $G = 77$  GPa, and the method of work and energy, determine the deflection of point  $A$



SOLUTION

Member AB

$$I = \frac{1}{12}(9)(24)^3 = 10.368 \times 10^3 \text{ mm}^4 = 10.368 \times 10^{-9} \text{ m}^4$$

$$E = 200 \times 10^9$$

$$M = 450x \quad M_B = 162 \text{ N}\cdot\text{m}$$

$$\begin{aligned} U_{AB} &= \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{(450)^2}{2EI} \int_0^{L_{AB}} x^2 dx \\ &= \frac{(450)^2 L_{AB}^3}{6EI} = \frac{(450)^2 (360 \times 10^{-3})^3}{(6)(200 \times 10^9)(10.368 \times 10^{-9})} \\ &= 0.75938 \text{ J} \end{aligned}$$

Member BC  $T = M_B = 162 \text{ N}\cdot\text{m} \quad L = 600 \times 10^{-3} \text{ m}$

$$J = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} \left(\frac{18}{2}\right)^4 = 10.306 \times 10^3 \text{ mm}^4 = 10.306 \times 10^{-9} \text{ m}^4$$

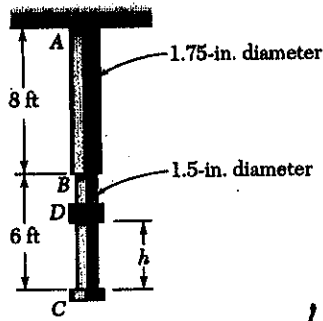
$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(162)^2 (600 \times 10^{-3})}{(2)(77 \times 10^9)(10.306 \times 10^{-9})} = 9.9213 \text{ J}$$

Total  $U = U_{AB} + U_{BC} = 10.681 \text{ J}$

$$\frac{1}{2} P \delta_A = U \quad \delta_A = \frac{2U}{P} = \frac{(2)(10.681)}{450} = 47.5 \times 10^{-3} \text{ m} = 47.5 \text{ mm} \downarrow$$

PROBLEM 11.144

11.144 The 75-lb collar  $D$  is released from rest in the position shown and is stopped by a plate attached at end  $C$  of the vertical rod  $ABC$ . Knowing that  $E = 29 \times 10^6$  psi for both portions of the rod, determine the distance  $h$  for which the maximum stress in the rod is 36 ksi.



SOLUTION

Portion BC:  $A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (1.5)^2 = 1.76715 \text{ in}^2$

$\sigma_{BC} = 30000 \text{ psi}$        $L_{BC} = 6 \text{ ft} = 72 \text{ in.}$

Force at C  $P = \sigma_{BC} A_{BC} = 53014 \text{ lb.}$

$U_{BC} = \frac{P^2 L_{BC}}{2EA_{BC}} = \frac{(53014)^2 (72)}{(2)(29 \times 10^6)(1.76715)} = 1974.3 \text{ in.}\cdot\text{lb.}$

Portion AB:  $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (1.75)^2 = 2.40528 \text{ in}^2$ .       $L_{AB} = 8 \text{ ft} = 96 \text{ in.}$

$U_{AB} = \frac{P^2 L_{AB}}{2EA_{AB}} = \frac{(53014)^2 (96)}{(2)(29 \times 10^6)(2.40528)} = 1934.0 \text{ in.}\cdot\text{lb.}$

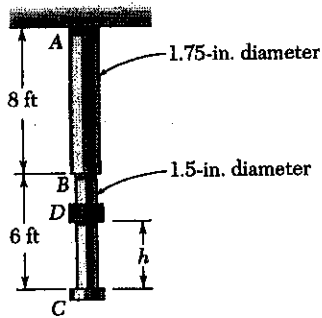
Total  $U = U_{AB} + U_{BC} = 3908.3 \text{ in.}\cdot\text{lb.}$

$\frac{1}{2} P s_c = U$        $s_c = \frac{2U}{P} = \frac{(2)(3908.3)}{53014} = 0.14744 \text{ in.}$

$W(h + s_c) = U$        $h = \frac{U}{W} - s_c = \frac{3908.3}{75} - 0.14744 = 52.0 \text{ in.}$

PROBLEM 11.145

11.145 The 75-lb collar  $D$  is released from rest when  $h = 20$  in. and is stopped by a plate attached at end  $C$  of the vertical rod  $ABC$ . Knowing that  $E = 29 \times 10^6$  psi for both portions of the rod, determine (a) the maximum deflection of end  $C$ , (b) the equivalent static load, (c) the maximum stress that occurs in the rod.



SOLUTION

Let  $P_m$  be the equivalent static load in lb.

Portion AB:  $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (1.75)^2 = 2.40528 \text{ in}^2$

$L_{AB} = 8 \text{ ft} = 96 \text{ in}$

$U_{AB} = \frac{P_m^2 L_{AB}}{2EA} = \frac{P_m^2 (96)}{(2)(29 \times 10^6)(2.40528)} = 688.14 \times 10^{-9} P_m^2$

Portion BC:  $A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (1.5)^2 = 1.76715 \text{ in}^2$        $L_{BC} = 6 \text{ ft} = 72 \text{ in}$

$U_{BC} = \frac{P_m^2 L_{BC}}{2EA_{BC}} = \frac{P_m^2 (72)}{(2)(29 \times 10^6)(1.76715)} = 702.48 \times 10^{-9} P_m^2$

Total:  $U = U_{AB} + U_{BC} = 1.39062 \times 10^{-6} P_m^2$

$\frac{1}{2} P_m \delta_m = U, \delta_m = \frac{2U}{P_m} = 2.78124 \times 10^{-6} P_m, P_m = 359.552 \times 10^3 \delta_m$

$U = \frac{1}{2} P_m \delta_m = 179.776 \times 10^3 \delta_m^2$

Work of falling weight  $W(h + \delta_m) = 75(20 + \delta_m) = 1500 + 75 \delta_m$

Equating work and energy  $1500 + 75 \delta_m = 179.776 \times 10^3 \delta_m^2$

$\delta_m^2 - 417.185 \times 10^{-6} \delta_m - 8.3437 \times 10^{-3} = 0$

(a)  $\delta_m = \frac{1}{2} \left\{ 417.185 \times 10^{-6} + \sqrt{(417.185 \times 10^{-6})^2 + (4)(8.3437 \times 10^{-3})} \right\}$

$= 0.091553 \text{ in.}$

$\delta_m = 0.0916 \text{ in.} \blacktriangleleft$

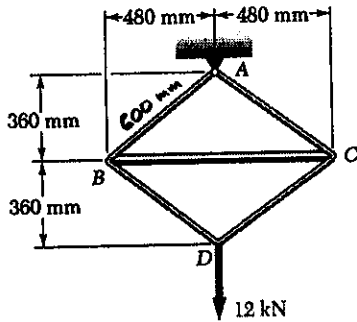
(b)  $P_m = (359.552 \times 10^3)(0.091553) = 32917 \text{ lb}$

$P_m = 32900 \text{ lb.} \blacktriangleleft$

$\sigma_m = \frac{P_m}{A_{min}} = \frac{32917}{1.76715} = 18630 \text{ psi} = 18.63 \text{ ksi} \blacktriangleleft$

PROBLEM 11.146

11.146 The steel rod BC has a 24-mm diameter and the steel cable ABDCA has a 12-mm diameter. Using  $E = 200$  GPa, determine the deflection of point D caused by the 12-kN load.



SOLUTION

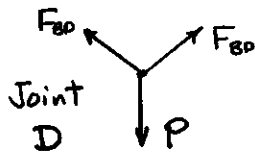
Owing to symmetry  $F_{AB} = F_{BD} = F_{DC} = F_{CA}$

$$U_{AB} = U_{BD} = U_{DC} = U_{CA}$$

$$U = 4 U_{BD} + U_{BC} = 4 \frac{F_{BD}^2 L_{BD}}{2EA_{BD}} + \frac{F_{BC}^2 L_{BC}}{2EA_{BC}}$$

Let  $P$  be the load at D

$$S_D = \frac{\partial U}{\partial P} = 4 \frac{F_{BD} L_{BD}}{EA_{BD}} \frac{\partial F_{BD}}{\partial P} + \frac{F_{BC} L_{BC}}{EA_{BC}} \frac{\partial F_{BC}}{\partial P}$$

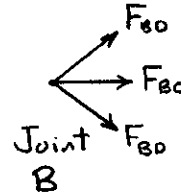


$$+\uparrow \sum F_y = 0$$

$$2 \frac{3}{5} F_{BD} - P = 0$$

$$F_{BD} = \frac{5}{6} P$$

$$\frac{\partial F_{BD}}{\partial P} = \frac{5}{6}$$



$$+\rightarrow \sum F_x = 0$$

$$F_{BC} + (2) \frac{4}{5} F_{BD} = 0$$

$$F_{BC} = -\frac{8}{5} F_{BD} = -\frac{4}{3} P$$

$$\frac{\partial F_{BC}}{\partial P} = -\frac{4}{3}$$

$$S_D = 4 \left(\frac{5}{6}\right)^2 \frac{P L_{BD}}{EA_{BD}} + \left(\frac{4}{3}\right)^2 \frac{P L_{BC}}{EA_{BC}} = \frac{P}{E} \left\{ \frac{25}{9} \frac{L_{BD}}{A_{BD}} + \frac{16}{9} \frac{L_{BC}}{A_{BC}} \right\}$$

Data:  $P = 12 \times 10^3$  N

$E = 200 \times 10^9$  Pa

$L_{BD} = 600 \times 10^{-3}$  m

$A_{BD} = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$

$L_{BC} = 960 \times 10^{-3}$  m

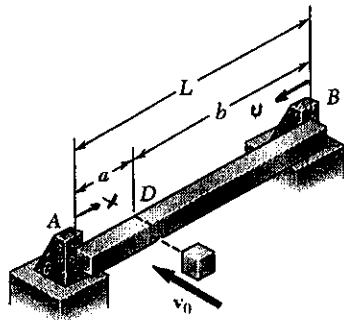
$A_{BC} = \frac{\pi}{4}(24)^2 = 452.39 \text{ mm}^2 = 452.39 \times 10^{-6} \text{ m}^2$

$$S_D = \frac{12 \times 10^3}{200 \times 10^9} \left\{ \frac{25}{9} \frac{600 \times 10^{-3}}{113.097 \times 10^{-6}} + \frac{16}{9} \frac{960 \times 10^{-3}}{452.39 \times 10^{-6}} \right\} = 1.111 \times 10^{-3} \text{ m}$$

$$= 1.111 \text{ mm} \downarrow$$

PROBLEM 11.147

11.147 The simply supported beam  $AB$  is struck squarely at  $D$  by a block of mass  $m$  moving horizontally with a velocity  $v_0$ . Show that the resulting maximum normal stress  $\sigma_m$  in the beam due to bending is independent of the location of point  $D$



SOLUTION

Let  $P_m$  be the equivalent static load at point  $D$

Reactions:  $R_A = \frac{P_m b}{L}$ ,  $R_B = \frac{P_m a}{L}$

Maximum bending moment  $M_m = R_A a = \frac{P_m a b}{L}$

Portion AD  $U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{(R_A x)^2}{2EI} dx = \frac{P_m^2 b^2}{2EI L^2} \int_0^a x^2 dx = \frac{P_m^2 b^2 a^3}{6EI L^2}$

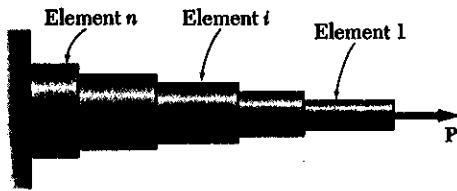
Portion DB  $U_{DB} = \int_0^b \frac{M^2}{2EI} du = \int_0^b \frac{(R_B u)^2}{2EI} du = \frac{P_m^2 a^2}{2EI L^2} \int_0^b u^2 du = \frac{P_m^2 a^2 b^3}{6EI L^2}$

Total  $U = \frac{P_m^2 a^2 b^2 (a+b)}{6EI L^2} = \frac{P_m^2 a^2 b^2}{6EI L} = \frac{M_m^2 L}{6EI}$

$\frac{1}{2} m v_0^2 = U = \frac{M_m^2 L}{6EI}$   $M_m = \sqrt{\frac{3EI m v_0^2}{L}}$

Stress  $\sigma_m = \frac{M_m c}{I} = \sqrt{\frac{3Em v_0^2 c^2}{IL}}$ , which is independent of  $a$  or  $b$ .

**PROBLEM 11.C1**



**11.C1** A rod consisting of  $n$  elements, each of which is homogeneous and of uniform cross section, is subjected to a load  $P$  applied at its free end. The length of element  $i$  is denoted by  $L_i$  and its diameter by  $d_i$ . (a) Denoting by  $E$  the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at the free end. (b) Use this program to determine the strain energy and deformation of the rods of Probs. 11.9 and 11.12.

**SOLUTION** ENTER: P AND E

FOR EACH ELEMENT

ENTER  $A_i$  AND  $D_i$

COMPUTE: NORMAL STRESS:  $\sigma_i = \frac{P}{A_i}$

STRAIN ENERGY:  $U_i = \frac{P^2 L_i}{2 A_i E}$

STRAIN ENERGY DENSITY:  $u = \frac{\sigma_i^2}{2E}$

TOTAL STRAIN ENERGY

UPDATE THROUGH  $n$  ELEMENTS

$U = U + U_i$

TOTAL DEFORMATION

$\frac{1}{2} P \Delta = U \quad ; \quad \Delta = \frac{2U}{P}$

PROGRAM OUTPUT

Problem 11.9

Axial load = 8.000 kips      Modulus of elasticity =  $29 \times 10^6$  psi

Element	Length in.	delta L in.	Stress ksi	Strain Energy in·lb	Strain Energy Density lb·in./in. <sup>3</sup>
1	24.000	0.022	26.08	86.32	11.72
2	36.000	0.022	18.11	89.92	5.65

Total Strain Energy = 176.24 in·lb  
Total Deformation = 0.0441 in.

Problem 11.12

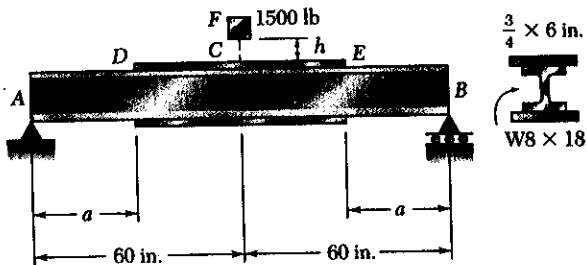
Axial load = 25.000 kN      Modulus of elasticity = 200 GPa

Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m <sup>3</sup>
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total Strain Energy = 12.1853 J  
Total Deformation = 0.9748 mm

**PROBLEM 11.C2**

**11.C2** Two 0.75 × 6-in. cover plates are welded to a W8 × 18 rolled-steel beam as shown. The 1500-lb block is to be dropped from a height  $h = 2$  in. onto the beam. (a) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of D and at the center of the beam for values of  $a$  from 0 to 60 in., using 5-in. increments. (b) From the values considered in part a, select the distance  $a$  for which the maximum normal stress is as small as possible. Use  $E = 29 \times 10^6$  psi.



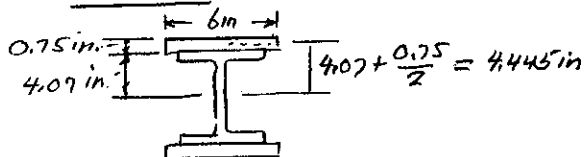
**SOLUTION**

COMPUTE AND ENTER MOMENTS OF INERTIA AND SECTION MODULI

FOR AD AND EB: W8X18

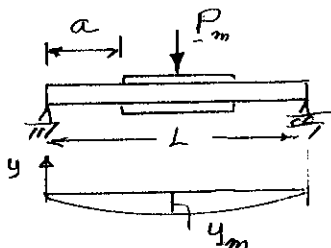
$$I_1 = 61.9 \text{ in}^4 \quad S_1 = 15.2 \text{ in}^3$$

FOR DCE: W8X18 PLUS COVER PLATES



$$I_2 = 61.9 + 2(6 \times 0.75)(4.445)^2 = 239.72 \text{ in}^4$$

$$S_2 = \frac{I_2}{(4.07 + 0.75)} = \frac{239.72}{4.82} = 49.7 \text{ in}^3$$



$$y_m = P_m \alpha$$

WHERE  $\alpha =$  INFLUENCE COEFFICIENT  
SEE NEXT PAGE FOR DETERMINATION OF  $\alpha$

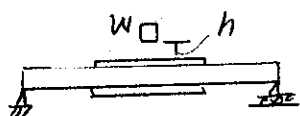
$P_m =$  EQUIVALENT STATIC LOAD

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

WORK DONE BY  $W$  IS  $W(h + y_m)$

$$\frac{1}{2} \frac{y_m^2}{\alpha} = Wh + Wy_m$$

$$\text{OR: } y_m^2 - 2W\alpha y_m - 2Wh\alpha \quad \textcircled{A}$$



POSITION 1



POSITION 2

PROGRAM SOLUTION OF  $\textcircled{A}$  FOR  $y_m$

ENTER  $L = 120 \text{ in.}$ ,  $h = 2 \text{ in.}$ ,  $W = 1500 \text{ lb}$ ,  $E = 29 \times 10^6 \text{ psi}$

FOR  $a = 0$  TO 60 in. STEP 5 in.:

SOLVE  $\textcircled{A}$  FOR  $y_m$ ,  $P_m = y_m / \alpha$ ,  $y_{ST} = W\alpha$

$$\sigma_D = \sigma_1 = \frac{1}{2} P_m a / S_1 \quad ; \quad \sigma_C = \sigma_2 = \frac{1}{4} P_m L / S_2$$

PRINT:  $a$ ,  $y_{ST}$ ,  $y_m$ ,  $P_m$ ,  $\sigma_1$ ,  $\sigma_2$ , AND  $(\sigma_1 - \sigma_2)$

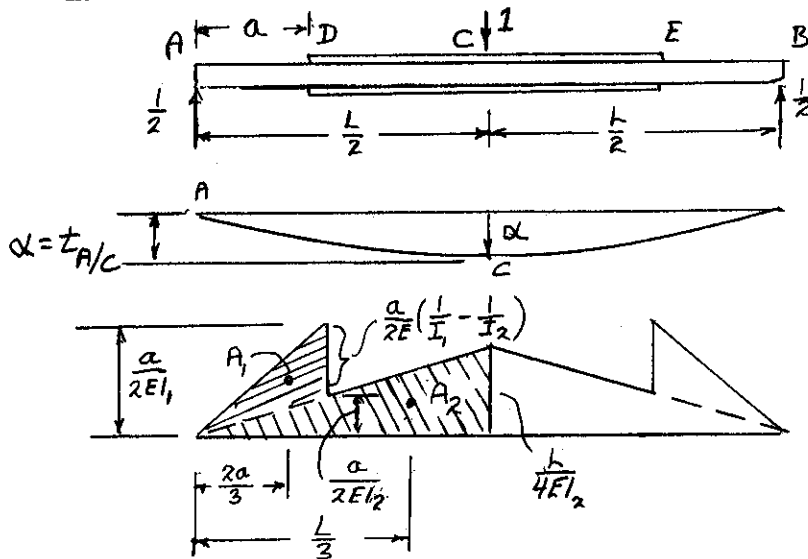
REPEAT WITH SMALLER INTERVALS TO FIND  $a$  FOR  $(\sigma_1 - \sigma_2) = 0$   
THIS IS THE DISTANCE  $a$  FOR  $\sigma_{max}$  AS SMALL AS POSSIBLE

CONTINUED



PROBLEM 11.C2 - CONTINUED

DETERMINATION OF  $\alpha$ :  $\alpha$  IS DEFLECTION AT C FOR A UNIT LOAD AT C.



$$\alpha = \delta_{A/C} = A_1 \left( \frac{2a}{3} \right) + A_2 \left( \frac{L}{3} \right) = \left[ \frac{1}{2} \frac{a}{EI_1} \left( \frac{1}{I_1} - \frac{1}{I_2} \right) \frac{a}{2} \right] \frac{2a}{3} + \left[ \frac{1}{2} \cdot \frac{L}{4EI_2} \cdot \frac{L}{2} \right] \frac{L}{3}$$

$$\alpha = \left[ \left( \frac{1}{I_1} - \frac{1}{I_2} \right) a^3 + \frac{1}{8I_2} L^3 \right] / 6E$$

PROGRAM OUTPUT

Beam = W 8x18 with two 6 by 0.75-in. cover plates  
 h = 2 in. W = 1500 lb L = 120 in.

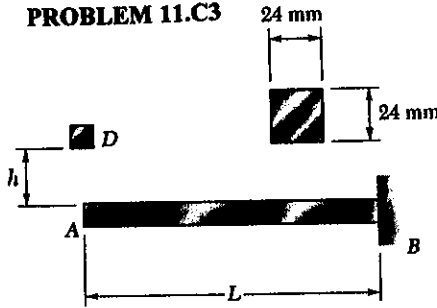
a in.	y <sub>stat</sub> in.	y <sub>max</sub> in.	P <sub>max</sub> lb	$\sigma_1$ ksi	$\sigma_2$ ksi	$\sigma_1 - \sigma_2$ ksi
0.00	0.00777	0.1842	35572	0.00	21.46	-21.46
5.00	0.00778	0.1844	35544	5.85	21.44	-15.59
10.00	0.00787	0.1855	35348	11.63	21.32	-9.69
15.00	0.00812	0.1885	34834	17.19	21.01	-3.82
20.00	0.00859	0.1942	33896	22.30	20.45	1.85
25.00	0.00938	0.2033	32509	26.73	19.61	7.13
30.00	0.01056	0.2163	30736	30.33	18.54	11.79
35.00	0.01220	0.2334	28706	33.05	17.32	15.73
40.00	0.01438	0.2546	26563	34.95	16.02	18.93
45.00	0.01718	0.2799	24436	36.17	14.74	21.43
50.00	0.02068	0.3090	22415	36.87	13.52	23.35
55.00	0.02496	0.3419	20550	37.18	12.40	24.78
60.00	0.03008	0.3783	18862	37.23	11.38	25.85

Use smaller increments to seek the smallest maximum normal stress

18.33	0.00840	0.1919	34259	20.657	20.665	-0.01
18.34	0.00840	0.1920	34257	20.667	20.664	0.00
18.35	0.00841	0.1920	34255	20.677	20.663	0.01

Max stress small as possible for a = 18.34in.  
 Smallest max stress = 20.67 ksi

**PROBLEM 11.C3**



11.C3 The 16-kg block D is dropped from a height  $h$  onto the free end of the steel bar AB. For the steel used  $\sigma_{all} = 120 \text{ MPa}$  and  $E = 200 \text{ GPa}$ . (a) Write a computer program to calculate the maximum allowable height  $h$  for values of the length  $L$  from 100 mm to 1.2 m, using 100-mm increments. (b) From the values considered in part a, select the length corresponding to the largest allowable height.

**SOLUTION**

ENTER  $\sigma_{all} = 120 \text{ MPa}$ ,  $E = 200 \text{ GPa}$ ,  $d = 0.024 \text{ m}$   
 $m = 16 \text{ kg}$ ,  $g = 9.81 \text{ m/s}^2$   
 $I = d^4/12$       $S = \frac{I}{c} = \frac{I}{d/2} = \frac{d^3}{6}$

FOR  $L = 100 \text{ m TO } 1200 \text{ m STEP } 100 \text{ mm}$

$L = L/1000$

$y_{st} = mgL^3/3EI$

$M_{max} = J_{all} S$

$P_{max} = M_{max}/L$

$y_{max} = P_{max} L^3/3EI$

FROM PROB. 11.69, PAGE 705

$y_m = y_{st} \left[ 1 + \sqrt{1 + \frac{2h}{y_{st}}} \right]$  SOLVE FOR

$h = \left[ \left( \frac{y_{max}}{y_{st}} - 1 \right)^2 - 1 \right] \frac{y_{st}}{2}$

PRINT:  $L$ ,  $y_{st}$ ,  $y_{max}$ ,  $P_{max}$ ,  $M_{max}$ ,  $h$   
 RETURN

PROGRAM OUTPUT

Problem 11.C3

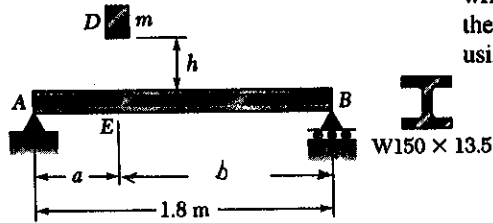
$m = 16.0 \text{ kg}$       $d = 24 \text{ mm}$       $\sigma = 120 \text{ MPa}$       $G = 200 \text{ GPa}$

L mm	y <sub>stat</sub> mm	y <sub>max</sub> mm	P <sub>max</sub> N	M <sub>max</sub> N·m	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	-0.289
1000	9.46181	16.667	276.5	276.48	-1.988
1100	12.59367	20.167	251.3	276.48	-4.020
1200	16.35000	24.000	230.4	276.48	-6.385

Use smaller increments to seek the largest height h

435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

PROBLEM 11.C4



11.C4 The block D of mass  $m = 8 \text{ kg}$  is dropped from a height  $h = 750 \text{ mm}$  onto the rolled-steel beam AB. Knowing that  $E = 200 \text{ GPa}$ , write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of  $a$  from 100 to 900 mm, using 100-mm increments.

SOLUTION

ENTER:  $L = 1.8 \text{ m}$ ,  $E = 200 \text{ GPa}$ ,  $h = 0.75 \text{ m}$   
 $m = 8 \text{ kg}$ ,  $g = 9.81 \text{ m/s}^2$   
 $I = 6.87 \times 10^{-6} \text{ m}^4$   
 $S = 91.6 \times 10^{-6} \text{ m}^3$

FOR  $a = 100 \text{ mm}$  TO  $900 \text{ mm}$  STEP  $100 \text{ mm}$

$a = a / 1000$

$b = L - a$

$y_{st} = m g a^2 b^2 / 3 E I L$

SEE PROB. 11.71, page 705 →

INFLUENCE COEFFICIENT FOR  $\Delta E$  } →  
 FOR UNIT LOAD AT E

$\alpha = a^2 b^2 / 3 E I L$

SEE PROB. 11.69, page 705 →

$y_m = y_{st} \left[ 1 + \sqrt{1 + \frac{2h}{y_{st}}} \right]$

$P_{max} = y_m / \alpha$

$M_{max} = P_{max} a b / L$

$\tau_{max} = M_{max} / S$

PRINT:  $\alpha$ ,  $y_{st}$ ,  $y_m$ ,  $P_{max}$ ,  $\tau_{max}$

RETURN

Problem 11.C4

Beam: W 150 x 13.5

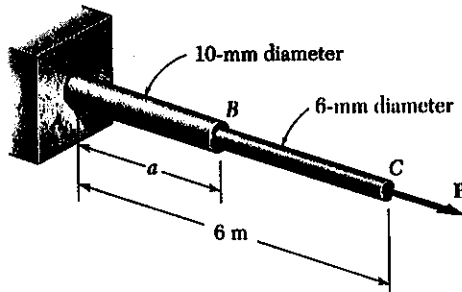
$I = 6.87 \times 10^{-6} \text{ m}^4$      $S = 91.6 \times 10^{-6} \text{ m}^3$

$L = 1.8 \text{ m}$      $h = 750 \text{ mm}$      $m = 8 \text{ kg}$      $g = 9.81 \text{ m/s}^2$

a mm	y <sub>stat</sub> mm	y <sub>max</sub> mm	P <sub>max</sub> N	σ <sub>max</sub> MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

NOTE: THE SMALL VARIATION IN  $\tau_{max}$ . THIS IS DUE TO THE ENERGY ACQUIRED BY THE MASS AS IT FALLS THROUGH  $y_{max}$ . SEE PROB. 11.147, page 731, FOR A CASE WHERE ENERGY DELIVERED IS CONSTANT AND  $\tau_{max}$  IS ALSO CONSTANT.

**PROBLEM 11.C5**



11.C5 The steel rods AB and BC are made of a steel for which  $\sigma_Y = 300 \text{ MPa}$  and  $E = 200 \text{ GPa}$ . (a) Write a computer program to calculate, for values of  $a$  from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (b) For each value of  $a$  considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

**SOLUTION**

ENTER:  $\sigma_Y = 300 \text{ MPa}$ ,  $E = 200 \text{ GPa}$ ,  $L = 6 \text{ m}$

$AREA_{AB} = \frac{\pi}{4} (0.010 \text{ m})^2$ ,  $AREA_{BC} = \frac{\pi}{4} (0.006 \text{ m})^2$

$P_m = \sigma_Y AREA_{BC}$

FOR  $a = 0$  TO  $6 \text{ m}$  STEP  $1 \text{ m}$

$U = \frac{P_m^2}{2E} \left( \frac{a}{AREA_{AB}} + \frac{L-a}{AREA_{BC}} \right)$

FOR UNIFORM ROD OF SAME VOLUME

$VOL = a (AREA_{AB}) + (L-a) (AREA_{BC})$

$d = \sqrt{\frac{4 VOL}{\pi L}}$

$AREA_{NEW} = \frac{\pi}{4} d^2$

$P_{NEW} = \sigma_Y (AREA_{NEW})$

$U_{NEW} = \frac{P_{NEW}^2 L}{2E (AREA_{NEW})}$

PRINT  $a$ ,  $U$ ,  $VOL$ ,  $d$ ,  $P_{NEW}$ ,  $U_{NEW}$   
RETURN

PROGRAM OUTPUT

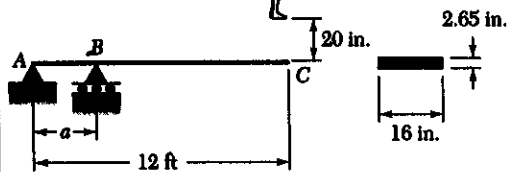
Problem 11C5

$\sigma_{mY} = 300 \text{ MPa}$ ,  $P_m = 8482 \text{ N}$ ,  $L = 6 \text{ m}$ ,  $E = 200 \text{ GPa}$

a m	U J	Vol m <sup>3</sup>	d mm	New P N	newU J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10995.58	49.48
2.00	30.03	270.18	7.57	13508.85	60.79
3.00	25.96	320.44	8.25	16022.12	72.10
4.00	21.88	370.71	8.87	18535.40	83.41
5.00	17.81	420.97	9.45	21048.67	94.72
6.00	13.74	471.24	10.00	23561.95	106.03

**PROBLEM 11.C6**

11.C6 A 160-lb diver jumps from a height of 20 in. onto end C of a diving board having the uniform cross section shown. Write a computer program to calculate for values of  $a$  from 10 to 50 in., using 10-in. increments, (a) the maximum deflection of point C, (b) the maximum bending moment in the board, (c) the equivalent static load. Assume that the diver's legs remain rigid and use  $E = 1.8 \times 10^6$  psi.

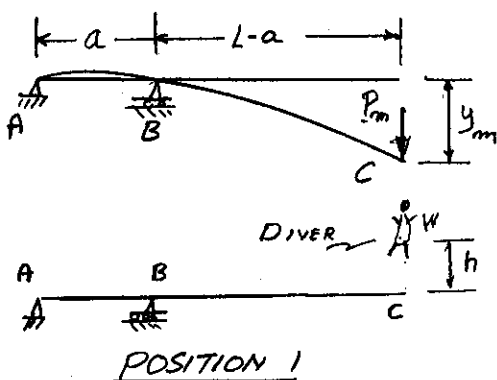


**SOLUTION**

ENTER:  $L = 12 \text{ ft}$ ,  $h = 20 \text{ in.}$ ,  $W = 160 \text{ lb}$   
 $E = 1.8 \times 10^6 \text{ psi}$

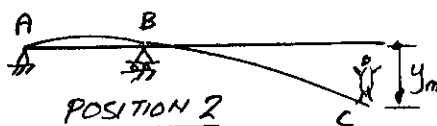
$$I = \frac{(16 \text{ in.})(2.65 \text{ in.})^3}{12}$$

$$S = \frac{(16 \text{ in.})(2.65 \text{ in.})^2}{6}$$



$y_m = P_m \alpha$  WHERE  $\alpha = \text{INFLUENCE COEFFICIENT}$   
 SEE BELOW FOR DETERMINATION OF  $\alpha$

WHERE  $P_m = \text{EQUIVALENT STATIC LOAD}$



$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

WORK =  $W(h + y_m)$

WORK =  $U_2$

$W(h + y_m) = \frac{1}{2} \frac{y_m^2}{\alpha}$  A

PROGRAM SOLUTION OF  $a$  FOR  $y_m$ . ENTER  $\alpha$   
 FOR  $a = 10 \text{ in.}$  TO  $50 \text{ in.}$  STEP  $10 \text{ in.}$

SOLVE A FOR  $y_m$ ,  $P_m = y_m / \alpha$

$$M_{\text{max}} = M_B = P_m (L - a)$$

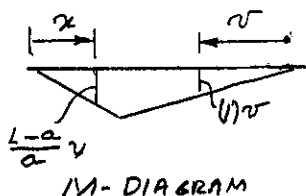
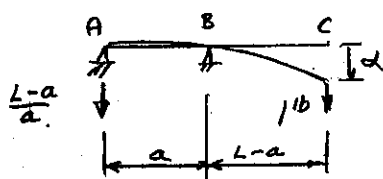
$$\sigma = M_{\text{max}} / S$$

PRINT  $a$ ,  $y_m$ ,  $P_m$ ,  $M_{\text{max}}$ ,  $\sigma$

PROGRAM OUTPUT

a in.	ym in.	Pm lb	Max M kip-in.	sigma psi
10	14.622	757.7	101.532	5422
20	13.262	802.6	99.519	5314
30	11.950	855.6	97.536	5208
40	10.683	919.1	95.583	5104
50	9.462	996.4	93.661	5001

DETERMINATION OF INFLUENCE COEFFICIENT  $\alpha$



$$U = \frac{1}{2} (1^{\text{lb}}) \alpha = \sum \int \frac{M^2}{2EI} dx$$

$$\frac{\alpha}{2} = \frac{1}{2EI} \left[ \int_0^a \left(\frac{L-a}{a}\right)^2 x^2 dx + \int_0^{L-a} \frac{x^2}{3} dx \right]$$

$$\alpha = \frac{1}{EI} \left[ \frac{(L-a)^2 a^3}{3} + \frac{(L-a)^3}{3} \right]$$

$$\alpha = \frac{1}{3EI} \left[ (L-a)^2 a + (L-a)^3 \right] \quad \triangle$$